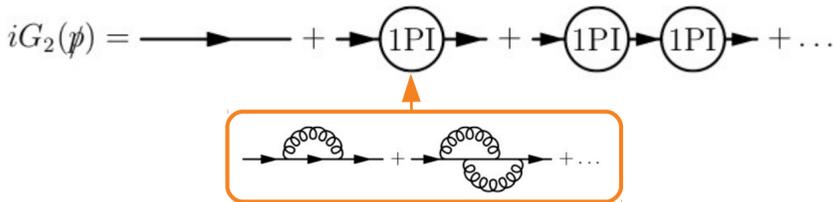


### Renormalization of quark masses



- Understanding quark masses is important for precision SM studies → *renormalization scheme and -scale dependent quantities*
- Theoretically well-defined masses extracted from cross sections → *proper scale setting mandatory*
- Interpretation of MC top mass  $m_t^{MC}$  includes additional uncertainty  $\mathcal{O}(0.5 \text{ GeV})$  in QCD
- Suitable *short-distance masses*:  
 $\overline{m}_t(\mu_m)$  ( $\overline{\text{MS}}$ ) or  $m_t^{\text{MSR}}(R)$  (MSR)  
For  $\mu_m \gtrsim \overline{m}_t(\overline{m}_t)$       For  $R \lesssim \overline{m}_t(\overline{m}_t)$

### The running of the top quark mass

- Implementation of running MSR and  $\overline{\text{MS}}$  masses into  $t\bar{t}$  production cross sections: **HATHOR** (*inclusive, NNLO*) & **MCFM v6.8** (*single-differential, NLO*)

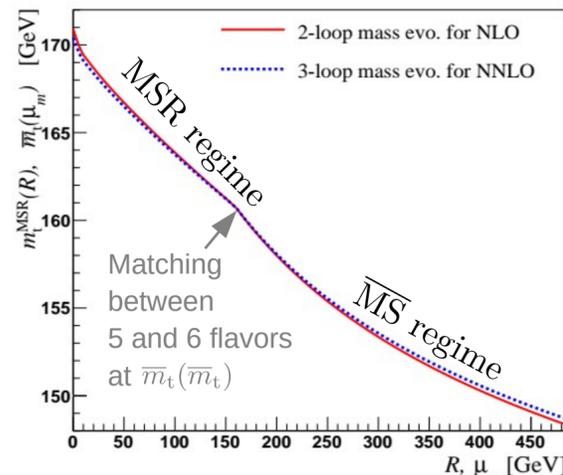
- The pole and  $\overline{\text{MS}}$  masses are related by  

$$m_t^{\text{pole}} = \overline{m}_t(\mu_m) \left( 1 + \sum_{n=1} \frac{\alpha_S(\mu_m)^n}{\pi^n} d_n(\mu_m) \right)$$
 *$\mu_m \neq \mu_r$  in general!*

- MSR introduces a mass renormalization scale  $R$ , so that  
 $m_t^{\text{MSR}}(R) \xrightarrow{R \rightarrow 0} m_t^{\text{pole}}$  and  
 $m_t^{\text{MSR}}(R) \xrightarrow{R \rightarrow \overline{m}_t(\overline{m}_t)} \overline{m}_t(\overline{m}_t)$ :

$$m_t^{\text{pole}} = m_t^{\text{MSR}}(R) + R \sum_{n=1} \frac{\alpha_S(R)^n}{\pi^n} d_n^{\text{MSR}}(R)$$

Decoupling coefficients: integrating the top quark out for  $R \lesssim \overline{m}_t(\overline{m}_t)$   
 → **Natural MSR (MSRn) [1]**



### The single-differential cross section

In the MSR regime:

$$\frac{d\sigma}{dm_{t\bar{t}}} = a_S(\mu_r)^2 \frac{d\sigma^{(0)}}{dm_{t\bar{t}}}(m_t^{\text{MSR}}(R), \mu_r)$$

#### Independent scales

- $\mu_f$ : factorization
- $\mu_r$ : renormalization
- $R, \mu_m$ : mass scales

$$+ a_S(\mu_r)^3 \frac{d\sigma^{(1)}}{dm_{t\bar{t}}}(m_t^{\text{MSR}}(R), \mu_r)$$

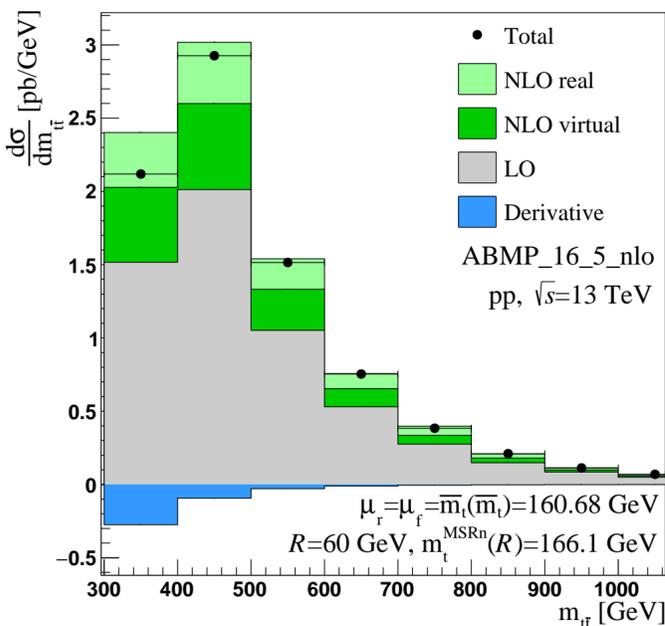
$$+ a_S(\mu_r)^3 d_1 R \frac{d}{dm_t} \left( \frac{d\sigma^{(0)}(m_t, \mu_r)}{dm_{t\bar{t}}} \right) \Big|_{m_t = m_t^{\text{MSR}}(R)}$$

In the  $\overline{\text{MS}}$  regime:

$$\frac{d\sigma}{dm_{t\bar{t}}} = a_S(\mu_r)^2 \frac{d\sigma^{(0)}}{dm_{t\bar{t}}}(\overline{m}_t(\mu_m), \mu_r)$$

$$+ a_S(\mu_r)^3 \frac{d\sigma^{(1)}}{dm_{t\bar{t}}}(\overline{m}_t(\mu_m), \mu_r)$$

$$+ a_S(\mu_r)^3 d_1(\mu_m) \overline{m}_t(\mu_m) \frac{d}{dm_t} \left( \frac{d\sigma^{(0)}(m_t, \mu_r)}{dm_{t\bar{t}}} \right) \Big|_{m_t = \overline{m}_t(\mu_m)}$$



Also  $d\sigma/dp_T(\bar{t})$  and  $d\sigma/dy(\bar{t})$  available

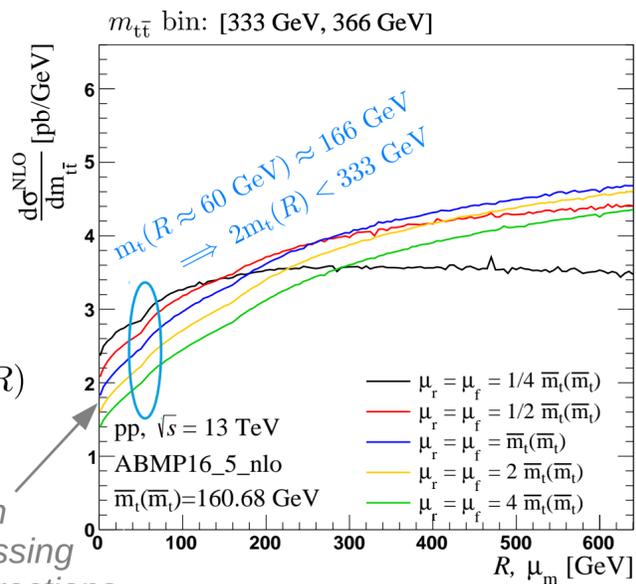
### Investigating independent scale behavior

- Studying  $d\sigma/dm_{t\bar{t}}$  as a function of  $R, \mu_m$  in small  $m_{t\bar{t}}$  bins and varying  $\mu_r, \mu_f$  permits novel investigations of dynamical scale dependency

- Low  $\mu_r, \mu_f$  lead to quick stabilization of NLO predictions near distribution peak

- Recommend scales  $R \sim 60 - 100 \text{ GeV}$  for extracting  $m_t^{\text{MSR}}(R)$   
 e.g.  $R = 80 \text{ GeV} \approx \overline{m}_t(\overline{m}_t)/2$

*Low-R region subject to missing Coulomb corrections*



### Extraction of the top quark MSR mass from CMS data at 13 TeV

- $m_t^{\text{MSR}}(R)$  extracted from the data of [2] at  $R = 80 \text{ GeV}$ , evolved to reference values at  $R \in \{1, 2\} \text{ GeV}$  and  $\overline{m}_t(\overline{m}_t)$  [1]
- Fit- and  $(\mu_r, \mu_f)$  uncertainties correspond to  $R = 80 \text{ GeV}$
- $R$ -uncertainty from difference in evolved values when starting  $R$  varied  $\pm 20 \text{ GeV}$
- Lowest  $m_{t\bar{t}}$  bin has highest mass sensitivity, but requires understanding of threshold effects (Coulomb + soft) for future precision studies
- At  $\mu_r = \mu_f = \mu \equiv \overline{m}_t^{\text{MSR}}(80)$ , agreement with CMS  $\overline{\text{MS}}$  [2] and ATLAS MSR [3]
- Stability investigations confirmed by setting  $\mu_r, \mu_f$  to  $\mu/2$  near threshold

Scales $(\mu_r, \mu_f)$ in bins (1,2,3,4) extracted from [2]	$m_t^{\text{MSR}}(80)$ [GeV]	$m_t^{\text{MSR}}(1)$ [GeV]	$m_t^{\text{MSR}}(2)$ [GeV]	$\overline{m}_t(\overline{m}_t)$ [GeV]	Fit [GeV]	$(\mu_r, \mu_f)$ [GeV]	$(R)$ [GeV]
$(\mu, \mu, \mu, \mu)$	167.7	173.2	173.0	163.3	+0.6	+0.4	+0.4
$(-, \mu, \mu, \mu)$	165.0	170.5	170.3	160.7	-0.6	-0.6	-0.5
$(\frac{\mu}{2}, \mu, \mu, \mu)$	169.3	174.8	174.6	164.8	+2.1	+6.7	+0.5
					-2.1	-9.8	-0.5
					+0.5	+0.2	+0.2
					-0.5	-0.4	-0.3

[1] A. Hoang et al., doi:10.1007/JHEP04(2018)003  
 [2] CMS Collaboration, doi:10.1016/j.physletb.2020.135263  
 [3] ATLAS Collaboration, ATL-PHYS-PUB-2021-034

**Dynamical scales decrease uncertainty**