

# Higher-order QCD corrections to the Higgs decay into bottom quarks from Padé approximants

ICHEP2022  
BOLOGNA

Diogo Boito<sup>1</sup>, Cristiane Yumi London<sup>1</sup>, Pere Masjuan<sup>2</sup>

<sup>1</sup>Instituto de Física de São Carlos, Universidade de São Paulo, Brazil

<sup>2</sup>Institut de Física d'Altes Energies, Universitat Autònoma de Barcelona, Spain

IFSC UNIVERSIDADE DE SÃO PAULO  
Instituto de Física de São Carlos

IFAE Institut de Física d'Altes Energies

$H \rightarrow b\bar{b}$

\* Optical Theorem

calculated for massless quarks

$$\Gamma(H \rightarrow b\bar{b}) = \frac{1}{v^2 m_H} \text{Im} \Pi(s) = \frac{1}{v^2 m_H} \frac{N_c}{8\pi} m_\mu^2 s \left[ 1 + \sum_{n=1}^{\infty} c_n a_\mu^n \right]$$

\* Scalar correlator — not directly related to physical observables

$$\Pi(s) = \frac{N_c}{8\pi^2} m_\mu^2 s \sum_{n=0}^{\infty} a_\mu^n \sum_{k=0}^{n+1} d_{n,k} L^k$$

$$L = \ln \left( -\frac{s}{\mu^2} \right)$$

\* Imaginary part known up to five loops [1]

$$\text{Im} \Pi(s) = \frac{N_c}{8\pi} m_\mu^2 s \left[ 1 + 5.667 a_\mu + 29.15 a_\mu^2 + 41.76 a_\mu^3 - 825.7 a_\mu^4 + \dots \right]$$

\* Perturbative series are asymptotic (factorially divergent) — it is useful to work with Borel transform [2]

$$B[R](u) = \sum_{n=0}^{\infty} \frac{r_n}{n!} u^n \quad \rightarrow \quad R(\alpha) = \int_0^{\infty} B[R](u) e^{-u/\alpha} du$$

Borel sum

\* Renormalons (singularities of Borel transform) govern the behavior of the series

## Padé Approximants

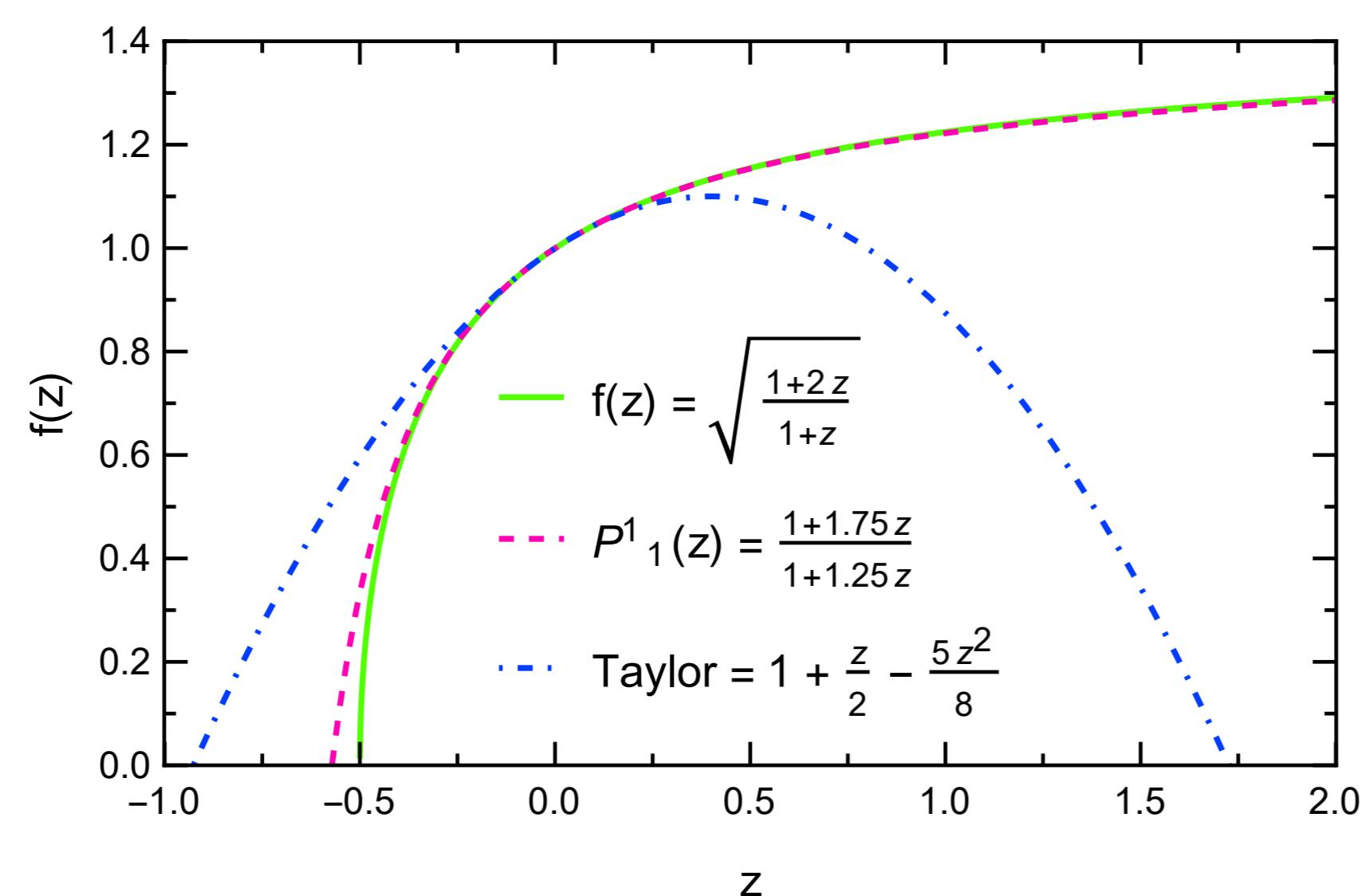
\* A Padé approximant (PA) is a ratio of two polynomials that approximates a function whose Taylor expansion is known [3]

$$f(z) = \sum_{n=0}^{\infty} f_n z^n \quad \rightarrow \quad P_N^M(z) = \frac{Q_M(z)}{R_N(z)} = \frac{a_0 + a_1 z + \dots + a_M z^M}{1 + b_1 z + \dots + b_N z^N}$$

$$\approx f_0 + f_1 z + \dots + f_{M+N} z^{M+N} + \mathcal{O}(z^{M+N+1})$$

\* Advantages of using Padés

- ♦ efficient approximation;
- ♦ partial reconstruction of analytic properties
- ♦ good predictions of higher orders.



## D-log Padé Approximants

\* For series with branch points or cuts, we can use D-log Padés [4]

$$f(z) = A(z) \frac{1}{(\mu - z)^\gamma} + B(z) \quad F(z) = \frac{d}{dz} \ln f(z) \approx \frac{\gamma}{\mu - z}$$

$$\text{Dlog}_N^M(z) = f_{\text{norm}}(0) \exp \left\{ \int dz' \bar{P}_N^M(z') \right\}$$

PA applied  
to F(z)

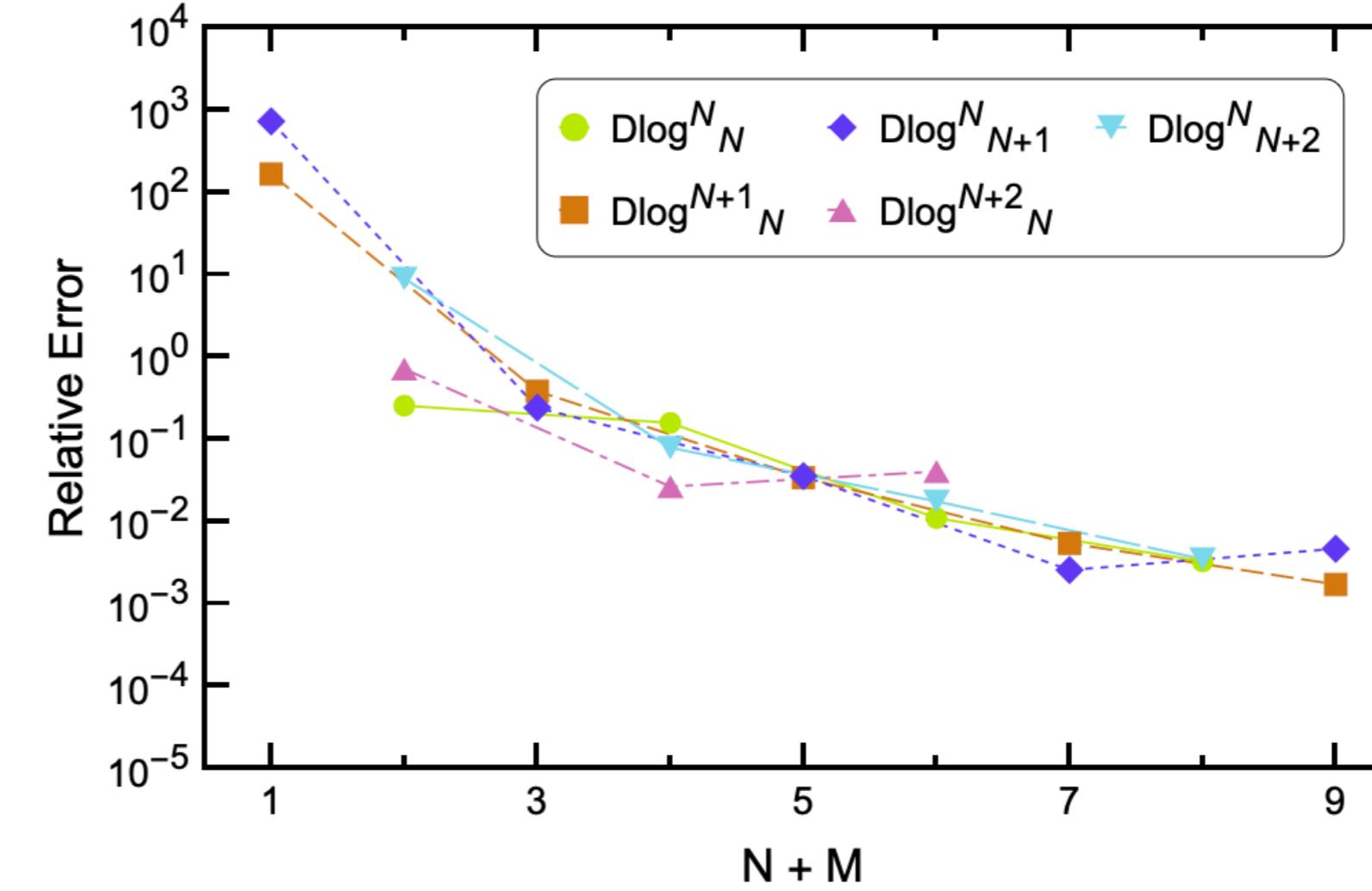
PAs and D-log PAs are able to reproduce the renormalons

Testing in the large- $\beta_0$  limit

\* The perturbative series is known to all orders in this limit [5]

$$\Pi_{L\beta}(s) = \frac{N_c}{4\pi} m_b^2 s \left[ 1 - \frac{L}{2} - \frac{1}{9} \sum_{n=1}^{\infty} \left( -\frac{\beta_0}{2} \right)^{n-1} H_{n+1}(L) a_s^n \right]$$

\* Systematic study of different strategies



Relative error  
decreases when order  
of the PA is increased

## Partial Conclusions

- ♦ Padés and D-log Padés sequences appear to converge
- ♦ Understanding the PAs and D-log PAs results in terms of Padé theory
- ♦ PAs that use only the first three coefficients as input are not sufficiently accurate — not enough information
- ♦ D-log Padés to  $B[\Pi''](u)$  were the best method

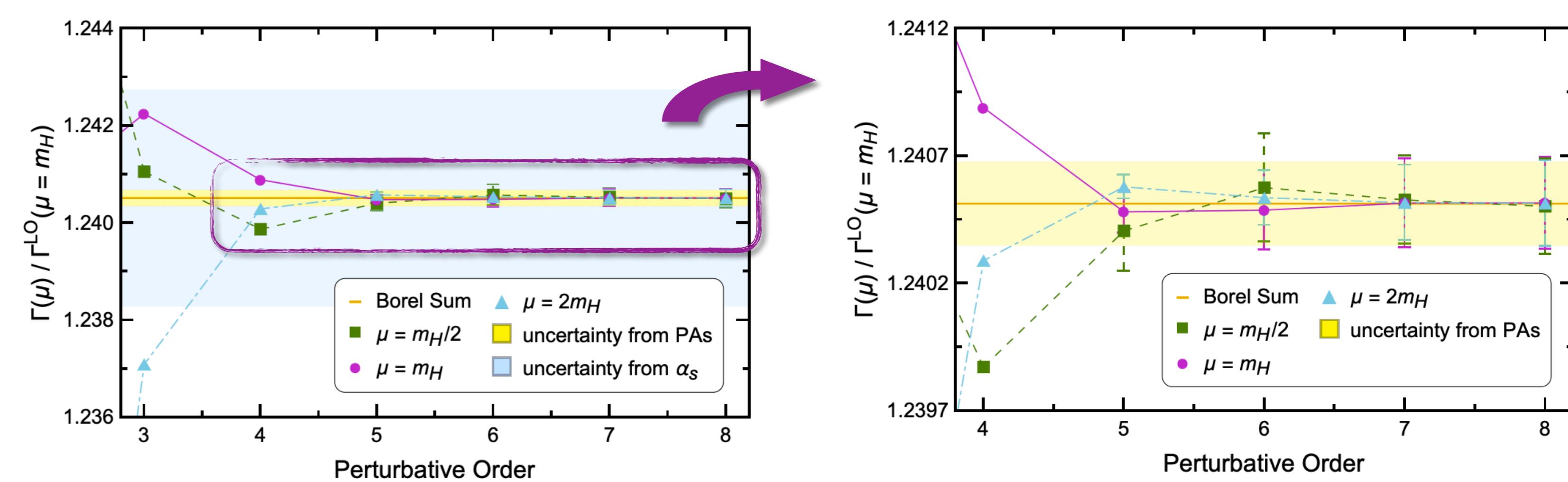
## Final Results in QCD

Final values [6]	$c_5$	$c_6$	$c_7$	$c_8$
	$-6900 \pm 1400$	$(0.3 \pm 3.5) \times 10^4$	$(3.7 \pm 2.5) \times 10^5$	$(0.2 \pm 2.4) \times 10^6$

predicted coefficient at six loops

## Estimated decay width [6]

$$\Gamma(H \rightarrow b\bar{b}) = 2.3806 \begin{pmatrix} +0.041 \\ -0.027 \end{pmatrix}_{m_b} \pm (0.0042)_{\alpha_s} \pm (0.0032)_{m_H} \pm \pm (0.0002)_{\mu} \pm (0.0003)_{\text{PAs}} \text{ MeV}$$



## Conclusions

- ♦ Model independent method:  $c_5 = -6900 \pm 1400$
- ♦ Truncation of the QCD series for  $\Gamma(H \rightarrow b\bar{b})$  is under control
- ♦ Limiting factors in the precision of  $\Gamma(H \rightarrow b\bar{b})$  are the uncertainty in  $m_b$ ,  $m_H$  and  $\alpha_s$

## Main References and Acknowledgements

[1] Baikov, Chetyrkin, Kühn. Phys. Rev. Lett. 96, 012003, 2006.

[2] Beneke. Phys. Rept., v. 317, p. 1-142, 1999

[3] Baker. *Essentials of Padé approximants*. Elsevier, 1975

[4] Boito, Masjuan, Oliani. JHEP 08, 075 (2018)

[5] Broadhurst, Kataev, Maxwell. Nucl. Phys., B592, p.247-293, 2001

[6] Boito, London, Masjuan. JHEP 01, 054 (2022)

FAPESP

CNPq