Transport coefficients of heavy mesons in a thermal medium

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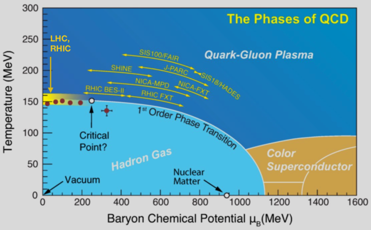
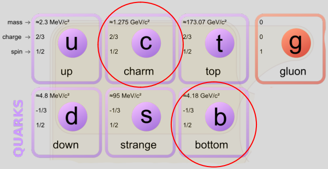
Introduction: Heavy flavor

(A. Bazavov *et al.*, 1904.09951)



- Infer QCD properties at high temperatures through final state of RHICs
- Find clean and solid observables to connect detections to early stages
- **Hard Probes:** Jets, hard electromagnetic emission, heavy flavor (quarkonia, open-heavy flavor hadrons...)

Heavy quarks: formed at the initial stage of the collision (short formation time) and difficult to equilibrate along their evolution (large relaxation time)



Interactions in a thermal medium?

Transport coefficients?

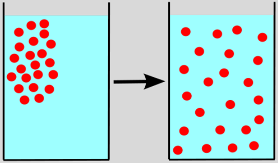
Introduction: Thermal propagation

- Heavy mesons (D, D^*, $\bar{B}$,...) at $T < 150$ MeV$/k_B$
- Interacting with an equilibrated light-meson gas ($\Phi = \{\pi, K, \bar{K}, \eta\}$)
- **Heavy-hadron mass is the dominant scale**

$$M_D \gg m_\Phi, T, \Lambda_{QCD}$$

- Picture: Brownian particle in a thermal bath
  B. Svetitsky, Phys. Rev. D37, 484 (1988)
- Transport properties: (Heavy-flavor) diffusion coefficient, D_s.

$$\vec{j} = -D_s(T, \mu_i)\vec{\nabla} n$$

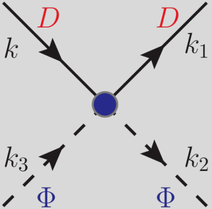


Effective field theory

Effective Lagrangian based on **chiral** and **heavy-quark spin-flavor** symmetries.

- **Chiral expansion** up to NLO
  : also explicitly broken due to light-meson masses ($\pi, K, \bar{K}, \eta$).
- **Heavy-quark mass expansion** up to LO
  : broken by heavy meson masses (D, D_s, D^*, D_s^*).

E.E. Kolomeitsev and M.F.M. Lutz *Phys.Lett. B582 (2004) 39*, J. Hofmann and M.F.M. Lutz *Nucl.Phys. A733 (2004) 142*, F.K.Guo, C.Hanhart. S. Krewald, U.G. Meissner *Phys.Lett. B666 (2008) 251*, L.S. Geng, N. Kaiser, J. Martin-Camalich and W. Weise *Phys.Rev.D82,05422 (2010)*, L.M. Abreu, D. Cabrera, F.J. Llanes-Estrada and JMT-R. *Annals Phys. 326 (2011) 2737...*



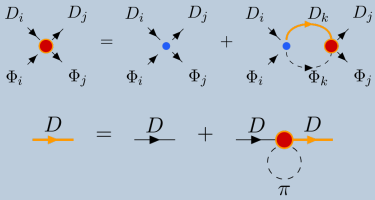
Elastic processes:
$D\pi$, DK, $D\bar{K}$, $D\eta$
$D_s\pi$, D_sK, $D_s\bar{K}$, $D_s\eta$
and their inelastic channels.

Self-consistency at finite temperature

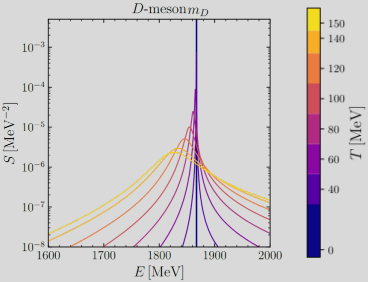
G. Montaña, À. Ramos, L. Tolos, JMT-R, PLB 806 (2020) 135464

G. Montaña, À. Ramos, L. Tolos, JMT-R, PRD 102 (2020) 9, 096020

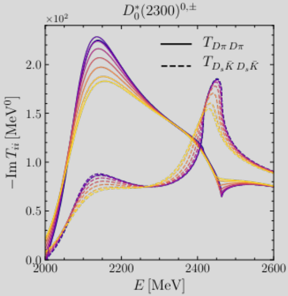
T-matrix equation



Spectral function



T-matrix elements



Kinetic theory with off-shell effects

↓

Kadanoff-Baym equations

L. Kadanoff, G.Baym, "Quantum statistical mechanics" 1962, P. Danielewicz, Annals Phys. 152, 239 (1984), W. Botermans and R. Malfliet, Phys. Rept. 198, 115 (1990), J.-P. Blaizot and E. Iancu, Nucl. Phys. B557, 183 (1999), J. Rammer "Quantum field theory of non-equilibrium states" (2007), W. Cassing, Eur. Phys. J.168, 3 (2009)

$$\overbrace{\left(k^{\mu} - \frac{1}{2}\frac{\partial \mathrm{Re}\ \Pi^{R}(X,k)}{\partial k_{\mu}}\right)\frac{\partial iG_{D}^{<}(X,k)}{\partial X^{\mu}}}^{\text{Advective term}} = \overbrace{\underbrace{\frac{1}{2}i\Pi^{<}(X,k)iG_{D}^{>}(X,k)}_{\text{Gain term}} - \underbrace{\frac{1}{2}i\Pi^{>}(X,k)iG_{D}^{<}(X,k)}_{\text{Loss term}}}^{\text{Collision term}}$$

Kadanoff-Baym ansatz:

$$iG_{D}^{<}(X,k) = 2\pi\ S_{D}(X,k)\ f_{D}(X,k^{0})$$

$$iG_{D}^{>}(X,k) = 2\pi\ S_{D}(X,k)\ \left[1 + f_{D}(X,k^{0})\right]$$

Fokker-Planck equation

Using $m_D \gg m_\Phi, T$:

Off-shell Fokker-Planck equation

$$\frac{\partial}{\partial t} G_D^<(t,k) = \frac{\partial}{\partial k^i}\left\{ \hat{A}(k;T) k^i G_D^<(t,k) + \frac{\partial}{\partial k^j}\left[\hat{B}_0(k;T)\Delta^{ij} + \hat{B}_1(k;T)\frac{k^i k^j}{\mathbf{k}^2}\right] G_D^<(t,k)\right\}$$

where $\Delta^{ij} = \delta^{ij} - k^i k^j/\mathbf{k}^2$

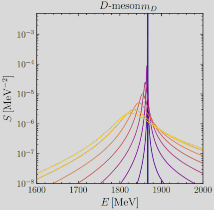
JMT-R, G. Montaña, À. Ramos, L. Tolos, Phys.Rev.C 105, 025203 (2022)

Off-shell Transport Coefficients

$$\begin{cases} \hat{A}(k^0,\mathbf{k};T) & \equiv \left\langle 1 - \frac{\mathbf{k}\cdot\mathbf{k}_1}{\mathbf{k}^2} \right\rangle \\ \hat{B}_0(k^0,\mathbf{k};T) & \equiv \frac{1}{4}\left\langle \mathbf{k}_1^2 - \frac{(\mathbf{k}\cdot\mathbf{k}_1)^2}{\mathbf{k}^2} \right\rangle \\ \hat{B}_1(k^0,\mathbf{k};T) & \equiv \frac{1}{2}\left\langle \frac{[\mathbf{k}\cdot(\mathbf{k}-\mathbf{k}_1)]^2}{\mathbf{k}^2} \right\rangle \end{cases}$$

Off-shell and thermal effects

Spectral function

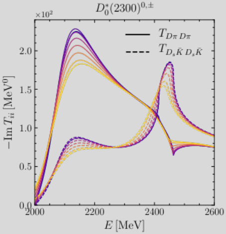


Energy-momentum conservation

$$\langle \mathcal{F}(\boldsymbol{k},\boldsymbol{k}_1)\rangle \equiv \frac{1}{2k^0}\sum_{\lambda,\lambda'=\pm}\lambda\lambda'\int dk_1^0\int\prod_{i=1}^{3}\frac{d^3k_i}{(2\pi)^3}\frac{1}{2E_2 2E_3}\,S_D(k_1^0,\mathbf{k}_1)(2\pi)^4\delta^{(3)}(\boldsymbol{k}+\boldsymbol{k}_3-\boldsymbol{k}_1-\boldsymbol{k}_2)$$

$$\times\,\delta(k^0+\lambda' E_3-\lambda E_2-k_1^0)|T(k^0+\lambda' E_3,\boldsymbol{k}+\boldsymbol{k}_3)|^2 f^{(0)}(\lambda' E_3)\tilde{f}^{(0)}(\lambda E_2) f^{(0)}(k_1^0)\,\mathcal{F}(\boldsymbol{k},\boldsymbol{k}_1)$$

Interaction

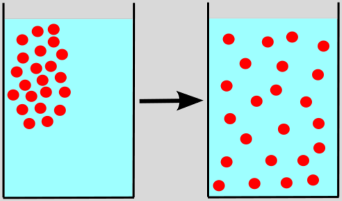


Equilibrium distribution functions

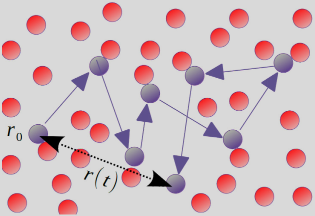
Diffusion Coefficient

Fick's diffusion law

$$\vec{j}_i = -\boldsymbol{D_s}\vec{\nabla} n_i$$



$\boldsymbol{D_s}$ depends on interaction and medium properties (T, μ_i)



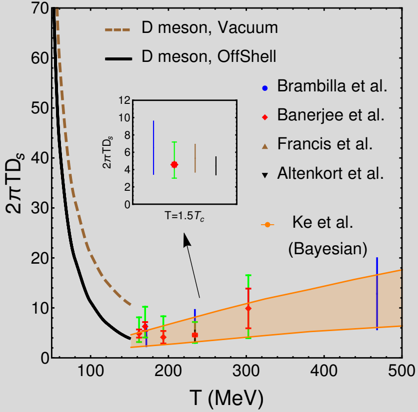
Brownian motion
Mean squared displacement

$$\langle [r(t) - r_0]^2 \rangle = 6\boldsymbol{D_s}t$$

Spatial diffusion coefficient

Spatial diffusion coefficient

$$2\pi T \boldsymbol{D_s}(T) = \frac{2\pi T^3}{B_0(k^0 = E_k, \boldsymbol{k} \to 0; T)}$$



JMT-R, G. Montaña, À. Ramos, L. Tolos, Phys.Rev.C 105, 025203 (2022)

Lattice-QCD calculations

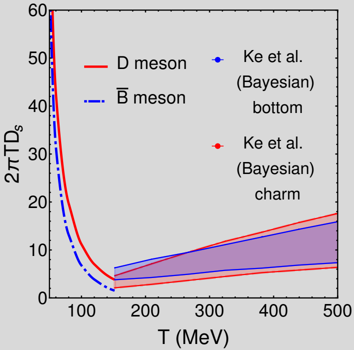
- N. Brambilla *et al.* Phys. Rev. D102, 074503 (2020)
- D. Banerjee *et al.* Phys. Rev. D85, 014510 (2012)
- A. Francis *et al.* Phys. Rev. D92, 116003 (2015)
- L. Altenkort *et al.* Phys. Rev. D103, 014511 (2021)

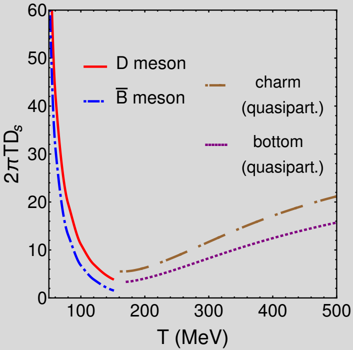
Bayesian study of RHICs

- W. Ke *et al.* Phys. Rev. C98, 064901 (2018)

$\overline{B}$-meson diffusion coefficient

JMT-R, G. Montaña, À. Ramos, L. Tolos (to be published)
G. Montaña (PhD thesis, July 2022)





Bayesian study of RHICs

- W. Ke *et al.*
  Phys. Rev. C98, 064901 (2018)

Quasiparticle model for QGP

- S.K. Das *et al.*
  Phys.Rev.D 94, 114039 (2016)

Conclusions

Summary

1. $D/\overline{B}$-meson **EFT** extended to **finite temperature** in a self-consistent way
2. Heavy-meson **kinetic theory** studied via the Kadanoff-Baym equations. We derived an **off-shell Fokker-Planck** equation
3. Heavy-flavor **transport coefficients** below T_c including thermal modifications and off-shell effects. Apparent matching to lattice-QCD and Bayesian analyses at $T \sim T_c$

References

1. G. Montaña, À. Ramos, L. Tolos, JMT-R, PLB 806 (2020) 135464
2. G. Montaña, À. Ramos, L. Tolos, JMT-R, PRD 102 (2020) 9, 096020
3. JMT-R, G. Montaña, À. Ramos, L. Tolos, PRC 105, 025203 (2022)

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E.E. Kolomeitsev and M.F.M. Lutz *Phys.Lett. B582 (2004) 39*, J. Hofmann and M.F.M. Lutz *Nucl.Phys. A733 (2004) 142*, F.K.Guo, C.Hanhart. S. Krewald, U.G. Meissner *Phys.Lett. B666 (2008) 251*, L.S. Geng, N. Kaiser, J. Martin-Camalich and W. Weise *Phys.Rev.D82,05422 (2010)*, L.M. Abreu, D. Cabrera, F.J. Llanes-Estrada and JMT-R. *Annals Phys. 326 (2011) 2737...*

$$\mathcal{L}_{\mathrm{LO}} = Tr[\nabla^\mu D \nabla_\mu D^\dagger] - m_D^2 Tr[D D^\dagger] - Tr[\nabla^\mu D^{*\nu} \nabla_\mu D_\nu^{*\dagger}] + m_{D^*}^2 Tr[D^{*\mu} D_\mu^{*\dagger}]$$

$$+ig Tr\left[\left(D^{*\mu} u_\mu D^\dagger - D u^\mu D_\mu^{*\dagger}\right)\right] + \frac{g}{2m_D} Tr\left[\left(D_\mu^* u_\alpha \nabla_\beta D_\nu^{*\dagger} - \nabla_\beta D_\mu^* u_\alpha D_\nu^{*\dagger}\right)\epsilon^{\mu\nu\alpha\beta}\right]$$

$$D = (D^0, D^+, D_s^+)$$

$$\nabla^\mu = \partial^\mu - \tfrac{1}{2}(u^\dagger \partial^\mu u + u \partial^\mu u^\dagger)$$

$$u^\mu = i(u^\dagger \partial^\mu u - u \partial^\mu u^\dagger)$$

$$u = \exp\left[\frac{i}{\sqrt{2}F}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}\right]$$

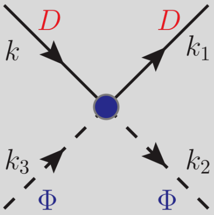
Perturbative potential

Perturbative amplitude

$$
\begin{aligned}
V(s,t,u) &= \frac{C_0}{4f_\pi^2}(s-u) + \frac{2C_1}{f_\pi^2}h_1 + \frac{2C_2}{f_\pi^2}h_3(k_2\cdot k_3) \\
&+ \frac{2C_3}{f_\pi^2}h_5[(k\cdot k_3)(k_1\cdot k_2)+(k\cdot k_2)(k_1\cdot k_3)]
\end{aligned}
$$

f_π: pion decay constant
Isospin coefficients: fixed by symmetry
Low-energy constants: fixed by experiment
or by underlying theory

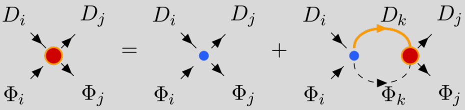
Z.-H. Guo *et al.* Eur. Phys. J.C79, 1, 13 (2019)



Elastic processes:
$D\pi$, DK, $D\bar{K}$, $D\eta$
$D_s\pi$, D_sK, $D_s\bar{K}$, $D_s\eta$ and their inelastic channels.

Resummation at finite temperature

At $T \neq 0$ **Imaginary Time Formalism** in a self-consistent approximation

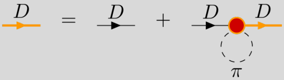


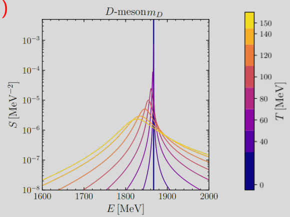
$$T_{ij} = [1 - G_{D\Phi} V]^{-1}_{ik} V_{kj}$$

$$G_{D\Phi}(E, \vec{p}; T) = \int \frac{d^3k}{(2\pi)^3} \int d\omega \int d\omega' \frac{S_D(\omega, \vec{k}; T) S_\Phi(\omega', \vec{p} - \vec{k}; T)}{E - \omega - \omega' + i\varepsilon} [1 + f(\omega, T) + f(\omega', T)]$$

$$S_D(\omega, \vec{k}; T) = -\frac{1}{\pi} \mathrm{Im}\, \Delta_D(\omega, \vec{k}; T) = -\frac{1}{\pi} \mathrm{Im} \left(\frac{1}{\omega^2 - \vec{k}^{\,2} - m_D^2 - \Pi_D(\omega, \vec{k}; T)} \right)$$

$$\Pi_D(\omega_n, \vec{k}; T) = T \int \frac{d^3p}{(2\pi)^3} \sum_m \Delta_\Phi(\omega_m - \omega_n, \vec{p} - \vec{k}\,) T_{D\Phi}(\omega_m, \vec{p}\,)$$



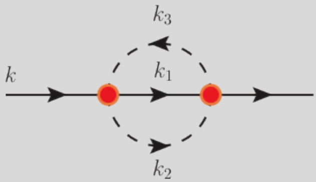


$T-$matrix approximation

Close kinetic equation employing the T**-matrix approximation**
(L. Kadanoff, G.Baym, "Quantum statistical mechanics" 1962, P. Danielewicz, Annals Phys. 152, 239 (1984), W. Botermans and R. Malfliet, Phys. Rept. 198, 115 (1990))

T-matrix approximation

$$i\Pi^{<}(X,k) = \sum_{\{a,b,c\}} \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \int \frac{d^4k_3}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(k_1 + k_2 - k_3 - k)$$
$$\times \left|T(k_1^0 + k_2^0 + i\epsilon, \mathbf{k}_1 + \mathbf{k}_2)\right|^2 iG^{<}_{D_a}(X,k_1) iG^{<}_{\Phi_b}(X,k_2) iG^{>}_{\Phi_c}(X,k_3)$$



Transport equation

$$\left(k^\mu - \frac{1}{2}\frac{\partial \mathrm{Re}\, \Pi^R(X,k)}{\partial k_\mu}\right)\frac{\partial i G_D^<(X,k)}{\partial X^\mu} = \frac{1}{2} i\Pi^<(X,k) i G_D^>(X,k) - \frac{1}{2} i\Pi^>(X,k) i G_D^<(X,k)$$

Kadanoff-Baym Ansätze:

$$iG_D^<(X,k) = 2\pi S_D(X,k) f_D(X,k^0)$$

$$iG_\Phi^<(X,k) = 2\pi S_\Phi^{(0)}(X,k)\ f_\Phi^{(0)}(X,k^0)$$

Approximation on light sector [Schenk (1993), Toublan (1997)]:

$$S_\Phi^{(0)}(k^0,\boldsymbol{k}) = \frac{1}{2E_k}[\delta(k^0 - E_k) - \delta(k^0 + E_k)]$$

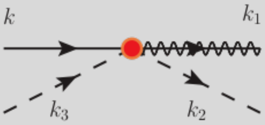
and

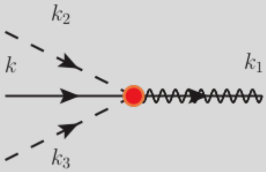
$$f_\Phi^{(0)}(k^0) = \frac{1}{e^{k^0/T} - 1}$$

using the faster equilibration of light sector

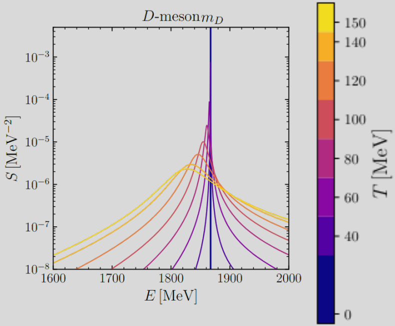
“Unitary” contribution: $\lambda' = +$

2 processes evaluated at $|T(k^0 + E_3, \boldsymbol{k} + \boldsymbol{k}_3)|^2$:



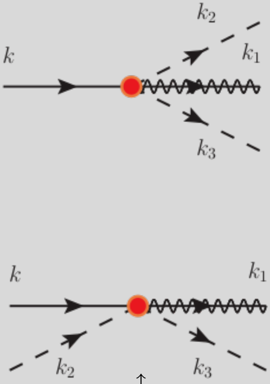


$3 \to 1$ process
only effective at $T \neq 0$
for off-shell D meson

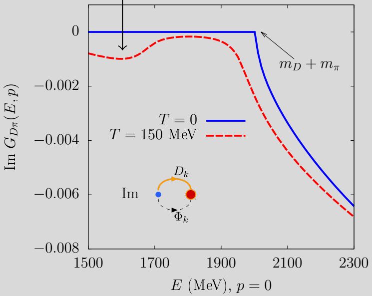


“Landau” contribution: $\lambda' = -$

2 processes evaluated at $|T(k^0 - E_3, \boldsymbol{k} + \boldsymbol{k}_3)|^2$:



$2 \to 2$ process **only effective at** $T \neq 0$ where Landau cut of $\mathrm{Im}\ G$ emerges



Not suppressed at high temperatures!