Neutrino mixing angle and neutrino oscillation in ALPs matter

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Neutrino effective Hamiltonian

Here we consider neutrino flavor oscillations in matter of axion-like particles (ALPs). Our case is 2-flavor neutrino (ν_e, ν_μ). All calculations are analogous to those performed in [1, 2]. The Lagrangian of interaction between neutrino and ALPs is very simiar to the Lagrangian of interactiob between leptons and ALPs [3]:

$$L_{int} = -i\frac{\partial_\mu a}{F}\bar{\nu}_i^{(f)}(g_V^{ij} + g_A^{ij}\gamma_5)\nu_j^{(f)}, \tag{1}$$

where $\nu_i^{(f)}$ – neutrino flavor state, a – ALP-s pseudo-scalar field, and g_V^{ij}, g_A^{ij} – vector-like and axial-like constants. Moreover:

$$g_V^{ij} = -g_V^{ji}, \tag{2}$$
$$g_A^{ij} = g_A^{ji}. \tag{3}$$

Let's transform neutrino flavor state to neutrino mass state with a rotation PMNS-matrix.

$$\nu^{(f)} = U\nu^{(p)}. \tag{4}$$

For the 2-flavor case PMNS-matrix would have the following form:

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{5}$$

After some transformations Lagrangian will take a move convenient form [3]:

$$L_{int} = -i\frac{a}{F}\bar{\nu}_i^{(p)}(g_V^{ij}(m_i - m_j) + g_A^{ij}(m_i + m_j)\gamma_5)\nu_j^{(p)}, \tag{6}$$

where we use the Dirac equation for the free neutrino field no neglect the derivative in ALPs-field. Also constants g_V^{ij}, g_A^{ij} will change for the masive form. Their symmetry and antysymmetry are saved.
Now, let's obtain effective Hamiltoian to build equation in a Schrödinger form. For this purpose, we should build the Dirac equqation

$$(\gamma_\mu \partial^\mu - m_i)\,\nu_i = \frac{a}{F}\left(g_V^{ij}(m_i - m_j) + g_A^{ij}(m_i + m_j)\gamma_5\right)\nu_j. \tag{7}$$

After multiplication on the left by γ_0 matrix, these equations will change in that way:

$$E\nu_i = (\gamma_0\boldsymbol{\gamma p} + m\gamma_0)\,\nu_i + \frac{a}{F}\left(g_V^{ij}(m_i - m_j) - g_A^{ij}(m_i + m_j)\gamma_5\right)\gamma_0\nu_j. \tag{8}$$

Now our equation has taken the Schrödinger form:

$$\hat{H}\nu_i^{(p)} = E\nu_i^{(p)}, \tag{9}$$

where $\hat{H}$ – Hamiltonian:

$$\hat{H} = \begin{pmatrix} \gamma_0\boldsymbol{\gamma p_1} + m_1\gamma_0 - 2g_A^{11}m_1\gamma_5\gamma_0 & g_V^{12}(m_1 - m_2)\gamma_0 - g_A^{12}(m_1 + m_2)\gamma_5\gamma_0 \\ g_V^{21}(m_2 - m_1)\gamma_0 - g_A^{21}(m_2 + m_1)\gamma_5\gamma_0 & \gamma_0\boldsymbol{\gamma p_2} + m_2\gamma^0 - 2g_A^{22}m_2\gamma_5\gamma_0 \end{pmatrix}. \tag{10}$$

Now we should reduild this Hamiltonian by averaging it over the mass states. The elements of rebuiled Hamiltonian would be written in the followoing form [2]:

$$\Delta_{ij} = \langle \nu_i | H | \nu_j \rangle. \tag{11}$$

Here we use spinors for neutrino free mass states

$$\nu_i^{(p)}(\mathbf{p}, h) = \sqrt{\frac{E_i + m_i}{2E_i}}\begin{pmatrix} \chi^{(h)} \\ \frac{\boldsymbol{\sigma}\mathbf{p}}{E+m_i}\chi^{(h)} \end{pmatrix}. \tag{12}$$

Here we use we the assumption, that neutrino is born like a left-handed particle and we don't have spin oscillations, so we have only $\chi^{(-)} = (0\ \ 1)^T$.

Substituting (12) into equation (11) we get

$$H = \begin{pmatrix} E_1 & H_{12} \\ H_{21} & E2 \end{pmatrix}, \tag{13}$$

where all addition of ALPs matter lie in the offdiagonal elements (here use assumption $\boldsymbol{p_1} = \boldsymbol{p_2} = \boldsymbol{p}$, $\boldsymbol{p_x} = \boldsymbol{p_y} = 0$):

$$\begin{gathered} H_{ij} = \tfrac{a}{F}\left[g_V^{ij}(m_i - m_j)\left(1 + \tfrac{p^2}{(E_i+m_i)(E_j+m_j)}\right) - g_A^{ij}(m_i + m_j)p\left(\tfrac{1}{E_i+m_i} - \tfrac{1}{E_j+m_j}\right)\right]\cdot \\ \cdot\sqrt{\tfrac{(E_i+m_i)(E_j+m_j)}{4E_iE_j}} = H_{ij}^V + H_{ij}^A \end{gathered} \tag{14}$$

where

$$H_{ij}^V = H_{ji}^V \tag{15}$$
$$H_{ij}^A = -H_{ji}^A \tag{16}$$

Neutrino flavor Hamiltonian

Let's turn (9) to the flavor basis.

$$i\frac{d}{dt}\nu_i^{(f)} = H'\nu_i^{(f)}, \tag{17}$$

where H' is

$$H' = U^\dagger H U \tag{18}$$

After this rotation Hamiltionian will take the following form (here we use approximation? that $E = \sqrt{p^2 + m^2} \approx |p| + \frac{m^2}{2|p|}$:

$$H' = \left(|p| + \frac{m_1^2 + m_2^2}{4|p|}\right) - \frac{\Delta}{4|p|}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + U^\dagger\begin{pmatrix} 0 & H_{12} \\ H_{21} & 0 \end{pmatrix}U, \tag{19}$$

where

$$\Delta = m_2^2 - m_1^2. \tag{20}$$

The last term in (19), which corresmonds to the interaction between neutrino and ALPs, could be expanded:

$$\begin{aligned} &\begin{pmatrix} (H_{12} + H_{21})\cos\theta\sin\theta & -H_{21}\sin^2\theta + H_{12}\cos^2\theta \\ H_{21}\cos^2\theta - H_{12}\sin^2\theta & -(H_{12} + H_{21})\cos\theta\sin\theta \end{pmatrix} = \\ &= \sigma_3\frac{H_{12} + H_{21}}{2}\sin 2\theta + \sigma_1(H_{12} + H_{21})\cos 2\theta + \begin{pmatrix} 0 & -H_{12} \\ -H_{21} & 0 \end{pmatrix} = \\ &= H_{12}^V(\sigma_3\sin 2\theta + \sigma_1\cos 2\theta) - iH_{12}^A\sigma_2 \end{aligned} \tag{21}$$

Connecting all parts of the Hamiltonian, we get (here we omit the term proportional to the identity matrix, because it has no effect on neutrino oscillation):

$$H' = \frac{\Delta}{4p}(\sigma_1\sin 2\theta - \sigma_3\cos 2\theta) + H_{12}^V(\sigma_1\cos 2\theta + \sigma_3\sin 2\theta) - iH_{12}^A\sigma_2 \tag{22}$$

Neutrino evolution and flavor oscillations

To simplify interaction term of Hamiltonian, we use approximation of ultrarelativictic neutrino ($E << m, E \approx p$). Then, the elements of interaction term will become much simplier:

$$H_{ij}^V = \tfrac{a}{F}g_{ij}^V(m_i - m_j), \tag{23}$$
$$H_{ij}^A = 0. \tag{24}$$

Therefore, we get simplified Hamiltonian:

$$H' = \sigma_1\left(\frac{\Delta}{4p}\sin 2\theta + H_{12}^V\cos 2\theta\right) + \sigma_3\left(H_{12}^V\sin 2\theta - \frac{\Delta}{4p}\cos 2\theta\right). \tag{25}$$

The formal solution for neutrino evolution will have the following form:

$$\begin{pmatrix} \nu_e(x) \\ \nu_\mu(x) \end{pmatrix} = \exp\left(-i\left[\sigma_1\left(\frac{\Delta}{4p}\sin 2\theta + H_{12}^V\cos 2\theta\right) + \sigma_3\left(H_{12}^V\sin 2\theta - \frac{\Delta}{4p}\cos 2\theta\right)\right]x\right)\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}. \tag{26}$$

Here we use exponent decomposition:

$$\exp\left[-i(\sigma_1 a + \sigma_3)\right] = \cos\sqrt{a^2 + b^2} - i\frac{\sigma_1 a + \sigma_3 b}{\sqrt{a^2 + b^2}}\sin\sqrt{a^2 + b^2}. \tag{27}$$

Substitute (27) in (26):

$$\begin{aligned} \begin{pmatrix} \nu_e(x) \\ \nu_\mu(x) \end{pmatrix} = \Bigg[\cos\sqrt{\left(\frac{\Delta}{4p}\right)^2 + (H_{12}^V)^2}x - i\Bigg(\sigma_1\left(\frac{\frac{\Delta}{4p}\sin 2\theta + H_{12}^V\cos 2\theta}{\sqrt{\left(\frac{\Delta}{4p}\right)^2 + (H_{12}^V)^2}}\right) + \\ +\sigma_3\left(\frac{H_{12}^V\sin 2\theta - \frac{\Delta}{4p}\cos 2\theta}{\sqrt{\left(\frac{\Delta}{4p}\right)^2 + (H_{12}^V)^2}}\right)\Bigg)\sin\sqrt{\left(\frac{\Delta}{4p}\right)^2 + (H_{12}^V)^2}x\Bigg]\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}. \end{aligned} \tag{28}$$

Here we have the interest only to offdiagonal elements, which defines propagation. The final expression for neutrino propagation:

$$\begin{aligned} P_{\nu_e \to \nu_\mu}(x) = |\langle \nu_\mu | \nu_e \rangle|^2 = \frac{\left(\frac{\Delta}{4p}\sin 2\theta + H_{12}^V\cos 2\theta\right)^2}{\left(\frac{\Delta}{4p}\right)^2 + (H_{12}^V)^2}\sin^2\sqrt{\left(\frac{\Delta}{4p}\right)^2 + (H_{12}^V)^2}x = \\ = \sin^2 2\theta_{\text{eff}}\sin^2\frac{\pi x}{L_{\text{eff}}}, \end{aligned} \tag{29}$$

where

$$\sin^2 2\theta_{\text{eff}} = \frac{\left(\frac{\Delta}{4p}\sin 2\theta + H_{12}^V\cos 2\theta\right)^2}{\left(\frac{\Delta}{4p}\right)^2 + (H_{12}^V)^2} = \sin^2 2(\theta + \theta_{\text{add}}), \quad \theta_{\text{add}} = \frac{1}{2}\arctan\frac{4pH_{12}^V}{\Delta}, \tag{30}$$

$$L_{\text{eff}} = \frac{\pi}{\sqrt{\left(\frac{\Delta}{4p}\right)^2 + (H_{12}^V)^2}}. \tag{31}$$

Thus, in the case of moving in ALPs matter, neutrino oscillations gets the dependence from the linear mass difference.

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