Drell–Yan lepton-pair production: q_T resummation at N^{3}LL accuracy and fiducial cross sections at N^{3}LO in QCD

Giancarlo Ferrera

Milan University & INFN, Milan

In collaboration with:

Stefano Camarda & Leandro Cieri

Phys.Rev.D 104 (2021) 11, L111503, e-Print:2103.04974
Eur.Phys.J.C 82 (2022) 6, 575, e-Print: 2111.14509

ICHEP – Bologna – July 7th 2022

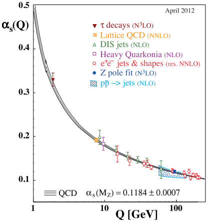
Motivations

Vector boson production is a benchmark process in hadron collider physics.

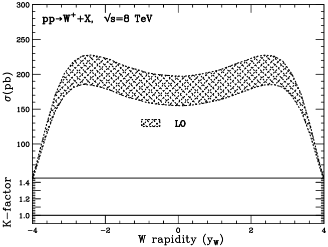
- Constraints of parton densities (PDFs).
- α_S determination $\rightarrow$ see S. Camarda talk.
- W-boson mass (M_W) and electroweak (EW) parameters determination.
- Beyond the Standard Model analyses.
- Perturbative QCD studies.

The above reasons and precise experimental data demands for accurate theoretical predictions $\Rightarrow$ computation of higher-order QCD corrections.

Higher-order calculations



The QCD coupling
$\alpha_S(Q) \sim 1/(\beta_0 \ln Q^2/\Lambda^2_{QCD}) \sim 0.1$
(for $Q \sim m_Z$).



- Factorization theorem

$$\sigma = \sum_{a,b} f_a(M^2) \otimes f_b(M^2) \otimes \hat{\sigma}_{ab}(\alpha_S) + \mathcal{O}\left(\frac{\Lambda}{M}\right)$$

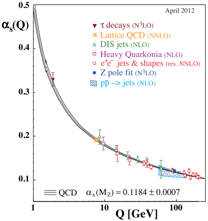
- Perturbation theory at **leading order (LO)**:

$$\hat{\sigma}(\alpha_S) = \hat{\sigma}^{(0)}$$

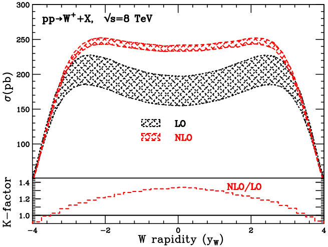
- **LO result:** only **order of magnitude** estimate.
  NLO: first reliable estimate.
  NNLO & beyond: precise prediction & **robust uncertainty**.

- Higher-order calculations **not an easy task** due to **infrared (IR) singularities** (impossible direct use of numerical techniques).

Higher-order calculations



The QCD coupling
$\alpha_S(Q) \sim 1/(\beta_0 \ln Q^2/\Lambda^2_{QCD}) \sim 0.1$
(for $Q \sim m_Z$).



- Factorization theorem

$$\sigma = \sum_{a,b} f_a(M^2) \otimes f_b(M^2) \otimes \hat{\sigma}_{ab}(\alpha_S) + \mathcal{O}\left(\frac{\Lambda}{M}\right)$$

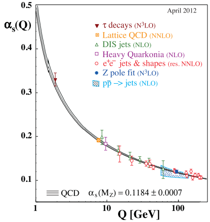
- Perturbation theory at **next order (NLO)**:

$$\hat{\sigma}(\alpha_S) = \hat{\sigma}^{(0)} + \alpha_S\, \hat{\sigma}^{(1)}$$

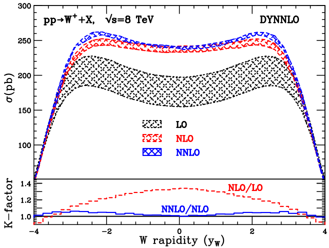
- **LO result:** only **order of magnitude** estimate.
  NLO: first reliable estimate.
  NNLO & beyond: precise prediction & **robust uncertainty**.

- Higher-order calculations **not an easy task** due to **infrared (IR) singularities** (impossible direct use of numerical techniques).

Higher-order calculations



The QCD coupling
$\alpha_S(Q) \sim 1/(\beta_0 \ln Q^2/\Lambda^2_{QCD}) \sim 0.1$
(for $Q \sim m_Z$).



- Factorization theorem

$$\sigma = \sum_{a,b} f_a(M^2) \otimes f_b(M^2) \otimes \hat{\sigma}_{ab}(\alpha_S) + \mathcal{O}\left(\frac{\Lambda}{M}\right)$$

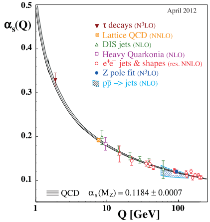
- Perturbation theory at **NNLO & beyond**:

$$\hat{\sigma}(\alpha_S) = \hat{\sigma}^{(0)} + \alpha_S\, \hat{\sigma}^{(1)} + \alpha_S^2\, \hat{\sigma}^{(2)} + \cdots$$

- **LO result:** only **order of magnitude** estimate.
  NLO: first reliable estimate.
  NNLO & beyond: precise prediction & robust uncertainty.

- Higher-order calculations **not an easy task** due to **infrared (IR) singularities** (impossible direct use of numerical techniques).

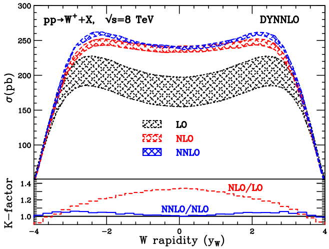
Higher-order calculations



The QCD coupling

$\alpha_S(Q) \sim 1/(\beta_0 \ln Q^2/\Lambda^2_{QCD}) \sim 0.1$

(for $Q \sim m_Z$).



- Factorization theorem

$$\sigma = \sum_{a,b} f_a(M^2) \otimes f_b(M^2) \otimes \hat{\sigma}_{ab}(\alpha_S) + \mathcal{O}\left(\frac{\Lambda}{M}\right)$$

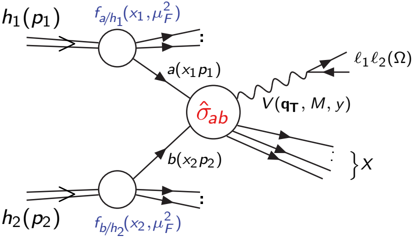
- Perturbation theory at **NNLO & beyond**:

$$\hat{\sigma}(\alpha_S) = \hat{\sigma}^{(0)} + \alpha_S\, \hat{\sigma}^{(1)} + \alpha_S^2\, \hat{\sigma}^{(2)} + \cdots$$

- **LO result:** only **order of magnitude** estimate.
  NLO: first reliable estimate.
  NNLO & beyond: precise prediction & robust uncertainty.

- Higher-order calculations **not an easy task** due to **infrared (IR) singularities** (impossible direct use of numerical techniques).

Drell–Yan $\mathbf{q_T}$ distribution



$$\mathrm{h_1(p_1) + h_2(p_2) \;\to\; V + X \;\to\; \ell_1 + \ell_2 + X}$$

where $\quad V = Z^0/\gamma^*, W^\pm$

QCD factorization formula:

$$\frac{d\sigma}{d^2\mathbf{q_T}\, dM^2\, dy\, d\Omega} = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\; f_{a/h_1}(x_1, \mu_F^2)\; f_{b/h_2}(x_2, \mu_F^2)\; \frac{d\hat{\sigma}_{ab}}{d^2\mathbf{q_T}\, dM^2\, d\hat{y}\, d\Omega}(\hat{s}; \alpha_S, \mu_R^2, \mu_F^2).$$

Fixed-order perturbative expansion reliable

only for $q_T \sim M$. When $q_T \ll M$:

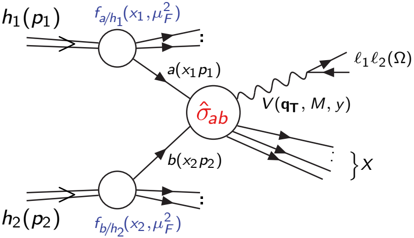
$$\int_0^{q_T^2} d\bar{q}_T^2 \frac{d\hat{\sigma}_{q\bar{q}}}{d\bar{q}_T^2} \sim 1 + \alpha_S \left[c_{12} L_{q_T}^2 + c_{11} L_{q_T} + \cdots \right]$$

$$+ \alpha_S^2 \left[c_{24} L_{q_T}^4 + \cdots + c_{21} L_{q_T} + \cdots \right] + \mathcal{O}(\alpha_S^3)$$

with $\alpha_S^n L_{q_T}^m \equiv \alpha_S^n \log^m (M^2/q_T^2) \gg 1$.

Resummation of logarithmic corrections mandatory.

Drell–Yan q_T distribution



$$\mathrm{h_1(p_1) + h_2(p_2) \;\to\; V + X \;\to\; \ell_1 + \ell_2 + X}$$

where $\quad V = Z^0/\gamma^*, W^\pm$

QCD factorization formula:

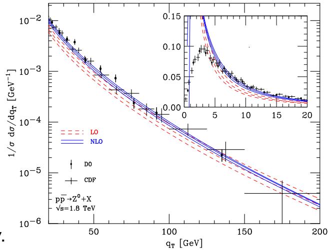
$$\frac{d\sigma}{d^2\mathbf{q_T}\, dM^2\, dy\, d\Omega} = \sum_{a,b}\int_0^1 dx_1 \int_0^1 dx_2\, f_{a/h_1}(x_1,\mu_F^2)\, f_{b/h_2}(x_2,\mu_F^2)\, \frac{d\hat{\sigma}_{ab}}{d^2\mathbf{q_T}\, dM^2\, d\hat{y}\, d\Omega}(\hat{s};\alpha_S,\mu_R^2,\mu_F^2).$$

Fixed-order perturbative expansion reliable only for $q_T \sim M$. When $q_T \ll M$:

$$\int_0^{q_T^2} d\bar{q}_T^2 \frac{d\hat{\sigma}_{q\bar{q}}}{d\bar{q}_T^2} \sim 1 + \alpha_S\left[c_{12} L_{q_T}^2 + c_{11} L_{q_T} + \cdots\right] + \alpha_S^2\left[c_{24} L_{q_T}^4 + \cdots + c_{21} L_{q_T} + \cdots\right] + \mathcal{O}(\alpha_S^3)$$

with $\alpha_S^n L_{q_T}^m \equiv \alpha_S^n \log^m(M^2/q_T^2) \gg 1$.

Resummation of logarithmic corrections mandatory.



$\mathbf{q_T}$ resummation in QCD [Catani, de Florian, Grazzini('01)] [Bozzi, Catani, de Florian, Grazzini('06)]

$$\frac{d\hat{\sigma}}{d^2\mathbf{q_T}\, dM^2\, d\hat{y}\, d\Omega} = \left[d\hat{\sigma}^{(res)}\right] + \left[d\hat{\sigma}^{(fin)}\right]; \qquad \int dq_T^2 \frac{d\hat{\sigma}^{(res)}}{dq_T^2} \overset{q_T\to 0}{\sim} \sum \alpha_S^n \log^m \frac{M^2}{Q_T^2} \qquad \int dq_T^2 \frac{d\hat{\sigma}^{(fin)}}{dq_T^2} \overset{q_T\to 0}{\sim} 0$$

Resummation holds in impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\left[d\hat{\sigma}^{(res)}\right] = \frac{d\hat{\sigma}^{(0)}}{d\Omega}\, \frac{1}{\hat{s}} \int \frac{d^2\mathrm{b}}{4\pi^2} e^{i\mathrm{b}\cdot\mathrm{q_T}}\, \mathcal{W}(b, M, \hat{y}, \hat{s}),$$

In the *double* Mellin space ($z_{1,2} = e^{\pm\hat{y}} M/\sqrt{\hat{s}}$) we have:

$$\mathcal{W}_{(N_1,N_2)}(b,M) = \mathcal{H}_{(N_1,N_2)}(\alpha_S) \times \exp\left\{\mathcal{G}_{(N_1,N_2)}(\alpha_S, \widetilde{L})\right\}$$

with $\widetilde{L} \equiv \log(Q^2 b^2 + 1)$ ($Q \sim M$ is the resummation scale)

$$\mathcal{G}(\alpha_S, \widetilde{L}) = \widetilde{L}\, g^{(1)}(\alpha_S \widetilde{L}) + g^{(2)}(\alpha_S \widetilde{L}) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S \widetilde{L}) + \cdots \qquad \mathcal{H}(\alpha_S) = 1 + \frac{\alpha_S}{\pi}\mathcal{H}^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{(2)} + \cdots$$

LL ($\sim \alpha_S^n \widetilde{L}^{n+1}$): $g^{(1)}$, $(\hat{\sigma}^{(0)})$; NLL ($\sim \alpha_S^n \widetilde{L}^{n}$): $g^{(2)}$, $\mathcal{H}^{(1)}$; NNLL ($\sim \alpha_S^n \widetilde{L}^{n-1}$): $g^{(3)}$, $\mathcal{H}^{(2)}$;

Resummed result at small q_T *matched* with fixed-order "finite" part at large q_T: *uniform accuracy* for $q_T \ll M$ and $q_T \sim M$.

q_T resummation in QCD [Catani, de Florian, Grazzini ('01)] [Bozzi, Catani, de Florian, Grazzini ('06)]

$$\frac{d\hat{\sigma}}{d^2\mathbf{q_T}\, dM^2\, d\hat{y}\, d\Omega} = \left[d\hat{\sigma}^{(res)}\right] + \left[d\hat{\sigma}^{(fin)}\right];$$

$$\int dq_T^2 \frac{d\hat{\sigma}^{(res)}}{dq_T^2} \overset{q_T\to 0}{\sim} \sum \alpha_S^n \log^m \frac{M^2}{Q_T^2}$$

$$\int dq_T^2 \frac{d\hat{\sigma}^{(fin)}}{dq_T^2} \overset{q_T\to 0}{\sim} 0$$

Resummation holds in impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\left[d\hat{\sigma}^{(res)}\right] = \frac{d\hat{\sigma}^{(0)}}{d\Omega}\, \frac{1}{\hat{s}} \int \frac{d^2\mathrm{b}}{4\pi^2} e^{i\mathrm{b}\cdot\mathrm{q_T}}\, \mathcal{W}(b, M, \hat{y}, \hat{s}),$$

In the *double* Mellin space ($z_{1,2} = e^{\pm\hat{y}} M/\sqrt{\hat{s}}$) we have:

$$\mathcal{W}_{(N_1,N_2)}(b,M) = \mathcal{H}_{(N_1,N_2)}(\alpha_S) \times \exp\left\{\mathcal{G}_{(N_1,N_2)}(\alpha_S, \widetilde{L})\right\}$$

with $\widetilde{L} \equiv \log(Q^2 b^2 + 1)$ ($Q \sim M$ is the resummation scale)

$$\mathcal{G}(\alpha_S, \widetilde{L}) = \widetilde{L}\, g^{(1)}(\alpha_S \widetilde{L}) + g^{(2)}(\alpha_S \widetilde{L}) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S \widetilde{L}) + \cdots \qquad \mathcal{H}(\alpha_S) = 1 + \frac{\alpha_S}{\pi}\mathcal{H}^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{(2)} + \cdots$$

LL ($\sim \alpha_S^n \widetilde{L}^{n+1}$): $g^{(1)}$, ($\hat{\sigma}^{(0)}$); NLL ($\sim \alpha_S^n \widetilde{L}^n$): $g^{(2)}$, $\mathcal{H}^{(1)}$; NNLL ($\sim \alpha_S^n \widetilde{L}^{n-1}$): $g^{(3)}$, $\mathcal{H}^{(2)}$;

Resummed result at small q_T *matched* with fixed-order "finite" part at large q_T: *uniform accuracy* for $q_T \ll M$ and $q_T \sim M$.

$\mathbf{q_T}$ resummation in QCD [Catani, de Florian, Grazzini ('01)] [Bozzi, Catani, de Florian, Grazzini ('06)]

$$\frac{d\hat{\sigma}}{d^2\mathbf{q_T}\, dM^2\, d\hat{y}\, d\Omega} = \left[d\hat{\sigma}^{(res)}\right] + \left[d\hat{\sigma}^{(fin)}\right];$$

$$\int dq_T^2 \frac{d\hat{\sigma}^{(res)}}{dq_T^2} \overset{q_T\to 0}{\sim} \sum \alpha_S^n \log^m \frac{M^2}{Q_T^2}$$

$$\int dq_T^2 \frac{d\hat{\sigma}^{(fin)}}{dq_T^2} \overset{q_T\to 0}{\sim} 0$$

Resummation holds in impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\left[d\hat{\sigma}^{(res)}\right] = \frac{d\hat{\sigma}^{(0)}}{d\Omega}\, \frac{1}{\hat{s}} \int \frac{d^2\mathrm{b}}{4\pi^2} e^{i\mathrm{b}\cdot\mathrm{q_T}}\, \mathcal{W}(b, M, \hat{y}, \hat{s}),$$

In the *double* Mellin space ($z_{1,2} = e^{\pm\hat{y}} M/\sqrt{\hat{s}}$) we have:

$$\mathcal{W}_{(N_1,N_2)}(b,M) = \mathcal{H}_{(N_1,N_2)}(\alpha_S) \times \exp\left\{\mathcal{G}_{(N_1,N_2)}(\alpha_S, \widetilde{L})\right\}$$

with $\widetilde{L} \equiv \log(Q^2 b^2 + 1)$ ($Q \sim M$ is the resummation scale)

$$\mathcal{G}(\alpha_S, \widetilde{L}) = \widetilde{L}\, g^{(1)}(\alpha_S \widetilde{L}) + g^{(2)}(\alpha_S \widetilde{L}) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S \widetilde{L}) + \cdots \qquad \mathcal{H}(\alpha_S) = 1 + \frac{\alpha_S}{\pi}\mathcal{H}^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{(2)} + \cdots$$

LL ($\sim \alpha_S^n \widetilde{L}^{n+1}$): $g^{(1)}$, ($\hat{\sigma}^{(0)}$); NLL ($\sim \alpha_S^n \widetilde{L}^n$): $g^{(2)}$, $\mathcal{H}^{(1)}$; NNLL ($\sim \alpha_S^n \widetilde{L}^{n-1}$): $g^{(3)}$, $\mathcal{H}^{(2)}$;

Resummed result at small q_T *matched* with fixed-order "finite" part at large q_T: *uniform accuracy* for $q_T \ll M$ and $q_T \sim M$.

q_T resummation in QCD [Catani,de Florian,Grazzini('01)] [Bozzi,Catani,de Florian,Grazzini('06)]

$$\frac{d\hat{\sigma}}{d^2\mathbf{q_T}\, dM^2\, d\hat{y}\, d\Omega} = \left[d\hat{\sigma}^{(res)}\right] + \left[d\hat{\sigma}^{(fin)}\right];$$

$$\int dq_T^2 \frac{d\hat{\sigma}^{(res)}}{dq_T^2} \overset{q_T\to 0}{\sim} \sum \alpha_S^n \log^m \frac{M^2}{Q_T^2}$$

$$\int dq_T^2 \frac{d\hat{\sigma}^{(fin)}}{dq_T^2} \overset{q_T\to 0}{\sim} 0$$

Resummation holds in impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\left[d\hat{\sigma}^{(res)}\right] = \frac{d\hat{\sigma}^{(0)}}{d\Omega}\,\frac{1}{\hat{s}} \int \frac{d^2\mathrm{b}}{4\pi^2} e^{i\mathrm{b}\cdot\mathrm{q_T}}\, \mathcal{W}(b, M, \hat{y}, \hat{s}),$$

In the *double* Mellin space ($z_{1,2} = e^{\pm\hat{y}} M/\sqrt{\hat{s}}$) we have:

$$\mathcal{W}_{(N_1,N_2)}(b,M) = \mathcal{H}_{(N_1,N_2)}(\alpha_S) \times \exp\left\{\mathcal{G}_{(N_1,N_2)}(\alpha_S, \widetilde{L})\right\}$$

with $\widetilde{L} \equiv \log(Q^2 b^2 + 1)$ ($Q \sim M$ is the resummation scale)

$$\mathcal{G}(\alpha_S, \widetilde{L}) = \widetilde{L}\, g^{(1)}(\alpha_S \widetilde{L}) + g^{(2)}(\alpha_S \widetilde{L}) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S \widetilde{L}) + \cdots \qquad \mathcal{H}(\alpha_S) = 1 + \frac{\alpha_S}{\pi}\mathcal{H}^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{(2)} + \cdots$$

LL ($\sim \alpha_S^n \widetilde{L}^{n+1}$): $g^{(1)}$, ($\hat{\sigma}^{(0)}$); NLL ($\sim \alpha_S^n \widetilde{L}^{n}$): $g^{(2)}$, $\mathcal{H}^{(1)}$; NNLL ($\sim \alpha_S^n \widetilde{L}^{n-1}$): $g^{(3)}$, $\mathcal{H}^{(2)}$;

Resummed result at small q_T *matched* with fixed-order "finite" part at large q_T: *uniform accuracy* for $q_T \ll M$ and $q_T \sim M$.

$\mathbf{q_T}$ resummation in QCD [Catani, de Florian, Grazzini ('01)] [Bozzi, Catani, de Florian, Grazzini ('06)]

$$\frac{d\hat{\sigma}}{d^2\mathbf{q_T}\, dM^2\, d\hat{y}\, d\Omega} = \left[d\hat{\sigma}^{(res)}\right] + \left[d\hat{\sigma}^{(fin)}\right];$$

$$\int dq_T^2 \frac{d\hat{\sigma}^{(res)}}{dq_T^2} \overset{q_T\to 0}{\sim} \sum \alpha_S^n \log^m \frac{M^2}{Q_T^2}$$

$$\int dq_T^2 \frac{d\hat{\sigma}^{(fin)}}{dq_T^2} \overset{q_T\to 0}{\sim} 0$$

Resummation holds in impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\left[d\hat{\sigma}^{(res)}\right] = \frac{d\hat{\sigma}^{(0)}}{d\Omega}\, \frac{1}{\hat{s}} \int \frac{d^2\mathrm{b}}{4\pi^2} e^{i\mathrm{b}\cdot\mathrm{q_T}}\, \mathcal{W}(b, M, \hat{y}, \hat{s}),$$

In the *double* Mellin space ($z_{1,2} = e^{\pm\hat{y}} M/\sqrt{\hat{s}}$) we have:

$$\mathcal{W}_{(N_1,N_2)}(b,M) = \mathcal{H}_{(N_1,N_2)}(\alpha_S) \times \exp\left\{\mathcal{G}_{(N_1,N_2)}(\alpha_S, \widetilde{L})\right\}$$

with $\widetilde{L} \equiv \log(Q^2 b^2 + 1)$ ($Q \sim M$ is the resummation scale)

$$\mathcal{G}(\alpha_S, \widetilde{L}) = \widetilde{L}\, g^{(1)}(\alpha_S \widetilde{L}) + g^{(2)}(\alpha_S \widetilde{L}) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S \widetilde{L}) + \cdots \qquad \mathcal{H}(\alpha_S) = 1 + \frac{\alpha_S}{\pi}\mathcal{H}^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{(2)} + \cdots$$

LL ($\sim \alpha_S^n \widetilde{L}^{n+1}$): $g^{(1)}$, ($\hat{\sigma}^{(0)}$); NLL ($\sim \alpha_S^n \widetilde{L}^{n}$): $g^{(2)}$, $\mathcal{H}^{(1)}$; NNLL ($\sim \alpha_S^n \widetilde{L}^{n-1}$): $g^{(3)}$, $\mathcal{H}^{(2)}$;

Resummed result at small q_T *matched* with fixed-order "finite" part at large q_T: *uniform accuracy* for $q_T \ll M$ and $q_T \sim M$.

q_T resummation in QCD [Catani, de Florian, Grazzini ('01)] [Bozzi, Catani, de Florian, Grazzini ('06)]

$$\frac{d\hat\sigma}{d^2\mathbf{q_T}\, dM^2\, d\hat y\, d\Omega} = \left[d\hat\sigma^{(res)}\right] + \left[d\hat\sigma^{(fin)}\right];$$

$$\int dq_T^2 \frac{d\hat\sigma^{(res)}}{dq_T^2} \overset{q_T\to 0}{\sim} \sum \alpha_S^n \log^m \frac{M^2}{Q_T^2}$$

$$\int dq_T^2 \frac{d\hat\sigma^{(fin)}}{dq_T^2} \overset{q_T\to 0}{\sim} 0$$

Resummation holds in impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\left[d\hat\sigma^{(res)}\right] = \frac{d\hat\sigma^{(0)}}{d\Omega}\, \frac{1}{\hat s} \int \frac{d^2 \mathrm{b}}{4\pi^2} e^{i\mathrm{b}\cdot \mathrm{q_T}}\, \mathcal{W}(b, M, \hat y, \hat s),$$

In the *double* Mellin space ($z_{1,2} = e^{\pm \hat y} M/\sqrt{\hat s}$) we have:

$$\mathcal{W}_{(N_1,N_2)}(b,M) = \mathcal{H}_{(N_1,N_2)}(\alpha_S) \times \exp\left\{\mathcal{G}_{(N_1,N_2)}(\alpha_S, \widetilde{L})\right\}$$

with $\widetilde{L} \equiv \log(Q^2 b^2 + 1)$ ($Q \sim M$ is the resummation scale)

$$\mathcal{G}(\alpha_S, \widetilde{L}) = \widetilde{L}\, g^{(1)}(\alpha_S \widetilde{L}) + g^{(2)}(\alpha_S \widetilde{L}) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S \widetilde{L}) + \cdots \qquad \mathcal{H}(\alpha_S) = 1 + \frac{\alpha_S}{\pi}\mathcal{H}^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{(2)} + \cdots$$

LL ($\sim \alpha_S^n \widetilde{L}^{n+1}$): $g^{(1)}$, $(\hat\sigma^{(0)})$; NLL ($\sim \alpha_S^n \widetilde{L}^{n}$): $g^{(2)}$, $\mathcal{H}^{(1)}$; NNLL ($\sim \alpha_S^n \widetilde{L}^{n-1}$): $g^{(3)}$, $\mathcal{H}^{(2)}$;

Resummed result at small q_T *matched* with fixed-order "finite" part at large q_T: *uniform accuracy* for $q_T \ll M$ and $q_T \sim M$.

$\mathbf{q_T}$ resummation in QCD [Catani, de Florian, Grazzini ('01)] [Bozzi, Catani, de Florian, Grazzini ('06)]

$$\frac{d\hat{\sigma}}{d^2\mathbf{q_T}\, dM^2\, d\hat{y}\, d\Omega} = \left[d\hat{\sigma}^{(res)}\right] + \left[d\hat{\sigma}^{(fin)}\right];$$

$$\int dq_T^2 \frac{d\hat{\sigma}^{(res)}}{dq_T^2} \overset{q_T \to 0}{\sim} \sum \alpha_S^n \log^m \frac{M^2}{Q_T^2}$$

$$\int dq_T^2 \frac{d\hat{\sigma}^{(fin)}}{dq_T^2} \overset{q_T \to 0}{\sim} 0$$

Resummation holds in impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\left[d\hat{\sigma}^{(res)}\right] = \frac{d\hat{\sigma}^{(0)}}{d\Omega}\, \frac{1}{\hat{s}} \int \frac{d^2\mathrm{b}}{4\pi^2} e^{i\mathrm{b}\cdot\mathrm{q_T}}\, \mathcal{W}(b, M, \hat{y}, \hat{s}),$$

In the *double* Mellin space ($z_{1,2} = e^{\pm\hat{y}} M/\sqrt{\hat{s}}$) we have:

$$\mathcal{W}_{(N_1,N_2)}(b,M) = \mathcal{H}_{(N_1,N_2)}(\alpha_S) \times \exp\left\{\mathcal{G}_{(N_1,N_2)}(\alpha_S, \widetilde{L})\right\}$$

with $\widetilde{L} \equiv \log(Q^2 b^2 + 1)$ ($Q \sim M$ is the resummation scale)

$$\mathcal{G}(\alpha_S, \widetilde{L}) = \widetilde{L}\, g^{(1)}(\alpha_S \widetilde{L}) + g^{(2)}(\alpha_S \widetilde{L}) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S \widetilde{L}) + \cdots \qquad \mathcal{H}(\alpha_S) = 1 + \frac{\alpha_S}{\pi}\mathcal{H}^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{(2)} + \cdots$$

LL ($\sim \alpha_S^n \widetilde{L}^{n+1}$): $g^{(1)}$, $(\hat{\sigma}^{(0)})$; NLL ($\sim \alpha_S^n \widetilde{L}^{n}$): $g^{(2)}$, $\mathcal{H}^{(1)}$; NNLL ($\sim \alpha_S^n \widetilde{L}^{n-1}$): $g^{(3)}$, $\mathcal{H}^{(2)}$;

Resummed result at small q_T *matched* with fixed-order "finite" part at large q_T: *uniform accuracy* for $q_T \ll M$ and $q_T \sim M$.

$\mathbf{q_T}$ recoil and lepton angular distribution

- The dependence of the resummed cross section on the leptonic variable Ω is

$$\frac{d\hat{\sigma}^{(0)}}{d\Omega} = \hat{\sigma}^{(0)}(M^2)\, F(\mathbf{q_T}/M; M^2, \Omega) \;\; , \;\; \text{with} \;\; \int d\Omega\, F(\mathbf{q_T}/M; \Omega) = 1 \; .$$

the q_T dependence arise as a *dynamical* q_T-recoil of the vector boson due to *soft* and *collinear* multiparton emissions.

- This dependence cannot be *unambiguously* calculated through resummation (it is not singular)

$$F(\mathbf{q_T}/M; M^2, \Omega) = F(0/M; M^2, \Omega) + \mathcal{O}(\mathbf{q_T^2}/M^2) \;\; ,$$

  - After matching between *resummed* and *finite* component the $\mathcal{O}(\mathbf{q_T^2}/M^2)$ ambiguity starts at $\mathcal{O}(\alpha_S^3)$ $(\mathcal{O}(\alpha_S^2))$ at NNLL+NNLO (NLL+NLO).
  - After integration over leptonic variable Ω the ambiguity *completely cancel*.

- A general procedure to treat the q_T recoil in q_T resummed calculations introduced in [Catani,de Florian,G.F.,Grazzini('15)].

- This procedure is directly related to the choice of a particular (among the infinite ones) vector boson rest frame to generate the lepton momenta: e.g. the Collins–Soper rest frame.

$\mathbf{q_T}$ recoil and lepton angular distribution

- The dependence of the resummed cross section on the leptonic variable Ω is

$$\frac{d\hat{\sigma}^{(0)}}{d\Omega} = \hat{\sigma}^{(0)}(M^2)\, F(\mathbf{q_T}/M; M^2, \Omega) \;\; , \quad \text{with} \quad \int d\Omega\, F(\mathbf{q_T}/M; \Omega) = 1 \; .$$

  the q_T dependence arise as a *dynamical* q_T-recoil of the vector boson due to *soft* and *collinear* multiparton emissions.

- This dependence cannot be *unambiguously* calculated through resummation (it is not singular)

$$F(\mathbf{q_T}/M; M^2, \Omega) = F(0/M; M^2, \Omega) + \mathcal{O}(\mathbf{q_T^2}/M^2) \;\; ,$$

  - After matching between *resummed* and *finite* component the $\mathcal{O}(\mathbf{q_T^2}/M^2)$ ambiguity starts at $\mathcal{O}(\alpha_S^3)$ $(\mathcal{O}(\alpha_S^2))$ at NNLL+NNLO (NLL+NLO).
  - After integration over leptonic variable Ω the ambiguity *completely cancel*.

- A general procedure to treat the q_T recoil in q_T resummed calculations introduced in [Catani,de Florian,G.F.,Grazzini('15)].

- This procedure is directly related to the choice of a particular (among the infinite ones) vector boson rest frame to generate the lepton momenta: e.g. the Collins–Soper rest frame.

$\mathbf{q_T}$ recoil and lepton angular distribution

- The dependence of the resummed cross section on the leptonic variable Ω is

$$\frac{d\hat{\sigma}^{(0)}}{d\Omega} = \hat{\sigma}^{(0)}(M^2)\, F(\mathbf{q_T}/M; M^2, \Omega) \ , \quad \text{with} \quad \int d\Omega\, F(\mathbf{q_T}/M; \Omega) = 1 \ .$$

  the q_T dependence arise as a *dynamical* q_T-recoil of the vector boson due to *soft* and *collinear* multiparton emissions.

- This dependence cannot be *unambiguously* calculated through resummation (it is not singular)

$$F(\mathbf{q_T}/M; M^2, \Omega) = F(0/M; M^2, \Omega) + \mathcal{O}(\mathbf{q_T^2}/M^2) \ ,$$

  - After matching between *resummed* and *finite* component the $\mathcal{O}(\mathbf{q_T^2}/M^2)$ ambiguity starts at $\mathcal{O}(\alpha_S^3)$ $(\mathcal{O}(\alpha_S^2))$ at NNLL+NNLO (NLL+NLO).
  - After integration over leptonic variable Ω the ambiguity *completely cancel*.

- A general procedure to treat the q_T recoil in q_T resummed calculations introduced in [Catani,de Florian,G.F.,Grazzini('15)].

- This procedure is directly related to the choice of a particular (among the infinite ones) vector boson rest frame to generate the lepton momenta: e.g. the Collins–Soper rest frame.

q_T resummation at N^{3}LL+N^{3}LO

- We have implemented the calculation in the publicly available code:

  DYTurbo: computes resummed and fixed-order fiducial cross section and related distributions it retains full kinematics of the vector boson and of its leptonic decay products [Camarda, Boonekamp, Bozzi, Catani, Cieri, Cuth, G.F., de Florian, Glazov, Grazzini, Vincter, Schott ('20)]

  https://dyturbo.hepforge.org.

- q_T resummation performed for Drell–Yan process up to N^{3}LL+N^{3}LO We have included
  - N^{3}LL logarithmic contributions to all orders (i.e. up to $\exp(\sim\alpha_S^n L^{n-2})$);
  - N^{3}LO corrections (i.e. up to $\mathcal{O}(\alpha_S^3)$) at small q_T;
  - NLO corrections (i.e. up to $\mathcal{O}(\alpha_S^2)$) at large q_T;

- Matching with NNLO corrections (i.e. up to $\mathcal{O}(\alpha_S^3)$) at large q_T from results in [Gehrmann-De Ridder et al.('16), Bizon et al.('19)];

- Results up to N^{3}LO (i.e. up to $\mathcal{O}(\alpha_S^3)$) recovered for the total cross section (from unitarity).

q_T resummation at N^{3}LL+N^{3}LO

- We have implemented the calculation in the publicly available code:

  DYTurbo: computes resummed and fixed-order fiducial cross section and related distributions it retains full kinematics of the vector boson and of its leptonic decay products [Camarda,Boonekamp,Bozzi,Catani,Cieri,Cuth,G.F.,de Florian,Glazov, Grazzini,Vincter,Schott('20)]

  https://dyturbo.hepforge.org.

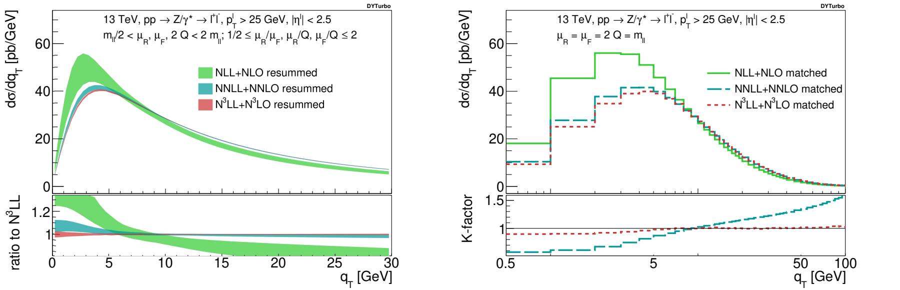
- q_T resummation performed for Drell–Yan process up to N^{3}LL+N^{3}LO We have included
  - N^{3}LL logarithmic contributions to all orders (i.e. up to $\exp(\sim\alpha_S^n L^{n-2})$);
  - N^{3}LO corrections (i.e. up to $\mathcal{O}(\alpha_S^3)$) at small q_T;
  - NLO corrections (i.e. up to $\mathcal{O}(\alpha_S^2)$) at large q_T;

- Matching with NNLO corrections (i.e. up to $\mathcal{O}(\alpha_S^3)$) at large q_T from results in [Gehrmann-De Ridder et al.('16), Bizon et al.('19)];

- Results up to N^{3}LO (i.e. up to $\mathcal{O}(\alpha_S^3)$) recovered for the total cross section (from unitarity).

Z/γ^* production at N^{3}LL+N^{3}LO

[Camarda,Cieri,G.F.('21)]



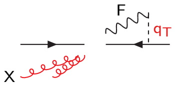
DYTurbo results. Resummed (left) and matched (right) NLL, NNLL and N^{3}LL bands for Z/γ^* q_T spectrum.
Lower panel: ratio with respect to the N^{3}LL central value.

Our results recently confirmed by [Chen et al.('22)].

The q_T-subtraction method

[Catani,Grazzini('07)]

$$h_1(p_1) + h_2(p_2) \;\rightarrow\; F(M, q_T) + X$$



- **Observation:** at LO the q_T of the F is exactly zero.

$$d\sigma^{F}_{N^nLO}|_{q_T\neq 0} = d\sigma^{F+\mathrm{jets}}_{N^{n-1}LO} \;,$$

for $q_T \neq 0$ the N^nLO IR sing. cancelled with the $N^{n-1}LO$ subtraction method.

- **Key point**: treat $q_T = 0$ exploiting universality q_T resummation formalism

$$d\sigma^{F}_{N^nLO} = \mathcal{H}^{F}_{N^nLO} \otimes d\sigma^{F}_{LO} + \left[d\sigma^{F+\mathrm{jets}}_{N^{n-1}LO} - d\sigma^{CT}_{N^{n-1}LO} \right] \;,$$

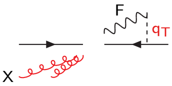
where

$$d\sigma^{CT}_{N^nLO} \;\xrightarrow{q_T\to 0}\; d\sigma^{F}_{LO} \otimes \sum_{n=1}^{\infty} \sum_{k=1}^{2n} \left(\frac{\alpha_S}{\pi}\right)^n \Sigma^{(n,k)} \frac{M^2}{q_T^2} \ln^{k-1} \frac{M^2}{q_T^2}\, d^2 q_T$$

$\mathcal{H}^{F}_{N^nLO}(\alpha_S)$ contains *multi-loop virtual* corrections via an *universal* factorization formula [Catani,Cieri,de Florian,G.F.,Grazzini('14)].

The q_T-subtraction method [Catani,Grazzini('07)]

$$h_1(p_1) + h_2(p_2) \;\rightarrow\; F(M, q_T) + X$$



- **Observation:** at LO the q_T of the F is exactly zero.

$$d\sigma^{F}_{N^nLO}|_{q_T \neq 0} = d\sigma^{F+\text{jets}}_{N^{n-1}LO} \quad ,$$

for $q_T \neq 0$ the N^nLO IR sing. cancelled with the $N^{n-1}LO$ subtraction method.

- **Key point**: treat $q_T = 0$ exploiting universality q_T resummation formalism

$$d\sigma^{F}_{N^nLO} = \mathcal{H}^{F}_{N^nLO} \otimes d\sigma^{F}_{LO} + \left[d\sigma^{F+\text{jets}}_{N^{n-1}LO} - d\sigma^{CT}_{N^{n-1}LO} \right] \quad ,$$

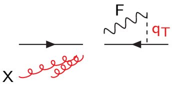
where

$$d\sigma^{CT}_{N^nLO} \xrightarrow{q_T \to 0} d\sigma^{F}_{LO} \otimes \sum_{n=1}^{\infty} \sum_{k=1}^{2n} \left(\frac{\alpha_S}{\pi} \right)^n \Sigma^{(n,k)} \frac{M^2}{q_T^2} \ln^{k-1} \frac{M^2}{q_T^2} \, d^2 q_T$$

$\mathcal{H}^{F}_{N^nLO}(\alpha_S)$ contains *multi-loop virtual* corrections via an *universal* factorization formula [Catani,Cieri,de Florian,G.F.,Grazzini('14)].

The q_T-subtraction method [Catani,Grazzini('07)]

$$h_1(p_1) + h_2(p_2) \rightarrow F(M, q_T) + X$$



- **Observation:** at LO the q_T of the F is exactly zero.

$$d\sigma^F_{N^nLO}|_{q_T\neq 0} = d\sigma^{F+\text{jets}}_{N^{n-1}LO} \quad ,$$

for $q_T \neq 0$ the N^nLO IR sing. cancelled with the $N^{n-1}LO$ subtraction method.

- **Key point**: treat $q_T = 0$ exploiting universality q_T resummation formalism

$$d\sigma^F_{N^nLO} = \mathcal{H}^F_{N^nLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+\text{jets}}_{N^{n-1}LO} - d\sigma^{CT}_{N^{n-1}LO} \right] \quad ,$$

where

$$d\sigma^{CT}_{N^nLO} \quad \xrightarrow{q_T \to 0} \quad d\sigma^F_{LO} \otimes \sum_{n=1}^{\infty} \sum_{k=1}^{2n} \left(\frac{\alpha_S}{\pi} \right)^n \Sigma^{(n,k)} \frac{M^2}{q_T^2} \ln^{k-1} \frac{M^2}{q_T^2} \, d^2 q_T$$

$\mathcal{H}^F_{N^nLO}(\alpha_S)$ contains *multi-loop virtual* corrections via an *universal* factorization formula [Catani,Cieri,de Florian,G.F.,Grazzini('14)].

Fiducial power corrections within the q_T subtraction

[Camarda,Cieri,G.F.('21)]

$$\sigma^F_{fid} = \int_{cuts} \mathcal{H}^F \otimes d\sigma^F_{LO} + \int_{cuts} \left[d\sigma^{F+\text{jets}}_{q_T > q_T^{cut}} - d\sigma^{CT}_{q_T > q_T^{cut}} \right] + \mathcal{O}\left((q_T^{cut}/M)^p \right)$$

- $d\sigma^{F+\text{jets}}$ and $d\sigma^{CT}$ are *separately* divergent, their sum is finite. A lower limit $q_T > q_T^{cut}$ is necessary with a power correction ambiguity, typically (standard fiducial cuts) linear $\mathcal{O}\left((q_T^{cut}/M) \right)$ [Alekhin et al.('21)]..
- The limit $q_T^{cut} \to 0$ leads to large cancellations and numerical uncertainties.
- **Key point**: "Fiducial" power corrections (FPC) absent including with

$$d\sigma^{FPC} = \left[d\widetilde{\sigma}^{CT}_{q_T < q_T^{cut}} - d\sigma^{CT}_{q_T < q_T^{cut}} \right]$$

  where $d\widetilde{\sigma}^{CT}$ is the q_T subtraction counteterm evaluated with q_T recoil defined in [Catani,de Florian,G.F.,Grazzini('15)].
- $d\sigma^{FPC}$ is *universal* and IR finite and can be treated as a *local* subtraction: integration for $q_T < q_T^{cut}$ extended at arbitrary small q_T (e.g. $q_T/M \sim 10^{-6}$ GeV)
- ("Recoil") q_T subtraction implemented in DYTurbo.
- Equivalent methods proposed by [Ebert et al.('20), Buonocore et al.('21)].

Fiducial power corrections within the q_T subtraction

[Camarda,Cieri,G.F.('21)]

$$\sigma^F_{fid} = \int_{cuts} \mathcal{H}^F \otimes d\sigma^F_{LO} + \int_{cuts} \left[d\sigma^{F+\mathrm{jets}}_{q_T > q_T^{cut}} - d\sigma^{CT}_{q_T > q_T^{cut}} \right] + \mathcal{O}\left((q_T^{cut}/M)^p \right)$$

- $d\sigma^{F+\mathrm{jets}}$ and $d\sigma^{CT}$ are *separately* divergent, their sum is finite. A lower limit $q_T > q_T^{cut}$ is necessary with a power correction ambiguity, typically (standard fiducial cuts) linear $\mathcal{O}\left((q_T^{cut}/M)\right)$ [Alekhin et al.('21)]..
- The limit $q_T^{cut} \to 0$ leads to large cancellations and numerical uncertainties.
- **Key point**: "Fiducial" power corrections (FPC) absent including with

$$d\sigma^{FPC} = \left[d\widetilde{\sigma}^{CT}_{q_T < q_T^{cut}} - d\sigma^{CT}_{q_T < q_T^{cut}} \right]$$

where $d\widetilde{\sigma}^{CT}$ is the q_T subtraction counteterm evaluated with q_T recoil defined in [Catani,de Florian,G.F.,Grazzini('15)].

- $d\sigma^{FPC}$ is *universal* and IR finite and can be treated as a *local* subtraction: integration for $q_T < q_T^{cut}$ extended at arbitrary small q_T (e.g. $q_T/M \sim 10^{-6}$ GeV)
- ("Recoil") q_T subtraction implemented in DYTurbo.
- Equivalent methods proposed by [Ebert et al.('20), Buonocore et al.('21)].

Fiducial power corrections within the q_T subtraction

[Camarda,Cieri,G.F.('21)]

$$\sigma^F_{fid} = \int_{cuts} \mathcal{H}^F \otimes d\sigma^F_{LO} + \int_{cuts} \left[d\sigma^{F+\text{jets}}_{q_T > q_T^{cut}} - d\sigma^{CT}_{q_T > q_T^{cut}} \right] + \int_{cuts} d\sigma^{FPC} + \mathcal{O}\left((q_T^{cut}/M)^2\right)$$

- $d\sigma^{F+\text{jets}}$ and $d\sigma^{CT}$ are *separately* divergent, their sum is finite. A lower limit $q_T > q_T^{cut}$ is necessary with a power correction ambiguity, typically (standard fiducial cuts) linear $\mathcal{O}\left((q_T^{cut}/M)\right)$ [Alekhin et al.('21)]..
- The limit $q_T^{cut} \to 0$ leads to large cancellations and numerical uncertainties.
- **Key point**: "Fiducial" power corrections (FPC) absent including with

$$d\sigma^{FPC} = \left[d\widetilde{\sigma}^{CT}_{q_T < q_T^{cut}} - d\sigma^{CT}_{q_T < q_T^{cut}} \right]$$

  where $d\widetilde{\sigma}^{CT}$ is the q_T subtraction counteterm evaluated with q_T recoil defined in [Catani,de Florian,G.F.,Grazzini('15)].
- $d\sigma^{FPC}$ is *universal* and IR finite and can be treated as a *local* subtraction: integration for $q_T < q_T^{cut}$ extended at arbitrary small q_T (e.g. $q_T/M \sim 10^{-6}$ GeV)
- ("Recoil") q_T subtraction implemented in DYTurbo.
- Equivalent methods proposed by [Ebert et al.('20), Buonocore et al.('21)].

Fiducial power corrections within the q_T subtraction

[Camarda,Cieri,G.F.('21)]

$$\sigma^F_{fid} = \int_{cuts} \mathcal{H}^F \otimes d\sigma^F_{LO} + \int_{cuts} \left[d\sigma^{F+\text{jets}}_{q_T > q_T^{cut}} - d\sigma^{CT}_{q_T > q_T^{cut}} \right] + \int_{cuts} d\sigma^{FPC} + \mathcal{O}((q_T^{cut}/M)^2)$$

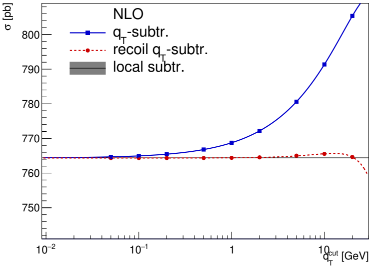
- $d\sigma^{F+\text{jets}}$ and $d\sigma^{CT}$ are *separately* divergent, their sum is finite. A lower limit $q_T > q_T^{cut}$ is necessary with a power correction ambiguity, typically (standard fiducial cuts) linear $\mathcal{O}\left((q_T^{cut}/M)\right)$ [Alekhin et al.('21)]..
- The limit $q_T^{cut} \to 0$ leads to large cancellations and numerical uncertainties.
- **Key point**: "Fiducial" power corrections (FPC) absent including with

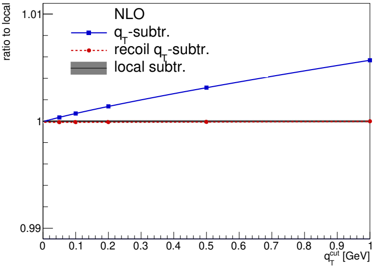
$$d\sigma^{FPC} = \left[d\widetilde{\sigma}^{CT}_{q_T < q_T^{cut}} - d\sigma^{CT}_{q_T < q_T^{cut}} \right]$$

  where $d\widetilde{\sigma}^{CT}$ is the q_T subtraction counteterm evaluated with q_T recoil defined in [Catani,de Florian,G.F.,Grazzini('15)].
- $d\sigma^{FPC}$ is *universal* and IR finite and can be treated as a *local* subtraction: integration for $q_T < q_T^{cut}$ extended at arbitrary small q_T (e.g. $q_T/M \sim 10^{-6}$ GeV)
- ("Recoil") q_T subtraction implemented in DYTurbo.
- Equivalent methods proposed by [Ebert et al.('20), Buonocore et al.('21)].

Fiducial power corrections at NLO

Z/γ^* production and decay at the LHC (13 TeV).
CUTS on leptons: $p_T > 25$ GeV, $|\eta| < 2.5$, $66 < M_{ll} < 116$ GeV, $q_T < 100$ GeV.

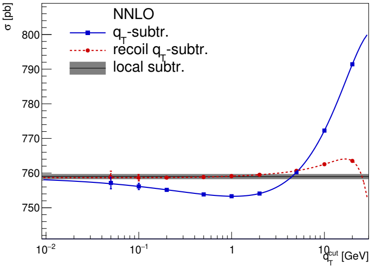


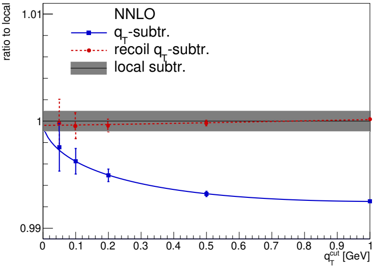


NLO results with the q_T subtraction method (blue squared points) and q_T subtraction method without FPC (red circled points) at various values of q_T^{cut}, and with a local subtraction method (black line).

Fiducial power corrections at NNLO

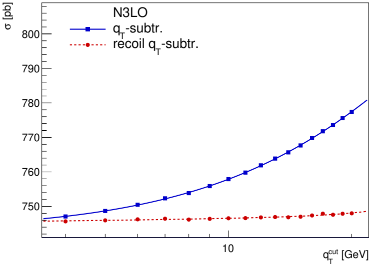
Z/γ^* production and decay at the LHC (13 TeV).
CUTS on leptons: $p_T > 25$ GeV, $|\eta| < 2.5$, $66 < M_{ll} < 116$ GeV,
$q_T < 100$ GeV.





NNLO results with the q_T subtraction method (blue squared points) and q_T subtraction method without FPC (red circled points) at various values of q_T^{cut}, and with a local subtraction method (black line).

Fiducial power corrections at N^3LO



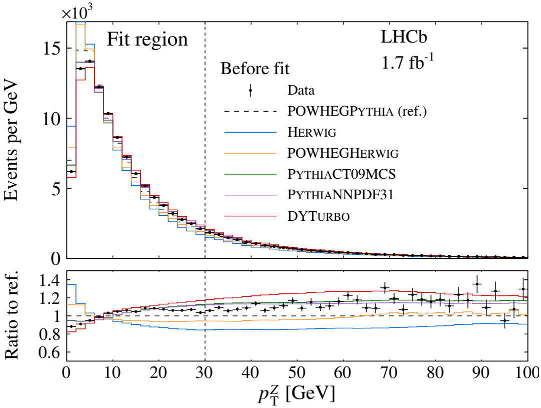
Z/γ^* production and decay at the LHC (13 TeV).

CUTS on leptons: $p_T > 25$ GeV, $|\eta| < 2.5$, $66 < M_{ll} < 116$ GeV, $q_T < 100$ GeV.

N^3LO results with the q_T subtraction method (blue squared points) and q_T subtraction method without FPC (red circled points) at various values of q_T^{cut}.

Modelling W and Z production for M_W determination

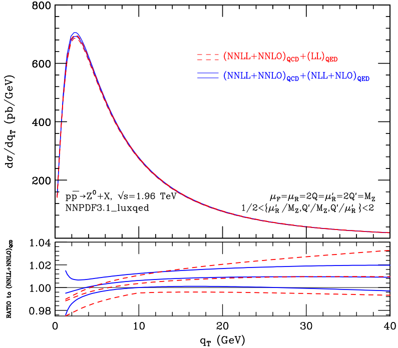
Theoretical predictions encoded in DYTurbo used for simulate W and Z boson events.



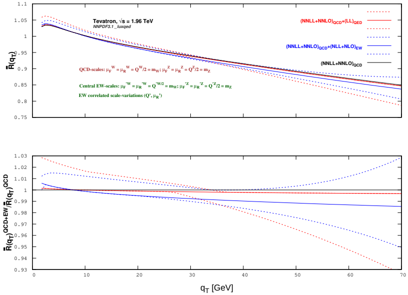
Z production at the LHC [LHCb Coll. ('22)]. LHCb data and Z q_T distribution for the different candidate models compared with LHCb data.

Combined QED and QCD q_T resummation for Z and W prod.

[Cieri,G.F.,Sborlini ('18)] [Autieri,Cieri,G.F.,Sborlini (in preparation)]



Z qT spectrum at the LHC. NNLL+NNLO QCD results combined with the LL (red dashed) and NLL+NLO (blue solid) QED effects together with the corresponding QED uncertainty bands.



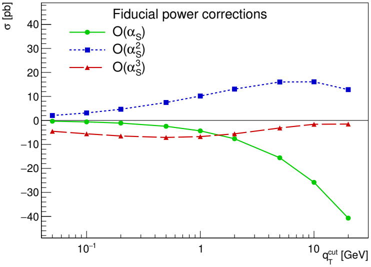
Ratio of the (normalized) W and Z q_T distrubution: NNLL+NNLO QCD results (black) combined with LL QED and NLL+NLO EW.

Conclusions

- To fully exploit the information contained in the experimental data, and **to increase the LHC discovery power**, precise theoretical predictions are necessary $\Rightarrow$ computation of higher-order pQCD corrections.
- Discussed formalisms necessary to perform fixed-order and q_T resummed predictions up to N^{3}LL+N^{3}LO and presented results for Drell–Yan production at the LHC.
- Presented a method to remove linear fiducial power corrections within the q_T-subtraction formalism.
- Computations encoded in the fast and numerically precise publicly available code `DYTurbo`:
  https://dyturbo.hepforge.org

Back up slides

Fiducial power corrections up to N^{3}LO

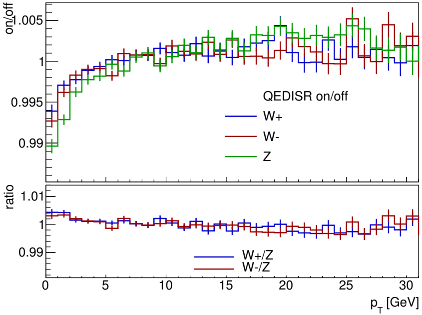


- Flip sign of the FPC with order. Alternating-sign "unphysical" factorial growth of the FO expansion due to symmetric cuts [Salam,Slade('21)].
- Unphysical behaviour can be removed within resummed perturbative predictions. *However* the goal of having precise FO calculations is very relevant.
- No reduction of FPC with higher orders. At N^{3}LO with $q_T^{cut} = 0.05\,\text{GeV}$ -0.4% (+0.3% from α_S^2 and a -0.7% α_S^3).
- Our method is crucial when *local* calculations are not available or when large numerical uncertainties are associated to the $q_T \to 0$ limit (e.g. at N^3LO).

Combining QED and QCD q_T resummation

LHC measurements for DY process sensitive to pure QED and mixed QCD-QED effects.

Pythia 8 QED ISR



October 2, 2017 Stefano Camarda 6

Combining QED and QCD q_T resummation

[Cieri,G.F.,Sborlini('18)]

We start considering QED contributions to the q_T spectrum in the case of colourless and **neutral** high mass systems, e.g. on-shell Z boson production

$$h_1 + h_2 \;\to\; Z^0 + X$$

In the impact parameter and Mellin spaces resummed partonic cross section reads:

$$\mathcal{W}_N(b,M) = \hat{\sigma}^{(0)}\,\mathcal{H}'_N(\alpha_S,\alpha) \times \exp\left\{\mathcal{G}'_N(\alpha_S,\alpha,L)\right\}$$

$$\mathcal{G}'(\alpha_S,\alpha,L) = \mathcal{G}(\alpha_S,L) + L\,g'^{(1)}(\alpha L) + g'^{(2)}(\alpha L) + \sum_{n=3}^{\infty}\left(\frac{\alpha}{\pi}\right)^{n-2} g'^{(n)}(\alpha L)$$

$$+\; g'^{(1,1)}(\alpha_S L,\alpha L) + \sum_{\substack{n,m=1\\ n+m\neq 2}}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^{n-1}\left(\frac{\alpha}{\pi}\right)^{m-1} g_N'^{(n,m)}(\alpha_S L,\alpha L)$$

$$\mathcal{H}'(\alpha_S,\alpha) \quad = \quad \mathcal{H}(\alpha_S) + \frac{\alpha}{\pi}\mathcal{H}'^{(1)} + \sum_{n=2}^{\infty}\left(\frac{\alpha}{\pi}\right)^{n}\mathcal{H}_N'^{(n)} + \sum_{n,m=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^{n}\left(\frac{\alpha}{\pi}\right)^{m}\mathcal{H}_N'^{F\,(n,m)}$$

LL QED ($\sim\alpha^n L^{n+1}$): $g'^{(1)}$; NLL QED ($\sim\alpha^n L^n$): $g'^{(2)}$, $\mathcal{H}'^{(1)}$;
LL mixed QCD-QED ($\sim\alpha_S^n\alpha^n L^{2n}$): $g'^{(1,1)}$;

The LL and NLL QED functions $g'^{(1)}$ and $g'^{(2)}$ has the same *functional* form of the QCD ones:

$$g'^{(1)}(\alpha L) = \frac{A_q'^{(1)}}{\beta_0'} \frac{\lambda' + \ln(1-\lambda')}{\lambda'} \quad ,$$

$$g_N'^{(2)}(\alpha L) = \frac{\widetilde{B}_{q,N}'^{(1)}}{\beta_0'} \ln(1-\lambda') - \frac{A_q'^{(2)}}{\beta_0'^2} \left(\frac{\lambda'}{1-\lambda'} + \ln(1-\lambda') \right) + \frac{A_q'^{(1)} \beta_1'}{\beta_0'^3} \left(\frac{1}{2} \ln^2(1-\lambda') + \frac{\ln(1-\lambda')}{1-\lambda'} + \frac{\lambda'}{1-\lambda'} \right) \; ,$$

the *novel* LL mixed QCD-QED function reads:

$$g'^{(1,1)}(\alpha_S L, \alpha L) = \frac{A_q^{(1)} \beta_{0,1}}{\beta_0^2 \beta_0'} h(\lambda, \lambda') + \frac{A_q'^{(1)} \beta_{0,1}'}{\beta_0'^2 \beta_0} h(\lambda', \lambda) \quad ,$$

$$h(\lambda, \lambda') = -\frac{\lambda'}{\lambda - \lambda'} \ln(1-\lambda) + \ln(1-\lambda') \left[\frac{\lambda(1-\lambda')}{(1-\lambda)(\lambda-\lambda')} + \ln\left(\frac{-\lambda'(1-\lambda)}{\lambda-\lambda'} \right) \right] - \mathrm{Li}_2\left(\frac{\lambda}{\lambda-\lambda'} \right) + \mathrm{Li}_2\left(\frac{\lambda(1-\lambda')}{\lambda-\lambda'} \right) ,$$

where $\lambda = \frac{1}{\pi} \beta_0 \, \alpha_S \, L$, $\lambda' = \frac{1}{\pi} \beta_0' \, \alpha \, L$, and β_0, β_0', β_1', $\beta_{0,1}$, $\beta_{0,1}'$ are the coefficients of the QCD and QED β functions.

Abelianization procedure

$$\frac{d\ln\alpha_S(\mu^2)}{d\ln\mu^2} = \beta(\alpha_S(\mu^2),\alpha(\mu^2)) = -\sum_{n=0}^{\infty}\beta_n\left(\frac{\alpha_S}{\pi}\right)^{n+1} - \sum_{n,m+1=0}^{\infty}\beta_{n,m}\left(\frac{\alpha_S}{\pi}\right)^{n+1}\left(\frac{\alpha}{\pi}\right)^{m},$$

$$\frac{d\ln\alpha(\mu^2)}{d\ln\mu^2} = \beta'(\alpha(\mu^2),\alpha_S(\mu^2)) = -\sum_{n=0}^{\infty}\beta'_n\left(\frac{\alpha}{\pi}\right)^{n+1} - \sum_{n,m+1=0}^{\infty}\beta'_{n,m}\left(\frac{\alpha}{\pi}\right)^{n+1}\left(\frac{\alpha_S}{\pi}\right)^{m}.$$

Novel QED coefficients obtained through an Abelianization algorithm

$$A_q'^{(1)} = e_q^2\,, \qquad A_q'^{(2)} = -\frac{5}{9}\,e_q^2\,N^{(2)} \qquad \widetilde{B}_{q,N}'^{(1)} = B_q'^{(1)} + 2\gamma_{qq,N}'^{(1)}\,,$$

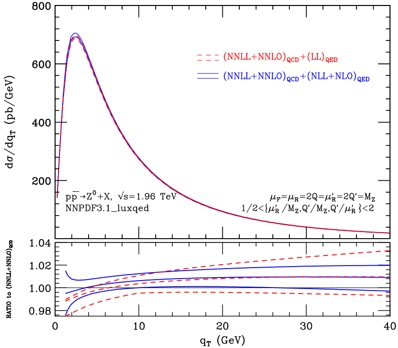
$$\text{with} \quad B_q'^{(1)} = -\frac{3}{2}\,e_q^2\,, \quad N^{(n)} = N_c\sum_{q=1}^{n_f}e_q^n + \sum_{l=1}^{n_l}e_l^n\,,$$

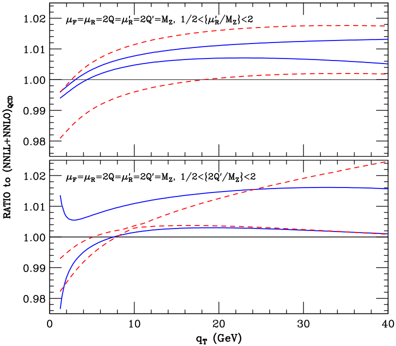
$$\gamma_{qq,N}'^{(1)} = e_q^2\left(\frac{3}{4} + \frac{1}{2N(N+1)} - \gamma_E - \psi_0(N+1)\right)\,, \quad \gamma_{q\gamma,N}'^{(1)} = \frac{3}{2}\,e_q^2\,\frac{N^2+N+2}{N(N+1)(N+2)}\,.$$

$$\mathcal{H}_{q\bar{q}\leftarrow q\bar{q},N}'^{F\,(1)} = \frac{e_q^2}{2}\left(\frac{2}{N(N+1)} - 8 + \pi^2\right)\,, \quad \mathcal{H}_{q\bar{q}\leftarrow \gamma q,N}'^{F\,(1)} = \frac{3\,e_q^2}{(N+1)(N+2)}\,,$$

Resummed result *matched* with corresponding finite $\mathcal{O}(\alpha)$ term.

Combined QED and QCD q_T resummation for Z production at the Tevatron [Cieri,G.F.,Sborlini ('18)]

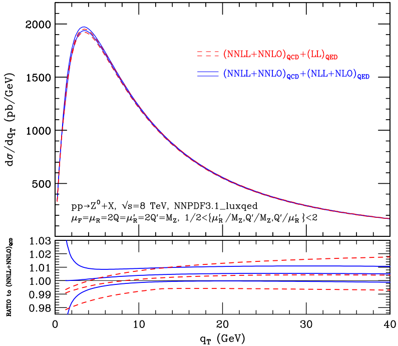


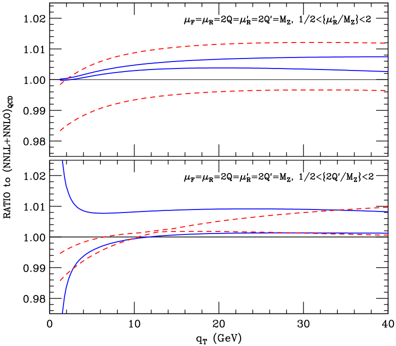


Z qT spectrum at the LHC. NNLL+NNLO QCD results combined with the LL (red dashed) and NLL+NLO (blue solid) QED effects together with the corresponding QED uncertainty bands.

Ratio of the resummation (upper panel) and renormalization (lower panel) QED scale-dependent results with respect to the central value NNLL+NNLO QCD result.

Combined QED and QCD q_T resummation for Z production at the LHC [Cieri,G.F.,Sborlini ('18)]





Z qT spectrum at the LHC. NNLL+NNLO QCD results combined with the LL (red dashed) and NLL+NLO (blue solid) QED effects together with the corresponding QED uncertainty bands.

Ratio of the resummation (upper panel) and renormalization (lower panel) QED scale-dependent results with respect to the central value NNLL+NNLO QCD result.

Combining QED and QCD q_T resummation for W production

[Cieri,G.F.,Sborlini (in preparation)]

We next consider QED contributions to the q_T spectrum in the case of colourless and **charged** high mass systems, e.g. on-shell $W^{\pm}$ boson production

$$h_1 + h_2 \;\rightarrow\; W^{\pm} + X$$

- Initial state QED emissions sensitive to different quark charges ($q\bar{q}' \rightarrow W^{\pm}$):

$$2e_q^2 \rightarrow e_q^2 + e_{\bar{q}'}^2$$

- Final state QED emissions: *abelianizion* of QCD resummation formula q_T resummation for $t\bar{t}$ production [Catani,Grazzini,Torre('14)]:

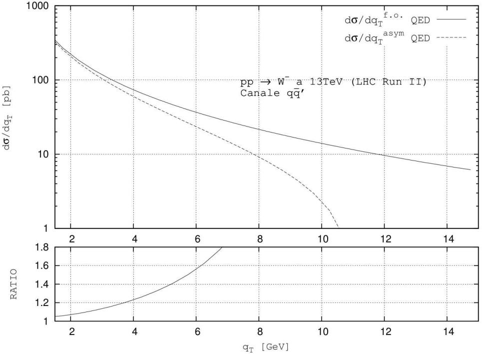
$$\Delta'(b, M) = \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} D'(\alpha(q^2)) \right\}$$

$$\text{with} \quad D'(\alpha) = \sum_{n=1}^{\infty} \left(\frac{\alpha}{\pi}\right)^n D'^{(n)}, \qquad \text{and} \qquad D'^{(1)} = -\frac{e^2}{2}\,.$$

- Factor $\Delta'(b, M)$ resums soft (non collinear) QED emissions from final state (and from initial-final interference). Effects from $D'(\alpha)$ start to contribute at NLL. Same functional dependence, in terms of $g'^{(i)}$ functions, as the $B'(\alpha)$ term.

Combined QED and QCD q_T resummation for W production at the LHC

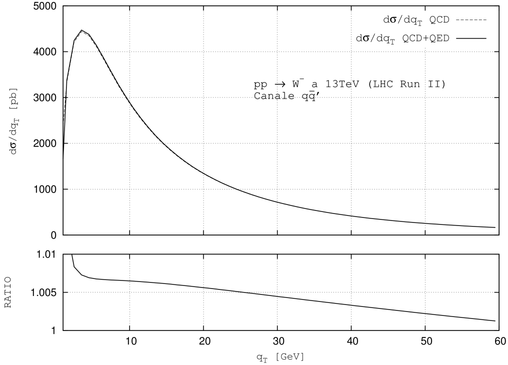
[S.Rota (degree thesis '18)]



W qT spectrum at the LHC (13 TeV). $\mathcal{O}(\alpha)$ fixed-order QED results compared with the asymptotic expansion of the resummed result.

Combined QED and QCD q_T resummation for W production at the LHC

[S.Rota (degree thesis '18)]



W qT spectrum at the LHC. NNLL QCD results combined with the NLL QED effects.

Universality in q_T resummation

The resummation formula is invariant under the *resummation scheme* transformations [Catani, de Florian, Grazzini('01)] (for $h_c(\alpha_S) = 1 + \sum_{n=1}^{\infty} \alpha_S^n h_c^{(n)}$):

$$
\begin{aligned}
H_c^F(\alpha_S) &\rightarrow H_c^F(\alpha_S)\, [\, h_c(\alpha_S)\,]^{-1} \,, \\
B_c(\alpha_S) &\rightarrow B_c(\alpha_S) - \beta(\alpha_S)\, \frac{d \ln h_c(\alpha_S)}{d \ln \alpha_S} \,, \\
C_{cb}(z, \alpha_S) &\rightarrow C_{cb}(z, \alpha_S)\, [\, h_c(\alpha_S)\,]^{1/2} \,.
\end{aligned}
$$

- This implies that H_c^F, S_c (B_c) and C_{cb} not unambiguously computable separately.
- Resummation scheme: define H_c^F (or C_{ab}) for *single* processes (one for $q\bar{q} \rightarrow F$ one for $gg \rightarrow F$) and unambiguously determine the process-dependent H_c^F and the universal (process-independent) S_c and C_{ab} for any other process.
- *DY/H resummation scheme:* $H_q^{DY}(\alpha_S) \equiv 1\,,\ \ H_g^H(\alpha_S) \equiv 1\,.$
  Hard resummation scheme: $C_{ab}^{(n)}(z)$ for $n \geq 1$ do not contain any $\delta(1-z)$ term (other than plus distributions).
- $H_c^F(\alpha_S) = 1$ (i.e. $h_c(\alpha_S) = H_c^F(\alpha_S)$) *does not* correspond to a resummation scheme (S_c^F and C_{ab}^F would be process dependent, [de Florian, Grazzini('00)]).

Universality in q_T resummation

The resummation formula is invariant under the *resummation scheme* transformations [Catani,de Florian,Grazzini('01)] (for $h_c(\alpha_S) = 1 + \sum_{n=1}^{\infty} \alpha_S^n h_c^{(n)}$):

$$
\begin{aligned}
H_c^F(\alpha_S) &\rightarrow H_c^F(\alpha_S)\,[\,h_c(\alpha_S)\,]^{-1}\ , \\
B_c(\alpha_S) &\rightarrow B_c(\alpha_S) - \beta(\alpha_S)\,\frac{d\ln h_c(\alpha_S)}{d\ln \alpha_S}\ , \\
C_{cb}(z,\alpha_S) &\rightarrow C_{cb}(z,\alpha_S)\,[\,h_c(\alpha_S)\,]^{1/2}\ .
\end{aligned}
$$

- This implies that H_c^F, S_c (B_c) and C_{cb} not unambiguously computable separately.
- Resummation scheme: define H_c^F (or C_{ab}) for *single* processes (one for $q\bar{q} \rightarrow F$ one for $gg \rightarrow F$) and unambiguously determine the process-dependent H_c^F and the universal (process-independent) S_c and C_{ab} for any other process.
- *DY/H resummation scheme:* $H_q^{DY}(\alpha_S) \equiv 1\,,\ \ H_g^{H}(\alpha_S) \equiv 1\,.$
  Hard resummation scheme: $C_{ab}^{(n)}(z)$ for $n \geq 1$ do not contain any $\delta(1-z)$ term (other than plus distributions).
- $H_c^F(\alpha_S) = 1$ (i.e. $h_c(\alpha_S) = H_c^F(\alpha_S)$) *does not* correspond to a resummation scheme (S_c^F and C_{ab}^F would be process dependent, [de Florian,Grazzini('00)]).

Universality in q_T resummation

The resummation formula is invariant under the *resummation scheme* transformations [Catani, de Florian, Grazzini ('01)] (for $h_c(\alpha_S) = 1 + \sum_{n=1}^{\infty} \alpha_S^n h_c^{(n)}$):

$$
\begin{aligned}
H_c^F(\alpha_S) &\rightarrow H_c^F(\alpha_S)\,[\,h_c(\alpha_S)\,]^{-1} \;, \\
B_c(\alpha_S) &\rightarrow B_c(\alpha_S) - \beta(\alpha_S)\,\frac{d\ln h_c(\alpha_S)}{d\ln \alpha_S} \;, \\
C_{cb}(z,\alpha_S) &\rightarrow C_{cb}(z,\alpha_S)\,[\,h_c(\alpha_S)\,]^{1/2} \;.
\end{aligned}
$$

- This implies that H_c^F, S_c (B_c) and C_{cb} not unambiguously computable separately.
- Resummation scheme: define H_c^F (or C_{ab}) for *single* processes (one for $q\bar{q} \rightarrow F$ one for $gg \rightarrow F$) and unambiguously determine the process-dependent H_c^F and the universal (process-independent) S_c and C_{ab} for any other process.
- *DY/H resummation scheme:* $H_q^{DY}(\alpha_S) \equiv 1\,, \quad H_g^H(\alpha_S) \equiv 1\,.$
  Hard resummation scheme: $C_{ab}^{(n)}(z)$ for $n \geq 1$ do not contain any $\delta(1-z)$ term (other than plus distributions).
- $H_c^F(\alpha_S) = 1$ (i.e. $h_c(\alpha_S) = H_c^F(\alpha_S)$) *does not* correspond to a resummation scheme (S_c^F and C_{ab}^F would be process dependent, [de Florian, Grazzini ('00)]).

Universality of hard factors at all orders

[Catani,Cieri,de Florian,G.F.,Grazzini('14)]

- *Process-dependence* is fully encoded in the hard-virtual factor $H_c^F(\alpha_S)$.
- However $H_c^F(\alpha_S)$ has an *all-order universal* structure: it can be directly related to the virtual amplitude of the corresponding process $c(\hat{p}_1) + \bar{c}(\hat{p}_2) \to F(\{q_i\})$.

$$\mathcal{M}_{c\bar{c}\to F}(\hat{p}_1, \hat{p}_2; \{q_i\}) = \alpha_S^k \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{2\pi}\right)^n \mathcal{M}^{(n)}_{c\bar{c}\to F}(\hat{p}_1, \hat{p}_2; \{q_i\}) ,$$

renormalized virtual amplitude (UV finite but IR divergent).

$$\tilde{I}_c(\epsilon, M^2) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{2\pi}\right)^n \tilde{I}_c^{(n)}(\epsilon) ,$$

IR subtraction *universal* operators (contain IR ϵ-poles and IR finite terms)

$$\widetilde{\mathcal{M}}_{c\bar{c}\to F}(\hat{p}_1, \hat{p}_2; \{q_i\}) = \left[1 - \tilde{I}_c(\epsilon, M^2)\right] \mathcal{M}_{c\bar{c}\to F}(\hat{p}_1, \hat{p}_2; \{q_i\}) ,$$

hard-virtual subtracted amplitude (IR finite).

Hard factor is directly related to the all-loop virtual amplitude:

$$\alpha_S^{2k}(M^2)\, H_q^F(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\to F}(x_1 p_1, x_2 p_2; \{q_i\})|^2}{|\mathcal{M}^{(0)}_{q\bar{q}\to F}(x_1 p_1, x_2 p_2; \{q_i\})|^2} ,$$

This formula extended up to N3LL in the case of threshold resummation.

Universality of hard factors at all orders

[Catani,Cieri,de Florian,G.F.,Grazzini('14)]

- *Process-dependence* is fully encoded in the hard-virtual factor $H_c^F(\alpha_S)$.
- However $H_c^F(\alpha_S)$ has an *all-order universal* structure: it can be directly related to the virtual amplitude of the corresponding process $c(\hat{p}_1) + \bar{c}(\hat{p}_2) \to F(\{q_i\})$.

$$\mathcal{M}_{c\bar{c}\to F}(\hat{p}_1, \hat{p}_2; \{q_i\}) = \alpha_S^k \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{2\pi}\right)^n \mathcal{M}^{(n)}_{c\bar{c}\to F}(\hat{p}_1, \hat{p}_2; \{q_i\}),$$

renormalized virtual amplitude (UV finite but IR divergent).

$$\tilde{I}_c(\epsilon, M^2) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{2\pi}\right)^n \tilde{I}_c^{(n)}(\epsilon),$$

IR subtraction *universal* operators (contain IR ϵ-poles and IR finite terms)

$$\widetilde{\mathcal{M}}_{c\bar{c}\to F}(\hat{p}_1, \hat{p}_2; \{q_i\}) = \left[1 - \tilde{I}_c(\epsilon, M^2)\right] \mathcal{M}_{c\bar{c}\to F}(\hat{p}_1, \hat{p}_2; \{q_i\}),$$

hard-virtual subtracted amplitude (IR finite).

Hard factor is directly related to the all-loop virtual amplitude:

$$\alpha_S^{2k}(M^2)\, H_q^F(x_1p_1, x_2p_2; \Omega; \alpha_S(M^2)) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\to F}(x_1p_1, x_2p_2; \{q_i\})|^2}{|\mathcal{M}^{(0)}_{q\bar{q}\to F}(x_1p_1, x_2p_2; \{q_i\})|^2},$$

This formula extended up to N3LL in the case of threshold resummation.

Universality of hard factors at all orders

[Catani,Cieri,de Florian,G.F.,Grazzini('14)]

- *Process-dependence* is fully encoded in the hard-virtual factor $H_c^F(\alpha_S)$.
- However $H_c^F(\alpha_S)$ has an *all-order universal* structure: it can be directly related to the virtual amplitude of the corresponding process $c(\hat{p}_1)+\bar{c}(\hat{p}_2)\to F(\{q_i\})$.

$$\mathcal{M}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\}) = \alpha_S^k \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{2\pi}\right)^n \mathcal{M}^{(n)}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\}) \,,$$

renormalized virtual amplitude (UV finite but IR divergent).

$$\tilde{I}_c(\epsilon, M^2) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{2\pi}\right)^n \tilde{I}_c^{(n)}(\epsilon) \,,$$

IR subtraction *universal* operators (contain IR ϵ-poles and IR finite terms)

$$\widetilde{\mathcal{M}}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\}) = \left[1-\tilde{I}_c(\epsilon,M^2)\right] \mathcal{M}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\}) \,,$$

hard-virtual subtracted amplitude (IR finite).

Hard factor is directly related to the all-loop virtual amplitude:

$$\alpha_S^{2k}(M^2)\, H_q^F(x_1p_1,x_2p_2;\Omega;\alpha_S(M^2)) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\to F}(x_1p_1,x_2p_2;\{q_i\})|^2}{|\mathcal{M}^{(0)}_{q\bar{q}\to F}(x_1p_1,x_2p_2;\{q_i\})|^2} \,,$$

This formula extended up to N3LL in the case of threshold resummation.

Universality of hard factors at all orders

[Catani,Cieri,de Florian,G.F.,Grazzini('14)]

- *Process-dependence* is fully encoded in the hard-virtual factor $H_c^F(\alpha_S)$.
- However $H_c^F(\alpha_S)$ has an *all-order universal* structure: it can be directly related to the virtual amplitude of the corresponding process $c(\hat{p}_1)+\bar{c}(\hat{p}_2)\to F(\{q_i\})$.

$$\mathcal{M}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\}) = \alpha_S^k \sum_{n=0}^{\infty}\left(\frac{\alpha_S}{2\pi}\right)^n \mathcal{M}^{(n)}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\}),$$

renormalized virtual amplitude (UV finite but IR divergent).

$$\tilde{I}_c(\epsilon, M^2) = \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{2\pi}\right)^n \tilde{I}_c^{(n)}(\epsilon),$$

IR subtraction *universal* operators (contain IR ϵ-poles and IR finite terms)

$$\widetilde{\mathcal{M}}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\}) = \left[1-\tilde{I}_c(\epsilon, M^2)\right]\mathcal{M}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\}),$$

hard-virtual subtracted amplitude (IR finite).

Hard factor is directly related to the all-loop virtual amplitude:

$$\alpha_S^{2k}(M^2)\, H_q^F(x_1p_1,x_2p_2;\Omega;\alpha_S(M^2)) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\to F}(x_1p_1,x_2p_2;\{q_i\})|^2}{|\mathcal{M}^{(0)}_{q\bar{q}\to F}(x_1p_1,x_2p_2;\{q_i\})|^2},$$

This formula extended up to N3LL in the case of threshold resummation.

Universality of hard factors at all orders

[Catani,Cieri,de Florian,G.F.,Grazzini('14)]

- *Process-dependence* is fully encoded in the hard-virtual factor $H_c^F(\alpha_S)$.
- However $H_c^F(\alpha_S)$ has an *all-order universal* structure: it can be directly related to the virtual amplitude of the corresponding process $c(\hat{p}_1) + \bar{c}(\hat{p}_2) \to F(\{q_i\})$.

$$\mathcal{M}_{c\bar{c}\to F}(\hat{p}_1, \hat{p}_2; \{q_i\}) = \alpha_S^k \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{2\pi}\right)^n \mathcal{M}^{(n)}_{c\bar{c}\to F}(\hat{p}_1, \hat{p}_2; \{q_i\}) \, ,$$

renormalized virtual amplitude (UV finite but IR divergent).

$$\tilde{I}_c(\epsilon, M^2) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{2\pi}\right)^n \tilde{I}_c^{(n)}(\epsilon) \, ,$$

IR subtraction *universal* operators (contain IR ϵ-poles and IR finite terms)

$$\widetilde{\mathcal{M}}_{c\bar{c}\to F}(\hat{p}_1, \hat{p}_2; \{q_i\}) = \left[1 - \tilde{I}_c(\epsilon, M^2)\right] \mathcal{M}_{c\bar{c}\to F}(\hat{p}_1, \hat{p}_2; \{q_i\}) \quad ,$$

hard-virtual subtracted amplitude (IR finite).

Hard factor is directly related to the all-loop virtual amplitude:

$$\alpha_S^{2k}(M^2)\, H_q^F(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\to F}(x_1 p_1, x_2 p_2; \{q_i\})|^2}{|\mathcal{M}^{(0)}_{q\bar{q}\to F}(x_1 p_1, x_2 p_2; \{q_i\})|^2} \, ,$$

This formula extended up to N3LL in the case of threshold resummation.

Soft gluon exponentiation

Sudakov resummation feasible when:
dynamics AND kinematics factorize
$\Rightarrow$ exponentiation.

- Dynamics factorization: general propriety of QCD matrix element for soft emissions.

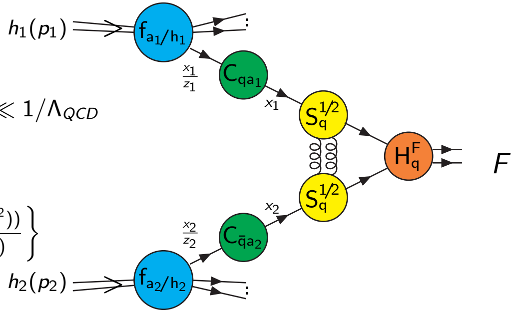
$$dw_n(q_1,\ldots,q_n) \simeq \frac{1}{n!}\prod_{i=1}^{n} dw_i(q_i)$$

- Kinematics factorization: not valid in general. For q_T distribution it holds in the impact parameter space (Fourier transform)

$$\int d^2\mathrm{q_T}\ \exp(-i\mathrm{b}\cdot\mathrm{q_T})\,\delta^{(2)}\left(\mathrm{q_T}-\sum_{j=1}^{n}\mathrm{q_{T_j}}\right) = \exp(-i\mathrm{b}\cdot\sum_{j=1}^{n}\mathrm{q_{T_j}}) = \prod_{j=1}^{n}\exp(-i\mathrm{b}\cdot\mathrm{q_{T_j}})\,.$$

- Exponentiation holds in the impact parameter space. Results have then to be transformed back to the physical space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$.

Transverse-momentum resummation formula



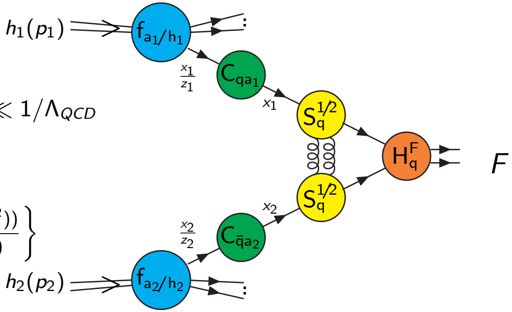
$$M \gg \Lambda_{QCD}\ , \quad b \gg 1/M\ , \quad b \ll 1/\Lambda_{QCD}$$

$$C(\alpha_S(b_0^2/b^2)) = C(\alpha_S(M^2)) \times \ \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2}\ \beta(\alpha_S(q^2))\ \frac{d\ln C(\alpha_S(q^2))}{d\ln\alpha_S(q^2)} \right\}$$

$$\frac{d\sigma_F^{(res)}}{d^2\mathbf{q_T}\, dM^2\, dy\, d\Omega} = \frac{M^2}{s}\left[d\sigma^{(0)}_{q\bar{q},F}\right] H_q^F(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) \sum_{a_1,a_2} \int \frac{d^2\mathbf{b}}{(2\pi)^2}\ e^{i\mathbf{b}\cdot\mathbf{q_T}}\ S_q(M,b)$$
$$\times \int_{x_1}^1 \frac{dz_1}{z_1}\ C_{qa_1}(z_1;\alpha_S(b_0^2/b^2))\ f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) \int_{x_2}^1 \frac{dz_2}{z_2}\ C_{\bar{q}\,a_2}(z_2;\alpha_S(b_0^2/b^2))\ f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

$$\tilde{F}_{q_f/h}(x,b,M) = \sum_a \int_x^1 \frac{dz}{z}\ \sqrt{S_q(M,b)}\,C_{q_f\,a}(z;\alpha_S(b_0^2/b^2))\ f_{a/h}(x/z, b_0^2/b^2)$$

Transverse-momentum resummation formula



$$M \gg \Lambda_{QCD}\ , \quad b \gg 1/M\ , \quad b \ll 1/\Lambda_{QCD}$$

$$C(\alpha_S(b_0^2/b^2)) = C(\alpha_S(M^2))$$
$$\times\ \exp\left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2}\ \beta(\alpha_S(q^2))\ \frac{d \ln C(\alpha_S(q^2))}{d \ln \alpha_S(q^2)} \right\}$$

$$\frac{d\sigma_F^{(res)}}{d^2\mathbf{q_T}\ dM^2\ dy\ d\Omega} = \frac{M^2}{s} \left[d\sigma^{(0)}_{q\bar{q},F} \right] H_q^F(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) \sum_{a_1,a_2} \int \frac{d^2\mathrm{b}}{(2\pi)^2}\ e^{i\mathrm{b}\cdot\mathbf{q_T}}\ S_q(M,b)$$
$$\times \int_{x_1}^{1} \frac{dz_1}{z_1}\ C_{qa_1}(z_1; \alpha_S(b_0^2/b^2))\ f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) \int_{x_2}^{1} \frac{dz_2}{z_2}\ C_{\bar{q}\,a_2}(z_2; \alpha_S(b_0^2/b^2))\ f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

$$\tilde{F}_{q_f/h}(x,b,M) = \sum_a \int_x^1 \frac{dz}{z}\ \sqrt{S_q(M,b)} C_{q_f\,a}(z; \alpha_S(b_0^2/b^2))\ f_{a/h}(x/z, b_0^2/b^2)$$

Distinctive features of the formalism [Catani at al ('01)], [Bozzi et al.('03,'06)]:

- Resummed effects exponentiated in a universal of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp\left\{-\int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2}\gamma_N(\alpha_S(q^2))\right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)],[Catani et al.('96)].
- Introduction of resummation scale $Q \sim M$: variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln(M^2 b^2) \to \ln(Q^2 b^2) + \ln(M^2/Q^2)$$

- Perturbative unitarity constraint:

$$\ln(Q^2 b^2) \to \widetilde{L} \equiv \ln(Q^2 b^2 + 1)$$

  - avoids unjustified higher-order contributions in the small-b region.
  - recover *exactly* the total cross-section (upon integration on q_T)

Distinctive features of the formalism [Catani at al ('01)], [Bozzi et al.('03,'06)]:

- Resummed effects exponentiated in a universal of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp\left\{-\int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2}\gamma_N(\alpha_S(q^2))\right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)],[Catani et al.('96)].
- Introduction of resummation scale $Q \sim M$: variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln\left(M^2b^2\right) \to \ln\left(Q^2b^2\right) + \ln\left(M^2/Q^2\right)$$

- Perturbative unitarity constraint:

$$\ln\left(Q^2b^2\right) \to \widetilde{L} \equiv \ln\left(Q^2b^2+1\right)$$

  - avoids unjustified higher-order contributions in the small-b region.
  - recover *exactly* the total cross-section (upon integration on q_T)

Distinctive features of the formalism [Catani at al ('01)], [Bozzi et al.('03,'06)]:

- Resummed effects exponentiated in a universal of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp\left\{-\int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2}\,\gamma_N(\alpha_S(q^2))\right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)],[Catani et al.('96)].
- Introduction of resummation scale $Q \sim M$: variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln(M^2 b^2) \rightarrow \ln(Q^2 b^2) + \ln(M^2/Q^2)$$

- Perturbative unitarity constraint:

$$\ln(Q^2 b^2) \rightarrow \widetilde{L} \equiv \ln(Q^2 b^2 + 1)$$

  - avoids unjustified higher-order contributions in the small-b region.
  - recover *exactly* the total cross-section (upon integration on q_T)

Distinctive features of the formalism [Catani at al ('01)], [Bozzi et al.('03,'06)]:

- Resummed effects exponentiated in a universal of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp\left\{-\int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2}\,\gamma_N(\alpha_S(q^2))\right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)],[Catani et al.('96)].
- Introduction of resummation scale $Q \sim M$: variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln(M^2 b^2) \rightarrow \ln(Q^2 b^2) + \ln(M^2/Q^2)$$

- Perturbative unitarity constraint:

$$\ln(Q^2 b^2) \rightarrow \widetilde{L} \equiv \ln(Q^2 b^2 + 1)$$

  - avoids unjustified higher-order contributions in the small-b region.
  - recover *exactly* the total cross-section (upon integration on q_T)

Distinctive features of the formalism [Catani at al ('01)], [Bozzi et al.('03,'06)]:

- Resummed effects exponentiated in a universal of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp\left\{-\int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2}\gamma_N(\alpha_S(q^2))\right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)],[Catani et al.('96)].
- Introduction of resummation scale $Q \sim M$: variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln(M^2 b^2) \to \ln(Q^2 b^2) + \ln(M^2/Q^2)$$

- Perturbative unitarity constraint:

$$\ln(Q^2 b^2) \to \widetilde{L} \equiv \ln(Q^2 b^2 + 1)$$

  - avoids unjustified higher-order contributions in the small-b region.
  - recover *exactly* the total cross-section (upon integration on q_T)

Distinctive features of the formalism [Catani at al ('01)], [Bozzi et al.('03,'06)]:

- Resummed effects exponentiated in a universal of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp\left\{ -\int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2}\gamma_N(\alpha_S(q^2))\right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)],[Catani et al.('96)].
- Introduction of resummation scale $Q \sim M$: variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln(M^2b^2) \to \ln(Q^2b^2) + \ln(M^2/Q^2)$$

- Perturbative unitarity constraint:

$$\ln(Q^2b^2) \to \widetilde{L} \equiv \ln(Q^2b^2+1)$$

  - avoids unjustified higher-order contributions in the small-b region.
  - recover *exactly* the total cross-section (upon integration on q_T)

Distinctive features of the formalism [Catani at al ('01)], [Bozzi et al.('03,'06)]:

- Resummed effects exponentiated in a universal of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp\left\{-\int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2}\gamma_N(\alpha_S(q^2))\right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)],[Catani et al.('96)].
- Introduction of resummation scale $Q \sim M$: variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln(M^2b^2) \rightarrow \ln(Q^2b^2) + \ln(M^2/Q^2)$$

- Perturbative unitarity constraint:

$$\ln(Q^2b^2) \rightarrow \widetilde{L} \equiv \ln(Q^2b^2+1) \Rightarrow \exp\{\alpha_S^n \widetilde{L}^k\}\big|_{b=0} = 1$$

  - avoids unjustified higher-order contributions in the small-b region.
  - recover *exactly* the total cross-section (upon integration on q_T)

Distinctive features of the formalism [Catani at al ('01)], [Bozzi et al.('03,'06)]:

- Resummed effects exponentiated in a universal of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp\left\{-\int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2}\,\gamma_N(\alpha_S(q^2))\right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)],[Catani et al.('96)].
- Introduction of resummation scale $Q \sim M$: variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln(M^2 b^2) \rightarrow \ln(Q^2 b^2) + \ln(M^2/Q^2)$$

- Perturbative unitarity constraint:

$$\ln(Q^2 b^2) \rightarrow \widetilde{L} \equiv \ln(Q^2 b^2 + 1) \quad \Rightarrow \quad \exp\{\alpha_S^n \widetilde{L}^k\}\big|_{b=0} = 1 \;\Rightarrow\; \int_0^\infty dq_T^2 \left(\frac{d\hat{\sigma}}{dq_T^2}\right) = \hat{\sigma}^{(tot)};$$

  - avoids unjustified higher-order contributions in the small-b region.
  - recover *exactly* the total cross-section (upon integration on q_T)