Absolute-mass threshold resummation for the production of four top quarks

Laura Moreno Valero

in collaboration with Melissa van Beekveld and Anna Kulesza

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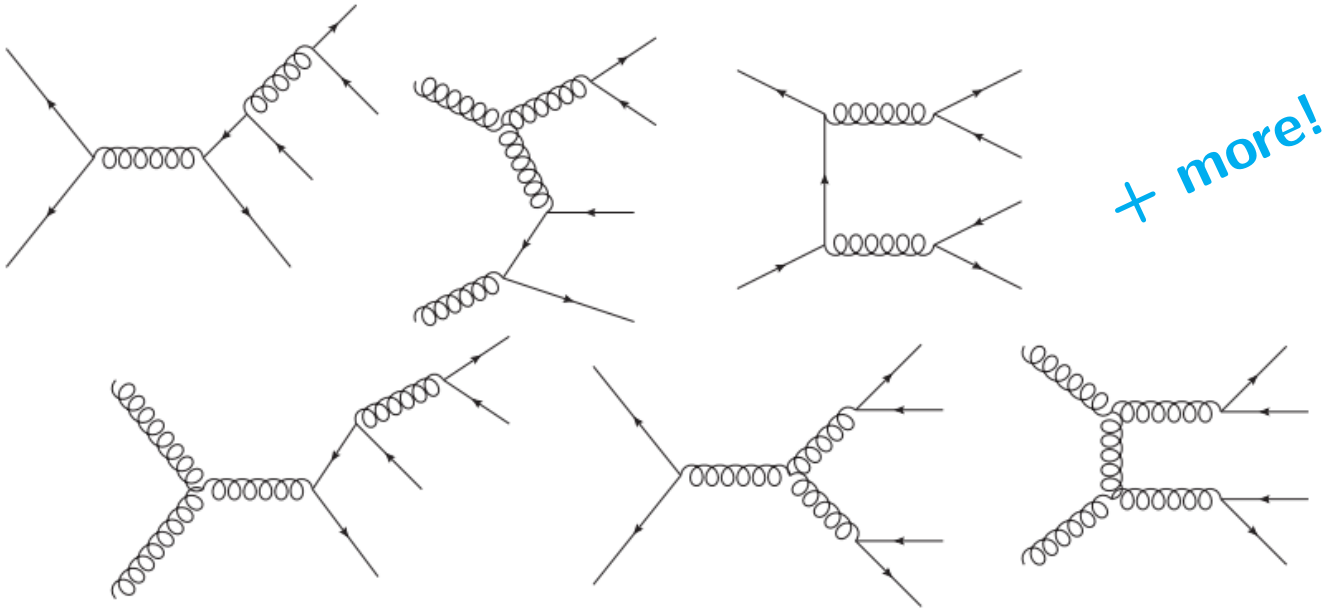
WHY 4 TOP?

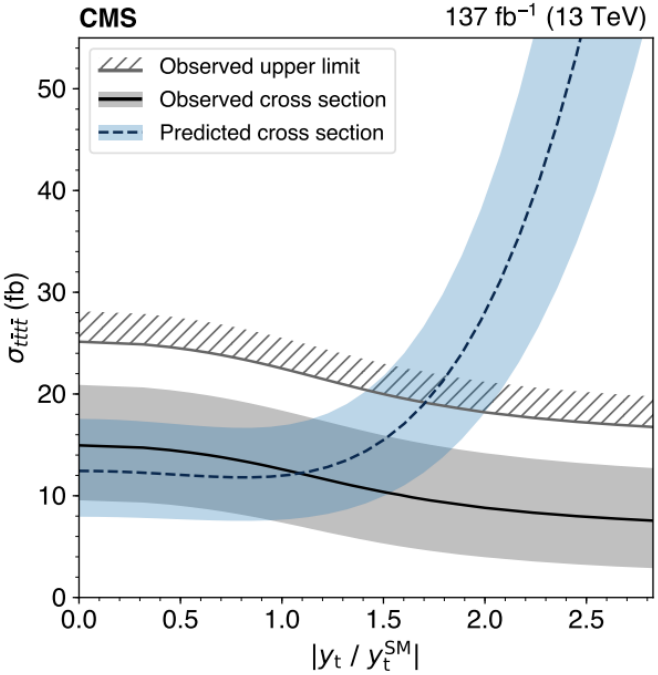


WHY 4 TOP?

- Four coloured massive particles in the final state!
- **Sensitive** to new physics (gluinos, scalar gluons, heavy scalar bosons, ...)
- First evidence:
  - ATLAS *[Eur. Phys. J. C 80, 1085; JHEP 11, 118; Phys. Rev. D 99, 052009]*
  - CMS *[Eur. Phys. J. C 80, 75; JHEP 11, 082]*

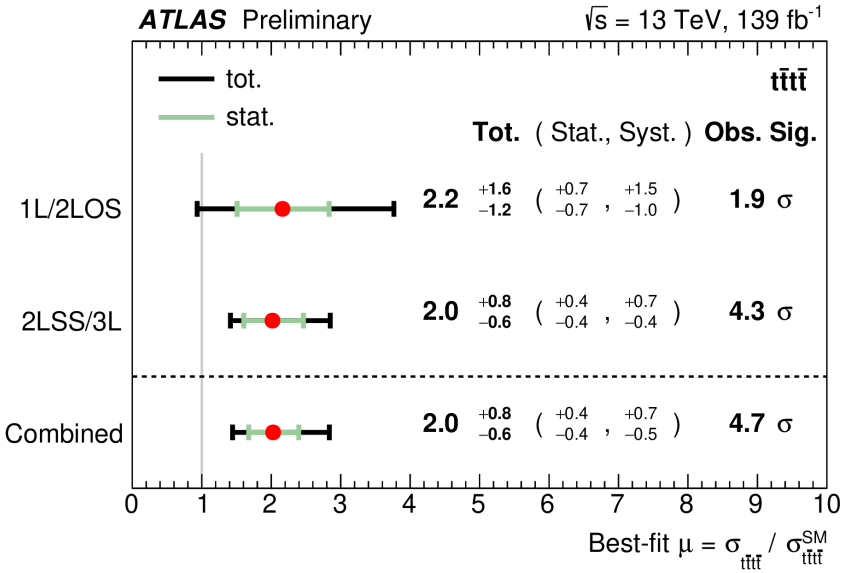
CMS

[Eur. Phys. J. C 80, 75]



ATLAS

[JHEP 11, 118]



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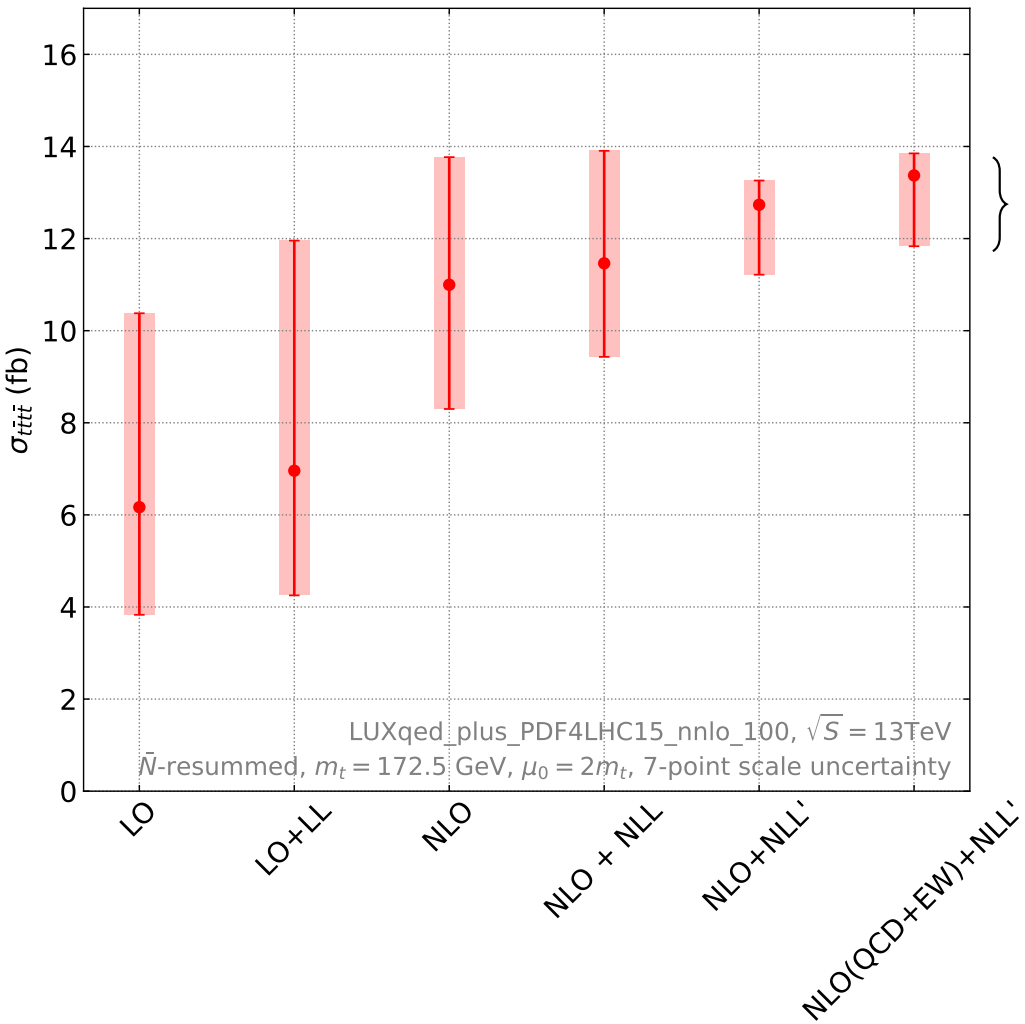
GOAL: extend the precision of theoretical predictions beyond NLO for $pp \to t\bar{t}t\bar{t}$ by means of **resummation** techniques



theory error due to scale uncertainty

Preliminary

NLO(QCD+EW): EW corrections up to $\mathcal{O}(\alpha^2)$

- Improving theoretical precision via soft gluon emission
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  - Gluons emitted from coloured particles
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  - Logarithmic terms grow as threshold is approached
  - Softness variable w ($\sim$ distance to the threshold)

$$w = 1 - \rho = 1 - M^2/s \quad \text{with } M = 4m_t \quad \text{(absolute mass threshold)}$$

$$\longrightarrow \quad \left[\frac{\log^k(1-\rho)}{1-\rho}\right]_+$$

$$\hat{\sigma}/\sigma_0 \sim 1 + \alpha_s (L+1) + \alpha_s^2 \left(L^3 + L^2 + L + 1\right) + ...$$

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- **Soft Gluon Resummation**

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- Large logs L can spoil predictive power of perturbative series
- **Soft Gluon Resummation**
  - Systematic treatment to all orders: resummation
  - Relies on: $|\mathcal{M}|^2$ and Phase Space factorization $\rightarrow$ Mellin space

$$L := \left[\frac{\log(1-\rho)}{1-\rho}\right]_+ \quad \longrightarrow \quad \tilde{L} := \log N$$

$$\hat{\sigma}^{\mathrm{res}}(N) \sim \mathcal{F}(\alpha_s)\exp\left[\tilde{L}\,g_1(\alpha_s\tilde{L}) + g_2(\alpha_s\tilde{L}) + \ldots\right]$$

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$$\hat{\sigma}^{\mathrm{res}}(N) \sim \mathcal{F}(\alpha_s)\exp\left[\underbrace{\tilde{L}\,g_1(\alpha_s\tilde{L})}_{\substack{\textbf{LL} \\ \alpha_s^n \log^{2n} N}} + \underbrace{g_2(\alpha_s\tilde{L})}_{\substack{\textbf{NLL} \\ \alpha_s^n \log^{2n-1} N}} + \ldots\right]$$

- Resummed partonic cross section for $pp \to t\bar{t}t\bar{t}$

$$\hat{\sigma}^{\text{res}}_{ij\to t\bar{t}t\bar{t}} = \text{Tr}\left[\ \mathbf{H}_{ij\to t\bar{t}t\bar{t}}\ \ \mathbf{S}_{ij\to t\bar{t}t\bar{t}}\ \right]\ \Delta_i\,\Delta_j$$

$$= \text{Tr}\left[\ \mathbf{H}_{ij\to t\bar{t}t\bar{t}}\ \ \bar{\mathbf{U}}_{ij\to t\bar{t}t\bar{t}}\,\tilde{\mathbf{S}}_{ij\to t\bar{t}t\bar{t}}\,\mathbf{U}_{ij\to t\bar{t}t\bar{t}}\ \right]\ \Delta_i\,\Delta_j$$

with

$$\mathbf{U}_{ij\to t\bar{t}t\bar{t}}(N, Q^2, \mu_F^2, \mu_R^2) = \text{Pexp}\left[\int_{\mu}^{Q/\bar{N}} \frac{dq}{q}\,\mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s(q^2)\right)\right]$$

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Incoming jet functions

$$= \text{Tr}\left[\, \mathbf{H}_{ij\to t\bar{t}t\bar{t}} \;\; \bar{\mathbf{U}}_{ij\to t\bar{t}t\bar{t}} \, \tilde{\mathbf{S}}_{ij\to t\bar{t}t\bar{t}} \, \mathbf{U}_{ij\to t\bar{t}t\bar{t}} \,\right] \boxed{\Delta_i \, \Delta_j}$$

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Hard Function **Soft Piece**

with

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Soft Anomalous Dimension Matrix

$$\mathbf{U}_{ij\to t\bar{t}t\bar{t}}(N, Q^2, \mu_F^2, \mu_R^2) = \text{Pexp}\left[\int_{\mu}^{Q/\bar{N}} \frac{dq}{q}\boxed{\mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s(q^2)\right)}\right]$$

- One-loop Soft Anomalous Dimension $\mathbf{\Gamma}^{(1)}_{ij\to t\bar{t}t\bar{t}}$

$$\mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}(\alpha_s) = \left(\frac{\alpha_s}{\pi}\right)\mathbf{\Gamma}^{(1)}_{ij\to t\bar{t}t\bar{t}} + \left(\frac{\alpha_s}{\pi}\right)^2\mathbf{\Gamma}^{(2)}_{ij\to t\bar{t}t\bar{t}} + \dots$$

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- In the absolute mass threshold (for $N_c = 3$)

$$\mathrm{Re}[\mathbf{\Gamma}^{(1)}_{q\bar{q}\to t\bar{t}t\bar{t}}] = \mathrm{diag}\left(0, 0, -\frac{3}{2}, -\frac{3}{2}, -\frac{3}{2}, -\frac{3}{2}\right)$$

$$\mathrm{Re}[\mathbf{\Gamma}^{(1)}_{gg\to t\bar{t}t\bar{t}}] = \mathrm{diag}\left(-4, -3, -3, -2, -\frac{3}{2}, -\frac{3}{2}, -\frac{3}{2}, -\frac{3}{2}, -\frac{3}{2}, -\frac{3}{2}, -\frac{3}{2}, -\frac{3}{2}, 0, 0\right)$$

[Quadratic Casimirs of the corresponding SU(3) representations]

$$\mathbf{H} = \mathbf{H}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\mathbf{H}^{(1)} + \ldots$$
$$\tilde{\mathbf{S}} = \tilde{\mathbf{S}}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\tilde{\mathbf{S}}^{(1)} + \ldots$$

- **NLL** accuracy: exponential functions at NLL together with

$$\mathrm{Tr}\left[\mathbf{H}\tilde{\mathbf{S}}\right] = \mathrm{Tr}\left[\mathbf{H}^{(0)}\tilde{\mathbf{S}}^{(0)}\right]$$

- **NLL**$'$ accuracy: exponential functions at NLL together with

$$\mathrm{Tr}\left[\mathbf{H}\tilde{\mathbf{S}}\right] = \mathrm{Tr}\left[\mathbf{H}^{(0)}\tilde{\mathbf{S}}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\mathbf{H}^{(1)}\tilde{\mathbf{S}}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\mathbf{H}^{(0)}\tilde{\mathbf{S}}^{(1)}\right]$$

$$\sigma_{t\bar{t}t\bar{t}}^{\mathrm{NLL}(')}(\tau) = \int_{\mathcal{C}} \frac{\mathrm{d}N}{2\pi i}\, \tau^{-N}\, f_i(N+1, \mu_F^2)\, f_j(N+1, \mu_F^2)\, \hat{\sigma}_{ij\to t\bar{t}t\bar{t}}^{\mathrm{res}}(N)$$

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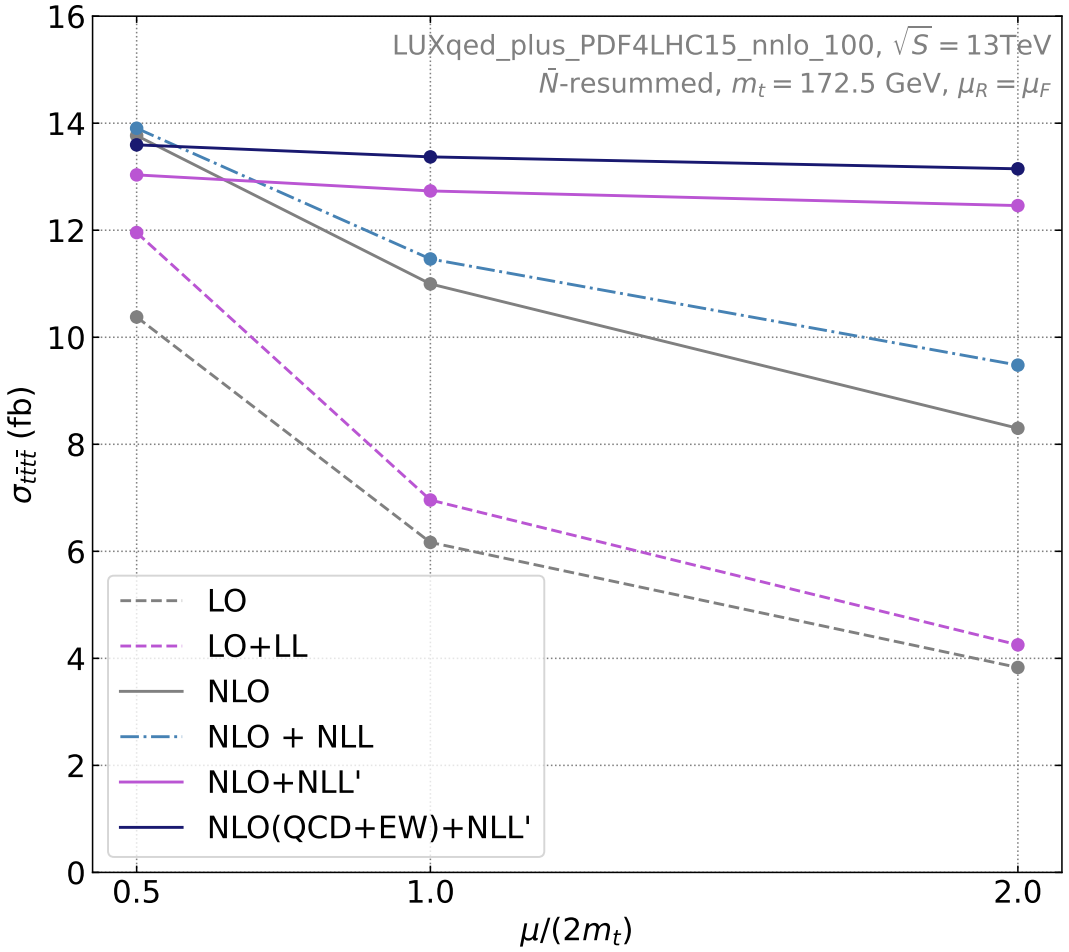
- Combination fixed-order + resummation $\Longrightarrow$ **Matching**

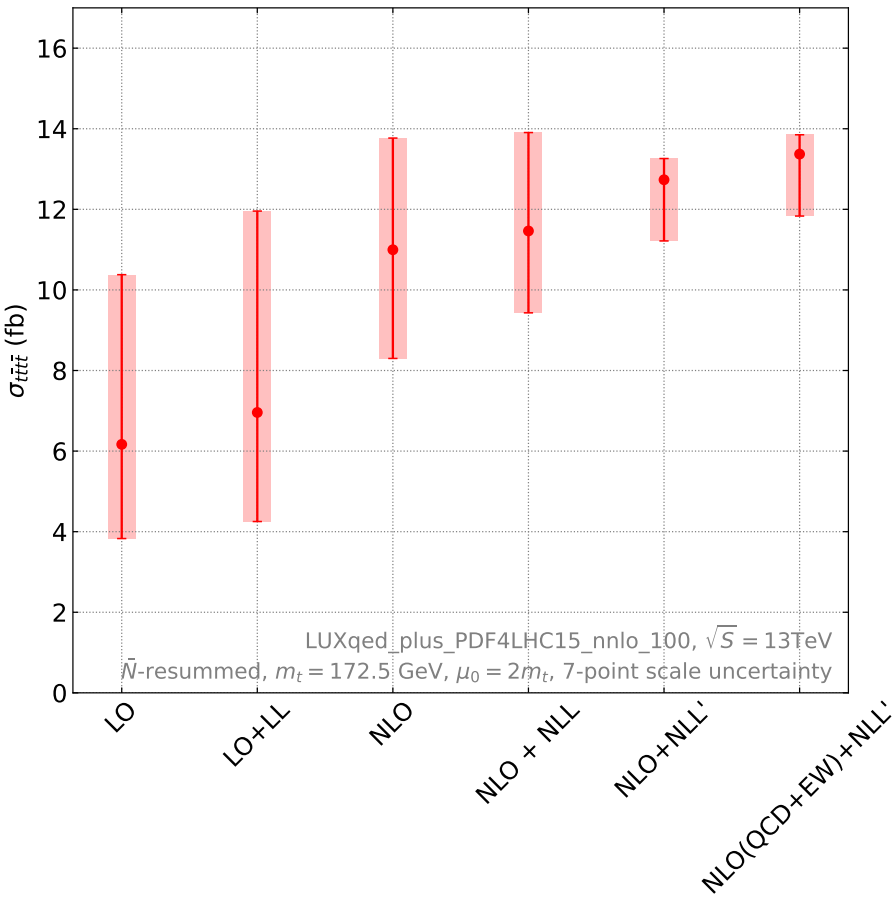
$$\sigma_{t\bar{t}t\bar{t}}^{\mathrm{NLO+NLL}(')}(\tau) = \sigma_{t\bar{t}t\bar{t}}^{\mathrm{NLO}}(\tau) + \int_{\mathcal{C}} \frac{\mathrm{d}N}{2\pi i}\, \tau^{-N}\, f_i(N+1,\mu_F^2)\, f_j(N+1,\mu_F^2) \times \underbrace{\left[\hat{\sigma}_{ij\to t\bar{t}t\bar{t}}^{\mathrm{res}}(N) - \hat{\sigma}_{ij\to t\bar{t}t\bar{t}}^{\mathrm{res}}(N)|_{\mathrm{NLO}}\right]}_{\text{avoid double counting with NLO!}}$$



- Big impact of resummation
- $\mu_F = \mu_R$

NLO(QCD+EW):
EW corrections up to $\mathcal{O}(\alpha^2)$

Preliminary



Preliminary

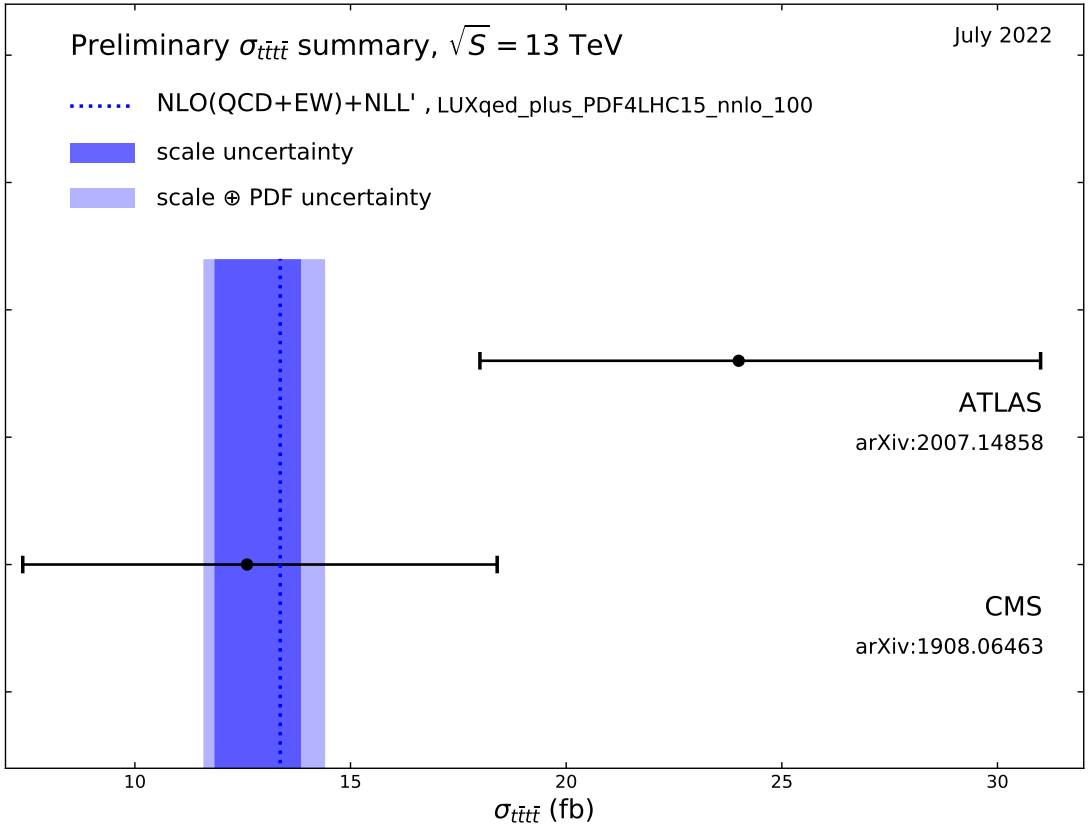
	$\sigma_{t\bar{t}t\bar{t}}$ [fb]	K-factor
NLO	$11.00^{+25.2\%}_{-24.5\%}$	
NLO+NLL	$11.46^{+21.3\%}_{-17.7\%}$	1.04
NLO+NLL′	$12.73^{+4.1\%}_{-11.8\%}$	1.16
NLO(QCD+EW)	$11.64^{+23.2\%}_{-22.8\%}$	
NLO(QCD+EW)+NLL′	$13.37^{+3.6\%}_{-11.4\%}$	1.15

7-point scale variation

NLO(QCD+EW):

EW corrections up to $\mathcal{O}(\alpha^2)$

PDF error: $^{+6.9\%}_{-6.9\%}$ NLO



Our result

NLO(QCD+EW)+NLL′

$$13.37\,(2)\,^{+3.6\%}_{-11.4\%}\,^{+6.9\%}_{-6.9\%}\ \text{fb}$$

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