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INFN
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Istituto Nazionale di Fisica Nucleare

Low Emittance Muon Accelerator LEMMA

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Dottorato di Ricerca in Fisica degli Acceleratori - XXXIV Ciclo

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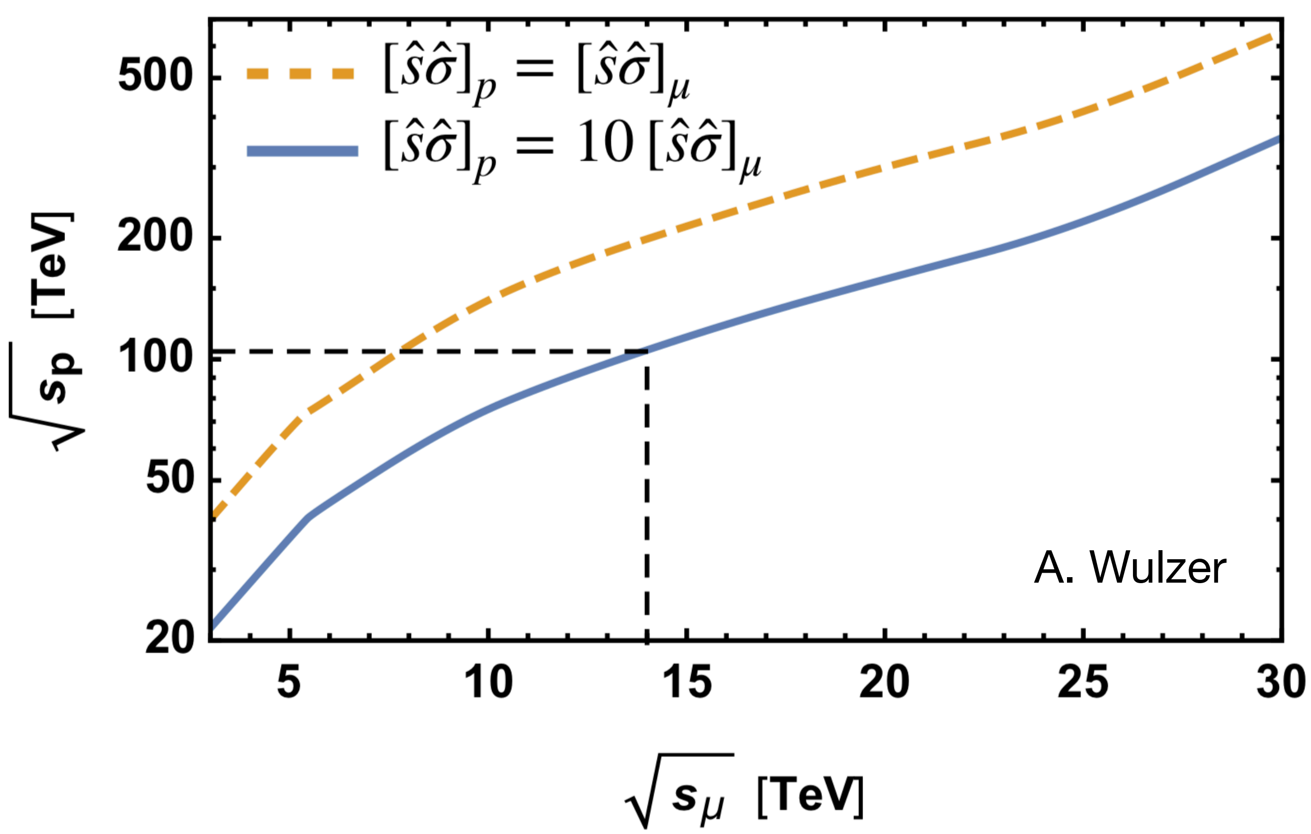
Muon Colliders

Muon based facilities have unique potential in HEP for **next-gen multi-TeV colliders**.

As $m_\mu \sim 200 m_e$ the energy emitted via Synchrotron radiation is $2 \cdot 10^9$ times smaller w.r.t. the electron and therefore **circular machines** can be used accelerate at very high energies.

$$U_0[GeV] = \begin{array}{ll} 8.85 \times 10^{-5} E^4[GeV]/\rho[m] & \text{for electrons} \\ 4.85 \times 10^{-14} E^4[GeV]/\rho[m] & \text{for muons} \\ 6.03 \times 10^{-18} E^4[GeV]/\rho[m] & \text{for protons} \end{array}$$

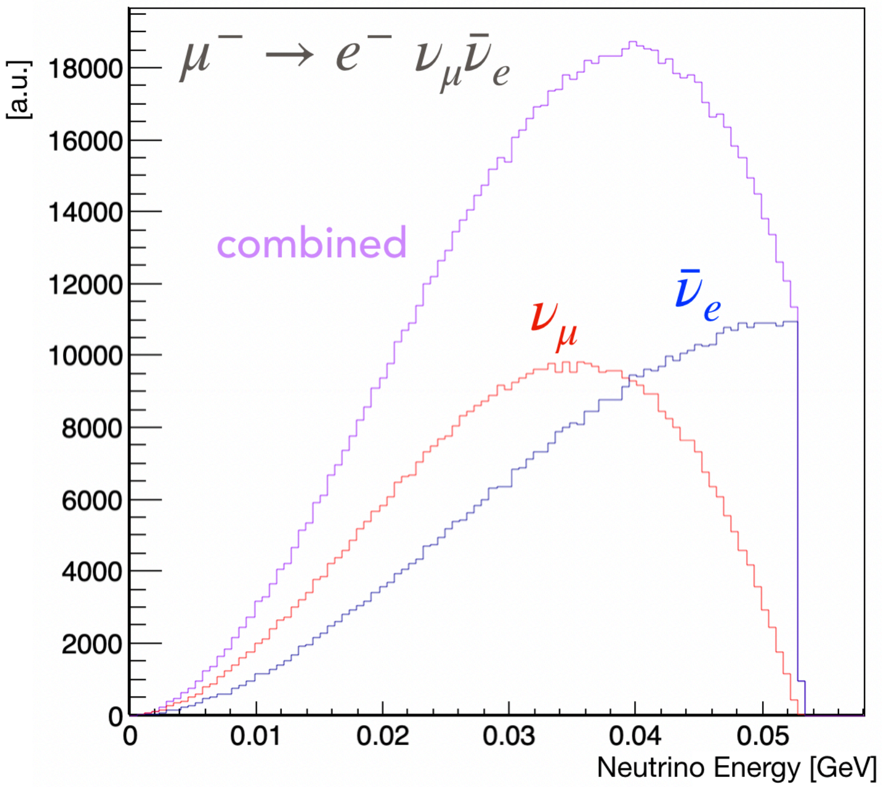


Lepton probe (elemental particle) can reach a given value of cross section at much **smaller $\sqrt{s}$** w.r.t. hadronic probes, even considering for the latter an enhancement due to strong channels

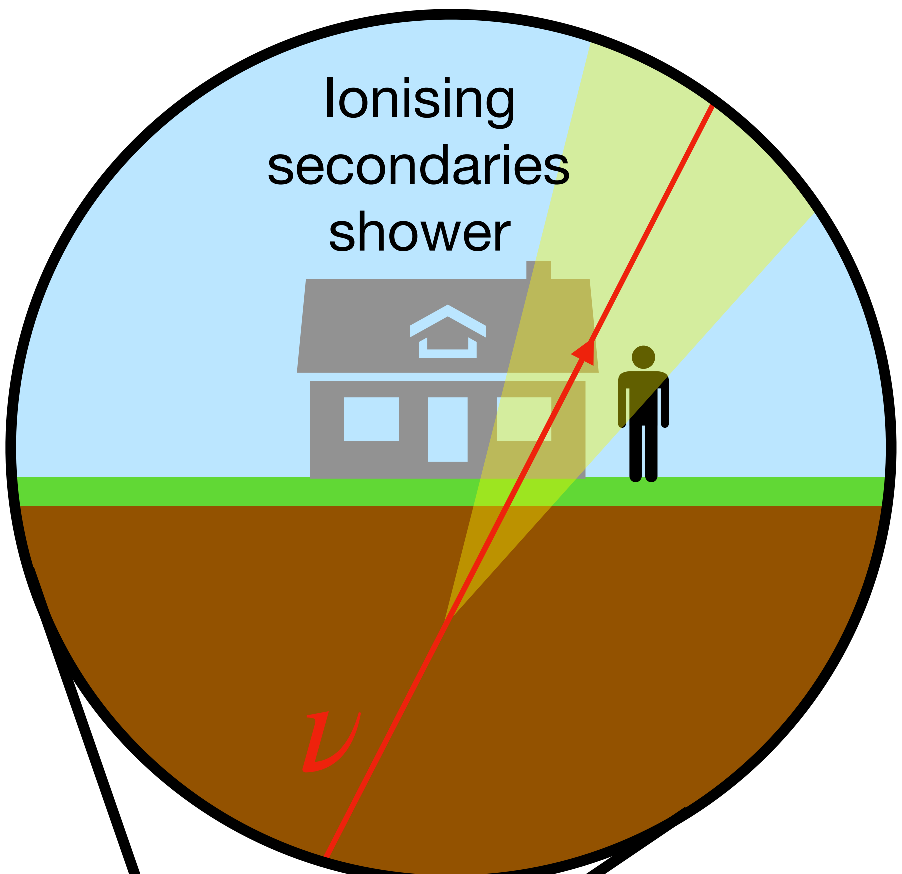
Challenges of designing a Muon Collider

- limited time from production to collision due to **muon lifetime**
- **Quality of the beam**: highly populated and low emittance muon beams $L \propto N_\mu^2/\epsilon$
- current limits due to **neutrino radiation hazard**

Neutrino Radiation Hazard



Constitute an **unshieldable radiation source** travelling through the Earth and producing a shower of ionising secondaries. In order to keep the effective dose for the population below safety values, limitation on the **muon production rate** must be applied.



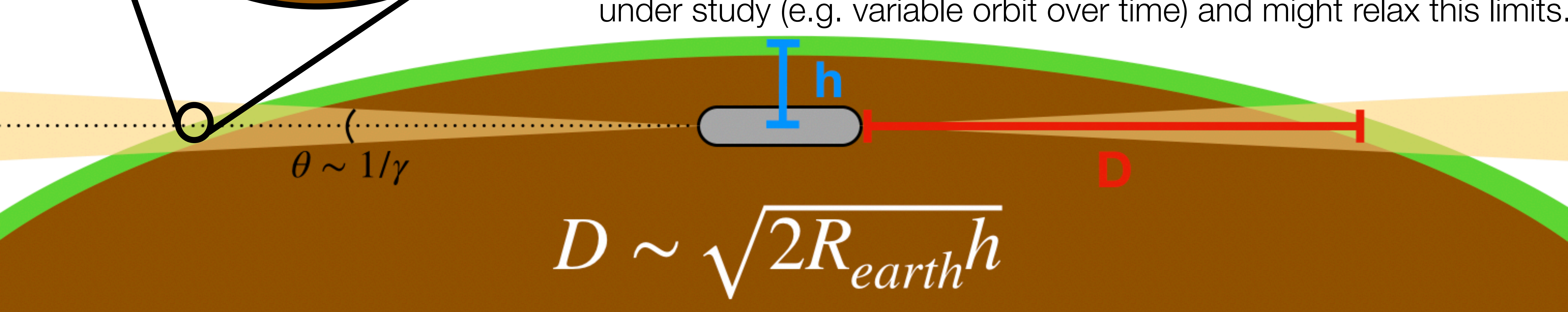
Neutrino flux

Effective Dose Conversion Factor

$$dose/yr = \Phi_\nu \cdot C(E_\nu) = 2\frac{R_\nu[\nu/yr]}{2\pi D\frac{D}{\gamma}} \cdot 10^{2log_{10}E_\nu - 15}$$

e.g. - Effective dose limit resulting from CERN activities for people outside the laboratory: $200\mu Sv/yr$

Options to **mitigate the dose** due to a muon collider are currently under study (e.g. variable orbit over time) and might relax this limits.



Muon Current Limits due to ν radiation

$$R_\mu^{limit}[\mu/yr] = \frac{1}{2} \cdot R_\nu^{limit}[\nu/yr] = 200\mu Sv/yr \frac{2\pi D \frac{D}{\gamma}}{4 \cdot 10^{2log_{10}E_\nu - 15}} \times \frac{3.1 \cdot 10^7 s}{10^7 s}$$

solar year

operational year

$$N_\mu^{limit}[\mu/bunch] = R_\mu^{limit}(E_\mu, D)[\mu/s] \ / \ r_{rep}[bunch/s]$$

$$\boxed{L^{limit} = \frac{N_\mu^{limit^2}(E_\mu, D, r_{rep}) \ n_b f_{rev}}{4\pi\beta^{IP}\epsilon_N/\gamma}}$$

Safety considerations on the neutrino radiation pose limits on the muon current, which in turns set the **maximum luminosity achievable** and therefore the expected number of events

The typical cross-section for EW processes in the multi-TeV

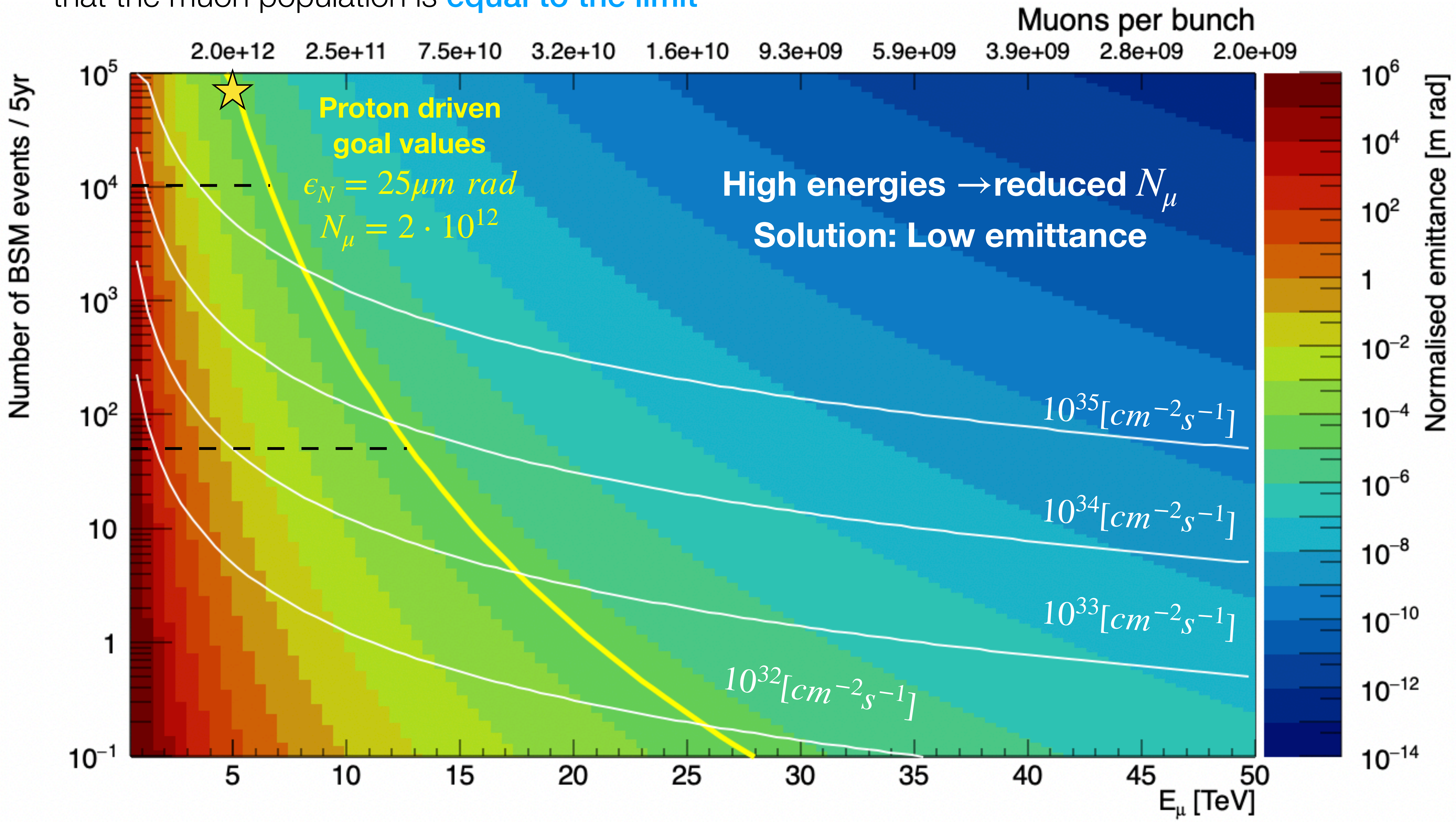
$$\sigma = \left(\frac{10TeV}{\sqrt{s_\mu}}\right)^2 \cdot 1fb \quad \Rightarrow \quad N_{events} = \int L^{limit} \ \sigma \ dt$$

Physics reach:

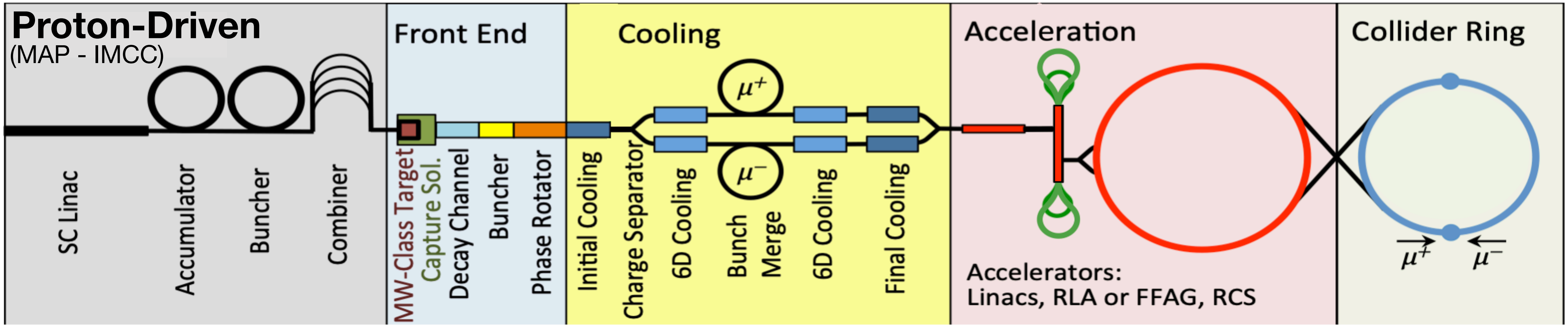
- **Direct discovery** of BSM particles: ~100 events
- High **precision measurements** of SM processes (indirect new physics): ~10^4 events

Muon Current Limits due to ν radiation

Considering a **16T collider** located 100m below the ground level, with 5Hz repetition rate and $\beta^* = 1mm$ the expected number of events / maximum luminosity achievable is given assuming that the muon population is **equal to the limit**

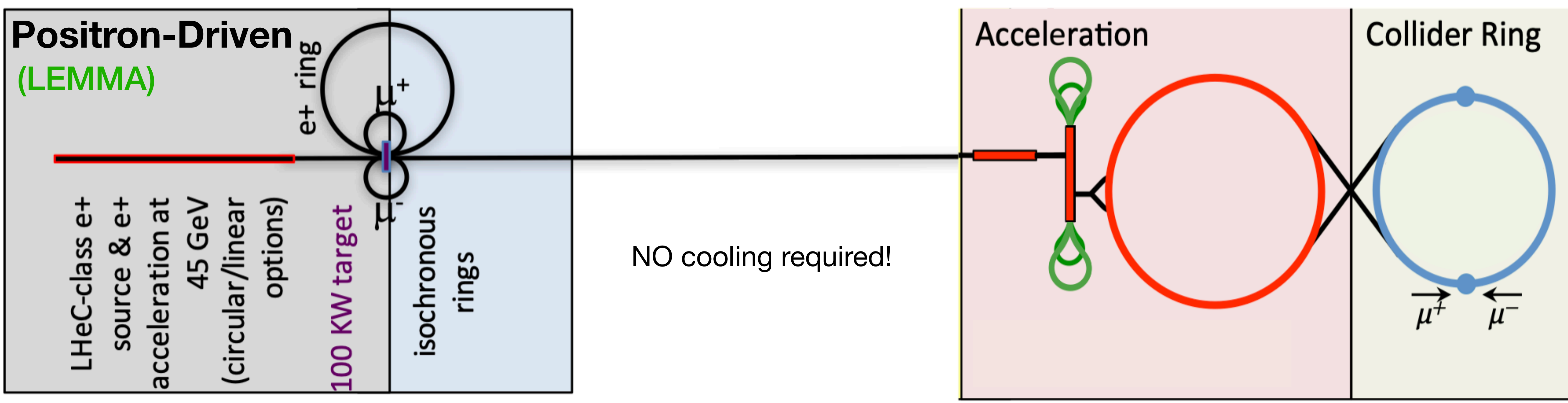


Possible options for the muon source



$$p + X \rightarrow \pi, K \rightarrow \mu$$

- large number of muons ($10^{13} - 10^{14} \mu/s$ MAP goal)
- large emittance →very efficient 6D cooling required (yet to be tested)



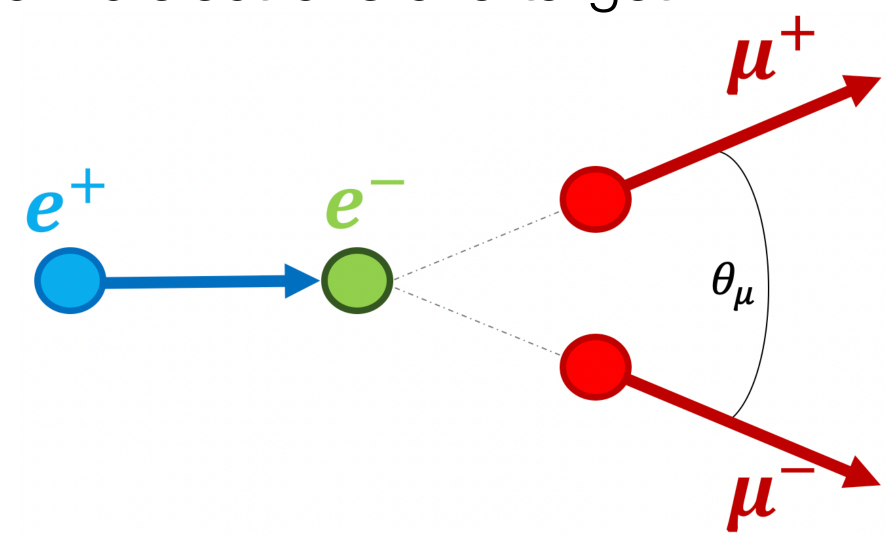
$$e^+e^- \rightarrow \mu^+\mu^-$$

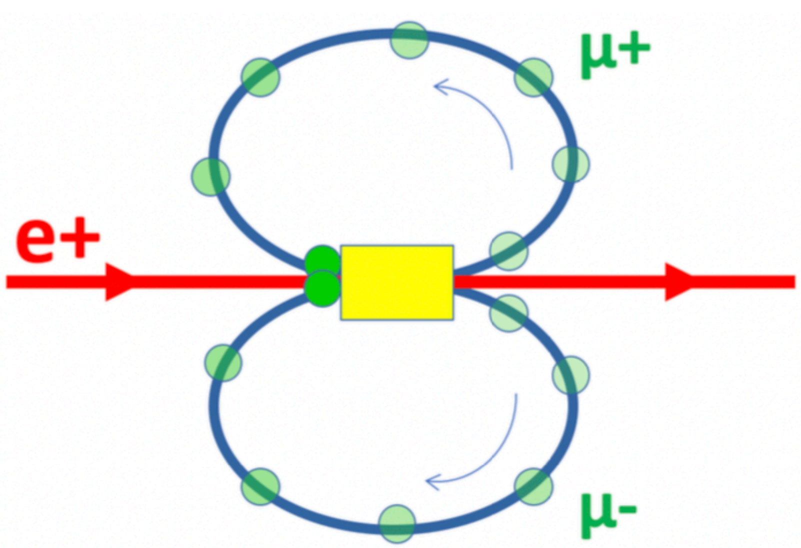
- low production emittance → NO cooling, plasma acceleration.
- low cross-section ($1\mu barn$) → accumulation system required

Low EMittance Muon Accelerator - LEMMA

LEMMA investigates the possibility of a **positron-driven** muon collider in the Multi-TeV range. Muon pairs are produced via **annihilation** of a positron beam on atomic electrons of a target.

- ✓ Low production emittance
- ✓ Muons produced with high boost due to asymmetric collision
- x Low production cross-section ($\sim 1\mu b$)



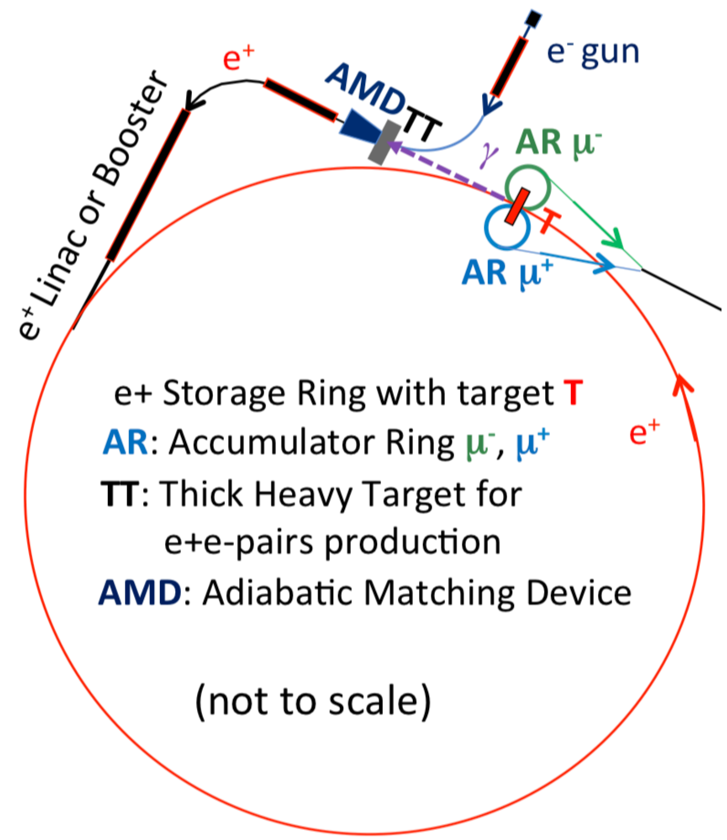


A dedicated muon **production** and **accumulation system** is one of the most important features of the LEMMA design.

Two rings are necessary in order to accumulate the muons over several iterations of positron bunches impinging on the target.

Muons are **recirculated** and arrive back to the target together with a new positron bunch, so that the new muons get produced in the **same phase space** of the accumulated bunch.

Multi-pass scheme

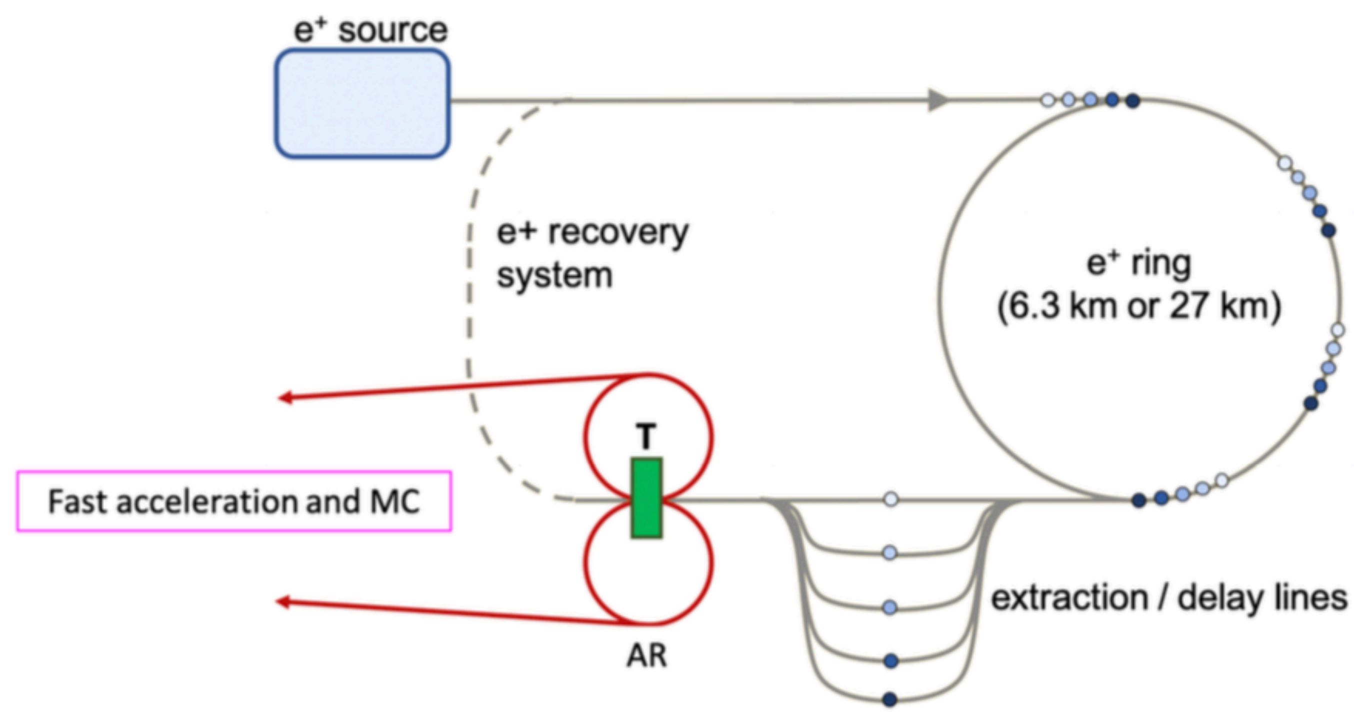


Target insertion embedded in the positron ring

Same e^+ bunch used for several prod. cycles, so $O(0.01X_0)$ target must be used to mitigate e^+ beam degradation (Multiple scattering and bremsstrahlung).

Currently in stand-by as no efficient recombination system was found to compensate the lower number of muons per bunch after accumulation

Single-pass scheme



Target and accumulators outside the e^+ ring

Each e^+ bunch passes once through the target, so $O(0.1 \sim 1X_0)$ targets can be used **increasing the number of produced muons** at the cost of **stronger requirements on the positron source**.

In this scheme it is also possible to use **extraction lines for recombination** to further enhance the muon production

Because luminosity is proportional to N_μ^2 the **single-pass scheme** has been taken in consideration for the studies on LEMMA

$$L = \frac{N^2 n_b f_{rev}}{4\pi\sqrt{\epsilon_x \epsilon_y \beta_x^{IP} \beta_y^{IP}}}$$

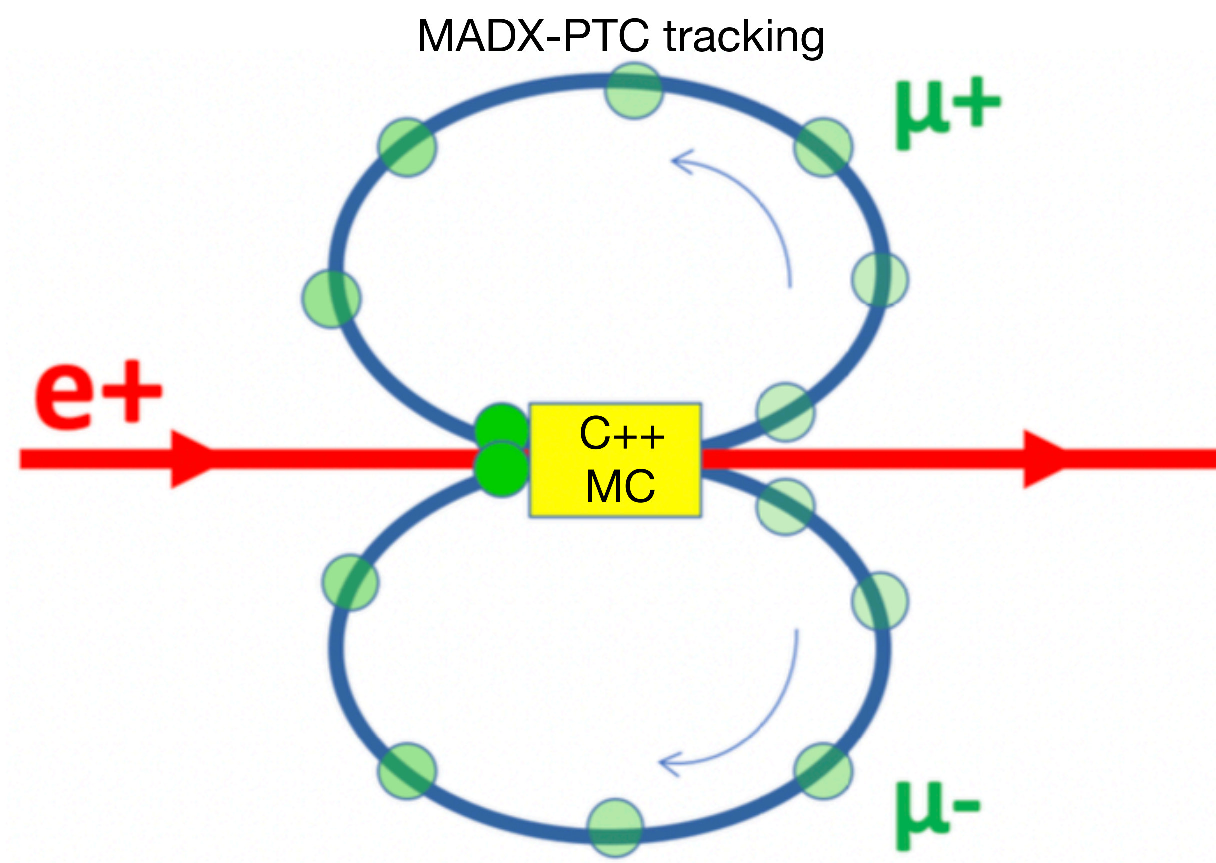
Start-to-end Simulations of the Production and Accumulation process

I have developed a dedicated simulation tool named **MUFASA** (MUon FAst Simulation Algorithm) to perform studies on the **muon beam dynamics** during the accumulation process.

This tool interfaces a C++ based MonteCarlo generator which includes the most relevant processes of **muon and electron interaction with matter**, with the particle tracking code MADX-PTC for the **6D particle tracking**.

This tool allows to perform **start-to-end simulations** to study the **dynamics of the stored beam** passing hundreds of times through the target during the accumulation process.

- Optimisation of the accumulator ring lattice
- Target material and thickness
- Positron beam energy



This note contains the description of the code and the results of its Benchmark against Geant4

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MUFASA: MUon FAst Simulation Algorithm

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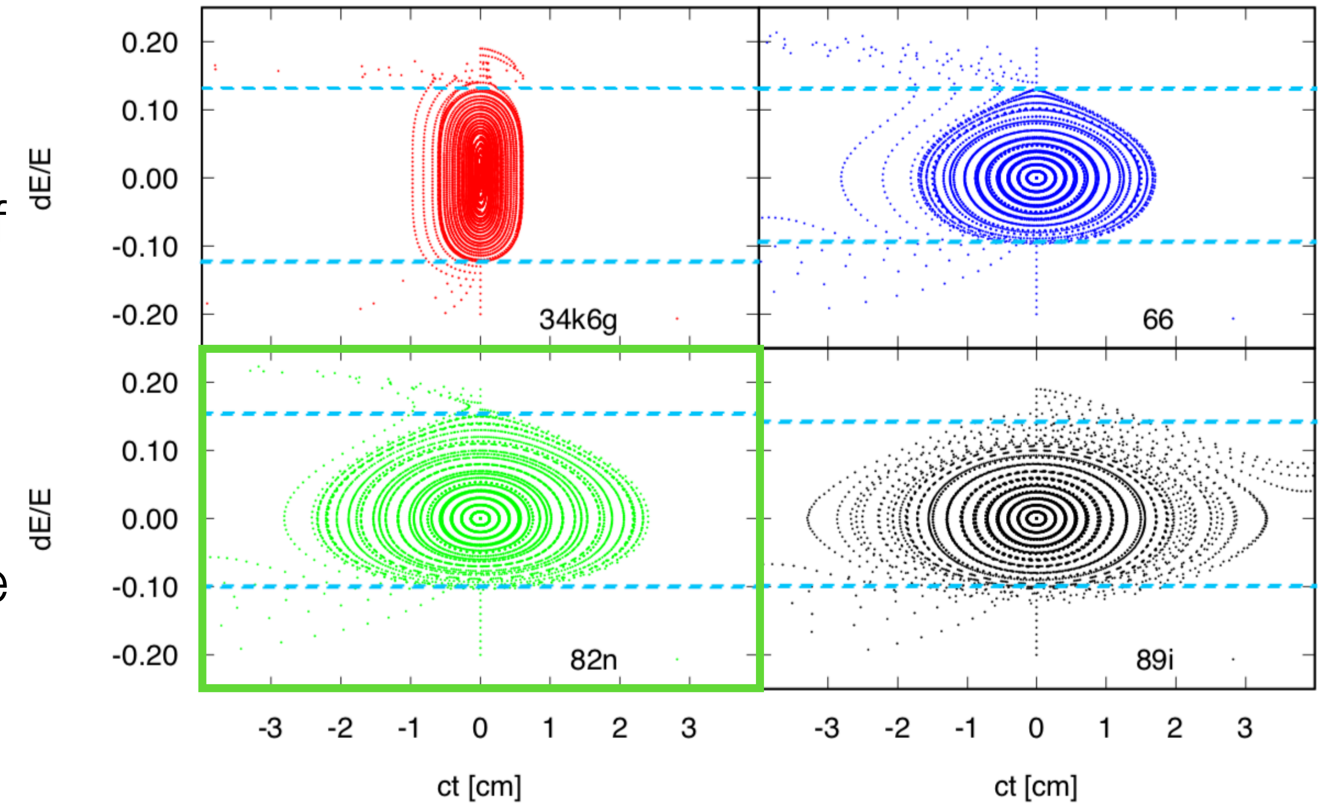
Muon Accumulator Ring Optics

Realistic lattice features: compactness and high energy acceptance

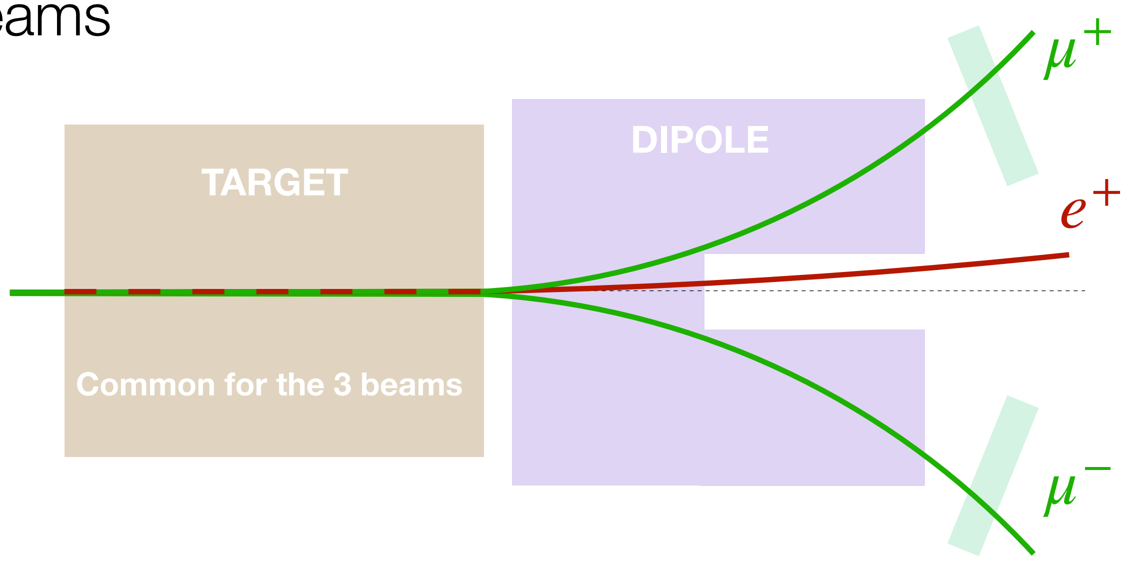
Dedicated optimisation studies of the lattice have been performed to correct for chromaticity and high order momentum compaction by dedicated families of sextupoles, resulting in a very large energy acceptance of -10%/+15%

The ring is composed by two symmetric arcs and two straight sections, one for the target insertion and one for the RF

Muon beam energy	GeV	22.5
Circumference	m	140
Number of cells		12
rf frequency	GHz	3.9
rf voltage	MV	200
Harmonic number		2100
Number of bunches		1
Horizontal betatron tune		8.84
Vertical betatron tune		3.73
Longitudinal tune		0.015
Momentum compaction		-7.12×10^{-5}
Natural horizontal chromaticity		-8.28
Natural vertical chromaticity		-10.37
Bunch length	cm	0.9
Ring energy acceptance		$-10\%, +15\%$

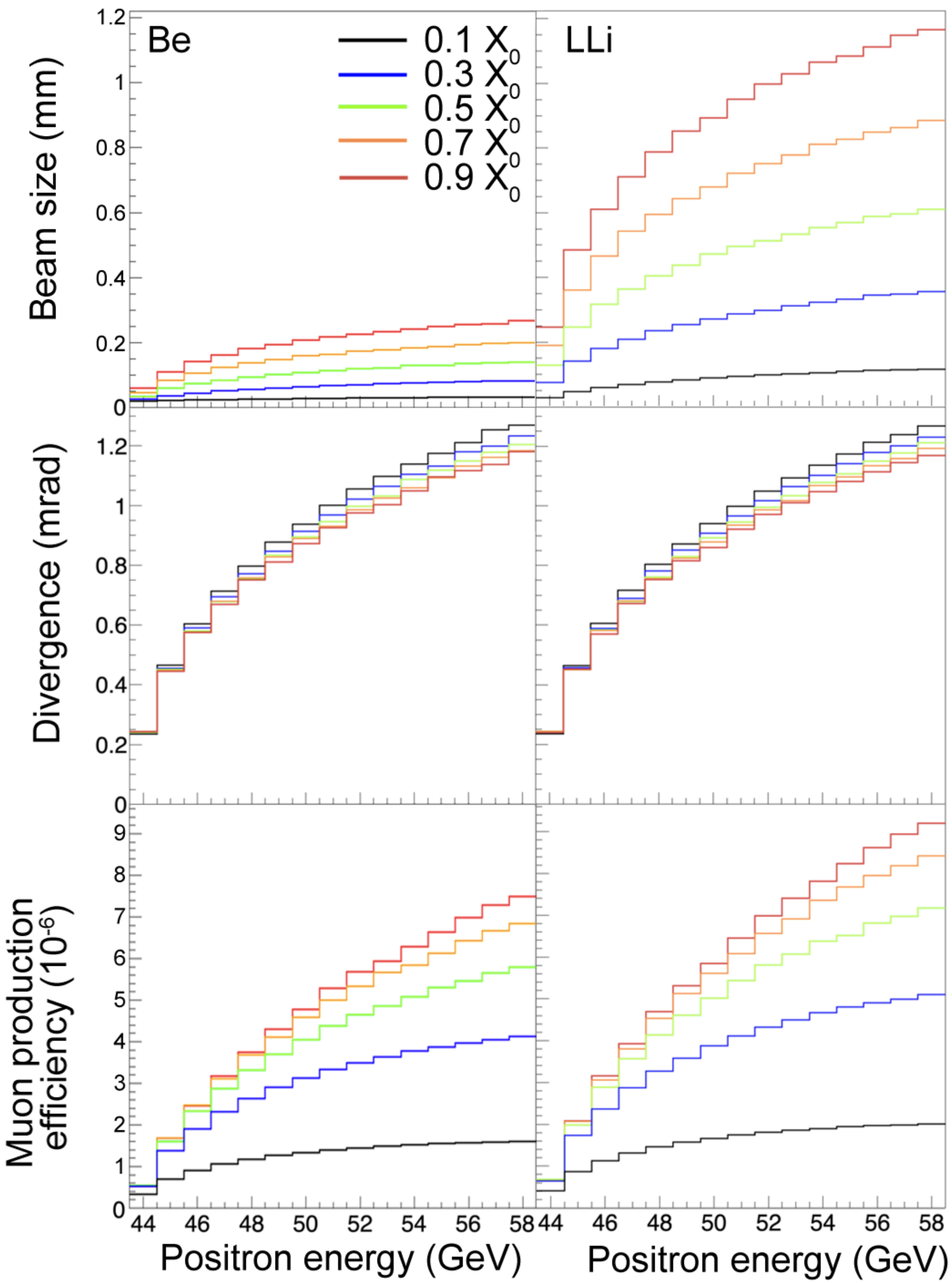


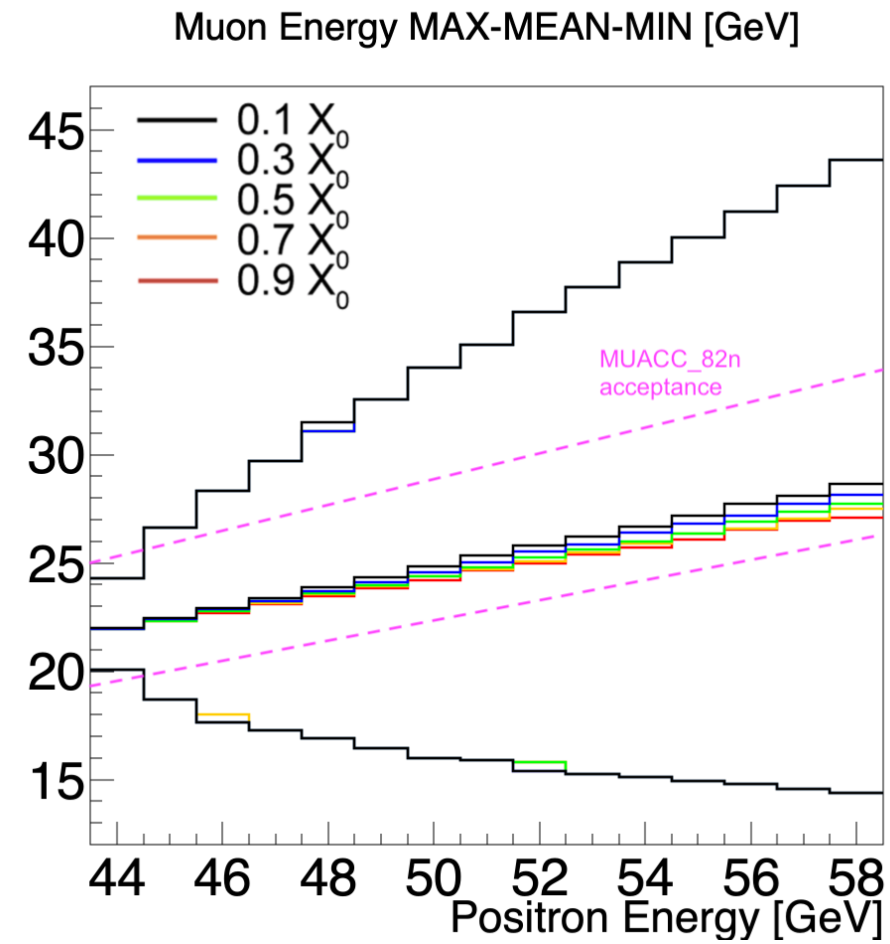
Since the target region is in common for the positrons and the two muon beams, a septum in the first bending magnet is used to separate the beams



Target specifics and e^+ beam

The muon ring energy acceptance sets the positron beam energy at **45GeV**.





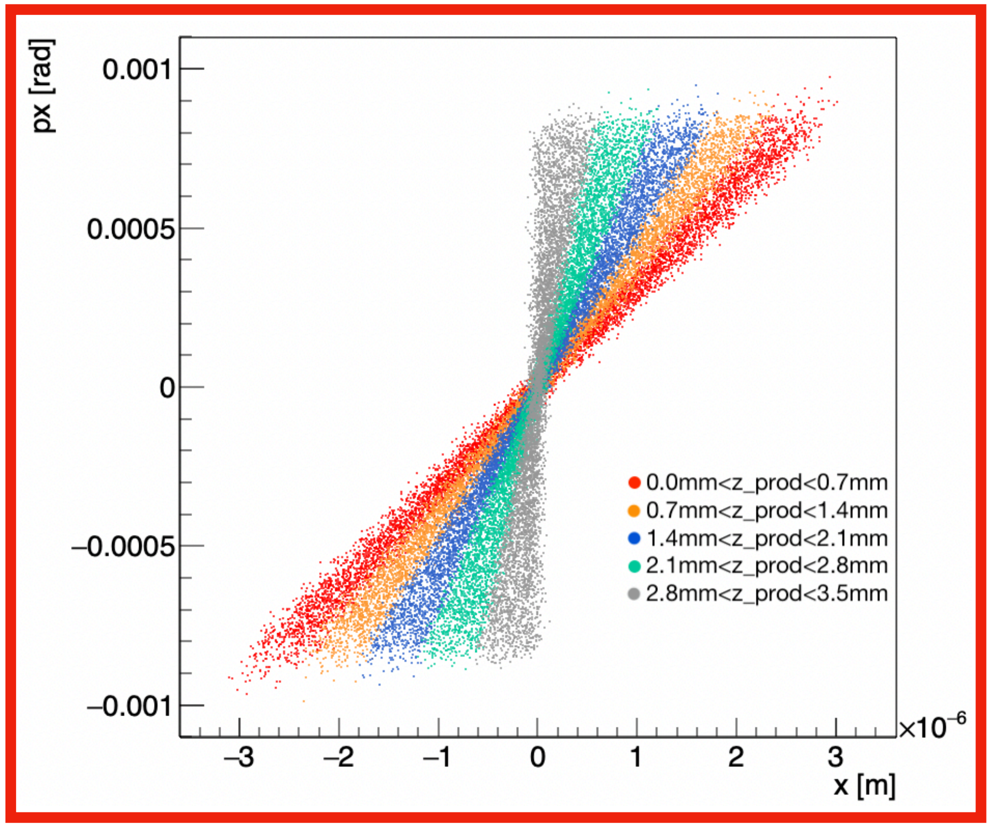
The candidate materials are **low-Z** in order to reduce the multiple scattering contribution.

The thickness of the target is decided in order to maximise production efficiency and minimising the emittance. For $E_{e^+} = 45GeV$ target thickness is chosen to be $L_{tgt} = 0.3X_0$

For accumulation **1500 bunches of 5x10^11 e+** are considered, $\epsilon_{e^+} = 0.7nm\ rad$.

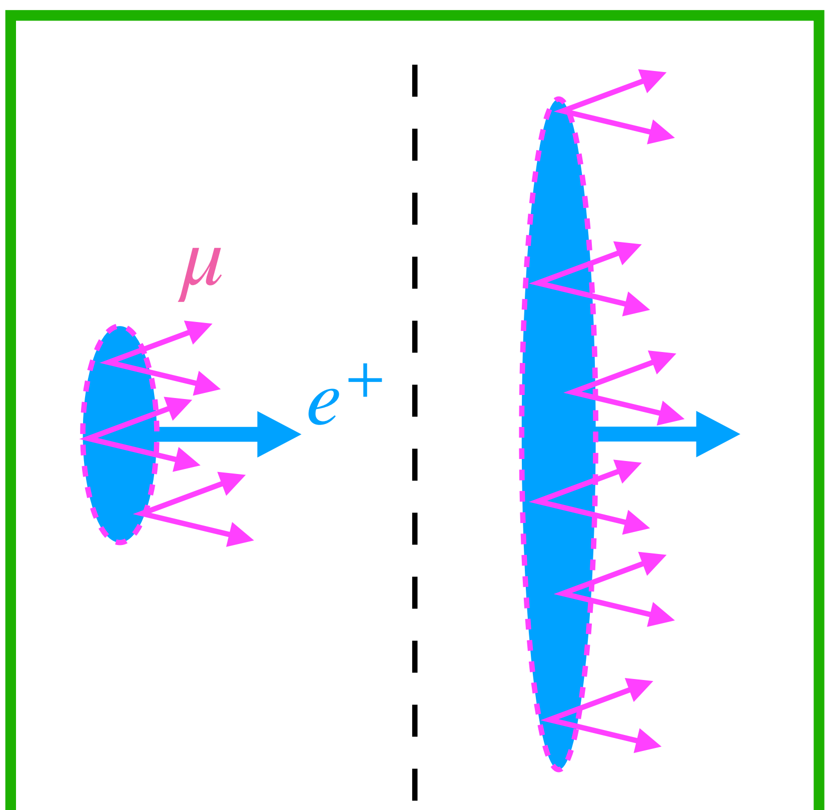
Contributions to muon emittance

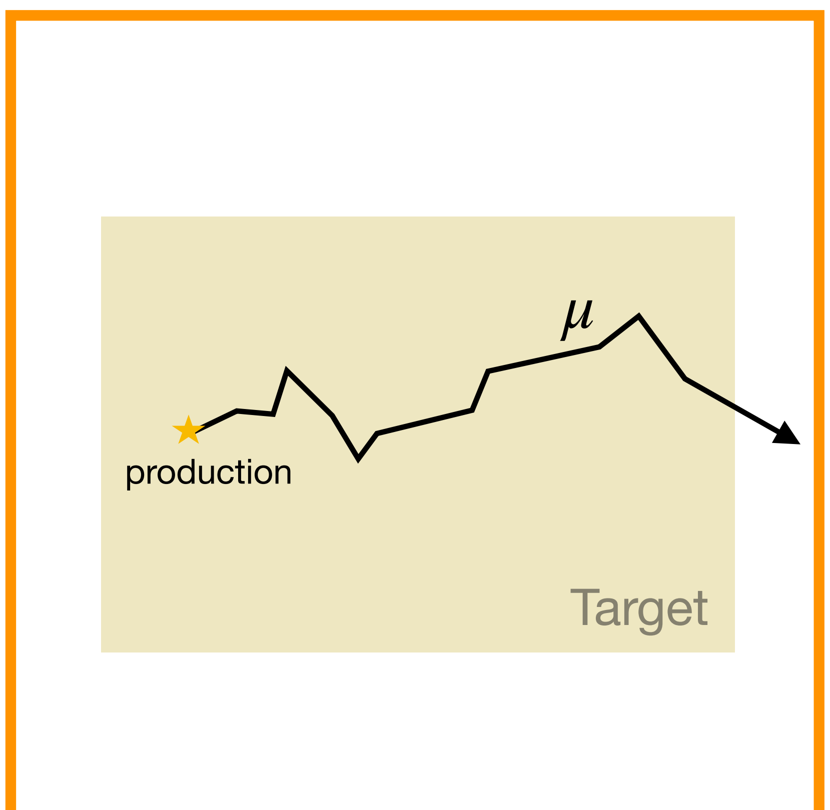
$$\epsilon_{final} = \epsilon_{prod} \oplus \epsilon_{e^+} \oplus \epsilon_{MSprod} \oplus \epsilon_{accum} \oplus \epsilon_{MSaccum}$$



Effects due to the production

- Drift through the target
- Positron beam size
- Multiple scattering to the end of the target





Contributions to muon emittance

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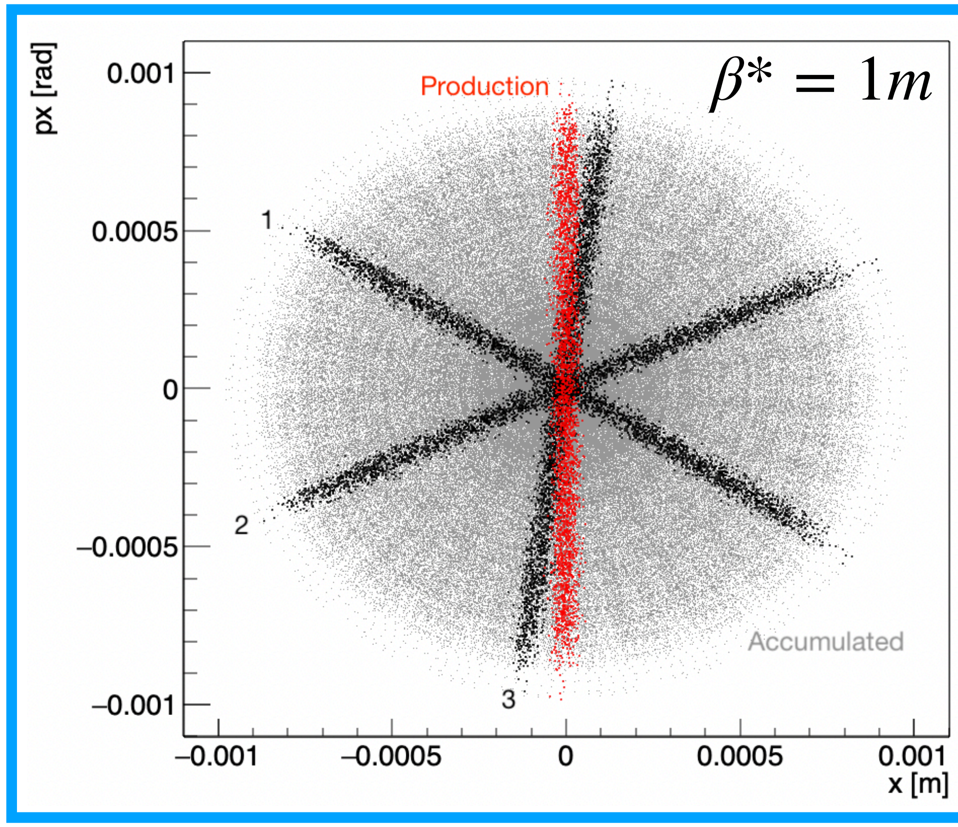
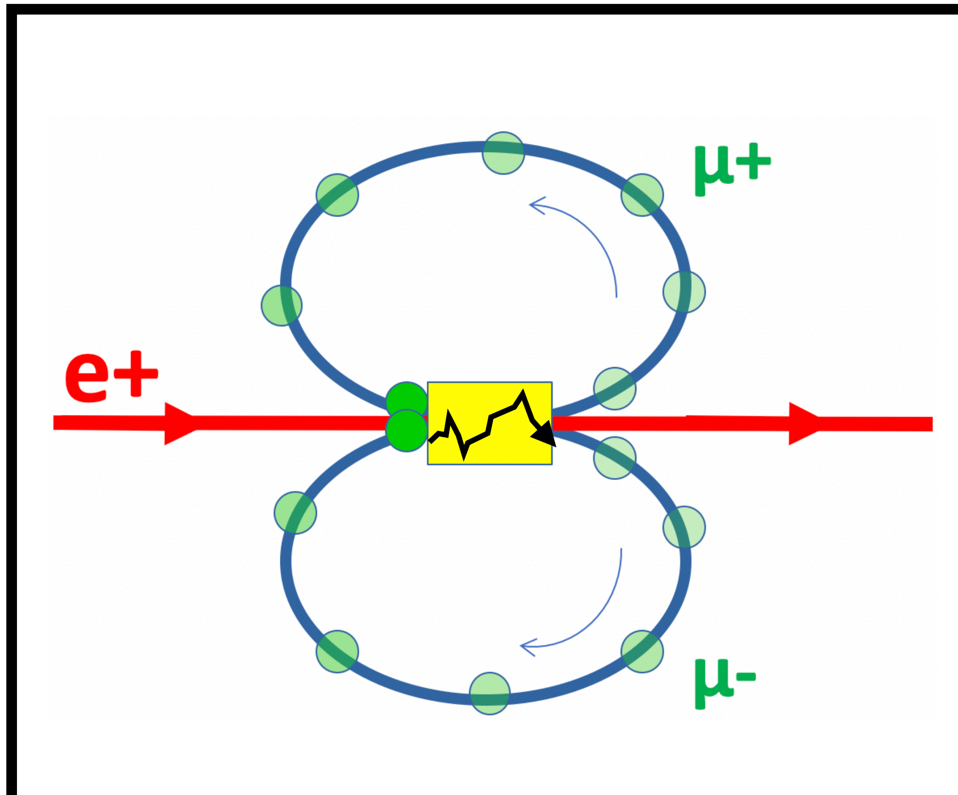


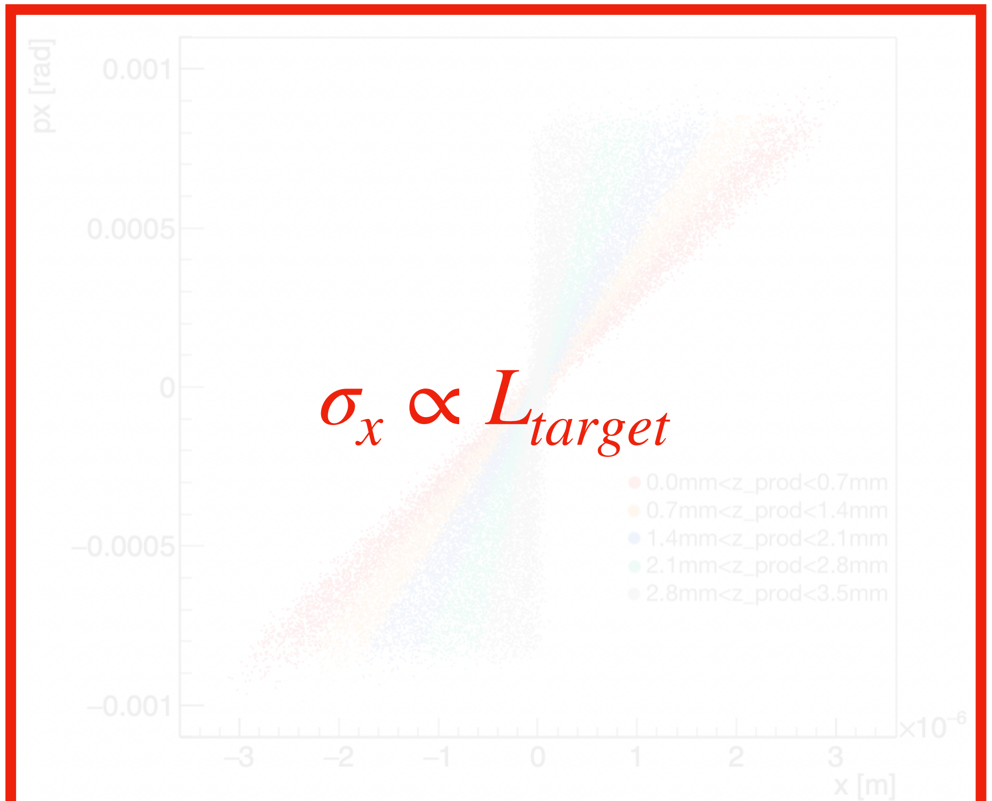
Effects due to the accumulation

- Mismatched β^* w.r.t. production emittance
- Repeated multiple scattering

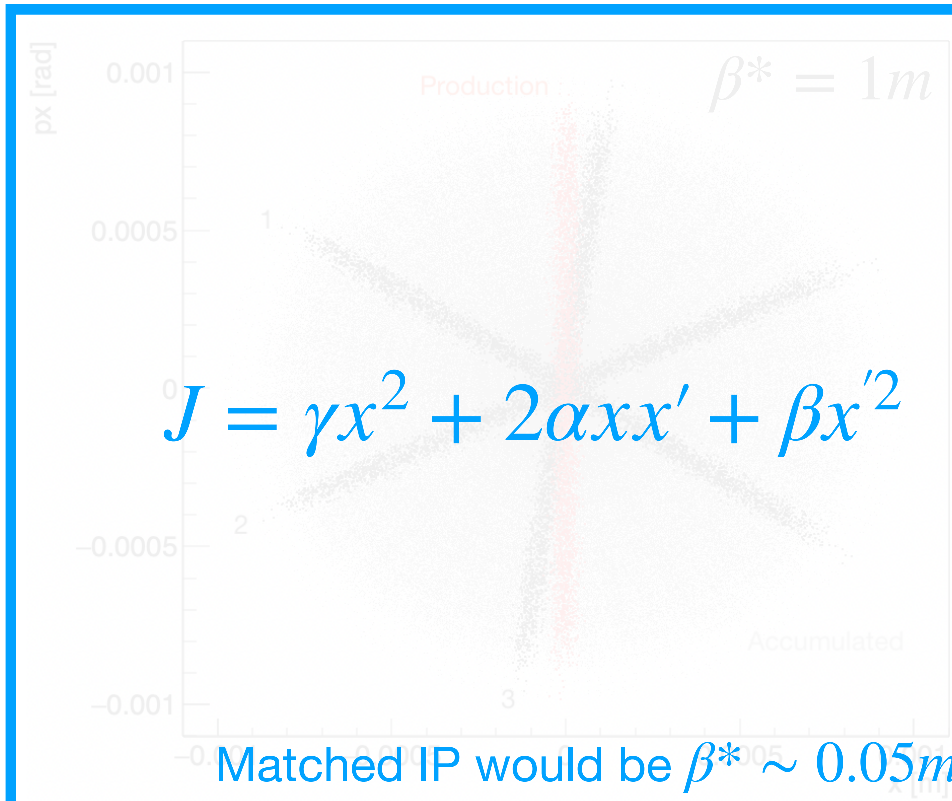


Contributions to muon emittance

$$\underbrace{\epsilon_{final}}_{} = \epsilon_{prod} \oplus \epsilon_{e^+} \oplus \epsilon_{MSprod} \oplus \epsilon_{accum} \oplus \epsilon_{MSacc}$$



$$\sigma_x \propto L_{target}$$



$$J = \gamma x^2 + 2\alpha x x' + \beta x'^2$$

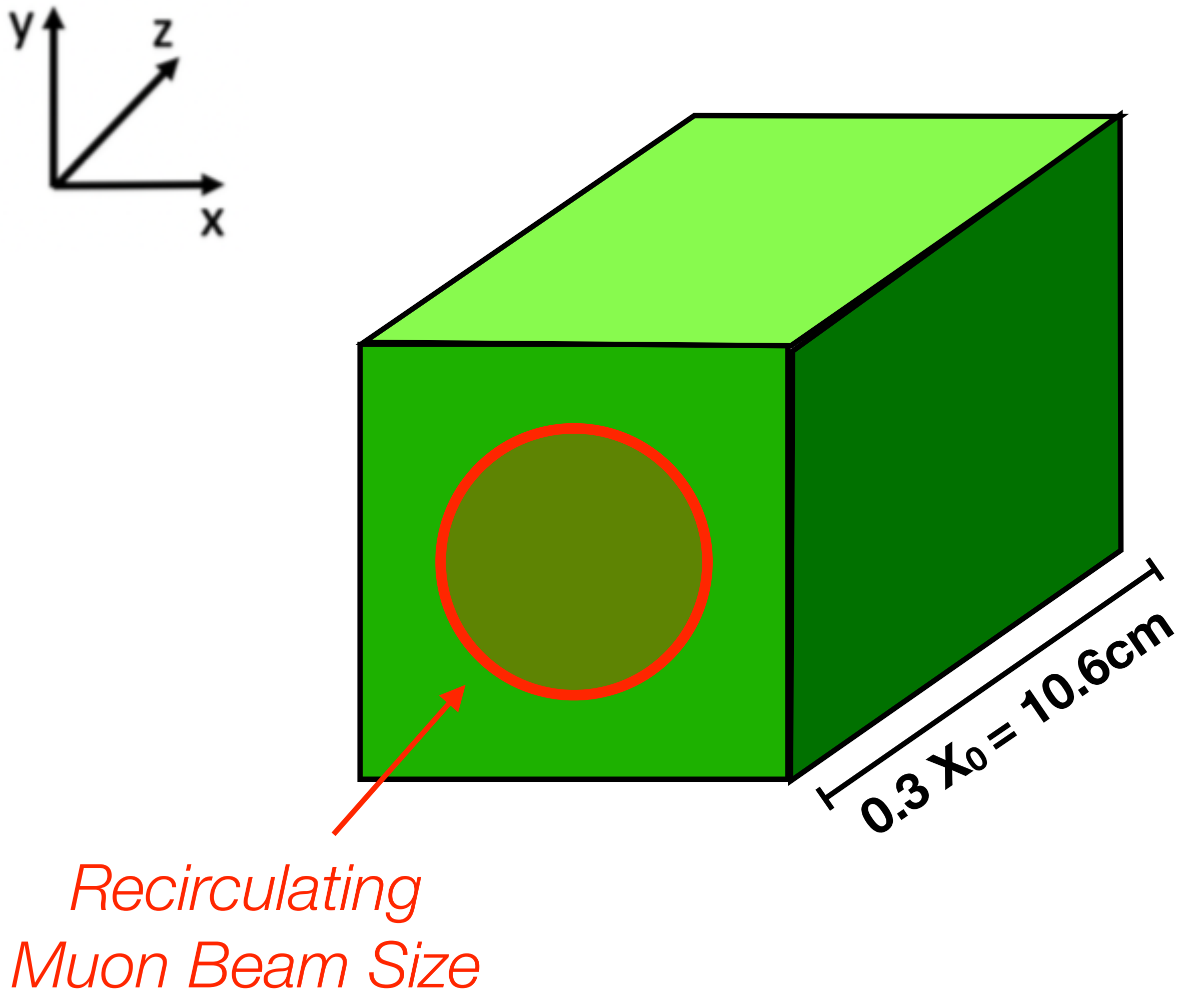
Matched IP would be $\beta^* \sim 0.05m$

$$\sigma^{\mu}_{x,0} = \sigma^{e^+}_{x,0}$$

$$\sigma_{MS} = \frac{0.0136}{E[GeV]}\sqrt{\frac{L}{X_0}}$$

$$\sigma_{x'}(N) = \sqrt{\sigma^2_{x',0} + N\sigma^2_{MS}}$$

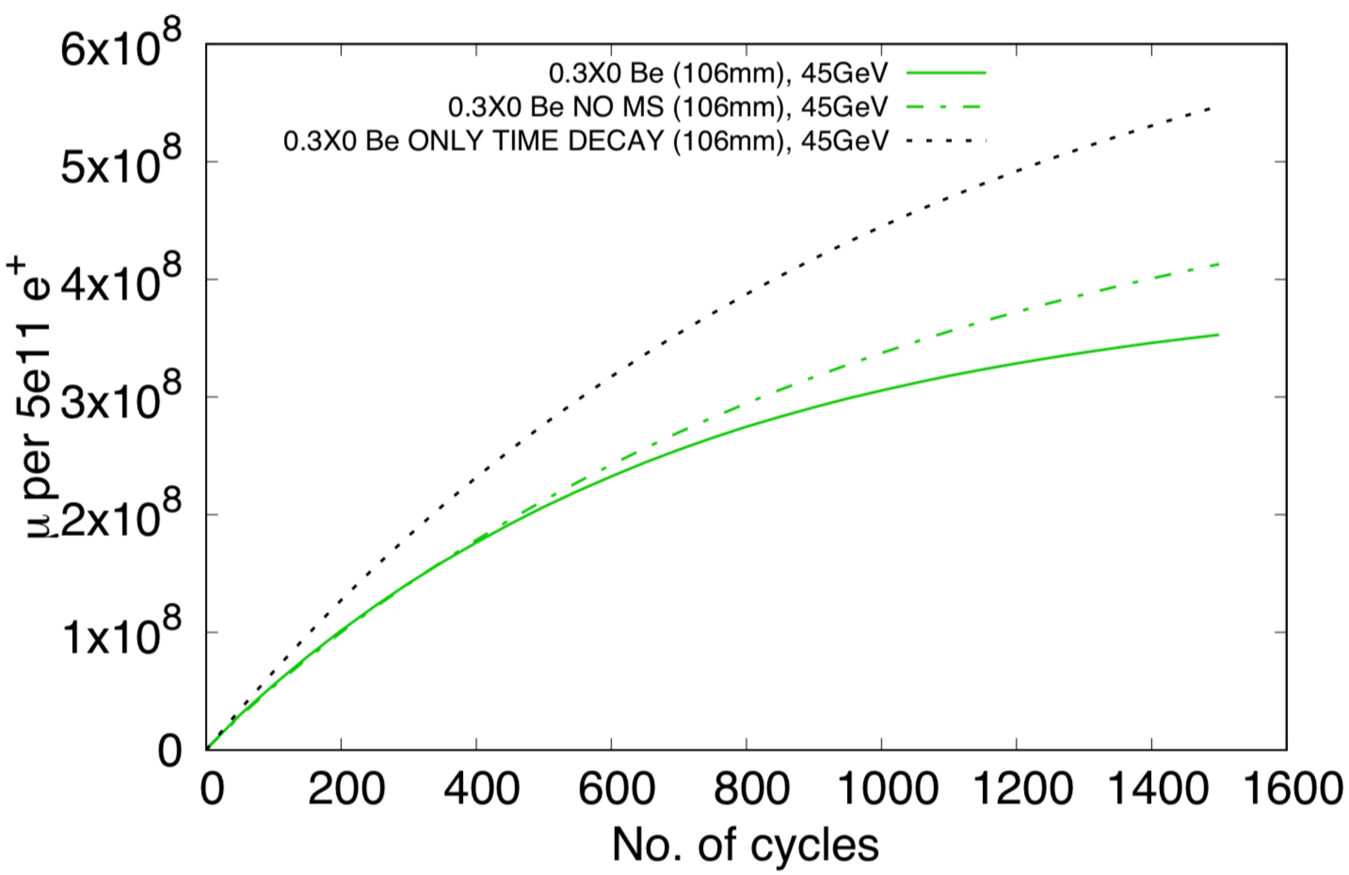
Beryllium Target

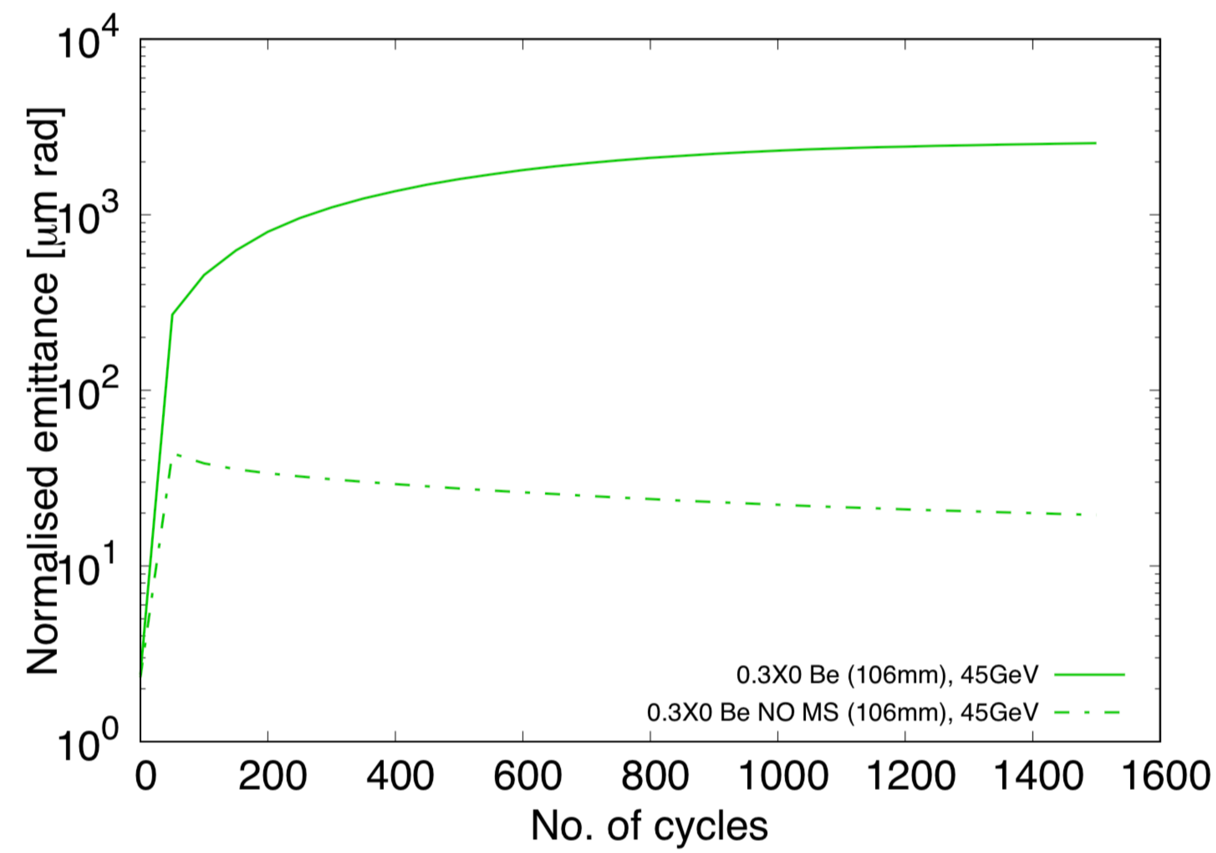


Beryllium is the **most efficient solid target** for muon pair production.

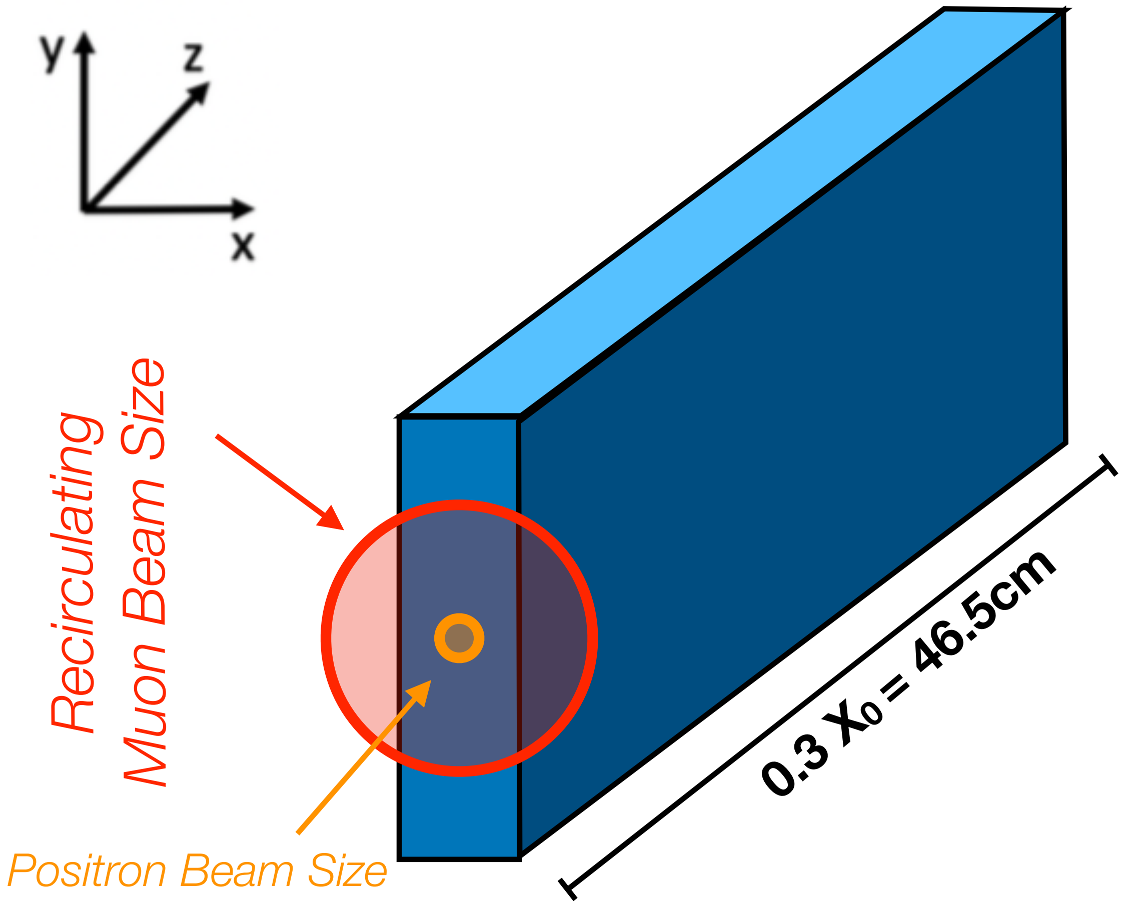
After 1500 turns (~1.5 lifetimes), **3.5×10^8 muon pairs** accumulated with $\epsilon_N = 2000\ \mu m\ rad$ considering 1500 impinging positron bunches of 5×10^{11} positrons per bunch.

Dominant contributions: repeated **Multiple Scattering** and **mismatched β^***





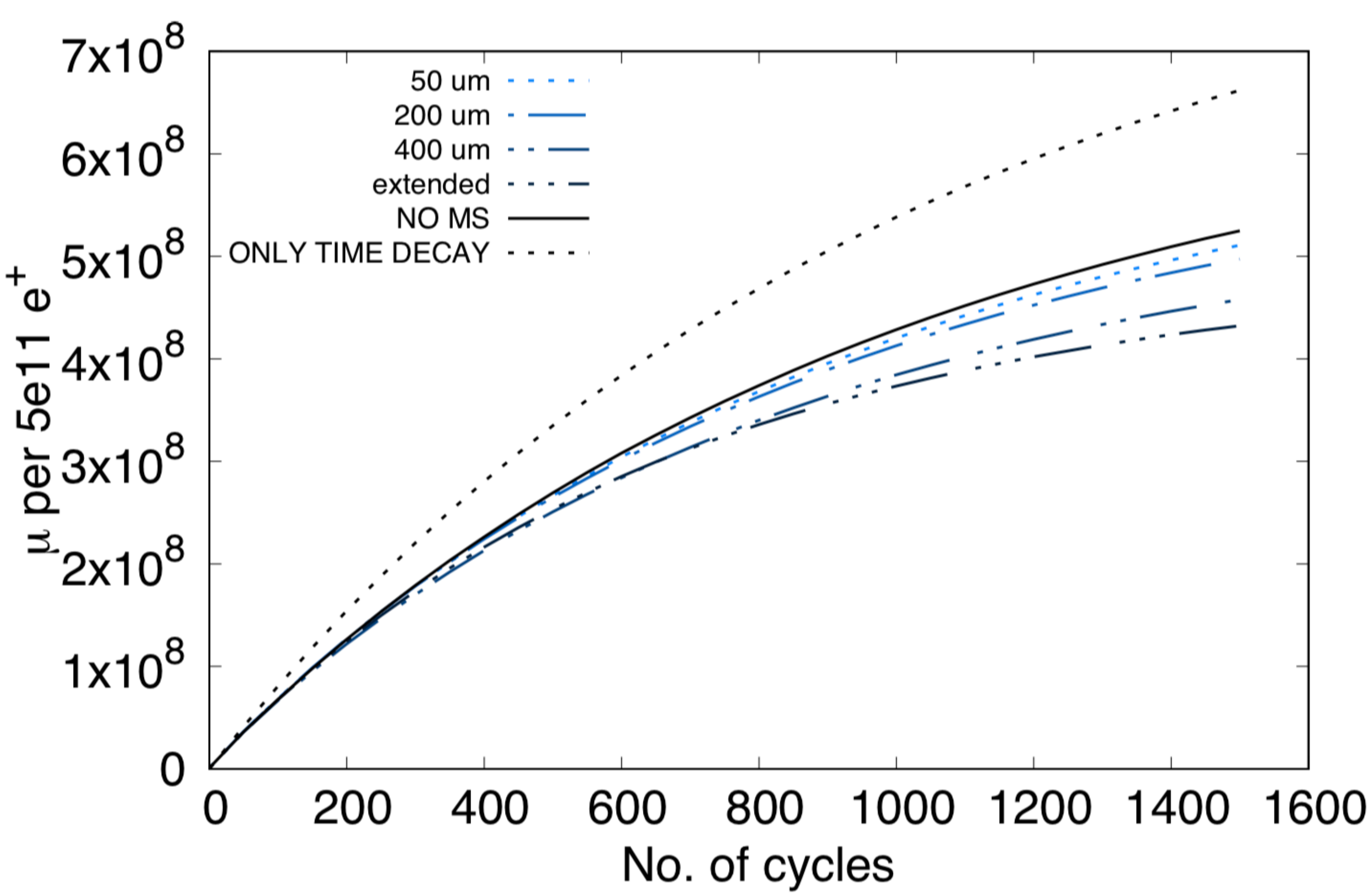
Liquid Lithium Target

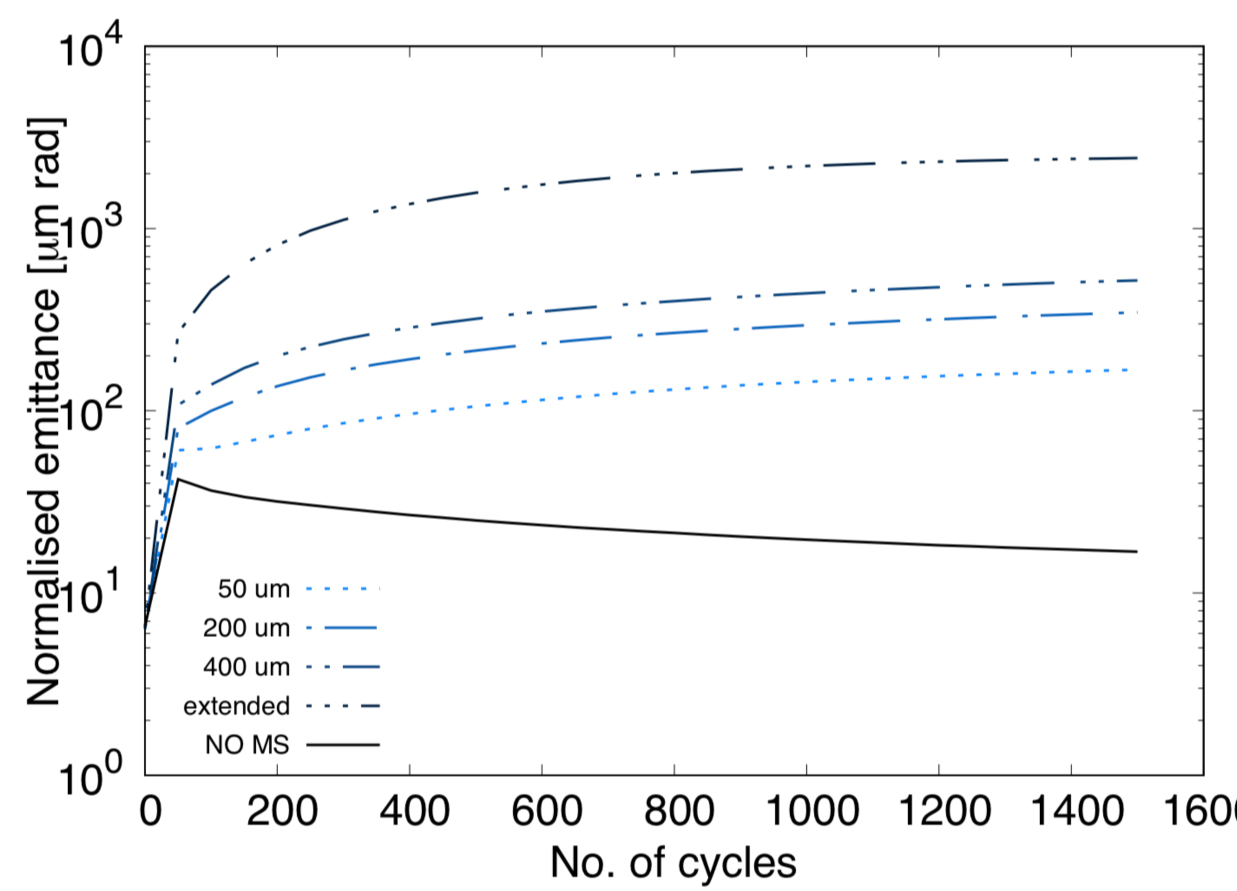


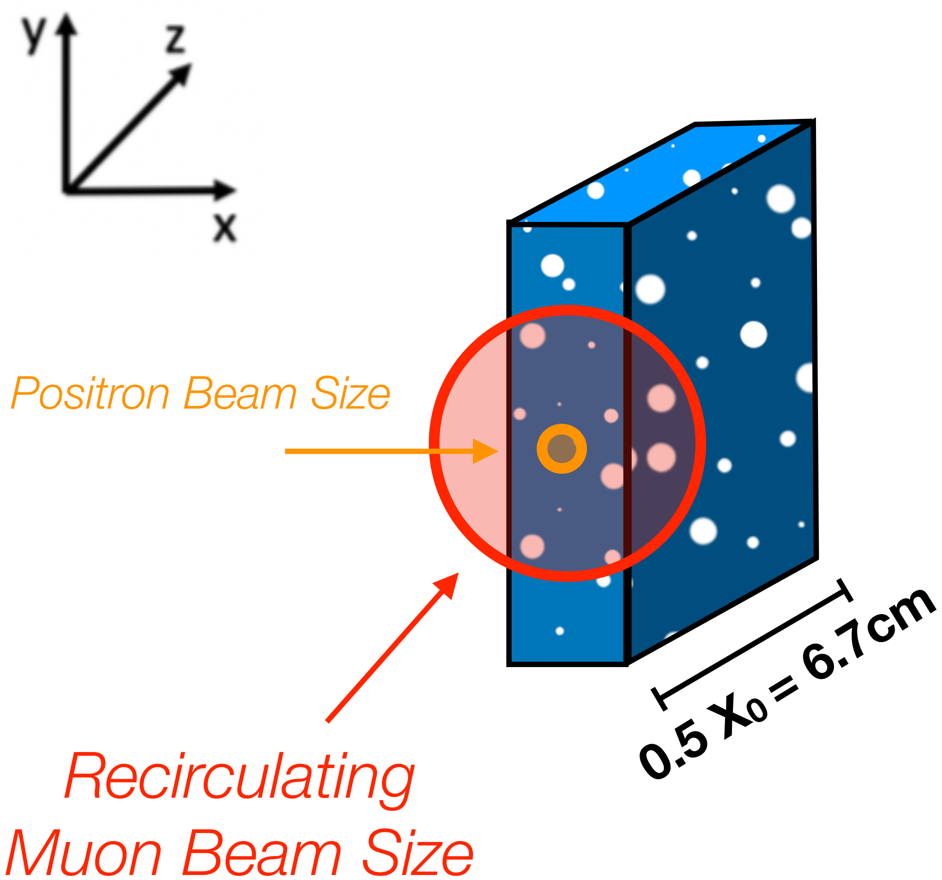
To **mitigate the effect of multiple scattering** during the accumulation a **thin film target** can be obtained using a jet of liquid Lithium.

If the transverse size of the target is much **smaller than the stored beam size**, muons will mostly not interact with matter. $\epsilon_N = 160\ \mu m\ rad$

Dominant contribution: **mismatched β^***





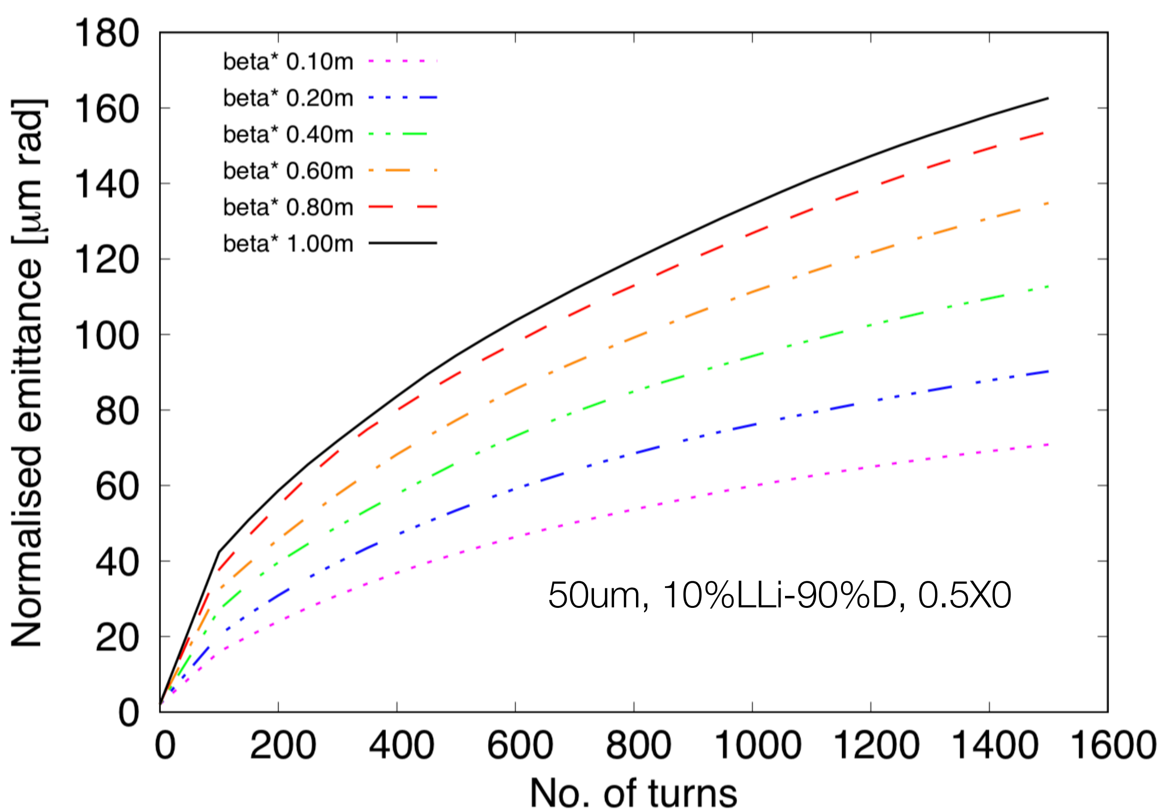


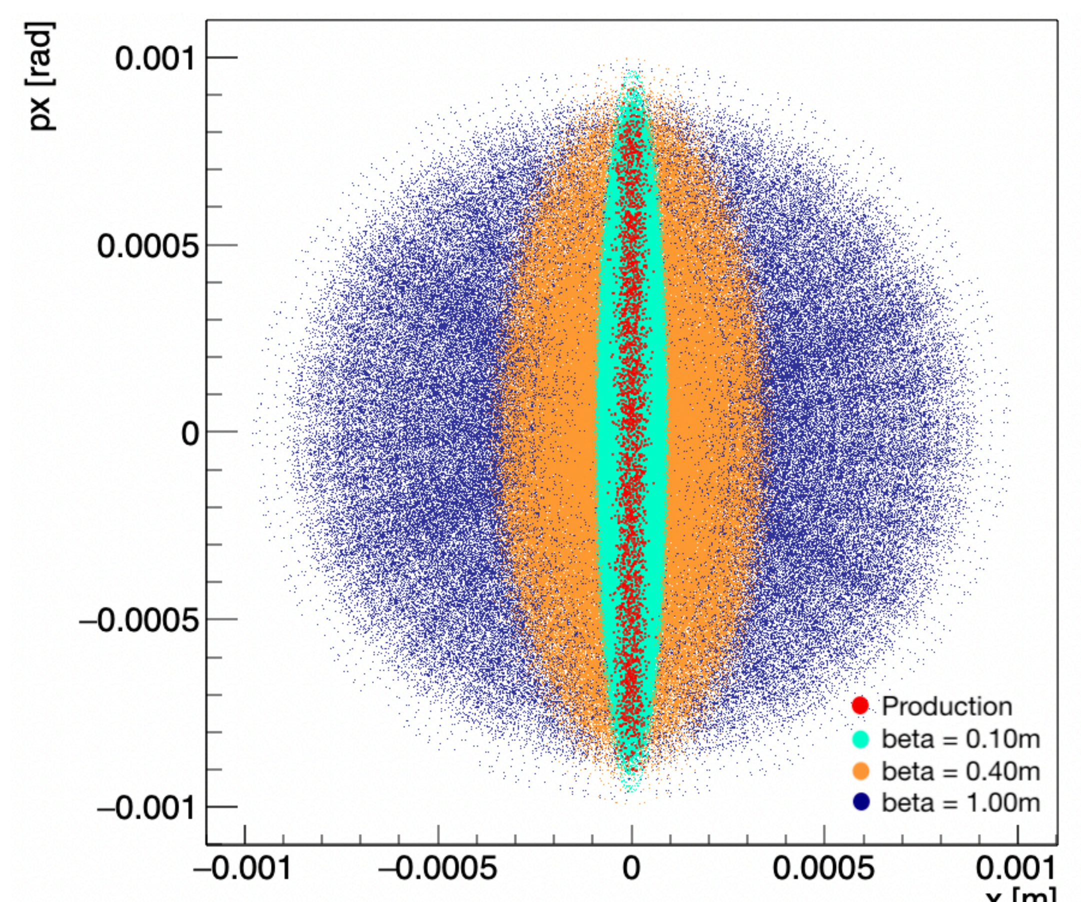
Liquid Lithium Target with Diamond dust

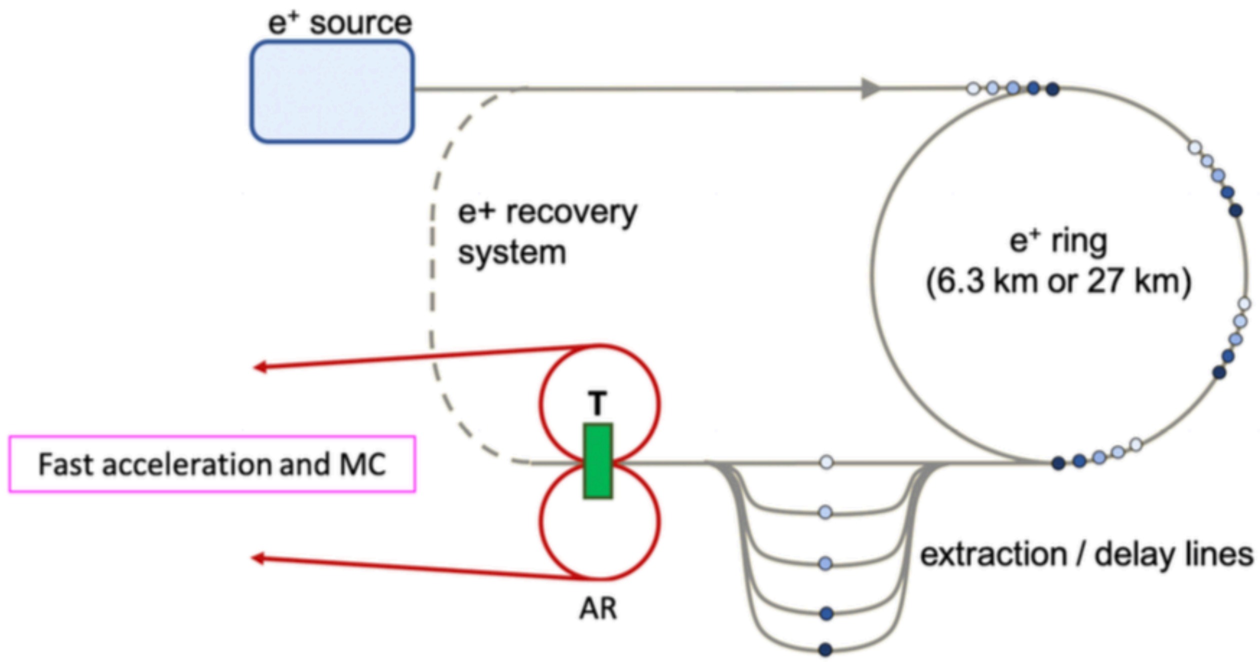
The film jet target length can be reduced by mixing **diamond micro-powder** to the liquid Lithium.

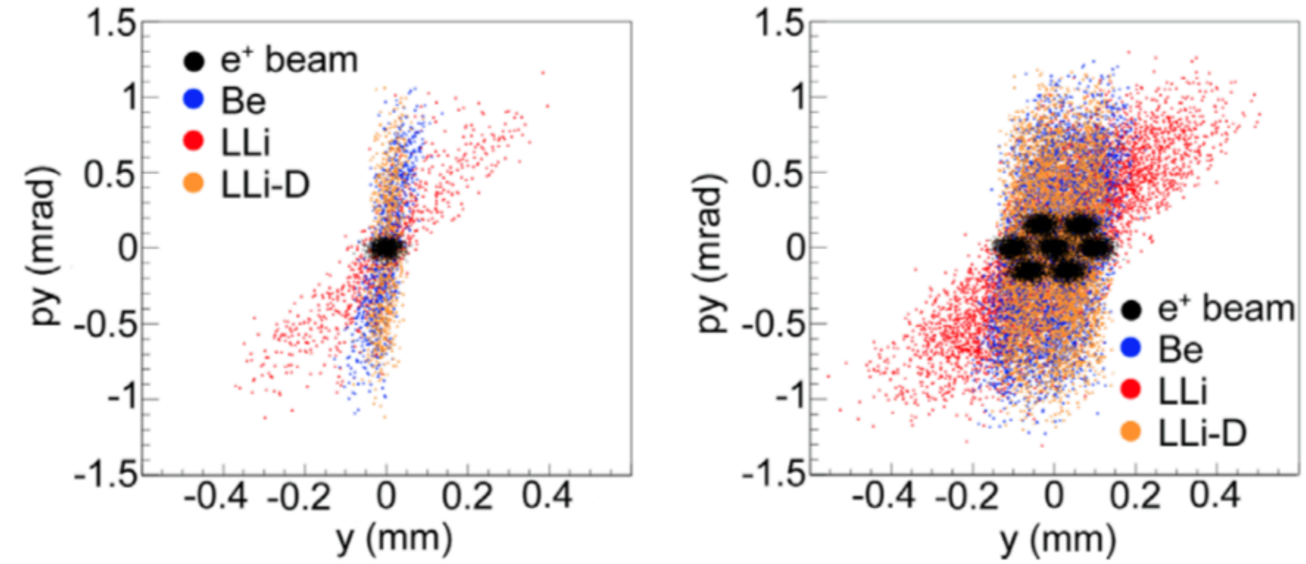
- multiple scattering strongly suppressed
- reduced target length

Using this target, a **scan on β^*** has been performed and it showed that a lower beta would **further reduce** the final muon beam emittance, **matching** the lattice action with the production emittance.







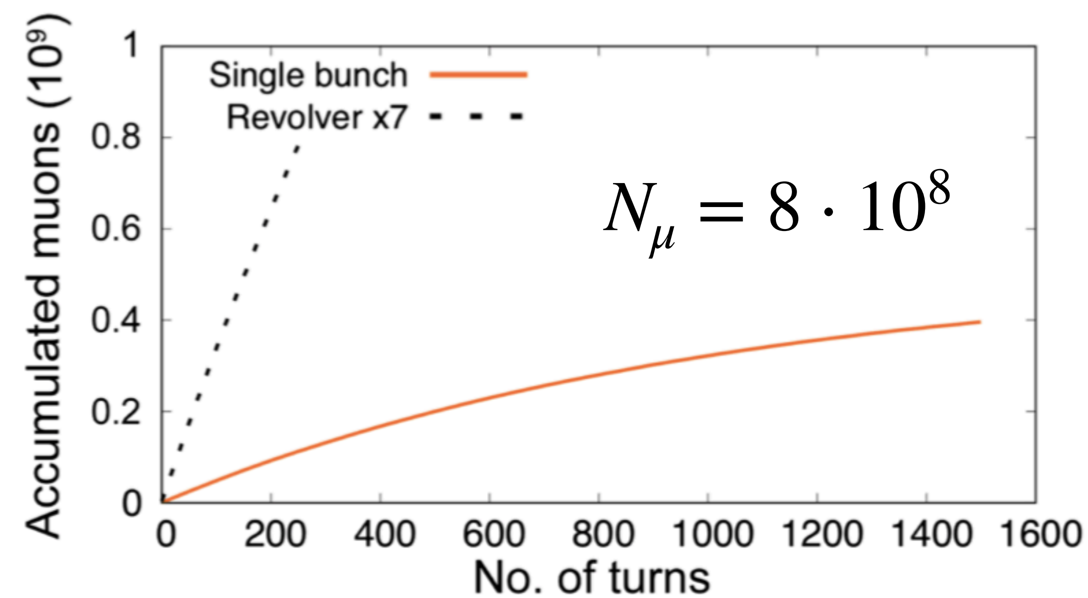


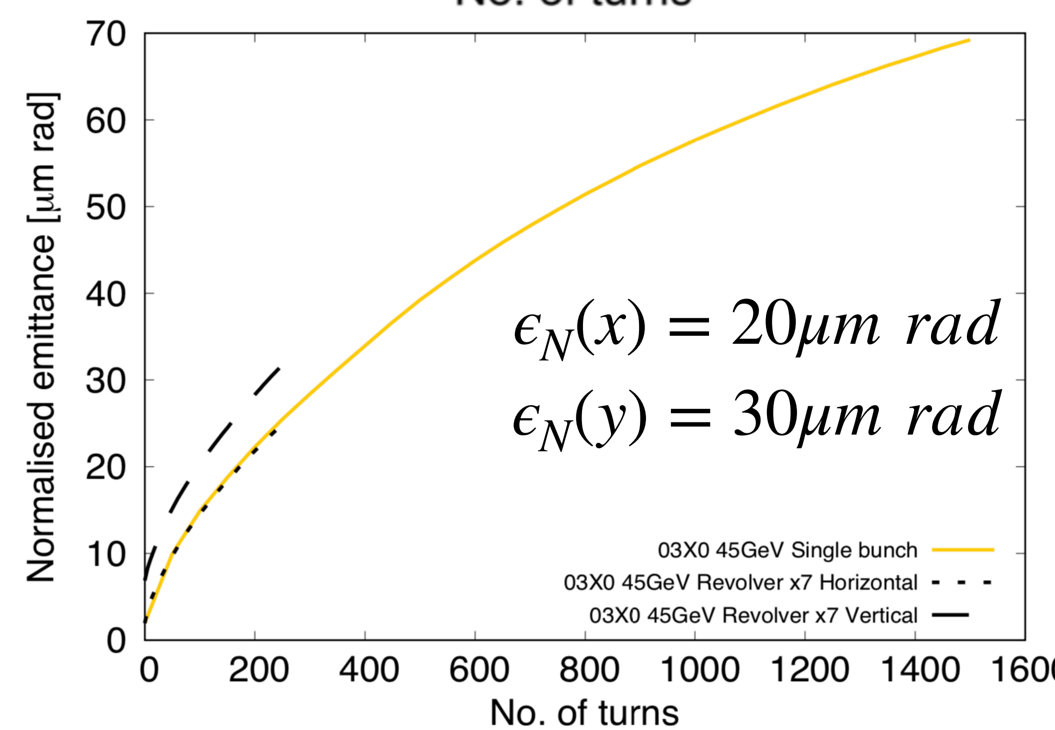
Positron bunch recombination at the target

Thanks to the small positron emittance it could be possible to simultaneously inject **multiple positron bunches** on the target by using a dedicated system of **delay lines**, spacing them on the vertical phase space.

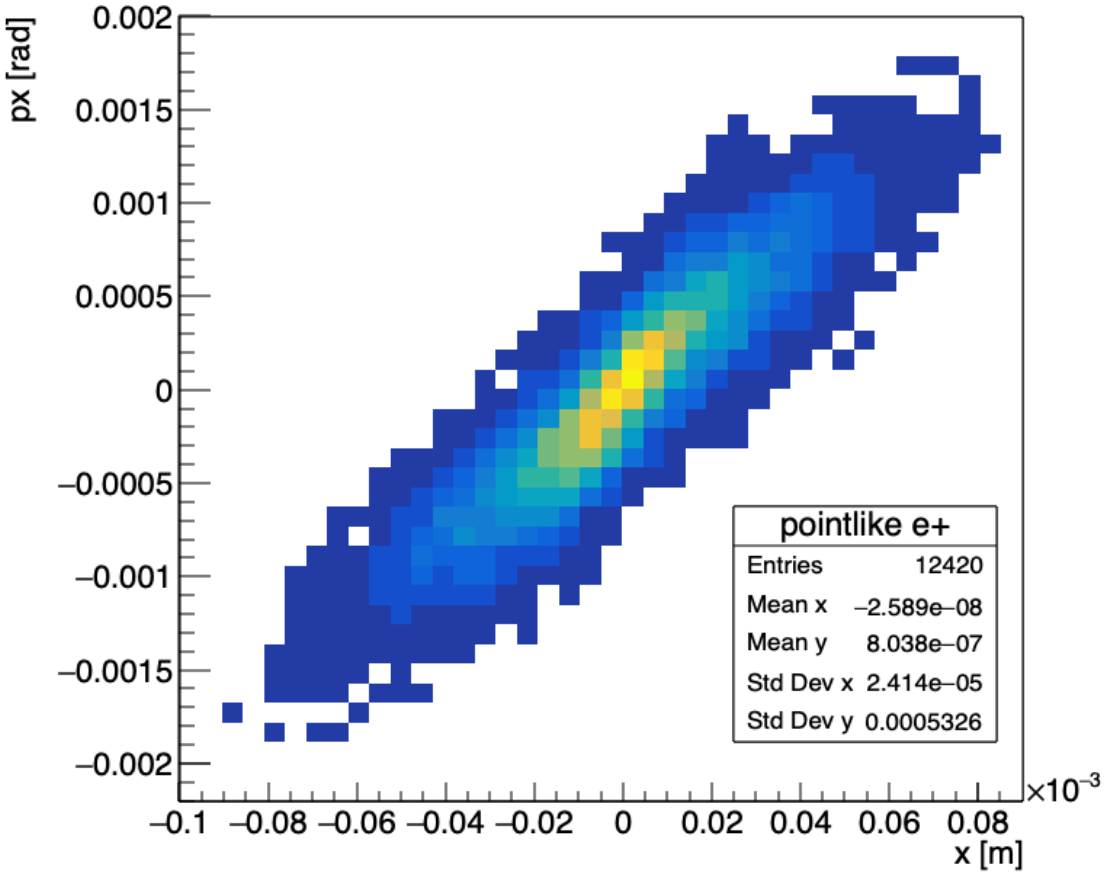
Fewer number of cycles required:

- reduced MS contribution to emittance
- preventing a lot of muons from decaying





Limit parameters for the Single-pass



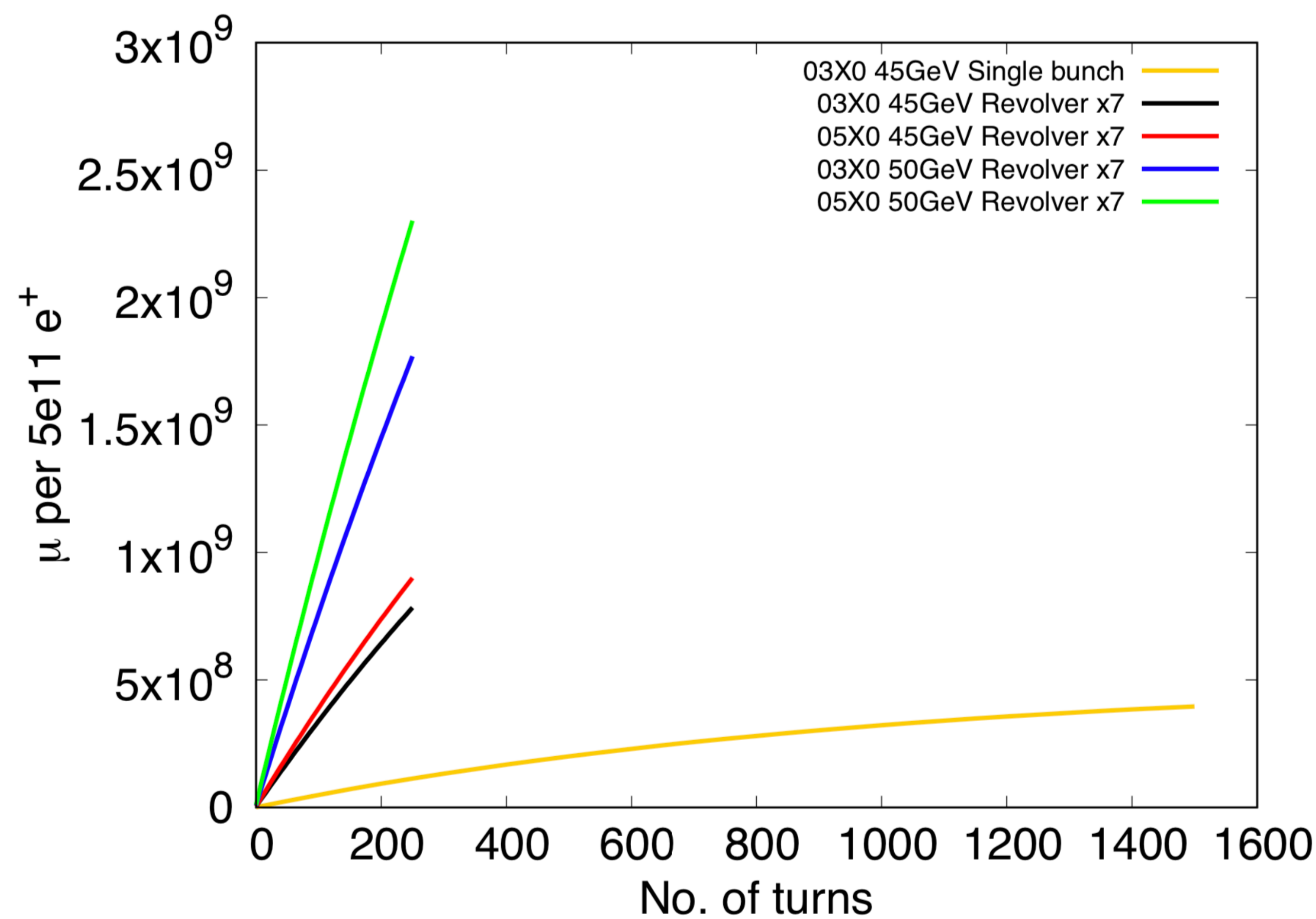
A possible improvement might be obtained by using a new lattice with **matched optical functions** in order to preserve the production emittance.

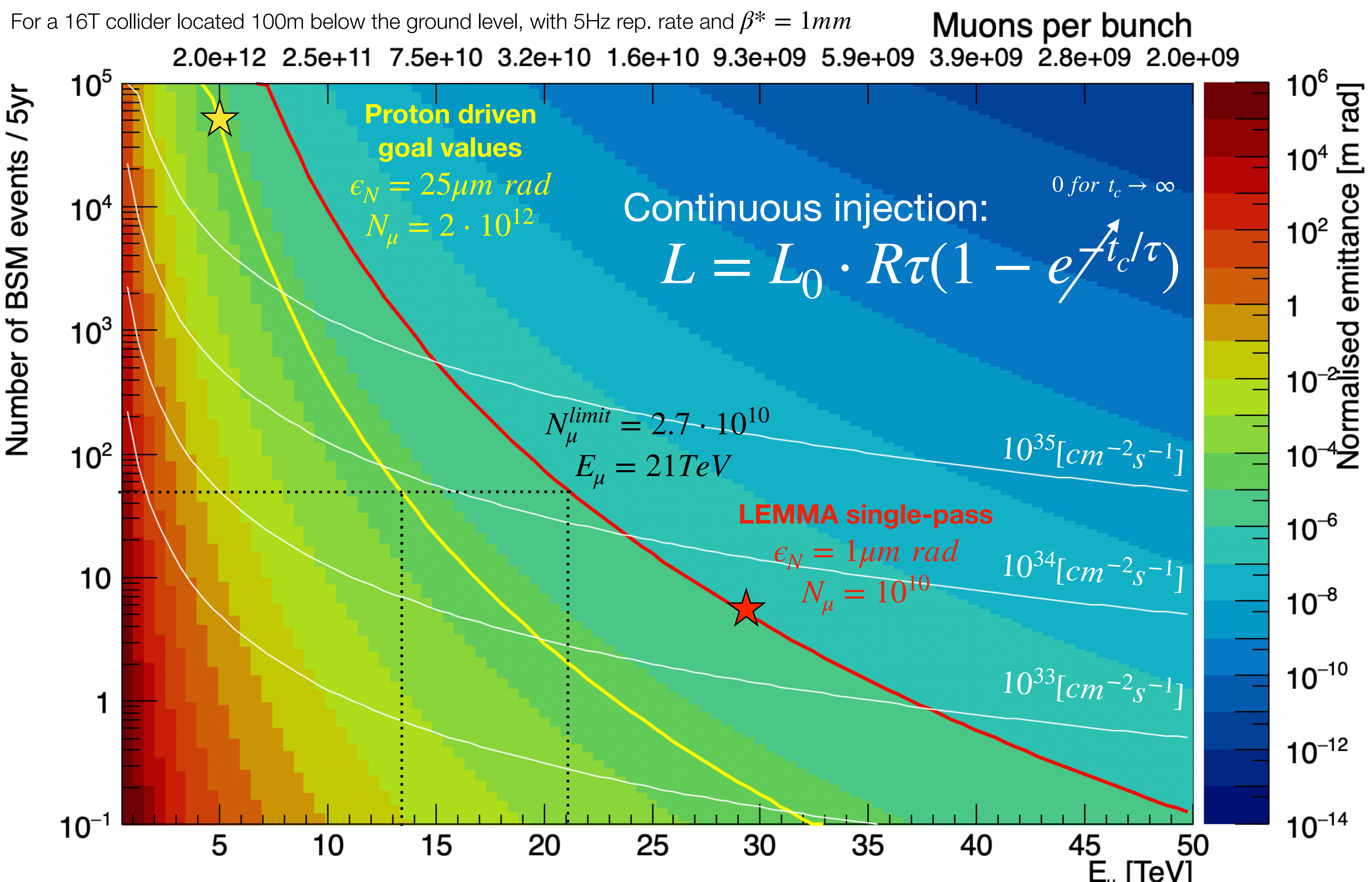
The contribution due to the **positron beam size** could be reduced, meaning a normalised production emittance of $\epsilon_N = 1\mu m \cdot rad$ for a 45GeV e^+ beam on a $0.5X_0$ LLi-D jet target, **without the requirement for 6D cooling.**

The muon population might be enhanced by **upgrading the e^+ source** (x2~x4).

New lattice with **larger energy acceptance** would allow a positron energy up to 50GeV, increasing the muon population of a factor ~3.

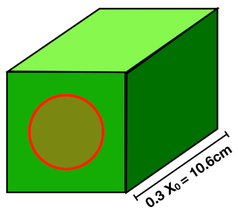
With these improvements the number of muons per bunch could rise up to $N_\mu = 10^{10}$

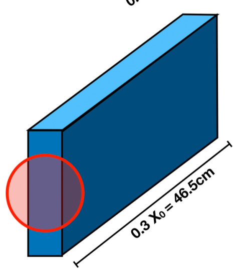


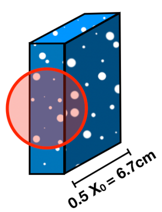


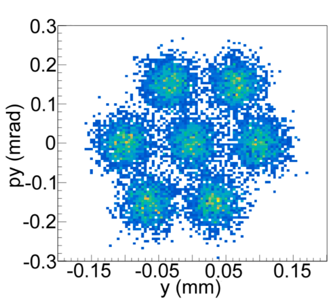
Considering working with continuous injections, only a **factor 3 in population** is required for LEMMA to be a discovery machine at $\sqrt{s} = 42TeV$. This could be further improved with **stronger magnets**, increasing the repetition rate or working in **top-up mode**

Single-pass scheme - Summary









Setup	Accumulated muons		$\epsilon[\mu m\ rad]$	$\epsilon_N[\mu m\ rad]$
Be $0.3X_0$ (106mm)	$3.5 \cdot 10^8 \mu$	ϵ_{prod}	0.015	3
		ϵ_{accum}	1.5	300
		ϵ_{final}	10	2'000
LLi $0.3X_0$ (465mm) Jet $50\mu m$	$5.0 \cdot 10^8 \mu$	ϵ_{prod}	0.025	5
		ϵ_{accum}	0.3	60
		ϵ_{final}	0.8	160
LLi-D $0.5X_0$ (67mm) Jet $50\mu m$ $\beta^* = 0.1m$	$4.2 \cdot 10^8 \mu$	ϵ_{prod}	0.011	2.2
		ϵ_{accum}	0.050	10
		ϵ_{final}	0.320	64
LLi-D $0.3X_0$ (40mm) Jet $50\mu m$ $\beta^* = 0.1m$ Revolver x7	$8.0 \cdot 10^8 \mu$	$\epsilon_{prod,x}$	0.009	1.8
		$\epsilon_{final,x}$	0.110	22
		$\epsilon_{prod,y}$	0.032	6.4
		$\epsilon_{final,y}$	0.150	30
Limit scenario pointlike 50GeV e^+ beam matched β^*	$10^{10}\mu$	ϵ_{limit}	0.005	1

The use of liquid jet targets and e^+ beam recombination allowed an increase of a **factor 2** in the muon population and a reduction of a **factor 100** in the normalised muon emittance w.r.t. the solid Beryllium target. Limit scenario foresees $10^{10}\mu/bunch$ and $\epsilon_N = 1\mu m\ rad$, result obtained **without the requirement of 6D cooling**.

Conclusions

In this thesis I have explored the possible reach of the *single*-pass LEMMA source

- Muon accumulator lattice has been optimised to reduce chromatic effects and have a **large energy acceptance -10%/+15%**
- The performance of the production and accumulation system has been studied by performing **start-to-end simulations** with the custom tool MUFASA.
- **Single contributions** to the muon emittance increase during the accumulation process (like the multiple scattering, energy loss, chromaticity,…) have been isolated and **independently studied.**
- The use of **liquid jet targets** allowed to reduce the multiple scattering contribution, and **positron beam recombination** reduced emittance (**x100**) and increased the number of muons per bunch (**x2**).
- **Best case achieved:** $\epsilon_N = 26[\mu m \ rad]$ and $8.0 \cdot 10^8 \mu/bunch$ **without the requirement of 6D cooling**.
- New accumulators lattice design and the positron source enhancement are necessary to achieve the limit **parameters** $\epsilon_N = 1[\mu m \ rad]$ and $N_\mu = 10^{10} \mu/bunch$
- With this configuration the limit for the LEMMA single-pass would be **less than one order of magnitude** from the minimum luminosity required to be a discovery machine at $\sqrt{s_\mu} \sim 40 TeV$, which could be reached by increasing the rep rate or working in **top-up mode**

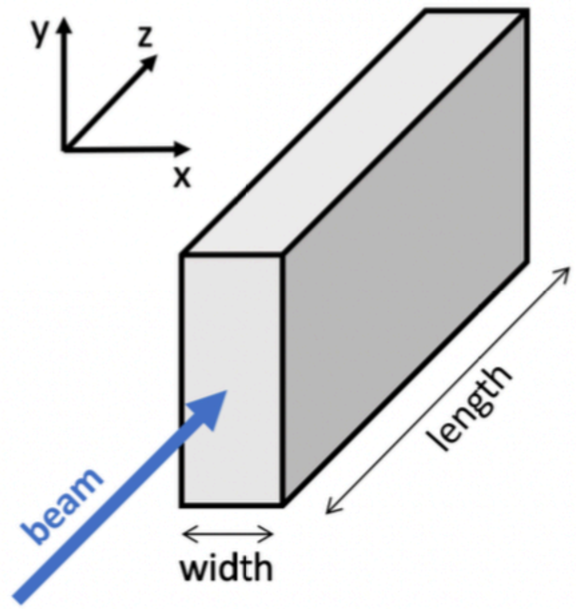
List of Publications

1. M. Boscolo, M. Antonelli, A. Ciarma, P. Raimondi. "Muon production and accumulation from positrons on target". *Phys. Rev. Accel. Beams 23, 051001* (2020)
2. A. Ciarma. "MUFASA: MUon FAst Simulation Algorithm" *Istituto Nazionale di Fisica Nucleare - Laboratori Nazionali di Frascati INFN-20-07/LNF* (2020).
3. A. Ciarma. "Study of the production of a low emittance muon beam for the LEMMA project" *Il Nuovo Cimento 43 C (2020) 58*.
4. O. R. Blanco-García, A. Ciarma. "Nanometric muon beam emittance from e^+ annihilation on multiple thin targets". *Phys. Rev. Accel. Beams 23, 091601 (2020)*
5. O. R. Blanco-García, A. Ciarma. "Optics studies of a Muon Accumulator Ring based on FFA cells". *arXiv 2011.11701 (2020)*
6. M.E. Biagini et al. "Positron driven muon source for a muon collider: recent development" *IPAC2019, Melbourne, Australia 10.18429/JACoW-IPAC2019- MOZZPLS2 (2019)*
7. O. R. Blanco-García et al. "Multi-target lattice for muon production from e+ beam annihilation on target" *IPAC2019, Melbourne, Australia 10.18429/JACoW-IPAC2019-MOPRB003 (2019)*
8. D. Alesini et al. "Positron driven muon source for a muon collider" *arXiv:1905.05747 (2019)*
9. G. Cesarini et al. "Theoretical Modeling for the Thermal Stability of Solid Targets in a Positron-Driven Muon Collider" *Int. J. Thermophys. 42, 163 (2021)*

Presentations at conferences

1. Incontri di Fisica delle Alte Energie IFAE19 - Napoli 8-10 Aprile 2019 - Awarded Best Presentation of the “New Technologies” section
2. 105° Congresso Nazionale della Società Italiana di Fisica - L’Aquila 23-27 Settembre 2019

Backup

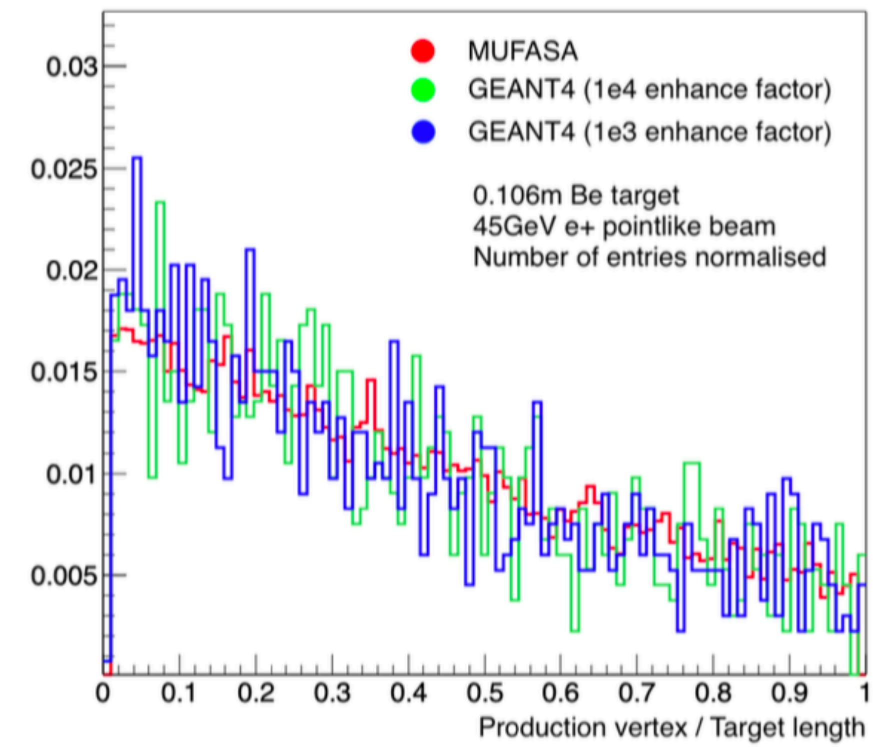


Target features are width, length and material.

```
// material[] = {X0, rho, A, Z, dE/dx};
double beryllium[] = {65.19,1.848,9.01218,4,0.2947};
```

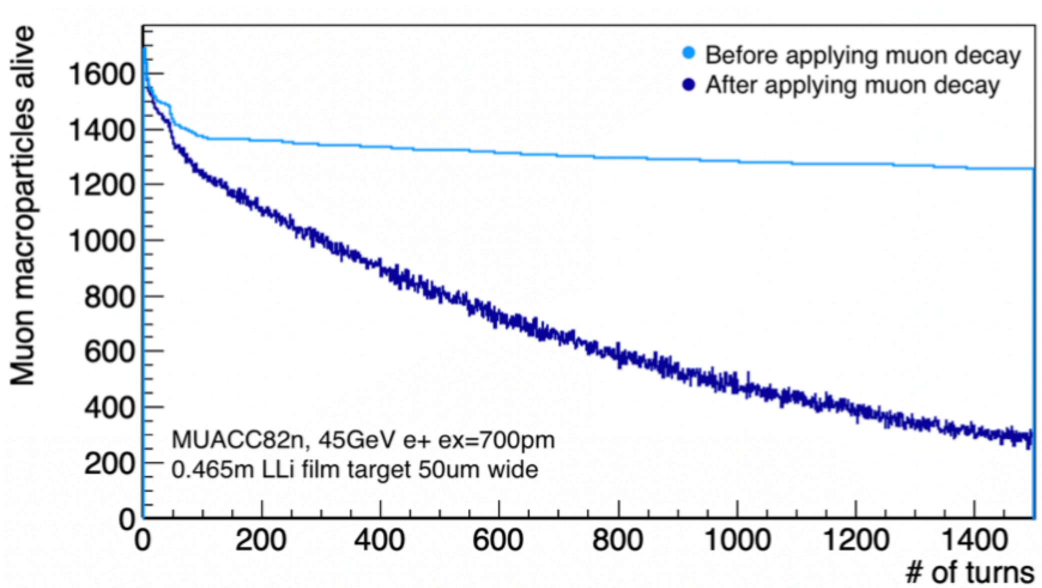
The simulations starts with e+ macroparticles at the beginning of the target. **Multiple scattering** and **energy loss** are evaluated at each step.

The **production vertex** z is extracted from the exponential distribution $f(z) = e^{-z/X_0}$



The positron is tracked *step-by-step* to z. Then if the positron energy is above threshold $E \geq 43.8 GeV$ a muon is produced and a **weight** is associated to it.

Muons are tracked *step-by-step* through the target, then tracked with MADX-PTC in the accumulator optics back to the target. This **cycle** is repeated for the whole accumulation process



At the end of the accumulation process, **muon survival probability** is evaluated by:

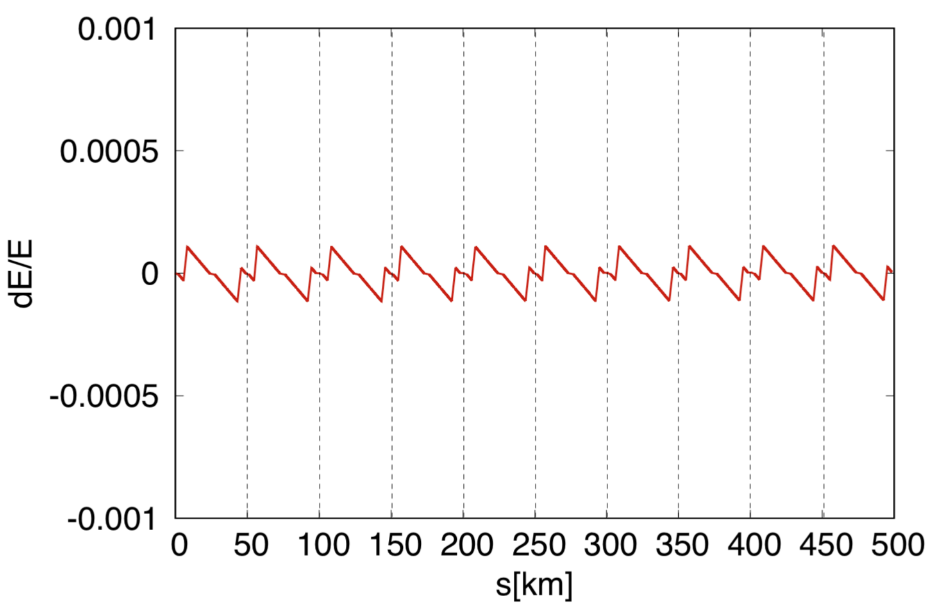
$$P = e^{-L/\tau\beta c}$$

where $\tau = \gamma \cdot 2.2 \mu s$ and $L = L_{accumulator} \times N_{turns}$

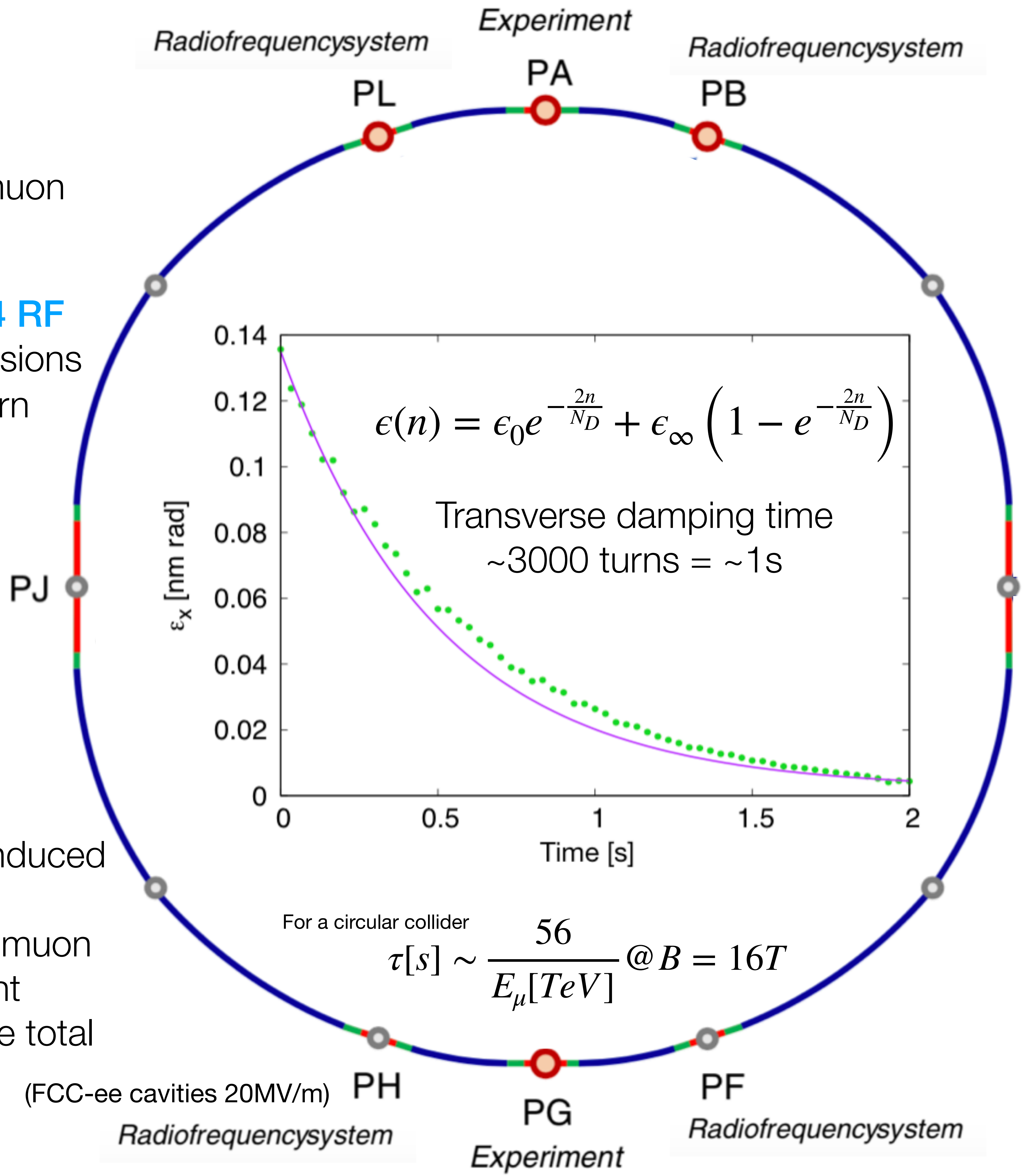
FCC-$\mu\mu$

Conceptual study for a 100TeV muon collider in the FCC tunnel

FCC-hh lattice modified to have **4 RF sections**, to have symmetric collisions and supply the 29GeV lost per turn



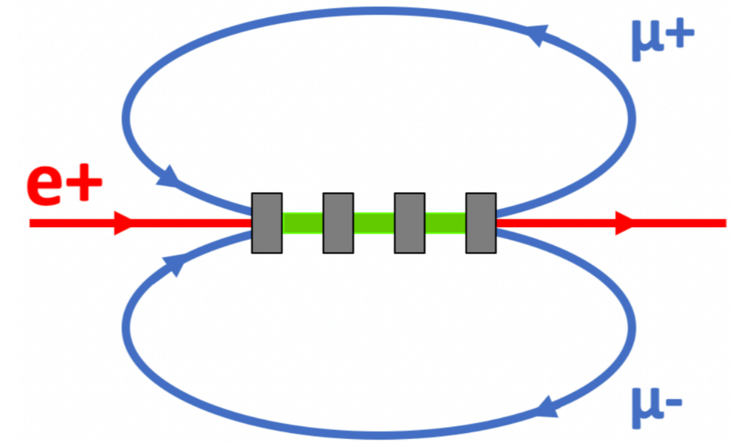
From a preliminary study the SR induced **damping** is appreciable, but the damping time corresponds to the muon lifetime. Enhancing this effect might allow **top-up mode**, increasing the total number of muons per bunch.



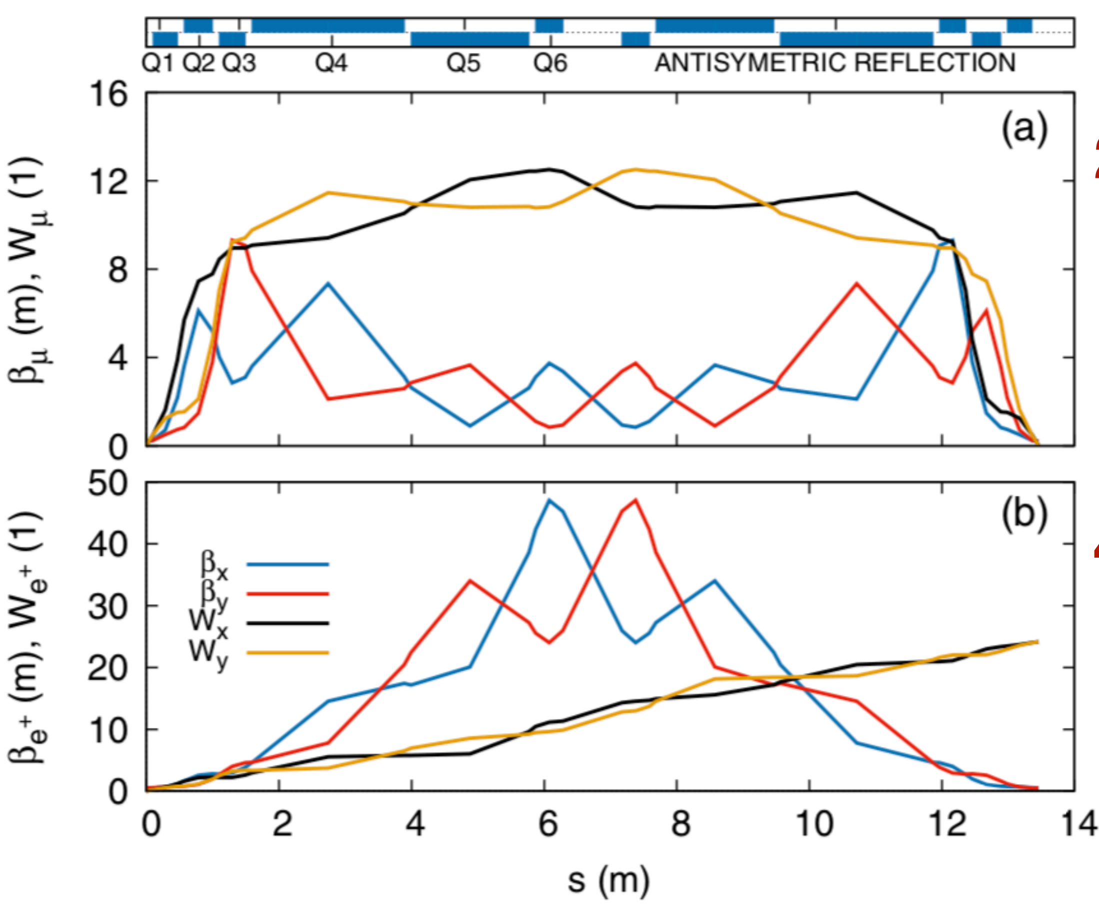
Transport lines for Multi-Target LEMMA option

Multiple thin targets $\mathcal{O}(0.01X_0)$ connected by a **transport line**:

- ✓ Reduced power per target
- ✓ Focusing the beams (instead of drift through thick target)
- X Longer production system



Only quadrupoles - max 525 T/m (CLIC 500 T/m)



The line must be able to **transport 3 beams** (e^+, μ^+, μ^-)

22.5GeV $\mu^+\mu^-$ beams

First order apochromatic line

$\beta* = 20cm$

$\pm 5\,\%$ energy acceptance

45GeV e^+ beams

$\beta* = 50cm$

Chromaticity not corrected

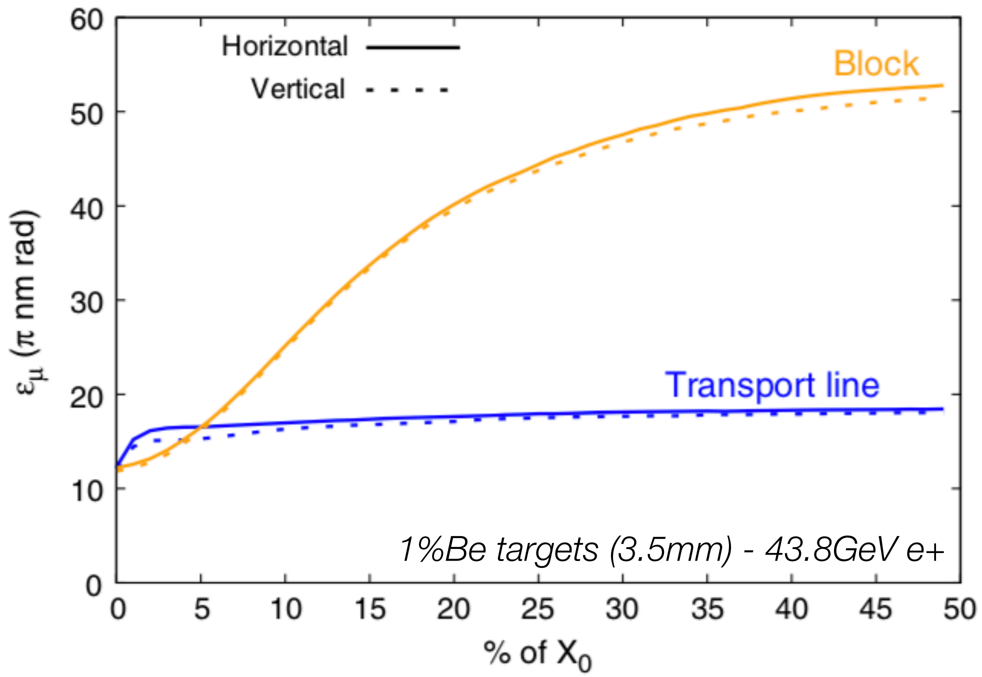
Positron beam $\delta E = 0.1\,\%$

A dedicated lattice for the accumulator ring has not been yet designed. In this paper the performances of the transport lines in use with thin targets have been studied.

PHYSICAL REVIEW ACCELERATORS AND BEAMS **23,** 091601 (2020)

Nanometric muon beam emittance from e^+ annihilation on multiple thin targets

O. R. Blanco-García* and A. Ciarma
INFN-LNF, Via E. Fermi 40, 00044 Frascati, Rome, Italy



The line proves **very effective** in transporting the muon beam through several targets while keeping the **emittance constant**.

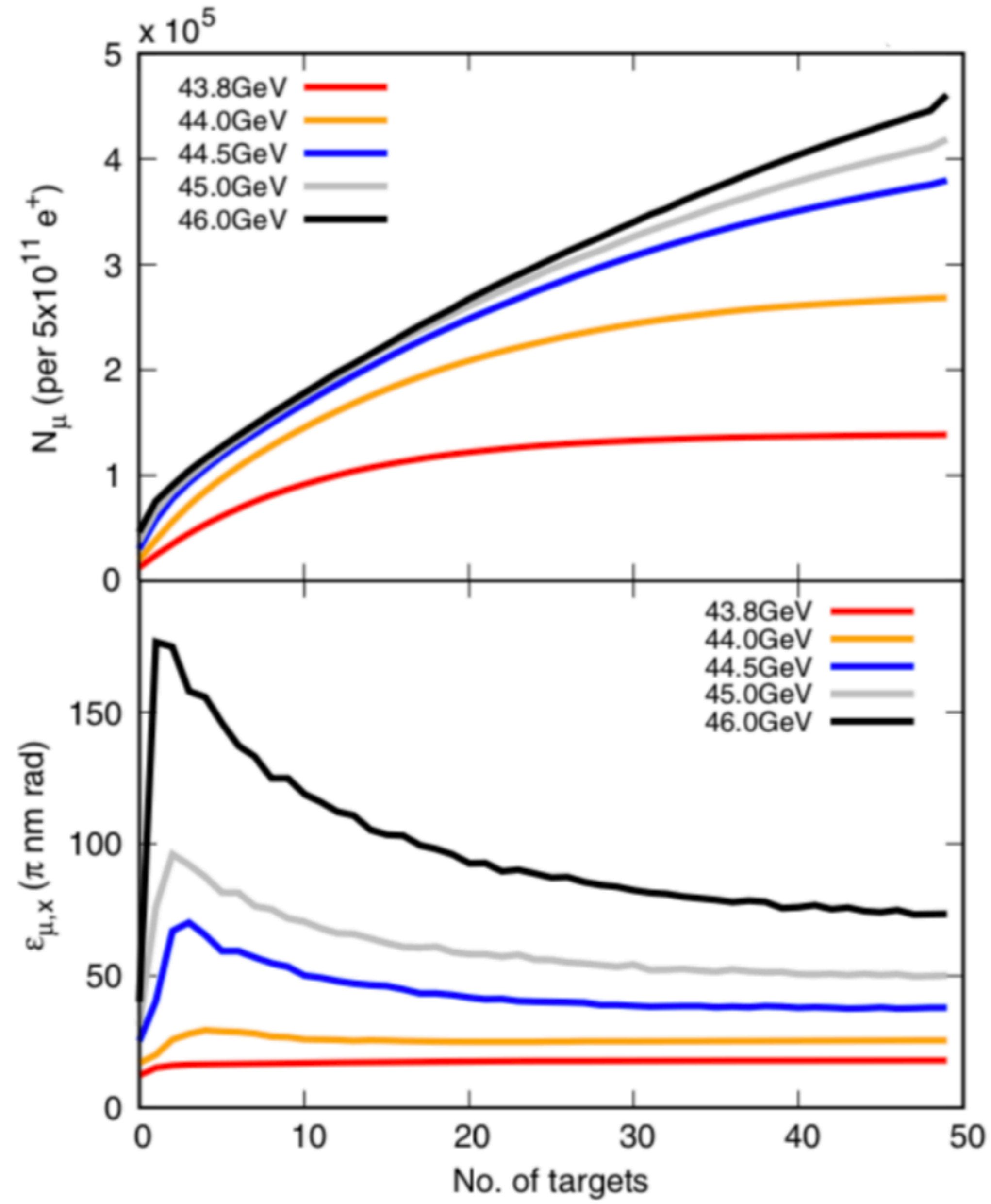
Changing the **target material** does not influence the trend as a function of the % of X_0

Small emittance increase (~30%) is due to higher order terms in the chromatic functions not corrected

Positron beam **energy scan** has been performed to find the optimised working point for this transport line.

Above 44.0GeV emittance starts to be degraded due to the **higher muon energy spread** at production, and increasing even more the energy saturates the population because muons are produced already outside the line energy acceptance.

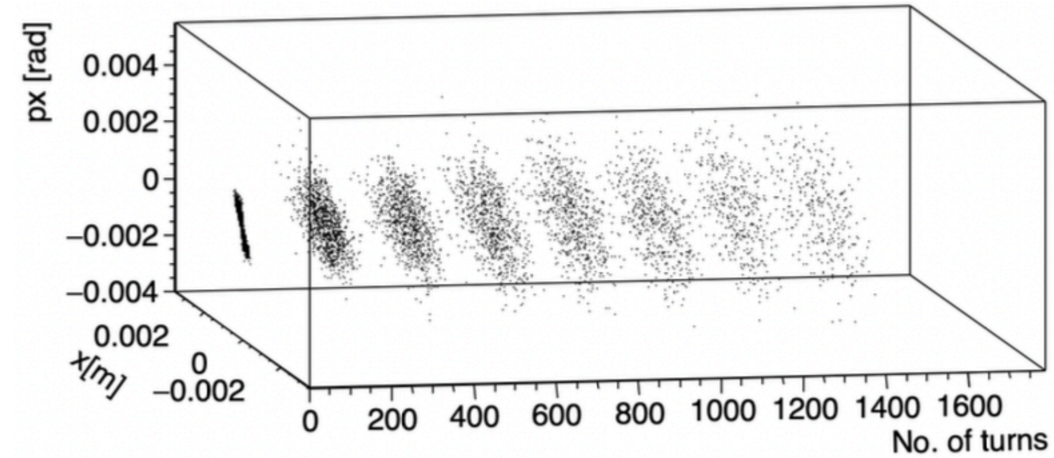
The working point is set to 44.0GeV achieving a production efficiency of 50×10^{-8} muon pairs per positron and an emittance of $25\pi\ nm\ rad$ after 50 Beryllium targets of $1\% X_0$



Multiple Scattering

$$\sigma_\theta[rad] = \frac{0.0136}{E[GeV]}\sqrt{\frac{L[m]}{X_0[m]}}$$

```
rnd = gRandom->Gaus(0,1);
x = x + L_step*px/2;
px = px + sigma_theta*rnd;
x = x + L_step*px/2;
```



Bremsstrahlung

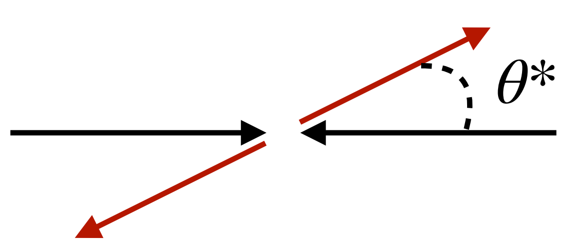
$$\frac{d\sigma}{dk} = \frac{A}{X_0 N_A k}\left(\frac{4}{3} - \frac{4}{3}y + y^2\right)$$

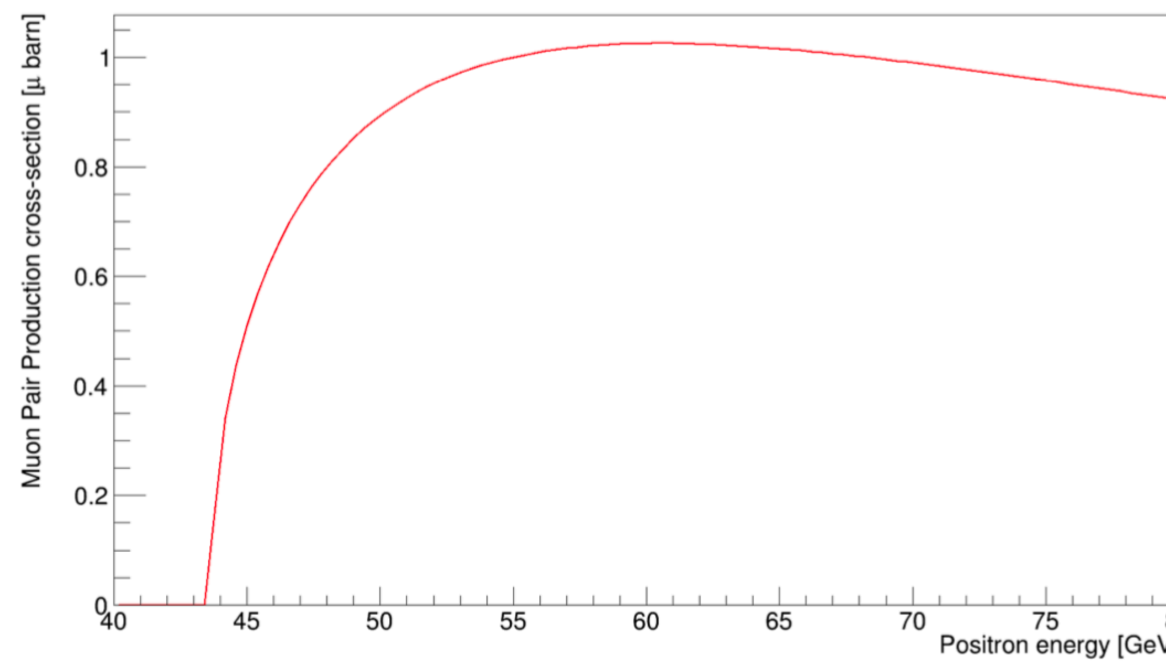
$$P = \frac{N_A \rho}{A} L_{step}\sigma_{tot} \sim \frac{L_{step}[m]}{X_0[m]}\frac{4}{3}log\left(\frac{k_{max}}{k_{min}}\right)$$

Muon Production

$$w = \frac{N_{e+}^{true}\ \rho N_A Z/A\ l_{tgt}\ \sigma_{e^+e^-\to\mu^+\mu^-}}{N_{e+}^{macro}}$$

$$f(\theta^*) = \left[1 + \frac{E_{th}}{E} + \left(1 - \frac{E_{th}}{E}\right)cos^2\theta^*\right]sin\theta^*$$





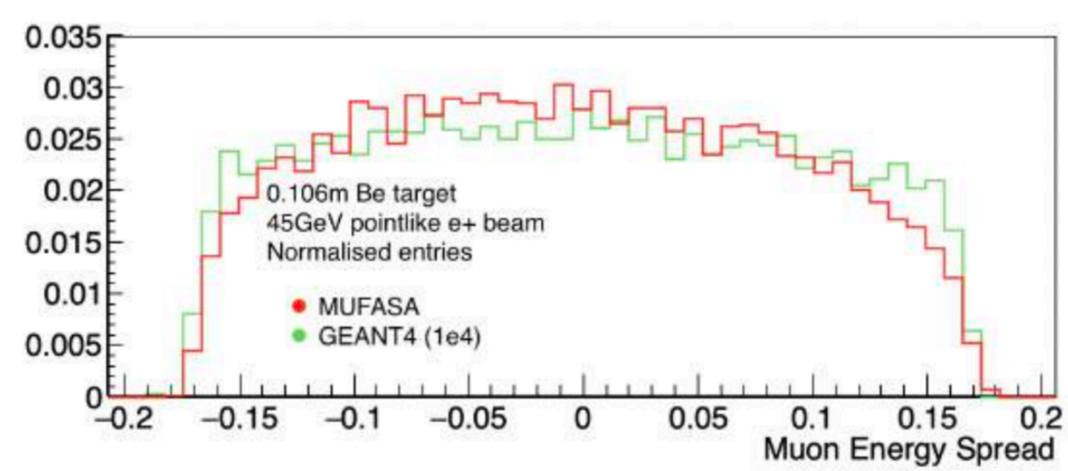


Figure 3.10. Energy distribution muons produced by a 45GeV positron beam on a 0.106m Beryllium target.

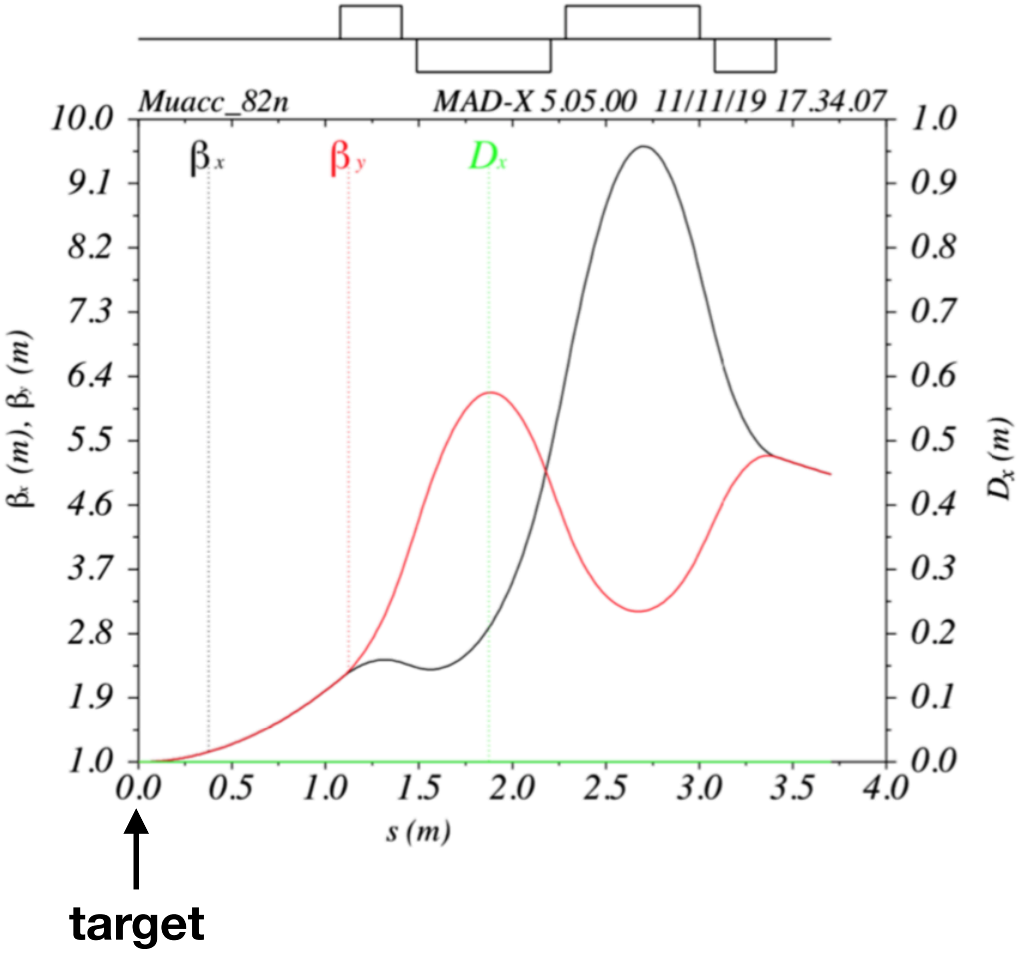
Tentative IMC 3 TeV
(based on MAP potential transmission factors)

IMC	3 TeV	Particle Transmission		Dilution/Cooling Factor: Transverse emittances		Dilution/Cooling Factor: Longitudinal emittances		Beam Energy	Number of bunches	Particles per bunch	Norm transv emittance	Norm. long. emittance	Bunch length	Beam Power
5	Rep rate (Hz)							GeV	#	E12	µrad-m	mrad-m	mm	W
Driver		0.153 at 8GeV						5	1	376.89			600 (2ns)	1.5E+06
Target & Front End		0.0956						0.255	12	36.04	15000	45	85.2	8.8E+04
Cooling	Initial Cooling	0.72	0.108	0.2	0.003	0.22	2.182	0.255	12	25.77	3000	10	85.2	6.3E+04
	Charge separator	0.90		1.05		1.05		0.255	12	23.19	3150	10	85.2	5.7E+04
	6D cooling before merge	0.72		0.5		0.20		0.255	12	16.58	1575	2	85.2	4.1E+04
	6D merge	0.88		2		4.00		0.255	1	14.59	3150	8	92.3	3.0E+03
	6D cooling after merge	0.44		0.067		0.23		0.255	1	6.42	211	2	92.3	1.3E+03
	Final cooling & Re-Accel	0.61		0.188		52.00		0.255	1	3.91	40	98	92.3	8.0E+02
Acceleration	Injector Linac	0.92	0.568	1.05	1.159	1.05	1.159	1.25	1	3.60	42	103	46.2	3.6E+03
	RLA1	0.92		1.02		1.02		5	1	3.32	42	105	23.1	1.3E+04
	RLA2	0.85		1.02		1.02		62.5	1	2.83	43	107	23.1	1.4E+05
	RCS1	0.90		1.02		1.02		303	1	2.54	44	109	23.1	6.2E+05
	RCS2	0.92		1.02		1.02		750	1	2.34	45	112	23.1	1.4E+06
	RCS3	0.95		1.02		1.02		1500	1	2.22	46	114	23.1	2.7E+06
Collider	IP	0.99		1.02		1.02		1500	1	2.20	47	116	5.0	2.6E+06
Front End to IP		6.10E-02		3.12E-03		2.58								

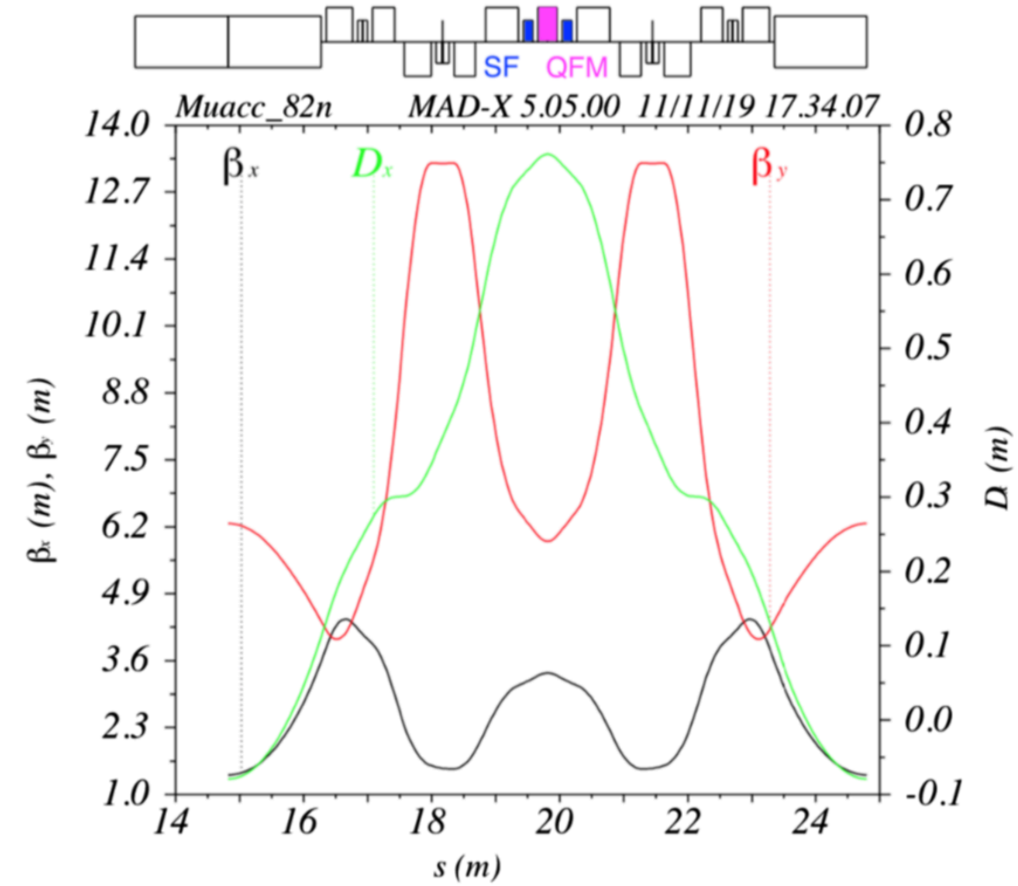
Proton beam power on target for 2.2E12 µ /5Hz at IP: 1.5 MW

IP transverse/longitudinal emittances: 47/116 mm-mrad

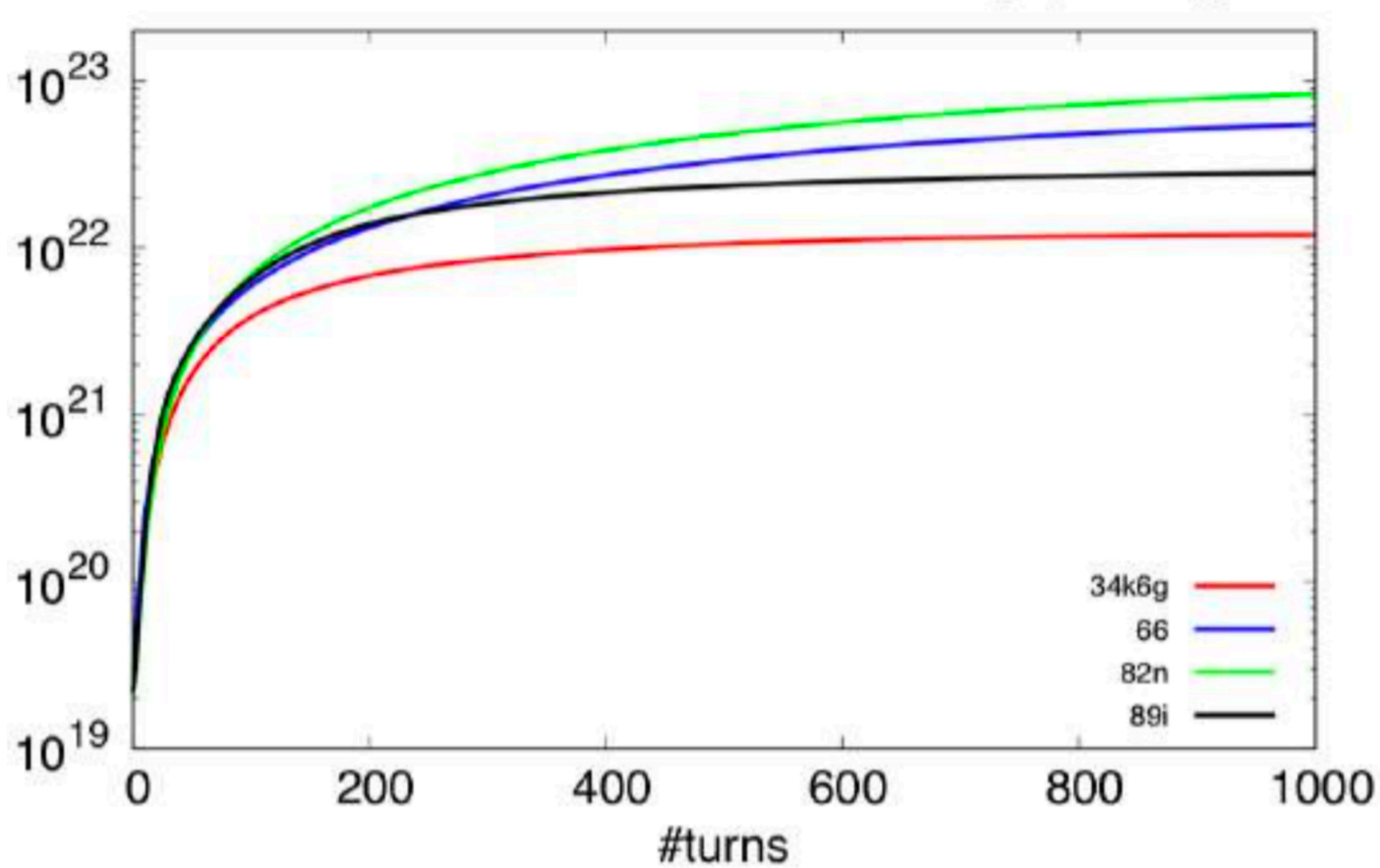
First Cell



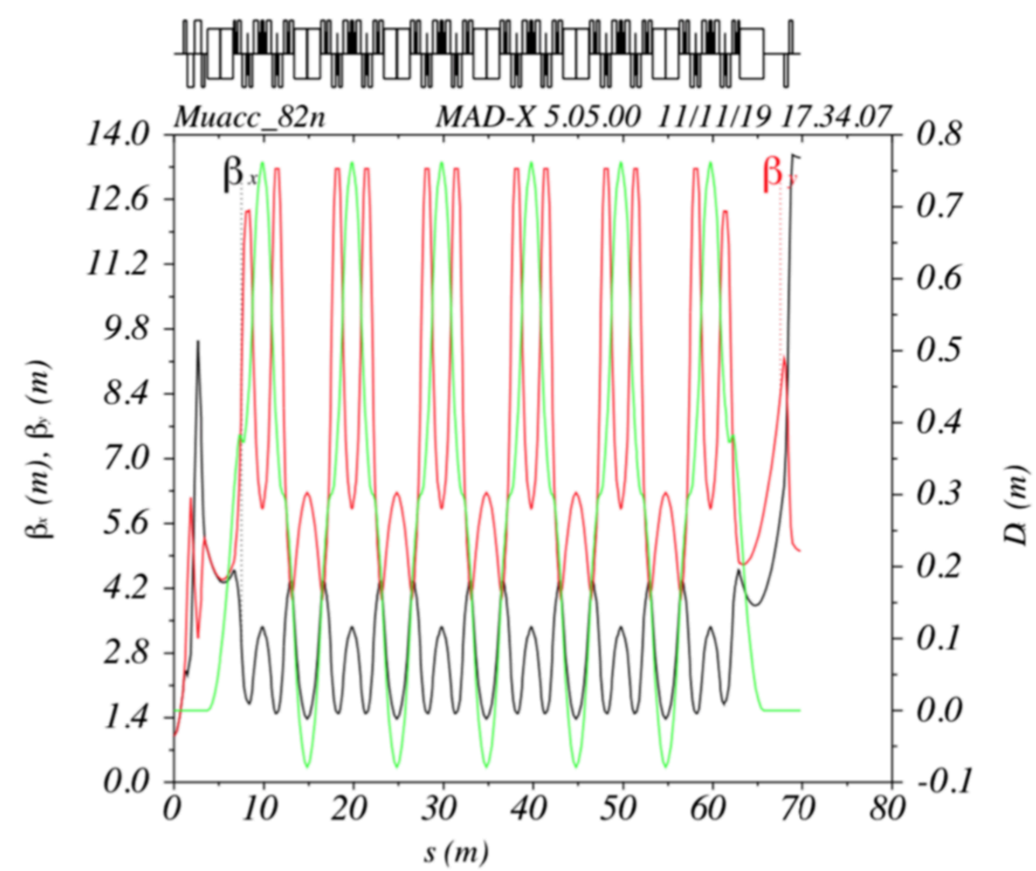
Central Cell

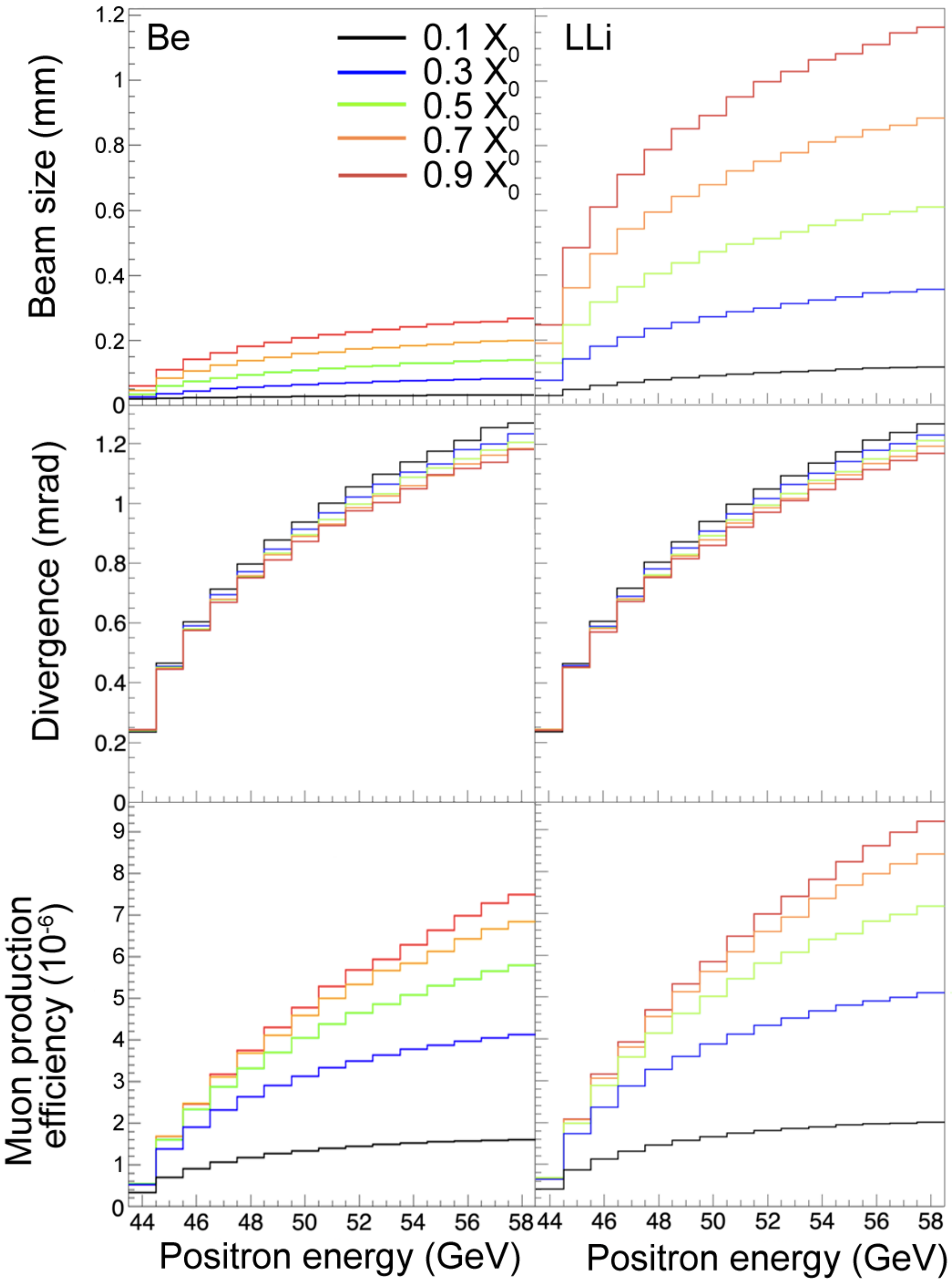


Accumulated muons N^2/emittance x [1/(m*rad)]



Half ring optics





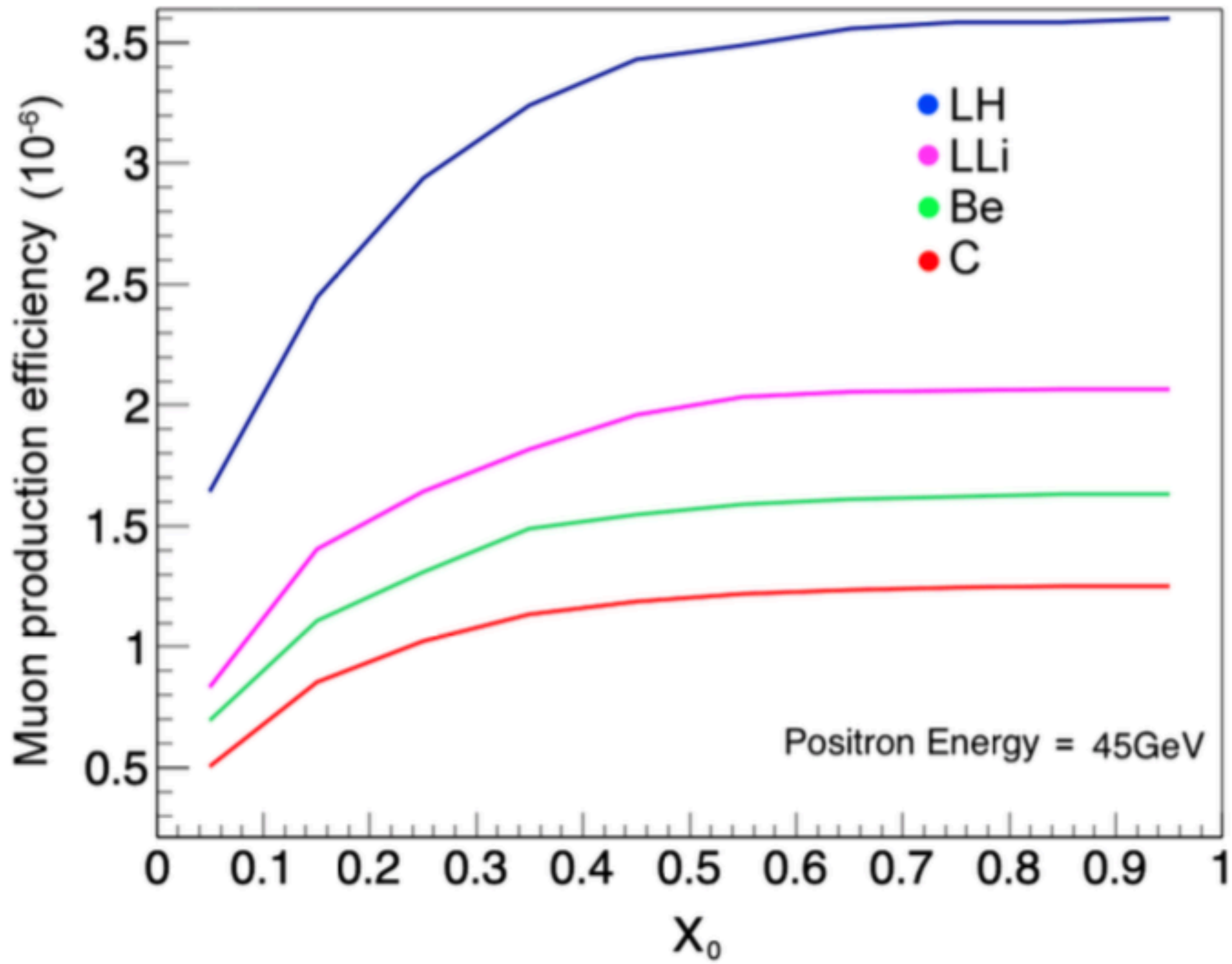
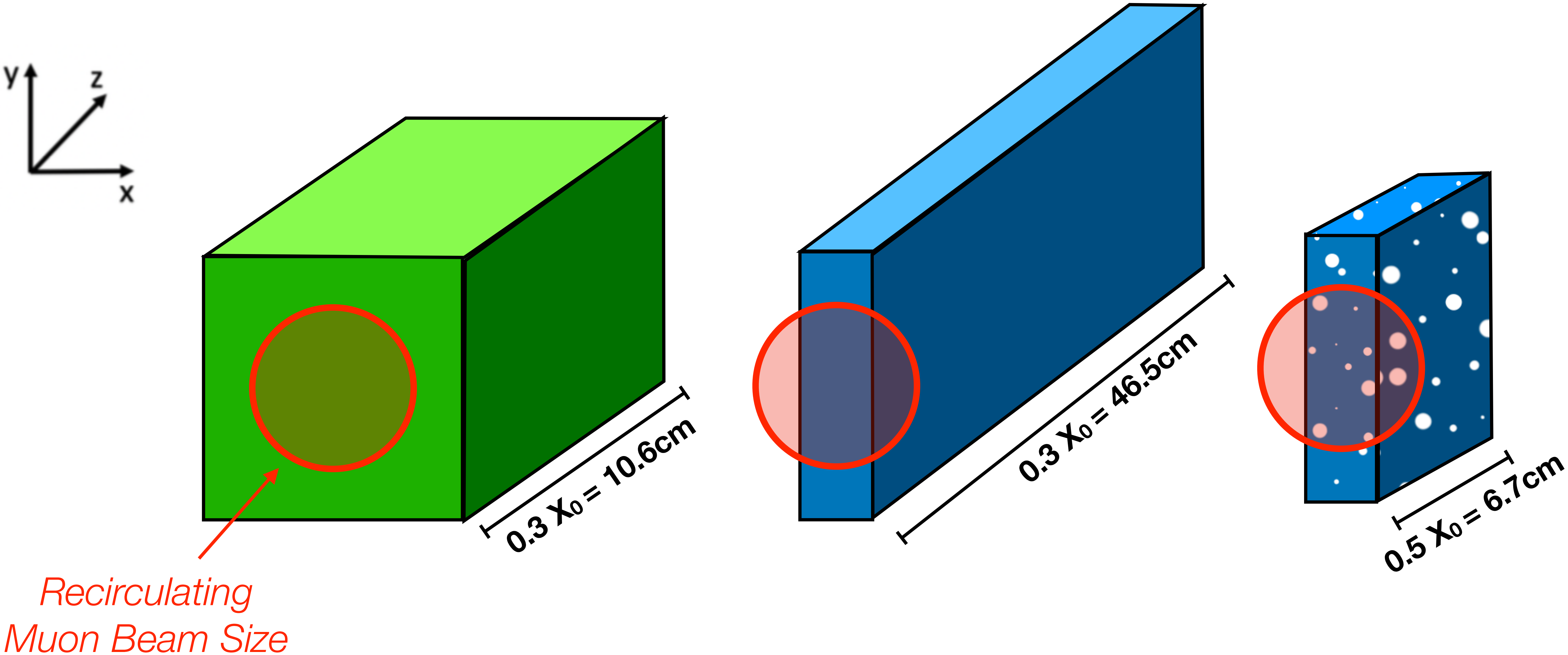
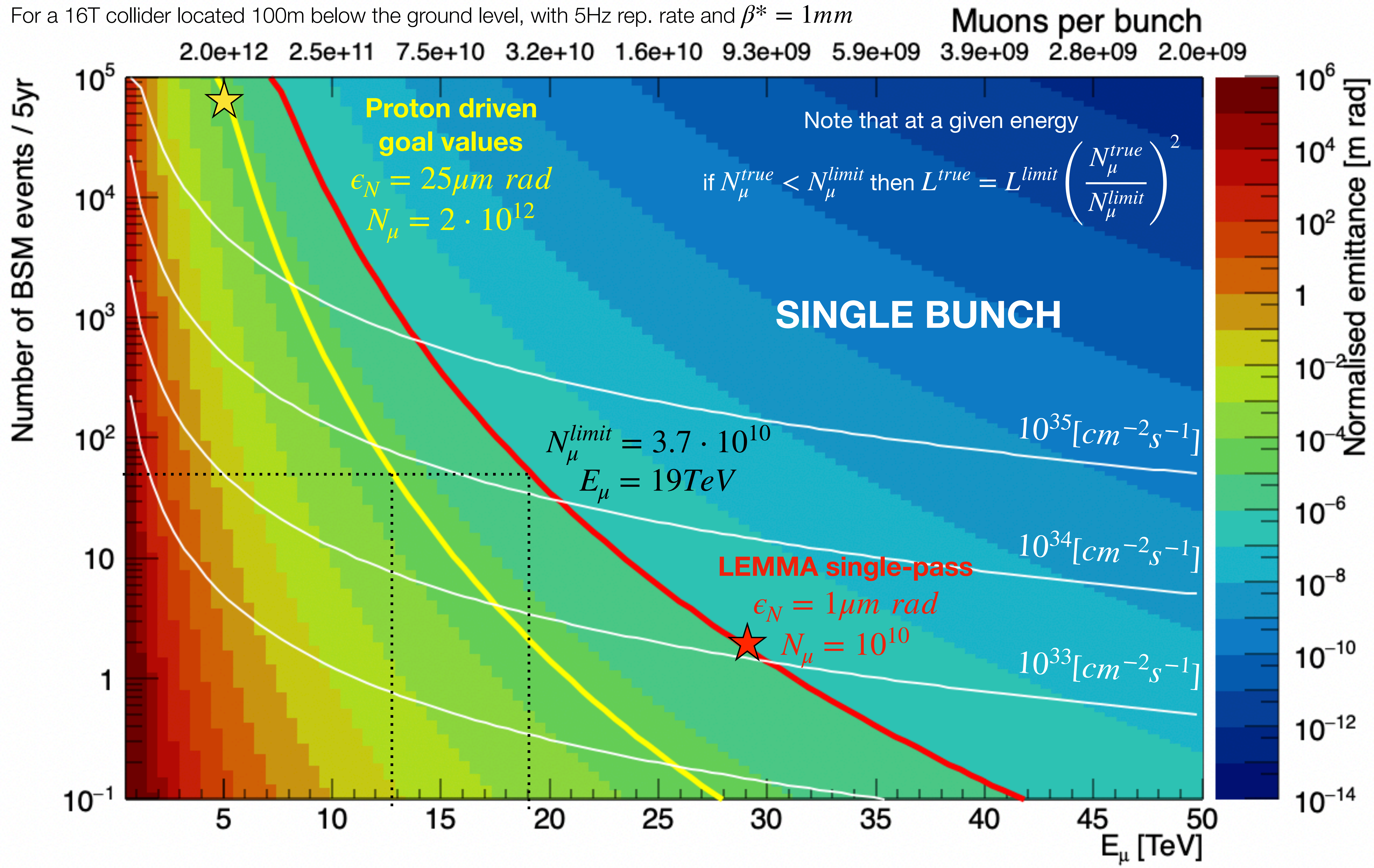


TABLE III. Properties of LLi–D compounds, for different fractions of liquid lithium $f(\mathrm{LLi})$, and diamond powder $f(\mathrm{D})$.

$f(\mathrm{LLi})$	$f(\mathrm{D})$	ρ[g cm^{-3}]	X_0 [g cm^{-2}]	X_0 [cm]
1.0	0.0	0.534	82.78	155.02
0.9	0.1	0.833	59.26	71.18
0.7	0.3	1.430	48.89	34.19
0.5	0.5	2.027	45.61	22.50
0.3	0.7	2.624	44.00	16.77
0.1	0.9	3.221	43.04	13.36
0.0	1.0	3.520	42.70	12.13

$$\frac{\rho}{X_0} = f(\mathrm{LLi})\frac{\rho\mathrm{LLi}}{X_0^{\mathrm{LLi}}} + f(\mathrm{D})\frac{\rho^{\mathrm{D}}}{X_0^{\mathrm{D}}}$$

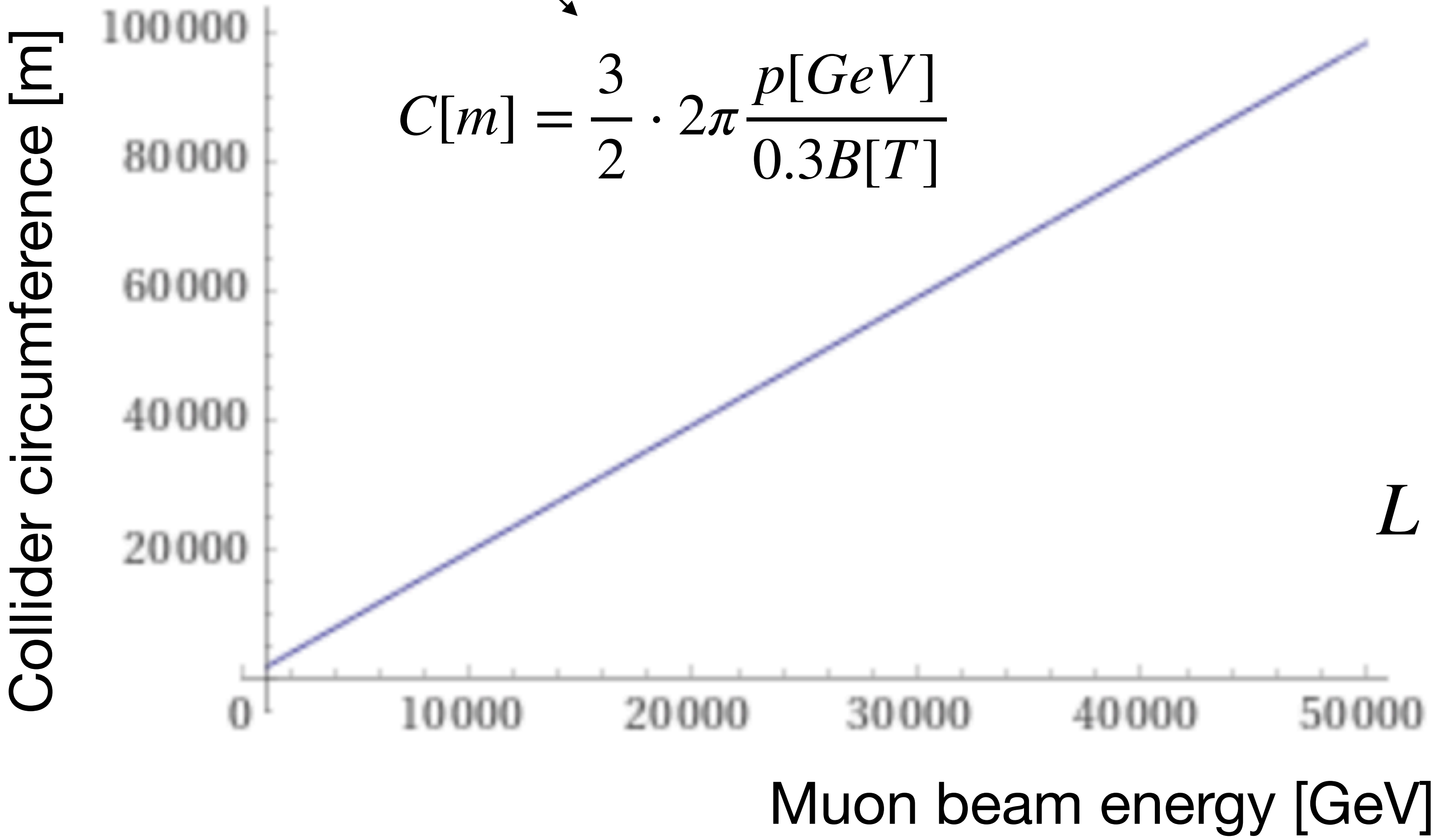

y
z
x
0.3 X_0 = 10.6cm
0.3 X_0 = 46.5cm
0.5 X_0 = 6.7cm
Recirculating
Muon Beam Size



LEMMA single-pass limit scenario: $\epsilon_N = 1\mu m\ rad$ without the need for cooling. A further factor 4 in population is required for LEMMA to be a discovery machine at $\sqrt{s} = 38TeV$, which could be achieved by reducing the collider circumference (16T→30km @17TeV), further upgrading the positron source or working in top-up mode.

Filling Factor = 2/3

$$C[m] = \frac{3}{2} \cdot 2\pi \frac{p[GeV]}{0.3B[T]}$$

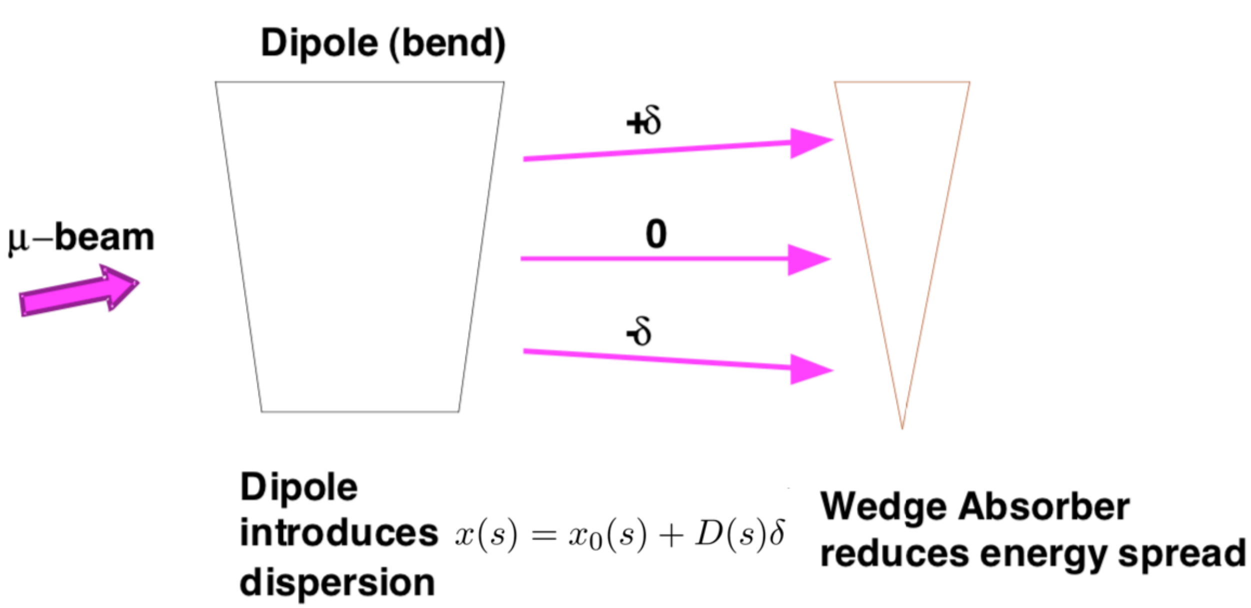


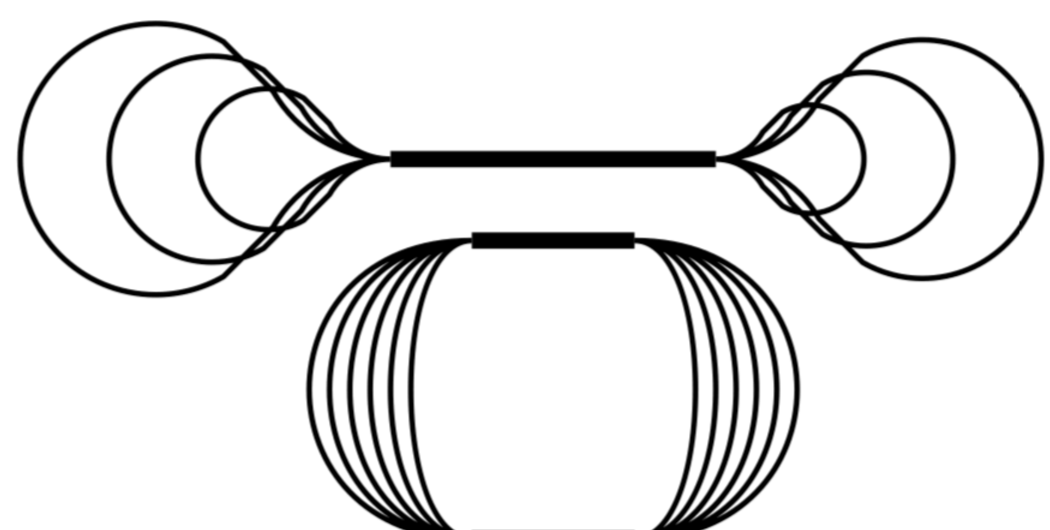
$$f_{rev} = \frac{c}{C}$$

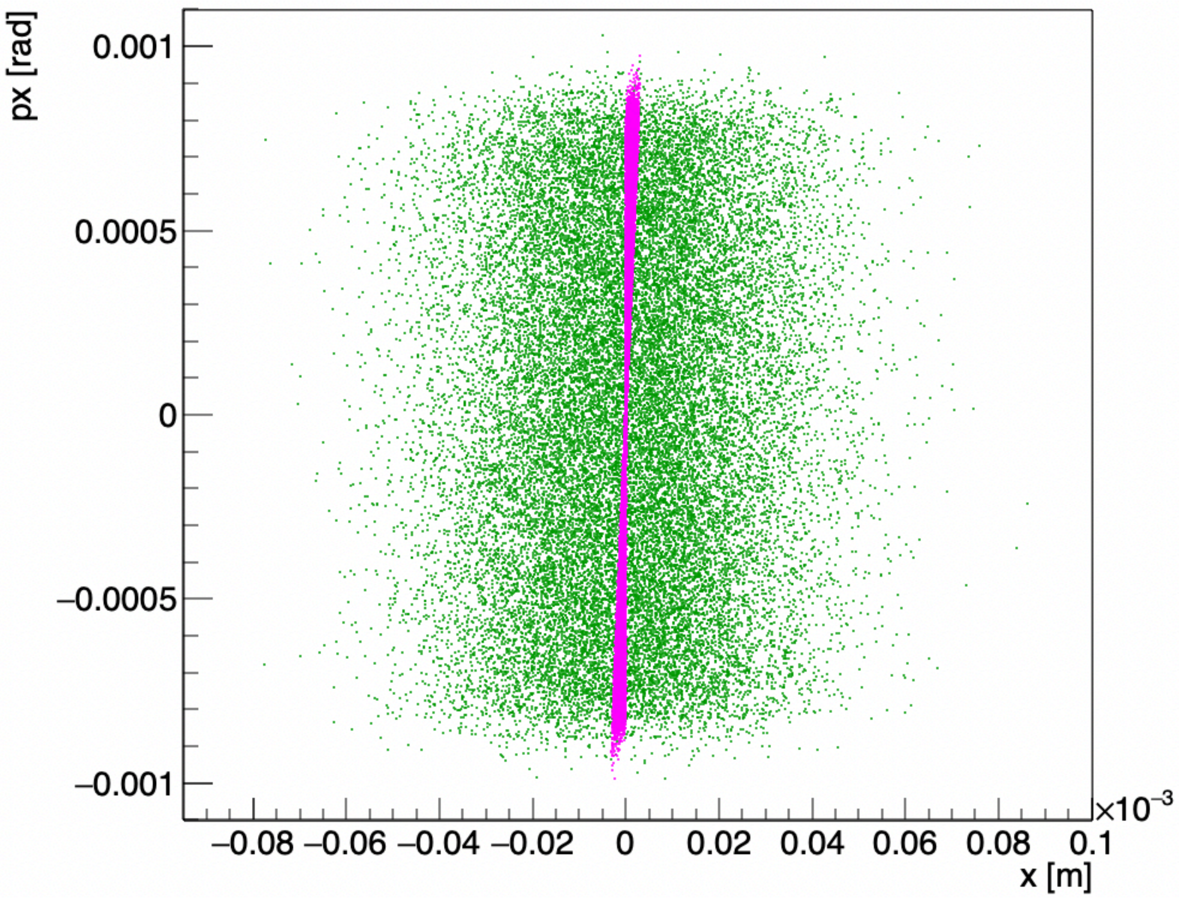
$$L = \frac{N^2 n_b f_{rev}}{4\pi\sqrt{\epsilon_x \epsilon_y \beta_x^{IP} \beta_y^{IP}}}$$

$$L = L_0 \frac{1 - e^{-\frac{t_c}{\tau}}}{\frac{t_c}{\tau} + \frac{t_f}{\tau}} = L_0 \frac{1 - e^{-\frac{t_c}{\tau}}}{\frac{1}{R\tau}}$$

$$\tau_{x,y}^{damping} = 2\frac{T_0 E}{U_0} = 2\frac{\frac{2\pi\rho \cdot \frac{3}{2}}{c} E}{4.85 \cdot 10^{-14} \frac{E^4}{\rho}} = 1.29 \cdot 10^6 \frac{\rho^2}{E^3} = 1.29 \cdot 10^6 \left(\frac{E}{0.3B}\right)^2 \frac{1}{E^3} = \frac{56}{E[TeV]}$$







Multi-pass 45GeV e+ on 3mm Be

Pointlike beam $\epsilon_N(\sigma_x = 0) = 44nm\ rad$

Realistic beam $\epsilon_N(\sigma_x = 20\mu m) = 1.9\mu m\ rad$

Single-pass 45GeV e+ on 0.5X0 LLi-D

Pointlike beam $\epsilon_N(\sigma_x = 0) = 1\mu m\ rad$

Realistic beam $\epsilon_N(\sigma_x = 20\mu m) = 2.2\mu m\ rad$