

Mixed QCD-electroweak corrections to the Drell-Yan process in the high invariant mass region

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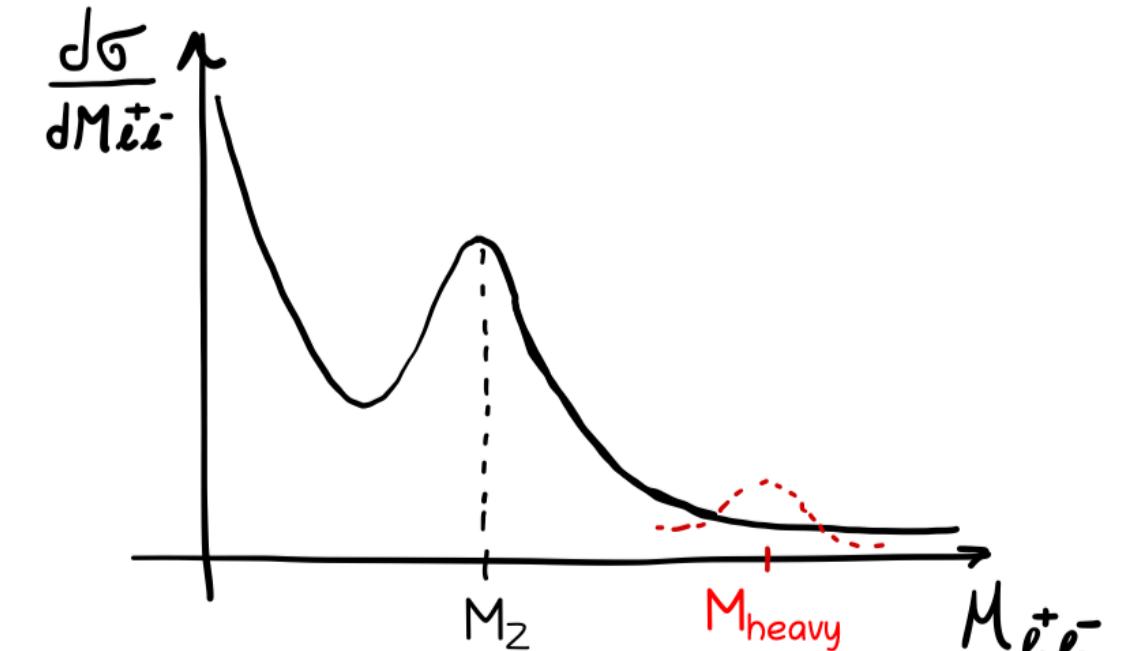
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In collaboration with: F. Buccioni, F. Caola, H. Chawdhry, F. Devoto, M. Heller, A. von Manteuffel, K. Melnikov, R. Röntsch
Based on: arXiv 2203.11237

Motivation: why Drell-Yan in the high invariant mass region?

- Hunting for New Physics (NP)

- Many extensions to the SM contain weakly-coupled states which can decay into leptons
 - Search for shape distortions in kinematic distributions



- Constrain heavy NP in a model-independent way using **SMEFT**

[[Barbieri, Pomarol, Rattazzi, Strumia '04](#)] [[Alioli, Farina, Pappadopulo, Ruderman '17](#)] [[Farina et al.'17](#)]

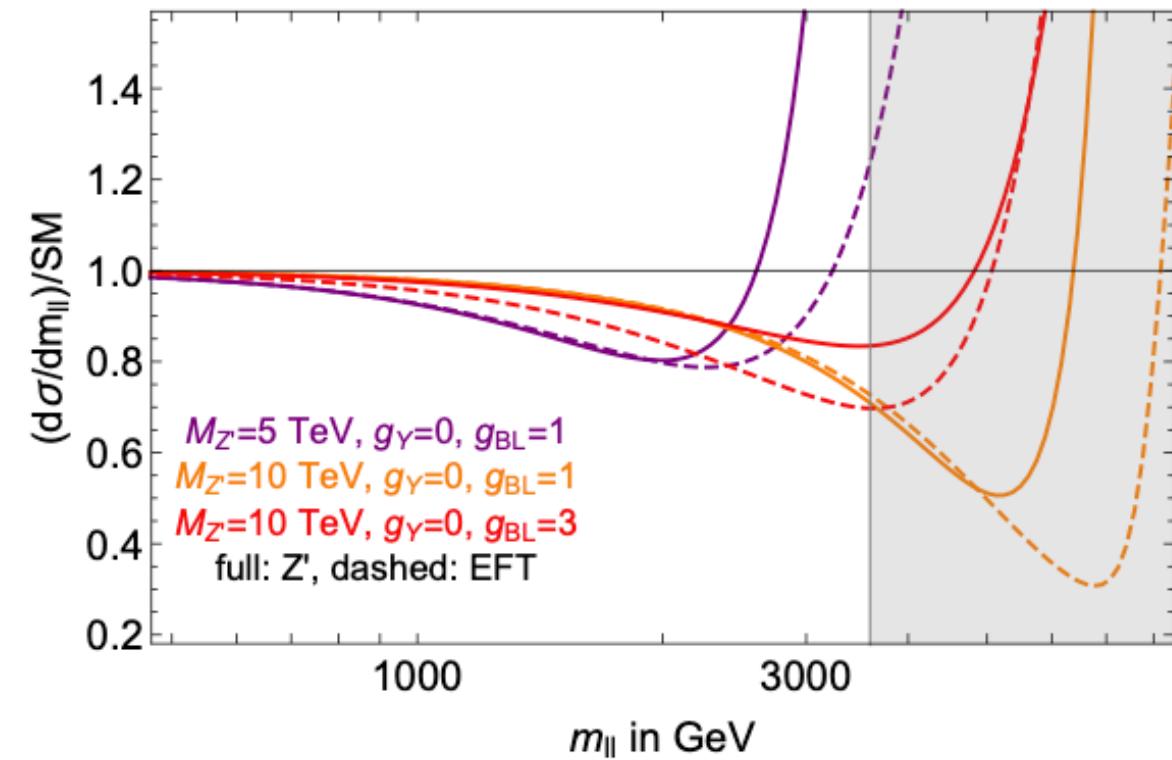
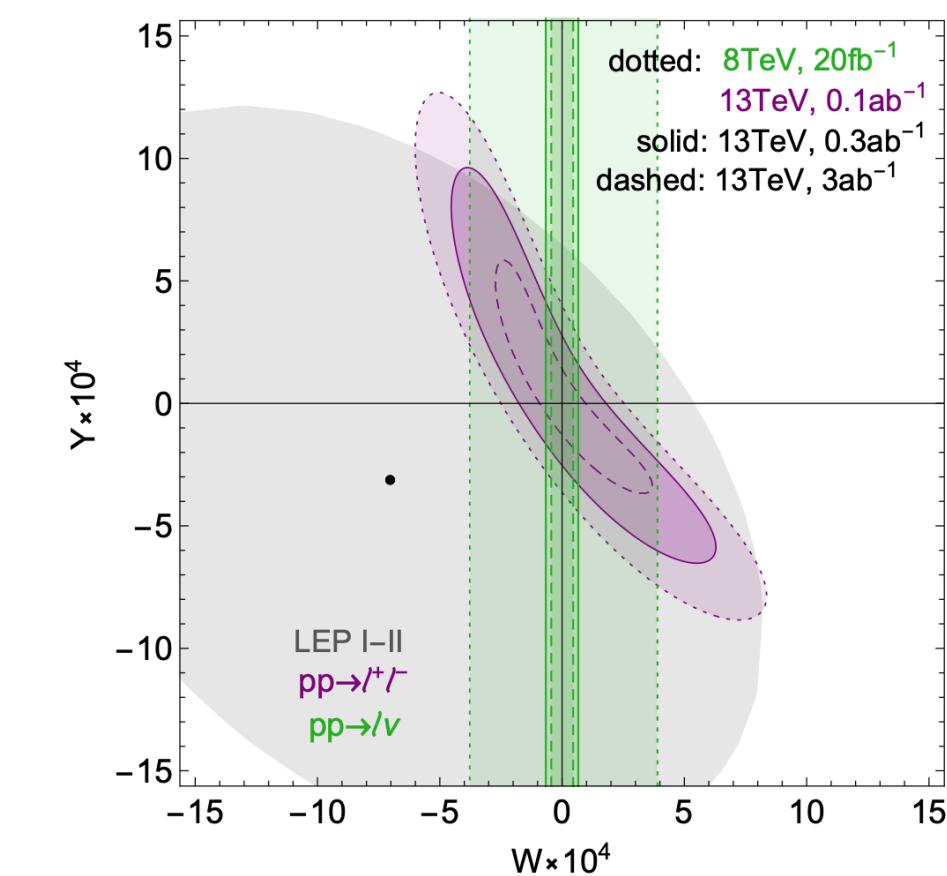
- Impact on *oblique parameters*, which are constrained at **permille** level with **LEP**
- Contribution of **dimension-6 operators** → *quadratic growth with energy*

LHC expected to reach only **percent-level precision**

BUT

Higher energy can compensate for the limited precision

→ enhancement factor ~ 150 for $\sqrt{s} \simeq 1\text{TeV}$



Motivation: why mixed QCD×EW corrections?

$$d\hat{\sigma}_{ij} = d\hat{\sigma}_{ij, \text{LO}} \left(1 + \alpha_s \Delta_{ij, \text{NLO}}^{QCD} + \alpha_{ew} \Delta_{ij, \text{NLO}}^{EW} + \alpha_s^2 \Delta_{ij, \text{NNLO}}^{QCD} + \alpha_s \alpha_{ew} \Delta_{ij, \text{NNLO}}^{QCD \otimes EW} + \alpha_s^3 \Delta_{ij, \text{N3LO}}^{QCD} + \dots \right)$$

Couplings: $\alpha_s \sim 0.1$, $\alpha_{ew} \sim 0.01$

Target precision: \sim few %

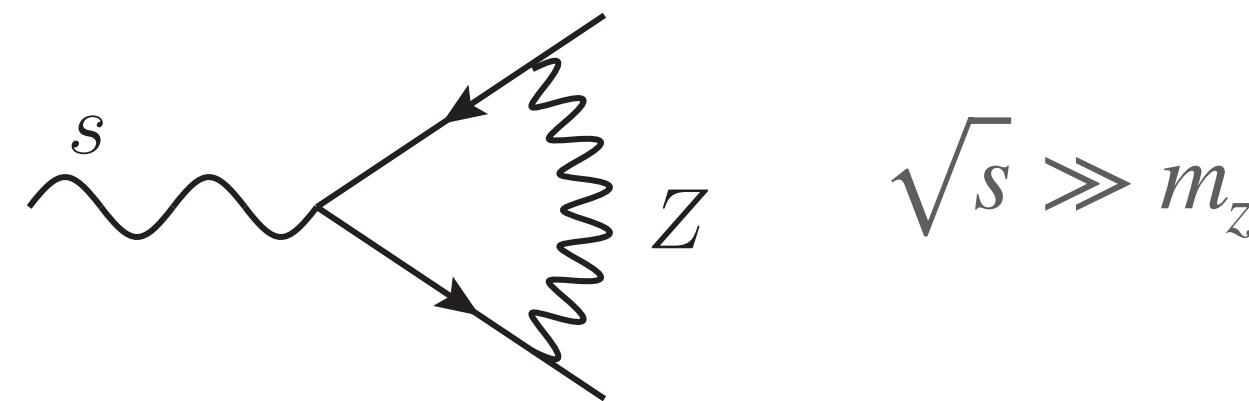
NLO EW corrections: $\sim \mathcal{O}(\alpha_{ew}) \sim 1\%$

Mixed QCD×EW corrections: $\sim \mathcal{O}(\alpha_s \alpha_{ew}) \sim 0.1\%$ 

Power counting predicts a permille effect

EW contributions enhanced by Sudakov logarithms:

[Kuhn, Penin, Smirnov '00][Ciafaloni, Ciafaloni, Comelli '01][Denner, Pozzorini '01]



$$\frac{\alpha_{ew}}{4\pi \sin^2 \theta_W} \log^2\left(\frac{s}{m_z^2}\right) \sim 10\%, \quad \frac{\alpha_{ew}}{4\pi \sin^2 \theta_W} \log\left(\frac{s}{m_z^2}\right) \sim 1.6\%, \quad \sqrt{s} = 2\text{TeV}$$

More accurate expectation: (20% QCD) \times (10% EW) 

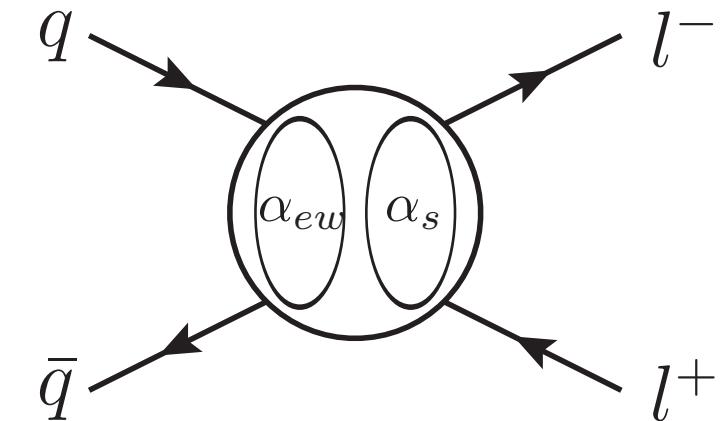
Potential 2% correction in the high invariant mass region

Challenges of the calculation

Fixed-order calculation of mixed corrections involves two main **technical challenges**:

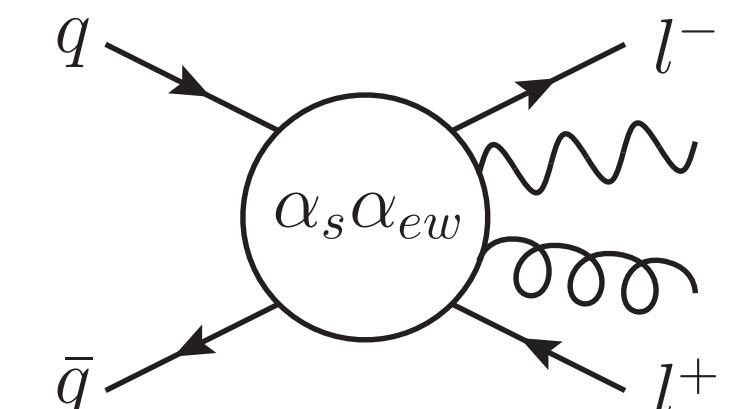
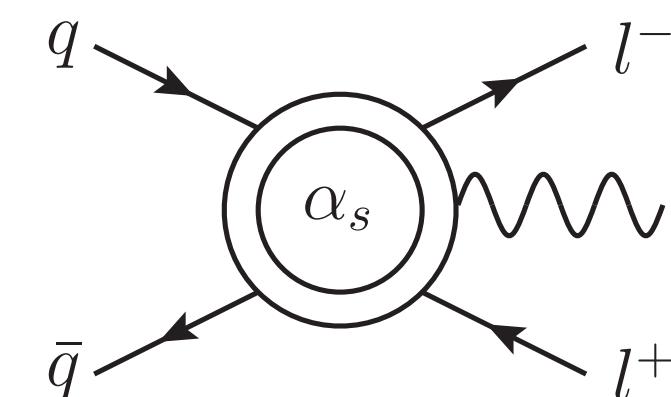
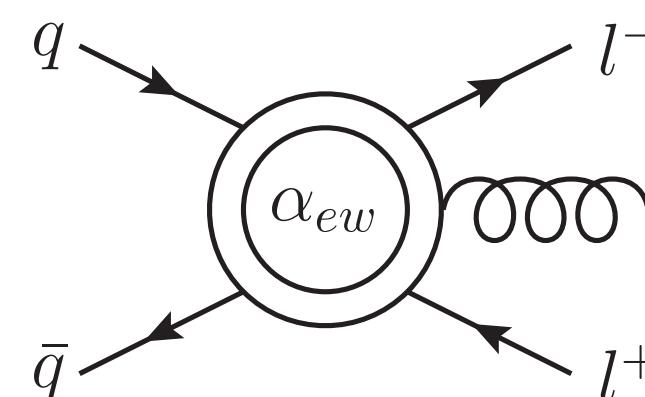
Amplitudes:

- **Two-loop integrals** involving **various masses**



IR singularities

- Extraction of **soft and collinear** singularities
→ fully differential predictions



❖ Two independent calculation → results qualitatively in agreement

	<i>Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini [2106.11953]</i>	<i>Buccioni, Caola, Chawdhry, Devoto, Heller, A. von Manteuffel, Melnikov, Röntsch, S-S [2203.11237]</i>
Two-loop amplitude	Semi-analytic MIs via series expansion	Fully analytic
IR singularities	qT slicing (implemented in MATRIX) massive leptons (mass as IR regulator)	Nested soft-collinear subtraction massless leptons (photon recombination)

Nested soft-collinear subtraction at NNLO: generalities

Extension of FKS subtraction to NNLO: originally introduced to treat pure QCD processes [Caola, Melnikov, Röntsch 1702.01352]

$$\int \text{---} d\Phi_g = \underbrace{\int \left[\text{---} - \text{---} \right] d\Phi_g}_{\text{Finite in } d=4, \text{ integrable numerically}} + \underbrace{\int \text{---} d\Phi_g}_{\text{exposes the same } 1/\epsilon \text{ poles as the virtual correction}}$$

- Fully local and fully analytic
- Transparent treatment of IR singularities

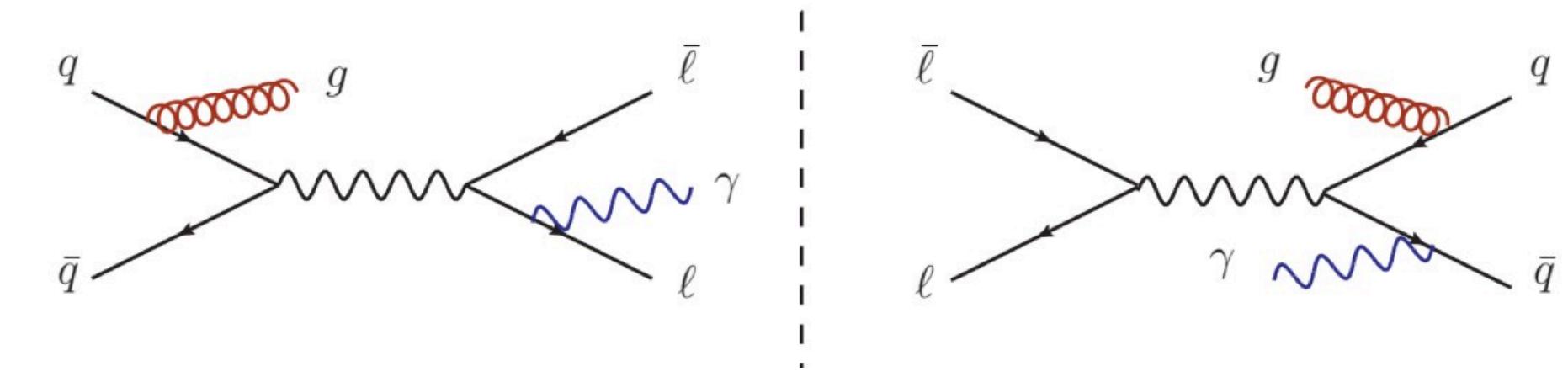
Independent subtraction of soft and collinear divergences → colour coherence

Collinear singularities disentangled → sector decomposition + phase space partition

- Flexibility
- Core structure depends only on the partons contributing to the process
- Modular building blocks**



Mixed corrections manifest some simplifications and some complications with respect to the pure QCD case

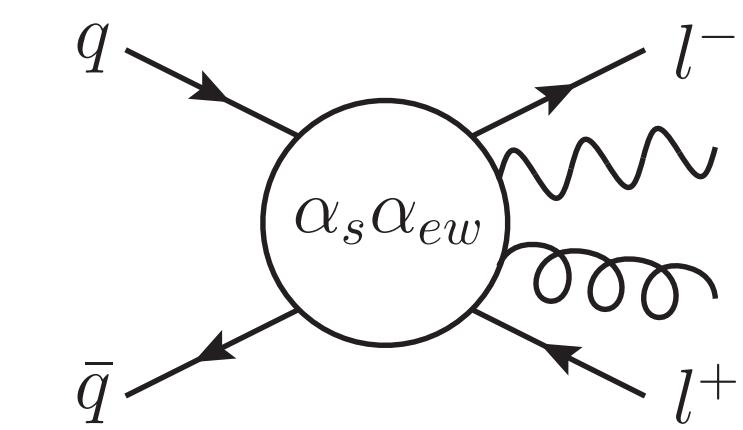


Mixed QCD-EW: differences with respect to NNLO QCD

Examples: $q\bar{q} \rightarrow Z \rightarrow e^-e^+ g\gamma$ [2203.11237]

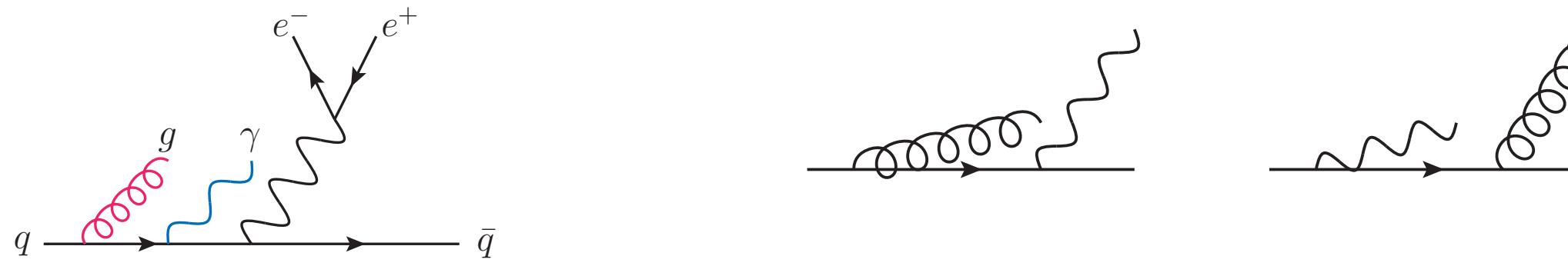
Soft limits:

- Double soft limit factorises into **NLO QCD** x **NLO EW**



Collinear limits:

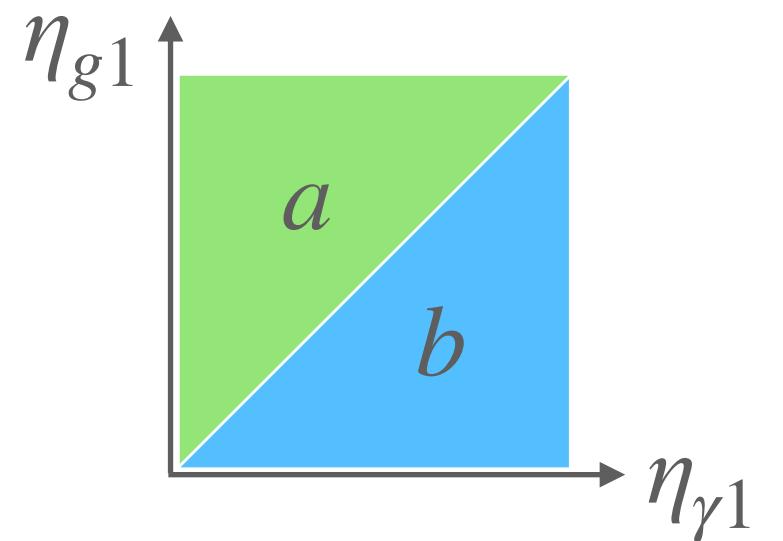
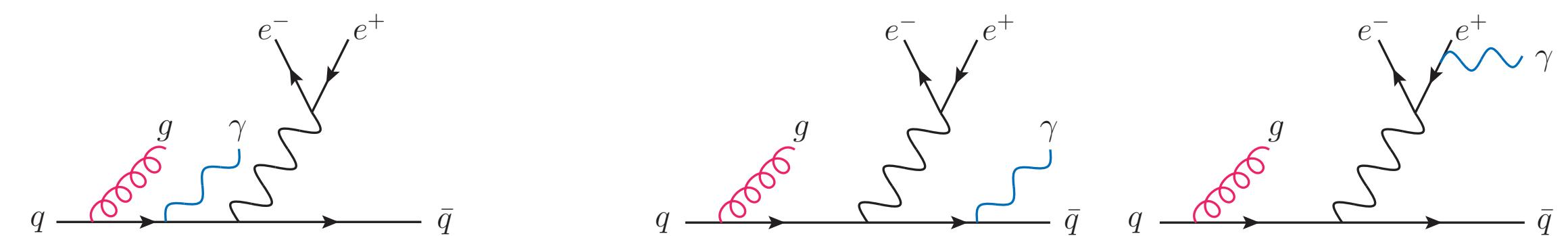
- Single, double and triple collinear limits to disentangle
→ **More sectors** to account for final state radiation
- Strongly-ordered limits to disentangle in triple collinear sectors
→ **BUT no photon-gluon** collinear singularity



→ Non-trivial structures to integrate

→ **BUT abelianization** of known results [de Florian, Der, Fabre '18][Delto, Jaquier, Melnikov, Röntsch '19]

$$\begin{aligned} 1 &= \theta_a + \theta_b \\ &= \theta(\eta_{g1} - \eta_{\gamma1}) + \theta(\eta_{\gamma1} - \eta_{g1}) \end{aligned}$$



Finite parts

Final result as combination of two- and one-loop corrections, double real radiation and pdf renormalisation:

- ✓ Check **pole cancellation** and finite parts calculation done analytically
- ✓ **Fully differential**
- ✓ **Fully local** → leads to a very stable numerical evaluation
- ✓ Cumbersome result for the finite parts
- ✓ In the **CoM reference frame, the result simplifies** remarkably
- ✓ Simple structures arise, **compact expressions**
- ✓ **Few main kinematic building blocks** can be easily identified

Phenomenology: fiducial cross section

Definition of the fiducial cross section:

$$\sqrt{s} = 13.6 \text{ TeV}$$

$$m_l = 0$$

$$m_{ll} > 200 \text{ GeV} \text{ (dressed leptons)}$$

$$R_{l\gamma} = 0.1 \text{ (dressed leptons)}$$

$$p_\perp^l > 30 \text{ GeV}$$

$$\sqrt{p_\perp^{l^+} p_\perp^{l^-}} > 35 \text{ GeV}$$

$$|y_l| < 2.5$$

$$\text{NNPDF31_nnlo_as_0118_luxqed}$$

$$\mu_F = \mu_R = m_{ll}/2 \text{ (dressed leptons)}$$

$$\sigma = \sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} + \delta\sigma^{(1,1)} + \dots$$

LO **NLO QCD** **NLO EW** **NNLO QCD** **NNLO QCDxEW**

$\sigma [\text{fb}]$	$\sigma^{(0,0)}$	$\delta\sigma^{(1,0)}$	$\delta\sigma^{(0,1)}$	$\delta\sigma^{(2,0)}$	$\delta\sigma^{(1,1)}$
$q\bar{q}$	1561.42	340.31	-49.907	44.60	-16.80
$\gamma\gamma$	59.645		3.166		
qg		0.060		-32.66	1.03
$q\gamma$			-0.305		-0.207
$g\gamma$					0.2668
gg				1.934	
sum	1621.06	340.37	-47.046	13.88	-15.71

What do we learn?

- ✓ NLO QCD $\sim +20\%$ $\delta^{\text{QCD}} \sim 8C_F \alpha_s/(2\pi) \sim 0.2$
- ✓ NLO EW $\sim -3\%$ $\delta^{\text{ew}} \sim \alpha_{ew}/\sin^2\theta_W \sim 0.03$
- ✓ NNLO QCD $\sim +0.8\%$ unexpected : $\alpha_s^2 \sim 0.014$
strong qq and qg cancellation
- ✓ QCDxEW $\sim -1\%$ unexpected : $\alpha_s \alpha_{ew} \sim 0.00089$

QCDxEW corrections larger than NNLO QCD ones!

Theoretical uncertainties

Envelope of QCD+EW related uncertainties

$$d\sigma = d\sigma^{(0,0)} \left(1 + \frac{\alpha_s}{2\pi} \delta^{(1,0)} + \left(\frac{\alpha_s}{2\pi} \right)^2 \delta^{(2,0)} + \frac{\alpha_{ew}}{2\pi} \delta^{(0,1)} + \frac{\alpha_s}{2\pi} \frac{\alpha_{ew}}{2\pi} \delta^{(1,1)} + \dots \right)$$



Reduction of EW
scheme dependence

- **QCD:** factor 2 rescaling (up and down) of central scale $\mu = m_{\ell\ell}/2$
- **EW:** variation of input parameters: G_μ scheme vs $\alpha(m_Z)$ scheme

$$\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} = 1928.3^{+1.8\%}_{-0.15\%} \text{ fb.}$$

The mixed QCD-EW corrections $\sim -1\%$
→ comparable in size to the theoretical uncertainties

$$\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} + \delta\sigma^{(1,1)} = 1912.6^{+0.65\%}_{-0\%} \text{ fb.}$$

Theoretical **uncertainty below percent** after inclusion of mixed corrections
→ pure **EW scheme uncertainty** reduced from $\sim 1\%$ to about $\sim 0.5\%$

Results do **not** include uncertainties from **PDFs**: uncertainty on $q\bar{q}$ luminosity $\sim 5\%$ for $m_{ll} \sim 2\text{TeV}$.

Phenomenology: m_{ll} windows

$$\sigma = \sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} + \delta\sigma^{(1,1)} + \dots$$

LO	NLO QCD	NLO EW	NNLO QCD	QCDxEW
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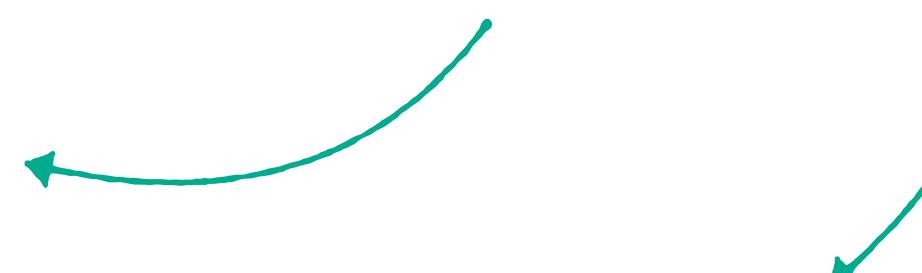
Different invariant **mass windows**: accuracy of the **factorised approximation**

$$\frac{\delta\sigma_{\text{fact.}}^{(1,1)}}{\sigma^{(0,0)}} = \frac{\delta\sigma^{(1,0)}}{\sigma^{(0,0)}} \cdot \frac{\delta\sigma^{(0,1)}}{\sigma^{(0,0)}} \quad \rightarrow \quad (\text{NLO QCD}) \cdot (\text{NLO EW})$$

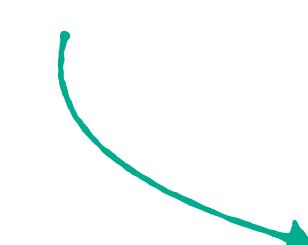
- $\Phi^{(1)}$: $200 \text{ GeV} < m_{\ell\ell} < 300 \text{ GeV}$,
- $\Phi^{(2)}$: $300 \text{ GeV} < m_{\ell\ell} < 500 \text{ GeV}$,
- $\Phi^{(3)}$: $500 \text{ GeV} < m_{\ell\ell} < 1.5 \text{ TeV}$,
- $\Phi^{(4)}$: $1.5 \text{ TeV} < m_{\ell\ell} < \infty$.

σ [fb]	$\sigma^{(0,0)}$	$\delta\sigma^{(1,0)}$	$\delta\sigma^{(0,1)}$	$\delta\sigma^{(2,0)}$	$\delta\sigma^{(1,1)}$	$\delta\sigma_{\text{fact.}}^{(1,1)}$
$\Phi^{(1)}$	1169.8	254.3	-30.98	10.18	-10.74	-6.734
$\Phi^{(2)}$	368.29	71.91	-11.891	2.85	-4.05	-2.321
$\Phi^{(3)}$	82.08	14.31	-4.094	0.691	-1.01	-0.7137
$\Phi^{(4)} \times 10$	9.107	1.577	-1.124	0.146	-0.206	-0.1946

flat QCD corr. $\sim 20\%$ LO



EW corr. grow from $\sim -3\%$ LO to $\sim -12\%$ LO
[expected from 1-loop Sudakov logs]



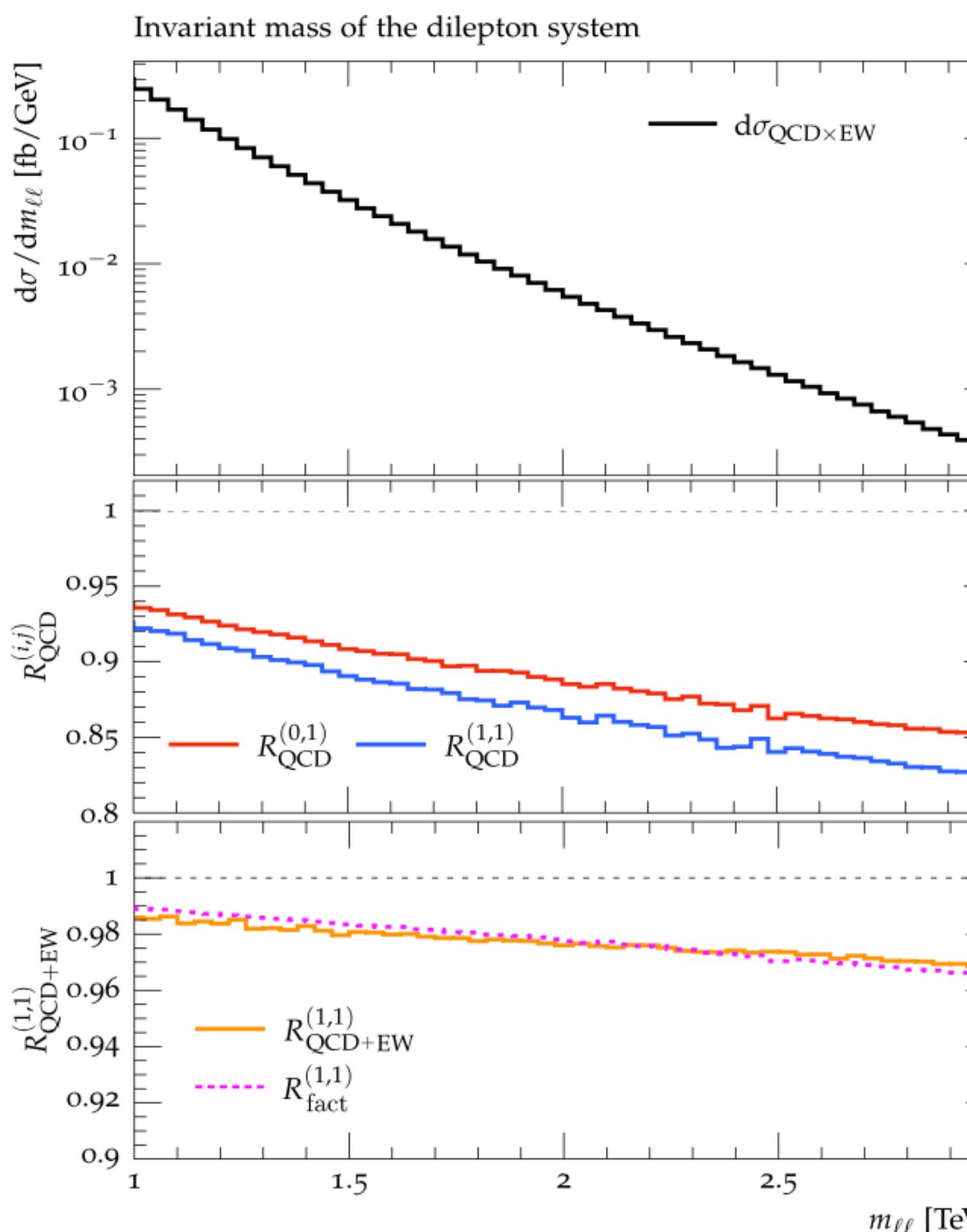
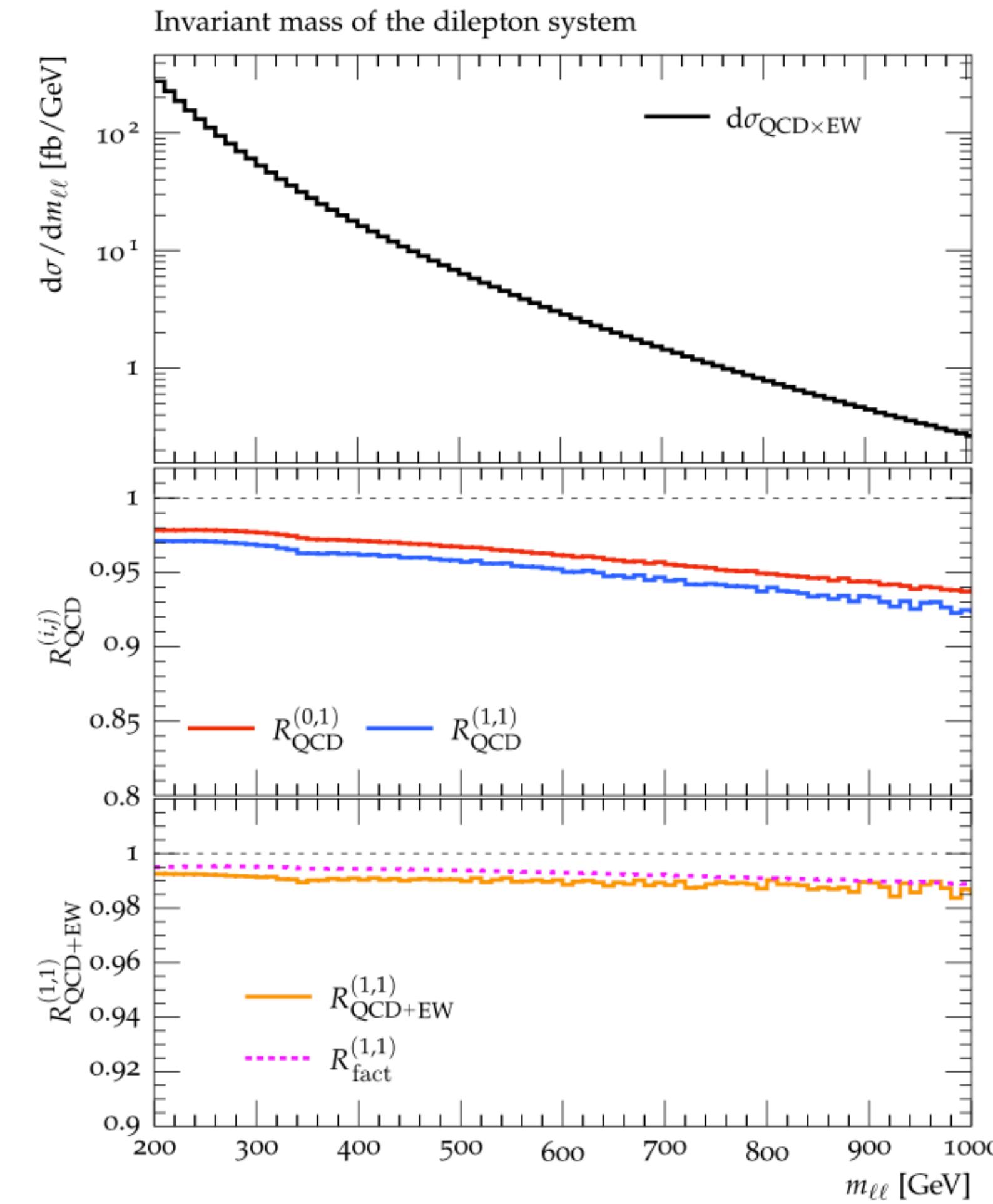
Factorised approx. improves as invariant mass grows

✓ At high invariant mass ($m_{ll} > 1.5 \text{ TeV}$) the **factorised approx.** captures more than **90%** of the **exact result**



→ Expected: factorised approx. correctly reproduces the leading Sudakov logs, which dominate at high invariant masses

Kinematic distributions: invariant mass



- ✓ Impact of **NLO EW** corr. on NLO QCD:
 - small at low invariant mass: $\sim -2\%$ @ 200GeV
 - reaches $\sim -15\%$ @ 3TeV

- ✓ Impact of **NLO EW+QCD×EW** corr. on NLO QCD:
 - similar shape as NLO EW corr.
 - reaches $\sim -18\%$ @ 3TeV

- ✓ Impact of **mixed QCD×EW** corr. on NLO (QCD+EW):
 - non entirely flat shape
 - large at low invariant mass: $\sim -0.8\%$ @ 200GeV
 - reaches $\sim -3\%$ @ 3TeV

- ✓ Factorised approximation:
 - under estimation of mixed corr. in $m_{\ell\ell} \in [200,1000]\text{GeV}$ region
 - better agreement in $m_{\ell\ell} \in [1,3]\text{TeV}$ region

Angular distribution and forward-backward asymmetry

Angular distributions can test quark to lepton interactions

Forward-backward asymmetry has been measured in the high invariant mass region
[CMS 2202.12327]

$$m_{ll} > 200\text{GeV} \text{ we found: } A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = 0.1580^{+0.15\%}_{-0.07\%}$$

Mixed QCD-EW corrections changes the value by about **2 permille**
→ comparable with the uncertainties

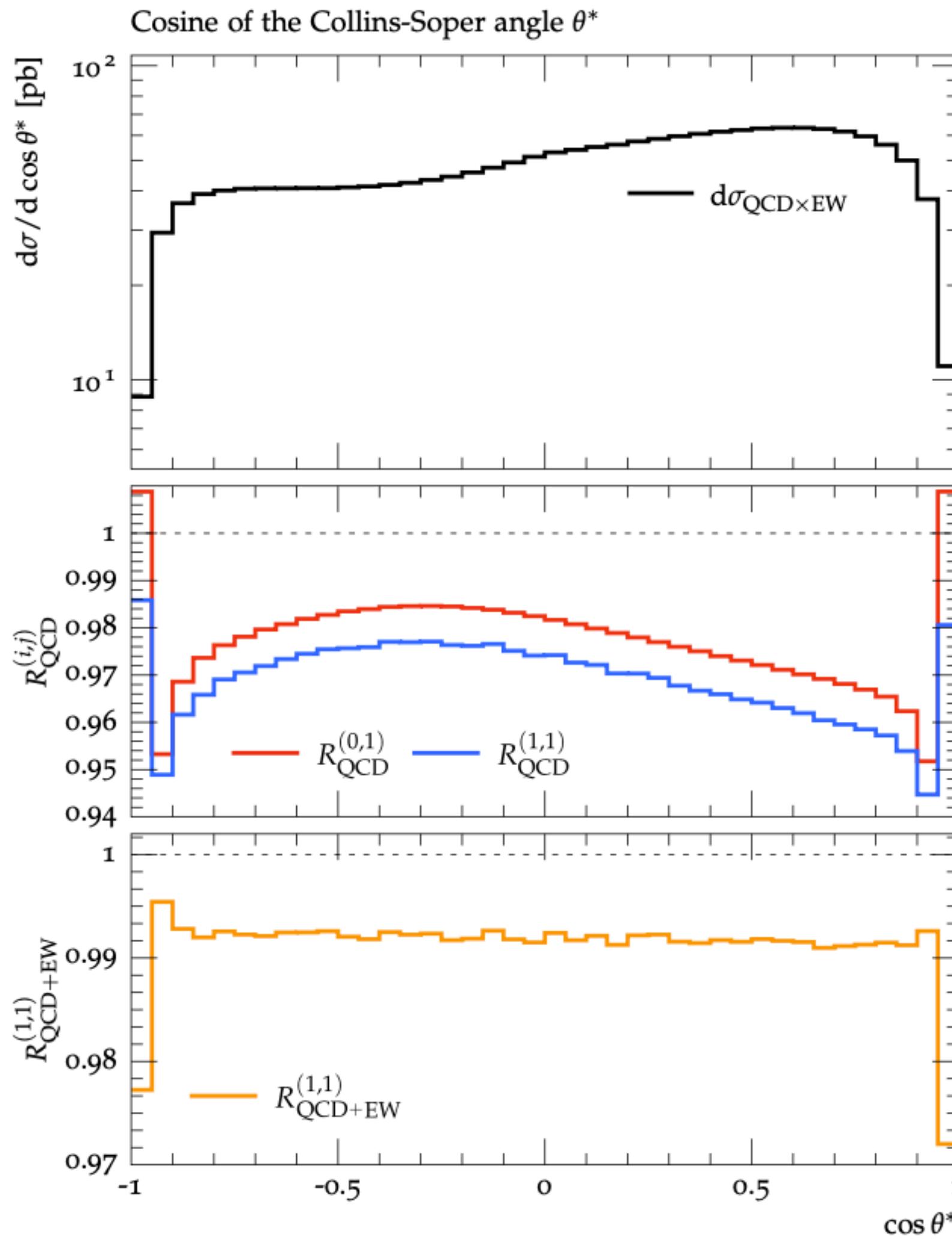
	\tilde{A}_{FB}	A_{FB}
$\Phi^{(1)}$	$0.1442^{+0.05\%}_{-0.31\%}$	$0.1440^{+0.11\%}_{-0.09\%}$
$\Phi^{(2)}$	$0.1852^{+0.08\%}_{-0.40\%}$	$0.1847^{+0.10\%}_{-0.19\%}$
$\Phi^{(3)}$	$0.2401^{+0.13\%}_{-0.64\%}$	$0.2388^{+0.06\%}_{-0.47\%}$
$\Phi^{(4)}$	$0.3070^{+0.49\%}_{-1.5\%}$	$0.3031^{+0.19\%}_{-1.2\%}$

Excluding mixed corrections

Including mixed corrections

Mixed QCD-EW corrections affect A_{FB} at the **percent** level for $m_{ll} \gtrsim 1\text{TeV}$

→ this shift should become observable at HL-LHC



Conclusions

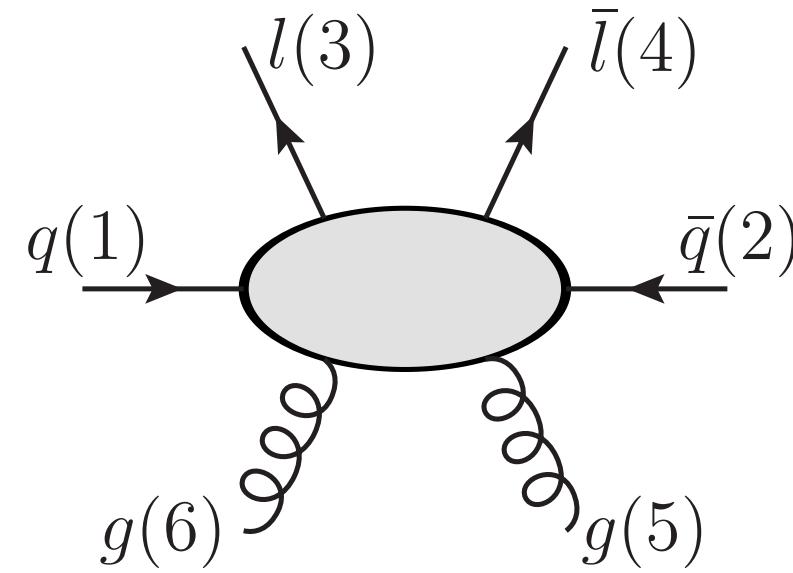
1. Mixed QCDxEW corrections to Drell-Yan are important to **search for NP in the high energy regime**
2. The **results** show a remarkably **simple structure**
3. Mixed QCD-EW amount to about -1% of the fiducial **LO** cross-section
 - **larger than expected from coupling magnitude**
 - **even with relative low cut on m_{ll}**
4. Good approximation by the **product of QCD and EW corrections in the TeV region**
5. Reduction of theoretical uncertainties related to EW input scheme

Thank you for your attention!

Backup

NNLO QCD difficulties and solutions

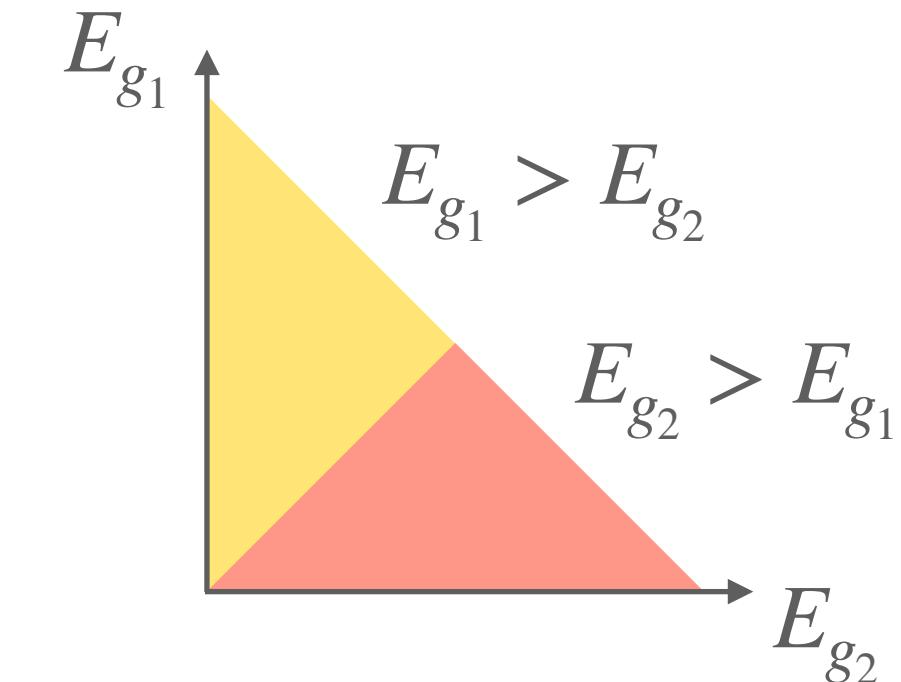
Examples: $q\bar{q} \rightarrow Z \rightarrow e^-e^+ gg$ [Caola, Melnikov, Röntsch 1702.01352]



Soft limits:

- Non-trivial structure of double soft eikonal
- Strongly-ordered limits to disentangle

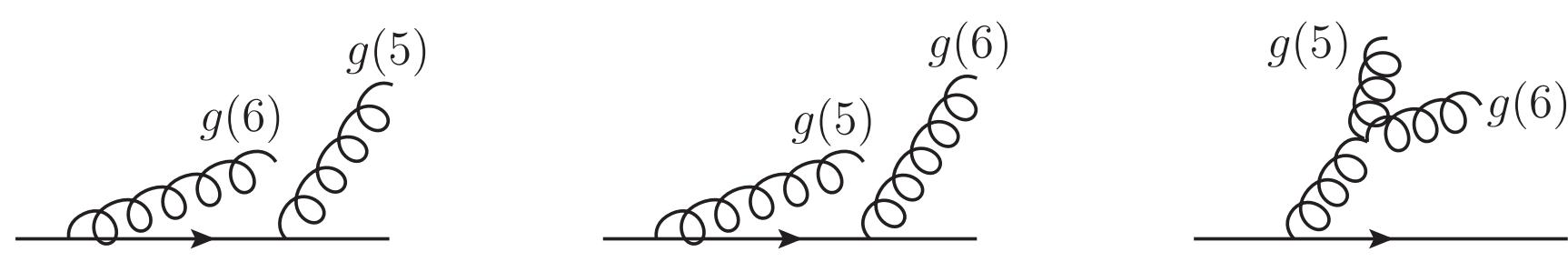
$$1 = \theta(E_{g_1} - E_{g_2}) + \theta(E_{g_2} - E_{g_1})$$



Collinear limits:

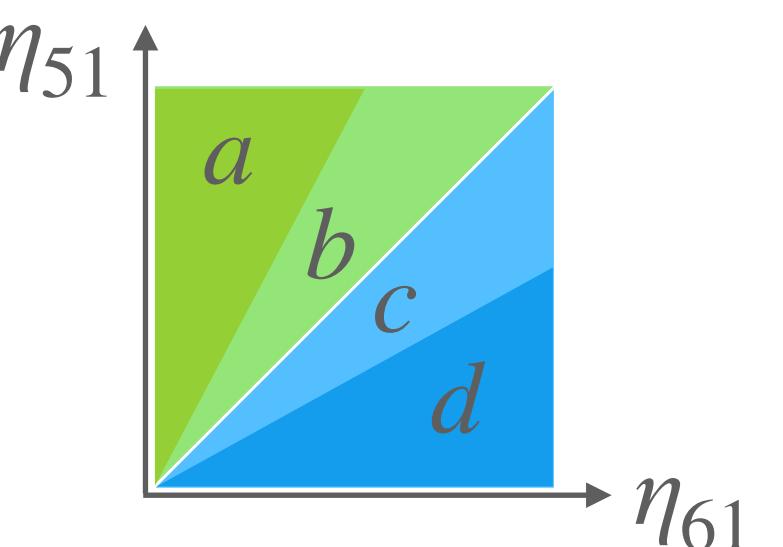
- Single, double and triple collinear limits to disentangle
- Strongly-ordered limits to disentangle in triple collinear sectors

$$1 = \sum_i \omega^i, \quad i \in \{(51,61), (52,62), (51,62), (52,61)\}$$



$$\eta_{ab} = \frac{1 - \cos \vartheta_{ab}}{2}$$

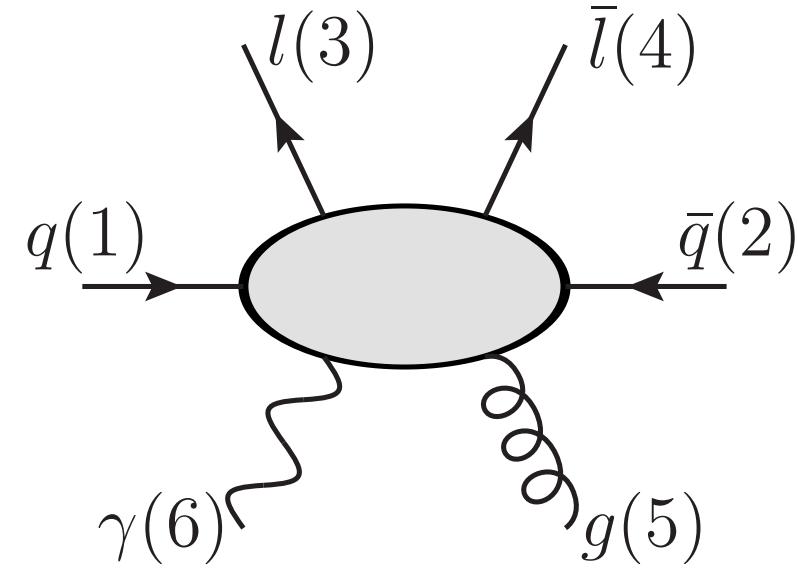
$$\omega^{51,61} = \omega^{51,61} (\theta_a + \theta_b + \theta_c + \theta_d)$$



→ Non-trivial structures to integrate → Reverse unitarity [Caola, Delto, Frellesvig Melnikov '18, Delto, Melnikov '19]

Mixed QCD-EW: differences with respect to NNLO QCD

Examples: $q\bar{q} \rightarrow Z \rightarrow e^-e^+ g\gamma$ [Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, Röntsch, [CSS](#)]



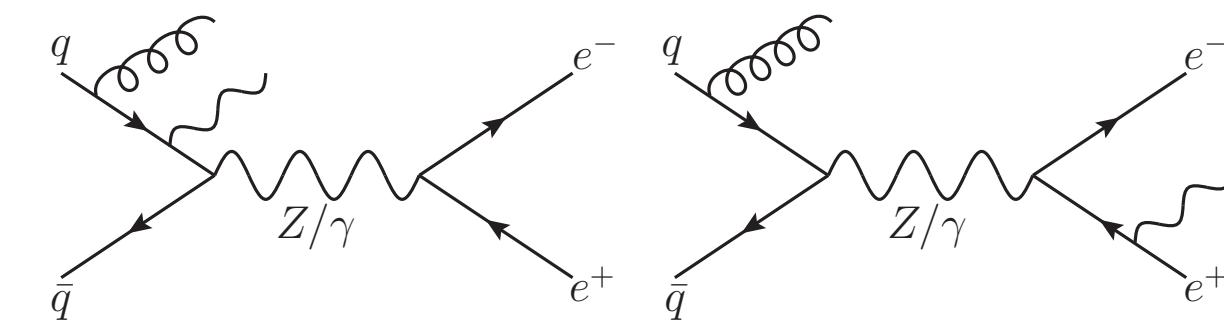
Soft limits:

- Double soft limit factorises into **NLO QCD** x **NLO EW** \rightarrow No need for energy ordering

$$\lim_{E_g, E_\gamma \rightarrow 0} |\mathcal{M}_{RR}|^2 = g_s^2 \text{Eik}(p_1, p_2; p_5) e^2 \sum_{i,j} Q_i Q_j \text{Eik}(p_i, p_j; p_6) |\mathcal{M}_B|^2$$

Collinear limits:

- Single, double and triple collinear limits to disentangle
 \rightarrow **More sectors** to account for final state radiation



$$1 = \sum_{i=1}^4 \sum_{j=1}^2 \omega^{\gamma i, g j}$$

Mixed QCD-EW: differences with respect to NNLO QCD

$$1 = \underbrace{\omega^{\gamma 1,g1} + \omega^{\gamma 2,g2}}_{\text{Triple collinear partition}} + \underbrace{\omega^{\gamma 1,g2} + \omega^{\gamma 2,g1} + \omega^{\gamma 3,g2} + \omega^{\gamma 3,g1} + \omega^{\gamma 4,g2} + \omega^{\gamma 4,g1}}$$

Triple collinear partition

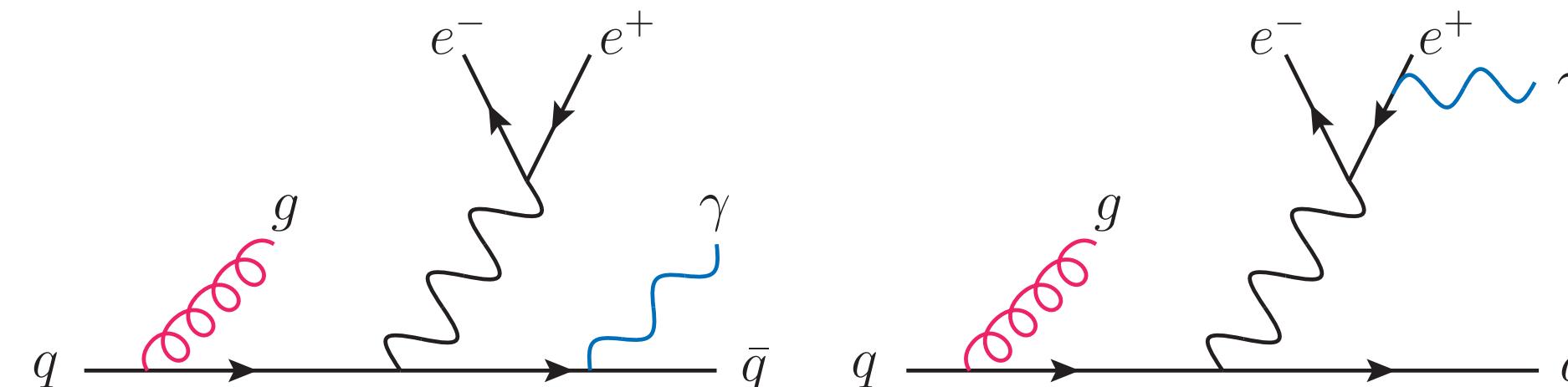
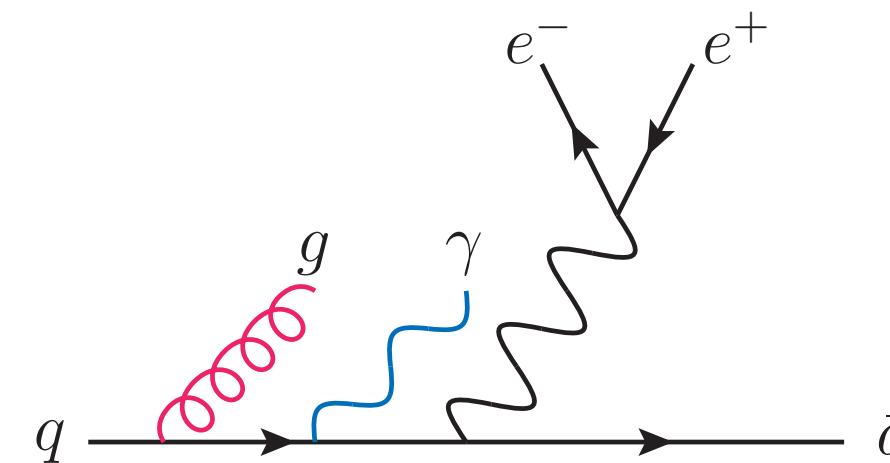
\sim NNLO \rightarrow difficult!

Overlapping singularities remain

Single collinear partition

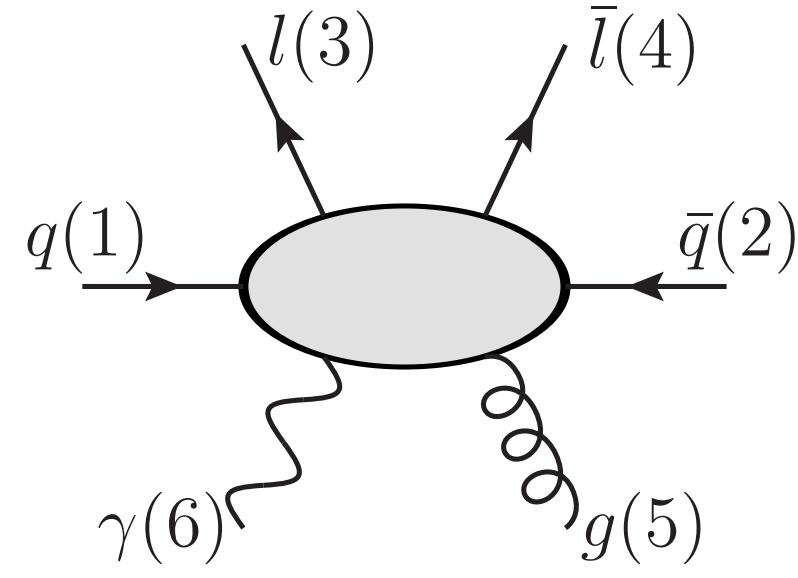
\sim NLO \times NLO \rightarrow simple!

Large rapidity difference in each sector



Mixed QCD-EW: differences with respect to NNLO QCD

Examples: $q\bar{q} \rightarrow Z \rightarrow e^-e^+ g\gamma$ [Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, Röntsch, CSS]



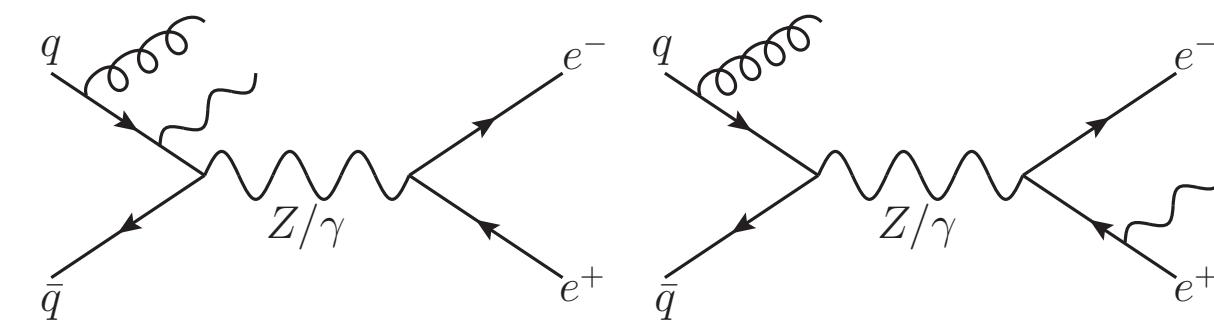
Soft limits:

- Double soft limit factorises into **NLO QCD** x **NLO EW** \rightarrow No need for energy ordering

$$\lim_{E_g, E_\gamma \rightarrow 0} |\mathcal{M}_{RR}|^2 = g_s^2 \text{Eik}(p_1, p_2; p_5) e^2 \sum_{i,j} Q_i Q_j \text{Eik}(p_i, p_j; p_6) |\mathcal{M}_B|^2$$

Collinear limits:

- Single, double and triple collinear limits to disentangle
 \rightarrow **More sectors** to account for final state radiation
- Strongly-ordered limits to disentangle in triple collinear sectors
 \rightarrow **BUT no photon-gluon collinear singularity**



$$1 = \sum_{i=1}^4 \sum_{j=1}^2 \omega^{\gamma i, g j}$$

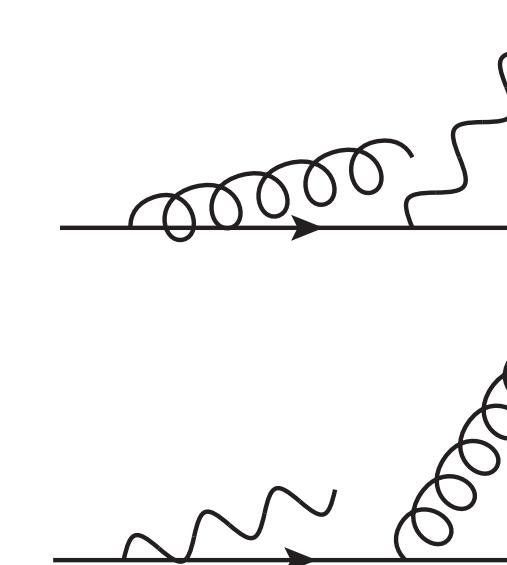
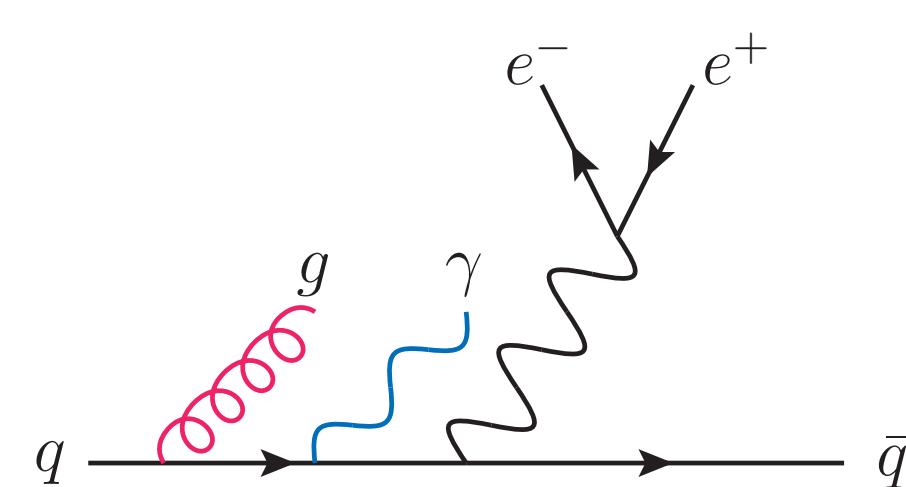
Mixed QCD-EW: differences with respect to NNLO QCD

$$1 = \underbrace{\omega^{\gamma 1,g1} + \omega^{\gamma 2,g2}}_{\text{Triple collinear partition}} + \omega^{\gamma 1,g2} + \omega^{\gamma 2,g1} + \omega^{\gamma 3,g2} + \omega^{\gamma 3,g1} + \omega^{\gamma 4,g2} + \omega^{\gamma 4,g1}$$

Triple collinear partition

\sim NNLO \rightarrow difficult!

Overlapping singularities remain

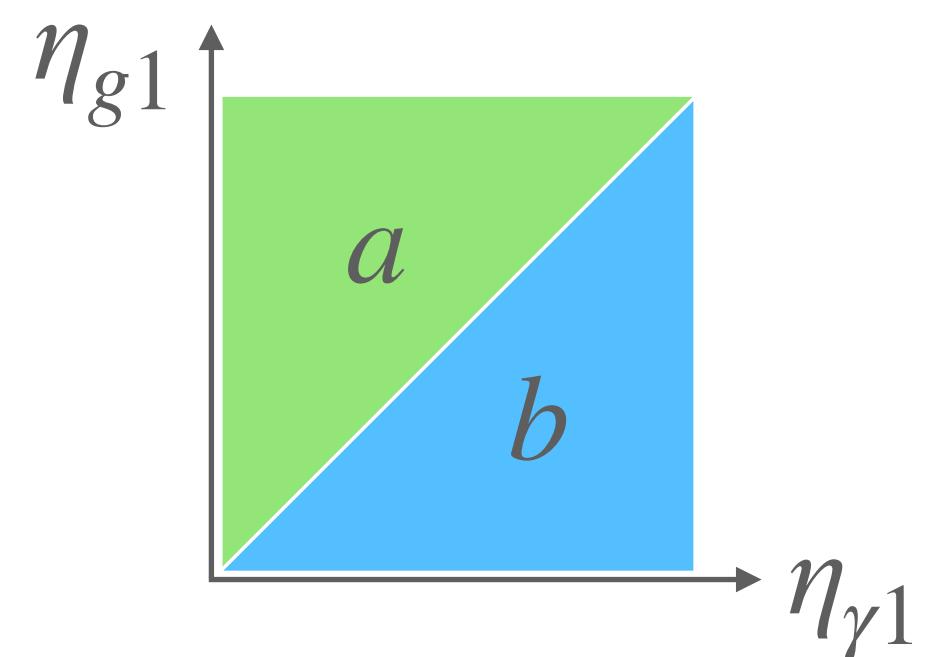


Triple collinear: $1 \parallel g \parallel \gamma$
Strongly-ordered: $1 \parallel g$

Triple collinear: $1 \parallel g \parallel \gamma$
Strongly-ordered: $1 \parallel \gamma$

Define **angular ordering** to separate singularities

$$1 = \theta_a + \theta_b = \theta(\eta_{g1} - \eta_{\gamma 1}) + \theta(\eta_{\gamma 1} - \eta_{g1})$$



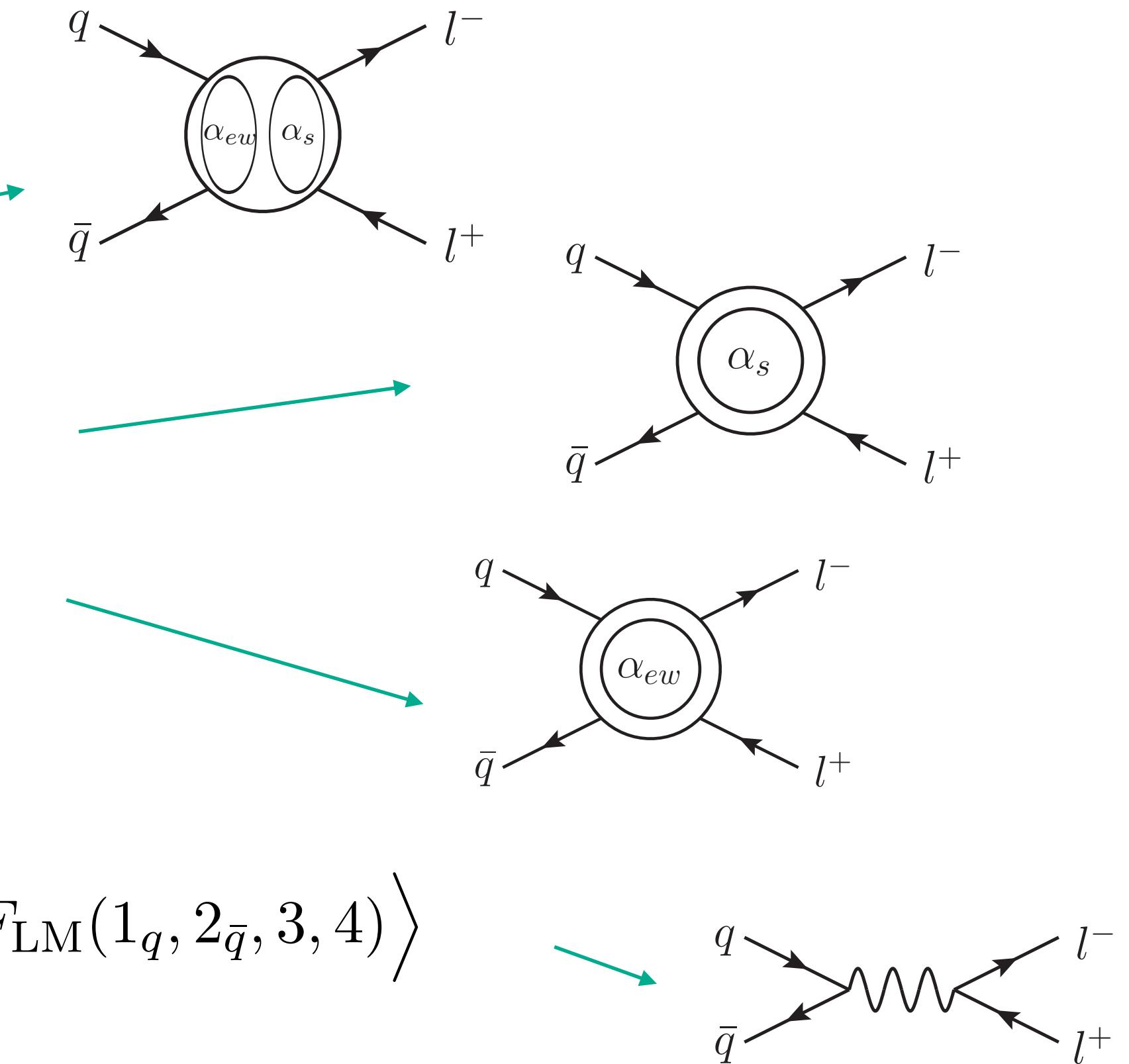
Finite parts II

$$d\hat{\sigma}_{\text{mix},g\gamma}^{q\bar{q}} = d\hat{\sigma}_{\text{el},g\gamma}^{q\bar{q}} + d\hat{\sigma}_{\text{bt},g\gamma}^{q\bar{q}} + d\hat{\sigma}_{\mathcal{O}_{\text{nlo}},g\gamma}^{q\bar{q}} + d\hat{\sigma}_{\text{reg},g\gamma}^{q\bar{q}}$$

Elastic contributions arise from:

- double-unresolved real emissions of photons and gluons
- finite remainders of virtual corrections

$$\begin{aligned} 2s \cdot d\hat{\sigma}_{\text{el},g\gamma}^{q\bar{q}} &= \langle F_{\text{LVV+LV}^2}^{(\text{QCD} \times \text{EW}), \text{fin}}(1_q, 2_{\bar{q}}, 3, 4) \rangle \\ &+ [\alpha] \left\langle \left[\mathcal{G}_{\text{EW}} + 3Q_q^2 \log\left(\frac{s}{\mu^2}\right) \right] F_{\text{LV}}^{(\text{QCD}), \text{fin}}(1_q, 2_{\bar{q}}, 3, 4) \right\rangle \\ &+ [\alpha_s] C_F \left[\frac{2}{3}\pi^2 + 3 \log\left(\frac{s}{\mu^2}\right) \right] \langle F_{\text{LV}}^{(\text{EW}), \text{fin}}(1_q, 2_{\bar{q}}, 3, 4) \rangle \\ &+ [\alpha] [\alpha_s] C_F \left\langle \left\{ Q_q^2 \left[-\frac{4\pi^4}{45} + (2\pi^2 + 32\zeta_3) \log\left(\frac{s}{\mu^2}\right) \right. \right. \right. \\ &\quad \left. \left. \left. + \left(9 - \frac{4\pi^2}{3}\right) \log^2\left(\frac{s}{\mu^2}\right) \right] + \mathcal{G}_{\text{EW}} \left(\frac{2\pi^2}{3} + 3 \log\left(\frac{s}{\mu^2}\right) \right) \right\} F_{\text{LM}}(1_q, 2_{\bar{q}}, 3, 4) \right\rangle \end{aligned}$$



$$F_{\text{LM}}(1_q, 2_{\bar{q}}, 3, 4) \propto \sum_{\text{col,pol}} \text{dLips}_{34} (2\pi)^d \delta^{(d)}(p_{12} - p_{34}) |M(\{p_i\})|^2$$

Recurring structure

$$\mathcal{G}_{\text{EW}} = Q_q^2 \frac{2\pi^2}{3} + Q_e^2 \left(13 - \frac{2\pi^2}{3} \right) + 2Q_q Q_e \left[3 \log\left(\frac{\eta_{13}}{\eta_{23}}\right) + 2 \text{Li}_2(1 - \eta_{13}) - 2 \text{Li}_2(1 - \eta_{23}) \right]$$

Finite parts III

$$d\hat{\sigma}_{\text{mix},g\gamma}^{q\bar{q}} = d\hat{\sigma}_{\text{el},g\gamma}^{q\bar{q}} + d\hat{\sigma}_{\text{bt},g\gamma}^{q\bar{q}} + d\hat{\sigma}_{\mathcal{O}_{\text{nlo}},g\gamma}^{q\bar{q}} + d\hat{\sigma}_{\text{reg},g\gamma}^{q\bar{q}}$$

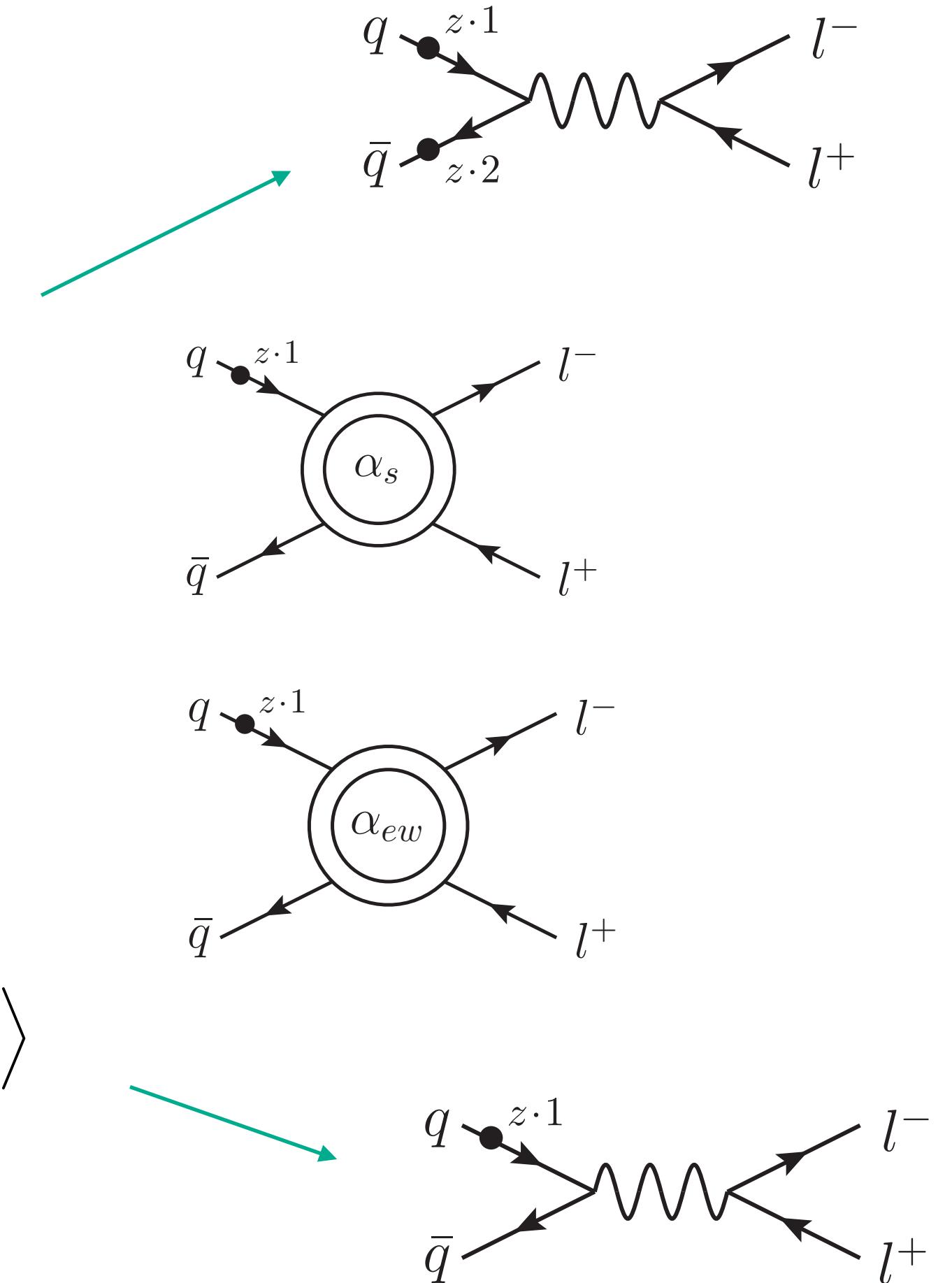
Boosted contributions arise from final states where either a gluon or a photon is collinear to incoming partons, or one of them is collinear and the other is soft.

$$\begin{aligned} 2s \cdot d\sigma_{\text{bt},g\gamma}^{q\bar{q}} &= [\alpha] [\alpha_s] 2C_F Q_q^2 \int_0^1 dz_1 dz_2 \tilde{P}_{qq}^{\text{NLO}}(z_1, E_c) \left\langle \frac{F_{\text{LM}}(z_1 \cdot 1, z_2 \cdot 2, 3, 4)}{z_1 z_2} \right\rangle \tilde{P}_{qq}^{\text{NLO}}(z_2, E_c) \\ &+ \sum_{i=1}^2 \int_0^1 dz \tilde{P}_{qq}^{\text{NLO}}(z, E_c) \left[[\alpha] Q_q^2 \left\langle F_{\text{LV}}^{(i),(\text{QCD}),\text{fin}}(1_q, 2_{\bar{q}}, 3, 4; z) \right\rangle \right. \\ &\quad \left. + [\alpha_s] C_F \left\langle F_{\text{LV}}^{(i),(\text{EW}),\text{fin}}(1_q, 2_{\bar{q}}, 3, 4; z) \right\rangle \right] \\ &+ [\alpha] [\alpha_s] C_F \sum_{i=1}^2 \int_0^1 dz \left\langle \left\{ Q_q^2 P_{qq}^{\text{NNLO}}(z, E_c) + \tilde{P}_{qq}^{\text{NLO}}(z, E_c) \right. \right. \\ &\quad \times \left. \left. \left[Q_e^2 G_{e^2} + 2Q_q Q_e \left(G_{eq}^{(1,2)} + (-1)^i \log\left(\frac{s_{i3}}{s_{i4}}\right) \log(z) \right) \right] \right\} F_{\text{LM}}^{(i)}(1_q, 2_{\bar{q}}, 3, 4; z) \right\rangle \end{aligned}$$

Recurring structure

$$\begin{aligned} G_{eq}^{(i,j)} &= \text{Li}_2(1 - \eta_{i3}) - \text{Li}_2(1 - \eta_{i4}) - \text{Li}_2(1 - \eta_{j3}) + \text{Li}_2(1 - \eta_{j4}) \\ &\quad + \left[\frac{3}{2} - \log\left(\frac{E_3}{E_c}\right) \right] \log\left(\frac{\eta_{i3}}{\eta_{j3}}\right) - \left[\frac{3}{2} - \log\left(\frac{E_4}{E_c}\right) \right] \log\left(\frac{\eta_{i4}}{\eta_{j4}}\right), \\ G_{e^2} &= 13 - \frac{2}{3}\pi^2 + \log^2\left(\frac{E_3}{E_4}\right) + \left[3 - 2\log\left(\frac{E_3 E_4}{E_c^2}\right) \right] \log(\eta_{34}) + 2\text{Li}_2(1 - \eta_{34}) \end{aligned}$$

$$\tilde{P}_{qq}^{\text{NLO}}(z, E) = 4\mathcal{D}_1(z) - 2(1+z)\log(1-z) + (1-z) + 2\log\left(\frac{2E_c}{\mu}\right)(2\mathcal{D}_0(z) - (1+z))$$



Finite parts IV

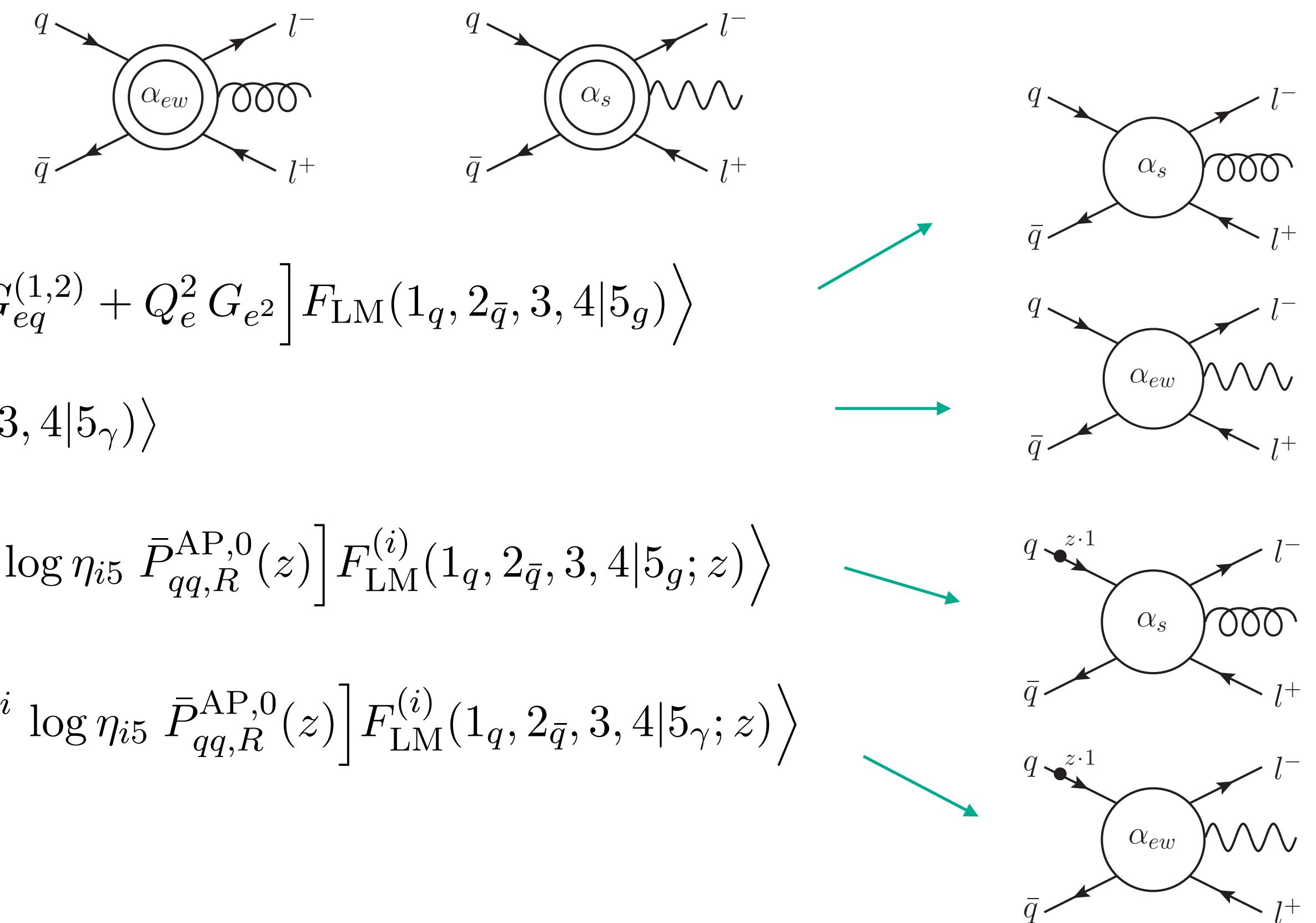
$$d\hat{\sigma}_{\text{mix},g\gamma}^{q\bar{q}} = d\hat{\sigma}_{\text{el},g\gamma}^{q\bar{q}} + d\hat{\sigma}_{\text{bt},g\gamma}^{q\bar{q}} + d\hat{\sigma}_{\mathcal{O}_{\text{nlo}},g\gamma}^{q\bar{q}} + d\hat{\sigma}_{\text{reg},g\gamma}^{q\bar{q}}$$

The \mathcal{O}_{nlo} terms describes NLO corrections to processes with an additional gluon or photon in the final state. They arise from virtual corrections to these final states and from remnants of $l^- l^+ g\gamma$ state in case either a gluon or a photon becomes unresolved.

$$\begin{aligned} 2s \cdot d\sigma_{\mathcal{O}_{\text{nlo}},g\gamma}^{q\bar{q}} &= \left\langle \mathcal{O}_{\text{nlo}}^g F_{\text{LV}}^{(\text{EW}), \text{fin}}(1_q, 2_{\bar{q}}, 3, 4|5_g) \right\rangle \\ &\quad + \left\langle \mathcal{O}_{\text{nlo}}^\gamma F_{\text{LV}}^{(\text{QCD}), \text{fin}}(1_q, 2_{\bar{q}}, 3, 4|5_\gamma) \right\rangle \\ &\quad + [\alpha] \left\langle \mathcal{O}_{\text{nlo}}^g \left[Q_q^2 \left(\frac{2}{3}\pi^2 + 3 \log \left(\frac{s}{\mu^2} \right) \right) + 2Q_q Q_e G_{eq}^{(1,2)} + Q_e^2 G_{e^2} \right] F_{\text{LM}}(1_q, 2_{\bar{q}}, 3, 4|5_g) \right\rangle \\ &\quad + [\alpha_s] C_F \left[\frac{2}{3}\pi^2 + 3 \log \left(\frac{s}{\mu^2} \right) \right] \left\langle \mathcal{O}_{\text{nlo}}^\gamma F_{\text{LM}}(1_q, 2_{\bar{q}}, 3, 4|5_\gamma) \right\rangle \\ &\quad + [\alpha] Q_q^2 \sum_{i=1}^2 \int_0^1 dz \left\langle \mathcal{O}_{\text{nlo}}^g \left[\tilde{P}_{qq}^{\text{NLO}}(z, E_c) + \tilde{\omega}_{\gamma||i}^{\gamma i, gi} \log \eta_{i5} \bar{P}_{qq,R}^{\text{AP},0}(z) \right] F_{\text{LM}}^{(i)}(1_q, 2_{\bar{q}}, 3, 4|5_g; z) \right\rangle \\ &\quad + [\alpha_s] C_F \sum_{i=1}^2 \int_0^1 dz \left\langle \mathcal{O}_{\text{nlo}}^\gamma \left[\tilde{P}_{qq}^{\text{NLO}}(z, E_c) + \tilde{\omega}_{g||i}^{\gamma i, gi} \log \eta_{i5} \bar{P}_{qq,R}^{\text{AP},0}(z) \right] F_{\text{LM}}^{(i)}(1_q, 2_{\bar{q}}, 3, 4|5_\gamma; z) \right\rangle \end{aligned}$$

Recurring structure

$$\bar{P}_{qq,R}^{\text{AP},0}(z) = 2\mathcal{D}_0(z) - (1+z),$$



Finite parts V

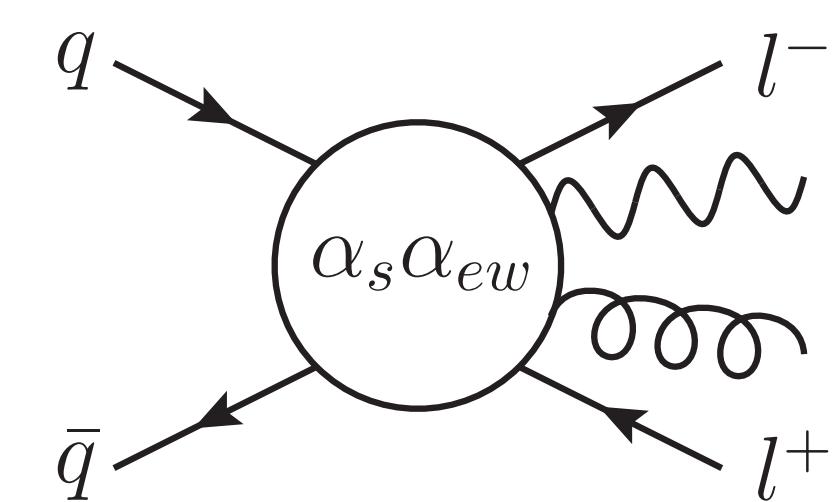
$$d\hat{\sigma}_{\text{mix},g\gamma}^{q\bar{q}} = d\hat{\sigma}_{\text{el},g\gamma}^{q\bar{q}} + d\hat{\sigma}_{\text{bt},g\gamma}^{q\bar{q}} + d\hat{\sigma}_{\mathcal{O}_{\text{nlo}},g\gamma}^{q\bar{q}} + d\hat{\sigma}_{\text{reg},g\gamma}^{q\bar{q}}$$

The regulated term is fully resolved and can be implemented numerically in $d = 4$.

Subtraction terms involve known eikonal contributions and splitting functions and are organised through partition functions

$$2s \cdot d\hat{\sigma}_{\text{reg},g\gamma}^{q\bar{q}} = \langle (I - S_g)(I - S_\gamma) \Omega_1^{q\bar{q}} F_{\text{LM}}(1_q, 2_{\bar{q}}, 3, 4|5_g, 6_\gamma) \rangle$$

$$\begin{aligned} \Omega_1^{q\bar{q}} = & (1 - C_{g\gamma,1})(1 - C_{g1}) \omega^{\gamma 1,g1} \theta_A + (1 - C_{g\gamma,1})(1 - C_{\gamma 1}) \omega^{\gamma 1,g1} \theta_B \\ & + (1 - C_{g\gamma,2})(1 - C_{g2}) \omega^{\gamma 2,g2} \theta_A + (1 - C_{g\gamma,2})(1 - C_{\gamma 2}) \omega^{\gamma 2,g2} \theta_B \\ & + (1 - C_{g2})(1 - C_{\gamma 1}) \omega^{\gamma 1,g2} + (1 - C_{g1})(1 - C_{\gamma 2}) \omega^{\gamma 2,g1} \\ & + (1 - C_{g2})(1 - C_{\gamma 3}) \omega^{\gamma 3,g2} + (1 - C_{g2})(1 - C_{\gamma 4}) \omega^{\gamma 4,g2} \\ & + (1 - C_{g1})(1 - C_{\gamma 3}) \omega^{\gamma 3,g1} + (1 - C_{g1})(1 - C_{\gamma 4}) \omega^{\gamma 4,g1}, \end{aligned}$$



Mixed QCD×EW corrections: state of the art

Theoretical developments

- ❖ Progress on two-loop master integrals [*Bonciani, Di Vita, Mastrolia, Schubert '16*][*Heller, von Manteuffel, Schabinger '19*][*Hasan, Schubert '20*]
- ❖ 2-loop amplitudes for $2 \rightarrow 2$ neutral current DY for massless leptons [*Heller, von Manteuffel, Schabinger, Spiesberger '20*]
- ❖ 2-loop amplitudes for $2 \rightarrow 2$ neutral current DY for massive leptons [*Armadillo, Bonciani, Devoto, Rana, Vicini '22*]

On-shell Z/W production ($2 \rightarrow 1$ process)

- ❖ Analytic mixed QCD-QED corrections to the inclusive production of an on-shell Z [*de Florian, Der, Fabre '18*]
- ❖ Fully differential mixed QCD-QED corrections to the production of an on-shell Z [*Delto, Jaquier, Melnikov, Röntsch '19*]
- ❖ Total on-shell Z production cross section in fully analytic with mixed QCD-EW corrections [*Bonciani, Buccioni, Rana, Vicini '20*]
- ❖ Fully differential on-shell Z and W production with mixed QCD-EW corrections [*Buccioni et al. '20*] [*Behring et al. '20*]

On-shell Z/W production ($2 \rightarrow 1$ process)

- ❖ Dominant mixed QCD-EW correction in the pole approximation NC and CC DY process [*Dittmaier, Huss, Schwinn '14, 15*]
- ❖ Approximate corrections available in parton showers based on factorised approach [*Balossini et al. '10*][*Bernaciak, Wackerlo '12*]
[*Barzè et al. '12, '13*] [*Calame et al. '17*]
- ❖ Neutrino-pair production including mixed QCD-QED corrections [*Cieri, de Florian, Der, Mazzitelli '21*]
- ❖ Neutrino-pair production including mixed QCD-EW corrections [*Buonocore, Grazzini, Kallweit, Savoini, Tramontano '21*]
- ❖ Complete mixed QCD-EW corrections to NC and CC DY process [*Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini '22*]

Phenomenology: m_{ll} windows

$$\sigma = \sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} + \delta\sigma^{(1,1)} + \dots$$

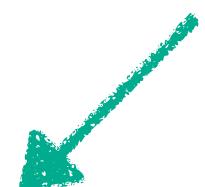
LO NLO QCD NLO EW NNLO QCD QCDxEW

Question: can we capture NNLO QCDxEW by only computing
 $(\text{NLO QCD}) \cdot (\text{NLO EW}) \rightarrow \delta\sigma_{\text{fact}}^{(1,1)}$?

- $\Phi^{(1)} : 200 \text{ GeV} < m_{\ell\ell} < 300 \text{ GeV},$
- $\Phi^{(2)} : 300 \text{ GeV} < m_{\ell\ell} < 500 \text{ GeV},$
- $\Phi^{(3)} : 500 \text{ GeV} < m_{\ell\ell} < 1.5 \text{ TeV},$
- $\Phi^{(4)} : 1.5 \text{ TeV} < m_{\ell\ell} < \infty.$

$\sigma [\text{fb}]$	$\sigma^{(0,0)}$	$\delta\sigma^{(1,0)}$	$\delta\sigma^{(0,1)}$	$\delta\sigma^{(2,0)}$	$\delta\sigma^{(1,1)}$	$\delta\sigma_{\text{fact.}}^{(1,1)}$
$\Phi^{(1)}$	1169.8	254.3	-30.98	10.18	-10.74	-6.734
$\Phi^{(2)}$	368.29	71.91	-11.891	2.85	-4.05	-2.321
$\Phi^{(3)}$	82.08	14.31	-4.094	0.691	-1.01	-0.7137
$\Phi^{(4)} \times 10$	9.107	1.577	-1.124	0.146	-0.206	-0.1946

What do we learn?



✓ At high invariant mass ($m_{ll} > 1.5 \text{ TeV}$) factorised approx. captures more than 90% of the exact result

$$\frac{\delta\sigma_{\text{fact.}}^{(1,1)}}{\sigma^{(0,0)}} = \left[\frac{\delta\sigma^{(1,0)}}{\sigma^{(0,0)}} \sim 0.17 \right] \cdot \left[\frac{\delta\sigma^{(0,1)}}{\sigma^{(0,0)}} \sim -0.12 \right] \sim -0.021$$

$$\frac{\delta\sigma^{(1,1)}}{\sigma^{(0,0)}} \sim -0.023$$

→ Expected: factorised approx. correctly reproduces the leading Sudakov logs, which dominate at high invariant mass

IR regularisation: subtraction vs slicing

$F(x)$ arbitrary complicated function

$$I = \lim_{\epsilon \rightarrow 0} \left[\int_0^1 \frac{dx}{x} x^\epsilon F(x) - \frac{1}{\epsilon} F(0) \right]$$

Goal: compute I without relying on the analytic evaluation of the integral

Slicing

$$I \sim \lim_{\epsilon \rightarrow 0} \left[F(0) \underbrace{\int_0^\delta \frac{dx}{x} x^\epsilon}_{\text{Slicing parameter } \delta \ll 1} + \int_\delta^1 \frac{dx}{x} x^\epsilon F(x) - \frac{1}{\epsilon} F(0) \right] = F(0) \log \delta + \int_\delta^1 \frac{dx}{x} x^\epsilon F(x)$$

Slicing parameter $\delta \ll 1 \rightarrow$ power dependence on the slicing parameter in the result

Subtraction

$$I = \lim_{\epsilon \rightarrow 0} \left[\underbrace{\int_0^1 \frac{dx}{x} x^\epsilon (F(x) - F(0))}_{\text{Regulated, finite for } \epsilon \rightarrow 0} + \underbrace{\int_0^1 \frac{dx}{x} x^\epsilon F(0) - \frac{1}{\epsilon} F(0)}_{\text{Extract } 1/\epsilon \text{ pole}} \right]$$

Counterterm: the definition may be involved!

Why is NNLO so difficult?

At NLO two main strategies have been implemented

Catani Seymour:

- Counterterm contribution: reproduces the **IR singularities** related to a dipole in **all of the phase space** [**complicated structure**]
- Full counterterm: sum of **contributions**, each **parametrised differently**
- **Analytic integration** of each term [**non trivial, complicated structure of the counterterm**]

FKS:

- **Partition** of the radiative phase space with sector functions
- **Different parametrisation** for each sector
- **Analytic integration**, after getting rid of sector functions [**non trivial, non optimised parametrisation**]

Detail informations of NNLO kernels also available ~ 20 years ago

(N3LO kernels partially available [[Catani, Colferai, Torrini 1908.01616](#), [Del Duca, Duhr, Haindl, Lazopoulos Michel 1912.06425](#),
[Dixon, Herrmann, Kai Yan, Hua Xing Zhu 1912.09370](#)[Yu Jiao Zhu 2009.08919](#)])

Why is NNLO so difficult?

1. Clear understanding of which singular configurations do actually contribute
2. Get to the point where the problem is well defined
3. Solve the phase space integrals of the relevant limits

1. Clear understanding of which singular configurations do actually contribute

$$\begin{array}{c}
 \text{Diagram: A gray oval representing a particle source emits three outgoing momentum vectors. The bottom vector is } k_1 + k_2 + k_3, \text{ the middle vector is } k_1 + k_2, \text{ and the top vector is } k_1. \\
 \text{The top vector } k_1 \text{ is shown as a wavy line segment with arrows at both ends, indicating it is a virtual particle exchange between two external lines.} \\
 \text{Equation: } \frac{1}{(k_1 + k_2)^2} \frac{1}{(k_1 + k_2 + k_3)^2} = \frac{1}{2k_1 \cdot k_2} \frac{1}{2k_1 \cdot k_2 + 2k_1 \cdot k_3 + 2k_2 \cdot k_3} \iff k_1 \rightarrow 0 \text{ and } k_2 \parallel k_3
 \end{array}$$

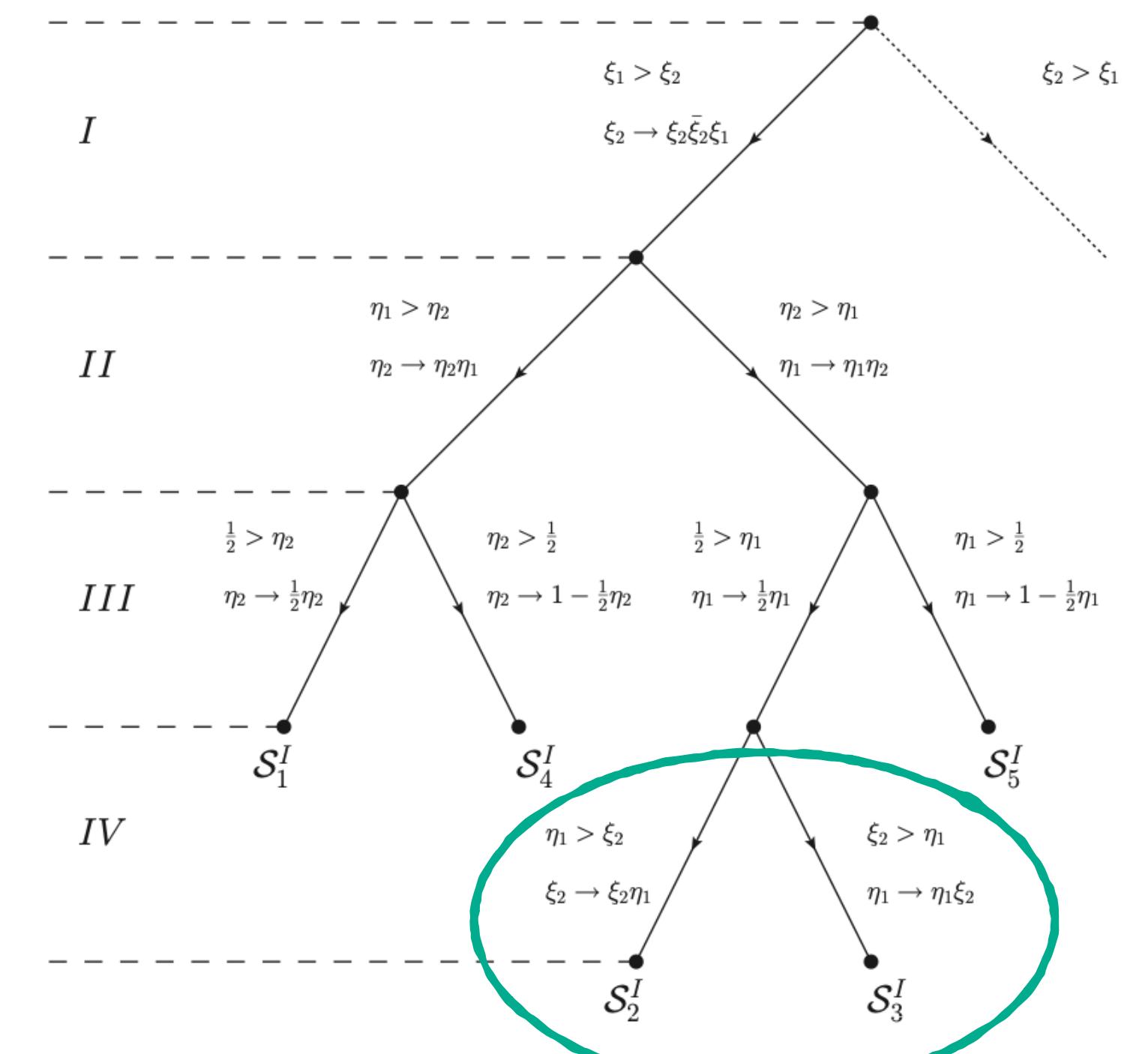
Entangled soft-collinear limits of diagrams can not be treated in a process-independent way.

Do non-commutative limits actually contribute?

STRIPPER was implemented taking into account all the possible choices of soft and collinear limits order -> redundant configurations were included

Gauge invariant amplitudes are free of entangled singularities
thanks to **color coherence**: soft parton does not resolve angles of the
collinear partons

Soft-collinear limits can be described by taking the known soft and collinear limits sequentially



2. Get to the point where the problem is well defined

- a) Identify the overlapping singularities
- b) Regulate them

The Feynman diagram shows a shaded oval representing an incoming particle with momentum $k_1 + k_2 + k_3$. It splits into two outgoing particles with momenta k_1 and k_2 , which then interact via a loop exchange to produce an outgoing particle with momentum k_3 .

The corresponding loop integral is given by:

$$\sim \frac{1}{E_1 E_2 (1 - \vec{n}_1 \cdot \vec{n}_2)} \frac{1}{E_1 E_2 (1 - \vec{n}_1 \cdot \vec{n}_2) + E_1 E_3 (1 - \vec{n}_1 \cdot \vec{n}_3) + E_2 E_3 (1 - \vec{n}_2 \cdot \vec{n}_3)}$$

The diagram is divided into three regions:

- Soft origin**: $E_1 \rightarrow 0, E_2 \rightarrow 0, E_1, E_2 \rightarrow 0$
- Collinear origin**: $\vec{n}_1 \parallel \vec{n}_2, \vec{n}_1 \parallel \vec{n}_2 \parallel \vec{n}_3$
- Includes strongly ordered configurations**

Arrows point from the soft and collinear regions to the following conditions:

- Soft origin**: $E_1 \ll E_2, E_2 \ll E_1$
- Collinear origin**: $\vec{n}_1 \cdot \vec{n}_2 < \vec{n}_1 \cdot \vec{n}_3$
- Strongly ordered configurations**: $\vec{n}_2 \cdot \vec{n}_3 < \vec{n}_1 \cdot \vec{n}_3$ and $\vec{n}_1 \cdot \vec{n}_3 < \vec{n}_2 \cdot \vec{n}_3$

Three diagrams illustrate the strongly ordered configurations for the collinear origin:

- Left: Vectors 1 and 2 are collinear and point in the same direction as vector 3.
- Middle: Vector 1 points in the same direction as vectors 2 and 3.
- Right: Vectors 2 and 3 are collinear and point in the same direction as vector 1.

Soft and collinear modes do not intertwine: soft subtraction can be done globally. Collinear singularities have still to be regulated.
Strongly ordered configurations have to be properly taken into account.

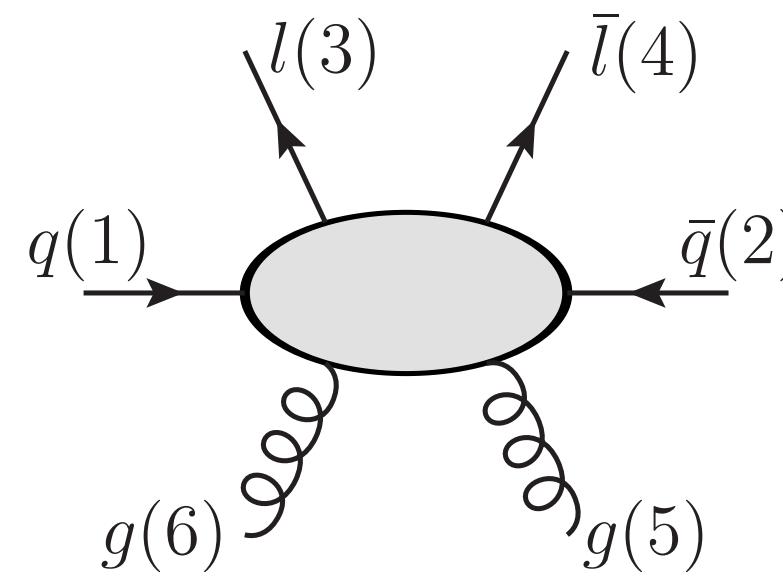
Phase space partitions

Efficient way to simplify the problem: introduce **partition functions** (following FKS philosophy):

- **Unitary partition**
- Select a **minimum number of singularities** in each sector
- Do not affect the **analytic integration** of the counterterms

Definition of partition functions benefits from remarkable degree of **freedom**: different approaches can be implemented

Examples: **Nested soft-collinear subtraction** $q\bar{q} \rightarrow Z \rightarrow e^-e^+ gg$ [Caola, Melnikov, Röntsch 1702.01352]



$$1 = \omega^{51,61} + \omega^{52,62} + \omega^{51,62} + \omega^{52,61}$$

$$\omega^{51,61} = \frac{\rho_{25}\rho_{26}}{d_5 d_6} \left(1 + \frac{\rho_{15}}{d_{5621}} + \frac{\rho_{16}}{d_{5612}} \right)$$

$$\omega^{52,62} = \frac{\rho_{15}\rho_{16}}{d_5 d_6} \left(1 + \frac{\rho_{25}}{d_{5621}} + \frac{\rho_{26}}{d_{5612}} \right)$$

$$\omega^{51,62} = \frac{\rho_{25}\rho_{16}\rho_{56}}{d_5 d_6 d_{5612}}$$

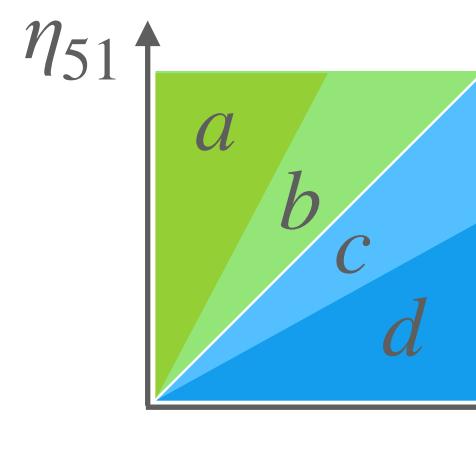
$$\omega^{52,61} = \frac{\rho_{15}\rho_{26}\rho_{56}}{d_5 d_6 d_{5621}}$$

$$\rho_{ab} = 1 - \cos \vartheta_{ab}, \eta_{ab} = \rho_{ab}/2$$

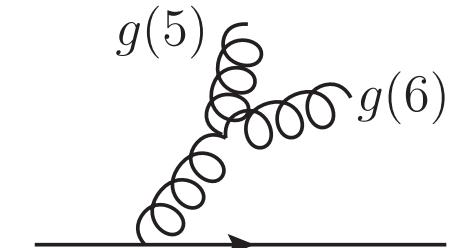
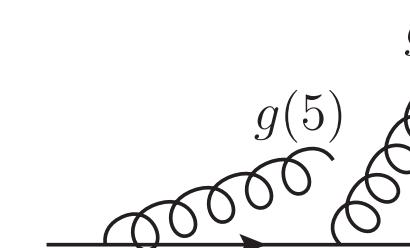
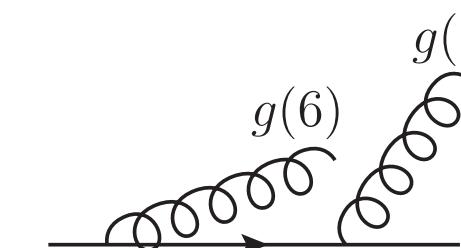
$$d_{i=5,6} = \rho_{1i} + \rho_{2i} = 2$$

$$d_{5621} = \rho_{56} + \rho_{52} + \rho_{61}$$

$$d_{5612} = \rho_{56} + \rho_{51} + \rho_{62}$$



$$\begin{aligned} 1 &= \theta\left(\eta_{61} < \frac{\eta_{51}}{2}\right) + \theta\left(\frac{\eta_{51}}{2} < \eta_{61} < \eta_{51}\right) + \theta\left(\eta_{51} < \frac{\eta_{61}}{2}\right) + \theta\left(\frac{\eta_{61}}{2} < \eta_{51} < \eta_{61}\right) \\ &= \theta^{(a)} + \theta^{(b)} + \theta^{(c)} + \theta^{(d)} \end{aligned}$$



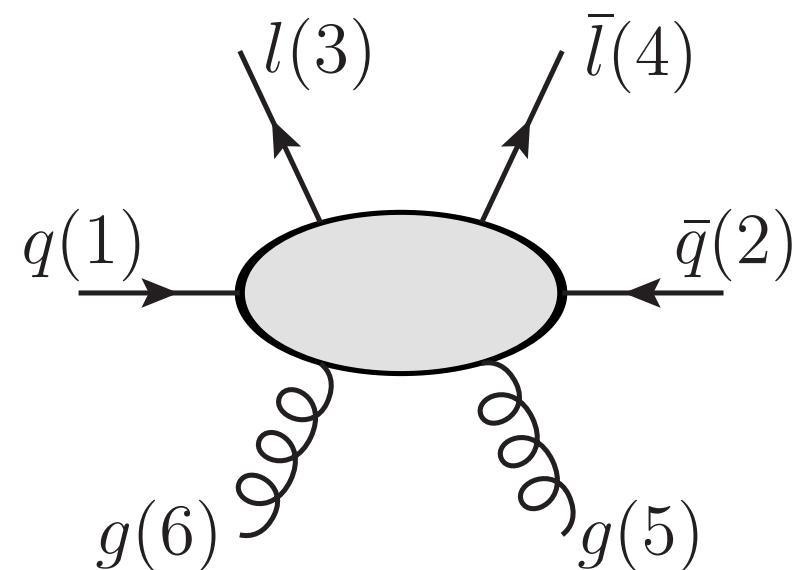
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Examples: **Nested soft-collinear subtraction** $q\bar{q} \rightarrow Z \rightarrow e^-e^+ gg$ [Caola, Melnikov, Röntsch 1702.01352]



Advantages:

1. Simple definition
2. Structure of collinear singularities fully defined
3. Same strategy holds for NNLO mixed QCDxEW processes
4. **Minimum number of sector**

Disadvantages:

1. Partition based on angular ordering -> Lorentz invariance not preserved
-> angles defined in a given reference frame
2. Theta function

3. Solve the PS integrals

The problem is now well defined:

A. **Singular kernels** and their nested limits have to be **subtracted from the double real correction** to get integrable object

$$\int d\Phi_{n+2} RR_{n+2} = \underbrace{\int d\Phi_{n+2} \left[RR_{n+2} - K_{n+2} \right]}_{\text{Fully regulated real emission contribution}} + \int d\Phi_{n+2} K_{n+2}$$

$K_{n+2} \supset C_{ij}, C_{kl}, S_i, S_{ij}, C_{ijk}$

—————> Numerical evaluation

Examples: **Nested soft-collinear subtraction** $q\bar{q} \rightarrow Z \rightarrow e^-e^+ gg$ [Caola, Melnikov, Röntsch]

$$[df_i] = \frac{d^d k_i}{(2\pi)^d} (2\pi) \delta_+(k_i^2)$$

$$\begin{aligned} d\hat{\sigma}_{\text{resolv.}}^{NNLO} = & \int \theta(E_5 - E_6) \theta(E_{\max} - E_5) \left\{ \sum_{i,j \in \{1,2\}, i \neq j} (1 - C_{5i}) (1 - C_{6j}) (1 - S_{56}) (1 - S_6) [dk_5] [dk_6] \omega^{5i,6j} B(\{k\}_{1\dots 6}) \right. \\ & + \sum_{i \in \{1,2\}} \left[\theta^{(a)}(1 - C_{i56}) (1 - C_{6i}) + \theta^{(b)}(1 - C_{i56}) (1 - C_{56}) \right. \\ & \quad \left. \left. + \theta^{(c)}(1 - C_{i56}) (1 - C_{5i}) + \theta^{(d)}(1 - C_{i56}) (1 - C_{56}) \right] [dk_5] [dk_6] \omega^{5i,6i} B(\{k\}_{1\dots 6}) \right\} \end{aligned}$$

Explicit expression depends on the scheme

3. Solve the PS integrals

The problem is now well defined:

A. **Singular kernels** and their nested limits have to be **subtracted from the double real correction** to get integrable object

$$\int d\Phi_{n+2} RR_{n+2} = \int d\Phi_{n+2} [RR_{n+2} - K_{n+2}] + \int d\Phi_{n+2} K_{n+2} \quad K_{n+2} \supset C_{ij}, C_{kl}, S_i, S_{ij}, C_{ijk}$$

B. **Counterterms** have to be **integrated over the unresolved phase space**

$$I = \int \text{PS}_{\text{unres.}} \otimes \text{Limit} \otimes \text{Constraints}$$

The ‘Limit’ component is universal and known. The phase space is well defined. Constraints may vary depending on the scheme.

Several kinematic structures have to be integrated **analytically** over a 6-dim PS.

Different approximations and techniques can be applied: the results assume different form depending on the adopted strategy

Two main structure are the most complicated ones and affect most of the physical processes:

- **Double soft**
- **Triple collinear**

Kernels integration

Examples: Nested soft-collinear subtraction $q\bar{q} \rightarrow Z \rightarrow e^-e^+ gg$ [Caola, Delto, Frellesvig, Melnikov 1807.05835, Delto, Melnikov 1901.05213]

Two soft parton (5,6) and two hard massless radiator (1,2): arbitrary relative angle between the three-momenta of the radiators

$$I_{12}^{(gg)(56)} = \frac{(1-\epsilon)(s_{51}s_{62} + s_{52}s_{61}) - 2s_{56}s_{12}}{s_{56}^2(s_{51} + s_{61})(s_{52} + s_{62})} + s_{12} \frac{s_{51}s_{62} + s_{52}s_{61} - s_{56}s_{12}}{s_{56}s_{51}s_{62}s_{52}s_{61}} \left[1 - \frac{1}{2} \frac{s_{51}s_{62} + s_{52}s_{61}}{(s_{51} + s_{61})(s_{52} + s_{62})} \right]$$

$$I_{S_{56}}^{(gg)} = \int [dk_5] [dk_6] \theta(E_{\max} - E_5) \theta(E_5 - E_6) I_{12}^{(gg)(56)}(k_1, k_2, k_5, k_6) \quad [df_i] = \frac{d^d k_i}{(2\pi)^d} (2\pi) \delta_+(k_i^2)$$

$$E_5 = E_{\max} \xi \quad E_6 = E_{\max} \xi z \quad 0 < \xi < 1, 0 < z < 1$$

Reverse unitarity: map phase space integrals onto loop integrals [Anastasiou, Melnikov 0207004]

after defining integral families, integration-by-part identities. Differential equations w.r.t. the ratio of energies of emitted gluons at fixed angle.

Boundary conditions for $z=0$, and arbitrary angle

Kernels integration

Examples: Nested soft-collinear subtraction $q\bar{q} \rightarrow Z \rightarrow e^-e^+ gg$ [Caola, Delto, Frellesvig, Melnikov 1807.05835, Delto, Melnikov 1901.05213]

$$\begin{aligned}
I_{S_{56}}^{(gg)} = & (2E_{\max})^{-4\epsilon} \left[\frac{1}{8\pi^2} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \right]^2 \left\{ \frac{1}{2\epsilon^4} + \frac{1}{\epsilon^3} \left[\frac{11}{12} - \ln(s^2) \right] \right. \\
& + \frac{1}{\epsilon^2} \left[2\text{Li}_2(c^2) + \ln^2(s^2) - \frac{11}{6} \ln(s^2) + \frac{11}{3} \ln 2 - \frac{\pi^2}{4} - \frac{16}{9} \right] \\
& + \frac{1}{\epsilon} \left[6\text{Li}_3(s^2) + 2\text{Li}_3(c^2) + \left(2\ln(s^2) + \frac{11}{3} \right) \text{Li}_2(c^2) - \frac{2}{3} \ln^3(s^2) \right. \\
& \quad \left. + \left(3\ln(c^2) + \frac{11}{6} \right) \ln^2(s^2) - \left(\frac{22}{3} \ln 2 + \frac{\pi^2}{2} - \frac{32}{9} \right) \ln(s^2) \right. \\
& \quad \left. - \frac{45}{4} \zeta_3 - \frac{11}{3} \ln^2 2 - \frac{11}{36} \pi^2 - \frac{137}{18} \ln 2 + \frac{217}{54} \right] \\
& + 4G_{-1,0,0,1}(s^2) - 7G_{0,1,0,1}(s^2) + \frac{22}{3} \text{Ci}_3(2\delta) + \frac{1}{3 \tan(\delta)} \text{Si}_2(2\delta) \\
& + 2\text{Li}_4(c^2) - 14\text{Li}_4(s^2) + 4\text{Li}_4 \left(\frac{1}{1+s^2} \right) - 2\text{Li}_4 \left(\frac{1-s^2}{1+s^2} \right) \\
& + 2\text{Li}_4 \left(\frac{s^2-1}{1+s^2} \right) + \text{Li}_4(1-s^4) + \left[10\ln(s^2) - 4\ln(1+s^2) \right. \\
& \quad \left. + \frac{11}{3} \right] \text{Li}_3(c^2) + \left[14\ln(c^2) + 2\ln(s^2) + 4\ln(1+s^2) + \frac{22}{3} \right] \text{Li}_3(s^2) \\
& + 4\ln(c^2)\text{Li}_3(-s^2) + \frac{9}{2}\text{Li}_2^2(c^2) - 4\text{Li}_2(c^2)\text{Li}_2(-s^2) + \left[7\ln(c^2) \ln(s^2) \right.
\end{aligned}$$

$$\begin{aligned}
& \quad \left. - \ln^2(s^2) - \frac{5}{2}\pi^2 + \frac{22}{3}\ln 2 - \frac{131}{18} \right] \text{Li}_2(c^2) + \left[\frac{2}{3}\pi^2 - 4\ln(c^2) \ln(s^2) \right] \times \\
& \quad \text{Li}_2(-s^2) + \frac{\ln^4(s^2)}{3} + \frac{\ln^4(1+s^2)}{6} - \ln^3(s^2) \left[\frac{4}{3}\ln(c^2) + \frac{11}{9} \right] \\
& \quad + \ln^2(s^2) \left[7\ln^2(c^2) + \frac{11}{3}\ln(c^2) + \frac{\pi^2}{3} + \frac{22}{3}\ln 2 - \frac{32}{9} \right] - \frac{\pi^2}{6} \ln^2(1+s^2) \\
& \quad + \zeta_3 \left[\frac{17}{2}\ln(s^2) - 11\ln(c^2) + \frac{7}{2}\ln(1+s^2) - \frac{21}{2}\ln 2 - \frac{99}{4} \right] + \ln(s^2) \times \\
& \quad \left[-\frac{7\pi^2}{2}\ln(c^2) + \frac{22}{3}\ln^2 2 - \frac{11}{18}\pi^2 + \frac{137}{9}\ln 2 - \frac{208}{27} \right] - 12\text{Li}_4 \left(\frac{1}{2} \right) \\
& \quad + \frac{143}{720}\pi^4 - \frac{\ln^4 2}{2} + \frac{\pi^2}{2}\ln^2 2 - \frac{11}{6}\pi^2 \ln 2 + \frac{125}{216}\pi^2 + \frac{22}{9}\ln^3 2 \\
& \quad \left. + \frac{137}{18}\ln^2 2 + \frac{434}{27}\ln 2 - \frac{649}{81} + \mathcal{O}(\epsilon) \right\},
\end{aligned}$$

$$\delta = \frac{\delta_{12}}{2}, s = \sin \frac{\delta_{12}}{2}, c = \cos \frac{\delta_{12}}{2}$$

$$\text{Ci}_n(z) = \frac{\text{Li}_n(e^{iz}) + \text{Li}_n(e^{-iz})}{2}, \text{Si}_n(z) = \frac{\text{Li}_n(e^{iz}) - \text{Li}_n(e^{-iz})}{2i}$$

Colour coherence and disentangled soft-collinear singularities

Parton q is soft and partons 1,2 are collinear [Catani, Grazzini 9908523]

$$\left| \mathcal{M}_{g,a_1,a_2,\dots,a_n}(q,p_1,p_2,\dots,p_n) \right|^2 \simeq -\frac{2}{s_{12}}(4\pi\mu^{2\epsilon}\alpha_s)^2 \left\langle \mathcal{M}_{a,a_3,\dots,a_n}(p,p_3,\dots,p_n) \left| \hat{\mathbf{P}}_{a_1a_2} [\mathbf{J}_{(12)\mu}^\dagger(q) \mathbf{J}_{(12)}^\mu(q)] \right| \mathcal{M}_{a,a_3,\dots,a_n}(p,p_3,\dots,p_n) \right\rangle$$

Mother parton:
 $a \rightarrow a_1 + a_2$

Altarelli-Parisi splitting functions:
spin correlations

Soft current: colour correlations

$$\mathbf{J}_{(12)\mu}^\dagger(q) \mathbf{J}_{(12)}^\mu(q) \simeq \sum_{i,j=3}^n \mathbf{T}_i \cdot \mathbf{T}_j \mathcal{S}_{ij}(q) + 2 \sum_{i=3}^n \mathbf{T}_i \cdot \mathbf{T}_{(12)} \mathcal{S}_{i(12)}(q)$$

$$\mathbf{T}_{(12)} = \mathbf{T}_1 + \mathbf{T}_2$$

$$\mathcal{S}_{ij}(q) = \frac{2 s_{ij}}{s_{iq} s_{jq}}$$

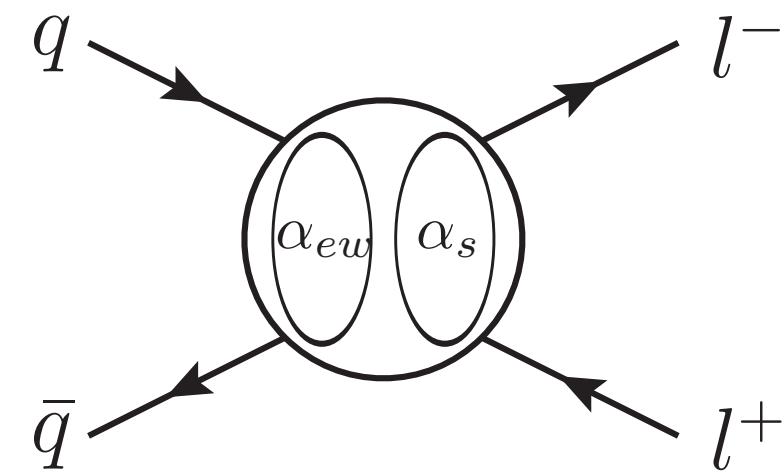
$$\mathcal{S}_{i(12)}(q) = \frac{2(s_{i1} + s_{i2})}{s_{iq} (s_{1q} + s_{2q})}$$

The soft-collinear limit at $\mathcal{O}(\alpha_s^2)$ is fully described in a factorised way, where the factors are the soft eikonal function and the Altarelli-Parisi splitting functions that control IR limits at $\mathcal{O}(\alpha_s)$.

This simplification, which is due to colour coherence, was not performed in FKS.

Ingredients for off-shell calculation at NNLO

- ❖ Fully differential description of mixed QCD-EW effects is a complicated problem



2-loop virtual + one-loop squared

$$\left\langle F_{LVV+LV^2}^{\text{QCDxEW}}(1,2,3,4) \right\rangle = \frac{\alpha_s(\mu)}{2\pi} \frac{\alpha(\mu)}{2\pi} \left[I^{\text{QCD}} \cdot I^{\text{QED}} + \frac{e^{\epsilon \gamma_E}}{\Gamma(1-\epsilon)} \frac{1}{\epsilon} H_{\text{QCDxEW}} \right] \left\langle F_{LM}(1,2,3,4) \right\rangle + \frac{\alpha_s(\mu)}{2\pi} I^{\text{QCD}} \left\langle F_{LV}^{\text{EW, fin}}(1,2,3,4) \right\rangle + \frac{\alpha(\mu)}{2\pi} I^{\text{QED}} \left\langle F_{LV}^{\text{QCD, fin}}(1,2,3,4) \right\rangle + \left\langle F_{LVV+LV^2}^{\text{QCDxEW, fin}}(1,2,3,4) \right\rangle$$

Universal IR operators
[Catani '98]

Finite part one-loop EW/QCD correction

Genuine NNLO hard-triple collinear contribution

$$H_{\text{QCDxEW}} = 2C_F Q_q^2 \left(\frac{\pi^2}{2} - 6\zeta_3 - \frac{3}{8} \right)$$

Finite part of the two-loop mixed
QCDxEW correction

- Handle chiral couplings in dimensional regularisation
 - > γ_5 really a 4-dimentional object > need for a prescription in d-dimension
- Solving master integrals
- Adapt the result for a fast Monte-Carlo integration



Ingredients for off-shell calculation at NNLO

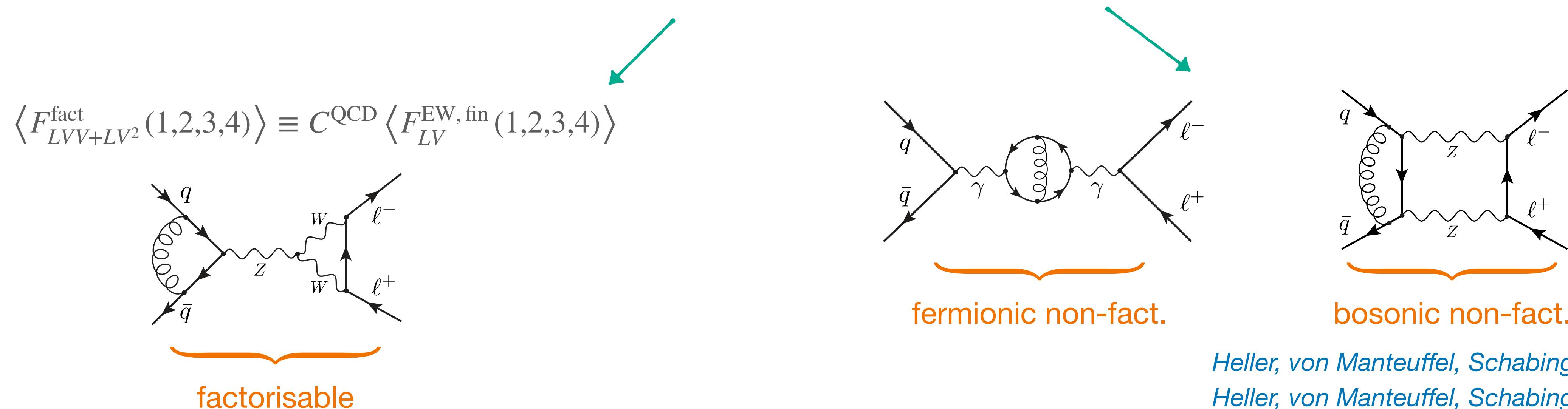
- ❖ Fully differential description of mixed QCD-EW effects is a complicated problem

- Given the Catani operator, the finite 1-loop QCD remainder reads

$$\langle F_{LV}^{\text{QCD, fin}}(1,2,3,4) \rangle = C^{\text{QCD}} \langle F_{LM}(1,2,3,4) \rangle, \quad C^{\text{QCD}} = -8C_F \frac{\alpha_s(\mu)}{2\pi}$$

- At **2-loop** we find it convenient to split the virtual correction into **factorisable** and **non-factorisable** contributions

$$\langle F_{LVV+LV^2}^{\text{QCDxEW, fin}}(1,2,3,4) \rangle = \langle F_{LVV+LV^2}^{\text{fact}}(1,2,3,4) \rangle + \langle F_{LVV+LV^2}^{\text{non-fact}}(1,2,3,4) \rangle$$



- Non-factorisable part is **CPU expensive**, **BUT** typically **1 order of magnitude smaller** than the **factorisable part**, across the entire phase space → can be determined to a much lower accuracy to obtain the cross section with a target precision

Ingredients for off-shell calculation at NNLO

- ❖ Fully differential description of mixed QCD-EW effects is a complicated problem
- Separation into factorisable/non-factorisable allows us to capture the **bulk of the virtual top-quarks contributions**
 - **neglected in the finite part of the bosonic non-factorisable contribution**
 - **kept in all the other contributions**
- **0.7s** per phase space point

Mixed QCDxEW corrections: VV and RV component

2-loop correction to $q(p_1) \bar{q}(p_2) \rightarrow e^+(p_3) e^-(p_4)$

$$\left\langle F_{LVV+LV^2}^{QCD \times EW}(1,2,3,4) \right\rangle = \frac{\alpha_s(\mu)}{2\pi} \frac{\alpha(\mu)}{2\pi} \left[I^{QCD} \cdot I^{QED} + \frac{e^{\epsilon \gamma_E}}{\Gamma(1-\epsilon)} \frac{1}{\epsilon} H_{QCD \times EW} \right] \left\langle F_{LM}(1,2,3,4) \right\rangle$$

$$+ \frac{\alpha_s(\mu)}{2\pi} I^{QCD} \left\langle F_{LV}^{EW, \text{fin}}(1,2,3,4) \right\rangle + \frac{\alpha(\mu)}{2\pi} I^{QED} \left\langle F_{LV}^{QCD, \text{fin}}(1,2,3,4) \right\rangle + \left\langle F_{LVV+LV^2}^{QCD \times EW, \text{fin}}(1,2,3,4) \right\rangle$$

Universal IR operators
[Catani '98]

Finite part one-loop EW/QCD correction

Genuine NNLO hard-triple collinear contribution
 $H_{QCD \times EW} = 2C_F Q_q^2 \left(\frac{\pi^2}{2} - 6\zeta_3 - \frac{3}{8} \right)$

Finite part of the two-loop mixed
QCDxEW correction

1-loop correction to $q(p_1) \bar{q}(p_2) \rightarrow e^+(p_3) e^-(p_4) \gamma(p_5) + e^+(p_3) e^-(p_4) g(p_5)$

IR subtraction proceeds as for **NLO**:

- soft singularities are extracted first
- collinear singularities are extracted from the soft-regulated term

$$2s \cdot d\sigma^{\text{RV}} = \left\langle S_g F_{LRV}^{\text{EW}}(1,2,3,4|5_g) \right\rangle + \left\langle S_\gamma F_{LRV}^{\text{QCD}}(1,2,3,4|5_\gamma) \right\rangle$$

$$+ \left\langle (I - S_g)(C_{g1} + C_{g2}) F_{LRV}^{\text{EW}}(1,2,3,4|5_g) \right\rangle + \sum_{k=1}^4 \left\langle (I - S_\gamma) C_{\gamma k} F_{LRV}^{\text{QCD}}(1,2,3,4|5_\gamma) \right\rangle$$

$$+ \left\langle \mathcal{O}_{\text{nlo}} F_{LRV}^{\text{EW}}(1,2,3,4|5_g) \right\rangle + \left\langle \mathcal{O}_{\text{nlo}} F_{LRV}^{\text{QCD}}(1,2,3,4|5_\gamma) \right\rangle$$

→ Integrable in the PS, explicit poles

The RR component: generalities

From NNLO QCD to mixed EWxQCD: the on-shell case as intermediate step

- > **Final state radiation** introduces additional soft and collinear singularities leading to new kinematic structure
- > **The phase space partition** has to be **extended**

BUT

Absence of gluon-gluon interactions

- > no true double-soft singularities
- > no spin correlations
- > pure abelian contributions
- > **abelianization** of known results

- Adapting the NNLO QCD computation is practically all you need to obtain the mixed QCDxEW correction in the resonance region.

[de Florian, Der, Fabre '18][Delto, Jaquier, Melnikov, Röntsch '19] [Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch '20]

- In the **high energy region, full control of the subtraction procedure** is crucial to be able to adapt the results from NNLO QCD.
A straightforward abelianization is not sufficient.