Linear Power Corrections in Collider Processes

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Works done in collaboration with Fabrizio Caola, Silvia Ferrario Ravasio, Kirill Melnikov, Paolo Nason, Melih A. Ozcelik

Introduction

- High precision era for Large Hadron Collider (LHC) physics
- The current absence of New Physics signals requires theoretical computations with high accuracy
- Need an input on Non-Perturbative (NP) (hadronization) corrections, scaling as $\mathcal{O}(\Lambda_{\text{QCD}}/Q)^p$, that may easily reach the percent level for theoretical expectations for hardness scales $Q \sim 100$ GeV
- Lack of a full general theory for estimating NP corrections for generic collider observables

Renormalons in QCD

- A generic observable in a renormalizable QFT $\quad D[\alpha] = \sum_{n=0}^{\infty} c_n \alpha^{n+1}$

This series diverges at large orders with factorial growth $\quad c_n = a^n n!$

It needs to be truncated at its minimum value $(n_{min} = 1/(|a|\alpha))$

$$c_{n_{\min}} \alpha^{n_{\min}+1} \simeq \sqrt{\frac{2\pi}{n_{\min}}} e^{-\frac{1}{|a|\alpha}}$$

- The ambiguity takes the form $\left(\beta(\alpha_s) = -b_0 \alpha_s^2 + \ldots, \quad b_0 = \frac{33-2n_f}{12\pi}\right)$

$$e^{-1/(a\alpha_s(Q))} = e^{\log(\Lambda_{\rm QCD}^2/Q^2)^{b_0/a}} = \left(\frac{\Lambda_{\rm QCD}}{Q}\right)^{2b_0/a} = \left(\frac{\Lambda_{\rm QCD}}{Q}\right)^{p}$$

The factorial growth of the perturbative series is related to power corrections

We talk about Infrared linear renormalons $(p = 1)$, arising from the low momentum region

Large-n_f limit

- NP terms need to be extracted with an all-order computation assuming an infinite number of quarks flavors n_f (**Large-n_f limit**)
- We consider the Abelian limit of QCD ($n_f \to -\infty$), decorating each gluon line with fermionic bubbles

$$\frac{-ig^{\mu\nu}}{k^2 + i\eta} \to \frac{-ig^{\mu\nu}}{k^2 + i\eta} \times \frac{1}{1 + \Pi(k^2 + i\eta, \mu^2, \epsilon) - \Pi_{\text{ct}}}$$

$$\Pi(k^2 + i\eta, \mu^2, \epsilon) - \Pi_{\text{ct}} = \alpha_s(\mu)\left(\frac{-n_f T_R}{3\pi}\right)\left[\log\left(\frac{|k^2|}{\mu^2}\right) - i\pi\theta(k^2) - \frac{5}{3}\right]$$

- Terms $(\alpha_s n_f)^k$ fully computable for each k
- At the end of the computation one recovers the non-Abelian limit by replacing

$$n_f \to \frac{11 C_A}{4 T_R} - n_l \quad (n_l \text{ the real number of QCD light flavors})$$

Large-b_0 approximation

Large-n_f limit

- The insertions of the fermion bubbles can be handled by considering radiative QCD corrections computed with a gluon of mass λ
- The average value of a generic IR-safe observable O is written as

$$\langle O \rangle = \langle O \rangle^{(0)} - \frac{1}{\alpha_s} \int \mathrm{d}\lambda \frac{\mathrm{d}\langle O \rangle_\lambda^{(1)}}{\mathrm{d}\lambda} \overbrace{\left[\frac{1}{\pi b_0} \arctan \frac{\pi b_0 \alpha_s}{1 + b_0 \alpha_s \log \lambda^2/\mu_C^2} \right]}^{\text{Beneke,'98}}$$

where

$$\begin{aligned} \langle O \rangle^{(0)} &= \frac{1}{\sigma} \int \mathrm{d}\Phi_B B(\Phi_B) O(\Phi_B) \\ \langle O \rangle_\lambda^{(1)} &= \underbrace{T_V(\lambda) + T_R(\lambda)}_{\text{Virtual and real corrections for a massive gluon}} + \underbrace{T_R^\Delta(\lambda)}_{\text{Nason,Seymour('95)}} \\ T_R^\Delta(\lambda) &= \frac{1}{\sigma} \frac{3\pi}{\alpha_s T_F} \lambda^2 \int \mathrm{d}\Phi_{q\bar{q}} R_{q\bar{q}}(\lambda) \delta(\lambda^2 - m_{q\bar{q}}^2) \left[O(\Phi_{q\bar{q}}) - O(\Phi_{(q\bar{q})}) \right] \end{aligned}$$

- All the logarithmically divergent terms as $\lambda \to 0$ cancel as O is IR-safe
- A linear term in λ in $\langle O \rangle_\lambda^{(1)}$ $\to$ IR linear renormalon

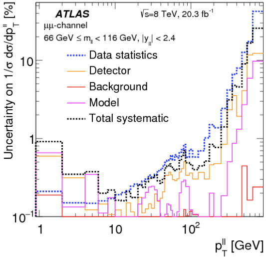
Large-n_f limit in literature

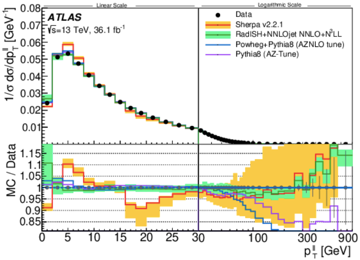
It provides a reliable framework for estimating renormalon corrections

- Beneke, Braun (1995): looking for power corrections in Drell-Yan total cross section, proving that claims about resummation as probe for linear power corrections were unfounded;
- Nason, Seymour (1995): issues about power corrections in shape variables observables;
- Dasgupta (1999): no linear renormalons in the rapidity distributions of DY pair;
- Ferrario Ravasio, Nason, Oleari (2019): leptonic observables in top production and decay are affected by IR linear renormalons;
- Ferrario Ravasio, GL, Nason (2020): absence of IR linear renormalons in the p_T distribution of a Z boson in hadronic collisions, in the large transverse momentum region, irrespective of rapidity cuts;
- Caola, Ferrario Ravasio, GL, Melnikov, Nason (2021): estimate of leading power corrections affecting Shape Variables in the 3-jet region;
- Caola, Ferrario Ravasio, GL, Melnikov, Nason, Ozcelik (2022): fully analytic approach to estimate leading NP corrections affecting Shape Variables in the 3-jet region (See Melih's talk)

The p_T of the Z

- One of the cleanest and best measured LHC observables
- Sub-percent level precision for normalized distributions measured at LHC (ATLAS and CMS ('15,'19))
- Theoretical uncertainties still at the percent level
- $Z + jet$ computed at NNLO in QCD (Boughezal, Campbell et al. ('16), Gehrmann-De Ridder, Gehrmann et al. ('16), Gehrmann-De Ridder et al ('18)). Current state of the art is NNLO + N^{3}LL with a large effect of resummation for small p_T^Z (Bizon et al. ('19))
- Very important implications for constraining α_s and PDFs at LHC (Boughezal et al. ('17))





The p_T of the Z

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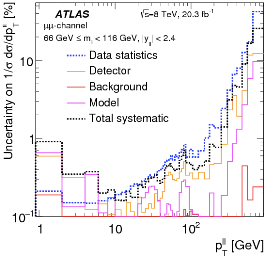
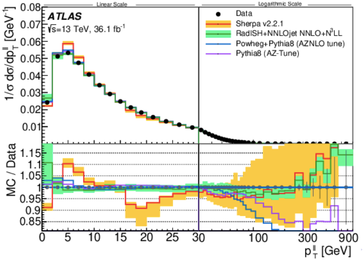
Motivation

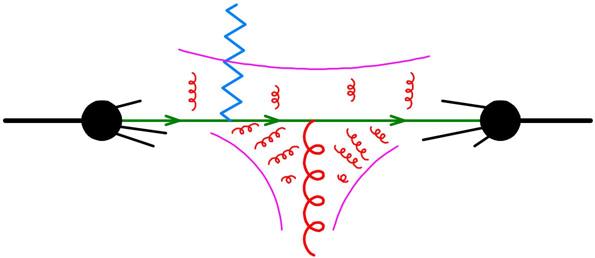
Given the high precision reached for this observable, it is crucial to look for the presence of IR linear renormalons in the moderately large transverse momentum region!

- Very important implications for constraining α_s and PDFs at LHC (Boughezal et al. ('17))





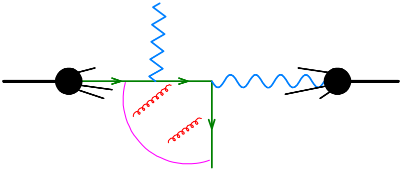
The p_T of the Z: a kinematic argument

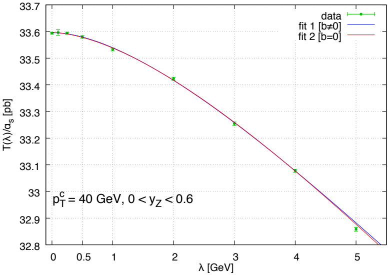


- The soft radiation pattern is not azimuthally symmetric
- A IR linear renormalon is strictly related to soft emissions

If we model a IR linear renormalon as due to the emission of a soft particle with transverse momentum $\sim \Lambda_{\text{QCD}}$, we may assume that it can also affect the p_T^Z by recoil!

- We consider the process $d(p_1)\gamma(p_2) \to Z(p_3)d(p_4)$ to work in the *Large-n_f* limit and to preserve the azimutal color asymmetry ($E_{CM} = 300$ GeV)





We (Ferrario Ravasio, GL, Nason ('20)) found

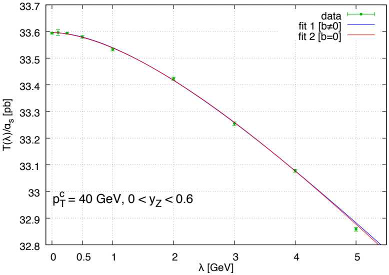
$$\langle O \rangle_\lambda^{(1)} \sim \left(\frac{\lambda}{p_T^c}\right)^2 \log\left(\frac{\lambda}{p_T^c}\right)$$

No numeric evidence of a IR linear renormalon for the transverse momentum of the Z boson!

- We consider the process $d(p_1)\gamma(p_2) \to Z(p_3)d(p_4)$ to

Question

Is it possible to provide an analytic argument about the presence (absence) of linear power corrections?



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Linear Power Corrections: an analytic argument

Question

Given a process involving the emission/exchange of a gluon with mass λ, under which hypotheses do the linear terms in λ appear/disappear?

- We (Caola, Ferrario Ravasio, GL, Melnikov, Nason ('21)) observed:
  1. For processes involving massless particles, virtual corrections cannot give rise to linear power corrections
  2. Evaluation of NLO corrections with a gluon with mass λ
     From collinear configurations we get
     $$\int \frac{\mathrm{d}^2\vec{k}_\perp}{\vec{k}_\perp^2 + \lambda^2} f(\eta, \phi)$$
     If $f(\eta, \phi) \sim e^{-|\eta|}$ after azimuthal integration (Thrust, C-parameter...) we can focus on soft emissions only to extract leading power corrections
  3. Real amplitude evaluated at Next-to-Leading term in k (gluon momentum)

Main Result

No linear terms in λ from an inclusive integration over the radiation phase space!

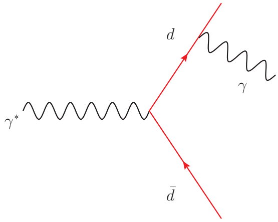
Linear Power Corrections for Shape Variables

- Shape Variables are routinely used to extract reliable values of α_s, from e^+e^- data, thanks to high precision calculations
- Need input on NP (hadronisation) corrections
- Both numeric (Monte Carlo event generator) and analytic approaches for estimating NP corrections
- Analytic NP methods (only valid in the 2-jet limit) give
  1. $\alpha_s = 0.1135 \pm 0.0010$ [1006.3080] from Thrust
  2. $\alpha_s = 0.1123 \pm 0.0015$ [1501.04111] from C-parameter
- Several standard deviations away from the PDG value: $\alpha_s = 0.1179 \pm 0.0010$
- The usual technique consisting of fitting the NP correction in the 2-jet region, and then extrapolating it in the 3-jet region has been proven to be unrealiable for the C-parameter (Luisoni, Monni, Salam ('20))

It is crucial to evaluate NP corrections in the 3-jet region, where α_s fits are performed!

Linear Power Corrections for Shape Variables

- Looking for linear power corrections for Shape Variables in the 3-jet region for the process $\gamma^*(q) \to d(p_1)\bar{d}(p_2)\gamma(p_3)$



- NLO corrections performed with a gluon with mass λ, also considering the splitting $g^*(k) \to q(l_1)\bar{q}(l_2)$
- The phase space can be factorized

$$\mathrm{d}\Phi_{3+2}\delta(\lambda^2 - (l_1 + l_2)^2) = \underbrace{\mathrm{d}\Phi_{3+1}}_{\gamma^* \to d\bar{d}\gamma g^*} \times \underbrace{\mathrm{d}\Phi_{\text{split}}}_{g^* \to q\bar{q}} = \underbrace{\mathrm{d}\Phi_3}_{\gamma^* \to d\bar{d}\gamma} \times \mathrm{d}\Phi_{\text{rad}} \times \mathrm{d}\Phi_{\text{split}}$$

- The mapping $\Phi_{3+1}(p'_{i=1,2,3}, k) \to (\Phi_3(p_{i=1,2,3}), \Phi_{\text{rad}}(k))$ needs to be smooth in k for small k

$$p'^{\mu} = p^{\mu} + K^{\mu}_{\nu}(p)k^{\nu} + \mathcal{O}(k^0)^2$$

Linear Power Corrections for Shape Variables

- For a generic Shape Variable O, vanishing in the 2-jet limit we evaluate the NLO correction

$$\langle O\rangle_\lambda^{(1)} = \frac{1}{\sigma_0}\int d\Phi_3\left\{V_\lambda O_3 + \int d\Phi_{\rm rad} M_{\mu\nu}(k,\lambda)\int d\Phi_{\rm split} P^{\mu\nu}_{\rm split} O_{3+2}\right\}$$

That can be manipulated as

$$\begin{aligned}\langle O\rangle_\lambda^{(1)} &= \frac{1}{\sigma_0}\int d\Phi_3\left\{\int d\Phi_{\rm rad} M_{\mu\nu}(k,\lambda)\overbrace{\left[\int d\Phi_{\rm split} P^{\mu\nu}_{\rm split} O_{3+2} + O_3 g^{\mu\nu}\right]}^{\text{The soft region is suppressed}}\right\}\\ &+ \frac{1}{\sigma_0}\int d\Phi_3\underbrace{\left\{\int d\Phi_{\rm rad} M_{\mu\nu}(k,\lambda)(-g^{\mu\nu}) + V_\lambda\right\}}_{\text{No linear terms from this integration!}} O_3\end{aligned}$$

- The term in the square bracket is suppressed in the soft limit, so we can use only the leading soft approximation for $M_{\mu\nu}$

The leading power correction can be extracted by computing $\langle O\rangle_\lambda^{(1)} - \langle O\rangle_0^{(1)}$

NP correction as a Shift in the Shape Variable

- We consider the cumulative distribution for a generic shape variable O

$$\Sigma(O) = \int_O \mathrm{d}O' \frac{\mathrm{d}\sigma}{\mathrm{d}O'}$$

- Non-Perturbative corrections show up as a shift in the Shape Observable

$$\Sigma^{\mathrm{NP}}(O) \sim \Sigma(O) - \delta O \Sigma^{'}(O) = \Sigma(O) - \delta O \frac{\mathrm{d}\sigma}{\mathrm{d}O}$$

- If the NP Correction is due to the emission of a soft gluon, we can write

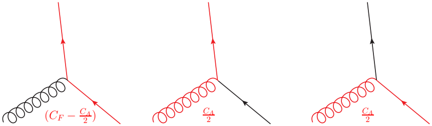
$$\delta O = \alpha_s \frac{\lambda}{Q} h_O \zeta(O)$$

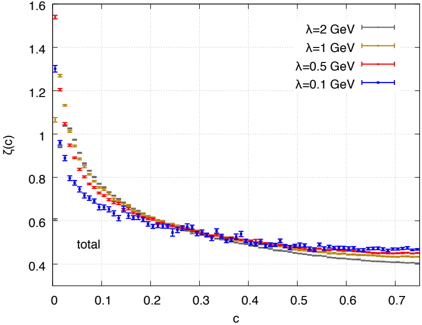
- h_O parametrises the Non-Perturbative correction in the 2-jet region
- $\zeta(O)$ describes the behaviour of the Non-Perturbative correction in the 3-jet region as a function of O

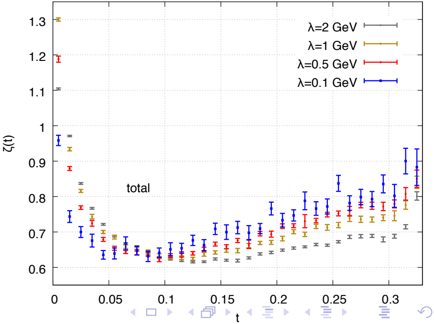
With our method we can easily evaluate the functional form of $\zeta(O)$ in the full phase space!

NP shift in the Shape Variable

- We extended our considerations to the realistic process $\gamma^* \to d\bar{d}g$
- The three contributions are additive
- $\zeta(O) = \frac{C_F - C_A/2}{C_F}\zeta_{q\bar{q}}(O) + \frac{C_A}{C_F}\zeta_{qg}(O)$







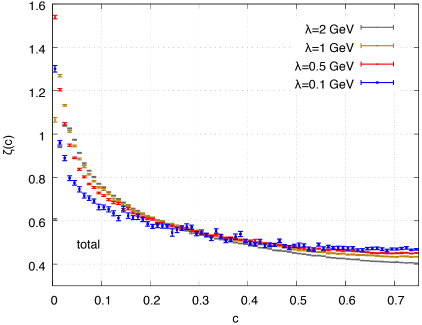
NP shift in the Shape Variable

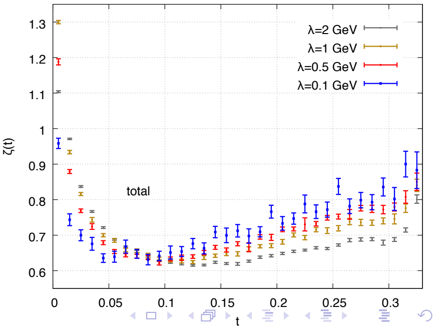
- We extended our

Result

For $\lambda = 0.1$ GeV and $Q = 100$ GeV we find $\zeta(c = 3/4) = 0.479(5)$, in excellent agreement with $\zeta_{\text{LMS}}(c = 3/4) = 0.476$

- $\zeta(O) = \frac{C_F - C_A/2}{C_F}\zeta_{q\bar{q}}(O) + \frac{C_A}{C_F}\zeta_{qg}(O)$





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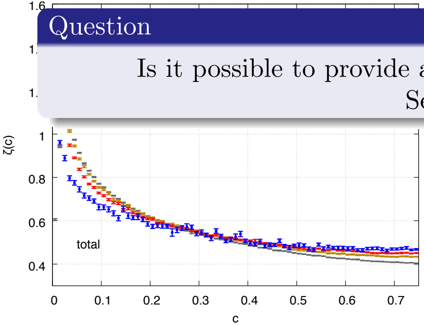
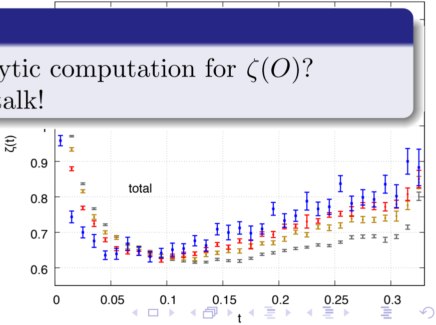




Question

Is it possible to provide a fully analytic computation for $\zeta(O)$?
See Melih's talk!

Conclusions and Outlooks

- Understanding Non-Perturbative corrections to collider processes is now crucial, given the high precision reached at LHC
- Large-n_f method is a reliable framework to investigate $\mathcal{O}(\Lambda_{\rm QCD}/Q)^p$, that can be related to $\mathcal{O}(\lambda)^p$ terms in a computation with a gluon with mass λ
- $\mathcal{O}(\lambda)$ terms in our abelian model without gluons can be exposed using the Next-to-eikonal expansion
- No linear terms if integrating inclusively over the radiation phase space: analytical explanation about the absence of IR linear renormalons in the p_T distribution of the Z boson, in hadronic collisions (Ferrario Ravasio, GL, Nason ('20))
- Simplified model to predict NP corrections for Thrust and C-parameter away from the two-jet region (Caola, Ferrario Ravasio, GL, Melnikov, Nason ('21))
- Next directions: extensions to other observables (e.g. heavy jet mass) and phenomenological applications

THANKS FOR THE ATTENTION!!!

BACKUP

Large-n_f limit and Power Corrections

If $\frac{\mathrm{d}\langle O\rangle_\lambda^{(1)}}{\mathrm{d}\lambda}\Big|_{\lambda=0} = A$ (constant), we study the region $\lambda < \mu_C$

$$-\frac{1}{b_0\alpha_s}\frac{\mathrm{d}\langle O\rangle_\lambda^{(1)}}{\mathrm{d}\lambda}\Big|_{\lambda=0}\int_0^{\mu_C}\frac{\mathrm{d}\lambda}{\pi}\arctan\frac{\pi b_0\alpha_s}{1+b_0\alpha_s\log\frac{\lambda^2}{\mu_C^2}}$$

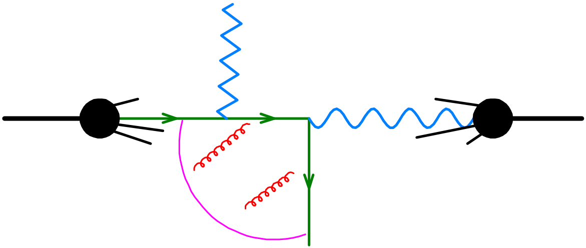
Putting $b_0\alpha_s = a$, $\frac{\lambda}{\mu_C} = l$

$$\int_0^1\frac{\mathrm{d}l}{\pi a}\arctan\frac{\pi a}{1+a\log(l^2)} = \overbrace{\frac{1}{\pi a}\arctan(\pi a)+\int_0^1\mathrm{d}z\frac{\pi a z\cos(\pi z/2)-\sin(\pi z/2)}{1+(z\pi a)^2}}^{\text{Analytic}}$$
$$+\underbrace{\frac{1}{\pi a}\mathrm{P}\int_0^\infty \mathrm{d}t\frac{\exp(-\frac{t}{2a})}{1-t}}_{\text{Borel Integral}}-\underbrace{\boxed{\frac{1}{a}\exp\left(-\frac{1}{2a}\right)}}_{\text{Ambiguity}}$$

- Replacing $a = b_0\alpha_s = 1/\log(\mu_C^2/\Lambda_{\mathrm{QCD}}^2)$ we get

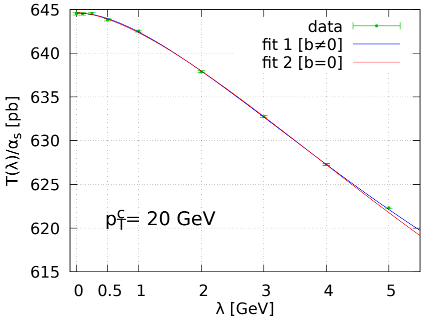
$$\boxed{\exp\left(-\frac{1}{2a}\right) = \frac{\Lambda_{\mathrm{QCD}}}{\mu_C}}$$

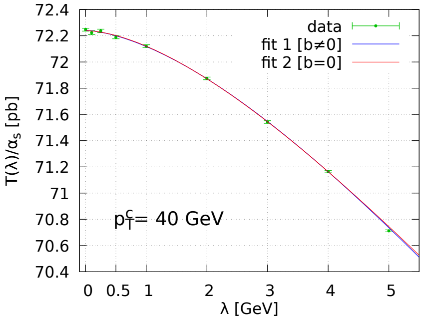
The p_T of the Z: working in the Large-n_f limit



- Evaluation of NLO corrections in α_s using a gluon with non-vanishing mass λ
- IR singularities associated with a soft or collinear gluon are regulated by λ and arise as $\log \lambda, \log^2 \lambda$ as $\lambda \to 0$
- DIS scheme to deal with ISR gluon emitted by the d quark
- Singularity associated with the collinear photon splitting into a $d\bar{d}$ pair handled with POWHEG-BOX
- Numeric integration in the regions regulated by the gluon mass has been performed through a dedicated Fortran code

The p_T of the Z: Results for an inclusive analysis





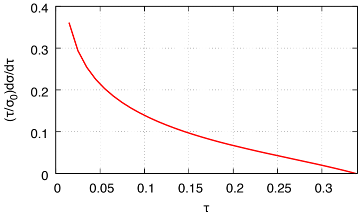
- Fit performed using the function (Excluding $\lambda = 5$ GeV)

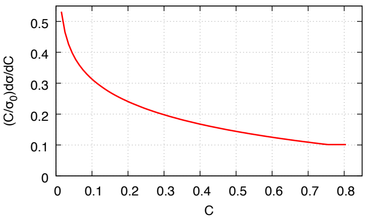
$$f(\lambda) = a\left[1 + b\left(\frac{\lambda}{p_T^c}\right) + c\left(\frac{\lambda}{p_T^c}\right)\log^2\left(\frac{\lambda}{p_T^c}\right) + d\left(\frac{\lambda}{p_T^c}\right)^2\log\left(\frac{\lambda}{p_T^c}\right)\right]$$

- We found $b = 0.009 \pm 0.004$ and $b = 0.024 \pm 0.0017$

Shape Variables: Details of the Computation

- Two shape variables O such that $\frac{\mathrm{d}\sigma}{\mathrm{d}O} = \sigma_0 \delta(O)$ at LO
  1. Thrust $(2/3 < T < 1)$
  $$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i E_i} \rightarrow \begin{cases} \tau = 0 & \text{2-jet region} \\ \tau = 1/3 & \text{3-jet symmetric point} \end{cases}$$
  2. C-parameter $(0 < C < 1)$
  $$3 - \frac{3}{2Q^2} \sum_{i,j} \frac{(p_i \cdot p_j)^2}{E_i E_j} \rightarrow \begin{cases} C = 0 & \text{2-jet region} \\ C = 3/4 & \text{3-jet symmetric point} \end{cases}$$





- The C-parameter has a Sudakov shoulder within the physical range $(C = 3/4)$

- Computation of

$$\langle O \rangle_{\lambda}^{(1)} = T_V(\lambda) + T_R(\lambda) + T_R^{\Delta}(\lambda)$$

  with

$$T_R^{\Delta}(\lambda) = \frac{1}{\sigma_0}\frac{3\pi}{\alpha_s T_F}\lambda^2 \int \mathrm{d}\Phi_{q\bar{q}} R_{q\bar{q}}(\lambda)\delta(\lambda^2 - m_{q\bar{q}}^2)\Big[O(\Phi_{q\bar{q}}) - O(\Phi_{(q\bar{q})})\Big]$$

- The integration diverges in the two-jet limit

$$F_{\mathrm{supp}} = C^2$$

- $T_V(\lambda)$:
  1. IR divergences regulated by the gluon mass λ
  2. UV divergences regulated in CDR ($d = 4 - 2\epsilon$) and canceled in the total
- $T_R(\lambda)$ evaluated in 4 dimensions:
  1. IR divergences arising as γ gets soft or collinear to either d or $\bar{d}$
  2. IR divergences when g gets collinear to either d or $\bar{d}$ (arising as $\log\lambda, \log^2\lambda$ singularities as $\lambda \to 0$)

$$T_R(\lambda) = \frac{1}{\sigma_0} \int \mathrm{d}\Phi_{3+1} R_{g^*}^{(\lambda)}(\Phi_{3+1}) O_{3+1}$$

- The real squared amplitude is divided in three regions

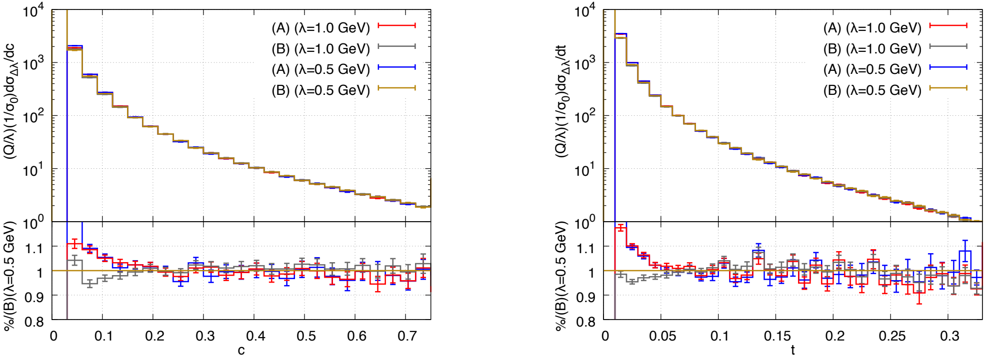
$$R = R^{(1)} + R^{(2)} + R^{(3)}$$

$$\begin{aligned}
R^{(1)} &= \frac{f_{d\gamma}^2 + f_{\bar{d}\gamma}^2}{f_{d\gamma}^2 + f_{\bar{d}\gamma}^2 + f_{dg}^2 + f_{\bar{d}g}^2} R \quad (\gamma \parallel d(\bar{d}), \gamma \text{ soft}) \\
R^{(2)} &= \frac{f_{dg}^2}{f_{d\gamma}^2 + f_{\bar{d}\gamma}^2 + f_{dg}^2 + f_{\bar{d}g}^2} R \quad (g \parallel d) \\
R^{(3)} &= \frac{f_{\bar{d}g}^2}{f_{d\gamma}^2 + f_{\bar{d}\gamma}^2 + f_{dg}^2 + f_{\bar{d}g}^2} R \quad (g \parallel \bar{d}) \\
f_{ij} &= \frac{E_i + E_j}{(k_i + k_j)^2} \quad (i, j = d, \bar{d}, \gamma, g)
\end{aligned}$$

- $R^{(1)}$ integrated within the `POWHEG-BOX`, $R^{(2)}, R^{(3)}$ with a separated Fortran code
- $\gamma^* \to d\bar{d}\gamma q\bar{q} \Rightarrow$ IR finite as $\lambda \to 0$, QED singularity from γ (`POWHEG-BOX`)

Shape Variables: Results for Kinematical Distributions

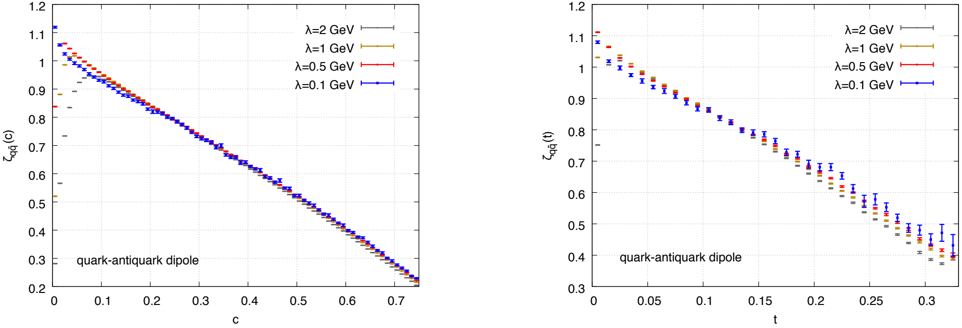
- $\langle O\rangle_\lambda^{(1)} - \langle O\rangle_0^{(1)}$, with $O = \delta(z - z(\Phi))$, for $\mathrm{t} = 1 -$ Thrust and C-parameter
- Computation for $\lambda = 0.5, 1$ GeV, for $Q = 100$ GeV
- Comparison between analytical approach (A) and Large-n_f limit (B)



- Behaviour in λ is nearly linear
- Excellent agreement between the two methods
- $\mathcal{O}(\lambda^2)$ entering for $C \lesssim 0.15$ and $\mathrm{t} \lesssim 0.07$

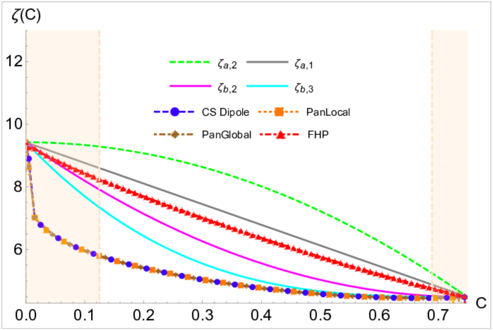
NP shift in the Shape Variable: $q\bar{q}$ dipole

- Evaluation of $\zeta_{q\bar{q}}$ in the abelian limit for Thrust and C-parameter for $\lambda = 0.1, 0.5, 1, 2$ GeV and $Q = 100$ GeV



- The C-parameter has a Sudakov shoulder in the 3-jet symmetric point ($c = 3/4$)
- We found $\zeta_{q\bar{q}}(c = 3/4) = 0.226(2)$ for $\lambda = 0.1$ GeV
- Good agreement with the result of Luisoni, Monni, Salam ('20) in the abelian limit: $\zeta_{\rm LMS}(c = 3/4) = \zeta_{q\bar{q}}(c = 3/4)/\zeta_{q\bar{q}}(c = 0) = 0.224$

NP shift for C-parameter (Luisoni, Monni, Salam ('20))



- Several interpolations among the 2-jet limit and 3-jet symmetric point
- Found good agreement with the $\zeta_{b,3}$ curve, leading to $\alpha_s = 0.117(3)$
- Much better agreement with the world average value $\alpha_s = 0.118(1)$ from PDG as compared to $\alpha_s = 0.112(2)$ obtained in [1501.04111]