

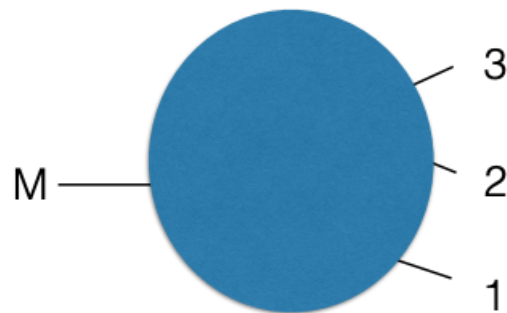
# Amplitude analyses on charm decays at the LHCb experiment

Fernanda Abrantes  
on behalf of the LHCb collaboration  
@ ICHEP meeting



# Dalitz plot analysis in 3 body decays

- Dalitz Plot has been a *fundamental tool* to study the dynamics of decay processes
- Provides important information of hadronic processes, such as:
  - Revealing and understanding resonances in different final states
  - Study the dynamics of the scalar sector (not well understood)
  - Search and study of CP violation in the beauty and charm sector
  - Study lineshapes and interference patterns



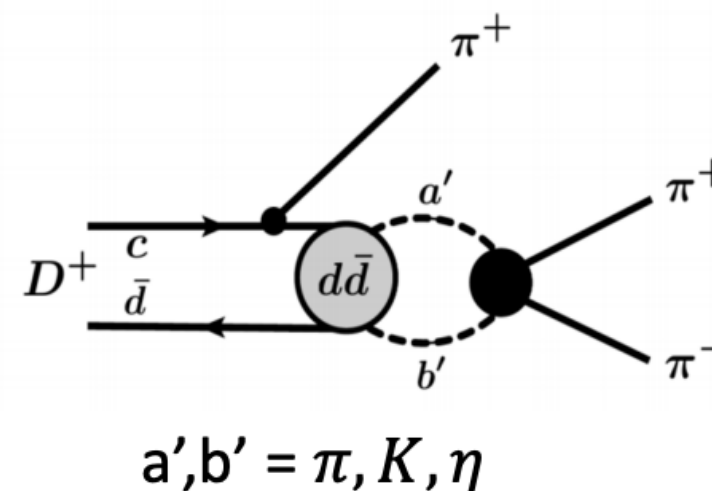
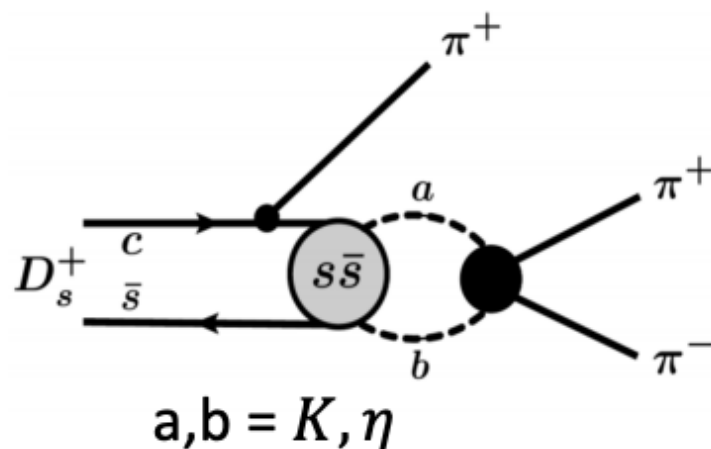
$$\begin{aligned} d\Gamma &= \left| \mathcal{M}_{fi} \right|^2 d\Phi \\ &= \left| \mathcal{M}_{fi} \right|^2 \left| \frac{\partial \Phi}{\partial (s_{12}, s_{13})} \right| ds_{12} ds_{13} \\ &= \frac{1}{(2\pi)^2 32 M^3} \left| \mathcal{M}_{fi} \right|^2 ds_{12} ds_{13} \end{aligned} \quad s_{ij} \equiv (p_i + p_j)^2 \equiv m_{ij}^2$$

*Focus on the latest LHCb charm meson amplitude analyses:*

Amplitude analysis of the  $D^+ \rightarrow \pi^- \pi^+ \pi^+$  and  $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$  decays and measurement of the  $\pi^- \pi^+$  S-wave amplitude

# $D^+ \rightarrow \pi^- \pi^+ \pi^+$ and $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$ decays

- Previous analyses:
  - ♦  $D^+$ : E791 (2001)<sup>[1]</sup>, FOCUS (2004)<sup>[2]</sup> and CLEO (2007)<sup>[3]</sup>
    - ♦ CLEO's ~2.6k events with 55% purity
  - ♦  $D_s^+$ : E791 (2001)<sup>[4]</sup>, BaBar (2009)<sup>[5]</sup>, BESIII (2021)<sup>[6]</sup>
    - ♦ BESIII sample ~13k events with 80% purity
- **S-wave measured to be the major contribution** in the  $3\pi$  final state
  - **Challenge:** understand the  $\pi^- \pi^+$  scalar sector with many overlapping resonances;
  - $f_0(980)$  accounts as ~50% of the total decay rate for  $D_s^+$  and  $f_0(500)$  with ~50% for  $D^+$
  - Comparison between  $D_s^+$  and  $D^+$  S-wave amplitudes
- LHCb data has larger samples allowing for unprecedented opportunity to study these channels and enlighten our knowledge of their dynamics
- **Quasi-Model Independent approach (QMIPWA)** for S-wave, and Isobar Model for P and D-waves
- Both analysis based on 2012 data sample (Run I)

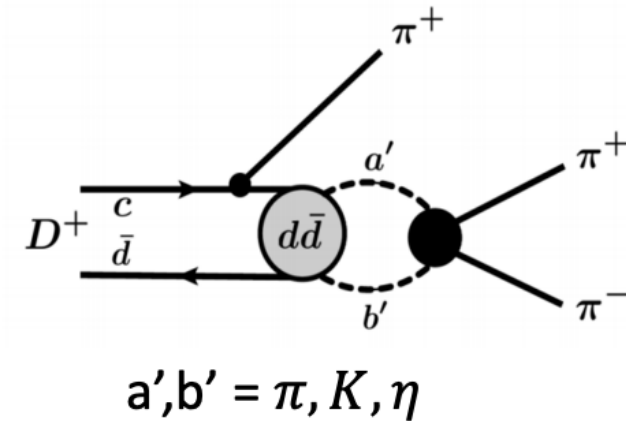


[1] PRL 86 (2001) 770  
 [2] PLB 585 (2004) 200  
 [3] PRD 76 (2007) 012001  
 [4] PRL 86 (2001) 765  
 [5] PRD 79 (2009) 032003  
 [6] arXiv:2108.10050

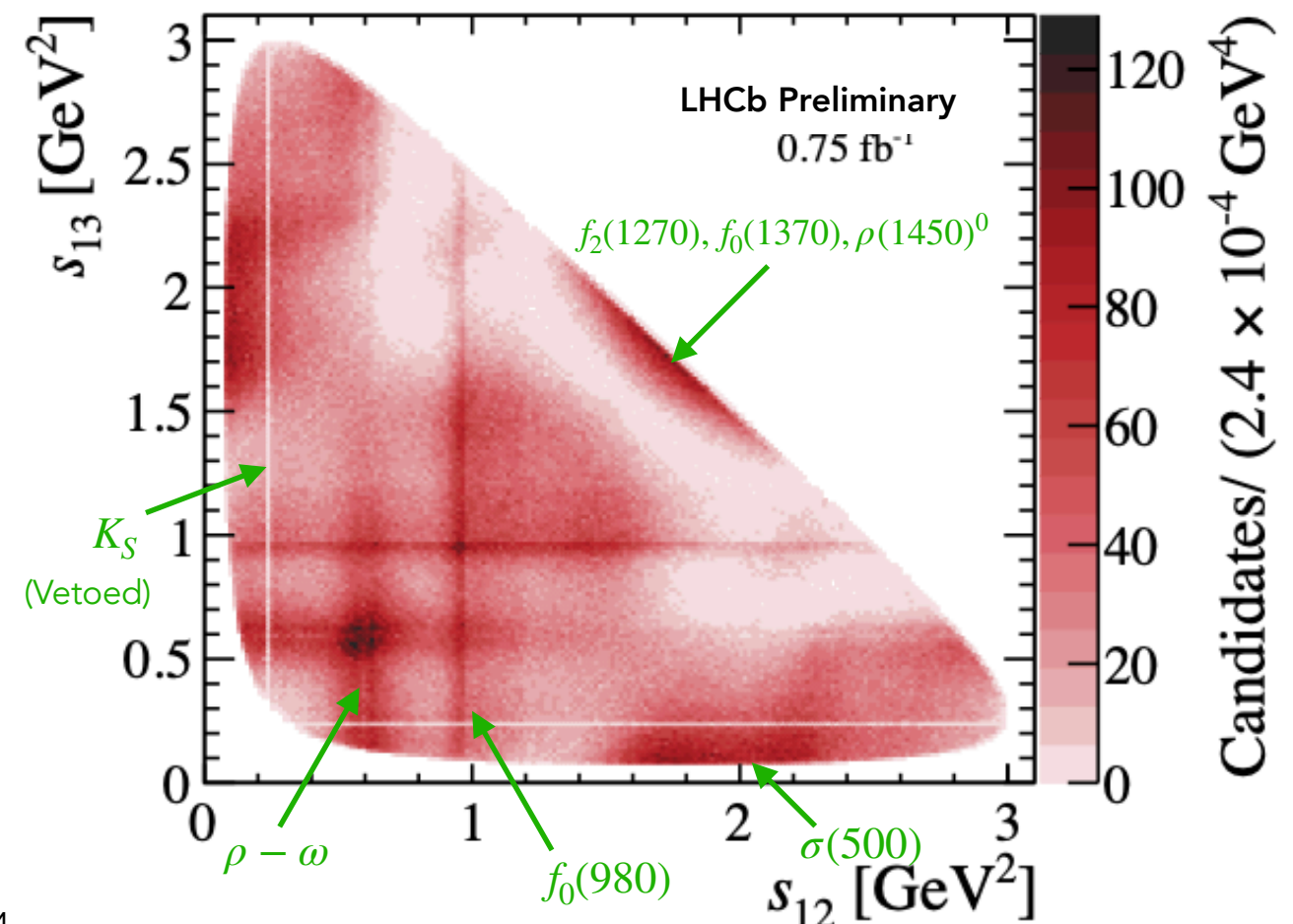
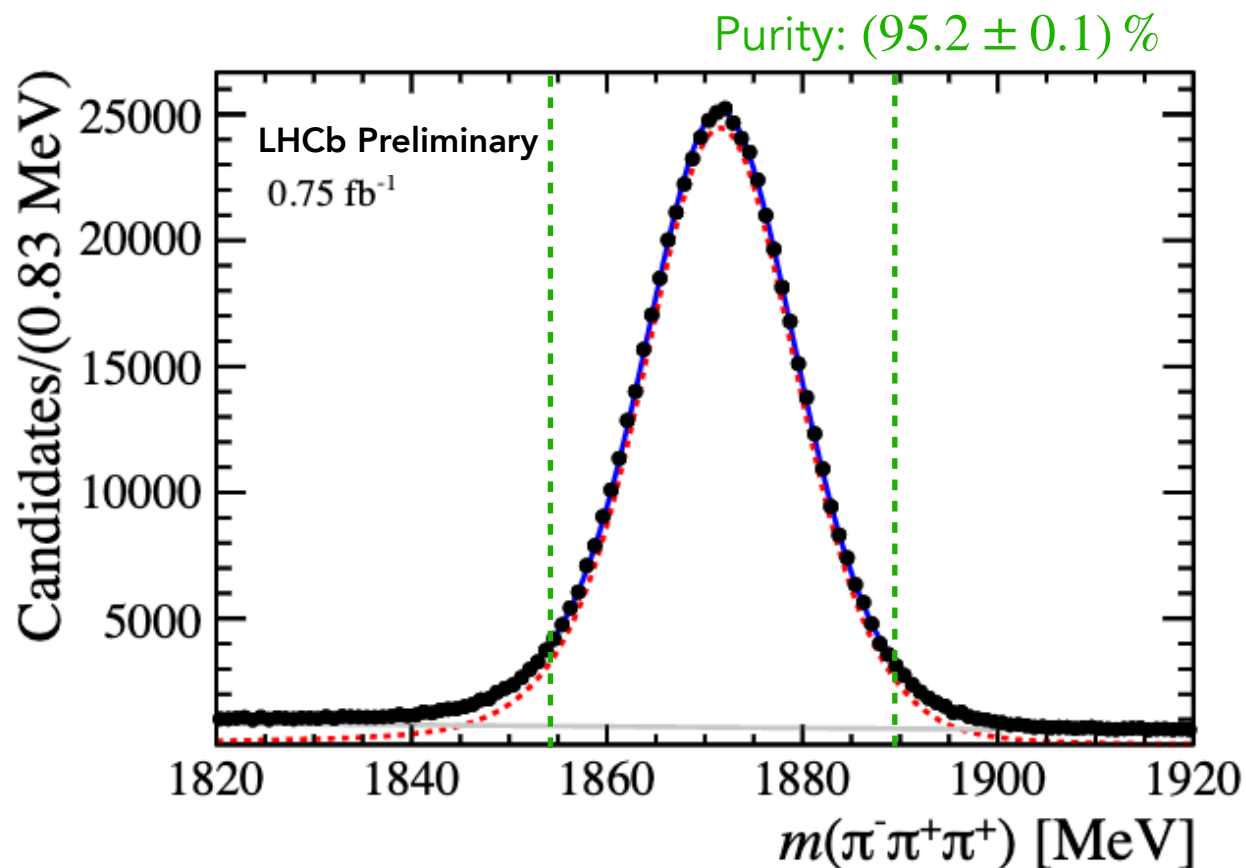
# $D^+ \rightarrow \pi^- \pi^+ \pi^+$ decays

LHCb-PAPER-2022-016 in preparation

- No new results since CLEO (15 years ago)!!
- First time performing a QMIPWA analysis!!
- High statistics, thus, sensitive to more details



- **Selection:** trigger+ offline pre selection + MVA selection (reduce combinatorial background)
- **Final sample:** ~600k events and ~95% purity (minimise effects of background misparametrisation)
- Use only events within the  $2\sigma$  window for all Dalitz fits



# Quasi Model-Independent Partial Wave Analysis - QMIPWA

- The  $m(\pi^-\pi^+)$  mass spectrum is divided into sub-intervals (knots)
- In each knot edge, the amplitude is determined by two real constants,  $a_k$  and  $\phi_k$

$$A_{S-wave}^k = a^k e^{i\phi^k} \quad k, l \rightarrow \text{knots in } m(\pi^-\pi^+)$$


$$A_{S-wave}^{k,l}(s_{12}, s_{13}) = A_{S-wave}^k(m_{12}) + A_{S-wave}^l(m_{13})$$

*A linear spline interpolation is used to get the S-wave amplitude at any point in  $m^2(\pi^-\pi^+)$*

- P- and D-waves assumed to be well parametrised and included via Isobar Model.
- GooFit: framework for maximum likelihood fits using GPU

**Total amplitude**

$$\mathcal{A}(s_{12}, s_{13}) = \left[ \mathcal{A}_{S-Wave} + \sum_{spin1, spin2} a_i e^{i\delta_i} \mathcal{A}_i \right] \quad (s_{12} \leftrightarrow s_{13})$$

 Bose-symmetrisation  
(2 identical  $\pi^+$ )

$$s_{12} = m^2(\pi_1^-\pi_2^+) \text{ and } s_{13} = m^2(\pi_1^-\pi_3^+)$$

**Free parameters:**

- Spin 1 and 2 contributions: magnitude  $a_i$  and phase  $\delta_i$
- S-Wave:  $a^k$  and  $\phi^k$

**Other information:**

- Fit quality via  $\chi^2$  test
- Comparison between models:  
FCN =  $-2 \log \mathcal{L}$

# Fit result: $D^+ \rightarrow \pi^- \pi^+ \pi^+$

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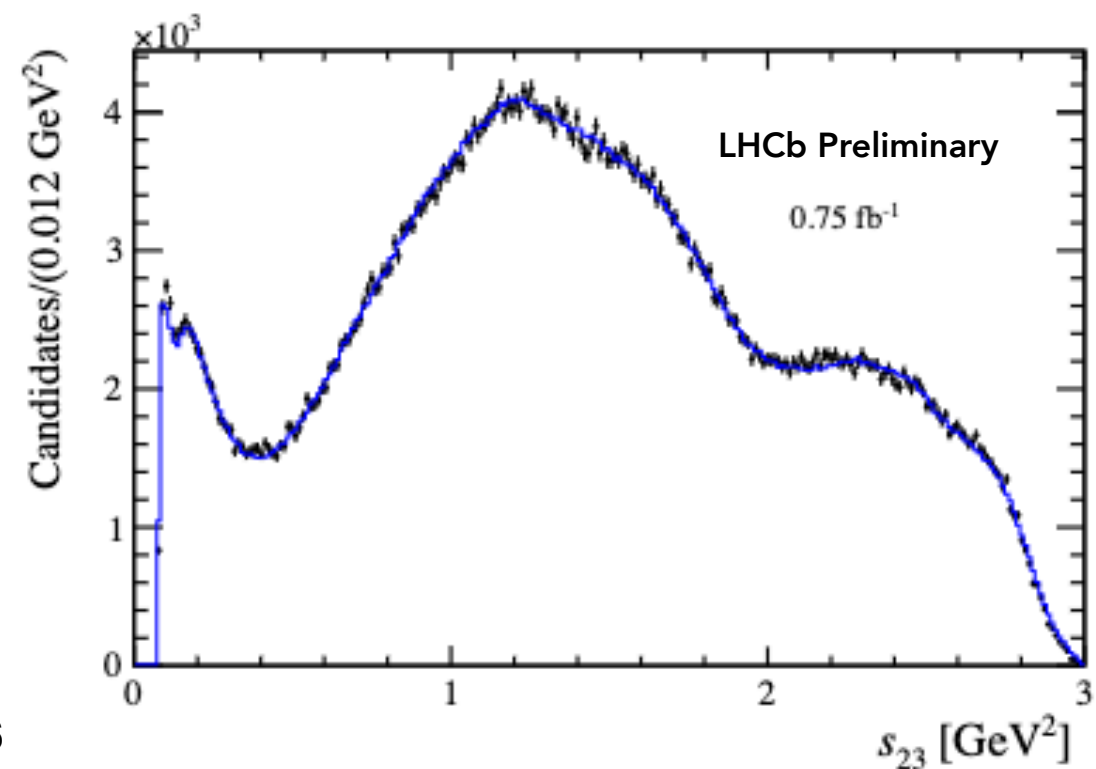
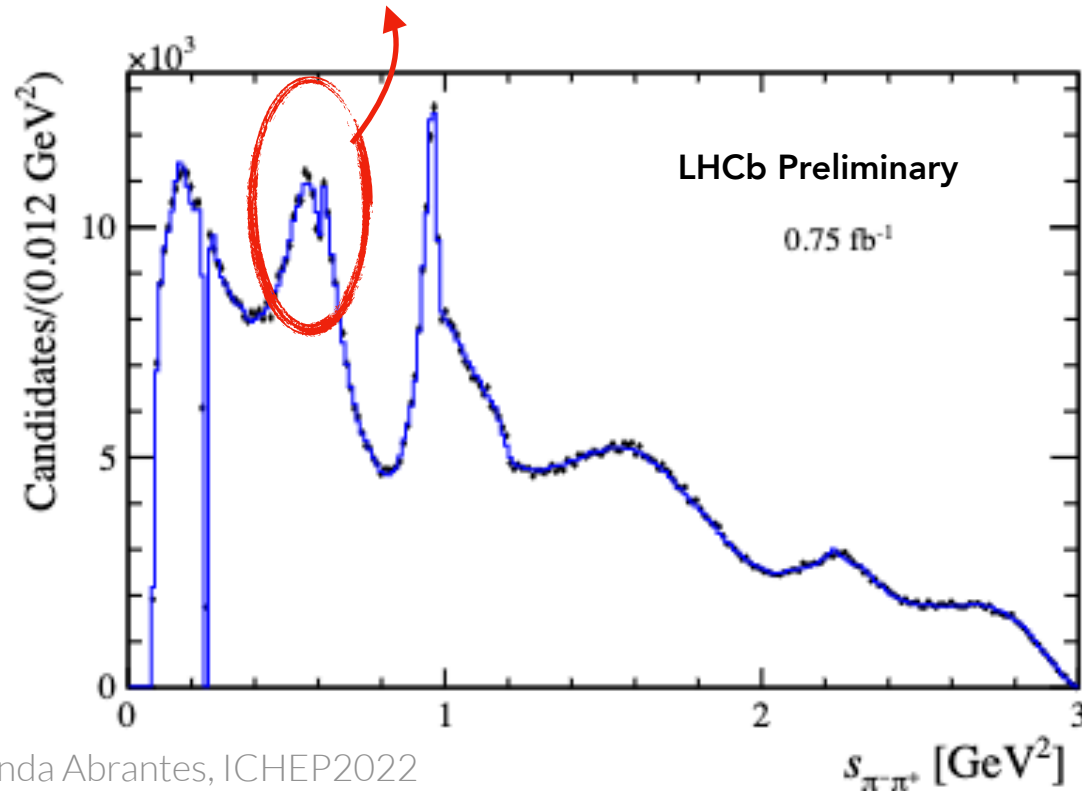
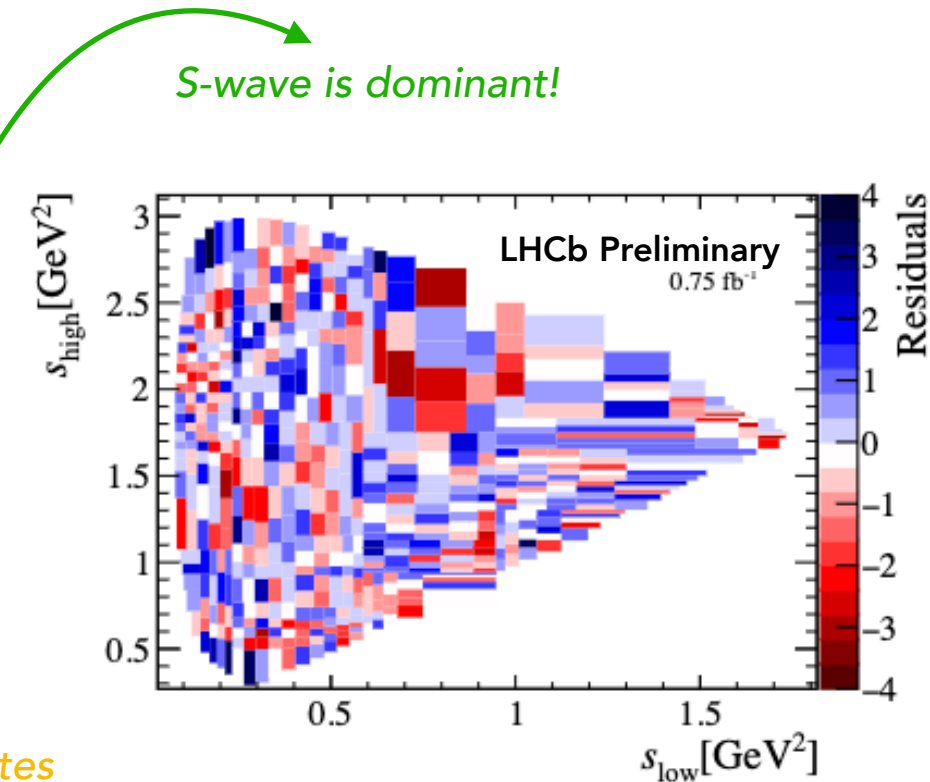
Component	Magnitude	Phase [°]	Fit fraction [%]
$\rho(770)^0 \pi^+$	1 [fixed]	0 [fixed]	26.0 $\pm$ 0.3 $\pm$ 1.6 $\pm$ 0.3
$\omega(782) \pi^+$	$(1.68 \pm 0.06 \pm 0.15 \pm 0.02) \times 10^{-2}$	$-103.3 \pm 2.1 \pm 2.6 \pm 0.4$	$0.103 \pm 0.008 \pm 0.014 \pm 0.002$
$\rho(1450)^0 \pi^+$	$2.66 \pm 0.07 \pm 0.24 \pm 0.22$	$47.0 \pm 1.5 \pm 5.5 \pm 4.1$	$5.4 \pm 0.4 \pm 1.3 \pm 0.8$
$\rho(1700)^0 \pi^+$	$7.41 \pm 0.18 \pm 0.47 \pm 0.71$	$-65.7 \pm 1.5 \pm 3.8 \pm 4.6$	$5.7 \pm 0.5 \pm 1.0 \pm 1.0$
$f_2(1270) \pi^+$	$2.16 \pm 0.02 \pm 0.10 \pm 0.02$	$-100.9 \pm 0.7 \pm 2.0 \pm 0.4$	$13.8 \pm 0.2 \pm 0.4 \pm 0.2$
S-wave			<b>61.8 <math>\pm</math> 0.5 <math>\pm</math> 0.6 <math>\pm</math> 0.5</b>
$\sum_i \text{FF}_i$			112.8
$\chi^2/\text{ndof}$ (range)	[1.47 - 1.78]		$-2 \log \mathcal{L} = 805622$

## Interference fit fractions

	$\omega(782) \pi^+$	$\rho(1450)^0 \pi^+$	$\rho(1700)^0 \pi^+$	$f_2(1270) \pi^+$	S-wave
$\rho(770)^0 \pi^+$	$-0.24 \pm 0.06$	<b><math>5.1 \pm 0.3</math></b>	<b><math>-5.8 \pm 0.4</math></b>	$-0.3 \pm 0.1$	$1.8 \pm 0.4$
$\omega(782) \pi^+$		$0.05 \pm 0.01$	$0.05 \pm 0.01$	$0.046 \pm 0.004$	$-0.04 \pm 0.01$
$\rho(1450)^0 \pi^+$			<b><math>-4.0 \pm 0.5</math></b>	$1.1 \pm 0.1$	$1.7 \pm 0.2$
$\rho(1700)^0 \pi^+$				$-0.8 \pm 0.1$	$-3.4 \pm 0.5$
$f_2(1270) \pi^+$					$-1.6 \pm 0.1$

$\rho - \omega$  interference: first time being observed in this channel!!!

Large interference between  $\rho$  states



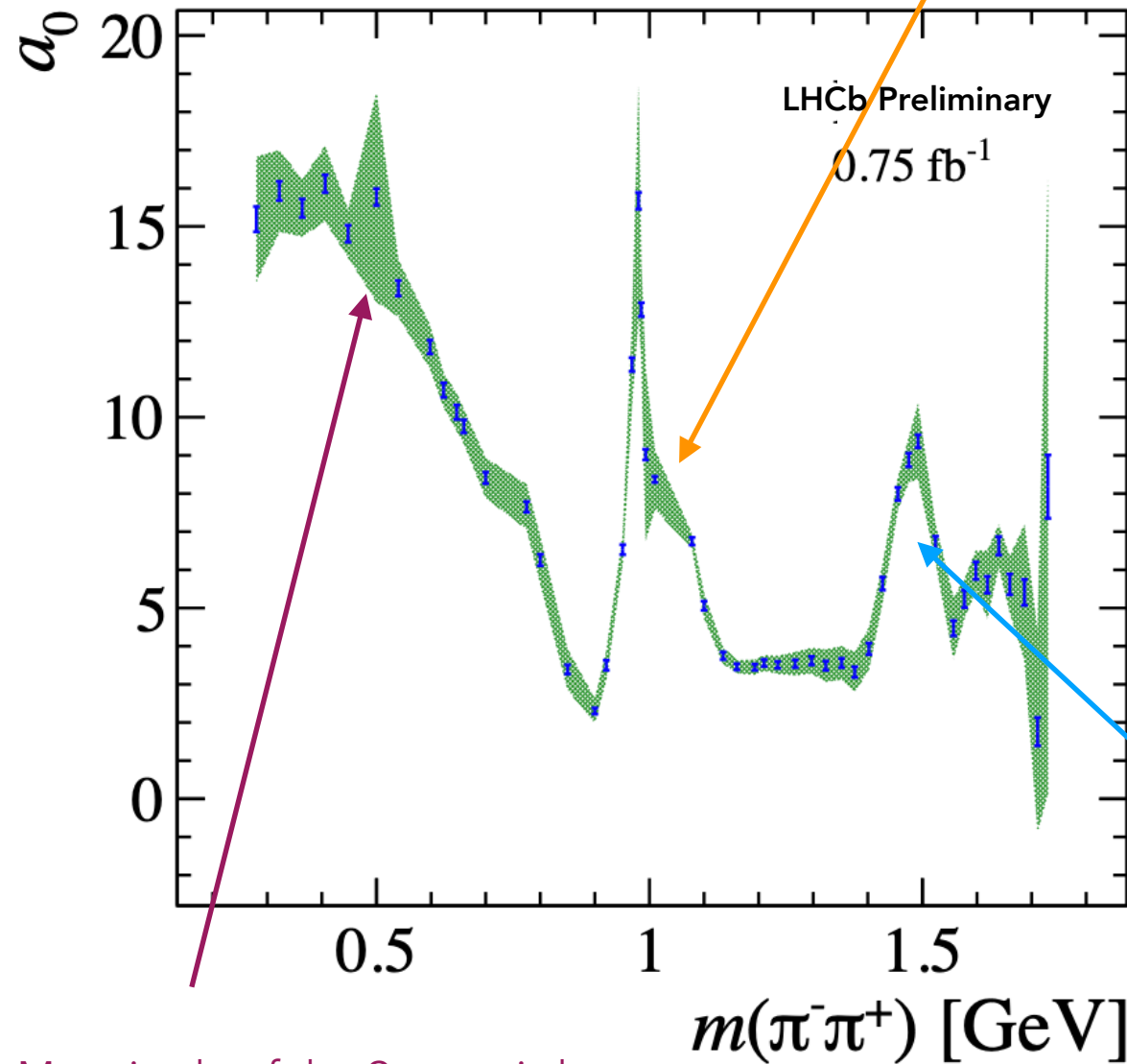


# $D^+ \rightarrow \pi^- \pi^+ \pi^+$ : Extracted $\pi^- \pi^+$ S-Wave amplitude

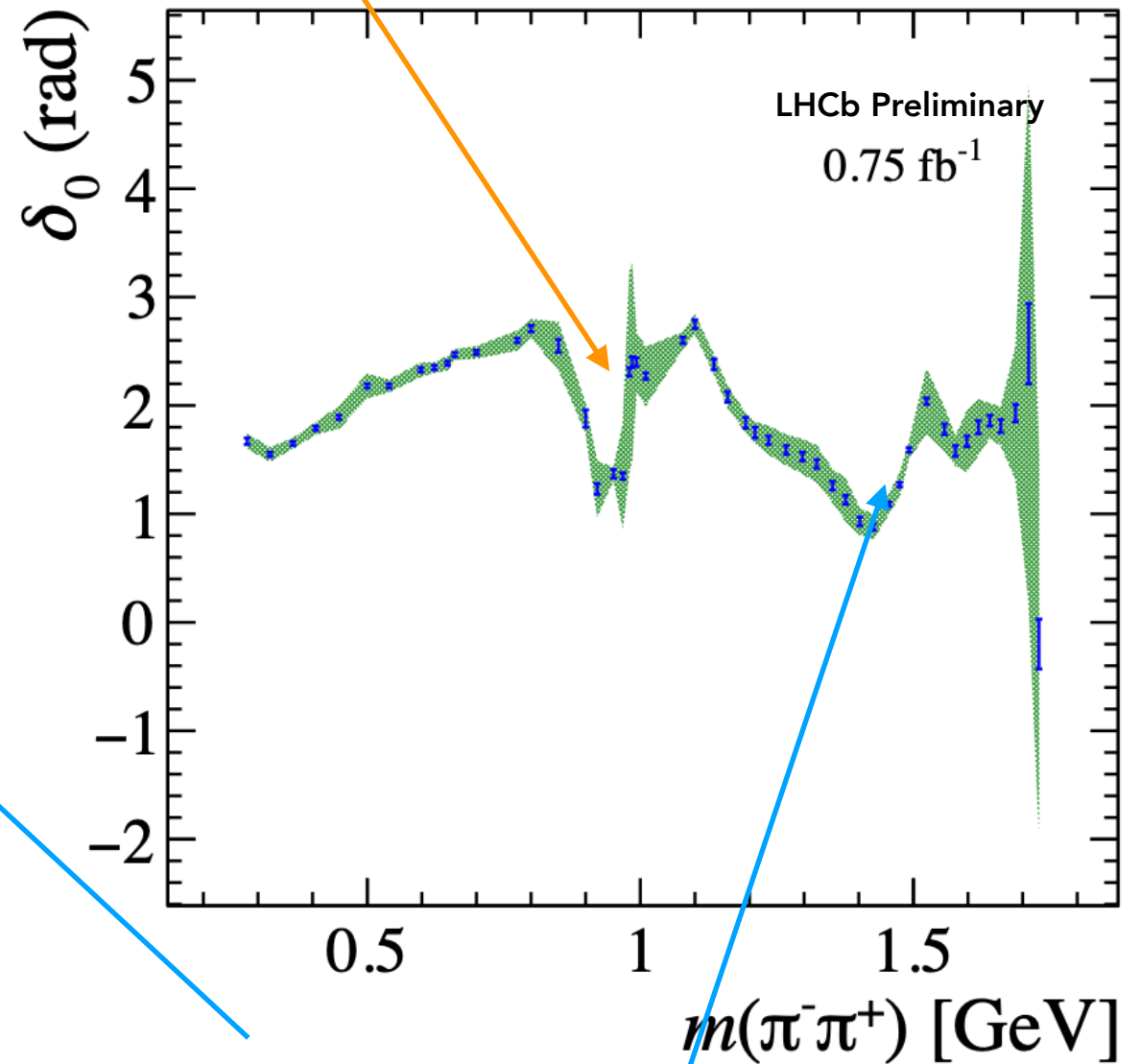
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Statistical uncertainties  
Stat + exp. syst + model  
syst uncertainties

Signature of  $f_0(980)$ : Peak in the magnitude and large phase variation near 1.0 GeV



Magnitude of the S-wave is larger close to the threshold indicating a dominant contribution from the  $f_0(500)$



Rapid growth of phase and amplitude towards the end of the spectrum indicates the presence of at least one more scalar resonance, for instance  $f_0(1500)$  (also enhanced by the opening of  $\eta\eta'$  channel)

# Summary $D^+ \rightarrow \pi^- \pi^+ \pi^+$

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## Principal contributions:

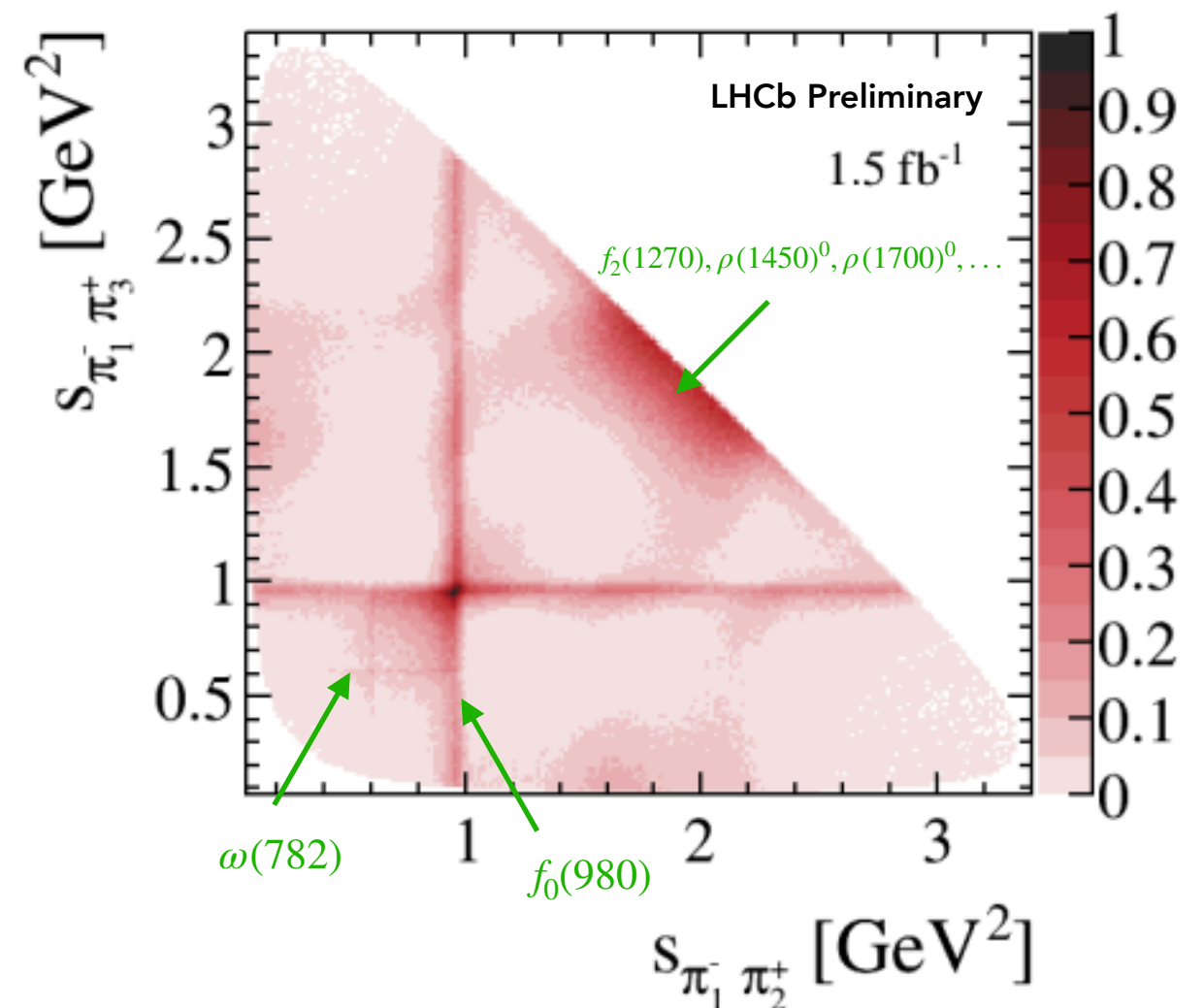
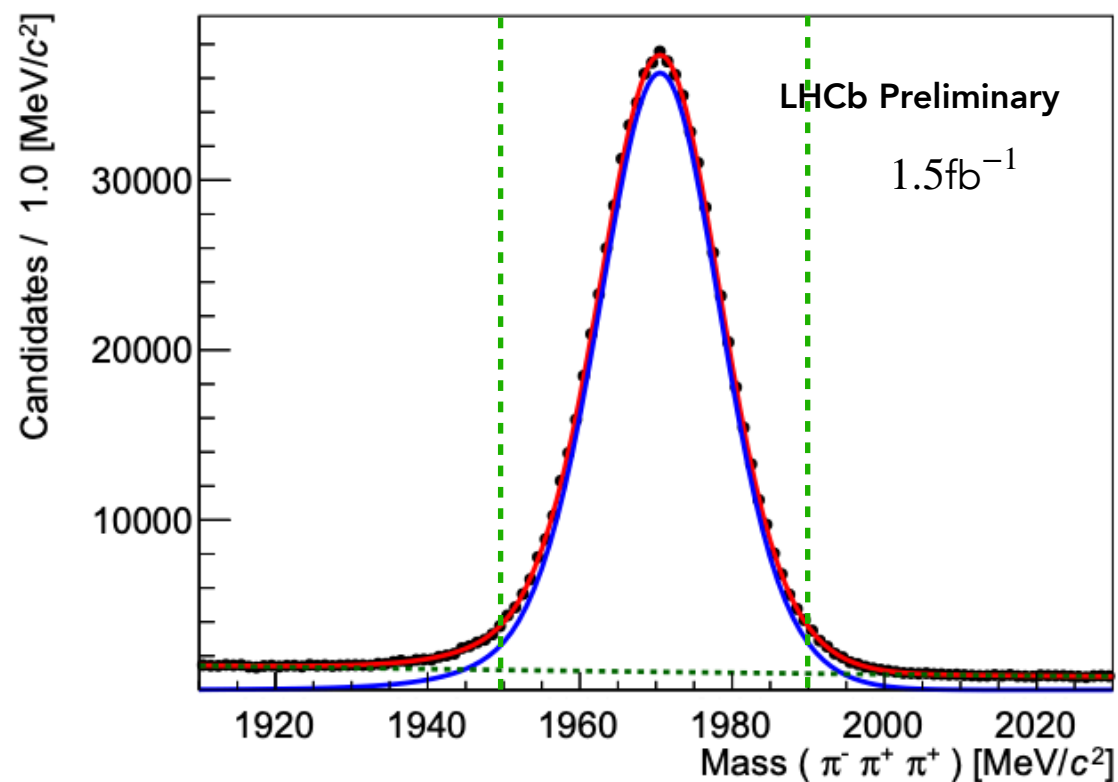
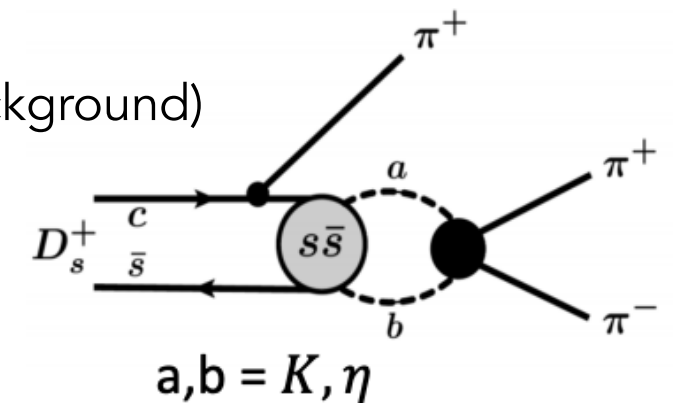
- First QMIPWA analysis for this decay channel!!
- Detailed measurement of the  $\pi^- \pi^+$  S-wave amplitude
- $\rho - \omega$ : first time observed in  $D^+ \rightarrow \pi^- \pi^+ \pi^+$ 
  - $\omega(782) \rightarrow \pi^- \pi^+$ :  $(0.103 \pm 0.008 \pm 0.014 \pm 0.002) \%$
  - $\omega(782) \rightarrow \pi^- \pi^+$  isospin-violating decay
- $\rho - \omega$  mixing lineshape also tested but no significant difference
  - $|B| = 0.522 \pm 0.019 \pm 0.047 \pm 0.006$  and  $\phi_B = (158.8 \pm 2.1 \pm 2.6 \pm 0.4)^\circ$
- S-wave found to be dominant  $\sim 62\%$  (in agreement with previous observations)
- Best fit result includes the  $\rho(1700)^0$  state
  - very significant change in the FCN! (-488 units compared to that of the model without it)
  - Only  $\rho^0(1450)$  is not enough to describe P wave in the high mass end



# $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$ decays

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- Latest results from BESIII with a QMIPWA analysis ( $\sim 1.3k$  events)
- Follows the same methodology as the  $D^+ \rightarrow \pi^- \pi^+ \pi^+$  analysis
- **Selection:** trigger + offline pre selection + MVA selection (reduce combinatorial background)
- Final sample with over 700k events and  $\sim 95\%$  purity
  - (minimise effects of background misparametrisation)



# Fit result: $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$

LHCb-PAPER-2022-030 in preparation

Resonance	Magnitude	Phase [°]	Fit Fraction (%)
$\rho(770)^0$	$0.1201 \pm 0.0030 \pm 0.0050 \pm 0.0062$	$79.4 \pm 1.8 \pm 7.8 \pm 4.4$	$1.038 \pm 0.054 \pm 0.097 \pm 0.11$
$\omega(782)$	$0.04001 \pm 0.00090 \pm 0.0018 \pm 0.00086$	$-109.9 \pm 1.7 \pm 0.94 \pm 1.4$	$0.360 \pm 0.016 \pm 0.034 \pm 0.016$
$\rho(1450)^0$	$1.277 \pm 0.026 \pm 0.023 \pm 0.48$	$-115.2 \pm 2.6 \pm 2.8 \pm 10$	$3.86 \pm 0.15 \pm 0.14 \pm 2.0$
$\rho(1700)^0$	$0.873 \pm 0.061 \pm 0.054 \pm 0.62$	$-60.9 \pm 6.1 \pm 6.7 \pm 12$	$0.365 \pm 0.050 \pm 0.045 \pm 0.34$
$f_2(1270)$	1 (fixed)	0 (fixed)	$13.69 \pm 0.14 \pm 0.22 \pm 0.49$
$f_2'(1525)$	$0.1098 \pm 0.0069 \pm 0.019 \pm 0.015$	$178.1 \pm 4.2 \pm 12 \pm 7$	$0.0528 \pm 0.0070 \pm 0.015 \pm 0.0087$
S-wave			$84.97 \pm 0.14 \pm 0.30 \pm 0.63$

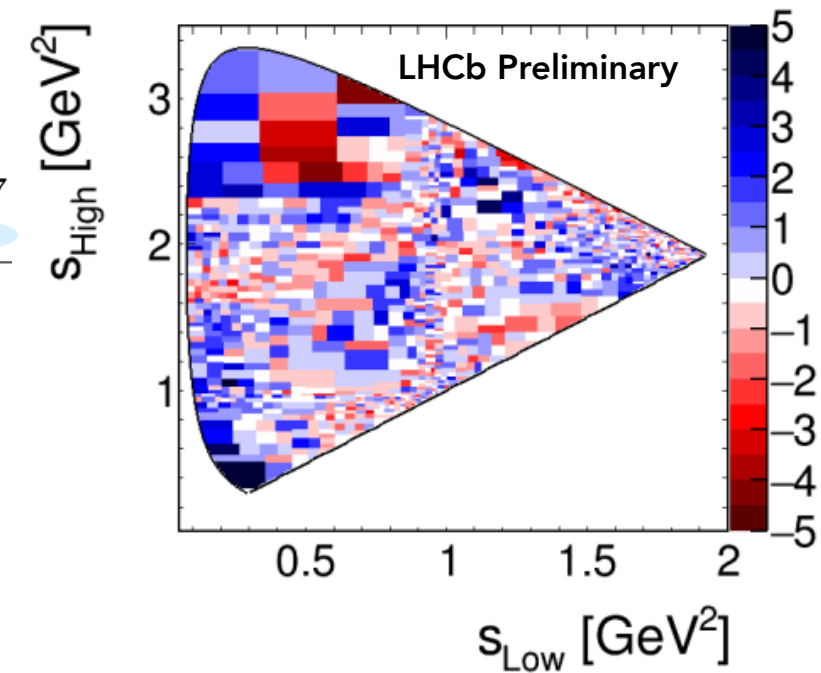
## Interference fit fractions

	$\omega(782)$	$\rho(770)^0$	$\rho(1450)^0$	$\rho(1700)^0$
$\omega(782)$	$0.360 \pm 0.016$			
$\rho(770)^0$	$0.128 \pm 0.013$	$1.038 \pm 0.054$		
$\rho(1450)^0$	$0.36 \pm 0.14$	$0.148 \pm 0.14$	$3.86 \pm 0.15$	
$\rho(1700)^0$	$0.089 \pm 0.010$	$-0.307 \pm 0.55$	$1.92 \pm 0.20$	$0.365 \pm 0.050$
$f_2(1270)$	$-0.1540 \pm 0.0040$	$0.280 \pm 0.029$	$-1.10 \pm 0.047$	$-0.376 \pm 0.047$
$f_2'(1525)$	$0.00827 \pm 0.00063$	$0.00283 \pm 0.0038$	$0.066 \pm 0.0021$	$0.0200 \pm 0.0021$
S-wave	$-0.053 \pm 0.0099$	$0.804 \pm 0.076$	$-1.520 \pm 0.086$	$-0.934 \pm 0.086$

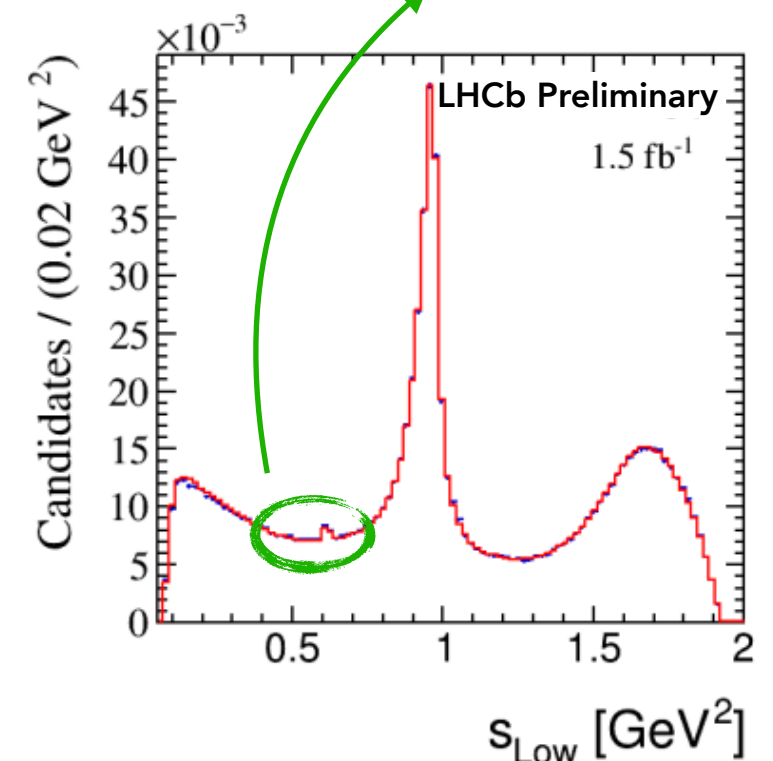
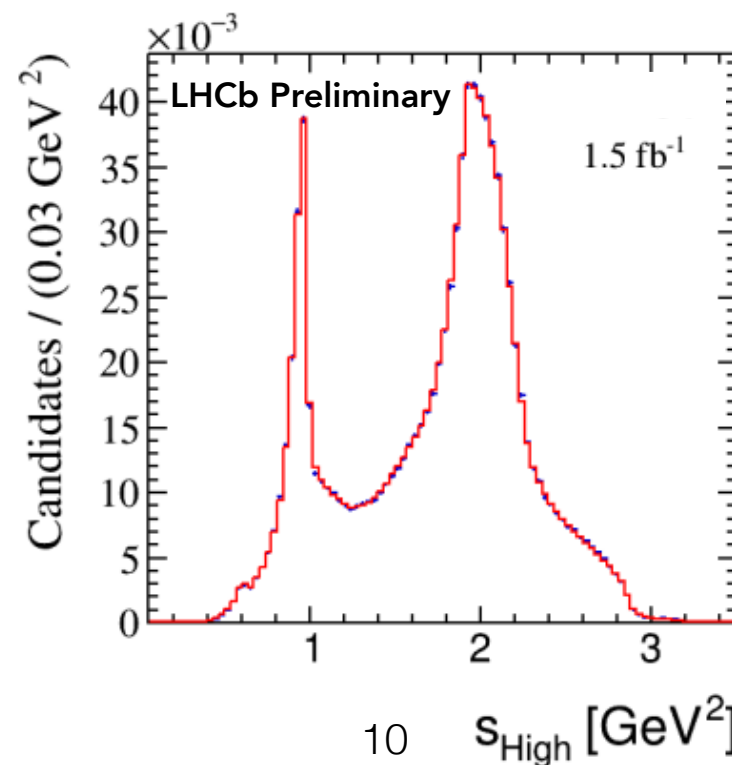
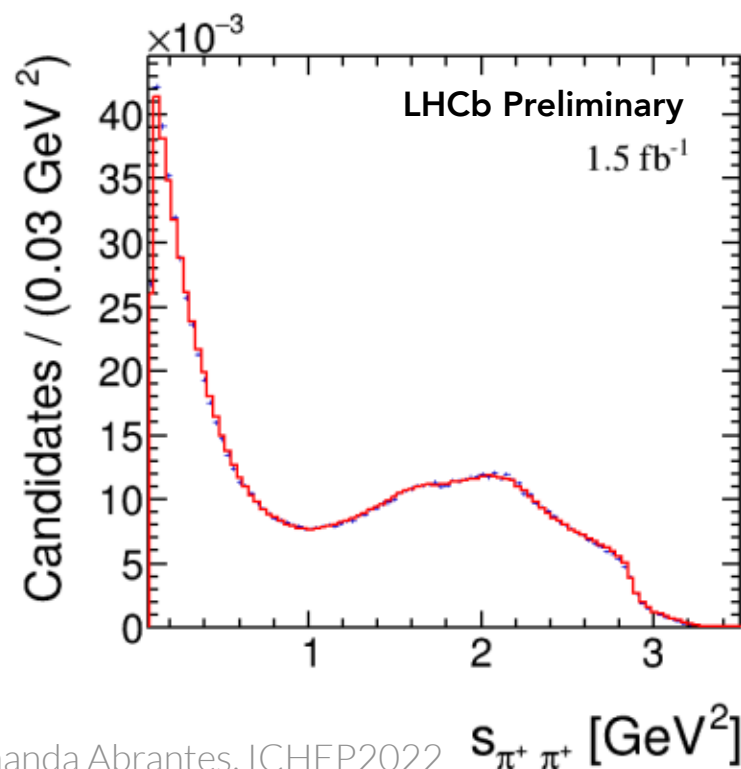
  

	$f_2(1270)$	$f_2'(1525)$	S-wave
$f_2(1270)$	$13.69 \pm 0.14$		
$f_2'(1525)$	$-0.429 \pm 0.072$	$0.0455 \pm 0.0070$	
S-wave	$-3.460 \pm 0.092$	$0.20 \pm 0.013$	$84.97 \pm 0.14$

S-wave is dominant!



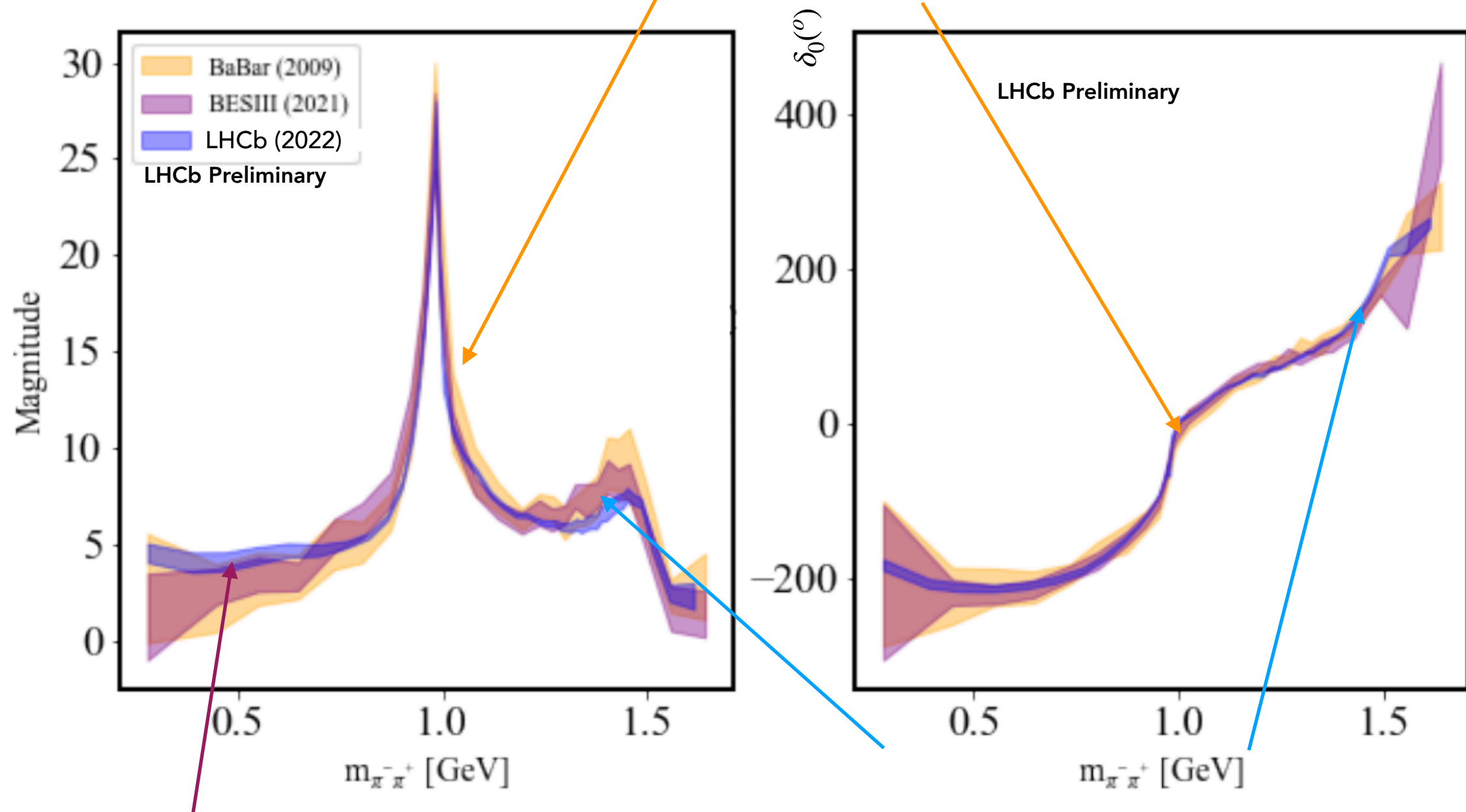
$\omega(782)$ : first time being observed in this channel!!



# $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$ : Extracted $\pi^- \pi^+$ S-Wave amplitude

*LHCb-PAPER-2022-030 in preparation*

Signature of  $f_0(980)$ : Peak in the magnitude and large phase variation near 1.0 GeV



Magnitude of the S-wave is small indicating no contribution from the  $f_0(500)$

Rapid growth of phase towards the end of the spectrum indicates the presence of at least one more scalar resonance, for instance  $f_0(1500)$  (also enhanced by the opening of  $\eta\eta'$  channel)

# Summary $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$

LHCb-PAPER-2022-030 in preparation

## Principal contributions:

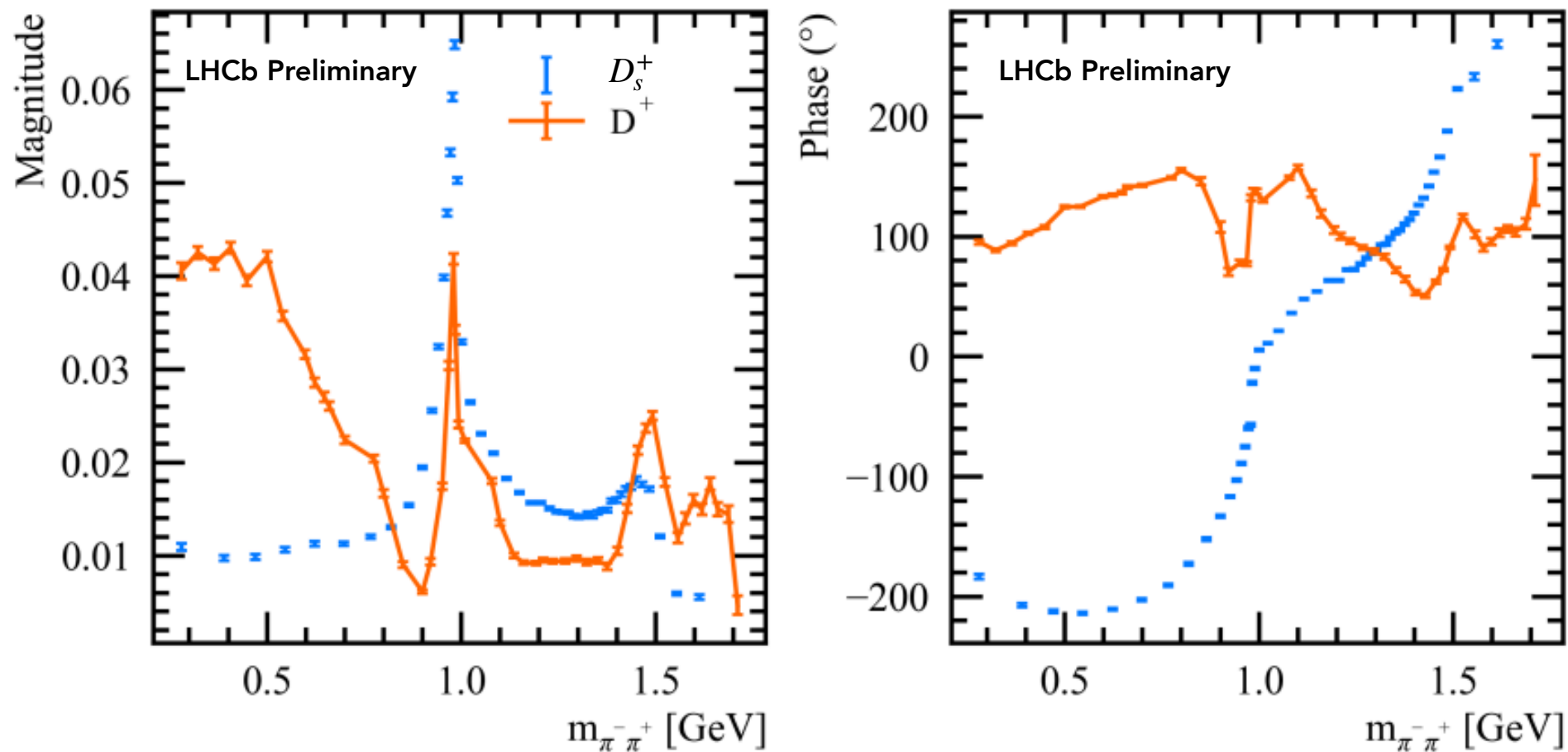
- Detailed measurement of the  $\pi^- \pi^+$  S-wave amplitude
  - Precision improved from previous analyses
- $D_s^+ \rightarrow (\omega(782) \rightarrow \pi^- \pi^+) \pi^+$ : *first time observed* in  $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$ 
  - $\omega(782) \rightarrow \pi^- \pi^+$ :  $(0.360 \pm 0.016 \pm 0.034 \pm 0.016) \%$
- $D_s^+ \rightarrow \rho(1700)^0 \pi^+$  and  $D_s^+ \rightarrow f_2'(1525) \pi^+$  also *observed for the first time* in  $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$
- S-wave also found to be dominant (in agreement with previous observations)  $\sim 85\%$
- Small contribution from  $\rho(770)^0 \pi^+$  (in agreement with previous analyses)
- Best fit result includes the  $\rho(1700)^0$  state
  - Combined fit fraction of the  $\rho(1450)^0$  and  $\rho(1700)^0$  amplitudes is stable

*Let's now compare the two channels...*

# $D_s^+$ vs $D^+$ : S-wave

LHCb-PAPER-2022-030 in preparation

(Statistical uncertainties only)



- S-wave as the **major** contribution ( $\sim 85\%$  for  $D_s^+$  and  $\sim 61\%$  for  $D^+$ )
- $f_0(980)$  is the most prominent contribution in  $D_s^+$  and  $f_0(500)$  in  $D^+$
- No indication of a scalar resonance at low  $\pi^-\pi^+$  mass for  $D_s^+$
- Indication of at least one scalar resonances near 1.5 MeV for both modes
  - Enhanced by the opening of the  $\eta\eta'$  channel
- *Different composition for  $D_s^+$  and  $D^+$ : S-wave produced from different sources...*

# $D_s^+$ vs $D^+$ : P- and D-waves

## P-wave

- $D_{(s)}^+ \rightarrow (\omega(782) \rightarrow \pi^- \pi^+) \pi^+$ : first time observed in both channels
  - $\omega(782)$  is produced by different mechanisms in both decays
  - $\rho(770)^0$  contributes with  $\sim 26\%$  for  $D^+$  and  $\sim 1\%$  for  $D_s^+$
- $D_{(s)}^+$ : Both  $\rho(1450)^0$  and  $\rho(1700)^0$  are necessary for a good fit
  - Combined contributions are very similar,  $(6.14 \pm 0.27) \%$  in  $D_s^+$ , and  $(7.1 \pm 0.8) \%$  in  $D^+$

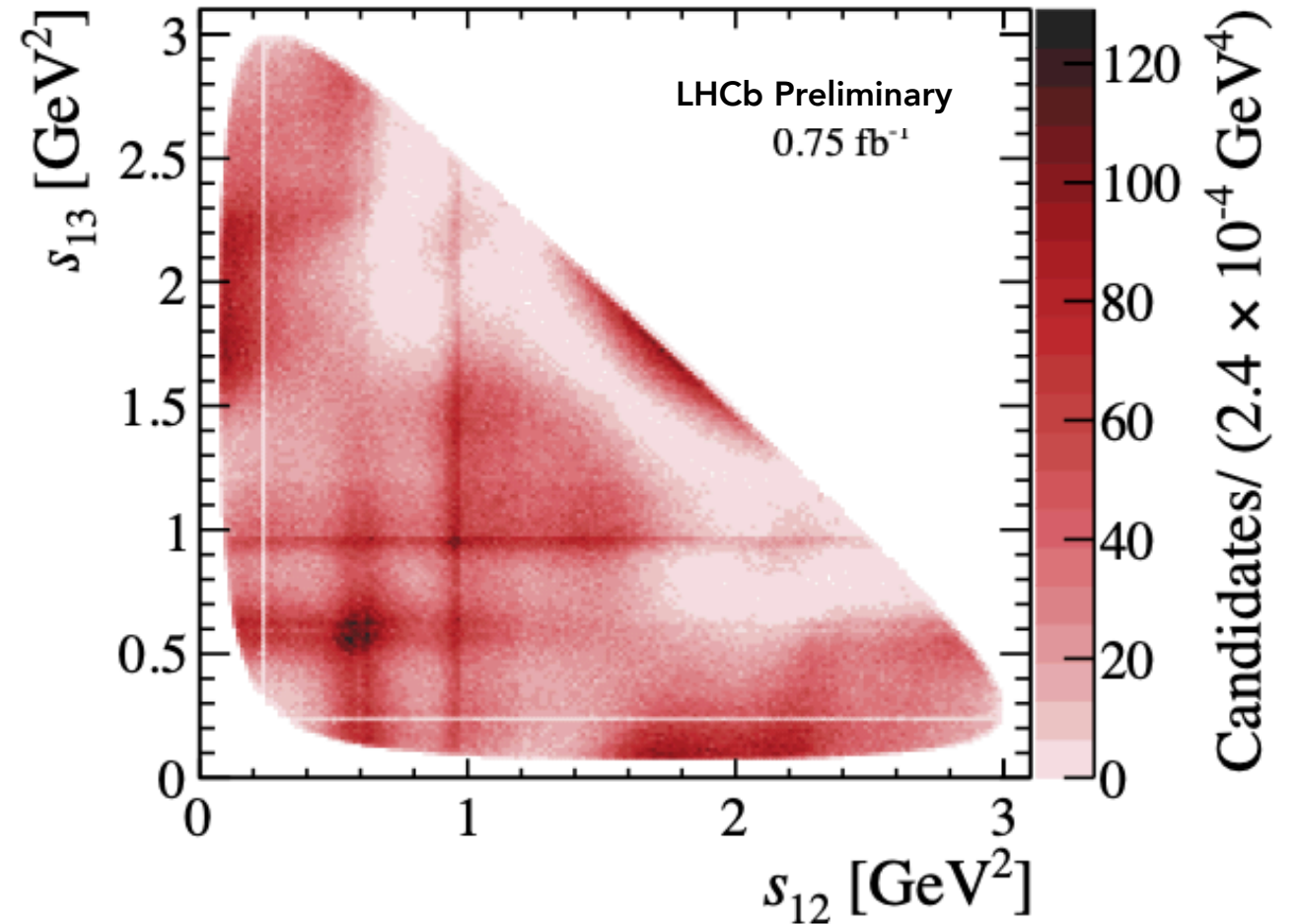
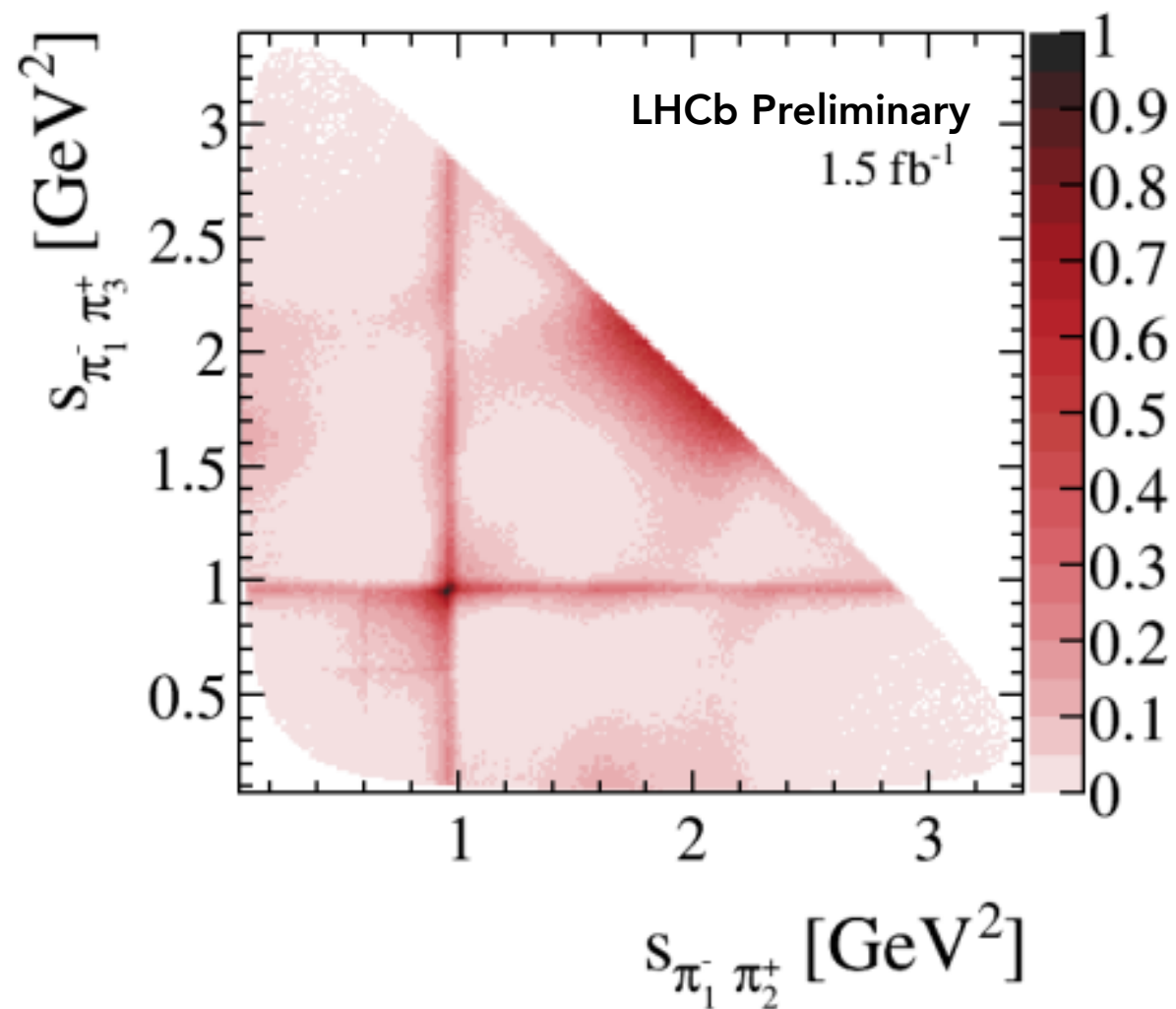
*Same resonances in both  $D_s^+$  and  $D^+$  modes but very different contributions!!*

## D-wave

- $f_2(1270)$  as the largest contribution for both channels
  - Identical contribution in both channels ( $\sim 13\%$ )
  - **Intriguing result!** Given its quark content, one would expect the production of the  $f_2(1270)$  at a higher rate from a  $d\bar{d}$  source ( $D^+$ ) than an  $s\bar{s}$  source as in  $D_s^+$
- $f_2'(1525)$  only seen in the  $D_s^+$  channel: dominant  $s\bar{s}$  and small  $(d\bar{d} + u\bar{u})$  component



# $D_s^+$ vs $D^+$ : Conclusions



- *Very different resonant structure in spite of the final state being the same*
- *Conclusion: Scalar resonances are produced by different mechanisms in these decays*

*LHCb-PAPER-2022-016 and LHCb-PAPER-2022-030 in preparation*

Thank you!

# Backup Slides

# $D^+$ : previous analyses

## E791

Mode	Fit 1 Fraction (%) magnitude phase	Fit 2 Fraction (%) magnitude phase
$\sigma\pi^+$	...	$46.3 \pm 9.0 \pm 2.1$
	...	$1.17 \pm 0.13 \pm 0.06$
	...	$(205.7 \pm 8.0 \pm 5.2)^\circ$
$\rho^0(770)\pi^+$	$20.8 \pm 2.4$	$33.6 \pm 3.2 \pm 2.2$
	1 (fixed)	1 (fixed)
	0 (fixed)	0 (fixed)
NR	$38.6 \pm 9.7$	$7.8 \pm 6.0 \pm 2.7$
	$1.36 \pm 0.20$	$0.48 \pm 0.18 \pm 0.09$
	$(150.1 \pm 11.5)^\circ$	$(57.3 \pm 19.5 \pm 5.7)^\circ$
$f_0(980)\pi^+$	$7.4 \pm 1.4$	$6.2 \pm 1.3 \pm 0.4$
	$0.60 \pm 0.07$	$0.43 \pm 0.05 \pm 0.02$
	$(151.8 \pm 16.0)^\circ$	$(165.0 \pm 10.9 \pm 3.4)^\circ$
$f_2(1270)\pi^+$	$6.3 \pm 1.9$	$19.4 \pm 2.5 \pm 0.4$
	$0.55 \pm 0.08$	$0.76 \pm 0.06 \pm 0.03$
	$(102.6 \pm 16.0)^\circ$	$(57.3 \pm 7.5 \pm 2.9)^\circ$
$f_0(1370)\pi^+$	$10.7 \pm 3.1$	$2.3 \pm 1.5 \pm 0.8$
	$0.72 \pm 0.12$	$0.26 \pm 0.09 \pm 0.03$
	$(143.2 \pm 9.7)^\circ$	$(105.4 \pm 17.8 \pm 0.6)^\circ$
$\rho^0(1450)\pi^+$	$22.6 \pm 3.7$	$0.7 \pm 0.7 \pm 0.3$
	$1.04 \pm 0.12$	$0.14 \pm 0.07 \pm 0.02$
	$(45.8 \pm 14.9)^\circ$	$(319.1 \pm 39.0 \pm 10.9)^\circ$

PRL 86 (2001) 770

## CLEO

PRD 76 (2007) 012001

Mode	Amplitude (a.u.)	Phase (°)	Fit fraction (%)
$\rho(770)\pi^+$	1 (fixed)	0 (fixed)	$20.0 \pm 2.3 \pm 0.9$
$f_0(980)\pi^+$	$1.4 \pm 0.2 \pm 0.2$	$12 \pm 10 \pm 5$	$4.1 \pm 0.9 \pm 0.3$
$f_2(1270)\pi^+$	$2.1 \pm 0.2 \pm 0.1$	$-123 \pm 6 \pm 3$	$18.2 \pm 2.6 \pm 0.7$
$f_0(1370)\pi^+$	$1.3 \pm 0.4 \pm 0.2$	$-21 \pm 15 \pm 14$	$2.6 \pm 1.8 \pm 0.6$
$f_0(1500)\pi^+$	$1.1 \pm 0.3 \pm 0.2$	$-44 \pm 13 \pm 16$	$3.4 \pm 1.0 \pm 0.8$
$\sigma$ pole	$3.7 \pm 0.3 \pm 0.2$	$-3 \pm 4 \pm 2$	$41.8 \pm 1.4 \pm 2.5$

## FOCUS

PLB 585 (2004) 200

$P$ -vector parameters	modulus	phase (deg)
$\beta_1$	1 (fixed)	0 (fixed)
$\beta_2$	$2.471 \pm 0.431$	$82.5 \pm 10.3$
$\beta_3$	$1.084 \pm 0.386$	$102.8 \pm 23.5$
$f_{11}^{\text{prod}}$	$2.565 \pm 0.737$	$155.4 \pm 18.3$
$f_{12}^{\text{prod}}$	$6.312 \pm 0.967$	$-160.0 \pm 8.7$

decay channel	fit fraction (%)	phase (deg)	amplitude coefficient
$(S\text{-wave})\pi^+$	$56.00 \pm 3.24 \pm 2.08$	0 (fixed)	1 (fixed)
$f_2(1270)\pi^+$	$11.74 \pm 1.90 \pm 0.23$	$-47.5 \pm 18.7 \pm 11.7$	$1.147 \pm 0.291 \pm 0.047$
$\rho^0(770)\pi^+$	$30.82 \pm 3.14 \pm 2.29$	$-139.4 \pm 16.5 \pm 9.9$	$1.858 \pm 0.505 \pm 0.033$
Fit C.L.	7.7 %		

# $D_s^+$ : previous analyses

PRL 86 (2001) 765

E791

	Fit A fraction (%) magnitude phase
$f_0(980)\pi^+$	$56.5 \pm 4.3 \pm 4.7$ 1 (fixed) 0 (fixed)
Nonresonant	$0.5 \pm 1.4 \pm 1.7$ $0.09 \pm 0.14 \pm 0.04$ ( $181 \pm 94 \pm 51$ )°
$\rho^0(770)\pi^+$	$5.8 \pm 2.3 \pm 3.7$ $0.32 \pm 0.07 \pm 0.19$ ( $109 \pm 24 \pm 5$ )°
$f_2(1270)\pi^+$	$19.7 \pm 3.3 \pm 0.6$ $0.59 \pm 0.06 \pm 0.02$ ( $133 \pm 13 \pm 28$ )°
$f_0(1370)\pi^+$	$32.4 \pm 7.7 \pm 1.9$ $0.76 \pm 0.11 \pm 0.03$ ( $198 \pm 19 \pm 27$ )°
$\rho^0(1450)\pi^+$	$4.4 \pm 2.1 \pm 0.2$ $0.28 \pm 0.07 \pm 0.01$ ( $162 \pm 26 \pm 17$ )°
$m_{f_0(980)}(\text{MeV}/c^2)$	$977 \pm 3 \pm 2$
$g_\pi$	$0.09 \pm 0.01 \pm 0.01$
$g_\kappa$	$0.02 \pm 0.04 \pm 0.03$
$m_{f_0(1370)}(\text{MeV}/c^2)$	$1434 \pm 18 \pm 9$
$\Gamma_{f_0(1370)}(\text{MeV}/c^2)$	$172 \pm 32 \pm 6$
$\chi^2/\nu$	71.8/68
C.L.	35%
$-2 \ln \mathcal{L}_{\text{max}}$	-3204

BaBar

PRD 79 (2009) 032003

Decay mode	Decay fraction (%)	Amplitude	Phase (rad)
$f_2(1270)\pi^+$	$10.1 \pm 1.5 \pm 1.1$	1.0 (fixed)	0.0 (fixed)
$\rho(770)\pi^+$	$1.8 \pm 0.5 \pm 1.0$	$0.19 \pm 0.02 \pm 0.12$	$1.1 \pm 0.1 \pm 0.2$
$\rho(1450)\pi^+$	$2.3 \pm 0.8 \pm 1.7$	$1.2 \pm 0.3 \pm 1.0$	$4.1 \pm 0.2 \pm 0.5$
$\mathcal{S}$ wave	$83.0 \pm 0.9 \pm 1.9$	Table II	Table II
Total	$97.2 \pm 3.7 \pm 3.8$		
$\chi^2/NDF$	$\frac{437}{422-64} = 1.2$		

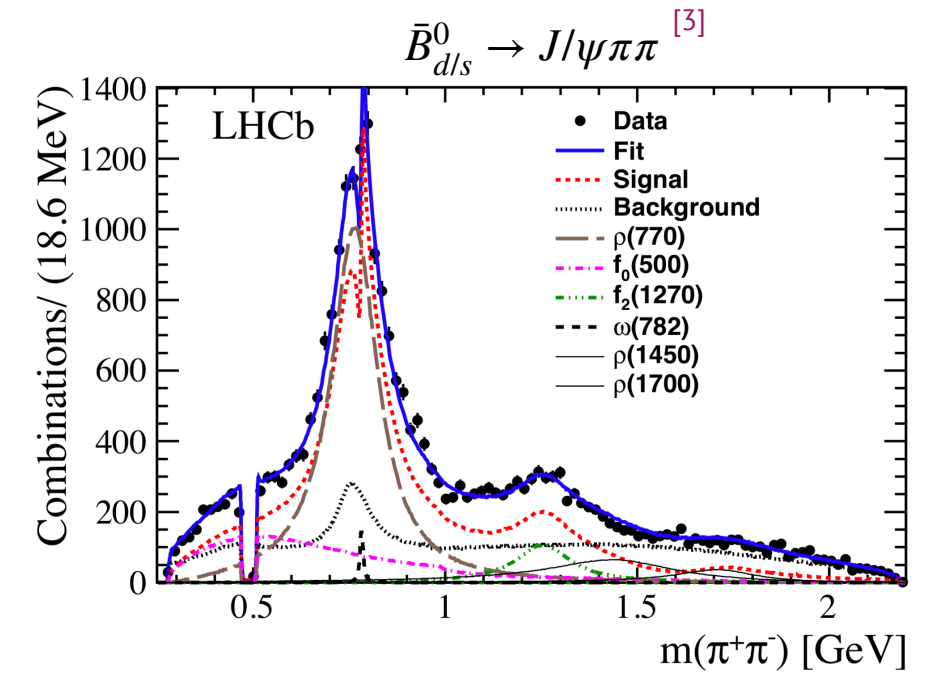
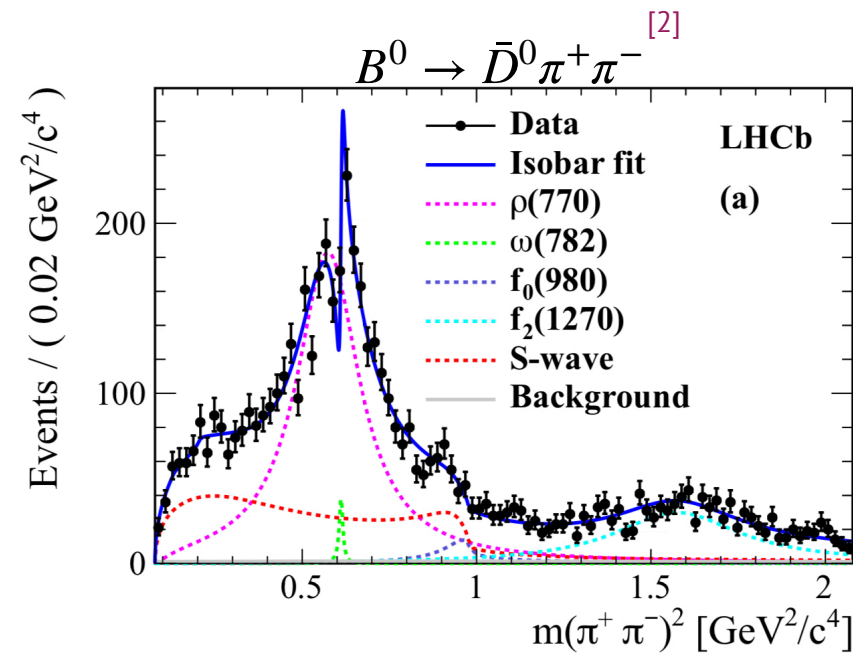
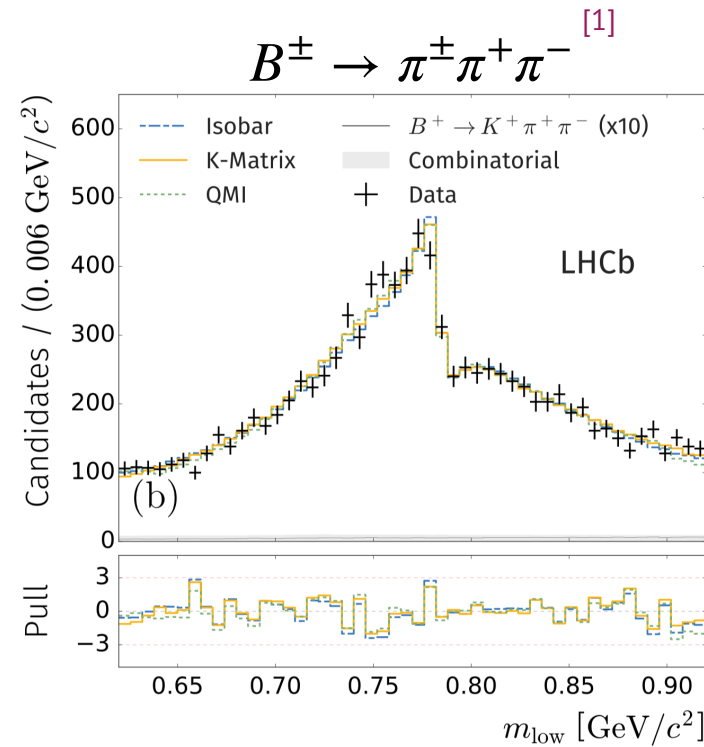
BESIII

[arXiv:2108.10050](https://arxiv.org/abs/2108.10050)

Decay mode	Fit fraction (%)	Magnitude	Phase (radians)
$f_2(1270)\pi^+$	$10.5 \pm 0.8 \pm 1.2$	1. (Fixed)	0. (Fixed)
$\rho(770)\pi^+$	$0.9 \pm 0.4 \pm 0.5$	$0.13 \pm 0.03 \pm 0.04$	$5.44 \pm 0.25 \pm 0.62$
$\rho(1450)\pi^+$	$1.3 \pm 0.4 \pm 0.5$	$0.91 \pm 0.16 \pm 0.22$	$1.03 \pm 0.32 \pm 0.51$
$\mathcal{S}$ wave	$84.2 \pm 0.8 \pm 1.3$	Table III	Table III
Total	$96.8 \pm 2.4 \pm 3.5$		

# $\rho - \omega$ interference

Different  $\rho$ - $\omega$  profiles depending on the decay channel!



Possible mixing parametrisation

$$A_{\rho-\omega} = A_\rho \left[ \frac{1 + A_\omega \Delta |B| \exp(i\phi_B)}{1 - \Delta^2 A_\rho A_\omega} \right] \quad \Delta = \delta(m_\rho + m_\omega)$$

$$\delta \approx 2.15 \text{ MeV}$$

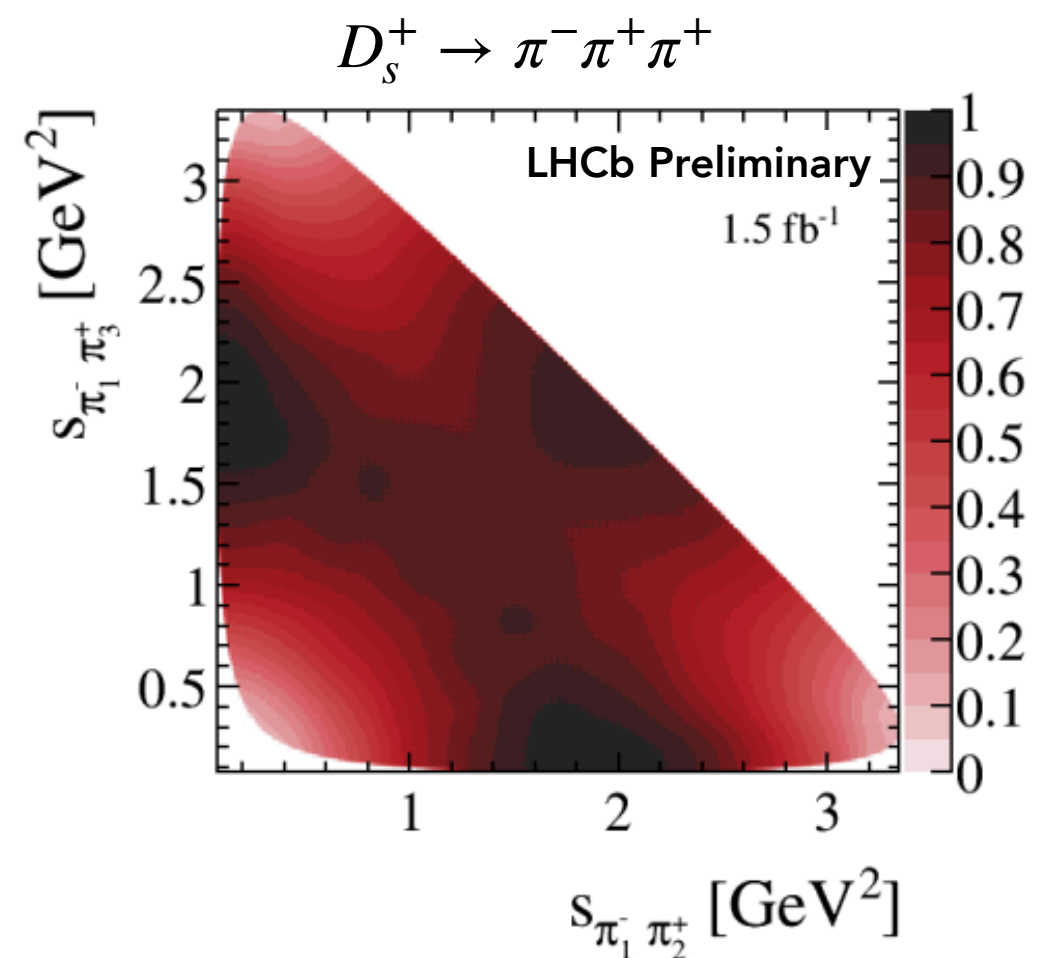
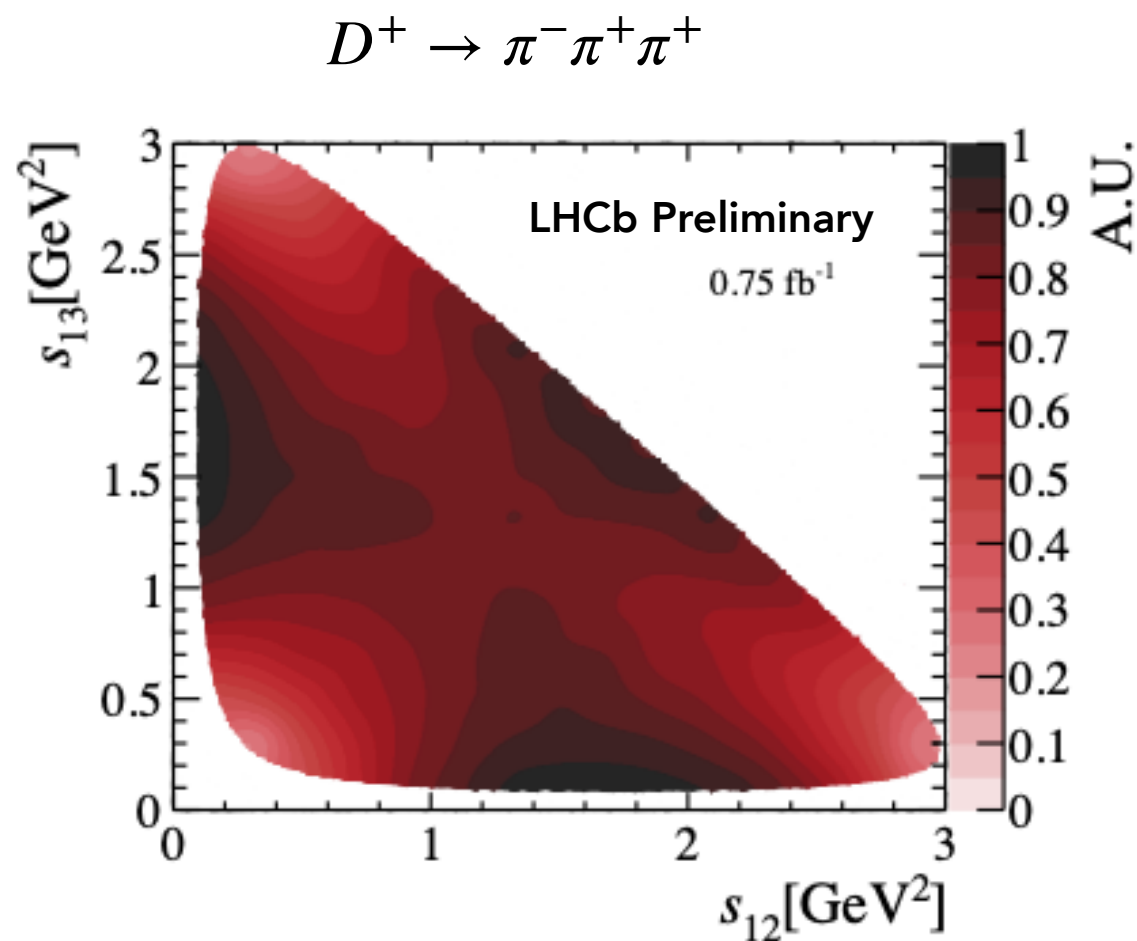
[1] *Phys. Rev. D* 101 (2020) 012006

[2] arXiv: 1505.01710v2 [hep-ex]

[3] *Phys. Lett. B* 742 (2015) 38

# Efficiency maps

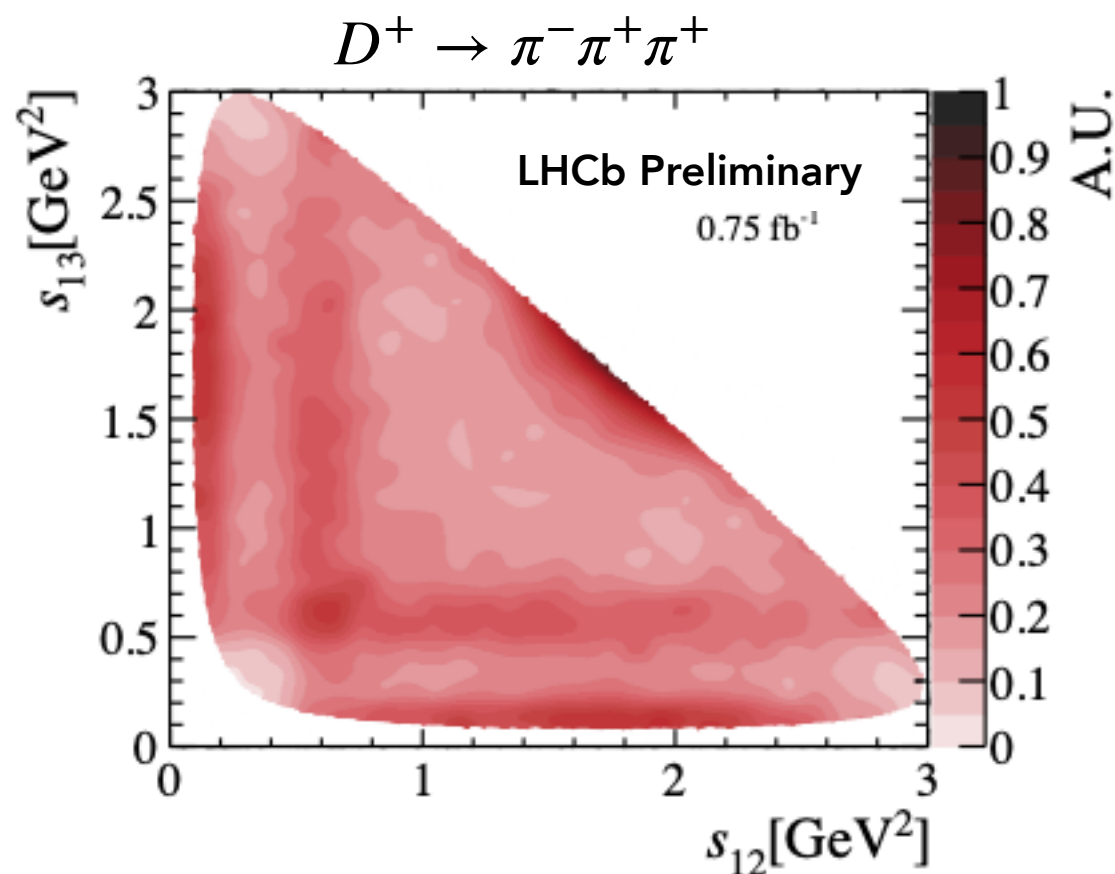
- Obtained from total simulated sample, which was generated as an uniform Dalitz Plot distribution
- Incorporates all steps of the selection process with except of PID
- PID incorporated through per-event weights to the simulated decays, from data calibration samples
- 15x15 bins
- 2D cubic spline based on the code LauCubicSpline from Laura++<sup>[1]</sup> to produce a high resolution smoothed histogram.





# $D^+$ : Background

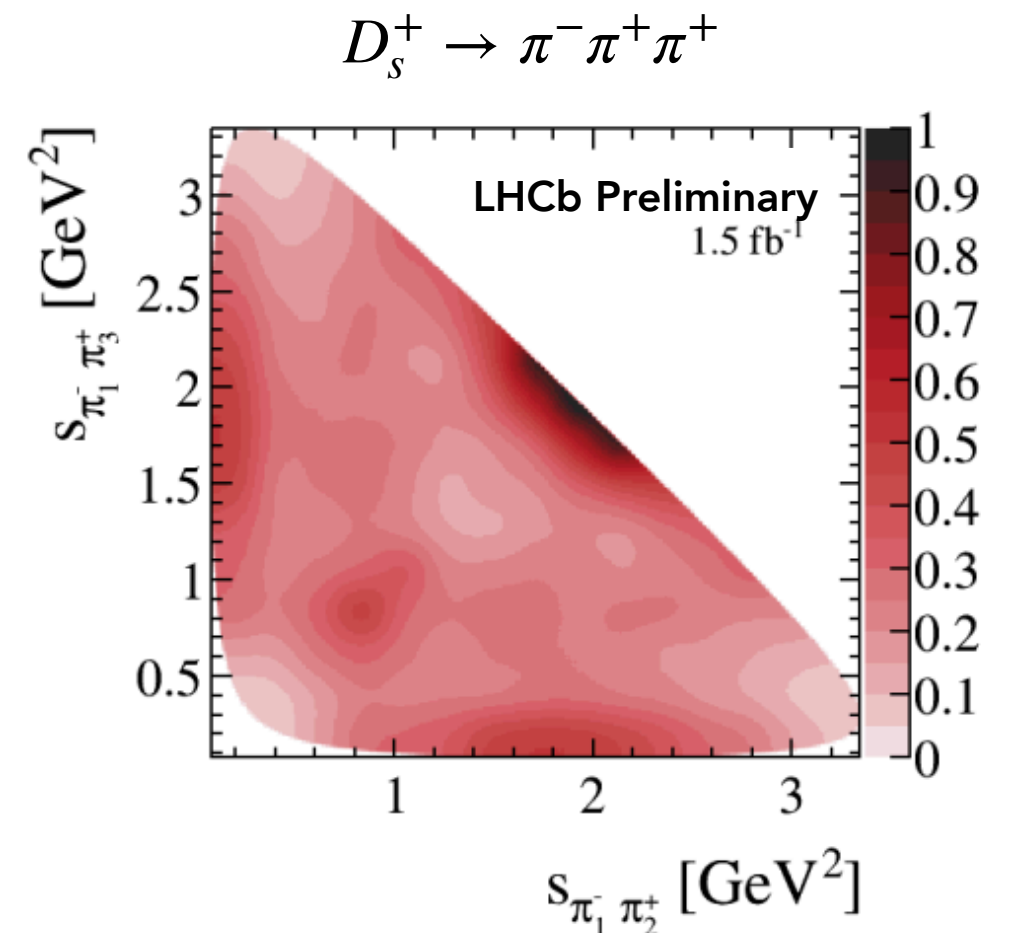
- Composition: 3-track random combinations +  $D_{(s)}^+ \rightarrow \eta(\prime)\pi^+$  + random  $\rho\pi$  combination + negligible partially reconstructed charm
- Approximately 5% of our final data sample
- Background within the signal region is assumed to be a composition of both sidebands
- Spline procedure based on Laura++ code



Events populating:

[1810,1830] MeV

[1910,1930] MeV



Events populating:

[1920,1940] MeV

[2000,2020] MeV

# Systematic Uncertainties

- Sources considered:

**Experimental uncertainties:** efficiency correction, background parametrisation, selection, finite detector resolution

**Model systematics:** uncertainties in lineshape parameters

## Efficiency:

- Finite size of simulated sample: vary bin content within its uncertainty
- PID efficiency: size of calibration sample. Vary values of efficiency and correction factors
- Binning scheme: 12x12, 15x15, 20x20

## Background:

- Purity: vary  $\pm 1\sigma$  according to the uncertainty from the mass fit
- Shape: only left or right wing as background model

## QMIPWA:

- Parametrisation of the S-wave: vary the number of knots used

## Fit bias and QMIPWA modelling:

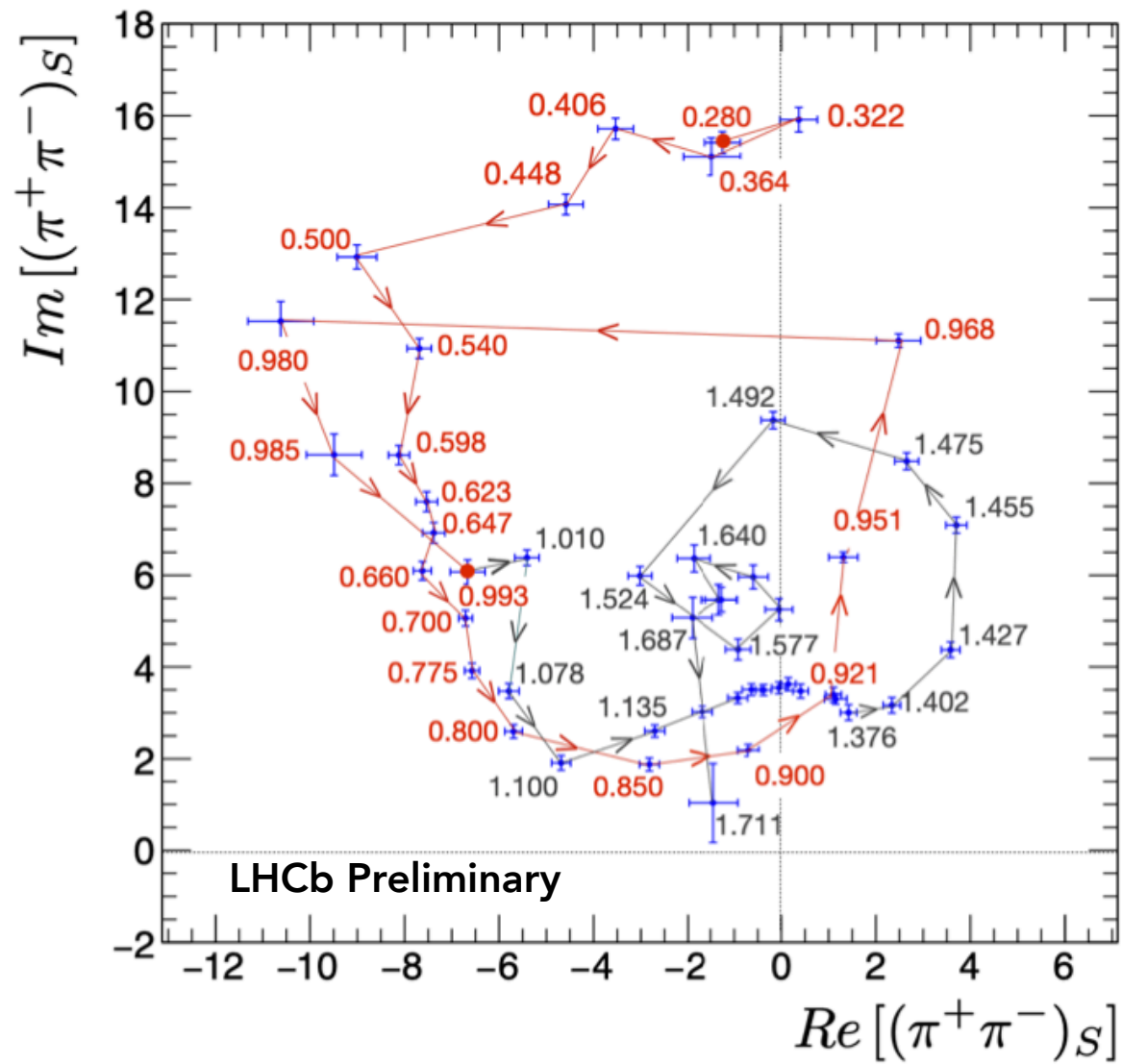
- Generate toys with a given binning scheme and fit each toy using the nominal binning. Observe the resulting distributions for each parameter and compare with the input.

## Model systematics:

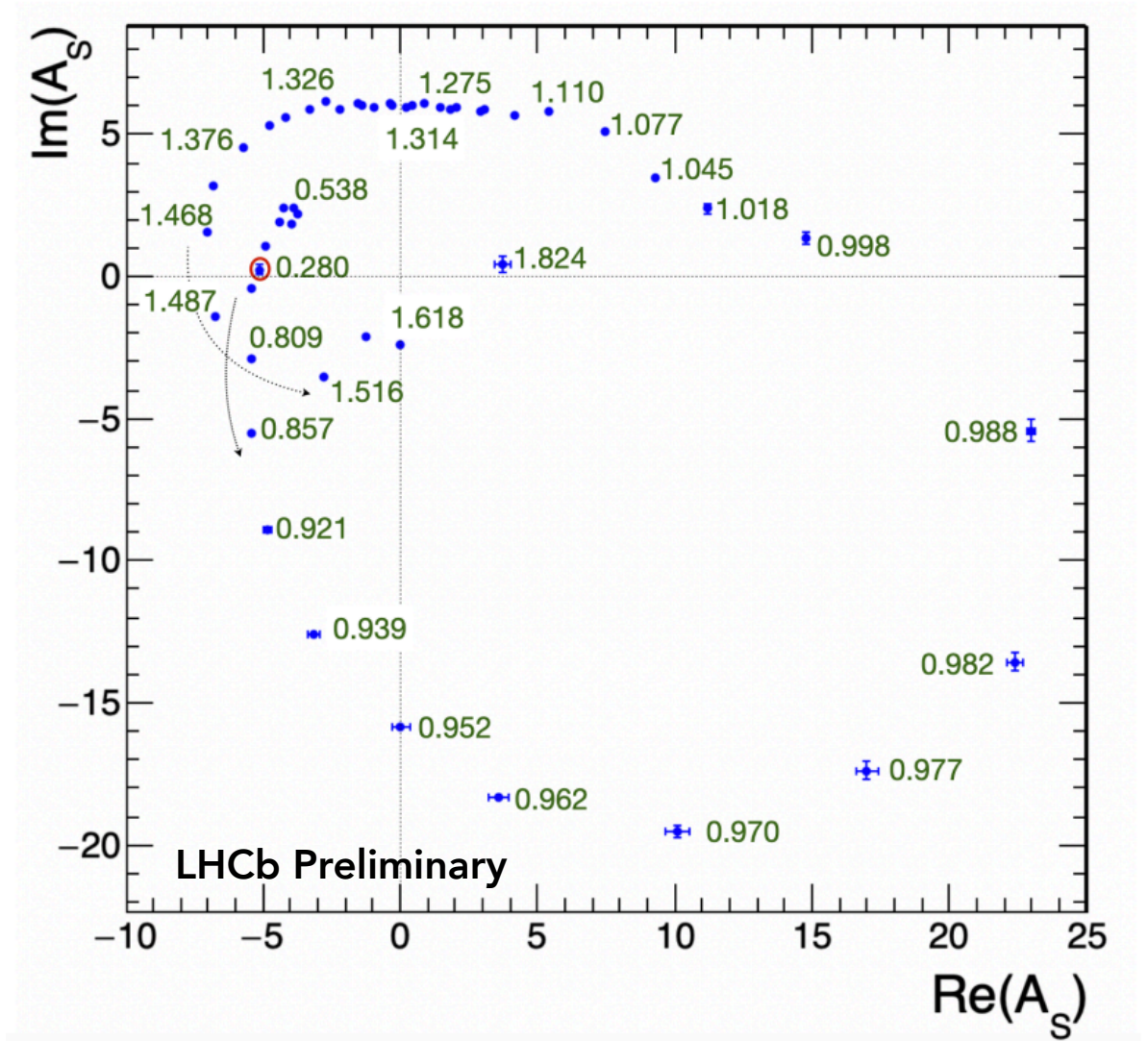
- Amplitude components: vary each lineshape parameter (masses and widths) within  $\pm 1\sigma$
- Effective barrier radii of mesons: vary Blatt Weisskopf radius  $r_{F_{D,Rp_3}} = 4.0, 5.0, 6.0 \text{ GeV}^{-1}$  and  $r_{F_{R,p_1p_2}} = 1.0, 1.5, 2.0 \text{ GeV}^{-1}$

# Argand plots

$$D^+ \rightarrow \pi^- \pi^+ \pi^+$$

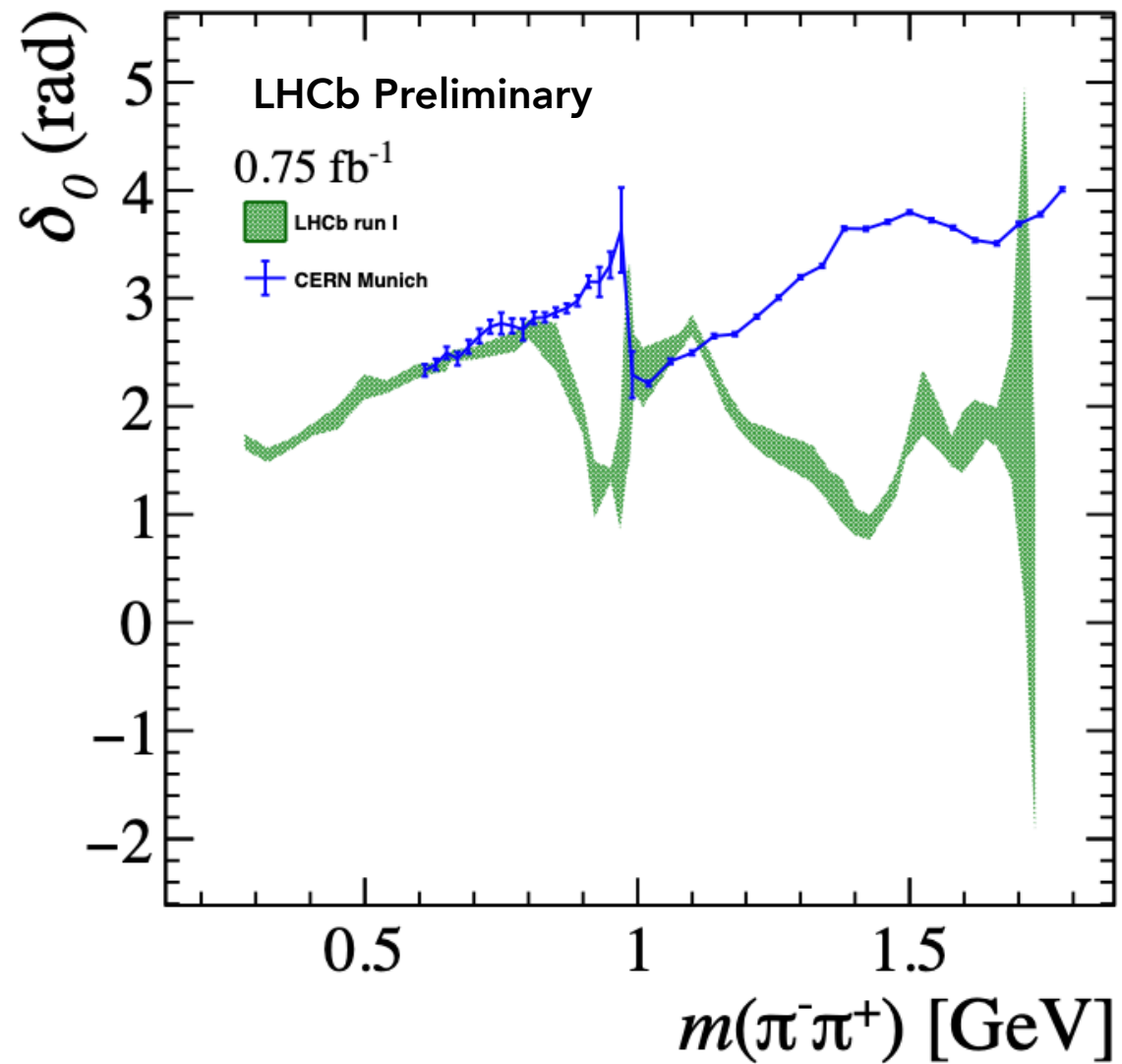


$$D_s^+ \rightarrow \pi^- \pi^+ \pi^+$$



# Comparison with scattering experiment

$$D^+ \rightarrow \pi^- \pi^+ \pi^+$$



$$D_s^+ \rightarrow \pi^- \pi^+ \pi^+$$

