Space Time Uncertainties and Quantum Decoherence

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Fundamental physics with exotic atoms and radiation detectors

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QT & dynamical reduction

- Why the quantum properties, most notably, superposition of different states at once, do not carry over to larger objects?

- The mechanism at the basis of the transition from Quantum to Classical behavior is not embedded in the original QT

- Superposition principle is a consequence of the linearity of the Schroedinger equation, which has to break down at a certain scale.

- Phenomenological dynamical models of w. f. collapse (Dios, Ghirardi, Rimini, Weber, Pearle, Adler, Penrose, Karojhazi, Lukacs, Milburn, Bassi ...): progressive reduction of the superposition, proportional to the increase of the mass of the system under consideration.

Dynamical reduction, the idea

modify the Schroedinger dynamics in one capable to describe the collapse.

1) Non linear ;
2) Stochastic ;
3) Change the dynamics at the level of the ket states .

$$|\psi\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}} \longrightarrow \rho = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

We want this state to evolve in:

$$\{50\% \ |+\rangle \ , \ 50\% \ |-\rangle\} \longrightarrow \rho = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Von Neumann reduction is not enough -> Heisenberg reduction

$$\rho = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{cases} \{50\% \ |+\rangle \ , \ 50\% \ |-\rangle\} \\ \left\{50\% \ \frac{|+\rangle + |-\rangle}{\sqrt{2}} \ , \ 50\% \ \frac{|+\rangle - |-\rangle}{\sqrt{2}}\right\} \end{cases}$$

Axioms of QM:

1) every physical system is associated to a Hilbet space, observables are self-adjoint operators, possible measurement outcomes are:

$$O\,|o_n\rangle = o_n\,|o_n\rangle$$

2) time evolution is governed by the Schördinger equation

$$i\hbar\frac{d}{dt}|\psi(t)\rangle = H\,|\psi(t)\rangle$$

first order in t → deterministic

$$|\psi(t)\rangle = U(t,t_0)\,|\psi(t_0)\rangle$$

linear → superposition principle

3) probability of getting a measurement outcome o_n at time t :

$$P[o_n] = |\langle o_n|\psi(t)\rangle|^2$$

4) wavepacket reduction principle (WPR):

$$|\psi(t)\rangle \text{ before measurement} \longrightarrow |o_n\rangle \text{ after measurement}$$

genuinely probabilistic, stochastic

non-linear

Does dynamical reduction emerge from space-time uncertainty?

- Decoherence means destruction of interference -> diminishes coherent dispersion

  large dispersion of an observable - Quantum ; small dispersion - Classical

- Decoherence should induce classicality in quantum systems

- Decoherence of various observables can be correlated or anticorrelated

  e.g. decoherence of local energy induces decoherence of position of massive objects

- But Nature does not tell us which observable is the primary, to induce decoherence on the others and, hence, classicality

Global time uncertainty and decoherence

Diosi, L. (2005), Braz. J. Phys. 35, 260, Diosi, L., and B. Lukacs (1987), Annalen der Physik 44, 488, Diosi, L. (1987), Physics Letters A 120, 377, A. Bassi et al.,Rev. Mod. Phys. 85,471

Initial state of a quantum system is a superposition of two eigenstates of total Hamiltonian

$$|\psi\rangle = c_1|\varphi_1\rangle + c_2|\varphi_2\rangle$$

time evolution

$$|\psi(t)\rangle = c_1 \exp(-i\hbar^{-1}E_1 t)|\varphi_1\rangle + c_2 \exp(i\hbar^{-1}E_2 t)|\varphi_2\rangle$$

Let us add an uncertainty to the time $t \to t + \delta t$

and assume that is distributed Gaussian, with zero mean, and dispersion which is proportional to the mean time, $\mathbf{M}[(\delta t)^2] = \tau t$ then the density matrix evolves as:

$$\begin{aligned} &\rho(t) \equiv \mathbf{M}\left[|\psi(t)\rangle\langle\psi(t)|\right] = \\ = \ &|c_1|^2|\varphi_1\rangle\langle\varphi_1| + |c_2|^2|\varphi_2\rangle\langle\varphi_2| + \\ + \ &\{c_1^\star c_2 \exp(i\hbar^{-1}\Delta E t)\mathbf{M}\left[\exp(i\hbar^{-1}\Delta E \delta t)\right]|\varphi_2\rangle\langle\varphi_1| + \\ + \ &\quad \text{h.c.} \quad \} \ . \end{aligned}$$

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$$\mathbf{M}[\exp(i\hbar^{-1}\Delta E \delta t)] = e^{-t/t_D}$$

$$t_D = \frac{\hbar^2}{\tau}\frac{1}{(\Delta E)^2}$$

Global time uncertainty and decoherence

The time evolution for the density matrix

$$\hat{\rho}(t+\tau)=\exp\left[\frac{-i\hat{H}\tau}{\hbar}\right]\hat{\rho}(t)\exp\left[\frac{i\hat{H}\tau}{\hbar}\right]$$

Described by the von Neumann equation

$$\frac{d\rho}{dt}=-i\hbar^{-1}[H,\rho]$$

turns to

$$\frac{d\rho}{dt}=-i\hbar^{-1}[H,\rho]-\frac{1}{2}\tau\hbar^{-2}[H,[H,\rho]]$$

G. J. Milburn Prys. Rev. A 44 5401 (1991)

Local time uncertainty and decoherence

To generalize the concept for a local time $t_{\mathbf{r}} \to t + \delta t_{\mathbf{r}}$

one defines the correlation $\mathbf{M}[\delta t_{\mathbf{r}} \delta t_{\mathbf{r}'}] = \tau_{\mathbf{r}\mathbf{r}'} t$

Galileo invariant spatial correlation function

If the total Hamiltonian is decomposed in the sum of the local ones

$$\frac{d\rho}{dt} = -i\hbar^{-1}[H,\rho] - \frac{1}{2}\hbar^{-2}\sum_{\mathbf{r},\mathbf{r}'} \tau_{\mathbf{r}\mathbf{r}'}[H_{\mathbf{r}},[H_{\mathbf{r}'},\rho]]$$

The master equation suppresses superpositions of eigenstates of local energy

Reminder .. proper time interval

In special relativity the Minkowski metric is

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

the coordinates of the arbitrary Lorentz frame are $(x^0, x^1, x^2, x^3) = (ct, x, y, z)$

the infinitesimal time-like interval is $ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 = \eta_{\mu\nu} dx^\mu dx^\nu$

due to invariance of the interval, if we consider the coordinates of an instantaneous rest frame

$$ds^2 = c^2 d\tau^2 - dx_\tau^2 - dy_\tau^2 - dz_\tau^2 = c^2 d\tau^2$$

Reminder .. proper time interval

The proper time interval is then the integral on the world-line

$$\Delta\tau = \int_P d\tau = \int \frac{ds}{c} \longrightarrow \Delta\tau = \int_P \frac{1}{c}\sqrt{\eta_{\mu\nu} dx^\mu dx^\nu}$$

In general relativity the analogous expression for the generic metric tensor yields

$$\Delta\tau = \int_P d\tau = \int_P \frac{1}{c}\sqrt{g_{\mu\nu}\, dx^\mu\, dx^\nu}$$

and when constant coordinates are chosen

$$\Delta\tau = \int_P d\tau = \int_P \frac{1}{c}\sqrt{g_{00}} dx^0$$

local time uncertainty and gravity

In the Newtonian limit $g_{00} = 1 + \frac{2\phi}{c^2}$

Here then comes the crucial point ... it is assumed that the gravitational potential should not be quantized

BUT that QM requires an absolute indeterminacy of the gravitational field.

I.E. the gravitational potential is a c-number stochastic variable, whose mean value is to be identified with the classical Newtonian potential.

Then local time fluctuation is relate to a fluctuation of the local gravitational potential

$$\delta t_{\mathbf{r}} \equiv \delta \int_0^t dt' g_{00}^{1/2}(\mathbf{r},t') \approx -c^{-2} \int_0^t dt' \Phi(\mathbf{r},t')$$

.. so correlations of local uncertainties of Newtonian gravity can lead to correlation of local time uncertainties.

Can the gravitational field be measured with unlimited precision?

Diosi and Lukacs [Ann. Phys. 44, 488 (1987)] apply the arguments of [N. Bohr and L. Rosenfeld, K. Dan. Vidensk. Selsk., Mat.-Fys. Medd. 12, 1 (1933)]: $\Delta\phi(\mathbf{r},t) = -4\pi G\rho(\mathbf{r},t)$ $\quad$ $\mathbf{g}(\mathbf{r},t) = -\nabla\phi$

The apparatus, obeying QM, is characterized by parameters m, R, T. In realistic measurements only a time-space averaged gravitational field is meaningful

$$\tilde{\mathbf{g}}(\mathbf{r},t) = \frac{1}{VT}\int \mathbf{g}(\mathbf{r}',t')d^3r'dt \qquad \text{with} \qquad |\mathbf{r}-\mathbf{r}'| < R \ , \ |t-t'| < T/2$$

The target is a point-like particle (of mass m) at rest at time t=0, immersed in the field g. Detector measures momentum changes. In the time T the momentum gain is $p = m\tilde{\mathbf{g}}T$

$$\delta p = \hbar/R \longrightarrow \sigma(\tilde{\mathbf{g}}) \sim \frac{\hbar}{mRT}$$

Can the gravitational field be measured with unlimited precision?

It's useless to increase R and T, since this would decrease the error on average field, not on the instantaneous local field of the Newtonian theory. m can be increased, till its own field does not perturb g, i.e. till:

$$\sigma(\tilde{\mathbf{g}}) \sim \frac{\hbar}{m\,R\,T} \qquad \delta\tilde{\mathbf{g}}_m \sim \frac{G\,m}{R^2}$$

Given the optimal mass choice then:

$$m_{\mathrm{opt}} \sim \left(\frac{\hbar\,R}{G\,T}\right)^{1/2} \qquad \sigma(\tilde{\mathbf{g}}) \sim \left(\frac{\hbar\,G}{V\,T}\right)^{1/2}$$

If the limitation is universal then the actual gravitational field is:

$$\mathbf{g}(\mathbf{r},t) = \mathbf{g}_N(\mathbf{r},t) + \mathbf{g}_S(\mathbf{r},t)$$

solution of Poisson Eq.

stochastic fluctuation

Uncorrelated gravitational field fluctuations

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If the limitation is universal then the actual gravitational field is: $\mathbf{g}(\mathbf{r},t) = \mathbf{g}_N(\mathbf{r},t) + \mathbf{g}_S(\mathbf{r},t)$

$$< \tilde{\mathrm{g}}_S >= 0 \qquad ; \qquad < \tilde{\mathrm{g}}_S^2 >= \frac{\hbar\,G}{V\,T}$$

The squared dispersion of the averaged g_S is inversely proportional to the space-time cell volume -> hence g_S is uncorrelated in time and space

$$< \mathbf{g}_S(\mathbf{r},t)\,\mathbf{g}_S(\mathbf{r}',t') >= \hbar\,G\,\delta(\mathbf{r}-\mathbf{r}')\,\delta(t-t')$$

Gravitational potential as a stochastic variable

In terms of the potential, this can be regarded as a stochastic variable, with momenta:

$$< \phi(\mathbf{r},t) > = \phi_N(\mathbf{r},t)$$

$$< \phi(\mathbf{r},t)\,\phi(\mathbf{r}',t') > - < \phi(\mathbf{r},t) >< \phi(\mathbf{r}',t') > \sim \frac{\hbar G}{|\mathbf{r}-\mathbf{r}'|}\,\delta(t-t')$$

The covariance function for the gravitational potential is not dependent on the parameters of the gedanken apparatus (m, T, R), which may suggest universality of the potential intrinsic fluctuation.

Going back to the searched correlation of the local time fluctuation $\mathbf{M}[\delta t_{\mathbf{r}}\delta t_{\mathbf{r}'}] = \tau_{\mathbf{r}\mathbf{r}'}t$

$$\delta t_{\mathbf{r}} \equiv \delta\int_0^t dt' g_{00}^{1/2}(\mathbf{r},t') \approx -c^{-2}\int_0^t dt' \Phi(\mathbf{r},t') \longrightarrow \tau_{\mathbf{r}\mathbf{r}'} = \mathrm{const}\times\frac{G\hbar}{|\mathbf{r}-\mathbf{r}'|}c^{-4}$$

Master equation

$$\tau_{\mathbf{r}\mathbf{r}'} = \text{const} \times \frac{G\hbar}{|\mathbf{r}-\mathbf{r}'|} c^{-4}$$

the local time correlation is extremely small

substituted in the master equation

$$\frac{d\rho}{dt} = -i\hbar^{-1}[H,\rho] - \frac{1}{2}\hbar^{-2}\sum_{\mathbf{r},\mathbf{r}'} \tau_{\mathbf{r}\mathbf{r}'}[H_{\mathbf{r}},[H_{\mathbf{r}'},\rho]]$$

yields

$$\frac{d\rho}{dt} = - \; i\hbar^{-1}[H,\rho] - \frac{G}{2}\hbar^{-1}\int\int \frac{d\mathbf{r}d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}[f(\mathbf{r}),[f(\mathbf{r}'),\rho]]$$

Master equation

Denote the configuration coordinates (classical and spin) of the dynamical system by X. The corresponding mass density at the point r is $f(\mathbf{r}|X)$

Given the coordinate eigenstate |x> we have $f(r|X)\delta(X'-X) \equiv \langle X'|\hat{f}(r)|X\rangle$

So if one introduces the damping time:

$$[\tau_d(X,X')]^{-1} = \frac{G}{2\hbar}\int\int d^3r\, d^3r' \times \frac{[f(\mathbf{r}|X) - f(\mathbf{r}|X')][f(\mathbf{r}'|X) - f(\mathbf{r}'|X')]}{|\mathbf{r}-\mathbf{r}'|}$$

the master equation becomes

$$\langle X|\dot{\hat{\rho}}(t)|X'\rangle = (-i/\hbar)\langle X|[\hat{H}_0,\hat{\rho}(t)]|X'\rangle - [\tau_d(X,X')]^{-1}\langle X|\hat{\rho}(t)|X'\rangle\ .$$

Energy decoherence

$$\langle X|\dot{\hat{\rho}}(t)|X'\rangle = (-i/\hbar)\langle X|[\hat{H}_0, \hat{\rho}(t)]|X'\rangle$$
$$- [\tau_d(X, X')]^{-1}\langle X|\hat{\rho}(t)|X'\rangle \, .$$

$$[\tau_d(X, X')]^{-1} = \frac{G}{2\hbar}\int\int d^3r\, d^3r' \times \frac{[f(\mathbf{r}|X) - f(\mathbf{r}|X')][f(\mathbf{r}'|X) - f(\mathbf{r}'|X')]}{|\mathbf{r} - \mathbf{r}'|}$$

If the difference between the mass distributions of two states |X> and |X'> in superposition becomes big

the corresponding damping time becomes short

the corresponding off-diagonal terms of the density operator vanish

this QM violating phenomenon is ENERGY DECOHERENCE

in Diosi approach.

Other theories of space-time uncertainty induced decoherence ..

an incomplete list

[1] R.P. Feynman, Lectures on gravitation (Caltech, 1962-63).
[2] F. K´arolyh´azi, Nuovo Cim. 42A, 390 (1966).
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Other theories of space-time uncertainty induced decoherence

- Milburn assumes that Planck-time is the smallest time,

- Adler derives quantum theory in the special limit of a hypothetical fundamental dynamics,

they share the same master Eq.

- Penrose focuses on the conceptual uncertainty of location in space-time,

Penrose and Diosi model share the same "decay time"

The theories have different mathematical apparatuses, interpretations, metaphysics, e.t.c., but have common divisors. "The fact that they are similar but not identical suggests that the involvement of gravity in wave-vector reduction is strongly indicated, but the exact mathematical treatment remains to be found." A. Bassi (referred to Gravity-related collapse)

The model of Penrose

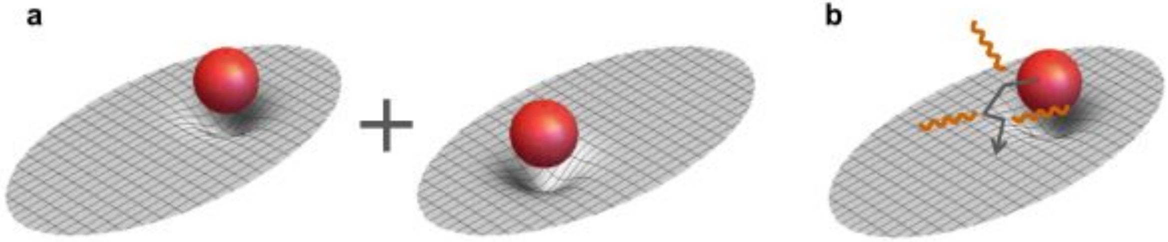
Consider a quantum system which consists of a linear superposition of two well-defined stationary states having the same energy E

$$|\psi\rangle = a|\alpha\rangle + b|\beta\rangle$$

If gravitation is ignored, as is done in standard quantum theory, the superposition is also stationary, with the same energy E

$$i\hbar\frac{\partial|\psi\rangle}{\partial t} = E.|\psi\rangle$$

BUT when gravitation is introduced in the play, there will be a nearly classical spacetime associated with the state $|\alpha\rangle$ and a Killing vector associated with it which represents the time displacement of stationarity, and the same for $|\beta\rangle$.The two Killing vectors can be identified with each other only if the two space-times can be identified point by point. BUT general covariance forbids that, since the matter distributions associated with the two states are different, in the presence of a background gravitational field.



The model of Penrose

On the other hand, unitary evolution in quantum theory requires and assumes the existence of a Schr□oedinger operator which applies to the superposition in the same way that it applies to the individual states.

Its action on the superposition is the superposition of its action on individual states.

↓

Conflict between the demands of QM and of General Relativity.

Imagine to make an approximate point-wise identification between the two spacetimes -> slight error in the identification of the Schr□oedinger operators for the two space-times -> slight uncertainty in the energy of the superposition. In the Newtonian approximation of the order of the gravitational self-energy of the mass distribution in the two superposed states.

Lifetime: $\hbar/E_G$ (the same as for Diosi model)

beyond which time the superposition will decay.

Are D-P models parameter free?

Unfortunately not!

E.g. consider a rigid, homogeneous sphere of radius R and mass m. Then the configuration X is the c.m. coordinate x, and the dumping time:

$$[\tau_d(X,X')]^{-1} = \frac{G}{2\hbar}\int\int d^3r\, d^3r' \times \frac{[f(\mathbf{r}|X) - f(\mathbf{r}|X')][f(\mathbf{r}'|X) - f(\mathbf{r}'|X')]}{|\mathbf{r}-\mathbf{r}'|}$$

The self-interaction is divergent for a point-like particle! -> the w.f. reduction should be instantaneous -> absurd

Penrose - which are the basic stable states to which the superposition decays? They are the stationary solutions of the Shroedinger-Newton equation:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi$$

In this case the Sh. dynamics is affected by the particle's own gravitational field! Moreover the dynamics seems to setup deterministically.

Where is the stochasticity which drives to the Born rule? If the evolution is deterministic & non-linear superluminar propagation should appear.

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Penrose - which are the basic stable states to which the superposition decays? They are the stationary solutions of the Shroedinger-Newton equation:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + m\Phi\Psi$$
$$\nabla^2\Phi = 4\pi Gm|\Psi|^2$$

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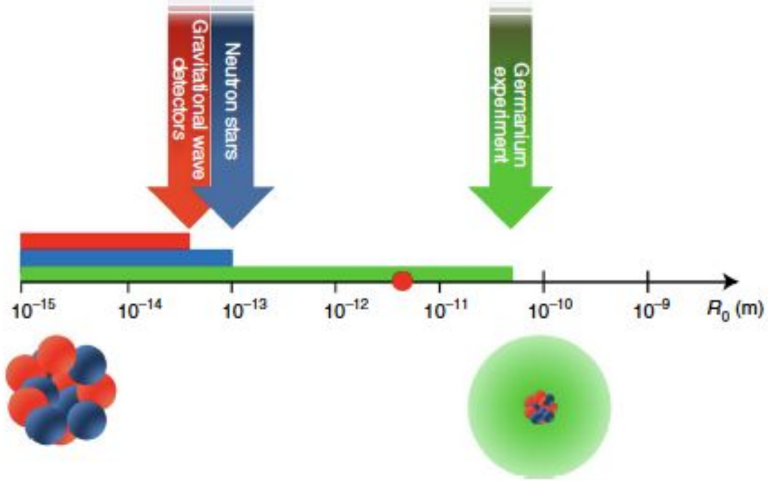
Are D-P models parameter free?

Penrose - which are the basic stable states to which the superposition decays? They are the stationary solutions of the Shroedinger-Newton equation.

Then the mass density of the particle is $\mu(\boldsymbol{r}) = m|\psi(\boldsymbol{r}, t)|^2$

We recently set an experimental lower bound $R_0 > 0.54 \cdot 10^{-10}$ m (90% c.l.) on the size of the particle's mass density which excludes the proposal of Penrose.
In our experimental situation the proposal of Penrose (mean square displacement of a nucleus in the lattice of the target material) corresponds to $0.05 \cdot 10^{-10}$ m.



Nature Physics 17, 74–78 (2021)

Are D-P models parameter free?

Diosi has a different proposal, he introduces a minimum length R_0 which limits the spatial resolution of the mass density, a short-length cutoff to regularize the mass density.

E_G becomes a function of R_0 the larger R_0 the longer the collapse time.

Diosi's proposal is still at stake.

Recently the need for the introduction of "colored" - i.e. non-white correlation in time - collapse models was rised

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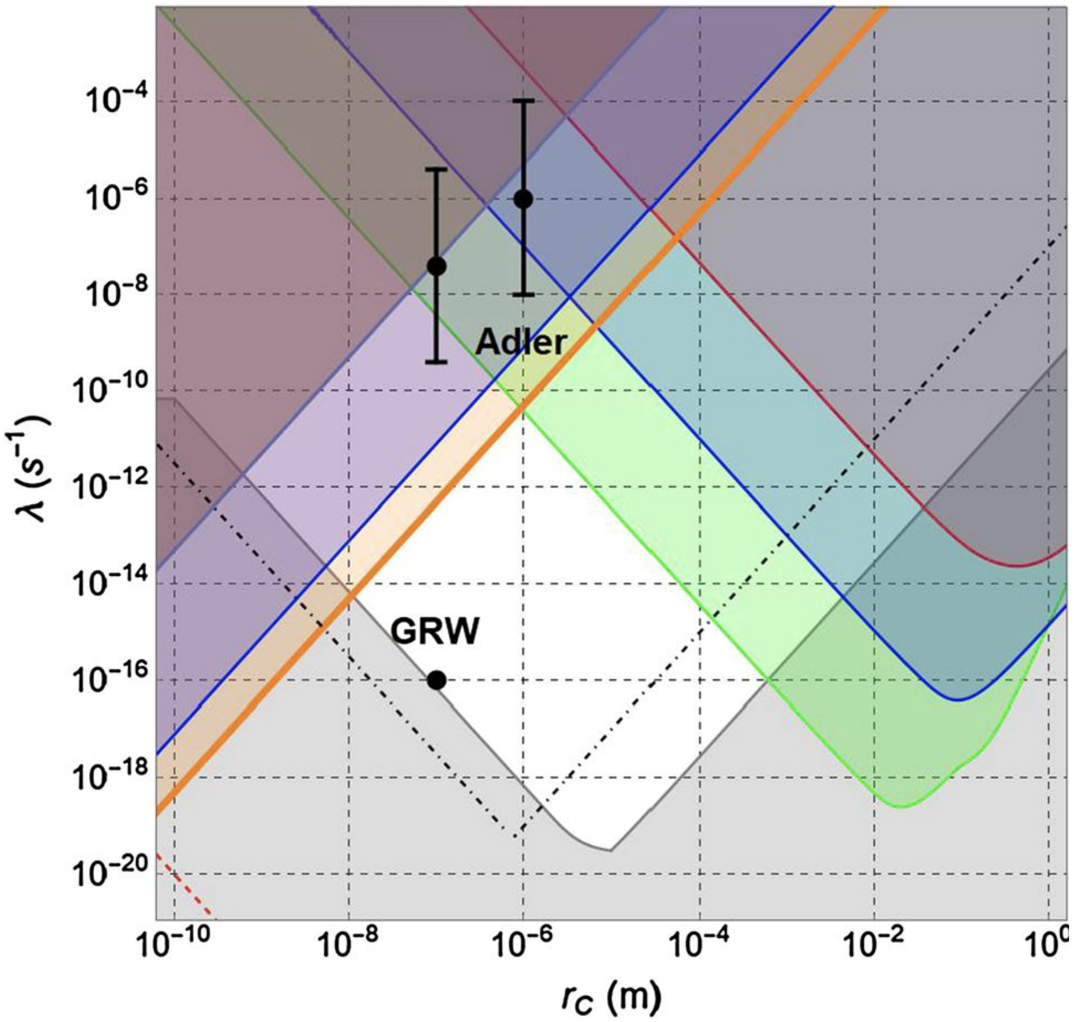

Fig. 4 Mapping of the $\lambda - r_C$ CSL parameters: the proposed theoretical values (GRW [6], Adler [24,25]) are shown as black points. The region excluded by theoretical requirements is represented in gray, and it is obtained by imposing that a graphene disk with the radius of 10 μm (about the smallest possible size detectable by human eye) collapses in less than 0.01 s (about the time resolution of human eye) [31]. Contrary to the bounds set by experiments, the theoretical bound has a subjective component, since it depends on which systems are considered as "macroscopic". For example, it was previously suggested that the collapse should be strong enough to guarantee that a carbon sphere with the diameter of 4000 Å should collapse in less than 0.01 s, in which case the theoretical bound is given by the dash-dotted black line [36]. A much weaker theoretical bound was proposed by Feldmann and Tumulka, by requiring the ink molecules corresponding to a digit in a printout to collapse in less than 0.5 s (red line in the bottom left part of the exclusion plot, the rest of the bound is not visible as it involves much smaller values of λ than those plotted here) [37]. The right part of the parameter space is excluded by the bounds coming from the study of gravitational waves detectors: Auriga (red), Ligo (Blue) and Lisa-Pathfinder (Green) [30]. On the left part of the parameter space there is the bound from the study of the expansion of a Bose–Einstein condensate (red) [28] and the most recent from the study of radiation emission from Germanium (purple) [22]. This bound is improved by a factor 13 by this analysis performed here, with a confidence level of 0.95, and it is shown in orange

CSL

averaged density matrix evolution can be derived from a standard Schroedinger equation with a random Hamiltonian. Such equation does not lead to the state vector reduction, because it is linear, but reproduce the same noise averaged density matrix evolution (photon emission rate ..)

$$H_{\mathrm{TOT}} = H - \hbar\sqrt{\gamma}\sum_{i}\frac{m_j}{m_0}\int N(\mathbf{y},t)\psi_j^{\dagger}(\mathbf{y})\psi_j(\mathbf{y})\,\mathrm{d}^3y$$

$$N(\mathbf{y},t) = \int g(\mathbf{y}-\mathbf{x})\xi_t(\mathbf{x})\,\mathrm{d}^3x,$$

and $\xi_t(\mathbf{x}) = \mathrm{d}W_t(\mathbf{x})/\mathrm{d}t$ is a white noise field (in the simplest case), with correlation function

$$\mathbb{E}[\xi_t(\mathbf{x})\xi_s(\mathbf{y})] = \delta(t-s)\delta(\mathbf{x}-\mathbf{y}).$$

So *N* is a Gaussian noise field, with zero mean, and correlation function:

$$\mathbb{E}[N(\mathbf{x},t)N(\mathbf{y},s)] = \delta(t-s)F(\mathbf{x}-\mathbf{y}), \quad F(\mathbf{x}) = \frac{1}{(\sqrt{4\pi}r_C)^3}\mathrm{e}^{-\mathbf{x}^2/4r_C^2}$$

The Hamoltonian density:

$$\mathcal{H}_{\mathrm{TOT}} = \mathcal{H}_{\mathrm{P}} + \mathcal{H}_{\mathrm{R}} + \mathcal{H}_{\mathrm{INT}}.$$

$$\mathcal{H}_{\mathrm{P}} = \frac{\hbar^2}{2m}\nabla\psi^\dagger \cdot \nabla\psi + V\psi^\dagger\psi - \hbar\sqrt{\gamma}\frac{m}{m_0}N\psi^\dagger\psi.$$

$$\mathcal{H}_{\mathrm{R}} = \frac{1}{2}\left(\varepsilon_0\mathbf{E}_\perp^2 + \frac{\mathbf{B}^2}{\mu_0}\right)$$

$$\mathcal{H}_{\mathrm{INT}} = \mathrm{i}\frac{\hbar e}{m}\psi^\dagger\mathbf{A}\cdot\nabla\psi + \frac{\mathrm{e}^2}{2m}\mathbf{A}^2\psi^\dagger\psi.$$

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perturbation terms

the calculation is performed at first order in $\sqrt{\gamma}$ and e.
So the first-order transition amplitude for a charged particle to emit a photon, as a consequence of the interaction with the noise field is calculated.

If the correlation function in time of the collapsing noise is a delta, the expected rate of radiation, as a consequence of the interaction of the non-relativistic particle with the noise filed (*spontaneous radiation*) is:

$$\frac{d\Gamma}{dp} = \frac{\lambda \hbar e^2}{2\pi^2 \varepsilon_0 c^3 m_0^2 r_c^2 p}$$

Emission rate in the non-white noise case

If a general correlation function in time is considered for the collapsing noise:

$$\mathbb{E}[N(\mathbf{x},t)N(\mathbf{y},s)] = f(t-s)F(\mathbf{x}-\mathbf{y})$$

the photon emission rate changes as:

$$\left.\frac{d\Gamma}{dp}\right|_{\text{NON-WHITE}} = \frac{1}{2}[\tilde{f}(0) + \tilde{f}(pc)] \times \left.\frac{d\Gamma}{dp}\right|_{\text{WHITE}}$$

Second term: the probability of emitting a photon with momentum *p* is proportional to the weight of the Fourier component of the noise corresponding to the frequency $\omega_p = pc$.

The first term is independent on the photon momentum

$$\left.\frac{d\Gamma}{dp}\right|_{\text{NON-WHITE}} = \frac{1}{2}[\tilde{f}(0) + \tilde{f}(pc)] \times \left.\frac{d\Gamma}{dp}\right|_{\text{WHITE}}$$

such term is un-physical. It arises because perturbation theory is formally not valid in the large time limit, since the effect of the noise accumulates continuously in time. Such terms disappears when adding higher terms in the perturbative expansion, or the perturbative calculation is “cured” by e.g. confining the noise.

Time correlation functions for the stochastic noise considered in literature:

$$f(t-s) = \frac{\Omega_c}{2} e^{-\Omega_c|t-s|}$$

whose Fourier transform is $\frac{\Omega_c^2}{\Omega_c^2 + \omega^2}$

or the Gaussian case:

$$f(t-s) = N e^{-\frac{\Omega_c^2 (t-s)^2}{2}}$$

$$e^{-\frac{1}{2}\left(\frac{\omega}{\Omega_c}\right)^2}$$

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Thank you !