Are Jets Narrowed or Broadened in e+A SIDIS?

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Based on Hannah Clayton, Matt Sievert, and WAH, Eur.Phys.J.C 82 (2022) 5, 437; arXiv:2110.14737

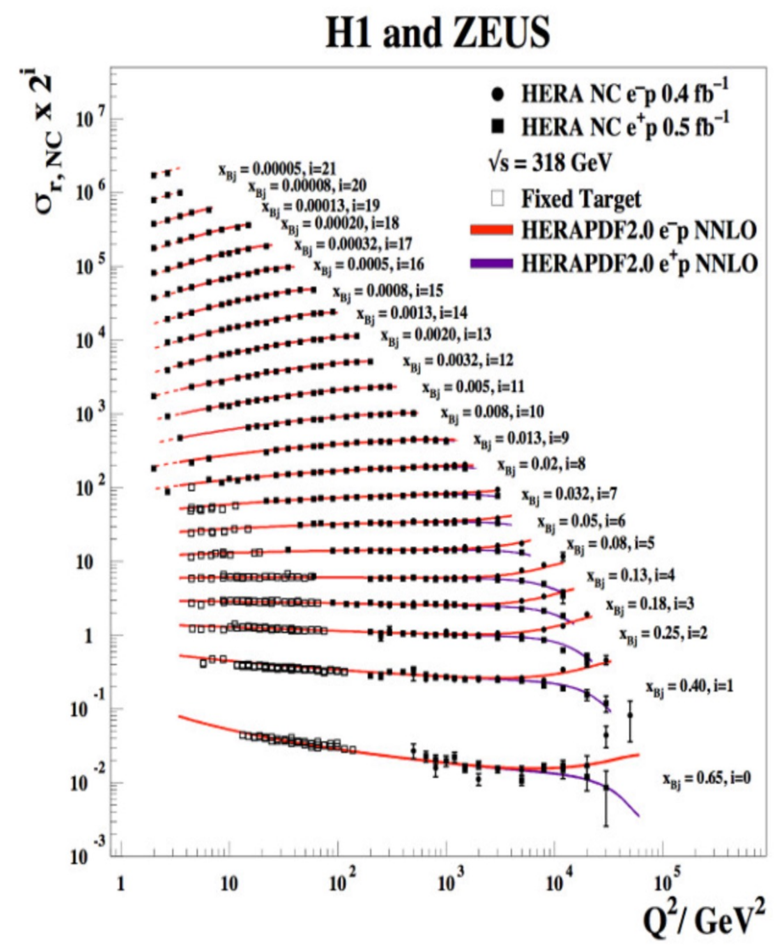
National Research Foundation

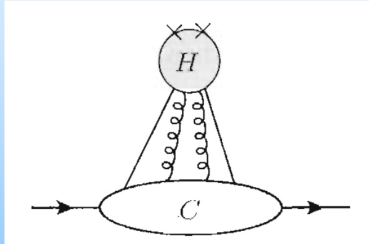
Motivation

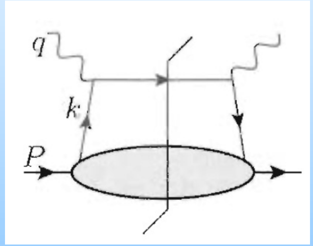
- Interested in emergent, many body dynamics of QCD
- High-p_T particles: powerful probe of such properties, ***<u>if</u>***
- we have control over the energy loss processes

Factorization ⇔ Rigorous QCD

- *Controlled* order-by-order expansion in perp/Q
  - Further controlled o-b-o expansion in α_s
- Clean: rigorous theorems, expansion parameters clear, error estimates from higher order effects
- Ex: e+p DIS, SIDIS, DY, …

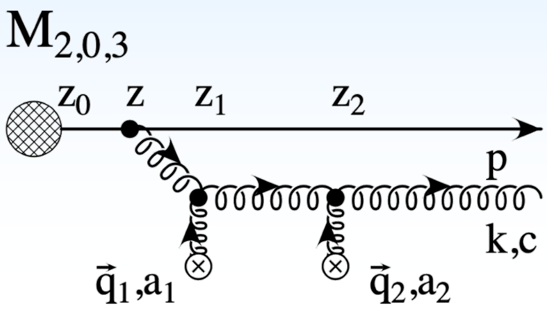






Factorization in E-Loss

- Most E-loss models *assume* factorization of production and in-medium E-loss
  - DGLV, BDMPS-Z, AMY, …
  - Medium modification of DGLAP for FF (Guo, Wang, Majumder, Vitev, et al.)



Djordjevic, PRC74 (2006)

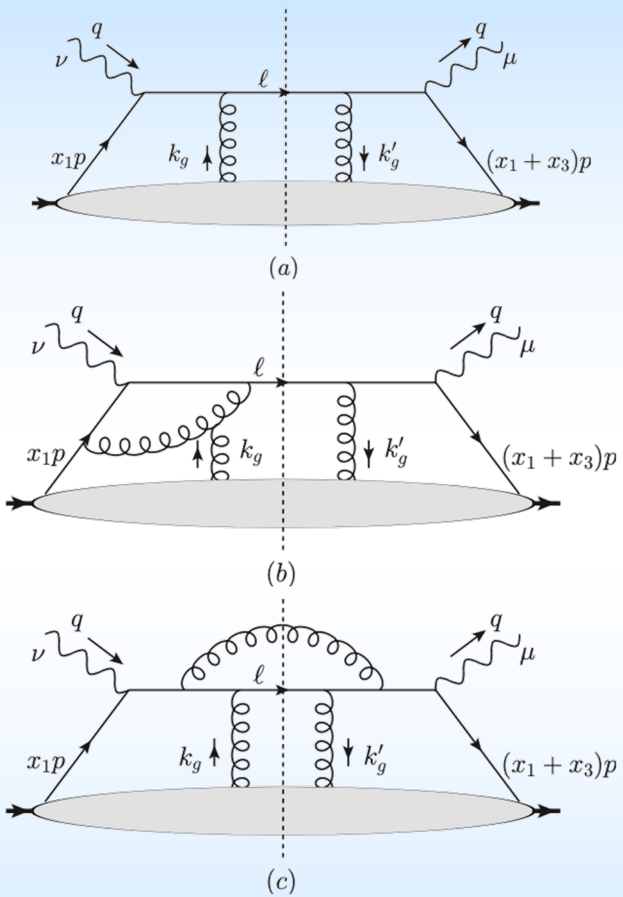
- eA SIDIS $<p_T^2>$ at Twist 4 (Kang, Wang, Wang, Xing)
  - Production and subsequent interaction on equal footing
  - Self-consistent to NLO; no factorization theorem yet
  - Make apples-to-apples comparison with DGLV? Quantify importance of ignoring production + E-loss interference in DGLV?

Twist 4 $<p_T^2>$ in SIDIS

$$\Delta\langle \ell_{hT}^2\rangle \approx \frac{d\langle \ell_{hT}^2\sigma^D\rangle}{d\mathcal{PS}} \Big/ \frac{d\sigma}{d\mathcal{PS}}$$

$$\frac{d\langle \ell_{hT}^2\sigma^D\rangle}{d\mathcal{PS}} = \sigma_h \sum_q e_q^2 \int_{x_B}^1 \frac{dx}{x} T_{qg}(x,0,0,\mu_f^2) \int_{z_h}^1 \frac{dz}{z} D_{h/q}(z,\mu_f^2)\delta(1-\hat{x})\delta(1-\hat{z})$$
$$+ \sigma_h \frac{\alpha_s}{2\pi} \sum_q e_q^2 \int_{z_h}^1 \frac{dz}{z} D_{h/q}(z,\mu_f^2) \int_{x_B}^1 \frac{dx}{x} \Big\{ \ln\left(\frac{Q^2}{\mu_f^2}\right) \Big[\delta(1-\hat{x}) P_{qq}(\hat{z}) T_{qg}(x,0,0,\mu_f^2)$$
$$+ \delta(1-\hat{z})\big(\mathcal{P}_{qg\to qg}\otimes T_{qg} + P_{qg}(\hat{x}) T_{gg}(x,0,0,\mu_f^2)\big)\Big]$$
$$+ H_{qg}^{C-R}\otimes T_{qg} + H_{qg}^{C-V}\otimes T_{qg} - H_{qg}^{A}\otimes T_{qg}^{A} + H_{gg}^{C}\otimes T_{gg}\Big\},$$

- Twist 4 *PDF*s
- Cross talk with IS
- *FF evolve with vacuum splitting functions*



KWWX, PRL (2014)

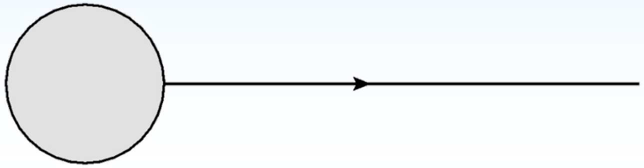
KWWX, PRL (2014); arXiv:1310.6759
XKWW, NPA (2014); arXiv:1407.8506
KWWX, PRD (2016); arXiv:1409.1315

Comparison to Opacity Expansion

- Twist 4:
  - Order-by-order expansion in perp/Q
    - Order-by-order expansion in α_s
  - Collinear factorization framework
- Opacity:
  - Order-by-order expansion in opacity
    - number of times there's a scattering off a medium quasi-particle
  - Captures destructive interference of LPM fully
  - ***Assumed*** factorization of initial hard scattering process with no subsequent interference
  - Often assumes
    - soft ($x << 1$)
    - collinear approx. ($k_T << x\, E$)
  - *Incorporates finite kinematics*
- How do the two compare at asymptotic E_{jet}?

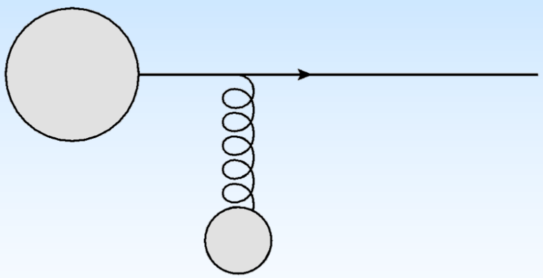
0^{th} Order in Opacity, LO α_s $\langle p_T^2 \rangle$

- Production is *assumed* factorized into blob



  - No medium scattering (0^{th} Order in Opacity)
  - No emissions (LO α_s)
- We seek medium modification, so $\Delta\langle p_T^2 \rangle = 0$

1^{st} Order in Opacity, LO α_s $<p_T^2>$



- ⇔ Elastic energy loss
- Calculation:
  - In medium Debye-screened scattering center

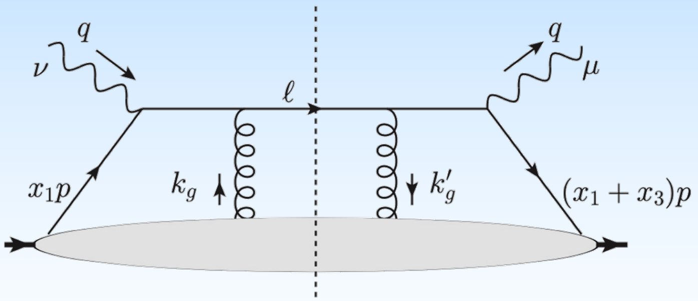
$$\left.\frac{d^2\sigma^{qg\to qg}}{d^2\mathbf{q}_\perp}\right|_1 = \frac{2\alpha_s^2}{(\mathbf{q}_\perp^2+\mu^2)^2}$$

  - Yields, assuming finite $q_{max}^2 \sim E * \mu$:

$$\langle p_\perp^2\rangle_{\mathrm{LO},\,1} \equiv \frac{L}{\lambda}\int d^2\mathbf{q}_\perp\,\mathbf{q}_\perp^2\,\frac{d^2\sigma^{qg\to qg}}{d^2\mathbf{q}_\perp}\Big/\int d^2\mathbf{q}_\perp\,\frac{d^2\sigma^{qg\to qg}}{d^2\mathbf{q}_\perp}$$
$$\approx \frac{L\mu^2}{\lambda}\log\left(\frac{E}{\mu}\right)$$
$$= \hat{q}L,$$

Twist 4 LO α_s $\langle p_T^2 \rangle$

- $\Leftrightarrow$ Elastic energy loss



$$\frac{d\langle \ell_\perp^2 \sigma\rangle}{dx_B dy dz_h} = \sigma_h e_q^2 \int_{x_B}^1 \frac{dx}{x} T_{qg}(x,0,0,\mu_f^2) \int_{z_h}^1 \frac{dz}{z} D_{h/q}(z,\mu_f^2)\delta(1-\hat{x})\delta(1-\hat{z})$$

- – Assume loosely bound nucleus

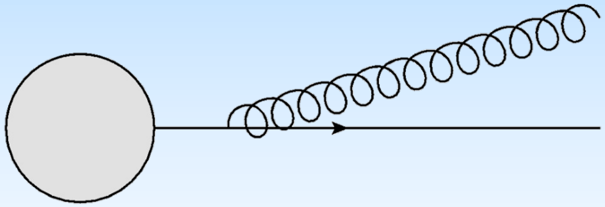
$$T_{qg}(x_B,0,\mu_f^2) \approx \frac{N_c}{4\pi^2\alpha_s} f_{q/A}(x_B,\mu_f^2)\hat{q}(\mu_f^2)L$$

- – Make FF trivial for comparison: $D_{h/q}(z,\mu_f^2) = \delta(1-z)$
- – Result, after dividing by 2=>2 x-scn

$$\langle p_T^2 \rangle = \hat{q}L,$$

  - Same as opacity expansion!

0^{th} Order in Opacity, NLO α_s $\langle p_T^2 \rangle$



- ⬄ Vacuum radiation
- Compute single inclusive rad. glu. dist.:

$$\frac{d^3 N_g^{(0)}}{dx d^2\mathbf{k}_\perp} = \frac{C_R \alpha_s}{\pi^2 x} \frac{\mathbf{k}_\perp^2}{(\mathbf{k}_\perp^2 + m_g^2 + M^2 x^2)^2}$$

- Finite kinematics $k_T < x\,E$:

$$\langle N_g \rangle_0 \equiv \int dx\, d^2\mathbf{k}_\perp \frac{d^3 N_g^{(0)}}{dx\, d^2\mathbf{k}_\perp},$$

$$\left.\frac{\Delta E}{E}\right|_{\text{NLO, 0}} = \int dx\, d^2\mathbf{k}_\perp \left(x \frac{d^3 N_g^{(0)}}{dx\, d^2\mathbf{k}_\perp} \right),$$

$$\langle p_T^2 \rangle_{\text{NLO, 0}} = \int dx\, d^2\mathbf{k}_\perp \left(k_\perp^2 \frac{d^3 N_g^{(0)}}{dx\, d^2\mathbf{k}_\perp} \right)$$

=>

$$\langle N_g \rangle_0 \approx \frac{\alpha_s C_R}{\pi} \ln^2 \frac{E}{\mu}$$

$$\left.\frac{\Delta E}{E}\right|_{\text{NLO, 0}} \approx \frac{2\alpha_s C_R}{\pi} \ln \frac{E}{\mu},$$

$$\langle p_T^2 \rangle_{\text{NLO, 0}} \approx 2 \frac{\alpha_s C_R}{\pi} E^2$$

1^{st} Order in Opacity, NLO α_s $\langle p_T^2\rangle$

- Calculation:

$$\frac{d^5N_g^{(1)}}{dx d^2\mathbf{k}_\perp d^2\mathbf{q}_\perp} = $$

$$= \frac{C_R\alpha_s}{\pi^3 x}\frac{L}{\lambda}\frac{1}{\mathbf{k}_\perp^2+m_g^2+M^2x^2}\frac{\mu^2}{(\mathbf{q}_\perp^2+\mu^2)^2}\times$$
$$\times\, 2\frac{\mathbf{k}_\perp\cdot\mathbf{q}_\perp(\mathbf{k}_\perp-\mathbf{q}_\perp)^2+(m_g^2+M^2x^2)\mathbf{q}_\perp\cdot(\mathbf{q}_\perp-\mathbf{k}_\perp)}{(\frac{4Ex}{L})^2+((\mathbf{k}_\perp-\mathbf{q}_\perp)^2+M^2x^2+m_g^2)^2}$$

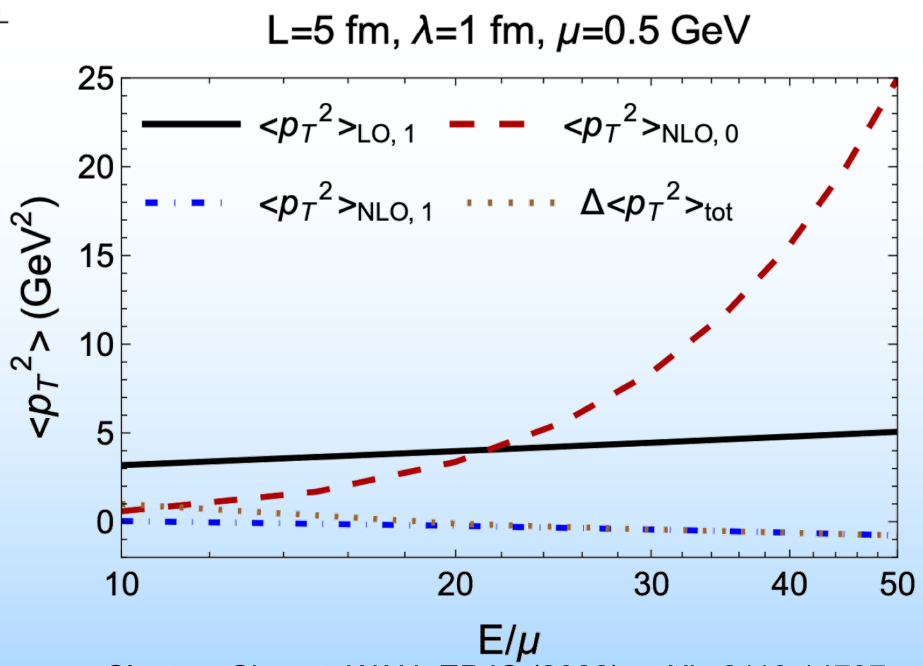
$$\langle N_g\rangle_1 \equiv \int dx\, d^2\mathbf{k}_\perp\, d^2\mathbf{q}_\perp\, \frac{d^5N_g^{(1)}}{dx\, d^2\mathbf{k}_\perp\, d^2\mathbf{q}_\perp}\,.$$

$$\left.\frac{\Delta E}{E}\right|_{\text{NLO, 1}} = \int dx\, d^2\mathbf{k}_\perp\, d^2\mathbf{q}_\perp\, x\, \frac{d^5N_g^{(1)}}{dx\, d^2\mathbf{k}_\perp\, d^2\mathbf{q}_\perp}\,,$$

$$\langle p_T^2\rangle_{\text{NLO, 1}} = \int dx\, d^2\mathbf{k}_\perp\, d^2\mathbf{q}_\perp\, (\mathbf{k}_\perp-\mathbf{q}_\perp)^2\, \frac{d^5N_g^{(1)}}{dx\, d^2\mathbf{k}_\perp\, d^2\mathbf{q}_\perp}\,,$$

- Full numerics for $\langle p_T^2\rangle$
  - 1^{st} LO ~ $\log(E/\mu)$
  - 0^{th} NLO ~ E^2
  - 1^{st} NLO ~ $-\log^2(E/\mu)$

!!!



Clayton, Sievert, WAH, EPJC (2022), arXiv:2110.14737

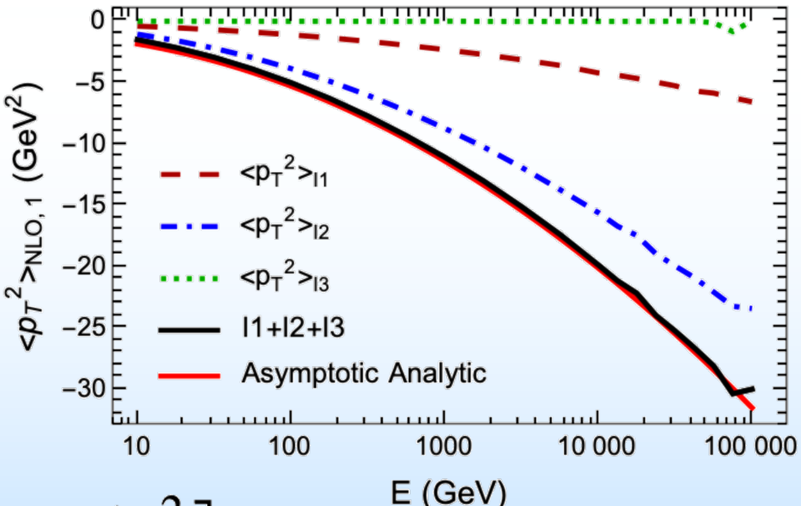
Jet Narrowing: Bad Numerics? No!

- Naïve infinite kinematics
  - Integrate over all k_T (amongst other sins)

$$\langle p_\perp^2 \rangle_{\mathrm{NLO},\,1} \sim \frac{L}{\lambda}\mu^2 \ln^2 \frac{E}{\mu}$$

$$\frac{\langle p_\perp^2 \rangle_{\mathrm{NLO},\,1}}{\langle p_\perp^2 \rangle_{\mathrm{LO},\,1}} = \frac{4\alpha_s}{3\pi} \ln \frac{E}{\mu}$$

- Careful treatment of <u>*finite*</u> kinematics (most esp. $k_T < x\,E$) in large E limit



$$\langle p_\perp^2 \rangle_{\mathrm{NLO},\,1} = -\frac{C_R\alpha_s}{4}\frac{L}{\lambda}\mu^2\left[\ln^2\left(\frac{4E}{\mu^2 L}\right) + \frac{5\pi^2}{12}\right]$$

Clayton, Sievert, WAH, EPJC (2022) arXiv:2110.14737

Twist 4 NLO α_s $\langle p_T^2 \rangle$

- More difficult to extract cf T-4 LO

$$\frac{d\langle \ell_\perp^2 \sigma\rangle}{dx_B dy dz_h}\bigg|_{\mathrm{NLO}} = \sigma_h \frac{\alpha_s}{2\pi} e_q^2 \log\left(\frac{Q^2}{\mu_f^2}\right) \int_{z_h}^1 \frac{dz}{z} D_{h/q}(z,\mu_f^2) \int_{x_B}^1 \frac{dx}{x} \Big\{ \delta(1-\hat{x}) P_{qq}(\hat{z}) T_{qg}(x,0,0,\mu_f^2) + \delta(1-\hat{z}) \big(P_{qg\to qg} \otimes T_{qg} + P_{qg}(\hat{x}) T_{gg}(x,0,0,\mu_f^2) \big) \Big\},$$

$$P_{qg\to qg} \otimes T_{qg} \equiv P_{qq}(\hat{x}) T_{qg}(x,0,0) + \frac{C_A}{2} \bigg\{ \frac{4}{(1-\hat{x})_+} T_{qg}(x_B, x-x_B, 0) - \frac{1+\hat{x}}{(1-\hat{x})_+} \big[T_{qg}(x,0,x_B-x) + T_{qg}(x_B, x-x_B, x-x_B) \big] \bigg\} + 2C_A \delta(1-\hat{x}) T_{qg}(x,0,0).$$

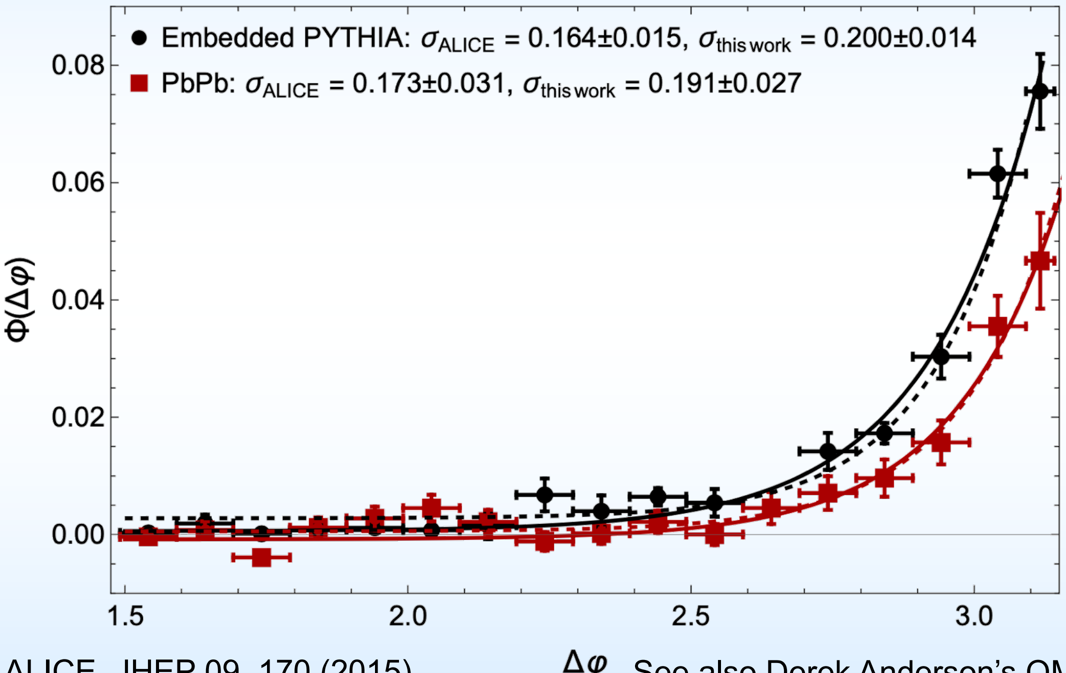
- Assume color triviality breaking terms are small, trivialize FF, loosely bound nucleus

$$\frac{\langle p_T^2 \rangle_{\mathrm{NLO},\,1}}{\langle p_T^2 \rangle_{\mathrm{LO},\,1}} \approx \frac{4\alpha_s}{3\pi} \ln\frac{E}{\mu} \times \frac{\int_{x_B}^1 \frac{dx}{x} \left[\frac{1+\hat{x}^2}{(1-\hat{x}_+} + \frac{3}{2}\delta(1-\hat{x}) \right] f_{q/A}(x,\mu^2)}{f_{q/A}(x_B,\mu^2)} > 0$$

  - NB: $4\alpha_s/3\pi$ identical to naïve inf. kin. opacity

Comparison to Data

- So do jets broaden (cf Twist 4) or narrow (cf Opacity)?



ALICE, JHEP 09, 170 (2015)
Clayton, Sievert, WAH, EPJC (2022)
arXiv:2110.14737

See also Derek Anderson's QM talk

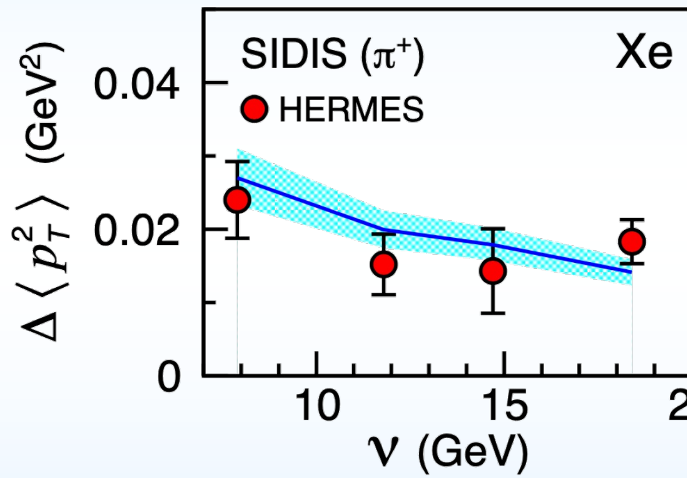


Figure:
Ru, Kang, Wang, Xing, Zhang, PRD 103 (2021) 3
Data: HERMES, PLB684, 114 (2010)

  - LHC data inconclusive; SIDIS suggests narrowing as E => infinity!

Conclusions (1/2)

- Seek precision jet tomography in HIC

  => Quantitative insight into many body QCD

- Asymptotic analysis of Twist 4 and Opacity
  - Twist 4: jet **broadening** for large E
  - Opacity: jet *narrowing* for large E
    - Due to v delicate destructive (LPM) interference
- Data: ambiguous, hints of narrowing
  - Difficult measurement, look forward to greater precision, range of systems, etc.

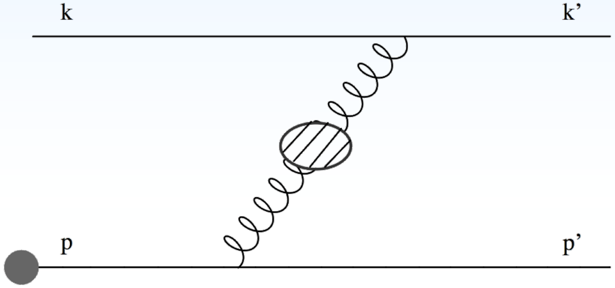
Discussion and Outlook (2/2)

- We saw:
  - Deep conceptual issues with Opacity Expansion vs Twist 4
    - How to categorize different classes of diagrams?
      - What's in the initial state vs final state vs …?
      - Medium modification cannot be in FF
    - How to incorporate LPM, finite kinematics?
      - How under control are kinematics?
      - Generalization of collinear factorization needed?
  - E loss less sensitive to finite kinematics
    - Try to compare Twist 4 and Opacity for another observable, closer to E loss?
    - Value of clean(er) e+A system!
- Much interesting work to do!

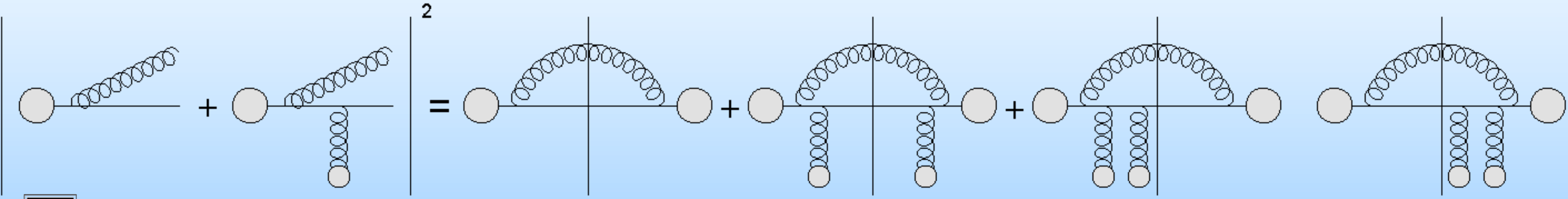
Bonus Material

Types of Energy Loss

- Two types of E-loss:
  - Collisional (elastic) 2➞2

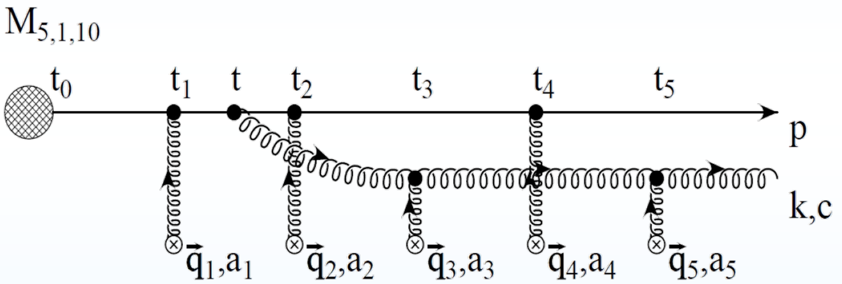


  - Radiative (inelastic) 2➞3
    - Scales => ~few scatterings, mult. coh. em. => LPM
    - Must include interference with production radiation



pQCD Rad Picture in Opacity Exp.

- Bremsstrahlung Radiation
  - Weakly-coupled plasma
    - Medium organizes into Debye-screened centers
  - T ~ 250 MeV, $g(2\pi T) \sim 2$
    - $\mu \sim gT \sim 0.5$ GeV
    - $\lambda_{mfp} \sim 1/g^2T \sim 1$ fm
    - $R_{Au} \sim 6$ fm
  - $1/\mu \ll \lambda_{mfp} \ll \tau_{form} \ll L$
    - mult. coh. em.



Gyulassy, Levai, and Vitev, NPB571 (2000)

- LPM

  $dp_T/dt \sim -LT^3 \log(p_T/M_q)$

- Bethe-Heitler

  $dp_T/dt \sim -(T^3/M_q^2)\, p_T$