Doubly charm tetraquark in DD^* scattering from lattice QCD

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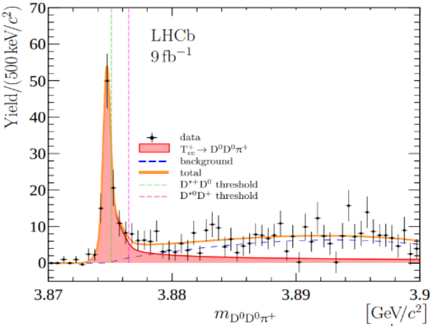
with **S. Prelovsek**. Based on article **2202.10110** [Phys.Rev.Lett.]

The motivation, T_{cc}^{+}



LHCb: 2109.01038, 2109.01056

$$\delta m \equiv m_{T_{cc}^{+}} - (m_{D^{*+}} + m_{D^0})$$

$$\begin{aligned} \delta m_{\text{pole}} &= -360 \pm 40\,^{+4}_{-0}\ \text{keV}/c^2\,, \\ \Gamma_{\text{pole}} &= 48 \pm 2\,^{+0}_{-14}\,\text{keV}\,. \end{aligned}$$

- The doubly charmed tetraquark T_{cc}^{+}, $I = 0$ and favours $J^P = 1^+$.
- No states observed in $D^0D^+\pi^+$: eliminates possibility of $I = 1$. Much likely a molecular D^0D^{*+} state.
- Near-threshold state: Demands pole identification to confirm existence.
- Several phenomenological predictions exists.
- Only two previous lattice investigations in this regard: HSC 2017, ILGTI 2018. No scattering amplitude determination involved.

Summary of the work

- Extract the near-threshold DD^* isoscalar s-wave scattering amplitude.

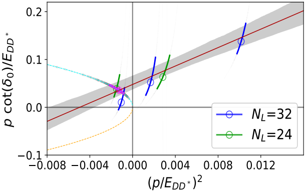
- Lattice QCD ensembles : CLS Consortium

  $m_\pi \sim$ 280 MeV, $m_K \sim$ 467 MeV, $a \sim 0.086$ fm

  Spatial volumes: $L \sim 2$ fm and $L \sim 2.7$ fm

  Charm quark masses ($m_c^?$): $m_D \sim$ 1762 MeV and 1927 MeV

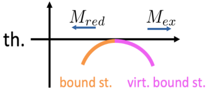
- We see a shallow virtual bound state related to T_{cc}.



$$\delta m \equiv m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0})$$

	m_D [MeV]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat. $m_c^{(h)}$	1927(1)	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
lat. $m_c^{(l)}$	1762(1)	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.
exp.	1864.85(5)	$-0.36(4)$	bound st.

$\delta m = E^p_{cm} - E_{th}$



- For $m_\pi > m_\pi^{phys}$, one expects weaker attraction and T_{cc} is expected to become a virtual bound state. $M_{ex} \propto m_\pi$

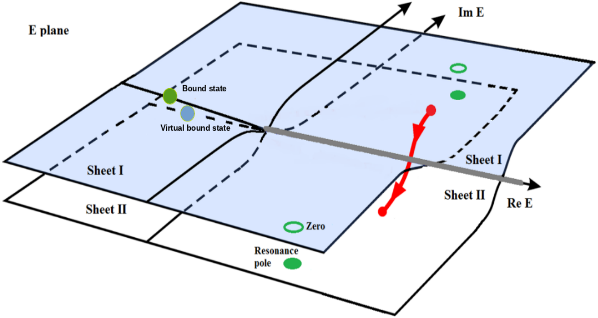
  At $m_\pi \sim$ 280 MeV, we indeed find a shallow virtual bound state.

- With increasing m_c, $|\delta m|$ of the virtual bound state becomes small.

  $M_{red} \propto m_c$

$E_{DD*} = m_D + m_{D*}$

Virtual bound states



- $T \;\propto\; (p cot\delta_0 - ip)^{-1}$. Bound state is a pole in T with $p = i|p|$. Virtual bound state is a pole in T with $p = -i|p|$.

- An example for virtual bound state: spin-singlet dineutron.

Hadron spectroscopy using lattice QCD

- Compute a matrices of two point correlation functions

$$C_{ji}(t) = \langle 0|\bar{O}_j(t)O_i(0)|0\rangle = \sum_n \frac{Z_i^n Z_j^{n*}}{2E_n} e^{-E_n(t)}$$

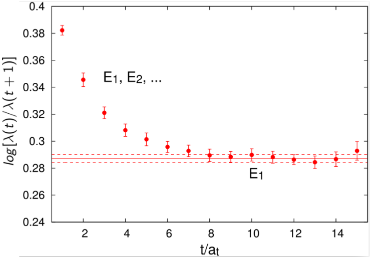
where $O_i(0)$ and $\bar{O}_j(t)$ are the desired interpolating operators.

- O can have any form that has the desired q. #s and can in principle couple with all the states that have these q. #s.

- $C(t)$ is Hermitian by construction.

- Diagonalize: The physical states represented by the eigensystem.

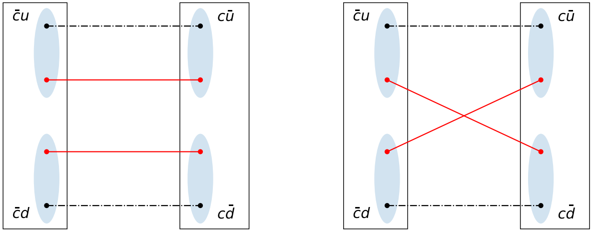
C. Michael (1985)



- Finite volume spectrum from non-linear fits to the large time behaviour of eigenvalues. $\lambda_n(t) \sim e^{-E_n t}(1 + \mathcal{O}(e^{-\partial E t}))$.

Interpolators and Wick contractions

- For doubly charmed four quark systems near DD^* threshold, we use 'only' operators of type $(\bar{q}\Gamma_1 c)_{1_c}(\bar{q}'\Gamma_2 c)_{1_c}$.
- Wick contractions to compute: $[c,\ q,\ q']$



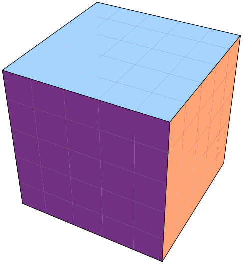
Reduced lattice symmetries

$O(3)$

$O_i^{J^{PC},M}$

$J = 0,1,2,3,\ldots$

O_h



$O_{i,\Lambda^{PC}}^{[J^{PC}]}$

$\Lambda = A_1, A_2, T_1, T_2, E$

Moving frames: Additional info from lattice

$O(3)$
$p^2 = 0$

$O_i^{J^{PC},M}$

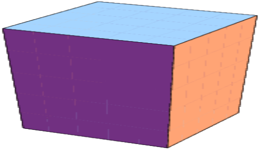
$J = 0,1,2,3,...$

$U(1)$
$p^2 \neq 0$

$O_i^{J^{PC},\lambda}$

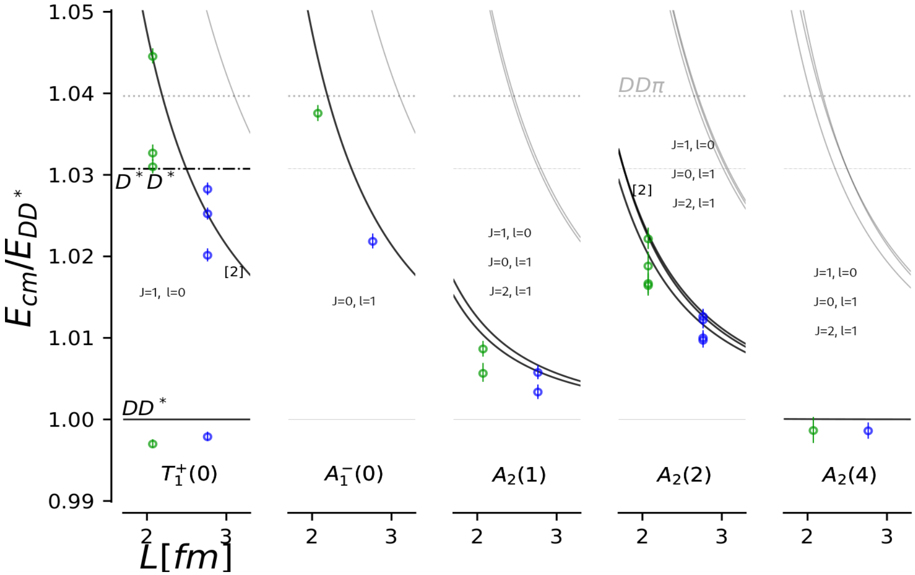
$|\lambda| = 0,1,2,3,...$

LG
$p^2 \neq 0$



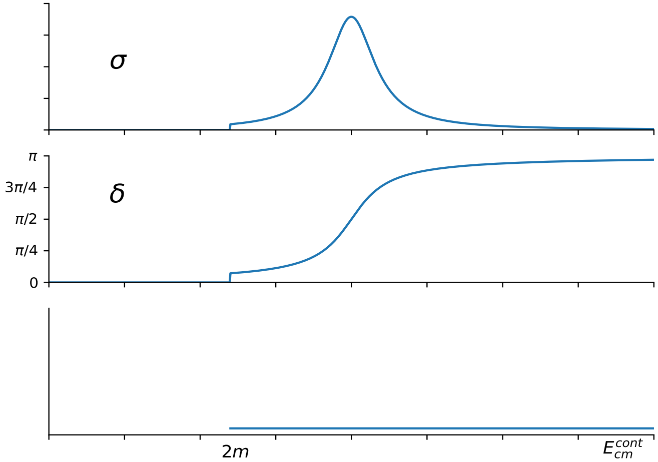
$O_{i,\Lambda^C}^{[J^{PC},|\lambda|]}$

Finite volume spectrum T_{cc}



Resonances in the infinite volume continuum

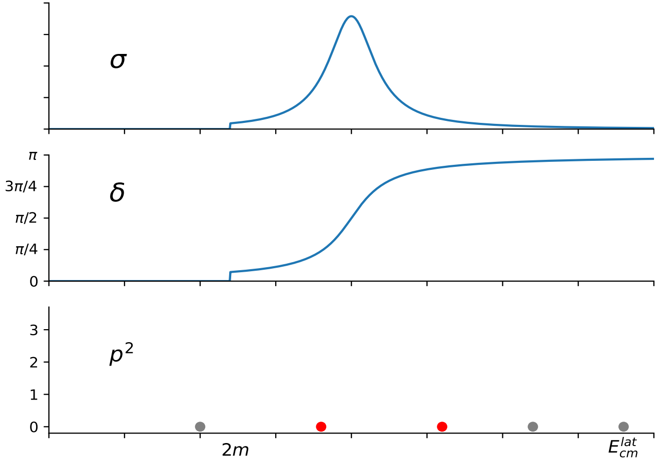
Scattering cross sections, phase shifts, branch cuts, Riemann sheets.



Schematic picture for illustration. Do not take it quantitatively.

Resonances on the lattice (elastic) : ??

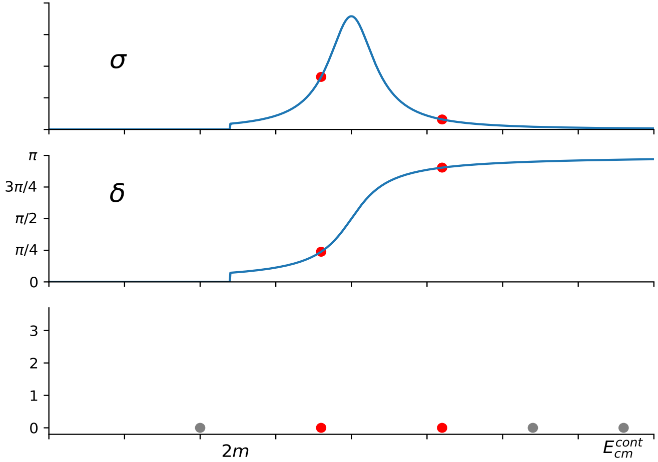
Discrete spectrum: No branch cuts, no Riemann sheets, no resonances!



Maiani-Testa no-go theorem [1990]

Resonances on the lattice (elastic) : Lüscher (1991)

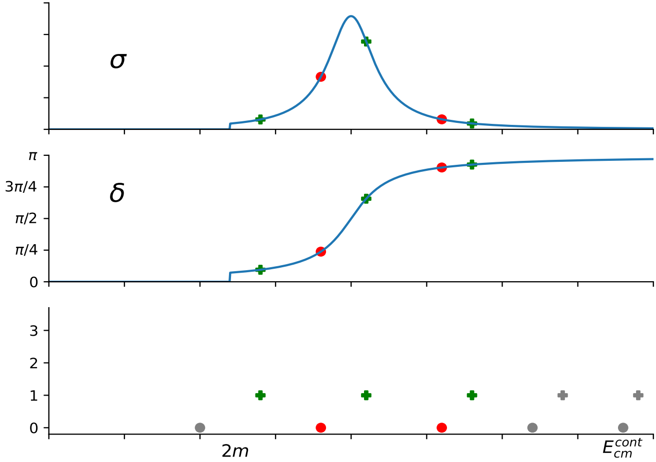
Infinite volume scattering amplitudes $\Leftrightarrow$ Finite volume spectrum



Lüscher [1991]

Resonances on the lattice (elastic) : Lüscher (1991)

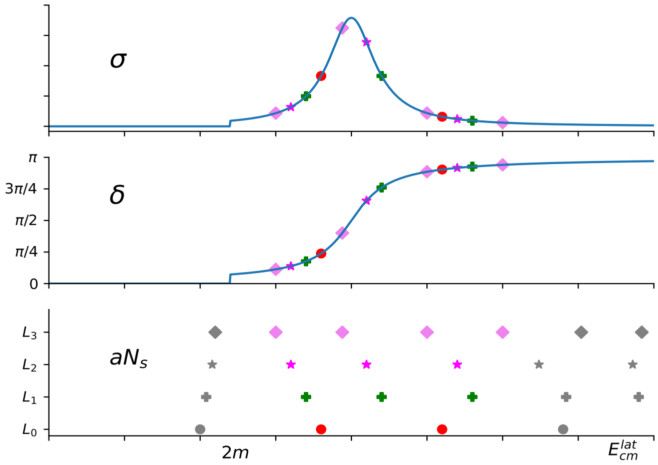
Infinite volume scattering amplitudes $\Leftrightarrow$ Finite volume spectrum



Different inertial frames can be utilized to extract more information

Resonances on the lattice (elastic) : Lüscher (1991)

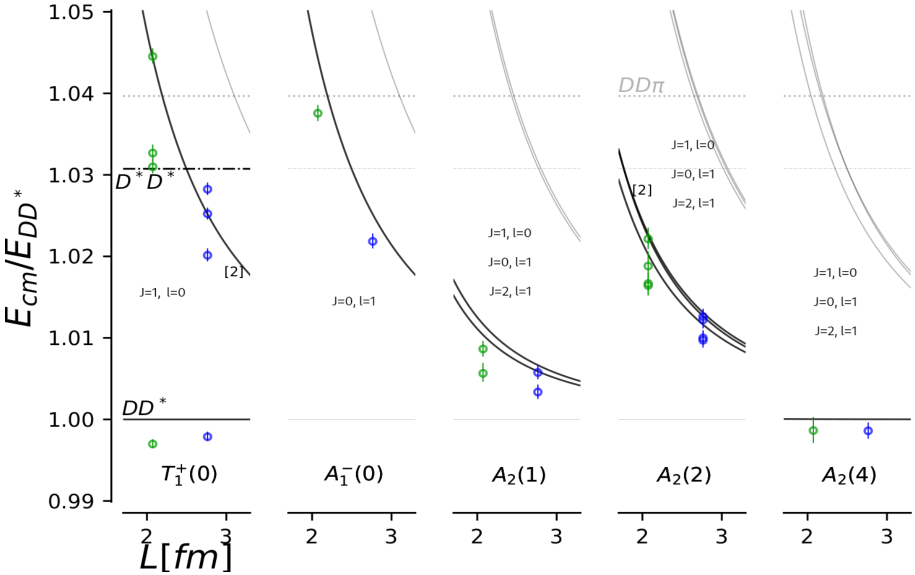
Infinite volume scattering amplitudes $\Leftrightarrow$ Finite volume spectrum



Multiple physical volumes can also be utilized to extract more information.

For generalizations of Lüscher framework, *c.f.* Briceño, Hansen 2014-15

Revisiting the finite volume spectrum T_{cc}



What do we learn till now?

- The inelastic threshold D^*D^* is sufficiently high.
  Assume elastic scattering near DD^* threshold.

- At $m_\pi \sim 280$ MeV, lowest three particle threshold $DD\pi$
  is higher up the lowest two particle inelastic threshold.

- Clear signatures for shifts from non-interacting scenario
  in s-wave and p-wave scattering.

- Lowest level with explicit $l = 2$ contribution consistent with
  the non-interacting level.

- Safe to assume no effects from $l \geq 2$.

Scattering amplitude and the parametrization

- Scattering amplitude:

$$S = 1 + i\frac{4p}{E_{cm}}t$$

- For the DD^* system [total spin equals 1], and assuming only $l < 2$, we have a 3×3 diagonal t matrix.

$$(t_l^{(J)})^{-1} = \frac{2(\tilde{K}_l^{(J)})^{-1}}{E_{cm}p^{2l}} - \frac{2p}{E_{cm}}, \quad (\tilde{K}_l^{(J)})^{-1} = p^{2l+1}\cot\delta_l^{(J)}$$

- Using an effective range expansion near-threshold, we have

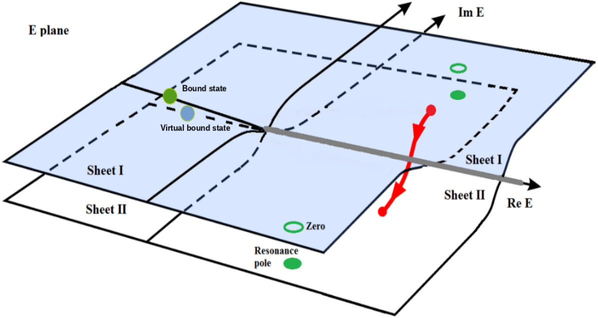
$$\tilde{K}^{-1} = \begin{array}{cc} \begin{bmatrix} \frac{1}{a_0^{(1)}} + \frac{r_0^{(1)}p^2}{2} & 0 & 0 \\ 0 & \frac{1}{a_1^{(0)}} + \frac{r_1^{(0)}p^2}{2} & 0 \\ 0 & 0 & \frac{1}{a_1^{(2)}} \end{bmatrix} & \begin{array}{ll} J{=}1 & l{=}0 \\ J{=}0 & l{=}1 \\ J{=}2 & l{=}1 \end{array} \end{array}$$

- Constraint on bound states:

$$p^{2l+1}cot(\delta_l) = -1^{\alpha}p^{2l}\sqrt{-p^2}$$

$$\alpha = 1(2) \text{ for a real (virtual) bound state.}$$

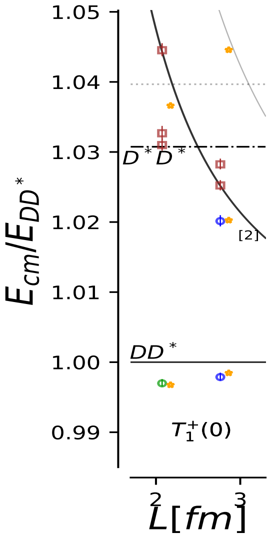
Virtual bound states

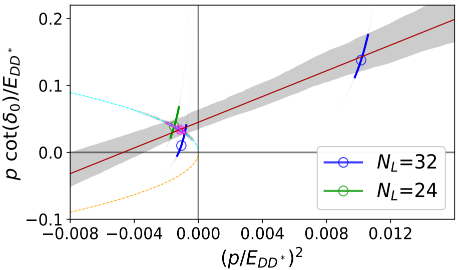


- $T \;\propto\; (p cot\delta_0 - ip)^{-1}$. Bound state is a pole in T with $p = i|p|$. Virtual bound state is a pole in T with $p = -i|p|$.

- An example for virtual bound state: spin-singlet dineutron.

DD^* scattering in s-wave @ $m_c^{(h)}$



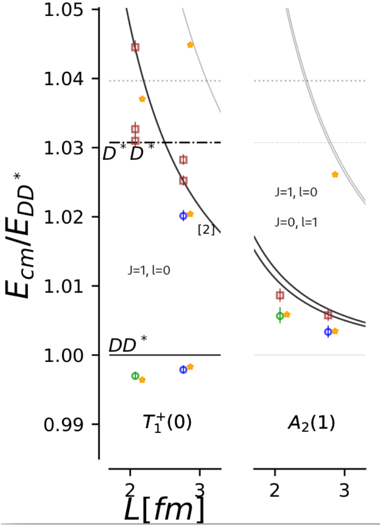


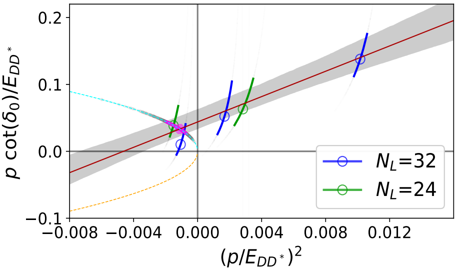
$m_\pi \sim 280$ MeV

- Fit quality:
  $$\chi^2/d.o.f. = 1.3/1.$$
- Fit parameters:
  $a_0^{(1)} = 1.10(^{+0.34}_{-0.31})$ fm $\quad r_0^{(1)} = 0.95(^{+0.24}_{-0.22})$ fm
- Binding energy:
  $$\delta m_{T_{cc}} = -9.1(^{+3.1}_{-6.9}) \text{ MeV}.$$

+/g refers to positive parity, -/u refers to negative parity.

DD^* scattering in s-wave @ $m_c^{(h)}$ *extended*



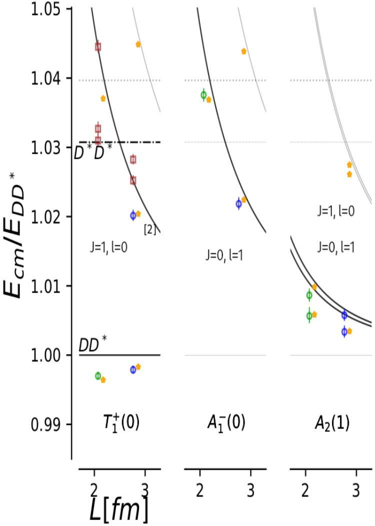


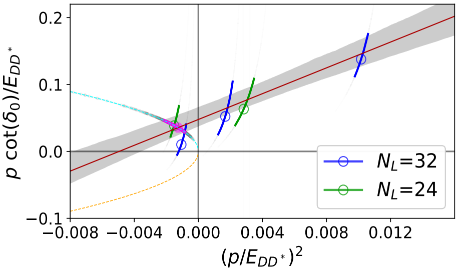
$m_\pi \sim 280$ MeV

- Fit quality:
  $\chi^2/d.o.f. = 1.3/3.$
- Fit parameters:
  $a_0^{(1)} = 1.13(^{+0.37}_{-0.34})$ fm $\quad r_0^{(1)} = 0.94(^{+0.19}_{-0.20})$ fm
- Binding energy:
  $\delta m_{T_{cc}} = -8.9(^{+3.2}_{-7.0})$ MeV.

+/g refers to positive parity, -/u refers to negative parity.

DD^* scattering in $I = 0, 1$ @ $m_c^{(h)}$

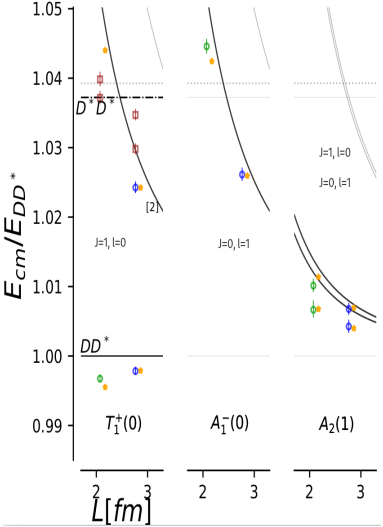


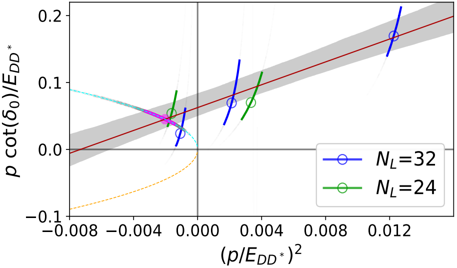


- Fit quality: $m_\pi \sim 280$ MeV
  $$\chi^2/d.o.f. = 3.7/5.$$
- Fit parameters:
  $$a_0^{(1)} = 1.04(0.29) \text{ fm} \;\&\; r_0^{(1)} = 0.96(^{+0.18}_{-0.20}) \text{ fm}$$
  $$a_1^{(0)} = 0.076(^{+0.008}_{-0.009}) \text{ fm}^3 \;\&\; r_1^{(0)} = 6.9(2.1) \text{ fm}^{-1}$$
- Binding energy:
  $$\delta m_{T_{cc}} = -9.9(^{+3.6}_{-7.2}) \text{ MeV}.$$

+/g refers to positive parity, -/u refers to negative parity.

DD^* scattering in $I = 0, 1$ @ $m_c^{(l)}$



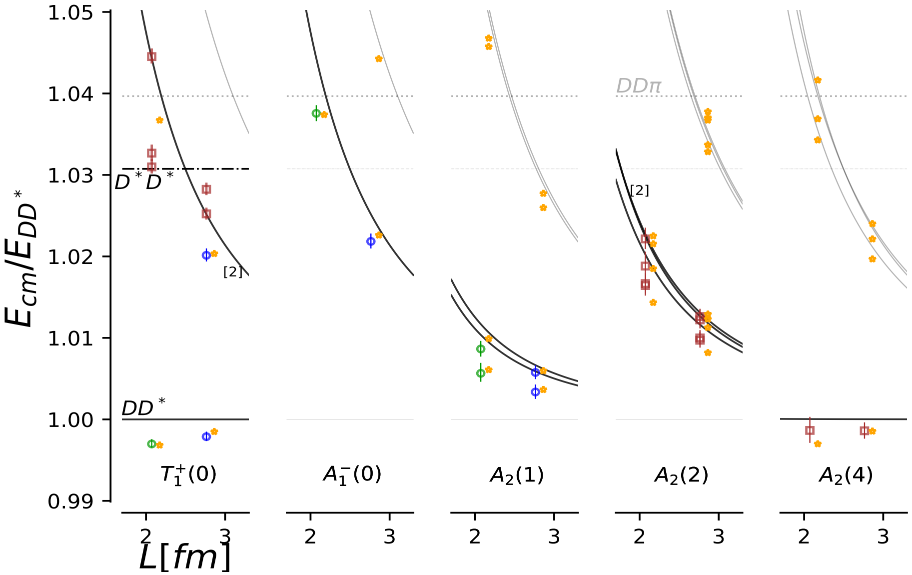


$m_\pi \sim 280$ MeV

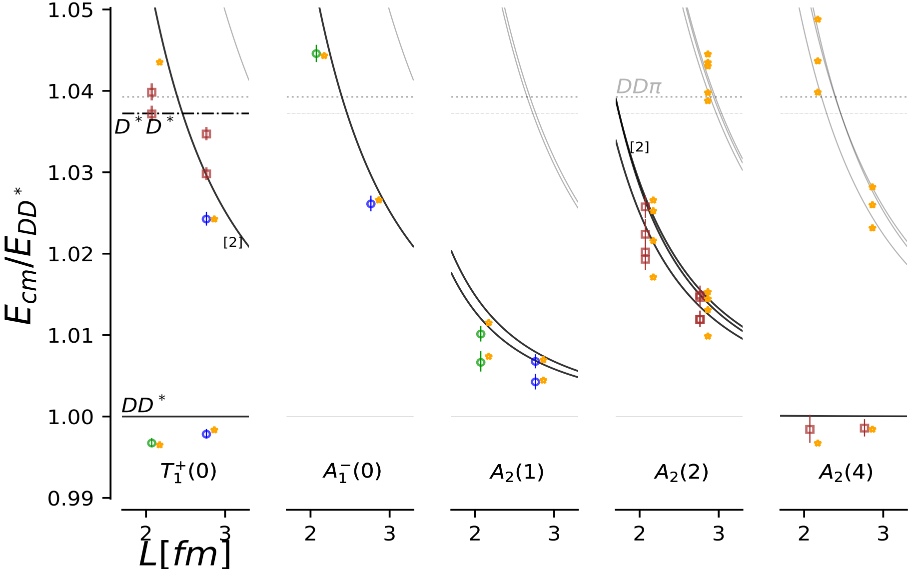
- **Fit quality:**
  $\chi^2/d.o.f. = 3.6/5.$
- **Fit parameters:**
  $a_0^{(1)} = 0.86(0.22)$ fm & $r_0^{(1)} = 0.92(^{+0.17}_{-0.19})$ fm
  $a_1^{(0)} = 0.117(^{+0.013}_{-0.014})$ fm^3 & $r_1^{(0)} = 8.6(^{+1.5}_{-1.1})$ fm^{-1}
- **Binding energy:**
  $\delta m_{T_{cc}} = -15.0(^{+4.6}_{-9.3})$ MeV.

+/g refers to positive parity, -/u refers to negative parity.

Predicting the finite volume spectrum @ $m_c^{(h)}$



Predicting the finite volume spectrum @ $m_c^{(l)}$

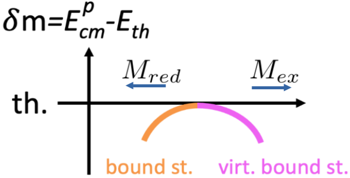


Our observations and inferences

- A shallow virtual bound state pole in s-wave related to T_{cc}.

	m_D [MeV]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(h)}$)	1927(1)	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(l)}$)	1762(1)	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.
exp.	1864.85(5)	$-0.36(4)$	bound st.

- For $m_\pi > m_\pi^{phys}$, T_{cc} is expected to become a virtual bound state. At $m_\pi \sim$ 280 MeV, we indeed find a shallow virtual bound state.
- Observations in line with the expected behaviour of a near-threshold molecular bound state pole in simple Quantum Mechanical potentials.



- $M_{red}(\propto m_c)$ is the reduced mass of the DD^* system.
- The mass of the particle exchanged during the interaction $M_{ex}(\propto m_{u/d})$.

Quark mass dependence: a QuanMech understanding

$$R \propto M_{red} \propto 1/M_{ex}$$

Summary one last time

- First observation of a shallow virtual bound state pole in the DD^* s-wave scattering amplitude related to T_{cc} from lattice QCD.

- With increasing m_π (equivalently $m_{u/d}$), T_{cc} is expected to be a virtual bound state. We find evidence for a virtual bound state at $m_\pi = 280$ MeV.

- With increasing m_c, $|\delta m|$ of the virtual bound state becomes small, and is expected eventually to become a bound state.

- The results are qualitatively robust to disretization effects for such a near-threshold pole, where these effects are expected to be the least.

- No features observed in p-wave in the energy region constrained by the finite-volume spectrum.

Outlook

- $m_{u,d}$ and extended m_c dependence.
- The diquark-antidiquark Fock component.
- Systematics.
- Other quantum channels.

Thank you

Back up slides

Theory predictions

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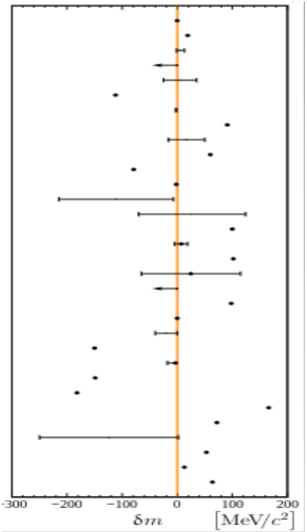
[65] J.-B. Cheng *et al.*, *Double-heavy tetraquark states with heavy diquark-antiquark symmetry*, arXiv:2008.00737. 2

[66] S. Noh, W. Park, and S. H. Lee, *The doubly-heavy tetraquarks $(qq'\bar{Q}\bar{Q}')$ in a constituent quark model with a complete set of harmonic oscillator bases*, arXiv:2102.09614. 2

[67] R. N. Faustov, V. O. Galkin, and E. M. Savchenko, *Heavy tetraquarks in the relativistic quark model*, Universe **7** (2021) 94, arXiv:2103.01763. 2

Theory predictions

Reference		Year	$\delta' m$ [MeV/c^2]
J. Carlson, L. Heller and J. A. Tjon	[36]	1987	~ 0
B. Silvestre-Brac and C. Semay	[37]	1993	$+19$
C. Semay and B. Silvestre-Brac	[38]	1994	$[-1, +13]$
S. Pepin, F. Stancu, M. Genovese and J. M. Richard	[39]	1996	< 0
B. A. Gelman and S. Nussinov	[40]	2002	$[-25, +35]$
J. Vijande, F. Fernandez, A. Valcarce, A. and B. Silvestre-Brac	[41]	2003	-112
D. Janc and M. Rosina	[42]	2004	$[-3, -1]$
F. Navarra, M. Nielsen and S. H. Lee	[43]	2007	$+91$
J. Vijande, E. Weissman, A. Valcarce	[44]	2007	$[-16, +50]$
D. Ebert, R. N. Faustov, V. O. Galkin and W. Lucha	[45]	2007	$+60$
S. H. Lee and S. Yasui	[46]	2009	-79
Y. Yang, C. Deng, J. Ping and T. Goldman	[47]	2009	-1.8
G.-Q. Feng, X.-H. Guo and B.-S. Zou	[48]	2013	-215
Y. Ikeda, B. Charron, S. Aoki, T. Doi, T. Hatsuda, T. Inoue, N. Ishii, K. Murano, H. Nemura and K. Sasaki	[49]	2013	$[-70, +124]$
S.-Q. Luo, K. Chen, X. Liu, Y.-R. Liu and S.-L. Zhu	[50]	2017	$+100$
M. Karliner and J. Rosner	[51]	2017	$7 \pm 12 \to 1$
E. J. Eichten and C. Quigg	[52]	2017	$+102$
Z. G. Wang	[53]	2017	$+25 \pm 90$
G. K. C. Cheung, C. E. Thomas, J. J. Dudek and R. G. Edwards	[54]	2017	$\lesssim 0$
W. Park, S. Noh and S. H. Lee	[55]	2018	$+98$
A. Francis, R. J. Hudspith, R. Lewis and K. Maltman	[56]	2018	~ 0
P. Junnarkar, N. Mathur and M. Padmanath	[57]	2018	$[-40, 0]$
C. Deng, H. Chen and J. Ping	[58]	2018	-150
M.-Z. Liu, T.-W. Wu, V. Pavon Valderrama, J.-J. Xie and L.-S. Geng	[59]	2019	-3^{+4}_{-15}
G. Yang, J. Ping and J. Segovia	[60]	2019	-149
Y. Tan, W. Lu and J. Ping	[61]	2020	-182
Q.-F. Lü, D.-Y. Chen and Y.-B. Dong	[62]	2020	$+166$
E. Braaten, L.-P. He and A. Mohapatra	[63]	2020	$+72$
D. Gao , D. Jia, Y.-J. Sun, Z. Zhang, W.-N. Liu and Q. Mei	[64]	2020	$[-250, +2]$
J.-B. Cheng, S.-Y. Li, Y.-R. Liu, Z.-G. Si, T. Yao	[65]	2020	$+53$
S. Noh, W. Park and S. H. Lee	[66]	2021	$+13$
R. N. Faustov, V. O. Galkin and E. M. Savchenko	[67]	2021	$+64$



Courtesy: Ivan Polyakov, EPS-HEP 2021

Finite-volume irreps

$\mathbf{p} = (0,0,0)$, O_h, $P = \pm$	
Λ (*dim*)	J
A_1 (1)	0, 4, ...
T_1 (3)	1, 3, 4, ...
T_2 (3)	2, 3, 4, ...
E (2)	2, 4, ...
A_2 (1)	3, 5, ...

$\mathbf{p} = (0,0,1)$, Dic_4		
Λ (*dim*)	$\lvert\lambda\rvert^{\tilde{\eta}}$	J^P (at rest)
A_1 (1)	0^+	0^+, 1^-, 2^+, 3^-, 4^+, ...
	4	$4^\pm$, ...
A_2 (1)	0^-	0^-, 1^+, 2^-, 3^+, 4^-, ...
	4	$4^\pm$, ...
E (2)	1	$1^\pm$, $2^\pm$, $3^\pm$, $4^\pm$, ...
	3	$3^\pm$, $4^\pm$, ...
B_1 (1)	2	$2^\pm$, $3^\pm$, $4^\pm$, ...
B_2 (1)	2	$2^\pm$, $3^\pm$, $4^\pm$, ...

$\mathbf{p} = (1,1,0)$, Dic_2		
Λ (*dim*)	$\lvert\lambda\rvert^{\tilde{\eta}}$	J^P (at rest)
A_1 (1)	0^+	0^+, 1^-, 2^+, 3^-, 4^+, ...
	2	$2^\pm$, $3^\pm$, $4^\pm$, ...
	4	$4^\pm$, ...
A_2 (1)	0^-	0^-, 1^+, 2^-, 3^+, 4^-, ...
	2	$2^\pm$, $3^\pm$, $4^\pm$, ...
	4	$4^\pm$, ...
B_1 (1)	1	$1^\pm$, $2^\pm$, $3^\pm$, $4^\pm$, ...
	3	$3^\pm$, $4^\pm$, ...
B_2 (1)	1	$1^\pm$, $2^\pm$, $3^\pm$, $4^\pm$, ...
	3	$3^\pm$, $4^\pm$, ...

- We investigate four inertial frames $p^2 = 0,\ 1,\ 2$ and 4.
- DD^* scattering in s-wave leads to $J^P = 1^+$.
  Our study focus on $T_1^+[0]$, $A_2[1]$, $A_2[2]$, and $A_2[4]$.
- Higher partial wave effects: Consider only up to $l \leq 2$ and $J \leq 2$.
  $l = 1 \rightarrow J^P = 0^-/2^- \Rightarrow A_2[1]$, $A_2[2]$, and $A_2[4]$.
  $l = 2 \rightarrow J^P = 1^+ \Rightarrow A_2[1]$, $A_2[2]$, and $A_2[4]$ and also introduce mixing.
  $l = 2 \rightarrow J^P = 2^+ \Rightarrow A_2[2]$.

+/g refers to positive parity, -/u refers to negative parity.

Irreps and interpolators

$\vec{P}$	LG	Λ^P	J^P	l	interpolators: $M_1(\vec{p}_1^{\,2})M_2(\vec{p}_2^{\,2})$
$(0,0,0)$	O_h	T_1^+	1^+	$0,2$	$D(0)D^*(0),\ D(1)D^*(1)\ [2],\ D^*(0)D^*(0)$
$(0,0,0)$	O_h	A_1^-	0^-	1	$D(1)D^*(1)$
$(0,0,1)\frac{2\pi}{L}$	Dic_4	A_2	$0^-,1^+,2^-$	$0,1,2$	$D(0)D^*(1),\ D(1)D^*(0)$
$(1,1,0)\frac{2\pi}{L}$	Dic_2	A_2	$0^-,1^+,2^-,2^+$	$0,1,2$	$D(0)D^*(2),\ D(1)D^*(1)\ [2],\ D(2)D^*(1)$
$(0,0,2)\frac{2\pi}{L}$	Dic_4	A_2	$0^-,1^+,2^-$	$0,1,2$	$D(1)D^*(1)$

- We assume contributions from $l \geq 2$ to be negligible.
  $l = 1 \rightarrow J^P = 0^-$ are constrained also considering $A_1^-[0]$.
- For the rest frame, $\vec{P} = \vec{0}$, we utilize partial wave projection

$$O^{|p|,J,m_J,L,S} = \sum_{m_L,m_S,m_{s1},m_{s2}} C^{Jm_J}_{Lm_L,Sm_S} C^{Sm_S}_{s_1m_{s1},s_2m_{s2}} \sum_{R\in O} Y^*_{Lm_L}(\widehat{Rp}) H^{(1)}_{m_{s1}}(Rp) H^{(2)}_{m_{s2}}(-Rp)$$

- For the moving frames, $\vec{P} \neq \vec{0}$, we utilize character projection
  $\sum_{R\in LG} \chi^{\Lambda}(R)\ R\ D(\vec{p}_1) D^*_k(\vec{p}_2)\ R^{-1}$

Prelovsek et al., 1607.06738

Resonances on the lattice & Lüscher's prescription

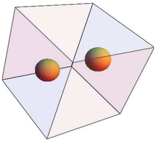
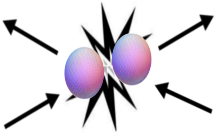
- On a finite volume Euclidean lattice : Discrete energy spectrum
  Maiani-Testa no-go theorem PLB**245** 585 (1990)
  No continuum of states $\Rightarrow$ No resonances, No scattering

- Non-interacting two-hadron levels are given by $E(L) = \sqrt{m_1^2 + \vec{p}_1^2} + \sqrt{m_2^2 + \vec{p}_2^2}$
  where $\vec{p}_{1,2} = \frac{2\pi}{L}(n_x, n_y, n_z)$.

- Switching on the interaction makes $\vec{p}_{1,2} \neq \frac{2\pi}{L}(n_x, n_y, n_z)$.
  The interactions induce a shift in the momentum,
  e.g. in 1D $\quad \vec{p}_{1,2} = \frac{2\pi}{L} n + \frac{2}{L}\delta(k)$.

- Lüscher's formula relates Lüscher, NPB**354** 531 (1991)

  finite volume level shifts $\Leftrightarrow$ infinite volume phase shifts.

$$\det[\delta_{ss'} B^{\vec{P},\Lambda}_{lJ;l'J'}(E_{cm}) - (\tilde{K}^{(J)}_{ls;l's'}(E_{cm}, \{a\}))^{-1}\delta_{JJ'}] = 0$$

Briceño 2014