Quantum information with Top quarks at the LHC

ICHEP 2022, Bologna, Italy

Based on **EPJP 136, 907 (2021)** and **2203.05582**

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08.07.2022

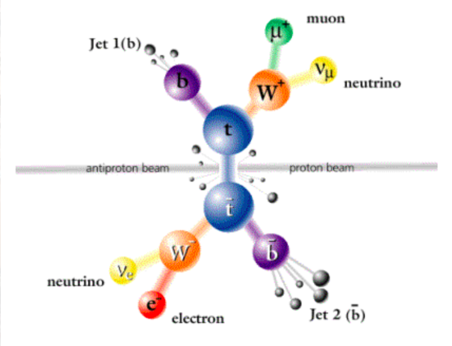
Overview

- The Standard Model is a QFT:
  - Special Relativity.
  - Quantum Mechanics.
- Fundamental properties of Quantum Mechanics can be tested via the Standard Model.

- Implementation of canonical techniques of Quantum Information at High-Energy Colliders.
- Fundamental Quantum Mechanics at the Frontier of known Physics.

Top-Quark

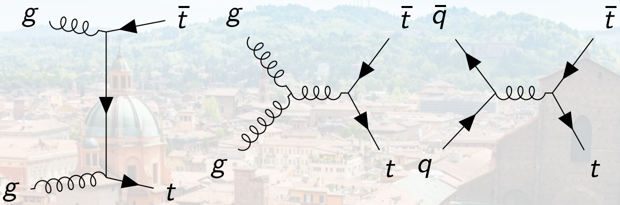
- **General:**
  - Hadronisation: $\sim 10^{-23}\mathrm{s}$.
  - Spin-decorrelation: $\sim 10^{-21}\mathrm{s}$.
- **Top quark:**
  - Lifetime: $\sim 10^{-25}\mathrm{s}$.
- Spin information $\rightarrow$ decay products.
- Spin-correlations between a pair of top-quarks can be measured.
- Considering leptonic decays.



Spin-Correlations between Top-Quark Pairs

- Studied extensively theoreticaly.
- Measured by the D0, CDF, ATLAS and CMS collaborations.
- No link between spin-correlations and quantum entanglement so far.
- **Spin-Correlations $\neq$ Quantum Entanglement!**
  However, Quantum Entanglement $\subset$ Spin-Correlations.

LO Analytical Calculation



- Analytical calculation at leading-order. The system is defined by:
  - $\hat{k}$: the direction of the top with respect to the beam axis.
  - The invariant mass $M_{t\bar{t}}$, $\beta = \sqrt{1 - \frac{4 \cdot m_t^2}{M_{t\bar{t}}^2}}$.
- Each one $I = q\bar{q}, gg$ gives rise to $\rho^I(M_{t\bar{t}}, \hat{k})$ with probability $w_I(M_{t\bar{t}}, \hat{k})$, which is PDF dependent.
- The spin density matrix: $\rho(M_{t\bar{t}}, \hat{k}) = \sum_{I=q\bar{q},gg} w_I(M_{t\bar{t}}, \hat{k})\rho^I(M_{t\bar{t}}, \hat{k})$.
- The total quantum state:
  $\rho(M_{t\bar{t}}) \equiv \int_{2m_t}^{M_{t\bar{t}}} \mathrm{d}M \int \mathrm{d}\Omega \; p(M, \hat{k})\rho(M, \hat{k}) = \int_{2m_t}^{M_{t\bar{t}}} \mathrm{d}M \; p(M)\rho_\Omega(M)$

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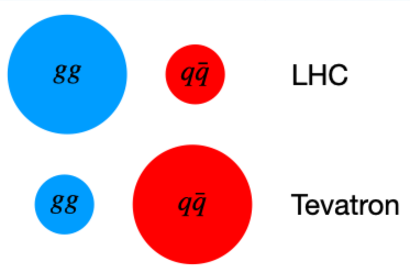

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Experimental Observables

Quantum Entanglement:

- **Concurrence** $C[\rho]$: quantitative measurement of entanglement.
- $0 \leq C[\rho] \leq 1$, $C[\rho] \neq 0$ iff the state is entangled.
- Here, $C[\rho] = \max(\Delta, 0)$; $\Delta = \frac{-C_{nn}+|C_{kk}+C_{rr}|-1}{2}$.

Non-Separable

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Bell's Inequality:

- A violation of the CHSH inequality: $|\mathbf{a}_1^{\mathrm{T}}\mathbf{C}\,(\mathbf{b}_1 - \mathbf{b}_2) + \mathbf{a}_2^{\mathrm{T}}\mathbf{C}\,(\mathbf{b}_1 + \mathbf{b}_2)\,| > 2$.
  - $\mathbf{C}$ - spin correlation matrix.
  - $\mathbf{a}_1, \mathbf{a}_2$ $(\mathbf{b}_1, \mathbf{b}_2)$ - axes in which we measure the spin of the top (antitop).
- Maximization: $2\sqrt{\mu_1 + \mu_2} \leq 2\sqrt{2}$ where $0 \leq \mu_i \leq 1$ are the eigenvalues of $\mathbf{C}^{\mathrm{T}}\mathbf{C}$.

Loopholes in a Collider Experiment

- Loopholes: experimental tests of Bell's inequality may not fulfill all hypotheses of the theorem.
- Collider experiment:
  - Free-will loophole: spin measurement directions should be free, independent from hidden-variables.
  - Detection loophole: only a subset of events is selected for the measurement, which can be biased.
- Collider experiments were not designed to test Bell's Inequality.

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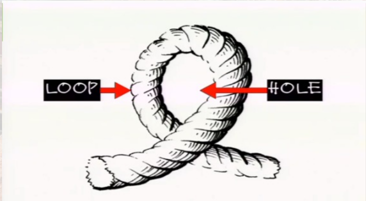
- Collider experiments were not designed to test Bell's Inequality.
- $\rightarrow$ Can only detect a *weak* violation of CHSH (Bell's) Inequality.



- **Bell's-Inequality $\subset$ Quantum Entanglement.**

Entanglement and Bell's Inequality Before Integration

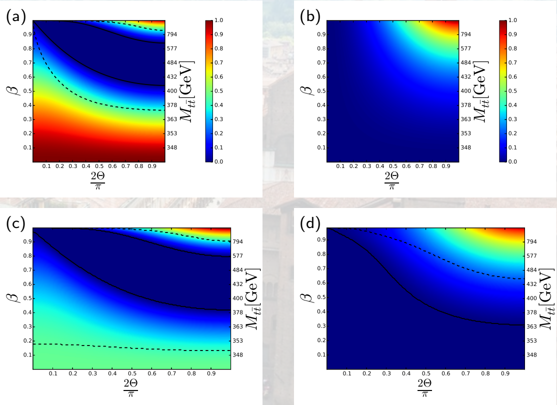
a) $gg \to t\bar{t}$ Concurrence.

b) $q\bar{q} \to t\bar{t}$ Concurrence.

c) Full LHC $\rho(M_{t\bar{t}}, \hat{k})$ Concurrence.

d) Full Tevatron $\rho(M_{t\bar{t}}, \hat{k})$ Concurrence.

- Solid line: entanglement limit; Dashed line: Bell's inequality limit.



- It is possible to control the $gg/q\bar{q}$ fraction by further selections, see Aguilar-Saavedra, Casas, 2205.00542.

Critical Values After Integration

- We focus on *pp* interactions.
- Clear motivation to restrict to selected regions of phase space.
- Plot is shown with integration only for $[2m_t, M_{t\bar{t}}]$.
- We focus on the region close to threshold. For high p_T see:
  - Fabbrichesi, Floreanini, Panizzo, PRL (2021).
  - Severi, Boschi, Maltoni, Sioli, EPJC (2022).

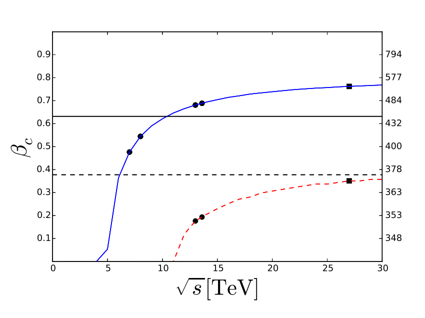


Figure: Critical values below which entanglement and CHSH violation can be observed, for different COM values.

Measurable Entanglement Marker

- Plots are shown with integration only for $[2m_t, M_{t\bar{t}}]$.
- In particular: $\frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi} = \frac{1}{2}(1 - D\cos\varphi)$ where φ is the angle between the lepton directions in each one of the parent top and antitop rest frames.
- $\Delta > 0 \Leftrightarrow D = \frac{\mathrm{tr}[\mathbf{C}]}{3} < -\frac{1}{3}$.
- Recently, D was measured with no selection on $M_{t\bar{t}}$ by the CMS collaboration in Phys. Rev. D 100, 072002:
  $D = -0.237 \pm 0.011 > -1/3$;
  $\Delta D/D = 4.6\%$.

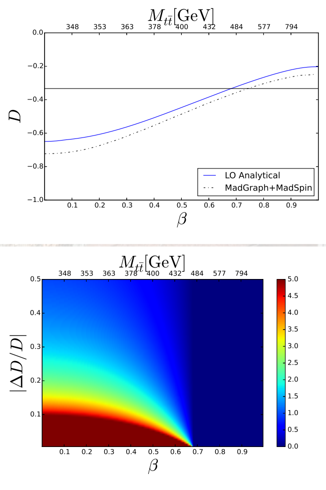


Figure: Up: the value of D; bottom: statistical deviation from the null hypothesis ($D = -1/3$).

Quantum Tomography

- **Quantum Tomography**: reconstruction of the quantum state from measurement of a set of expectation values.
- Spin polarizations $\mathbf{B}^{\pm}$ and spin correlation matrix $\mathbf{C}$ can be extracted from cross-section $\sigma_{\ell\bar{\ell}}$ of dileptonic decay:

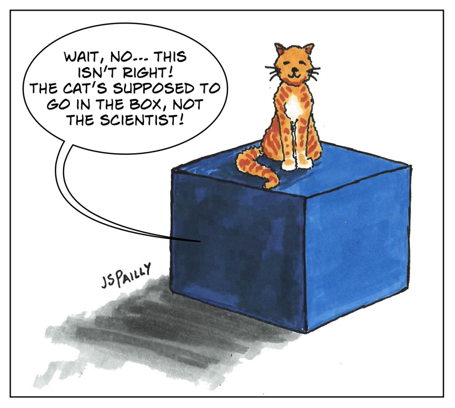
$$\frac{1}{\sigma_{\ell\bar{\ell}}}\frac{\mathrm{d}\sigma_{\ell\bar{\ell}}}{\mathrm{d}\Omega_+\mathrm{d}\Omega_-}=\frac{1}{(4\pi)^2}\left[1+\mathbf{B}^+\cdot\hat{\ell}_+-\mathbf{B}^-\cdot\hat{\ell}_--\hat{\ell}_+\cdot\mathbf{C}\cdot\hat{\ell}_-\right]$$

- Symmetry around beam axis:
  - 2 spin correlations $C_\perp, C_z$.
  - 2 individual spin (longitudinal) polarizations $B_z^{\pm}$.
- No assumption on the particular form of the quantum state:
  - 9 spin correlations C_{ij}.
  - 6 individual spin polarizations $B_i^{\pm}$.

Conclusions and outlook

- Implementation of ABC in quantum information at hadron colliders, in particular at the LHC: interdisciplinary, huge potential and great interest.
- Quantum Information perspective: new system to test quantum foundations at the highest-energy scale so far. Genuinely relativistic, exotic symmetries and interactions, fundamental nature.
- High-Energy perspective: quantum information techniques can inspire new approaches to test physics beyond the Standard Model:
  - See e.g. Aoude, Madge, Maltoni, Mantani, 2203.05619 and next talk by Luca Mantani.

Thank You


WAIT, NO... THIS ISN'T RIGHT! THE CAT'S SUPPOSED TO GO IN THE BOX, NOT THE SCIENTIST!
JSPAILLY

Backup Slides

Backup

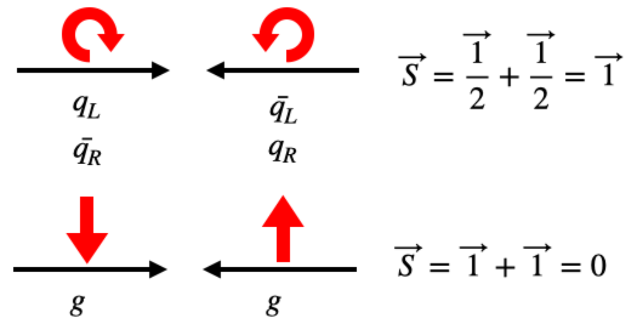
High Energy Physics Example

- At B-Factories, e^+e^- collisions can be properly adjusted in order to create $\Upsilon(4S)(b\bar{b})$.
- $\Upsilon(4S)(b\bar{b})$ decays to $B^0 + \bar{B}^0$, where we have $|B^0\rangle = |\bar{b}d\rangle, |\bar{B}^0\rangle = |b\bar{d}\rangle$.
- We get an entangled state: $\frac{1}{\sqrt{2}}(|B^0\rangle|\bar{B}^0\rangle - |\bar{B}^0\rangle|B^0\rangle)$.

BELLE

Intuition: Spin States at Threshold

- The state is determined by the initial spins.
- $q\bar{q}$: $\rho^{q\bar{q}} = \left(\left|\uparrow_{\hat{p}}\uparrow_{\hat{p}}\right\rangle \left\langle\uparrow_{\hat{p}}\uparrow_{\hat{p}}\right| + \left|\downarrow_{\hat{p}}\downarrow_{\hat{p}}\right\rangle \left\langle\downarrow_{\hat{p}}\downarrow_{\hat{p}}\right|\right)/2$.
- gg: $\rho^{gg} = \left|\Psi_0\right\rangle\left\langle\Psi_0\right|$, with $\left|\Psi_0\right\rangle = \left(\left|\uparrow_{\hat{p}}\downarrow_{\hat{p}}\right\rangle - \left|\downarrow_{\hat{p}}\uparrow_{\hat{p}}\right\rangle\right)/\sqrt{2}$.
- $q\bar{q} \to$ correlated, not entangled; $gg \to$ correlated, entangled.



Experimental Entanglement

- Entanglement has been observed in a wide variety of systems
- Testing entanglement in any new system is highly interesting by itself!

Experimental long-lived entanglement of two macroscopic objects

Brian Julsgaard, Alexander Kozhekin & Eugene S. Polzik

Institute of Physics and Astronomy, University of Aarhus, 8000 Aarhus, Denmark

Entanglement is considered to be one of the most profound features of quantum mechanics[1,2]. An entangled state of a system consisting of two subsystems cannot be described as a product of the quantum states of the two subsystems[3–6]. In this sense, the entangled system is considered inseparable and non-local. It is generally believed that entanglement is usually manifest in systems consisting of a small number of microscopic particles. Here we demonstrate experimentally the entanglement of two macroscopic objects, each consisting of a caesium gas sample containing about 10^{12} atoms. Entanglement is generated via

Measurement of the Entanglement of Two Superconducting Qubits via State Tomography

Matthias Steffen,* M. Ansmann, Radoslaw C. Bialczak, N. Katz, Erik Lucero, R. McDermott, Matthew Neeley, E. M. Weig, A. N. Cleland, John M. Martinis†

Demonstration of quantum entanglement, a key resource in quantum computation arising from a nonclassical correlation of states, requires complete measurement of all states in varying bases. By using simultaneous measurement and state tomography, we demonstrated entanglement between two solid-state qubits. Single qubit operations and capacitive coupling between two superconducting phase qubits were used to generate a Bell-type state. Full two-qubit tomography yielded a density matrix showing an entangled state with fidelity up to 87%. Our results demonstrate a high degree of unitary control of the system, indicating that larger implementations are within reach.

Entangling Macroscopic Diamonds at Room Temperature

K. C. Lee,[1*] M. R. Sprague,[1*] B. J. Sussman,[2] J. Nunn,[1] N. K. Langford,[1] X.-M. Jin,[1,3] T. Champion,[1] P. Michelberger,[1] K. F. Reim,[1] D. England,[1] D. Jaksch,[1,3] I. A. Walmsley[1]†

Quantum entanglement in the motion of macroscopic solid bodies has implications both for quantum technologies and foundational studies of the boundary between the quantum and classical worlds. Entanglement is usually fragile in room-temperature solids, owing to strong interactions both internally and with the noisy environment. We generated motional entanglement between vibrational states of two spatially separated, millimeter-sized diamonds at room temperature. By measuring strong nonclassical correlations between Raman-scattered photons, we showed that the quantum state of the diamonds has positive concurrence with 98% probability. Our results show that entanglement can persist in the classical context of moving macroscopic solids in ambient conditions.

nature | Vol 440|20 April 2006|doi:10.1038/nature04627

LETTERS

Experimental determination of entanglement with a single measurement

S. P. Walborn[1], P. H. Souto Ribeiro[1], L. Davidovich[1], F. Mintert[1,2] & A. Buchleitner[3]

LETTER

https://doi.org/10.1038/s41586-018-0038-x

Stabilized entanglement of massive mechanical oscillators

C. F. Ockeloen-Korppi[1], E. Damskägg[1], J.-M. Pirkkalainen[1], M. Asjad[2], A. A. Clerk[3], F. Massel[2], M. J. Woolley[4] & M. A. Sillanpää[1]*

nature physics | ARTICLES | PUBLISHED ONLINE: 15 AUGUST 2016 | DOI: 10.1038/NPHYS3863

Observation of quantum Hawking radiation and its entanglement in an analogue black hole

Jeff Steinhauer

nature physics | LETTERS | PUBLISHED ONLINE: 14 OCTOBER 2012 | DOI: 10.1038/NPHYS2444

Demonstration of entanglement-by-measurement of solid-state qubits

Wolfgang Pfaff[1], Tim H. Taminiau[1], Lucio Robledo[1], Hannes Bernien[1], Matthew Markham[2], Daniel J. Twitchen[2] and Ronald Hanson[1]*

PRL 99, 131802 (2007) | PHYSICAL REVIEW LETTERS | week ending 28 SEPTEMBER 2007

Measurement of Einstein-Podolsky-Rosen-Type Flavor Entanglement in $\Upsilon(4S) \to B^0 \bar{B}^0$ Decays

VOLUME 79 | 7 JULY 1997 | NUMBER 1

Generation of Einstein-Podolsky-Rosen Pairs of Atoms

E. Hagley, X. Maître, G. Nogues, C. Wunderlich, M. Brune, J. M. Raimond, and S. Haroche

Laboratoire Kastler Brossel, Département de Physique de l'Ecole Normale Supérieure, 24 rue Lhomond, F-75231 Paris Cedex 05, France*

(Received 6 March 1997)

Pairs of atoms have been prepared in an entangled state of the Einstein-Podolsky-Rosen (EPR) type.

PHYSICAL REVIEW LETTERS 122, 113602 (2019)

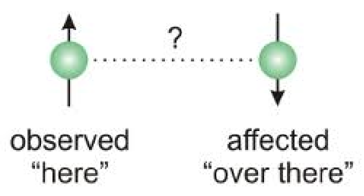
Editors' Suggestion | Featured in Physics

On-Demand Semiconductor Source of Entangled Photons Which Simultaneously Has High Fidelity, Efficiency, and Indistinguishability

Hui Wang,[1,2] Hai Hu,[3] T.-H. Chung,[1,2] Jian Qin,[1,2] Xiaoxia Yang,[3] J.-P. Li,[1,2] R.-Z. Liu,[1,2] H.-S. Zhong,[1,2] Y.-M. He,[1,2] Xing Ding,[1,2] Y.-H. Deng,[1,2] Qing Dai,[3,*] Y.-H. Huo,[1,2,†] Sven Höfling,[1,4,5] Chao-Yang Lu,[1,2‡] and Jian-Wei Pan[1,2,§]

What is Quantum Entanglement?

- Quantum state of one particle cannot be described independently from another particle.
- $\Rightarrow$ **Correlations** of observed physical properties of both systems.
- $\Rightarrow$ **Measurement** performed on one system seems to be influencing other systems entangled with it.



- Observed in photons, atoms, superconductors, mesons, analog Hawking radiation, nitrogen-vacancy centers in diamond and even macroscopic diamond.

Basis Selection

- Helicity basis: $\{\hat{k}, \hat{r}, \hat{n}\}$:
  - $\hat{k}$ - direction of the top in the $t\bar{t}$ CM frame.
  - $\hat{p}$ - direction of the beam.
  - $\cos\Theta = \hat{k}\cdot\hat{p}$.
  - $\hat{r} = (\hat{p} - \cos\Theta\hat{k})/\sin\Theta$.
  - $\hat{n} = \hat{r}\times\hat{k}$.
- Beam basis: $\{\hat{x}, \hat{y}, \hat{z}\}$:
  - $\hat{z}$ along the beam axis.
  - $\hat{x}, \hat{y}$ transverse directions to the beam.
  - After averaging: $C_x = C_y = C_\perp$.

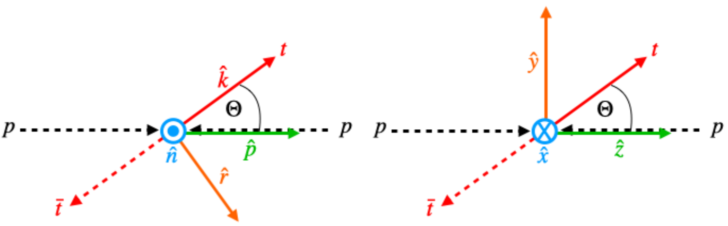


Figure: Helicity and beam bases.

Quantum Tomography: One Qubit

- Qubit: quantum system with two states (e.g., spin-1/2 particle).
- Most general density matrix for a qubit:

$$\rho = \frac{1 + \sum_i B_i \sigma^i}{2} = \frac{1}{2} \begin{bmatrix} 1 + B_3 & B_1 - iB_2 \\ B_1 + iB_2 & 1 - B_3 \end{bmatrix}$$

- Only 3 parameters B_i $\rightarrow$ Quantum tomography is the measurement of spin polarization **B**:

$$B_i = \langle \sigma^i \rangle = \mathrm{tr}(\sigma^i \rho)$$

Quantum Tomography: Two Qubits

- Most general density matrix for 2 qubits:

$$\rho = \frac{1 + \sum_i \left(B_i^+ \sigma^i + B_i^- \bar{\sigma}^i \right) + \sum_{i,j} C_{ij} \sigma^i \bar{\sigma}^j}{4}$$

- 15 parameters $B_i^{\pm}, C_{ij} \rightarrow$ Quantum tomography=Measurement of individual spin polarizations $\mathbf{B}^{\pm}$ and spin correlation matrix $\mathbf{C}$:

$$B_i^+ = \langle \sigma^i \rangle, \; B_i^- = \langle \bar{\sigma}^i \rangle, \; C_{ij} = \langle \sigma^i \bar{\sigma}^j \rangle$$

Quantum State

- **Pure state:** can be described by wave-functions $\sum_i \alpha_i \cdot |\psi_i\rangle$.

Quantum State

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- **Mixed state:** can be described by a density matrix: $\rho = \sum_i p_i \cdot |\psi_i\rangle \langle\psi_i|$.
  - Example: at the LHC we cannot control the initial state.

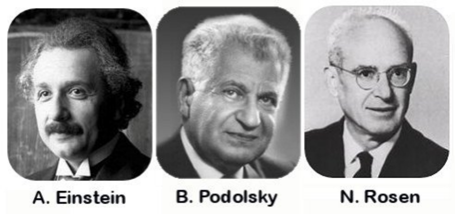
Quantum Entanglement

- Two different systems A and B: $\mathcal{H} = \mathcal{H}_a \otimes \mathcal{H}_b$.
- Separable: $\rho = \sum_n p_n \rho_n^a \otimes \rho_n^b$.
- $\rho_n^{a,b}$ are quantum states in A, B, $\sum_n p_n = 1,\ p_n \geq 0$
- Classically correlated state in $\mathcal{H}$ $\rightarrow$ can be written in this form.
- Non-separable state is called entangled and hence, it is a non-classical state.

Separable

Non-Separable

EPR Paradox



MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

EPR Paradox

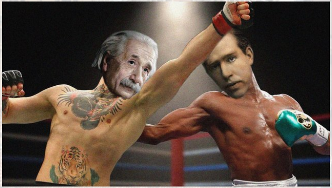
- Entanglement: "spooky action at a distance" (A. Einstein).

- Assuming two particles with spacial distance.
- When a measurement is done on one of the particles, the other one "knows" about it immediately.
- Information travel faster than light?
- Contradicts the theory of relativity.
- **Conclusion:** the theory of Quantum Mechanics is incomplete.

Hidden Variables

- By EPR, each particle "carries" variables that knows the state before the measurement.
- $\Rightarrow$ There are some hidden variables that are missing in order to have a full theory.
- The Copenhagen Interpretation: superposition of states until a measurement was done.
- Bohr Vs. Einstein.

"God does not play at dice with the universe".



"Quit telling God what to do!"

- Who is right?

Bell's Inequality

III.5 ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

JOHN S. BELL†

- If local hidden variables hold, they should satisfy some inequality.
- $C(x, y)$ are the correlations between different measurements at different detectors.
- The parameters a,b,c are different directions for the measurement.
- Original form: $1 + C(b, c) \geq |C(a, b) - C(a, c)|$.

Recent Related Measurement

- Recently, D was measured with no selection on $M_{t\bar{t}}$ by the CMS collaboration.
- Results:
  $D = -0.237 \pm 0.011 > -1/3$;
  $\Delta D/D = 4.6\%$.
- No evidence of quantum entanglement.

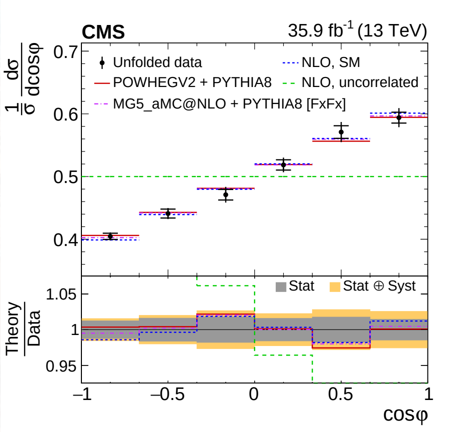


Figure: Distribution of $cos\varphi$. Figure is from Phys. Rev. D 100, 072002.