The strong coupling at the tau mass from a new tau vector isovector spectral function

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with Maarten Golterman, Kim Maltman, Santi Peris, Marcus Rodrigues and Wilder Schaaf

DB, Golterman, Maltman, Peris, Rodrigues, Schaaf, arXiv:2012.10440, PRD 103 (2021)

ICHEP 2022 BOLOGNA

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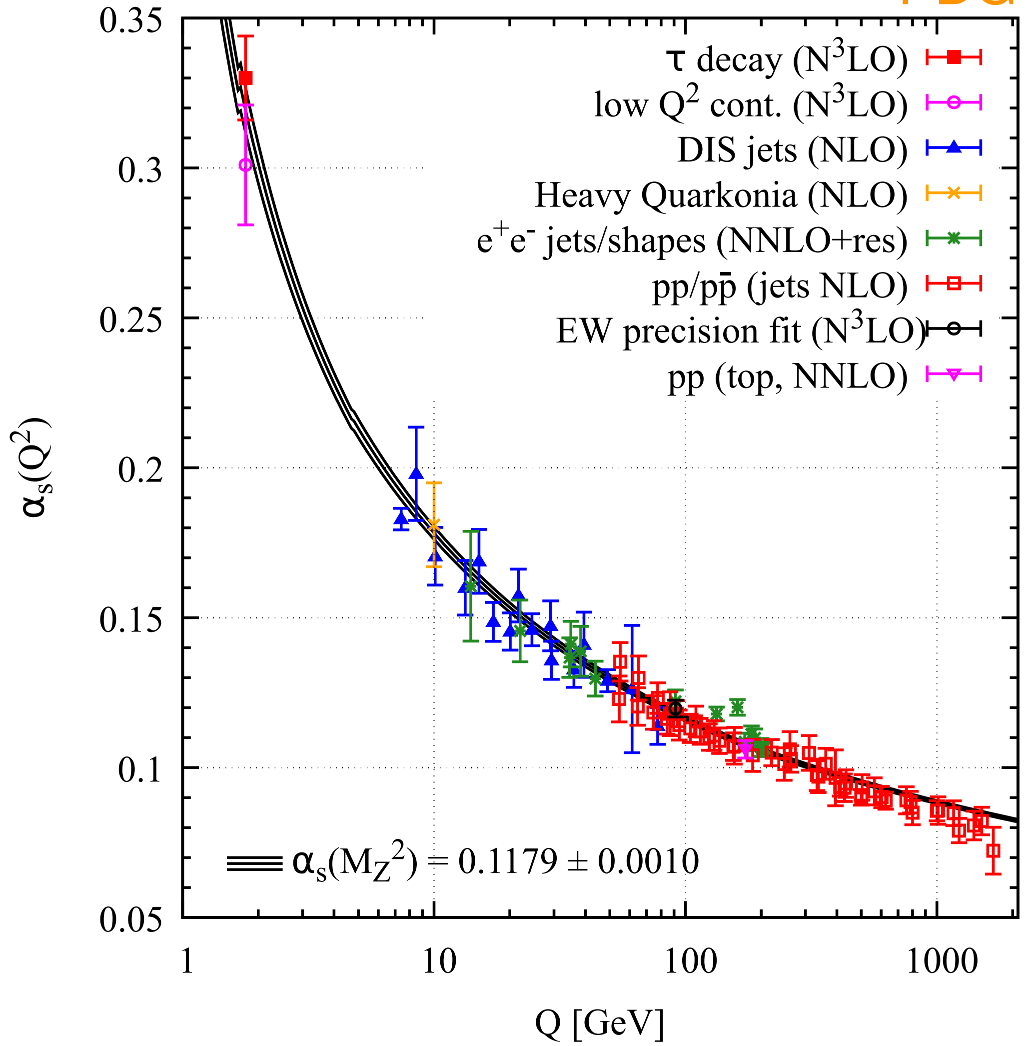
strong coupling from tau decays

PDG



Lower energies

Larger coupling, more sensitivity to QCD corrections.

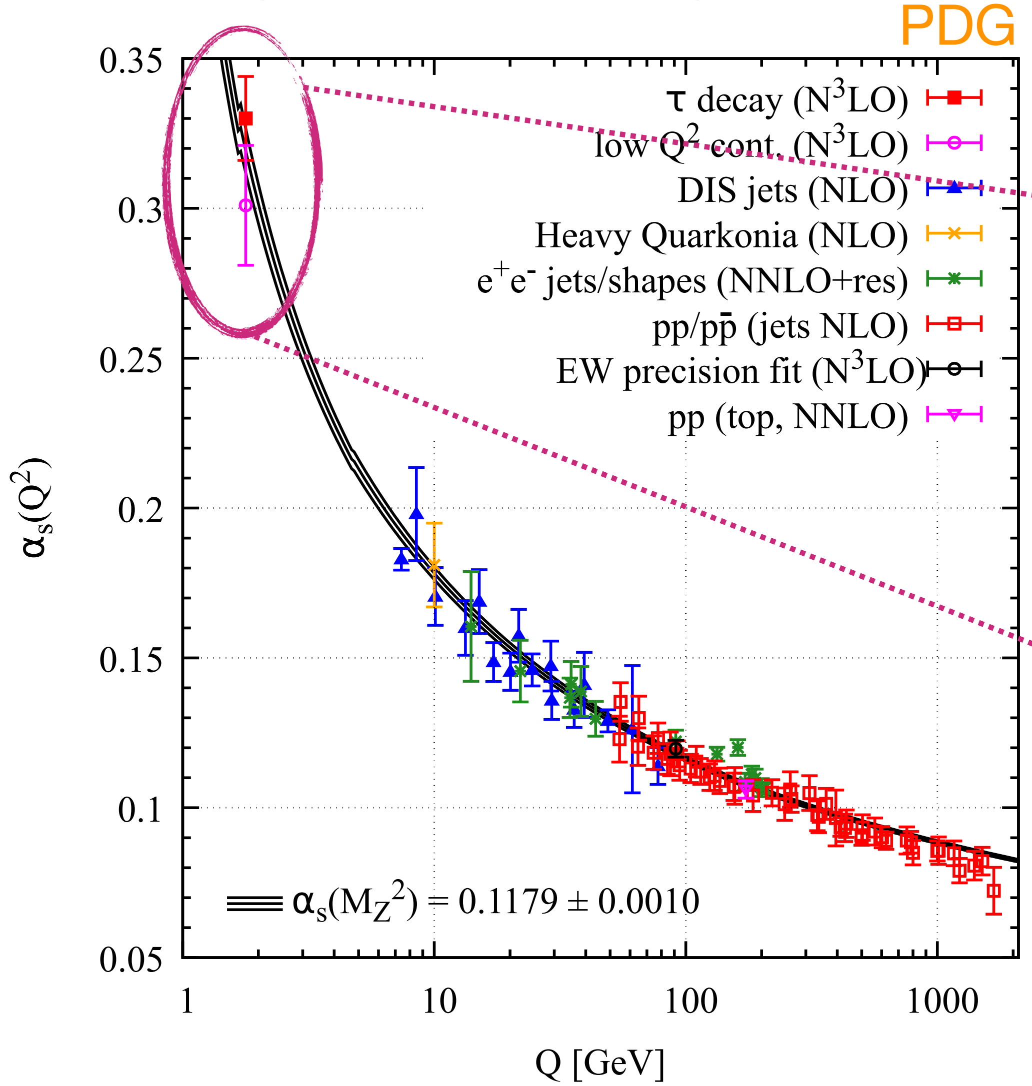
Larger non-perturbative physics (OPE, DVs), Problems with pt. theory (renormalons,...).

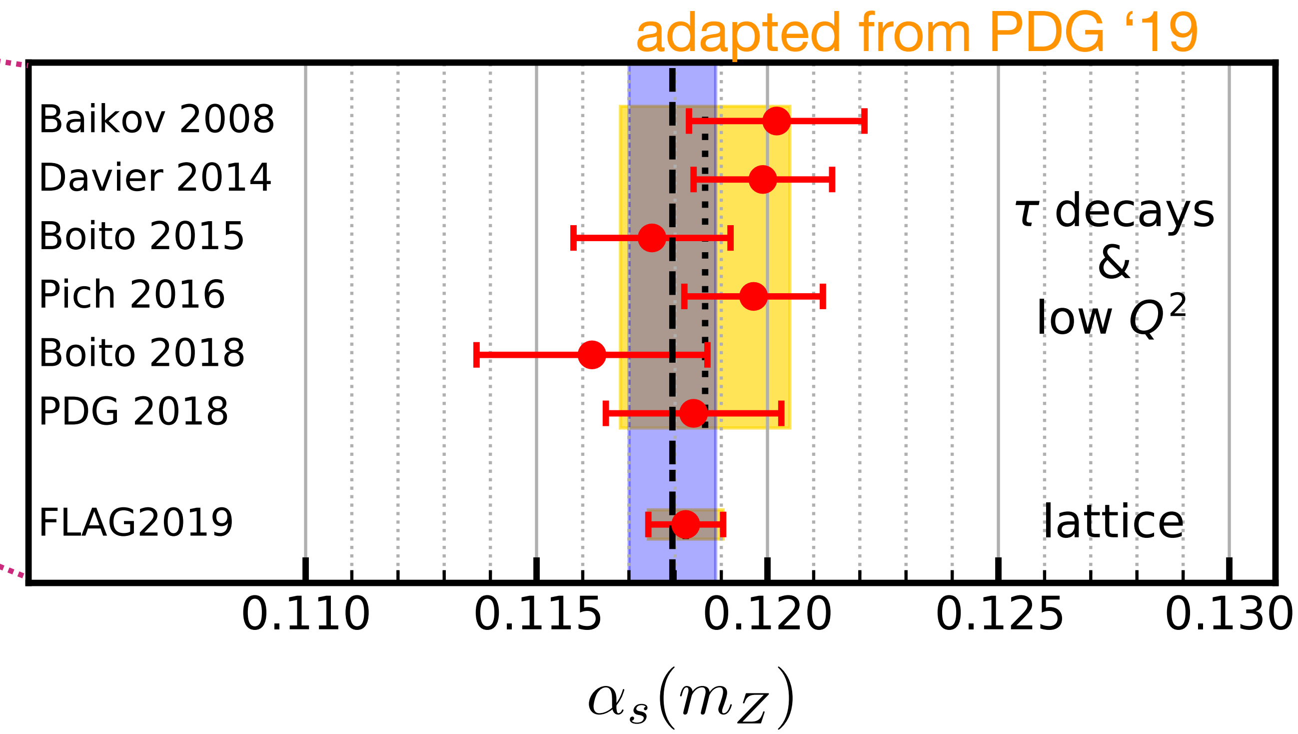
Higher energies

Smaller coupling, less sensitive to QCD corrections, more precision required from exp.

Small contamination from non-perturbative physics, pt. series is "almost" convergent

strong coupling from tau decays





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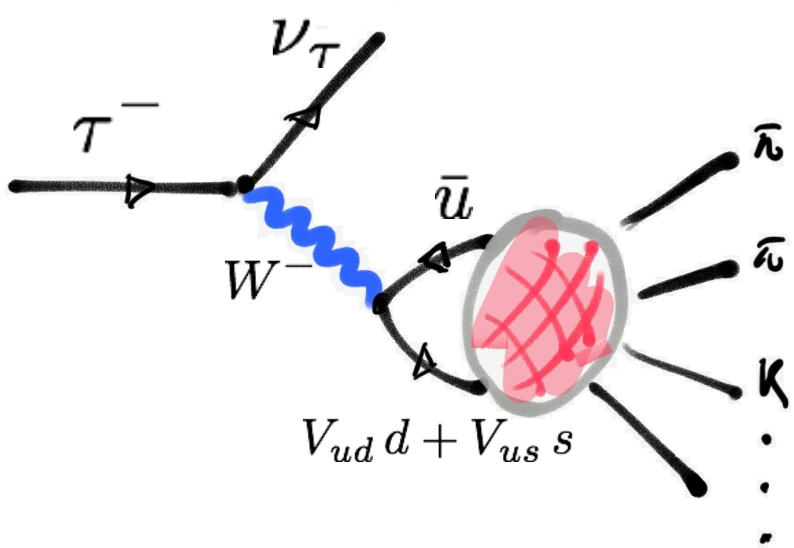
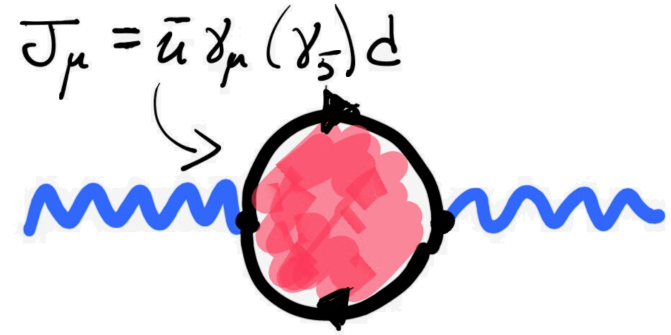


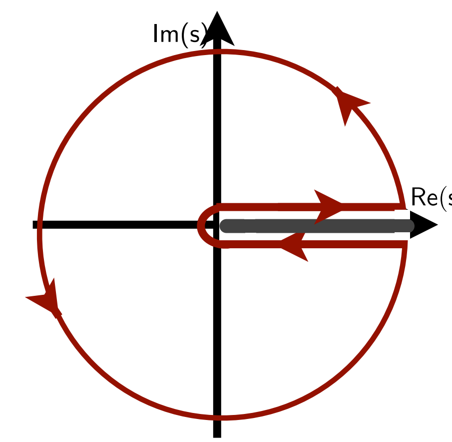


$$\Pi_{\mu\nu}(q) = i\int d^4x\, e^{iqx}\langle 0|T\{J_\mu(x)J_\nu(0)^\dagger\}|\rangle$$

Massless (V&A) correlators

Braaten, Narison, and Pich '92

Sum rules (using Cauchy's theorem)

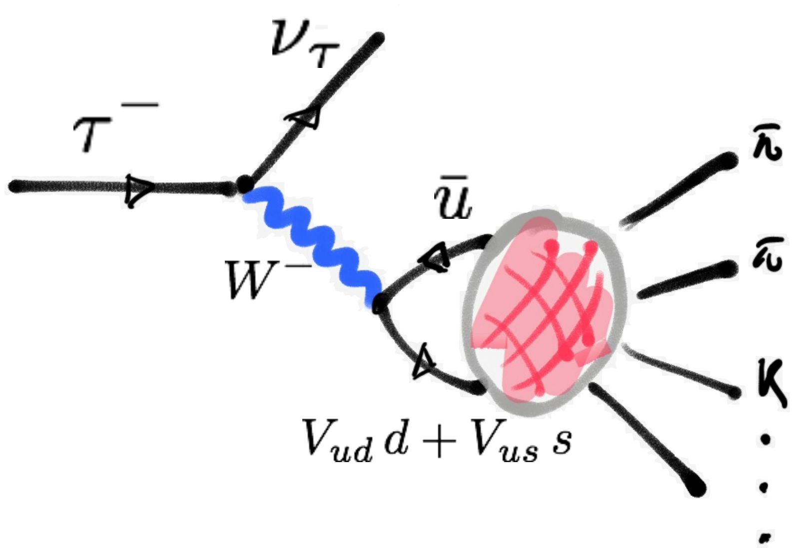


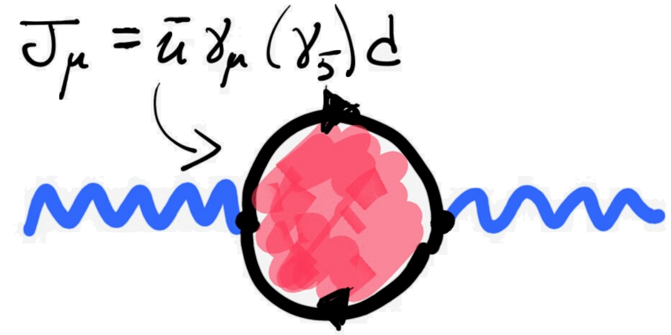
$$\frac{1}{s_0}\int_0^{s_0} ds\, w(s)\frac{1}{\pi}\mathrm{Im}\Pi(s) = -\frac{1}{2\pi i s_0}\oint_{|z|=s_0} dz\, w(z)\Pi(z)$$

spectral function

$$\rho(s) = \frac{1}{\pi}\mathrm{Im}\Pi(s)$$

strong coupling from tau decays





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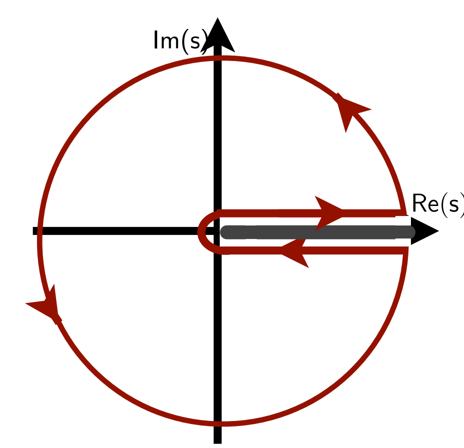

Braaten, Narison, and Pich '92

Sum rules (using Cauchy's theorem)

experiment

$$\frac{1}{s_0}\int_0^{s_0} ds\, w(s)\frac{1}{\pi}\mathrm{Im}\Pi(s) = -\frac{1}{2\pi i s_0}\oint_{|z|=s_0} dz\, w(z)\Pi(z)$$

spectral function

$$\rho(s) = \frac{1}{\pi}\mathrm{Im}\Pi(s)$$

theory

theory overview

$$\frac{-1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\, \Pi(z) \approx S_{\rm EW}\, N_c (1 + \delta^{(0)} + \delta_{\rm EW} + \delta_{\rm OPE} + \delta_{\rm DVs})$$

theory

Perturbation theory (OPE) ⟶ $$\sum_{n=0}^{4}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{k=0}^{n+1} c_{n,k} \log^k\left(\frac{-s}{\mu^2}\right) + \frac{C_4}{Q^4} + \frac{C_6}{Q^6} + \frac{C_8}{Q^8} + \cdots$$

Gorishnii, Kataev, Larin '91
Surguladze&Samuel '91

Baikov, Chetyrkin, Kühn '08

α_s^1 α_s^2 α_s^3 α_s^4

pt. correction is ~20%

$$\delta^{(0)}_{\rm FO} = 0.1012 + 0.0533 + 0.0273 + 0.0133 \;=\; 0.1952$$

(fixed order perturbation theory)

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Perturbation theory (OPE) →

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Gorishnii, Kataev, Larin '91
Surguladze&Samuel '91 → α_s^3

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Duality Violations →

$$\rho_{\rm DV}(s) = e^{-\delta-\gamma s}\sin(\alpha+\beta s)$$

Ansatz based on widely accepted assumptions about QCD: Regge behaviour and large-N_c. Main expected corrections: logarithmic and powers of *1/s*.

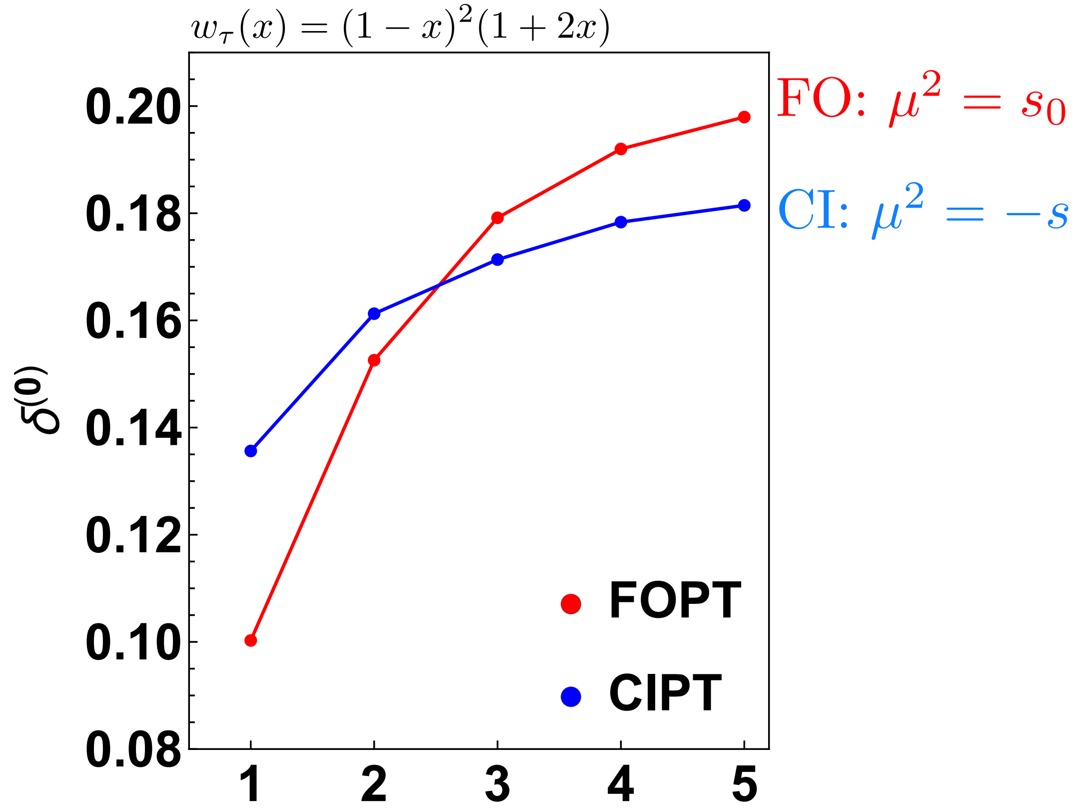
DB, Caprini, Golterman, Maltman, Peris, PRD '18

theory: FOPT vs CIPT

Fixed Order (FO) or Contour Improved (CI) lead to different α_s values

theoretical uncertainty?

$$\begin{array}{lccccl} & \alpha_s^1 & \alpha_s^2 & \alpha_s^3 & \alpha_s^4 & \\ \delta^{(0)}_{\rm FO} = & 0.1012 + & 0.0533 + & 0.0273 + & 0.0133 & = 0.1952 \\ \delta^{(0)}_{\rm CI} = & 0.1375 + & 0.0262 + & 0.0104 + & 0.0072 & = 0.1814 \end{array}$$



Discrepancy between FOPT and CIPT (asymptotic separation): linked to an incompatibility of CIPT with the standard form of the OPE.

Hoang and Regner '20.'21

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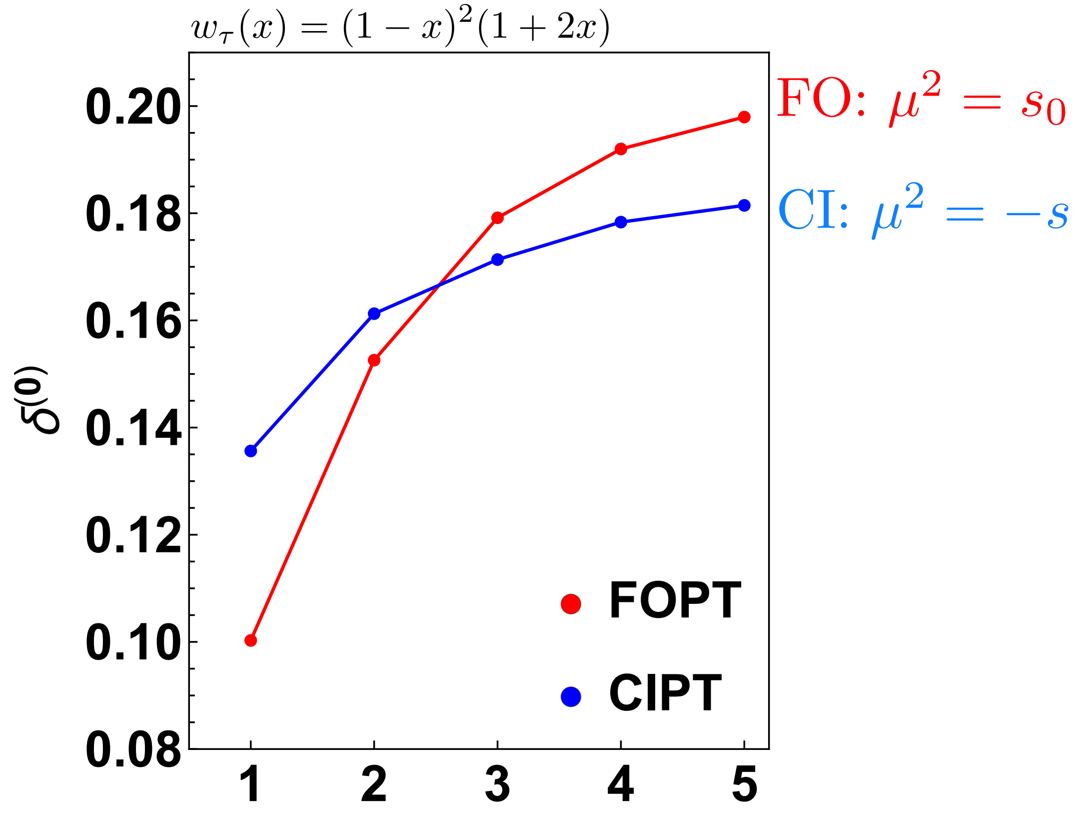

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Hoang and Regner ’20.‘21

Resolution to this problem: subtraction of the leading IR renormalon (Gluon condensate) which gives leading contribution to asymptotic separation

Benitez-Rathgeb, DB, A. Hoang, M. Jamin, JHEP (2022), 2202.10957

Renormalon-free gluon-condensate scheme (RF GC Scheme) results:

Benitez-Rathgeb, DB, A. Hoang, M. Jamin, 2207.01116

analysis strategy

$$\frac{-1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\, \Pi(z) \approx S_{\rm EW}\, N_c (1 + \delta^{(0)} + \delta_{\rm EW} + \delta_{\rm OPE} + \delta_{\rm DVs})$$

theory

Desired properties from the choice of weights

1. Good perturbative behaviour.
2. Small condensate contributions.
3. Suppression of DVs.

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theory

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Choice of weights

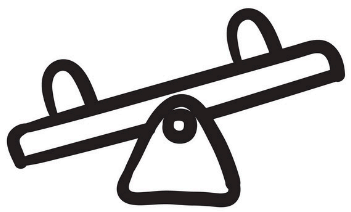
$w_0(y) = 1$	Tiny condensate contributions, sensitive to DVs
$w_2(y) = 1 - y^2$	Only D=6
$w_3(y) = (1-y)^2(1+2y)$	Only D=6 and 8 Tau kinematical Moment (R_τ)
$w_4(y) = (1-y^2)^2$	Only D=6 and 10

DB, Cata, Golterman, Jamin, Maltman '11, Beneke, DB, Jamin '12, DB, M. Golterman, K. Maltman, S. Peris '16
DB F Oliani '20

analysis strategy

Suppression of DVs comes with the price of additional (unknown) higher dim. contributions from the OPE.

DV strategy



Truncated OPE strategy

DB, M. Golterman, K. Maltman, S. Peris,
M. V. Rodrigues and W. Schaaf, 2012.10440

- Accept some DVs, strongly suppress contamination on the OPE side.

A Pich, A. Rodriguez-Sanchez 1605.06830
Davier, Höcker, Malaescu, Yuan, Zhang 1312.1501

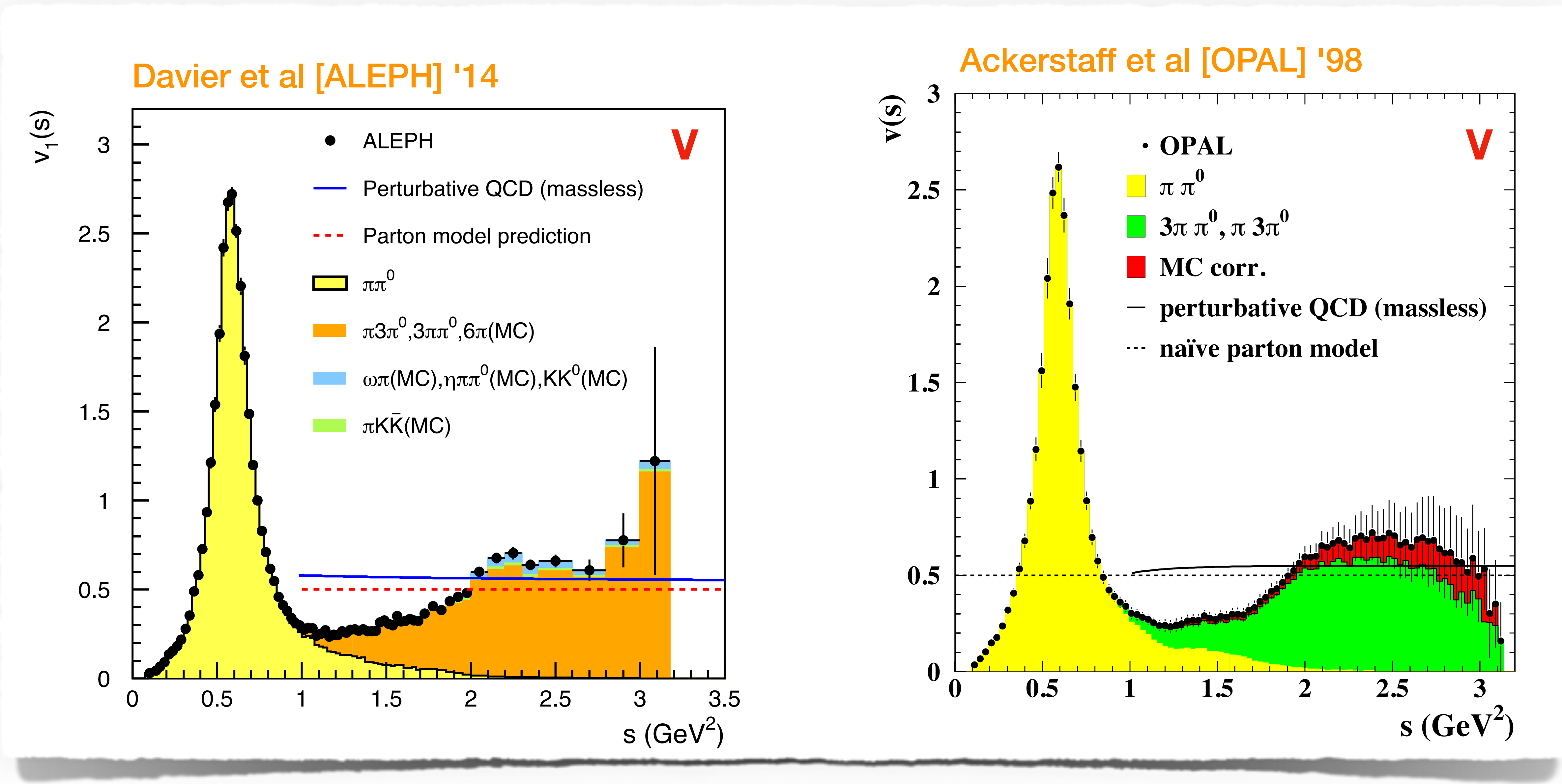
- Suppress DVs but need to ignore the higher order contributions on the OPE side (too many parameters).

(Serious issues with the truncation of the OPE)
DB, M. Golterman, K. Maltman, S. Peris '16 '19

Data

anatomy of the ALEPH and OPAL data sets

- V channel dominated by $\tau \to 2\pi + \nu_\tau$ and $\tau \to 4\pi + \nu_\tau$
- "Residual" channels subdominant (but important for α_s!)
- Monte Carlo (MC) inputs for several channels



Recently measured channels in e^+e^- can be used to improve the vector channel

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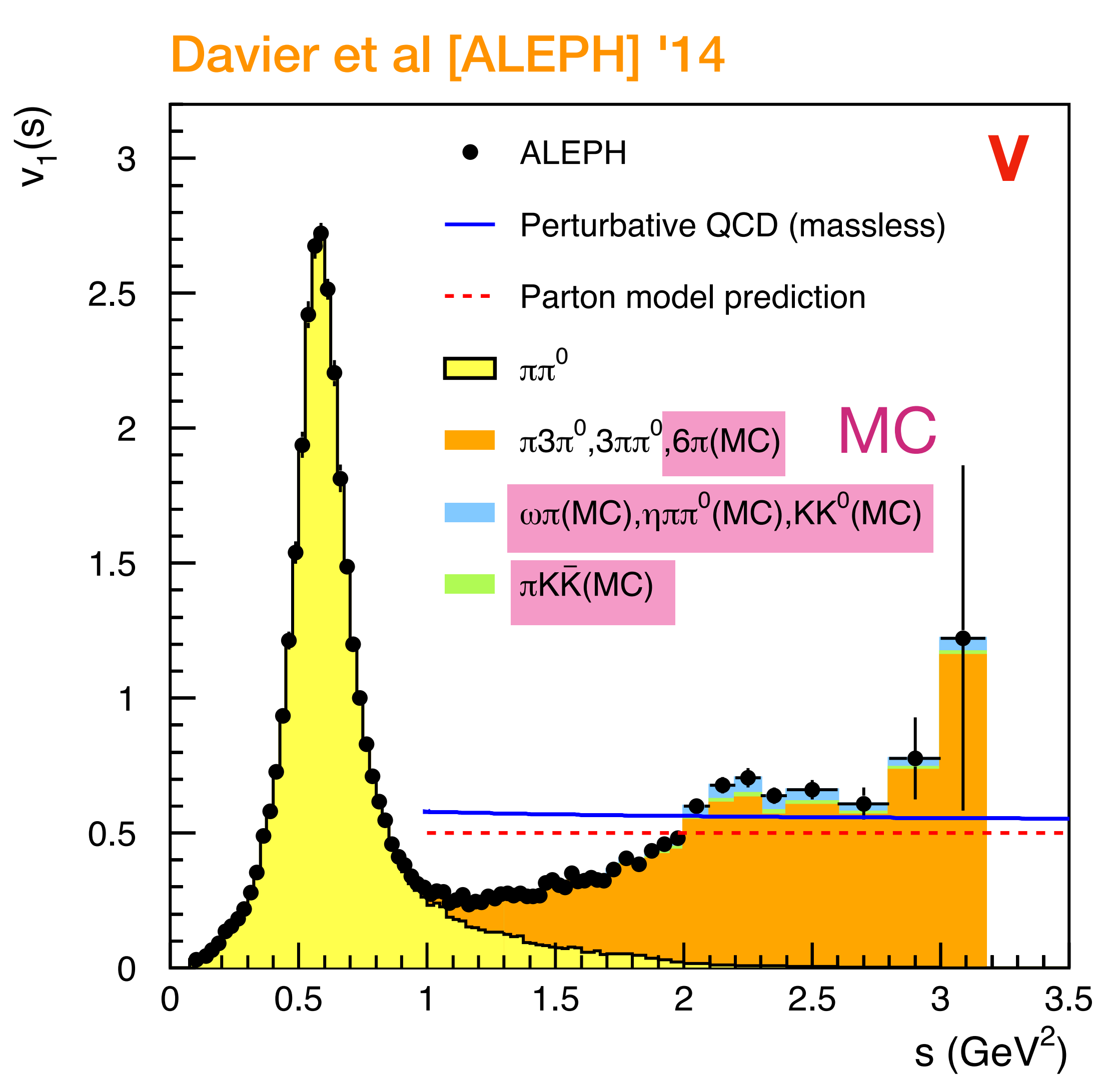
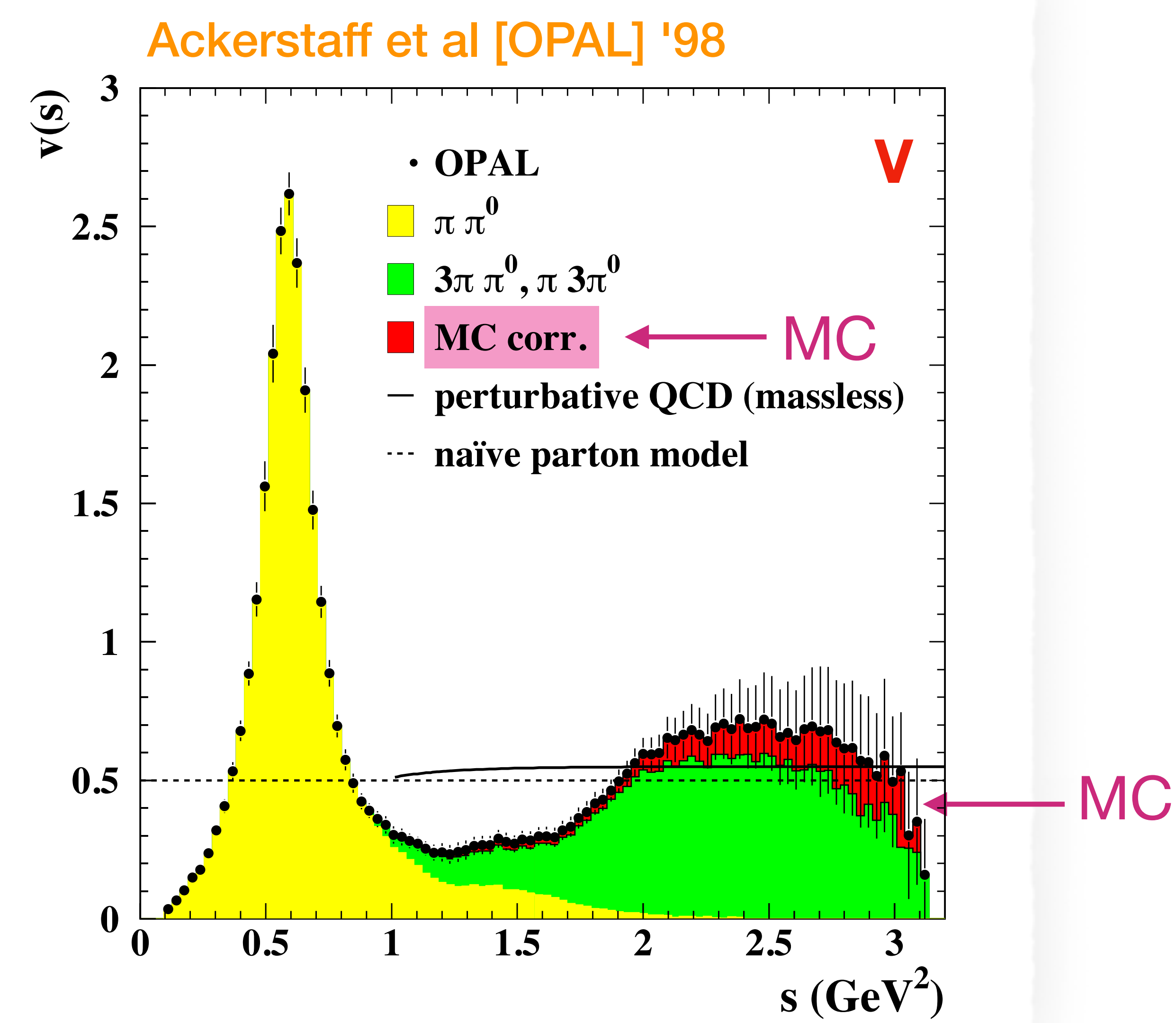




Recently measured channels in e^+e^- can be used to improve the vector channel

new vector isovector spectral function

- Combined data for 2π and 4π channels from ALEPH & OPAL

Data combination: same algorithm used in R-data combination for muon g-2.

Keshavarzi, Nomura, Teubner '18

- Exp. data only: 7 residual channels from e^+e^- using CVC (conserved vector current) and BaBar data for $\tau \to K K_S \nu_\tau$

Data sets from: BABAR, SND and CMD-3 (last ~5 yrs)

No Monte Carlo inputs; IB corrections to CVC negligible

- Results updated for recent branching ratio measurements

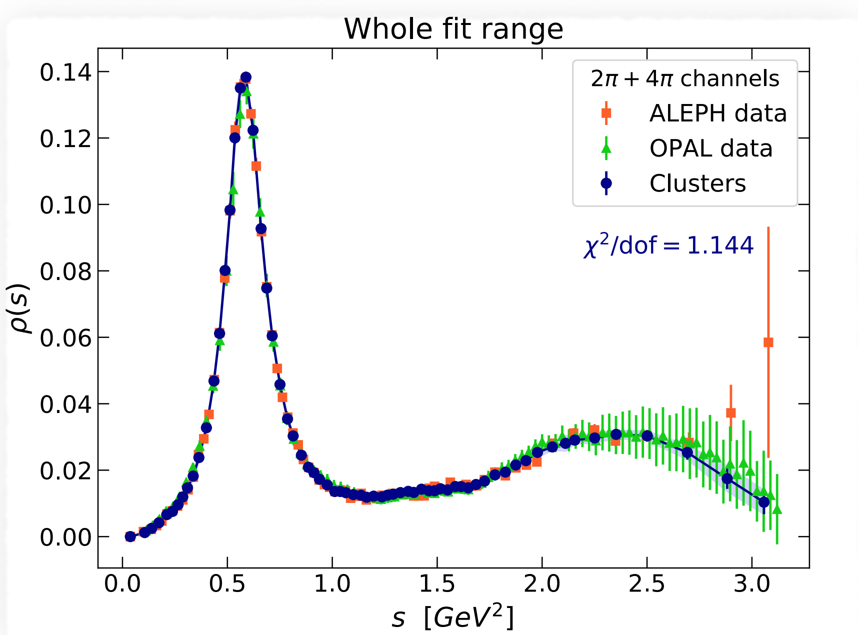
improved vector isovector spectral function

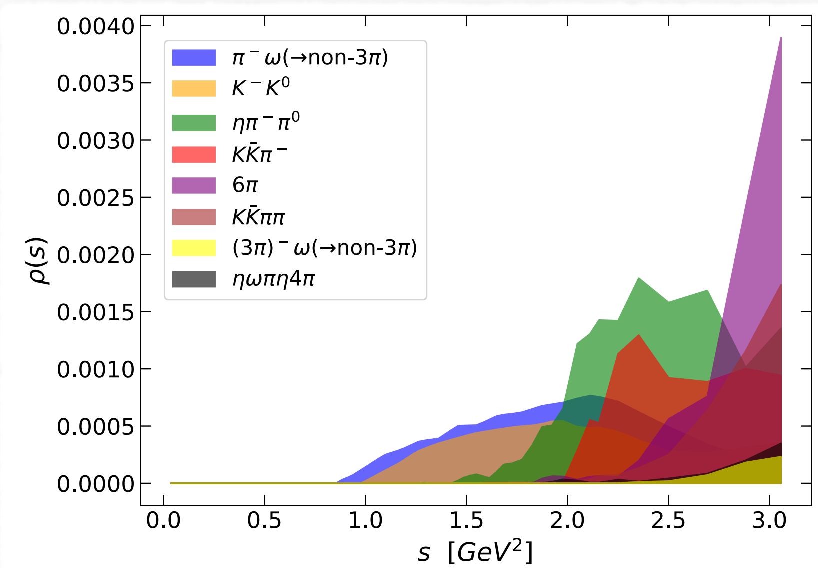
Combination of 2π $+4\pi$ channels

Good χ^2 both locally and globally, no χ^2 inflation needed

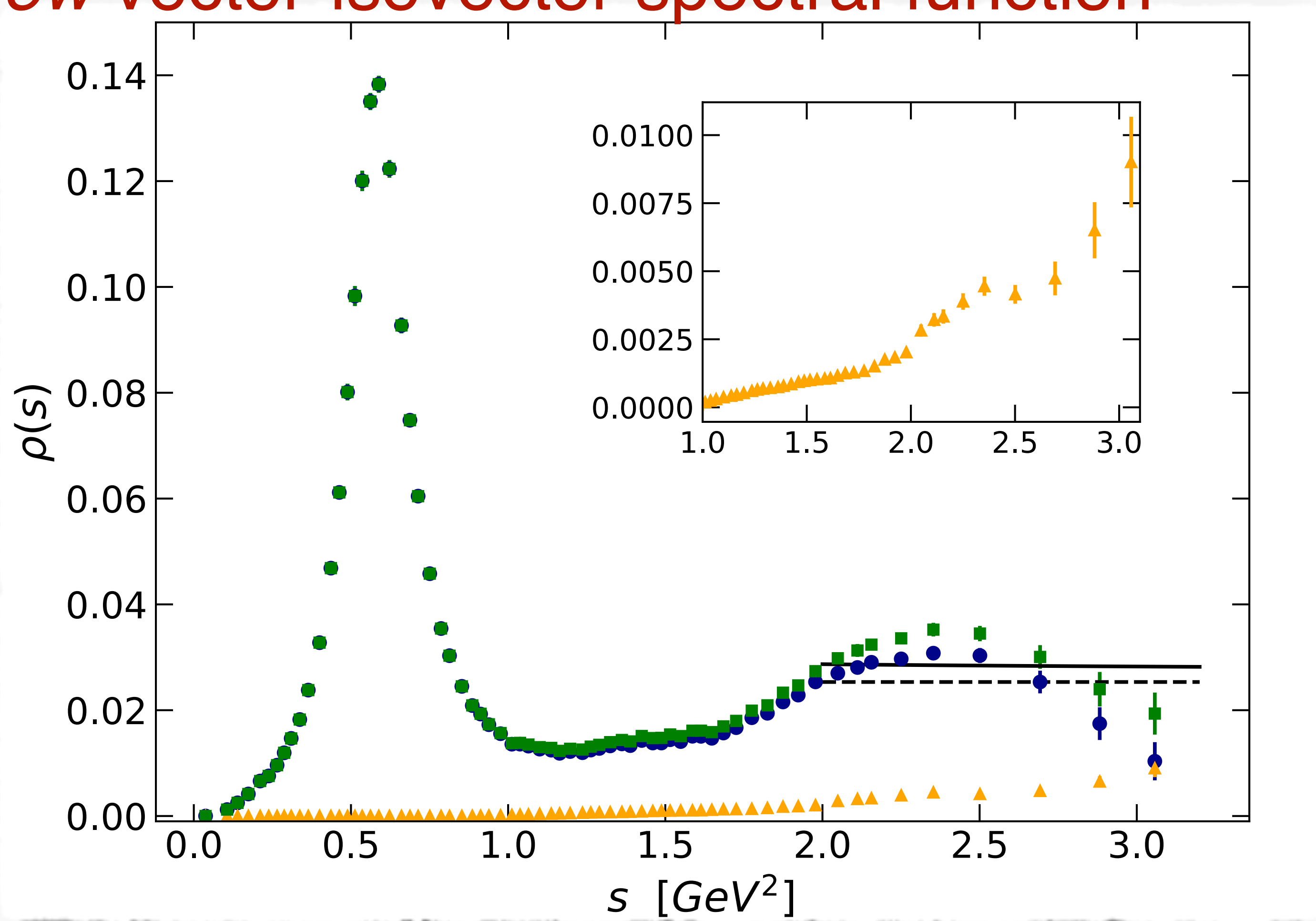
No Monte Carlo input

Original data sets from: BABAR, SND and CMD-3





new vector-isovector spectral function



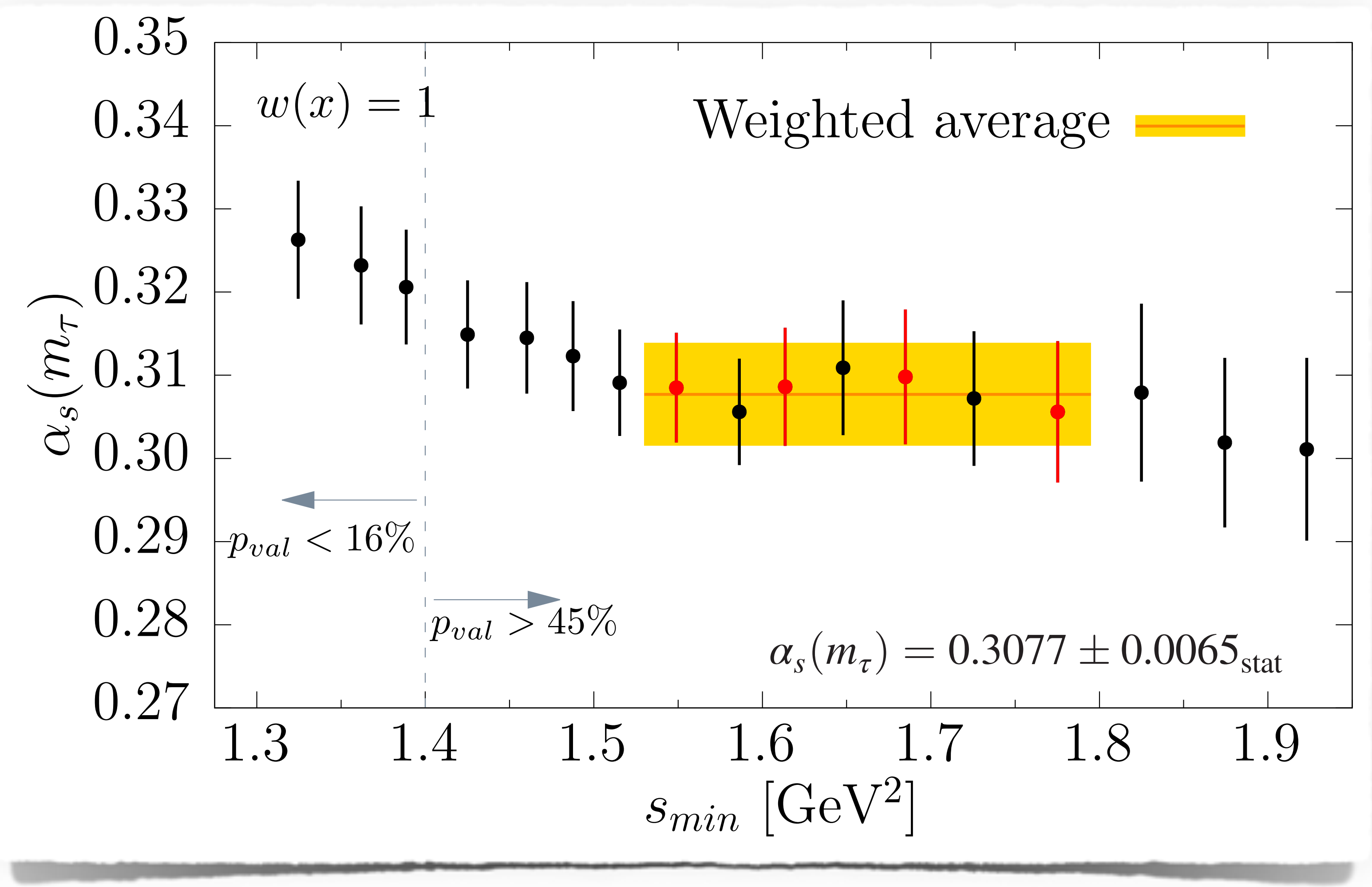
Results

strong coupling from the new spectral function

Several fits, single moments or in combination

Many fit windows: $[s_{\min}, m_\tau^2]$

Consistency between different fits (α_s , condensates, DV params.)



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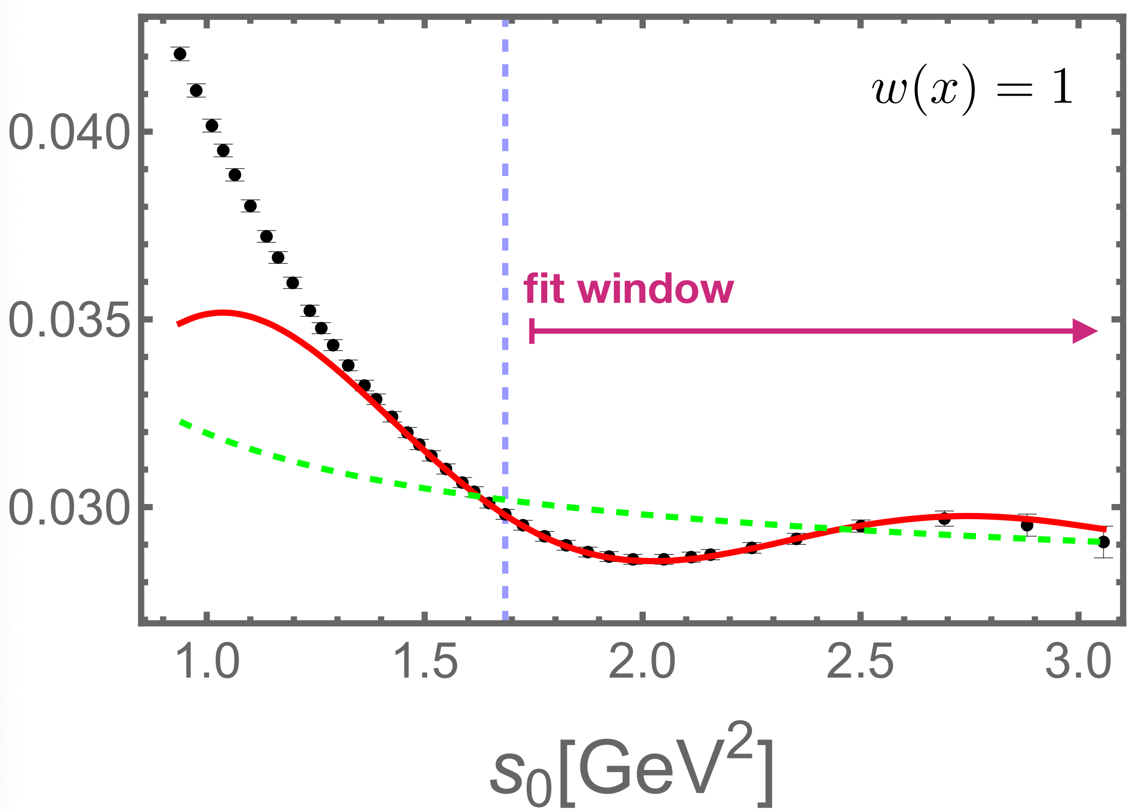

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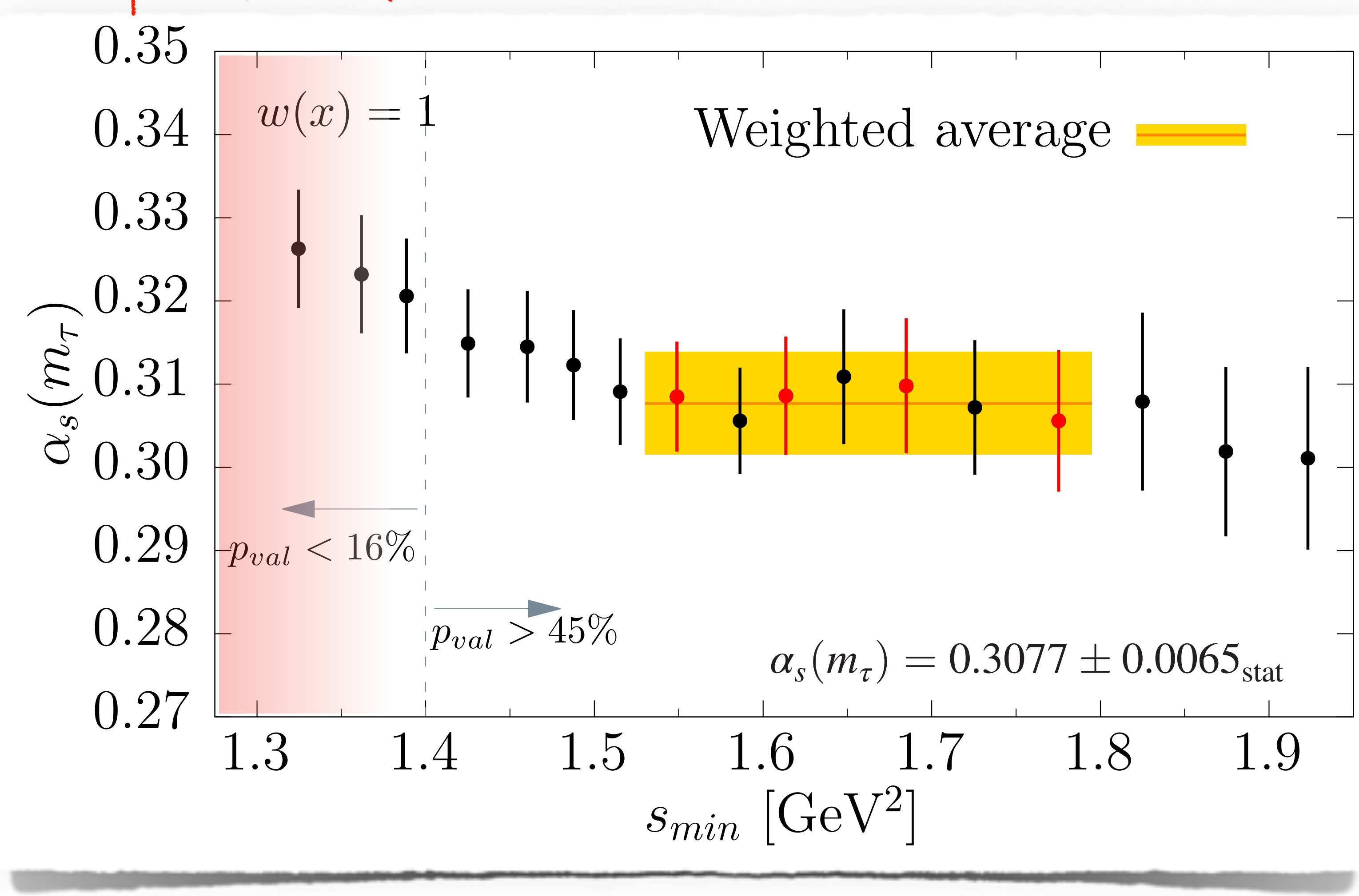
nonperturbative QCD



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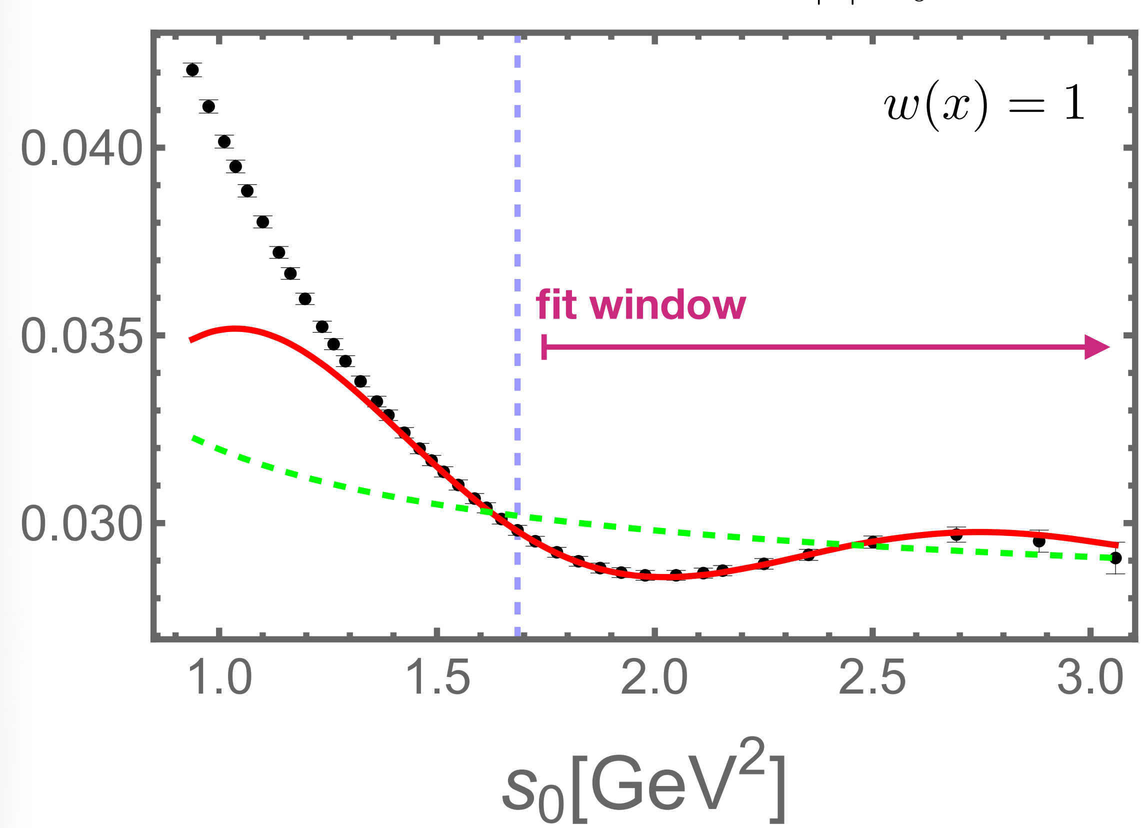

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Consistency between different fits

mom.	α_s	$c_6[\mathrm{GeV}^6]$
w_0	$0.3077(65)$	$--$
$w_0\&w_2$	$0.3091(69)$	$-0.0059(13)$
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Final value

pt. series truncation, scale variation

$$\alpha_s(m_\tau) = 0.3077 \pm 0.0065_{\mathrm{stat}} \pm 0.0038_{\mathrm{pert}}$$
$$= 0.3077 \pm 0.0075 \qquad (n_f = 3,\ \mathrm{FOPT})$$

stability of the DV ansatz

leading corrections to DV ansatz

$$\rho_{\rm DV}(s) = \left(1 + \frac{c}{s} + \cdots\right) e^{-\delta-\gamma s}\sin\left(\alpha + \beta s\right)$$

DB, Caprini, Golterman, Maltman, Peris, PRD '18

Fits including the leading DV correction (scan of fits with fixed value of *c*)

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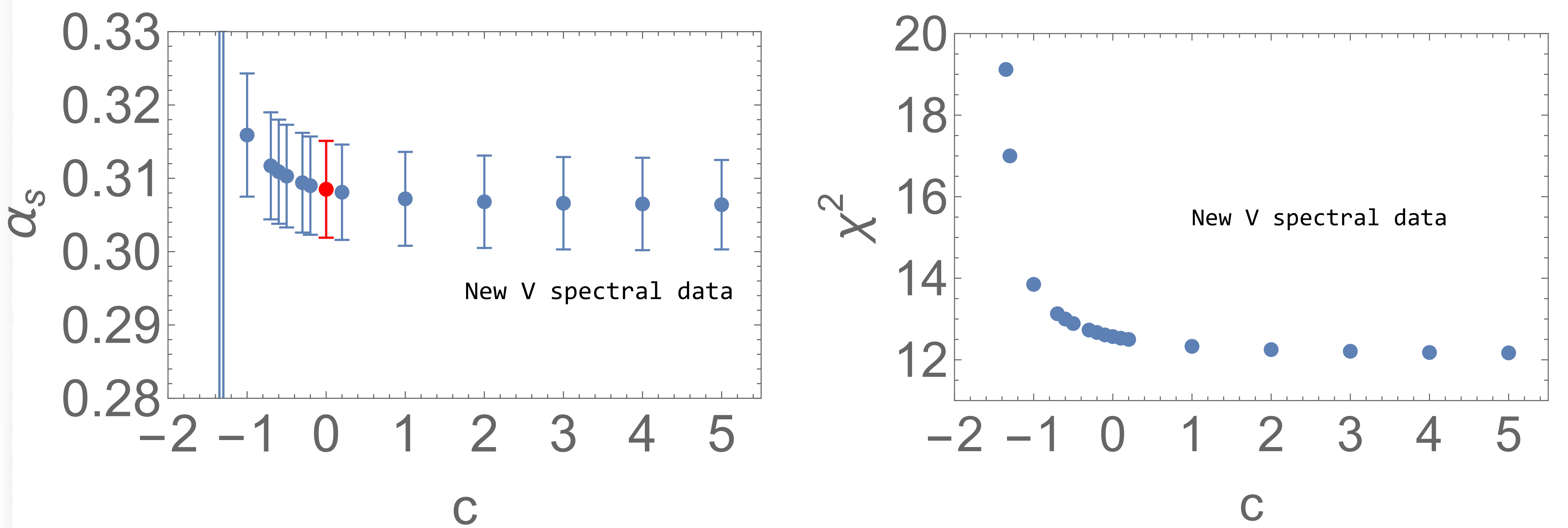

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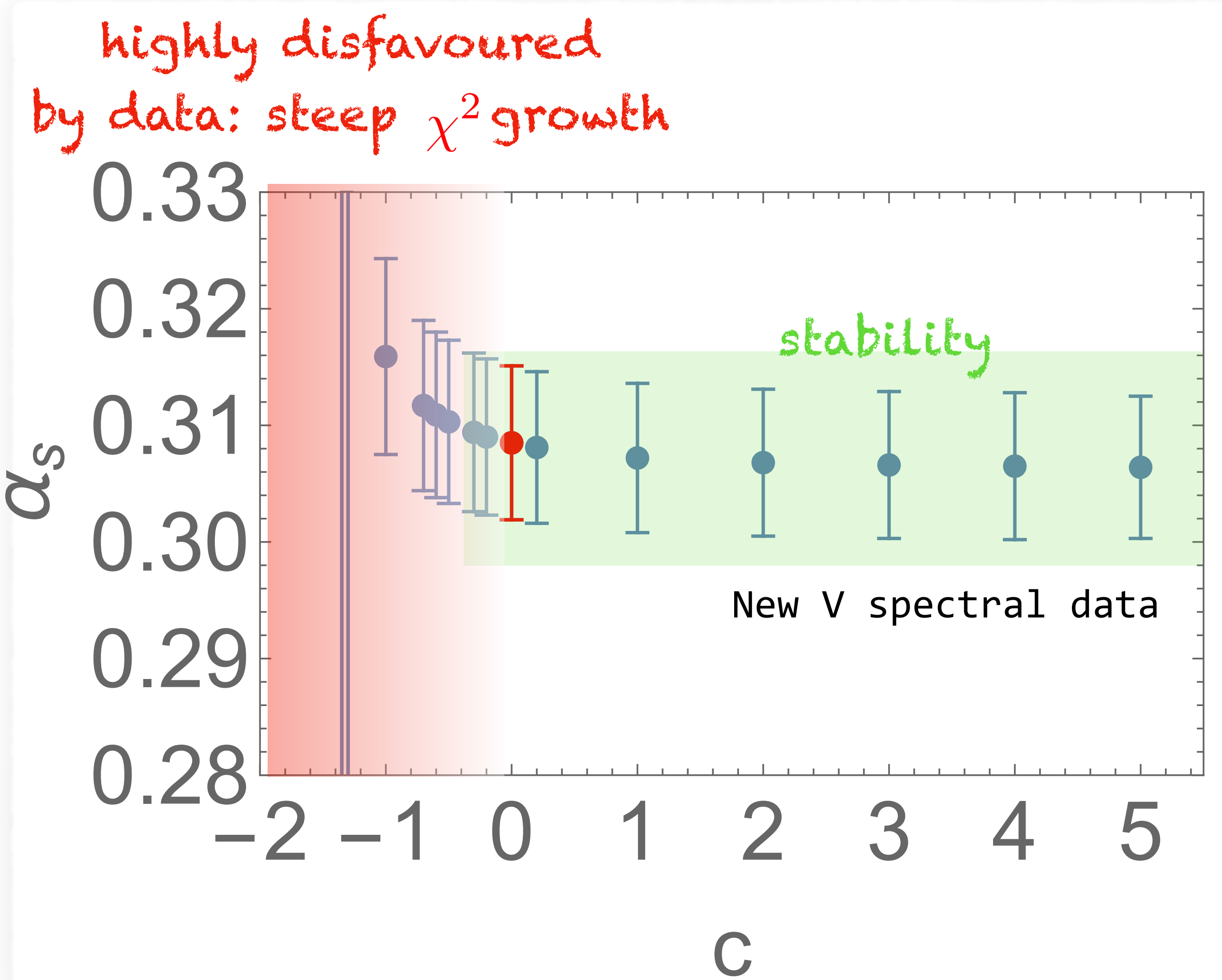
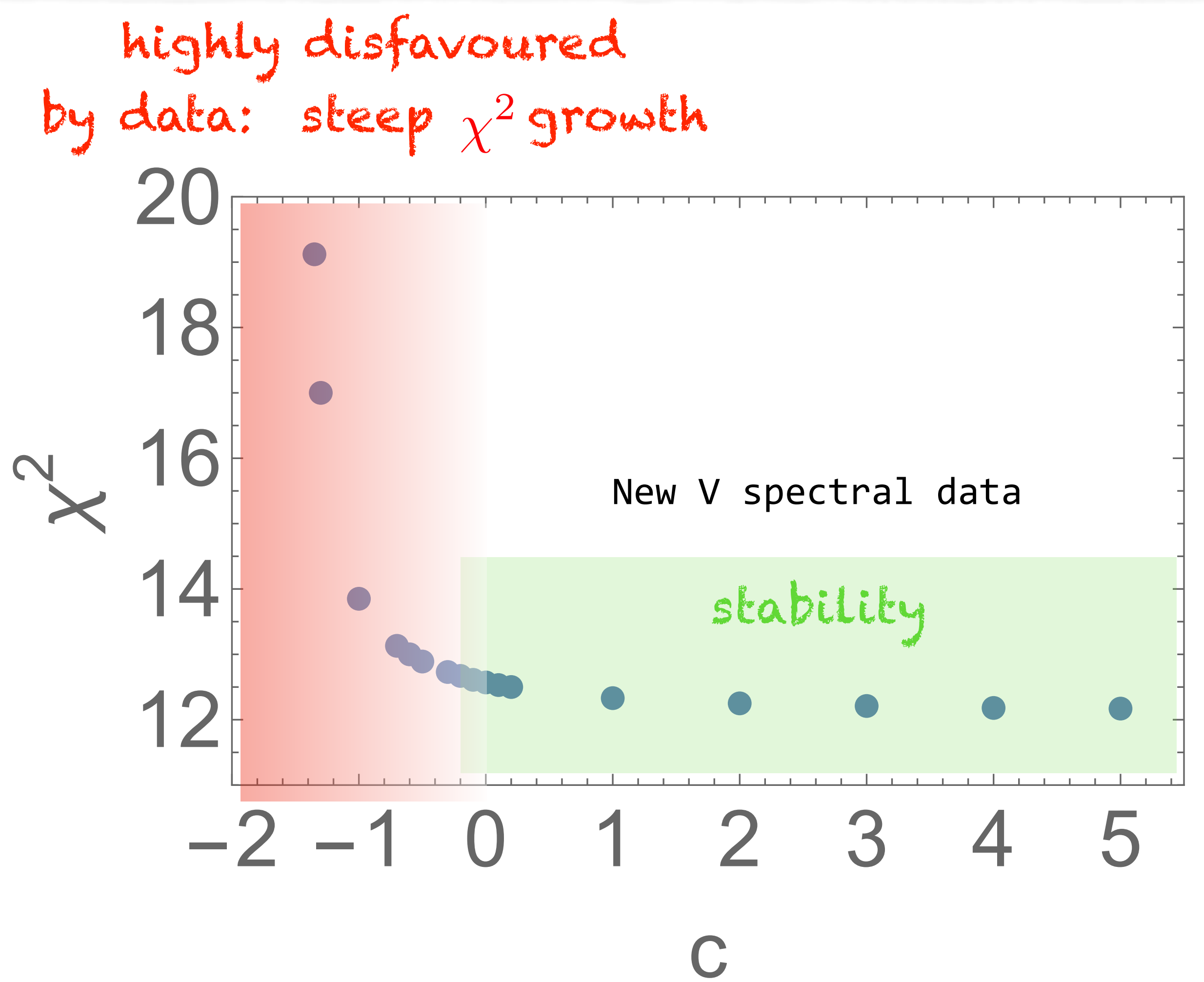




Results are very stable against this modification of the Ansatz

Final result

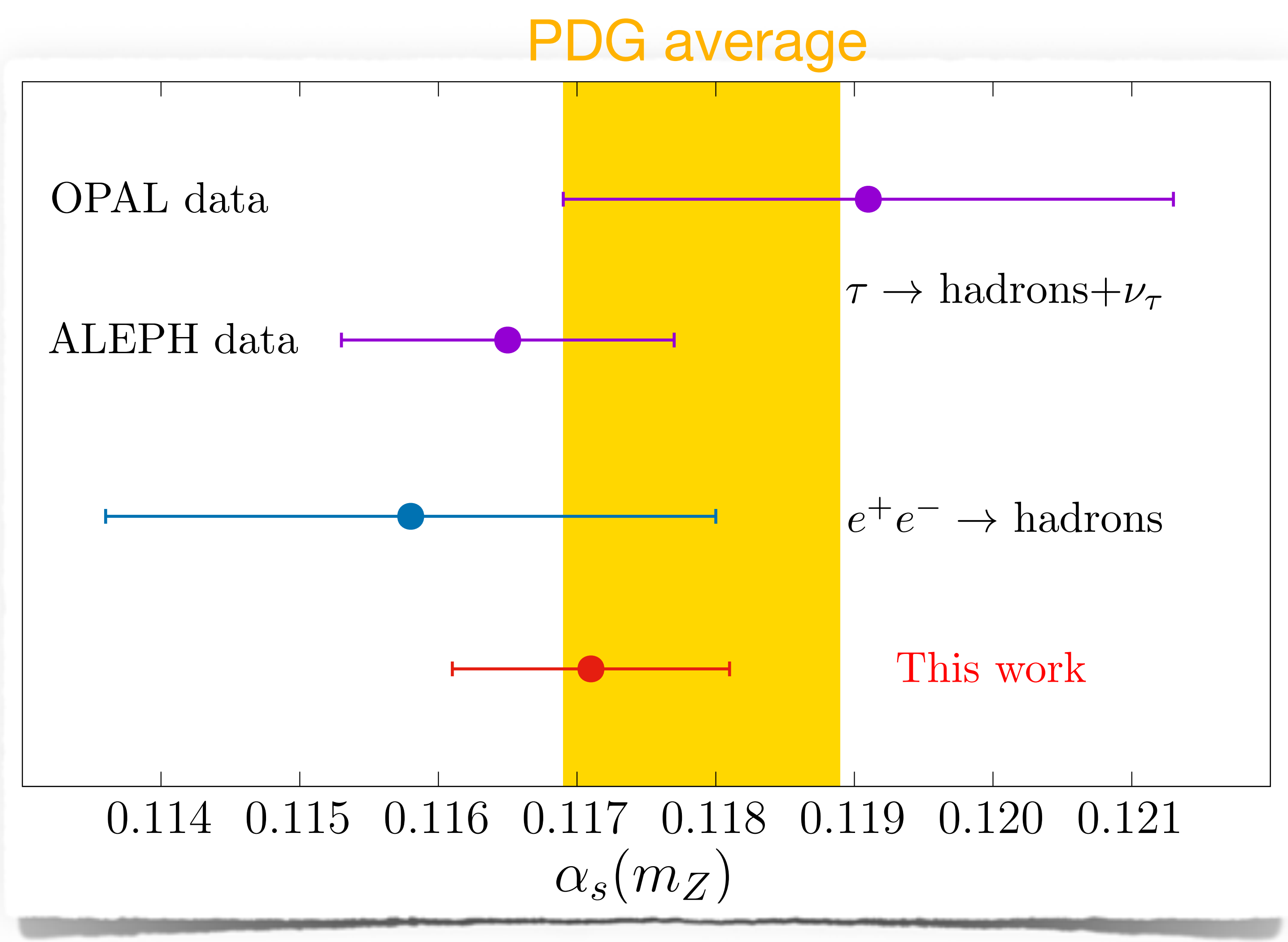
Results at m_τ

$$\alpha_s(m_\tau) = 0.3077 \pm 0.0065_{\rm stat} \pm 0.0038_{\rm pert}$$
$$= 0.3077 \pm 0.0075 \qquad (n_f = 3,\ \text{FOPT})$$

Results evolved to m_Z

$$\alpha_s(m_Z) = 0.1171 \pm 0.0010$$

$(\overline{\rm MS}, N_f = 5)$



DB, Golterman, Jamin, Mahdavi, Maltman, Osborne, Peris, '12

DB, Golterman, Maltman, Osborne, Peris, '15

DB, Golterman, Keshavarzi, Maltman, Nomura, Peris, Teubner '18

DB, Golterman, Maltman, Peris, Rodrigues, Schaaf, '21

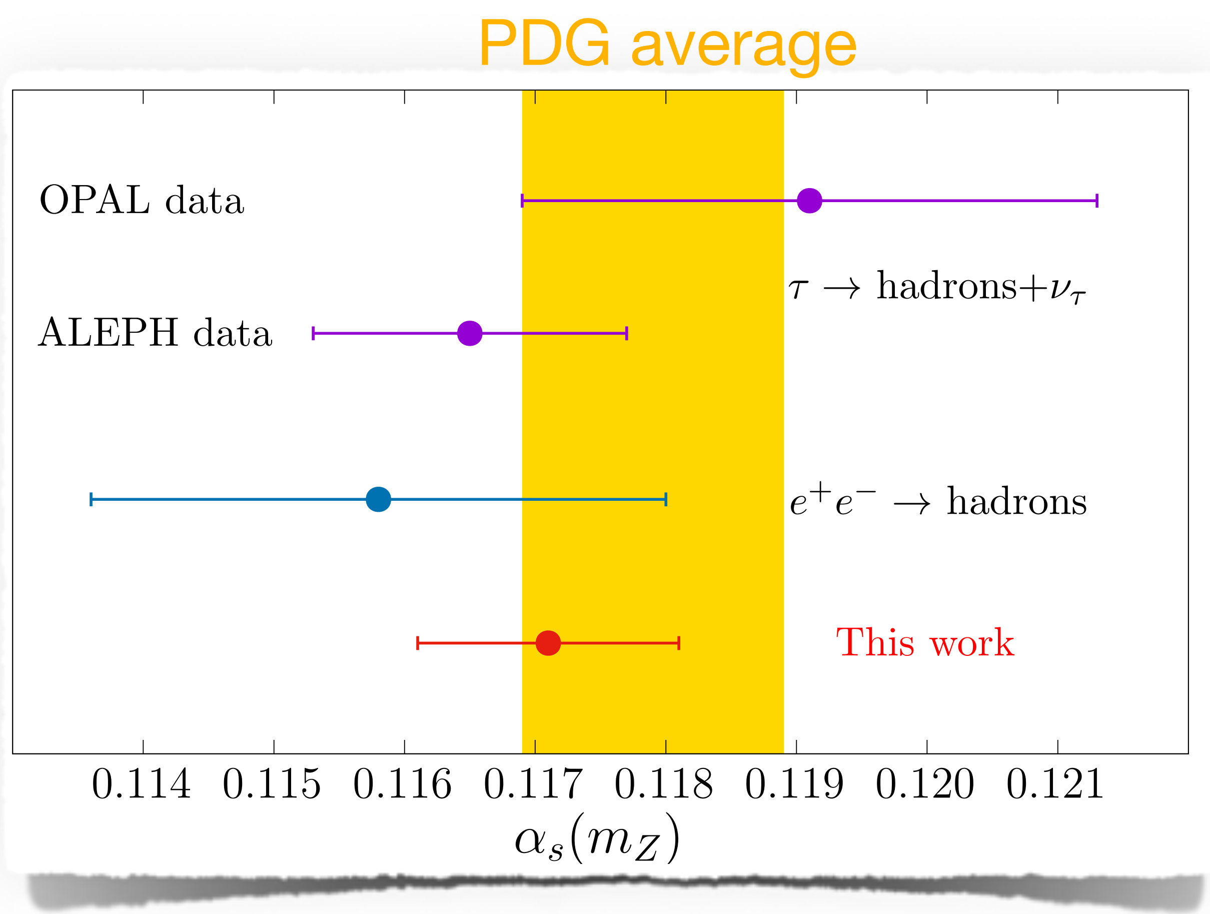
conclusions

- Vector channel is special: CVC allows improvement near tau kin. end point.
- New vector isovector spectral function purely based on data, no MC input.
- Analysis can be improved with new data for the $2\pi + 4\pi$ channels only!

Belle II

- Improvements of this type not possible for the axial channel (no axial photon).
- Final result from the new vector spectral function is competitive.

$$\alpha_s(m_Z) = 0.1171 \pm 0.0010$$



Extra

reconciling FOPT and CIPT: renormalon free (RF) scheme for the Gluon cond.

$\bar{a}_Q \equiv \frac{\beta_1}{2\pi}\alpha_s(Q)$

General structure of the gluon condensate (GC) pole is known in QCD at NLO

normalization not determined by theory (app. known)

$$B_{4,0}(u) = \left[1 + \bar{c}^{(1)}_{4,0}\,\bar{a}_Q\right] \frac{N_{4,0}}{(2-u)^{1+4\hat{b}_1}} \longmapsto N_{4,0}\left(1 + \bar{c}^{(1)}_{4,0}\bar{a}_Q\right)\sum_{\ell=1}^{\infty} r_\ell^{(4,0)}\,\bar{a}_Q^\ell$$

$$r_\ell^{(4,0)} = \left(\frac{1}{2}\right)^{\ell+4\hat{b}_1}\frac{\Gamma(\ell+4\hat{b}_1)}{\Gamma(1+4\hat{b}_1)}$$

determined on general grounds from QCD

contribution of the GC singularity to the perturbative series

coefficients that diverge factorially are known

Infrared-subtracted scheme for the GC condensate ("short distance scheme")

Benitez-Rathgeb, DB, A. Hoang, M. Jamin, 2202.10957

$$\langle \bar{G}^2\rangle^{(n)} \equiv \langle G^2\rangle(R^2) - R^4\sum_{\ell=1}^{n} N_g\, r_\ell^{(4,0)}\,\bar{a}_R^\ell$$

reconciling FOPT and CIPT: renormalon free (RF) scheme for the Gluon cond.

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IR scale

to be expanded (coherently) in perturbation theory

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normalization not determined by theory (app. known)

$$B_{4,0}(u) = \left[1 + \bar{c}_{4,0}^{(1)}\bar{a}_Q\right]\frac{N_{4,0}}{(2-u)^{1+4\hat{b}_1}} \longmapsto N_{4,0}\left(1+\bar{c}_{4,0}^{(1)}\bar{a}_Q\right)\sum_{\ell=1}^{\infty} r_\ell^{(4,0)}\bar{a}_Q^\ell$$

$$r_\ell^{(4,0)} = \left(\frac{1}{2}\right)^{\ell+4\hat{b}_1}\frac{\Gamma(\ell+4\hat{b}_1)}{\Gamma(1+4\hat{b}_1)}$$

determined on general grounds from QCD

contribution of the GC singularity to the perturbative series

coefficients that diverge factorially are known

Infrared-subtracted scheme for the GC condensate ("short distance scheme")

Benitez-Rathgeb, DB, A. Hoang, M. Jamin, 2202.10957

IR scale

to be expanded (coherently) in perturbation theory

$$\langle \bar{G}^2\rangle^{(n)} \equiv \langle G^2\rangle(R^2) - R^4\sum_{\ell=1}^{n} N_g\, r_\ell^{(4,0)}\,\bar{a}_R^\ell$$

Its more convenient to work with scale invariant GC

"tree level" (unexpanded) contribution

$$\langle \bar{G}^2\rangle^{(n)} \equiv \langle G^2\rangle^{\rm RF} - R^4\sum_{\ell=1}^{n} N_g\, r_\ell^{(4,0)}\,\bar{a}_R^\ell + N_g\,\bar{c}_0(R^2)$$

$$\bar{c}_0(R^2) \equiv R^4 \;\mathrm{PV}\int_0^\infty \frac{\mathrm{d}u\; e^{-\frac{u}{\bar{a}_R}}}{(2-u)^{1+4\hat{b}_1}}$$

$$\frac{d}{d\log R^2}\langle G^2\rangle^{\rm RF} = 0$$

scale invariant

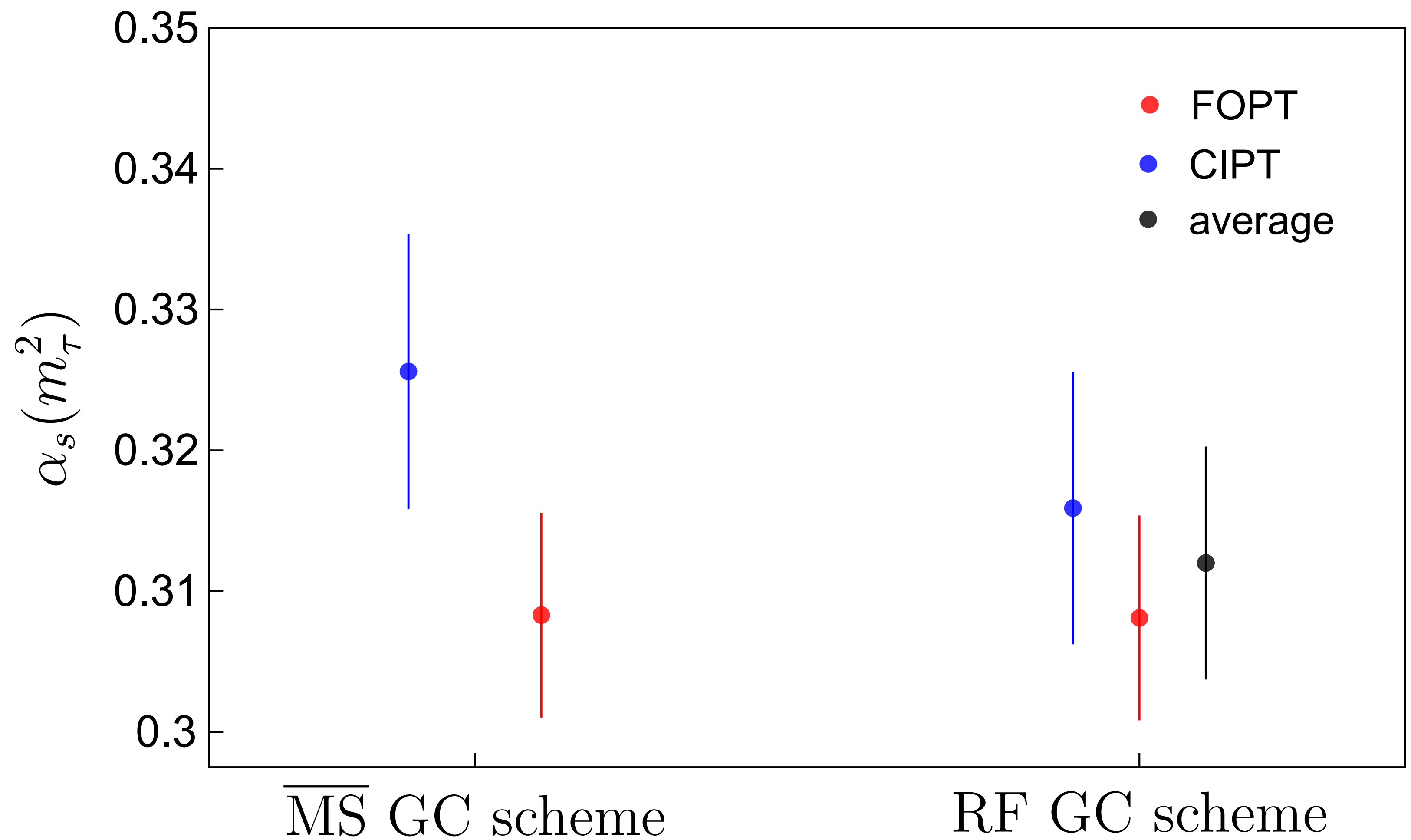
Borel sum unchanged, for any value of the norm. Minimal scheme.

Diogo Boito

reconciling FOPT and CIPT

The renormalon-free scheme for the gluon condensate is able to reconcile FO and CIPT results

Benitez-Rathgeb, DB, A. Hoang, M. Jamin, 2207.01116

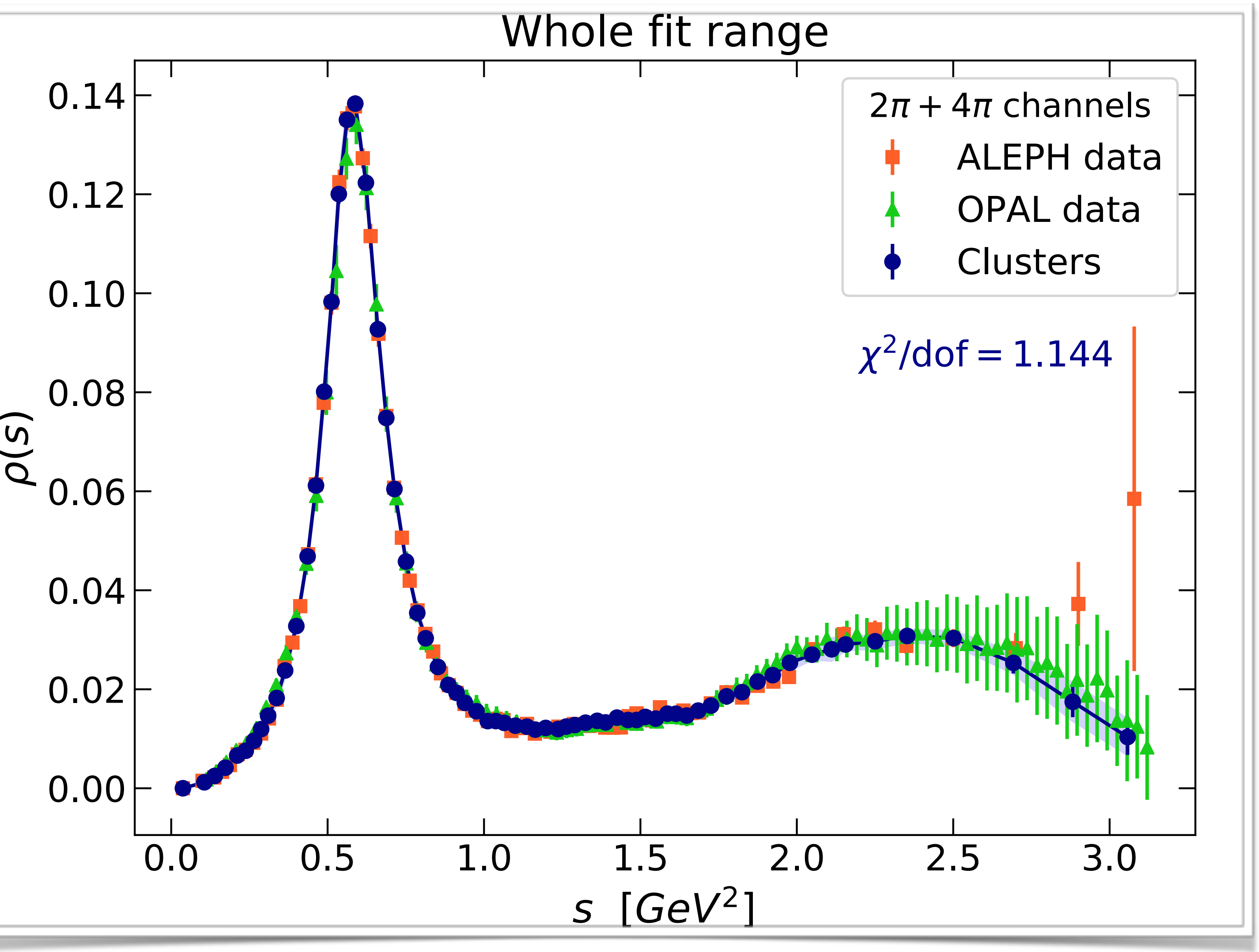


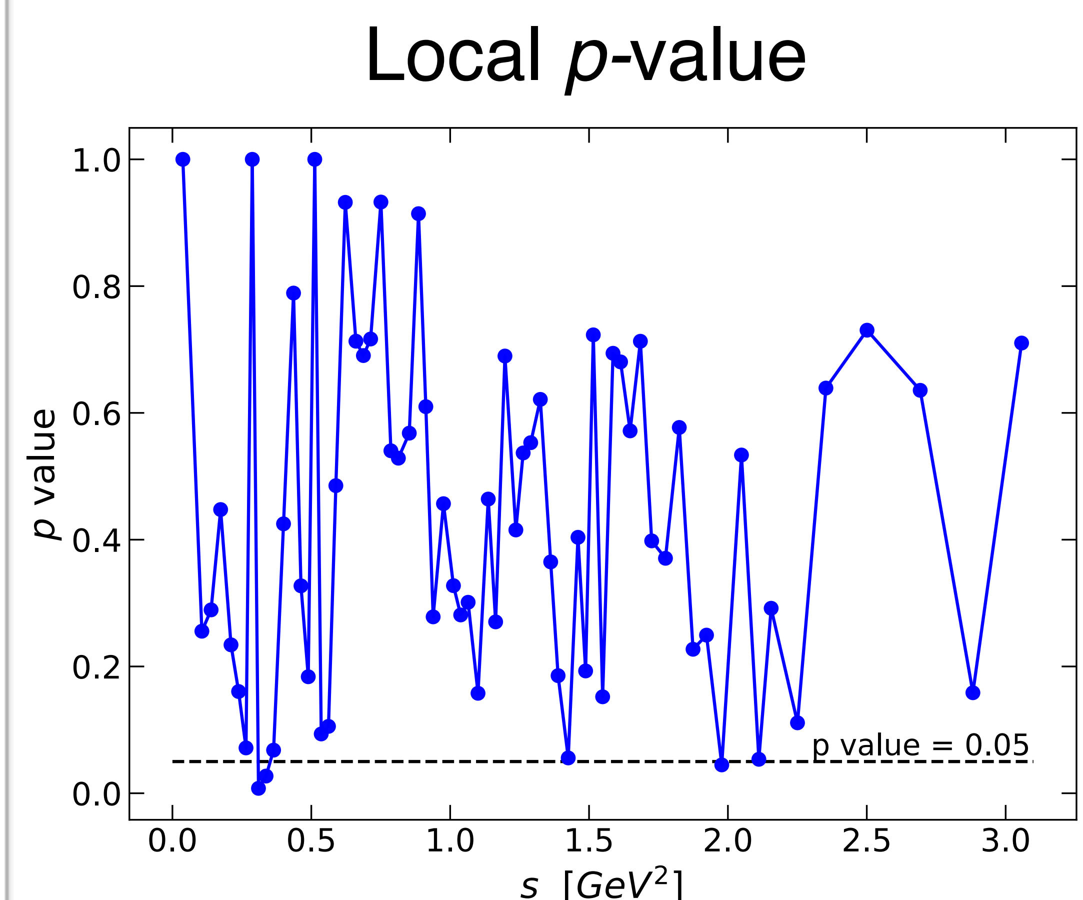
We can now consistently average the two results to obtain

$$\alpha_s(m_\tau) = 0.3120 \pm 0.0082$$

new vector isovector spectral function

Combination of $2\pi + 4\pi$ channels
Good χ^2 both locally and globally, no χ^2 inflation needed





DB, Golterman, Maltman, Peris, Rodrigues and Schaaf, arXiv:2012.10440

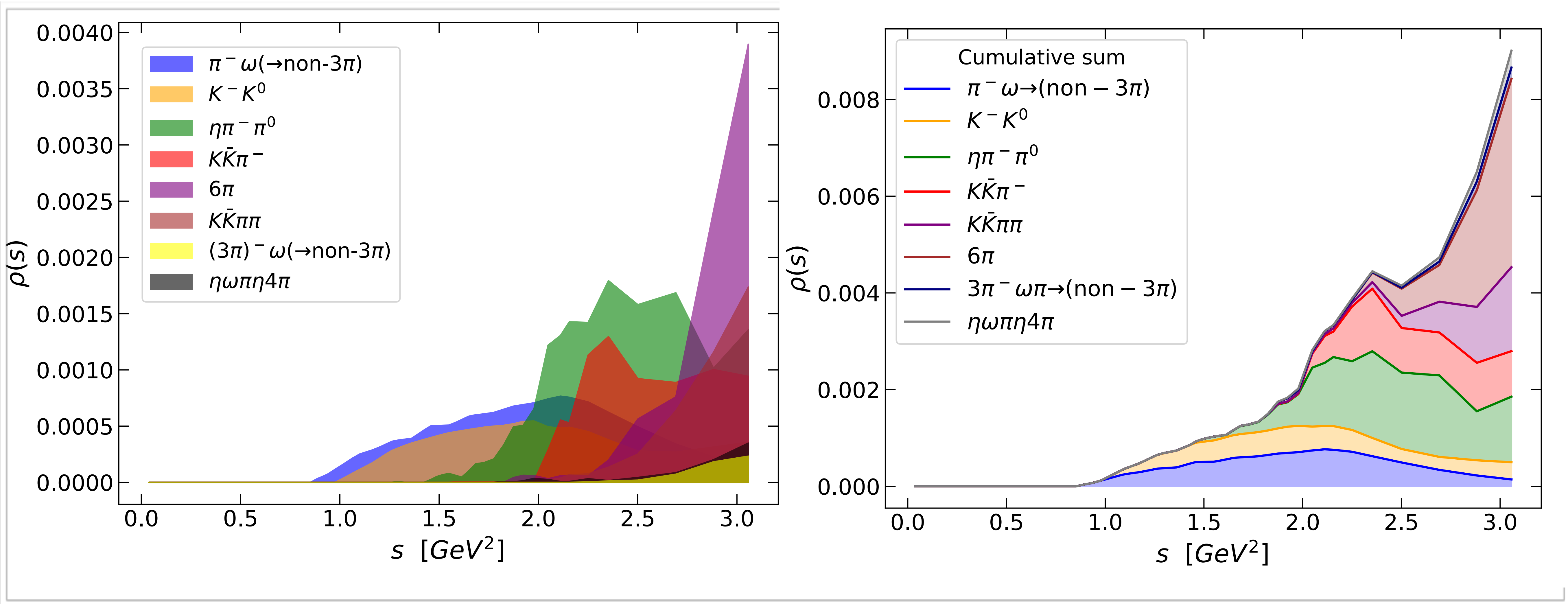
new vector isovector spectral function

7 residual channels extracted from e^+e^- data + BaBar data for $\tau \to K K_S \nu_\tau$

Dramatic improvement in errors for higher multiplicity modes (near end point)

No Monte Carlo input

Original data sets from: BABAR, CMD-3 and SND (results from 16 papers)

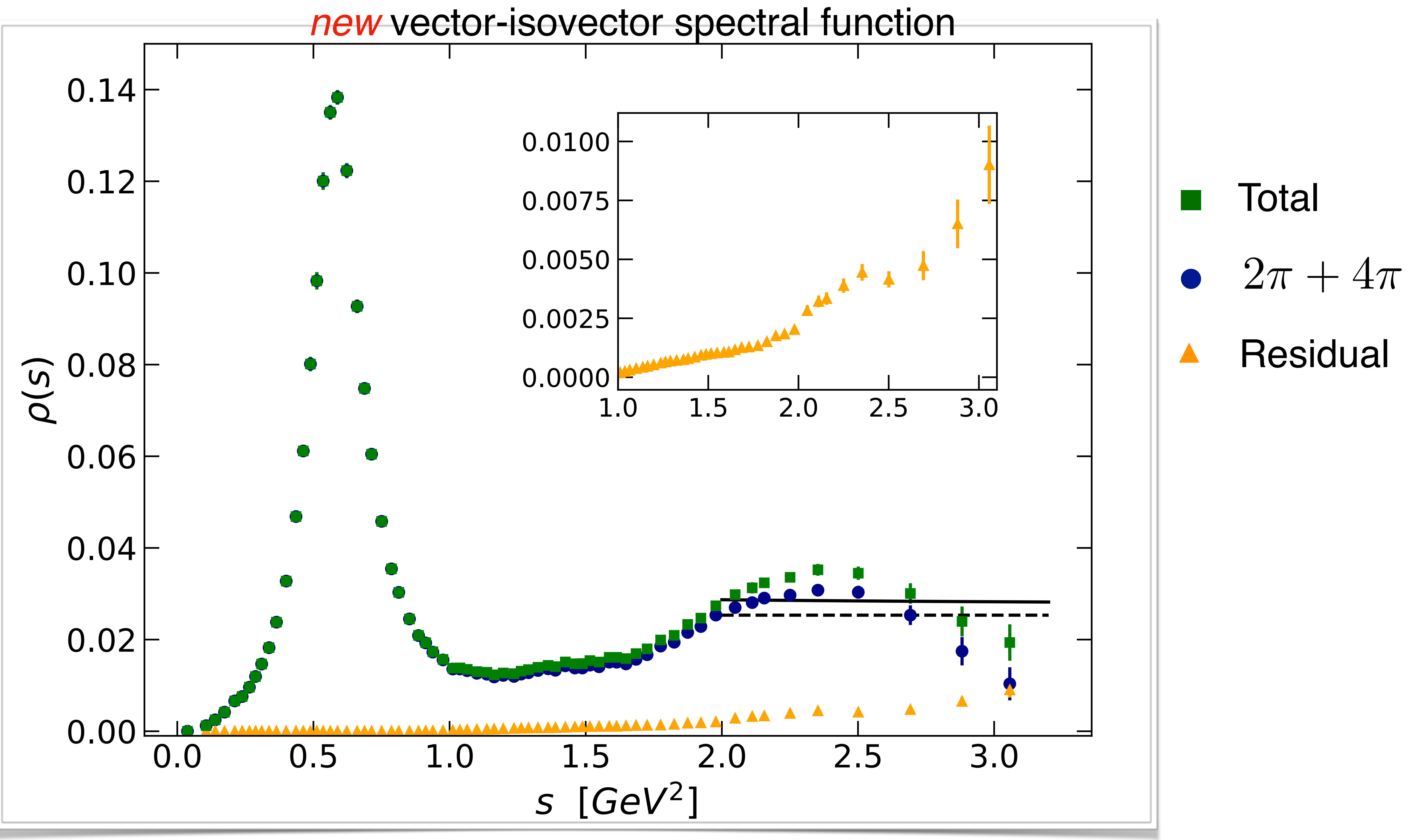


DB, Golterman, Maltman, Peris, Rodrigues and Schaaf, arXiv:2012.10440

new vector isovector spectral function

Combined $2\pi + 4\pi$ (ALEPH and OPAL) + residual channels from data

99.95% of the Branching Fraction covered



stability of fit parameters