National
Technical
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University of Science and Technology of China

# Modified Gravity Cosmology

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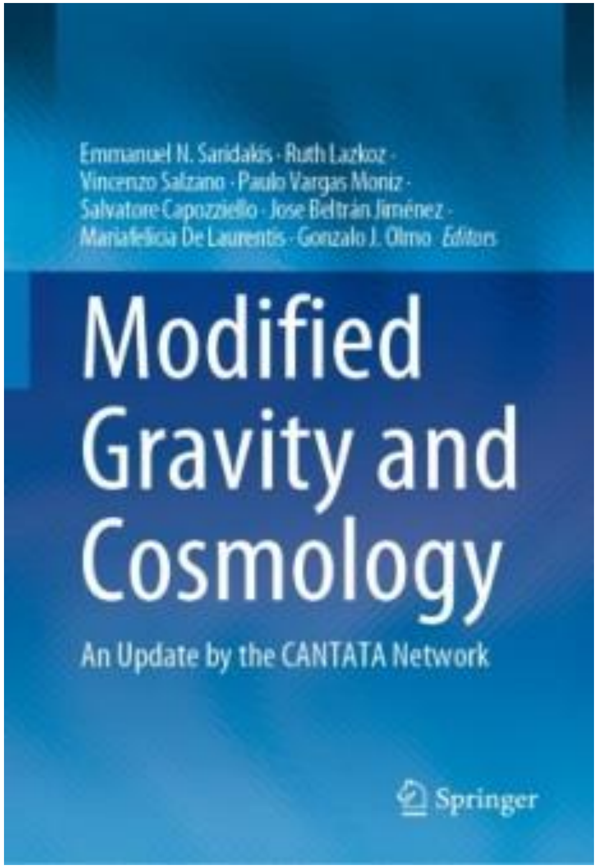
## *SIGRAV 2022*

CANTATA
Cosmology and Astrophysics Network for
Theoretical Advances and Training Actions

QUANTUM · GRAVITY · MULTI · MESSENGER

# Bibliography



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Modified Gravity and Cosmology

An Update by the CANTATA Network

Springer

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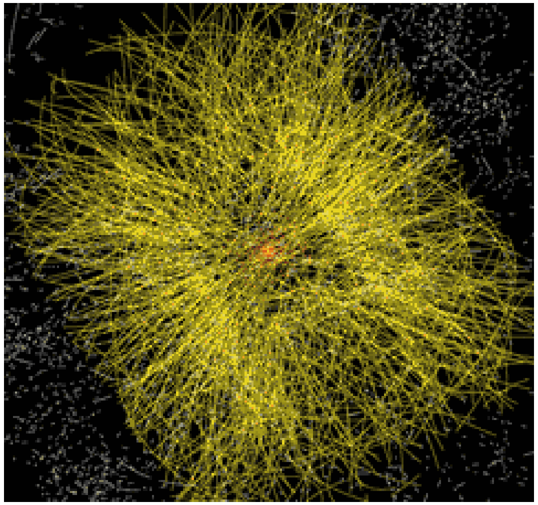
- Gravity is the most interesting interaction in Nature, and the one we know less about
- Gravity determines the Universe evolution: Cosmology

# Accelerators: CERN

- We study interactions in accelerators: CERN the world´s largest.



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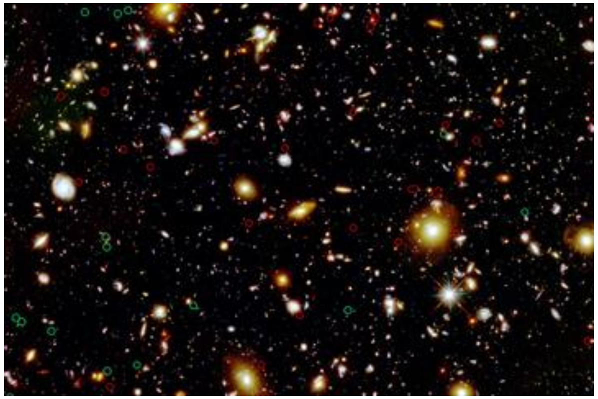
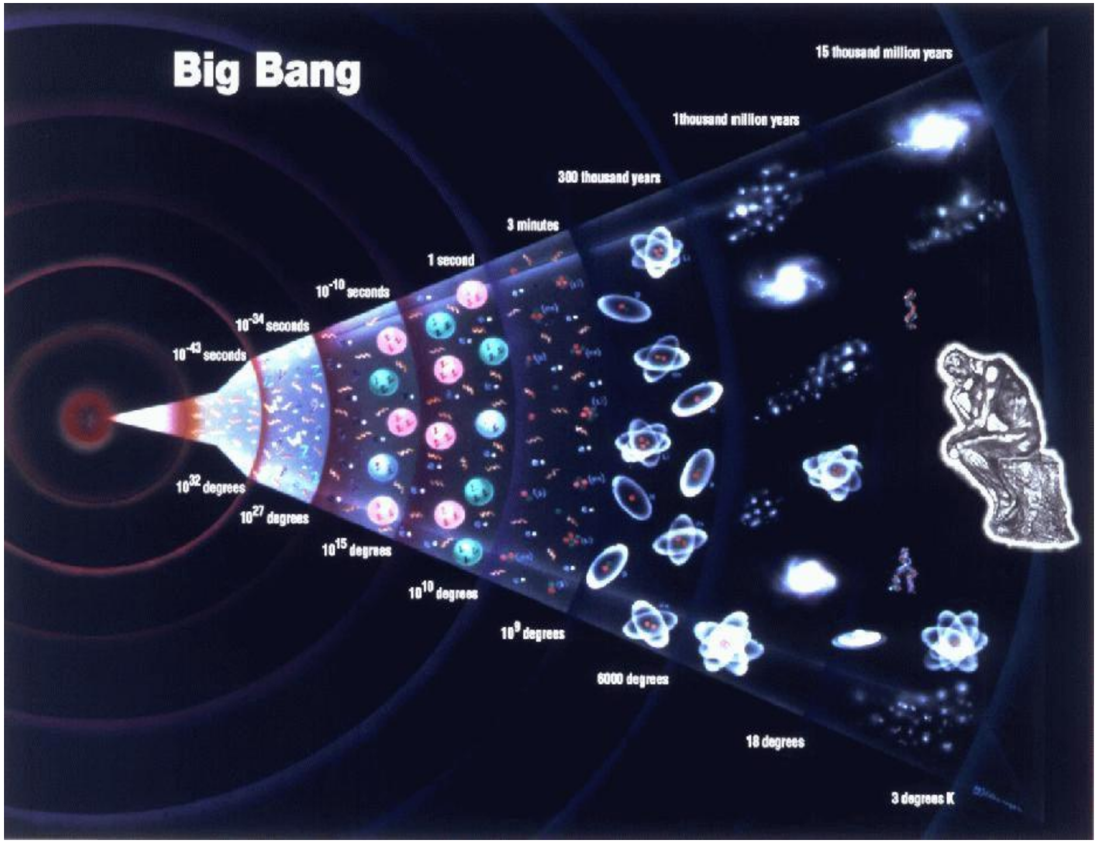
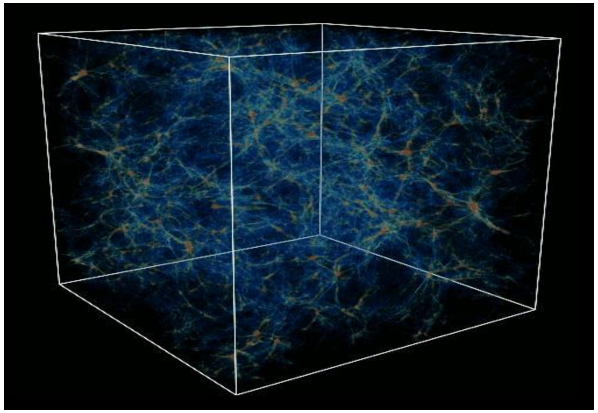
# Cosmology: the Lab of gravity

- Gravity cannot appear in accelerators. So we need to observe it in the Universe: Cosmology





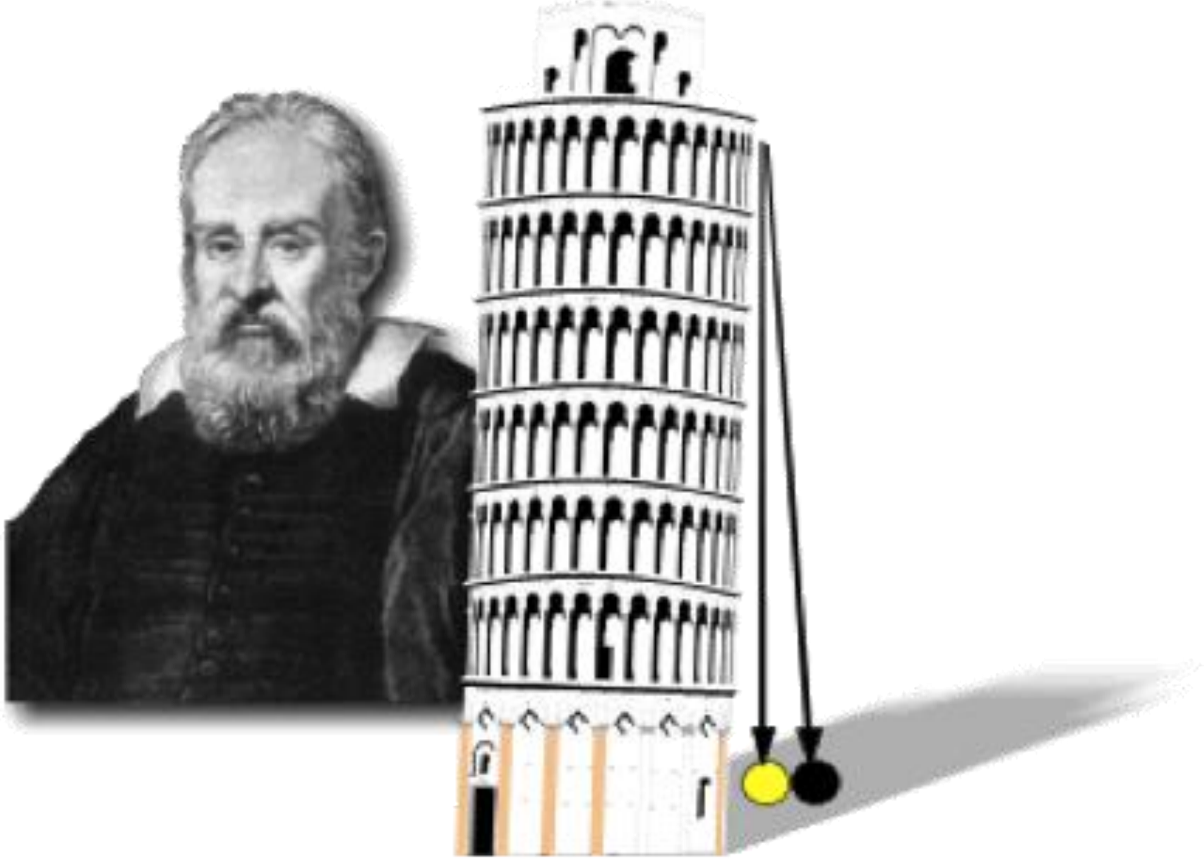


# Aristotle - 350 BC

- According to Aristotle heavier bodies fall faster.
- Bodies fall in order to com back to thei “initial state”.

# Galileo - 1600

- Bodies fall with the same speed, independently from their weight.



# Brahe, Kepler- 1600

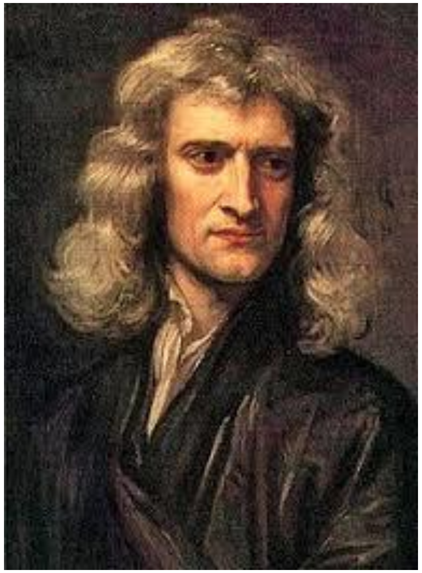
- Heliocentrism, elliptical Orbits

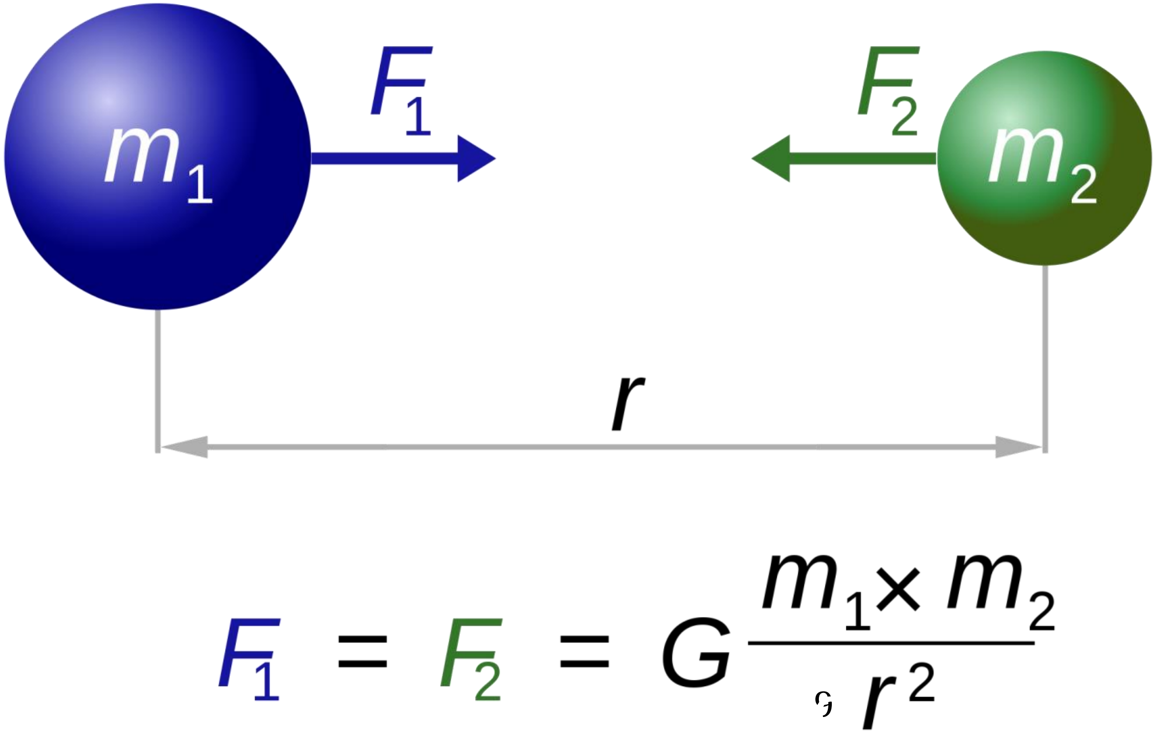
# Newton - 1700

- Law of Universal Gravitation:

  All bodies (either apples or planets) attract mutually.

  First time that gravity is related to astronomy

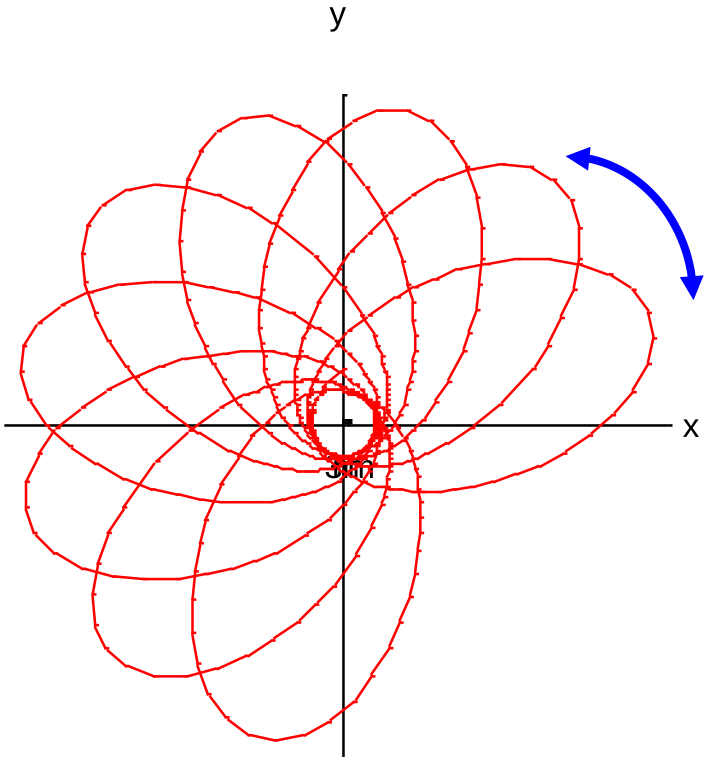




$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

# Mercury periliheimum - 1859

- *The true orbits of planets, even if seen from the SUN are not ellipses. They are rather curves of this type:*

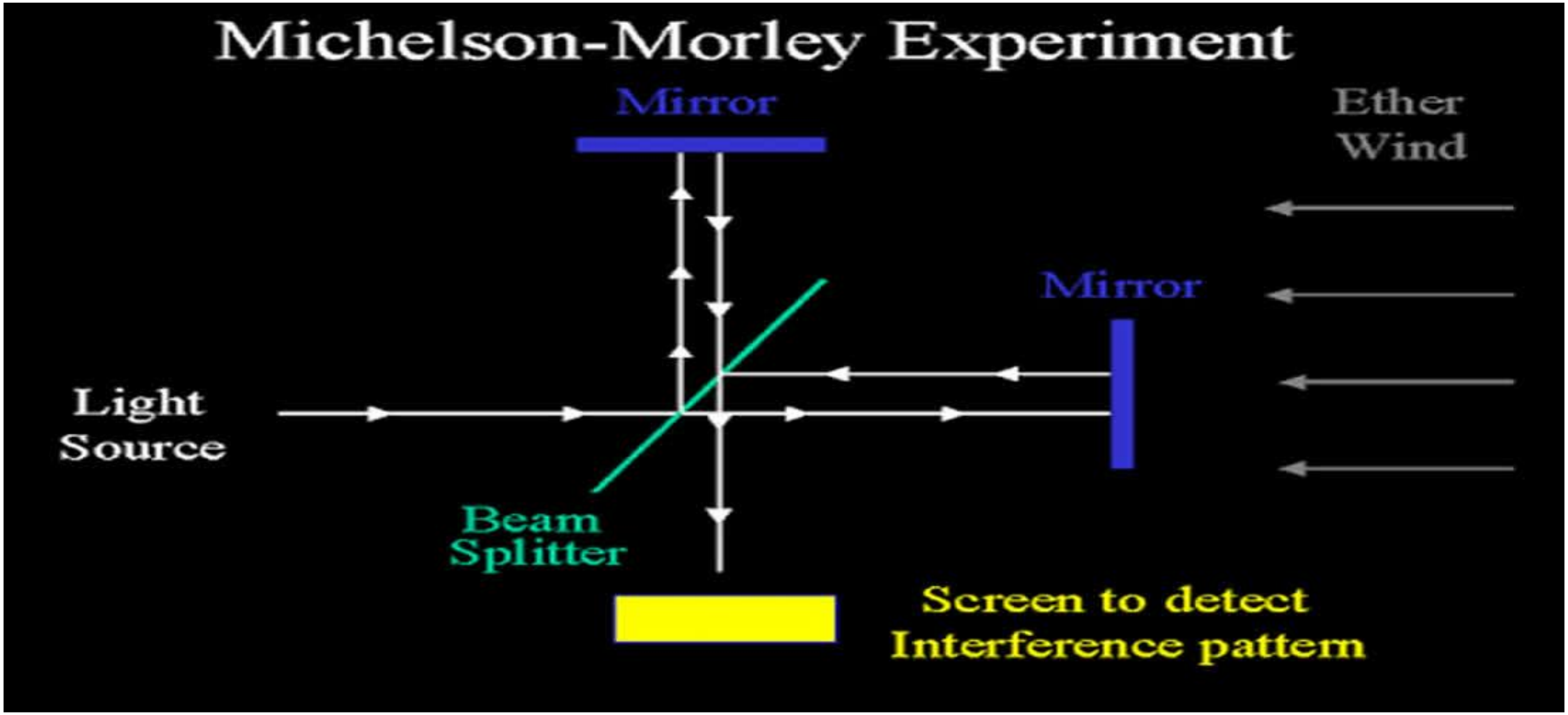


This angle is the perihelion advance, predicted by G.R.

For the planet Mercury it is

$$\Delta\varphi = 43'' of\ arc\ per\ century$$

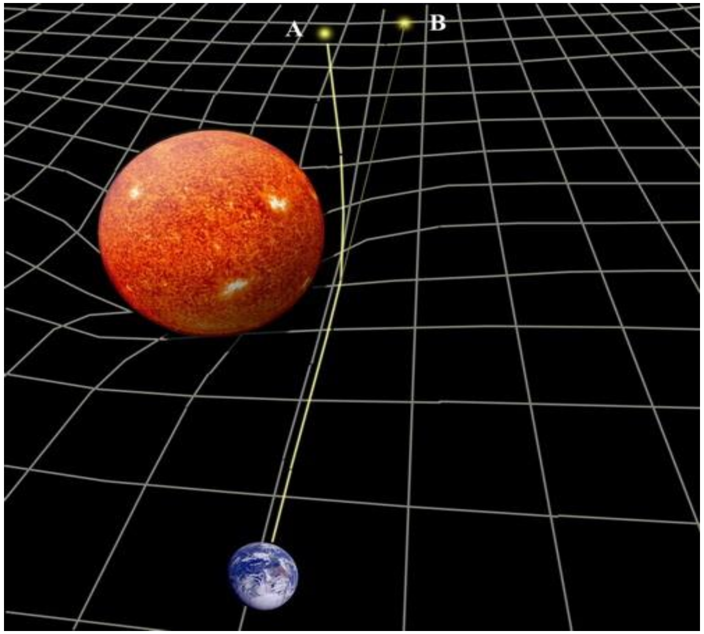
# Michelson–Morley experiment - 1887



# Gravity: General Relativity

- Matter tells spacetime how to curve

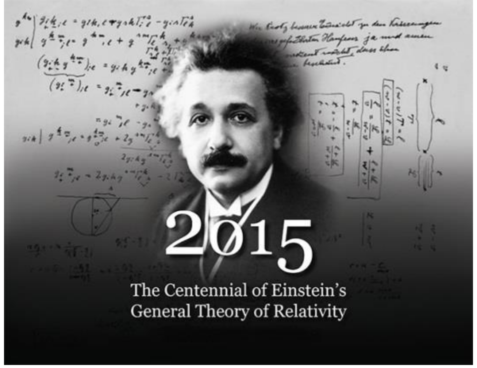
  Curved spacetime tells matter how to move



- It seems weird but it has been verified everywhere (Satellites, GPS, etc)

# General Relativity

- Einstein 1915: **General Relativity**:



energy-momentum source of spacetime Curvature

$$\boxed{S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,[R-2\Lambda]+\int d^4x\ L_m\left(g_{\mu\nu},\psi\right)}$$

$$\Rightarrow \quad R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+g_{\mu\nu}\Lambda=8\pi G\ T_{\mu\nu}$$

with $$T^{\mu\nu}\equiv\frac{2}{\sqrt{-g}}\frac{\delta L_m}{\delta g_{\mu\nu}}$$

$$
\begin{aligned}
\delta S_{\mathrm{EH}} &= \delta \int \sqrt{-g} R \, \mathrm{d}^4 x \\
&= \int \mathrm{d}^4 x \delta \Big( \sqrt{-g} g^{ab} R_{ab} \Big) \\
&= \int \mathrm{d}^4 x \sqrt{-g} g^{ab} \delta R_{ab} + \int \mathrm{d}^4 x \sqrt{-g} R_{ab} \delta g^{ab} + \int \mathrm{d}^4 x R \delta \sqrt{-g}.
\end{aligned}
$$

Now we have three terms of variation that

$$\delta S_{\mathrm{EH}} = \delta S_{\mathrm{EH}(1)} + \delta S_{\mathrm{EH}(2)} + \delta S_{\mathrm{EH}(3)} \tag{4.3}$$

The variation of first term is

$$\delta S_{\mathrm{EH}(1)} = \int \mathrm{d}^4 x \sqrt{-g} g^{ab} \delta R_{ab}. \tag{4.4}$$

$$S = \frac{1}{16\pi G} S_{\mathrm{EH}} + S_{\mathrm{M}} \tag{4.24}$$

where $S_{\mathrm{M}}$ is the action for matter. We normalize the gravitational action so that we get the right answer. Following the above equation we have

$$\frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g^{ab}} = \frac{1}{16\pi G} \Big( R_{ab} - \frac{1}{2} g_{ab} R \Big) + \frac{1}{\sqrt{-g}} \frac{\delta S_{\mathrm{M}}}{\delta g^{ab}} = 0.$$

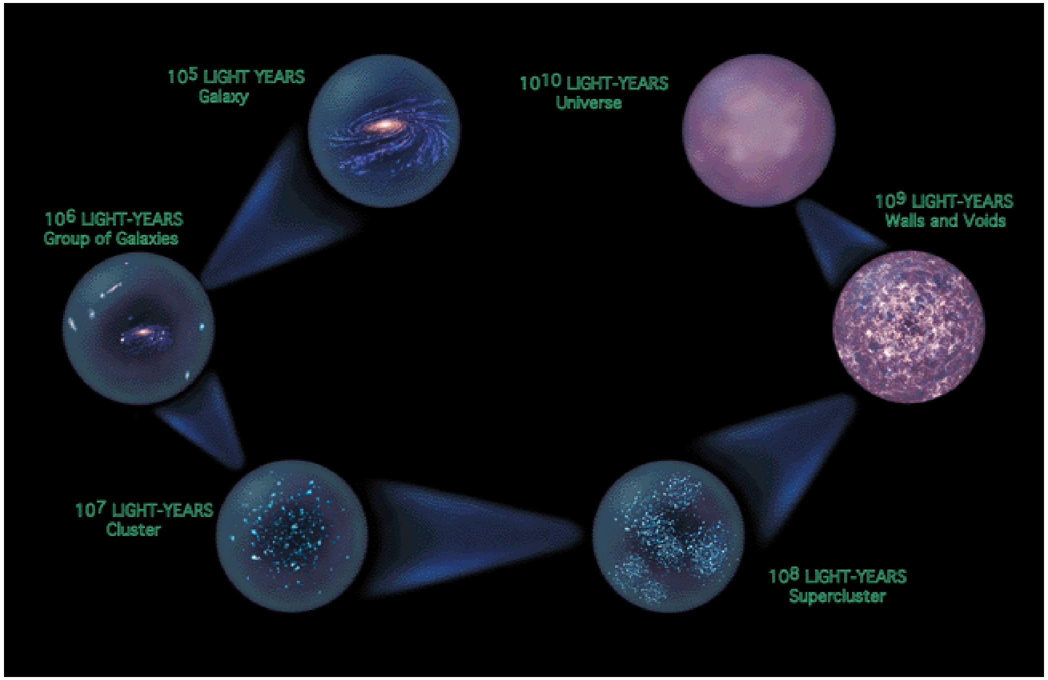
We now define the energy-momentum tensor as

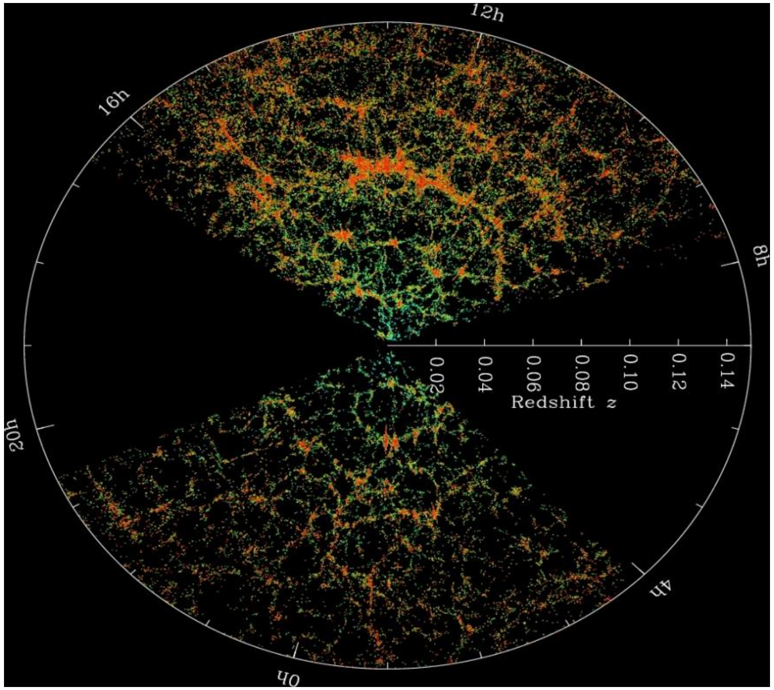
$$T_{ab} = -2 \frac{1}{\sqrt{-g}} \frac{\delta S_{\mathrm{M}}}{\delta g^{ab}}. \tag{4.25}$$

This allows us to recover the complete Einstein's equation,

$$R_{ab} - \frac{1}{2} R g_{ab} = 8\pi G T_{ab}. \tag{4.26}$$

# Observations





- SDSS (Sloan Digital Sky Survey) 2004: ~ clusters ”above and below the galactic plane” up to 1 Gpc

# Observations

- As the scale we observe the Universe increases, it looks as homogeneous and isotropic.

- Cosmological Principle: "axiom" (indirect result)

  I) We know that earth is an isotropic observation point.

  II) An anisotropic system has up to one isotropic observation point.

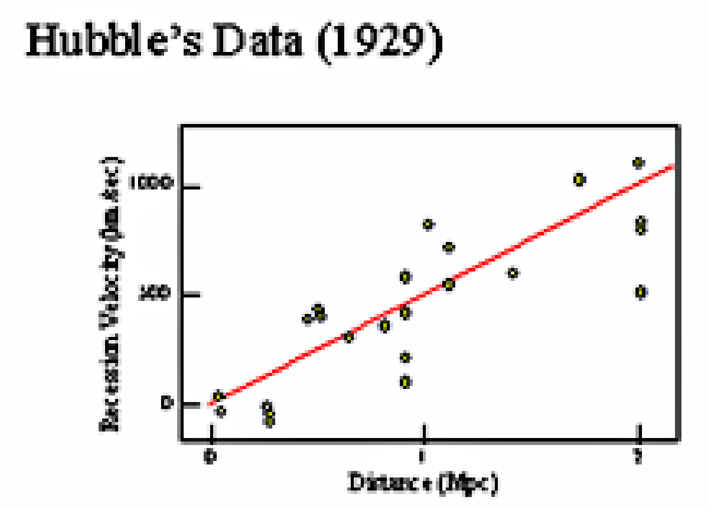
- Hence, either we lie in the only isotropic observation point in an anisotropic Universe, or all its points are isotropic observation points.

- Thus, the Universe is homogeneous and isotropic (isotropic and inhomogeneous is not possible)

# Observations

- Hubble 1929: The Universe expands

Hubble excelled in every course at school (except spelling), but was better known for his athletic prowess. He was a star player in football, baseball, and basketball, and ran track in high school and at the University of Chicago, where he earned a Bachelor of Science in 1910.



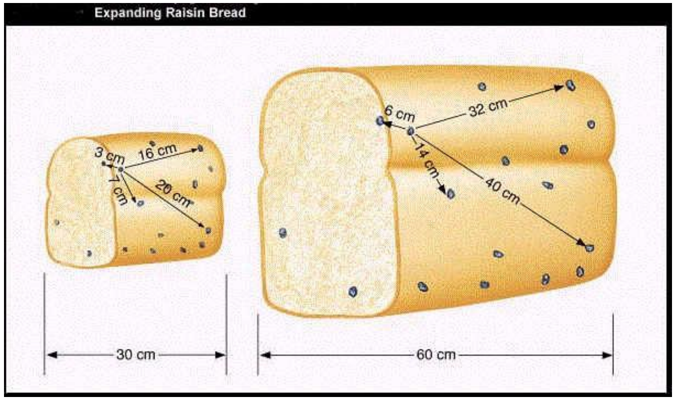
Hubble's Data (1929)

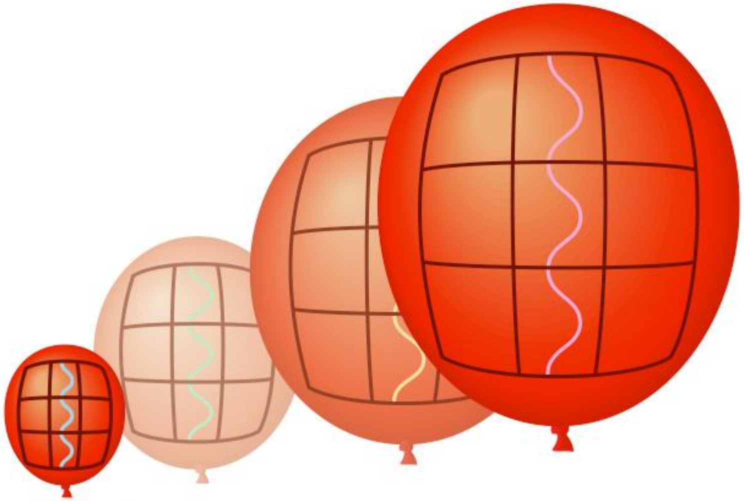
$$\boxed{v = H\ r}$$

$$H_0 \approx 70\ km\ s^{-1}\ Mpc^{-1}$$

# Expansion

$$z = \frac{\Delta\lambda}{\lambda} = \sqrt{\frac{c+v}{c-v}} - 1 \quad \approx \frac{v}{c}$$





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# Observations

- Since the Universe expands it is reasonable that it originates from a "too tiny" and "too dense" "primordial atom" (Lemaitre 1927)

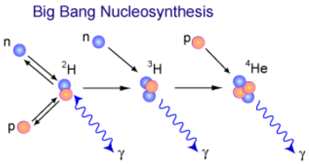
- Alpher, Bethe, Gamow (1948): The Universe begun to expand from a very high-density and high-temperature state towards less dense and hot states. Hoyle named the theory "The Big Bang Theory".

$$t_U = \frac{r}{v} = \frac{r}{Hr} = \frac{1}{H} = \frac{1}{70}\left[\frac{Mpc}{km}\right] s \approx 14\ Gy$$

- **Prediction I**: Nucleosynthesis has primordial origin, namely at first 3 minutes ($\sim 10^9$ K) (giving 25% Helium) and not in stars (1-4%) As observed.

# Observations

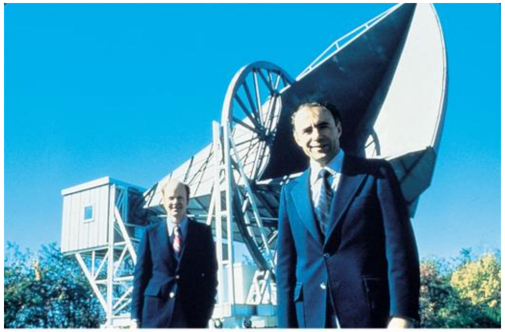
- **Prediction II**: The primordial Universe became full of high-energy photons



$$\lambda \approx 7\cdot 10^{-12}\ cm,$$

380.000 years after (~3000K) they decouple from electrons (Recombination era). Black body radiation (today ~2.7 K)

- 1965 Penzias και Wilson



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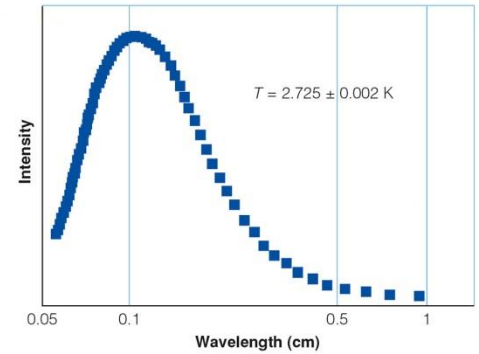

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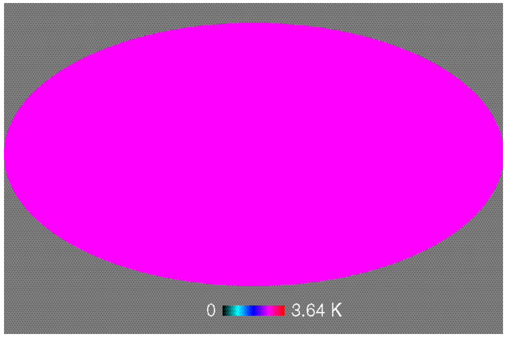


# Theoretical arguments

- Big Bang Theory explained: Olbers paradox (1826) (why night sky is not bright), Ryle (1970) (Radio galaxies density increases with redshift), Element abundance, CMB, etc

- Theoretical Problems:
- I) Horizon problem: Why points at opposite directions have the same properties
- II) Flatness problem: Why the universe is today almost spatially flat $\Omega_k$ ~0.001. It must have started with $\sim 10^{-50}$!
- Monopole problem: They are not observed.

# Inflation

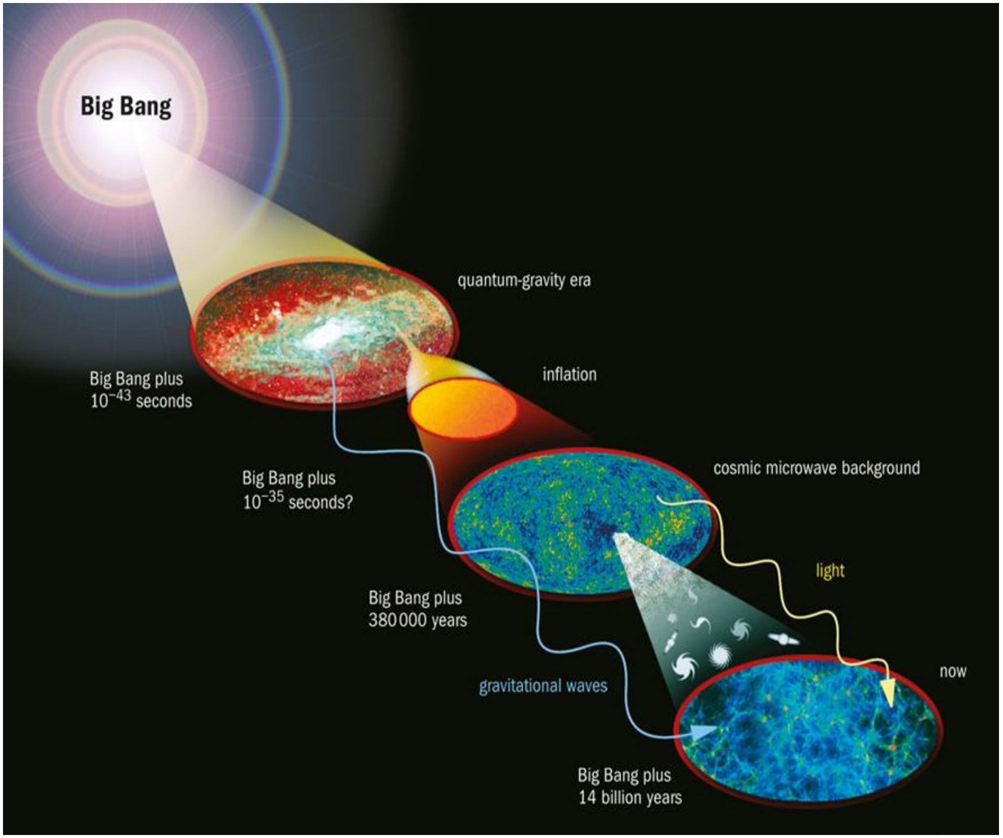
- Kazanas, Guth, Linde (1982): The Universe $\boxed{10^{-36}\,\text{sec}}$ after the Big Bang, through some mechanism went into an exponential expansion up to $\boxed{10^{-32}\,\text{sec}}$ increasing in size ~ $\boxed{10^{30}}$ times: Inflation.

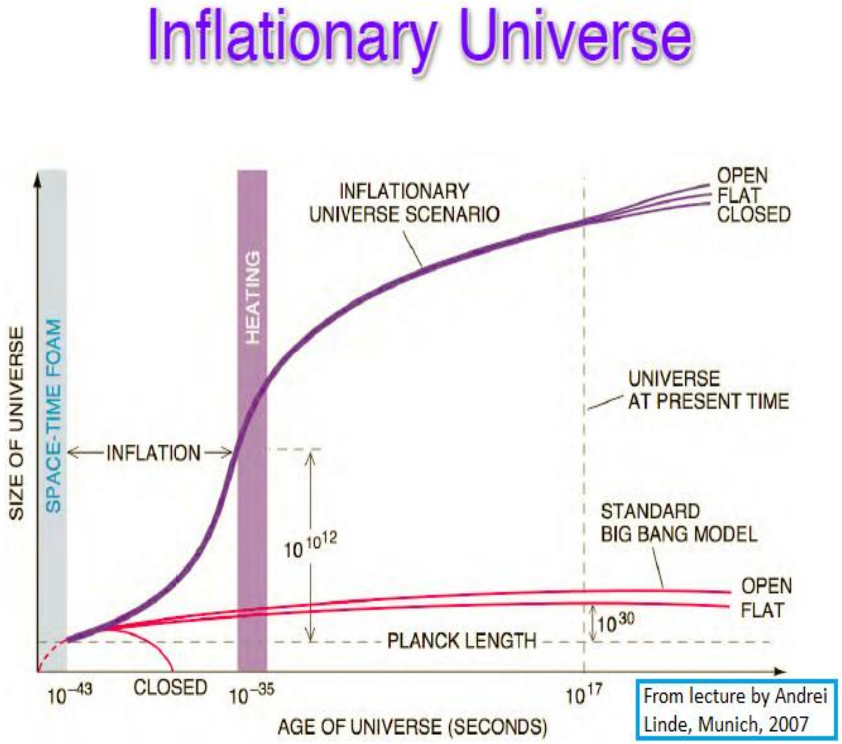
- I) The observable Universe is a tiny part of the total one, and originates from a small, causally connected region.

- II) Due to the huge expansion, the spatial curvature became almost zero.

- III) Due to the huge expansion the monopoles spread in all regions, and thus our own, observable universe, has at most one.

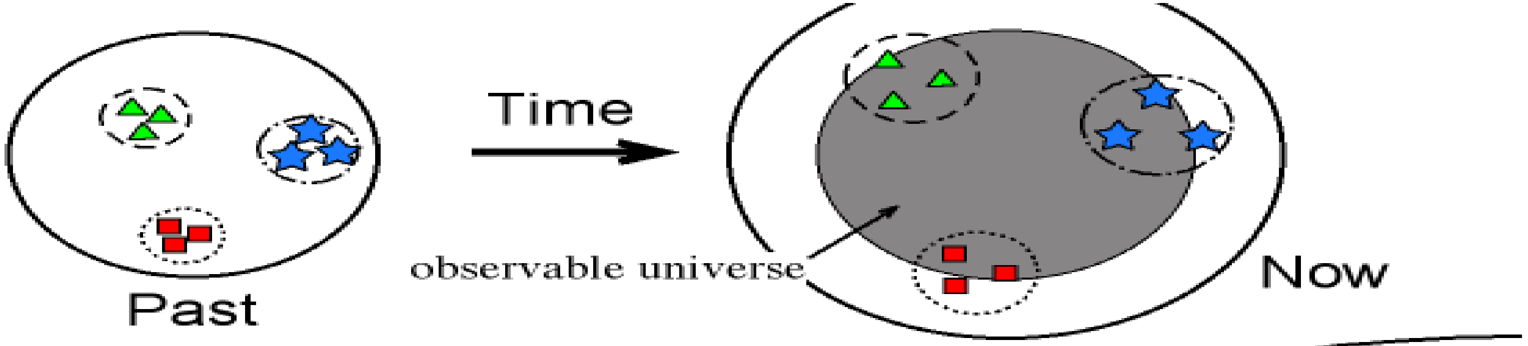
# Inflation



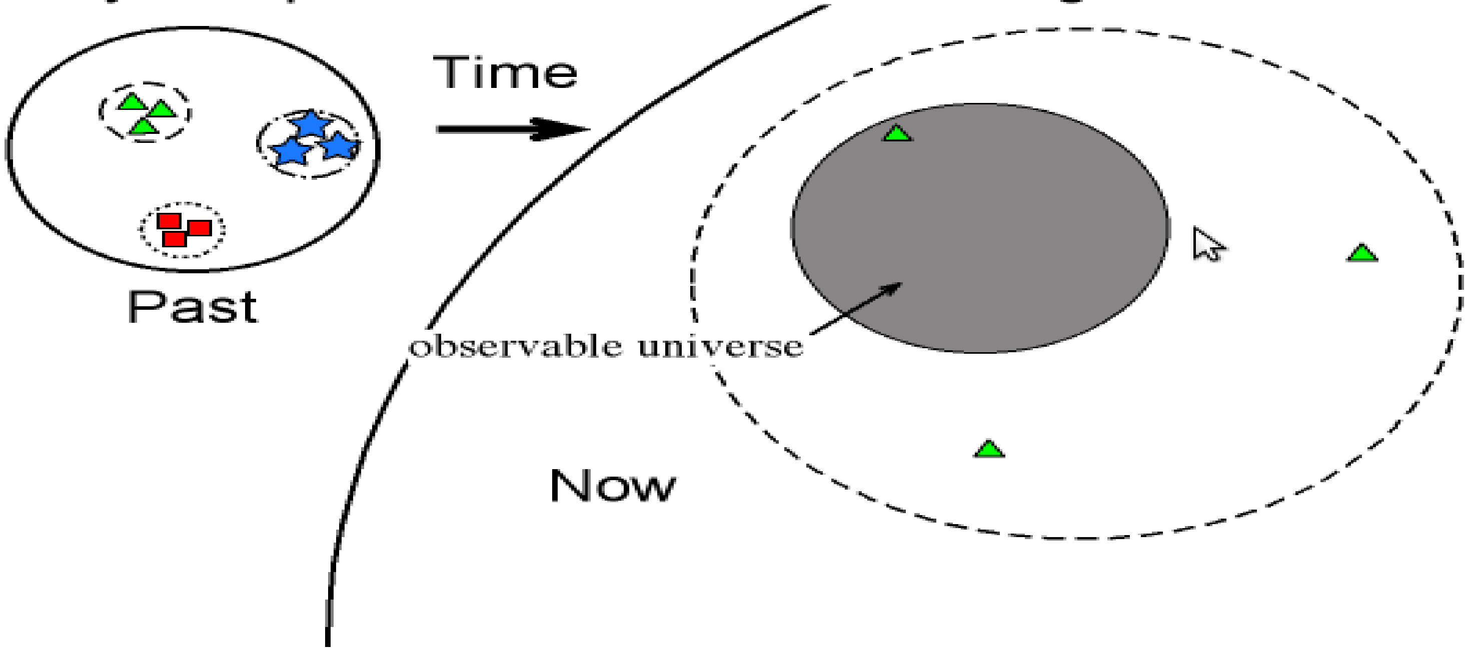


From lecture by Andrei Linde, Munich, 2007

NO inflation: observable universe (shaded) includes parts that are different from each other



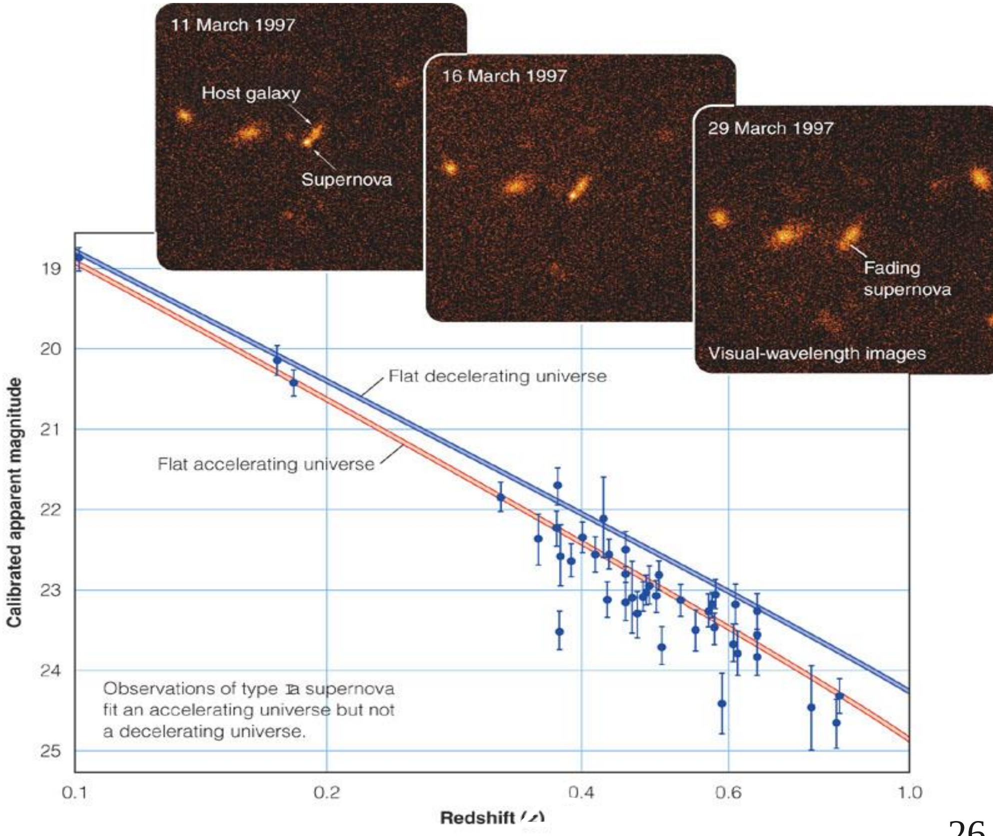
Inflation: observable universe (shaded) includes only one part that is the same throughout



# Dark Energy

- The Supernovae type Ia (explosions of binaries with one being white dwarf) are "standard candles", since their absolute magnitude M can be determined.

- In 1998 οι Perlmutter, Schmidt, Riess observed that 50 SnIa had smaller apparent magnitude than expected hence light traveled more, and thus the Universe today expands faster than before!



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# Dark Energy

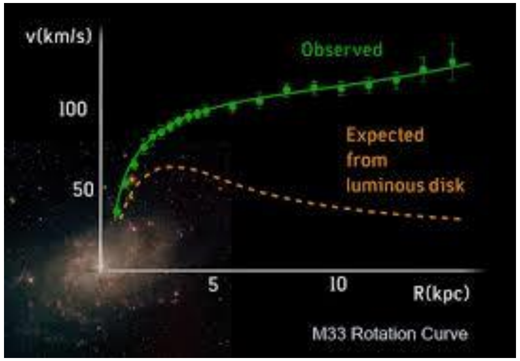
- The accelerated expansion is verified by independent observations, Cosmic Microwave Background (CMB), Baryon Acoustic Oscillations (BAO), Large Scale Structure (LSS), etc

- Around 70% of the total energy density of the Universe is this unknown dark energy (it does not interact electromagnetically).

- Possible explanation: The cosmological constant Λ (Einstein's "greatest blunder"). A term that produces the extra "repulsion".

# Dark Matter

- Galaxy rotation curves:



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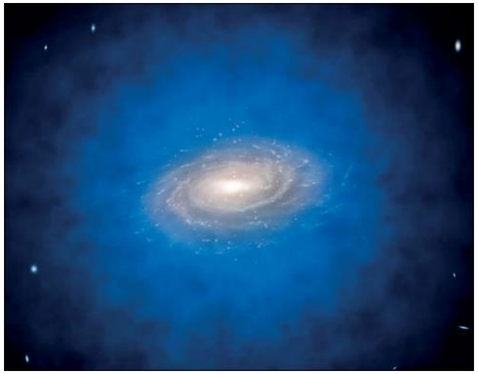
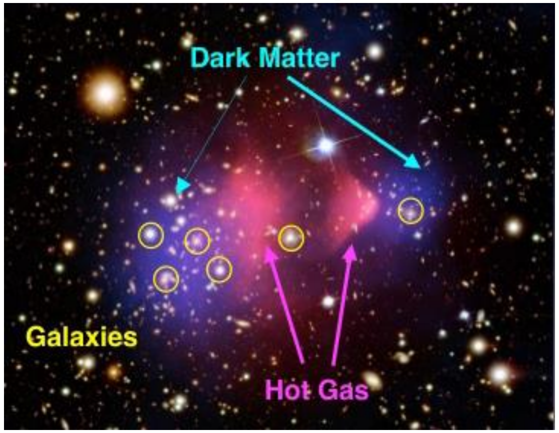


- Bullet cluster (collision of two galaxy clusters)



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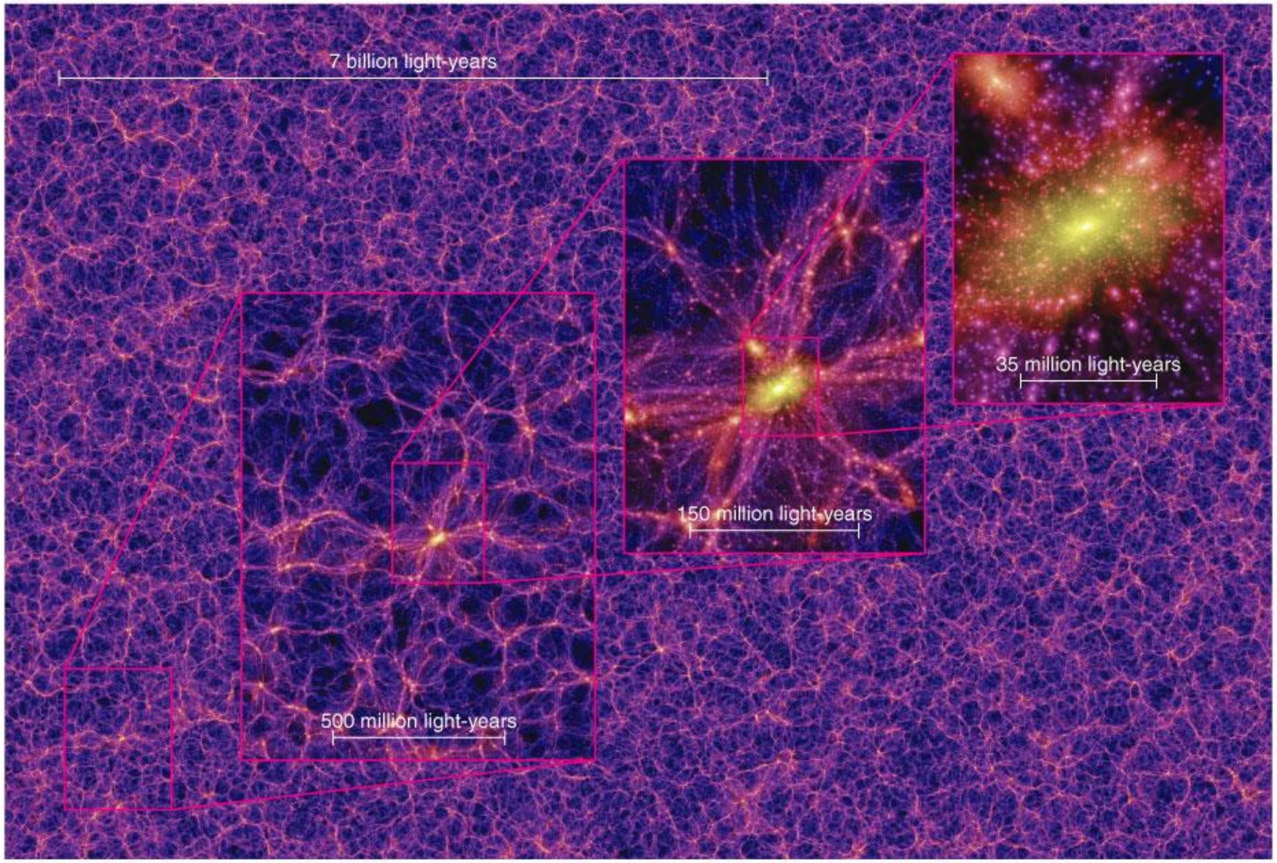
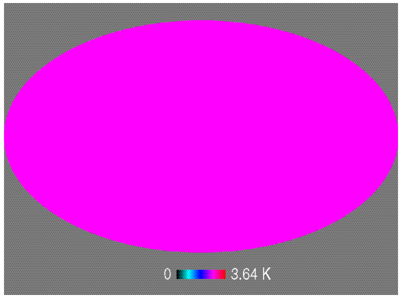
- 80% of matter is an "unknown" dark matter (it does not interact electromagnetically)!

# Dark Matter


7 billion light-years
500 million light-years
150 million light-years
35 million light-years

# Cosmic Microwave Background radiation

- Since 1989, COBE, WMAP και Planck satellites show that CMB has small fluctuations:



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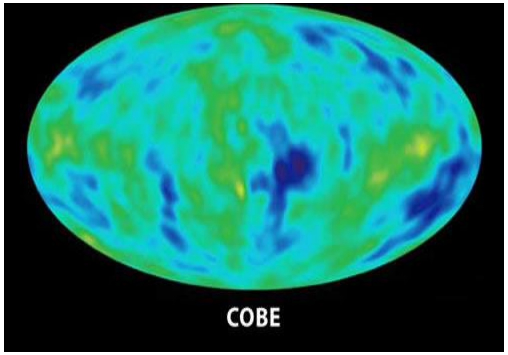

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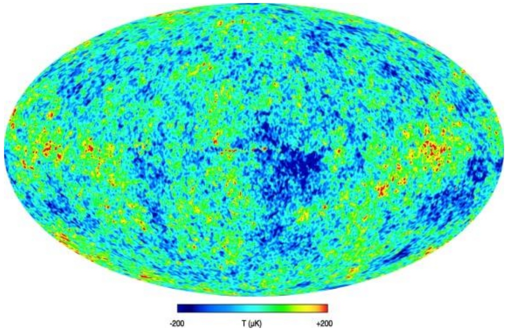

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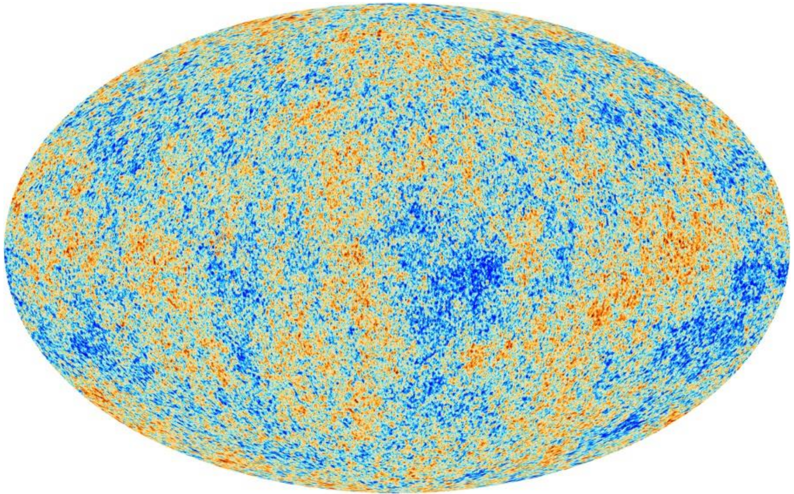
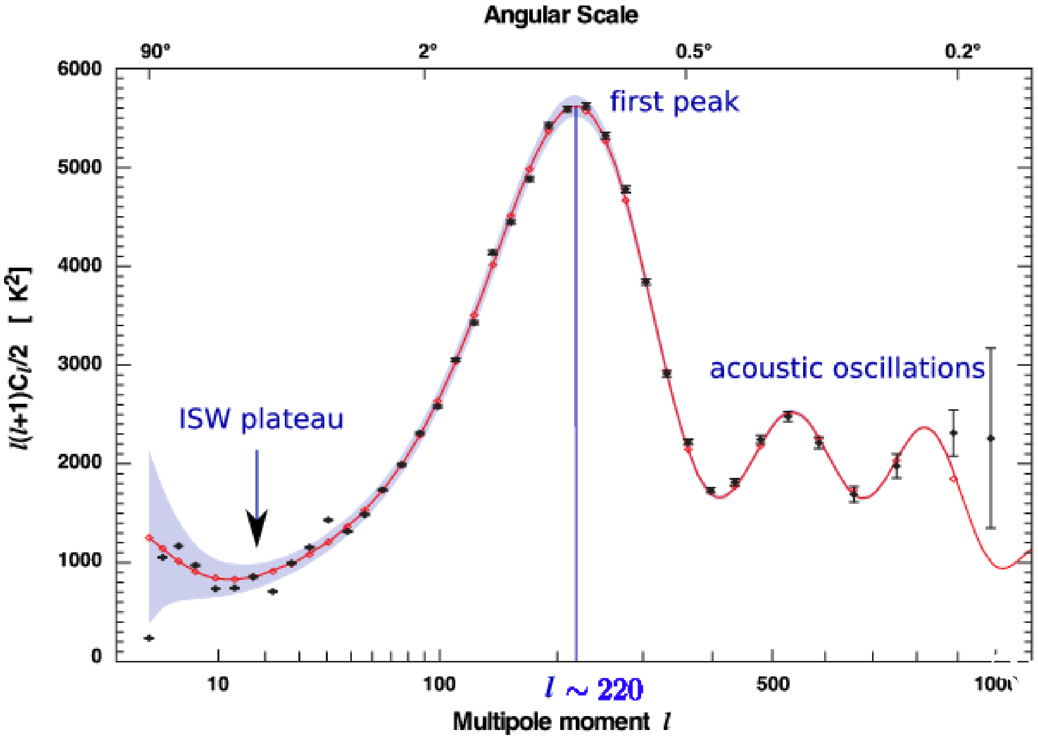


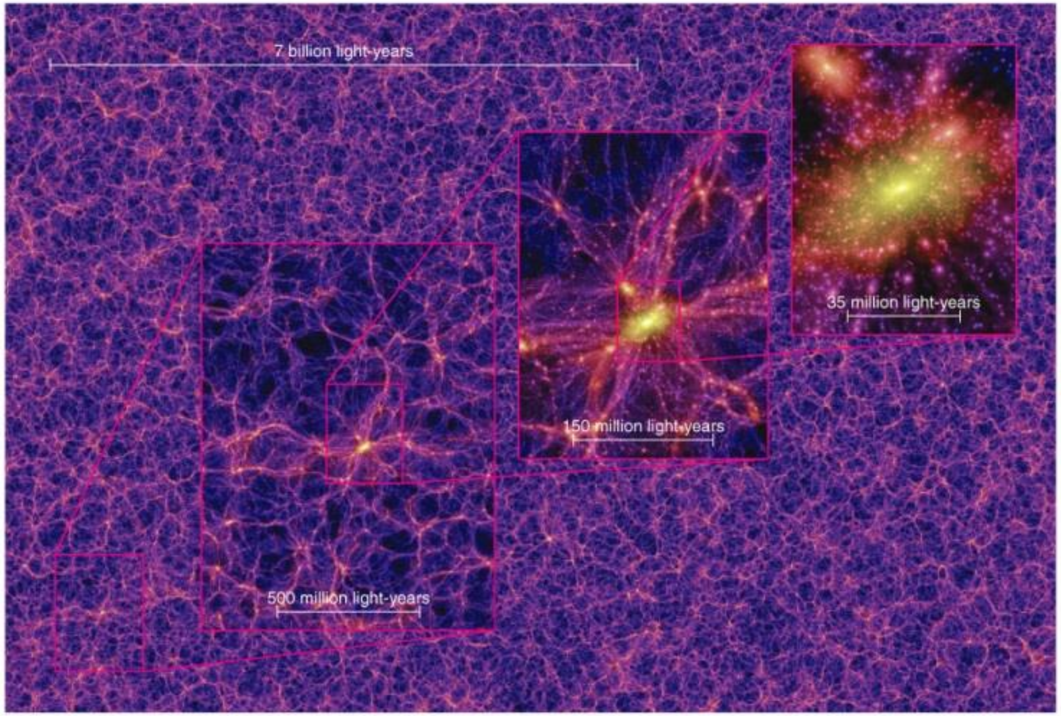
# Cosmic Microwave Background radiation

- From the fluctuation spectrum we extract information: The first peak provides the spatial curvature (it results to flat universe), the second peak the baryon energy density parameter, the third peak the dark matter energy density parameter, etc.



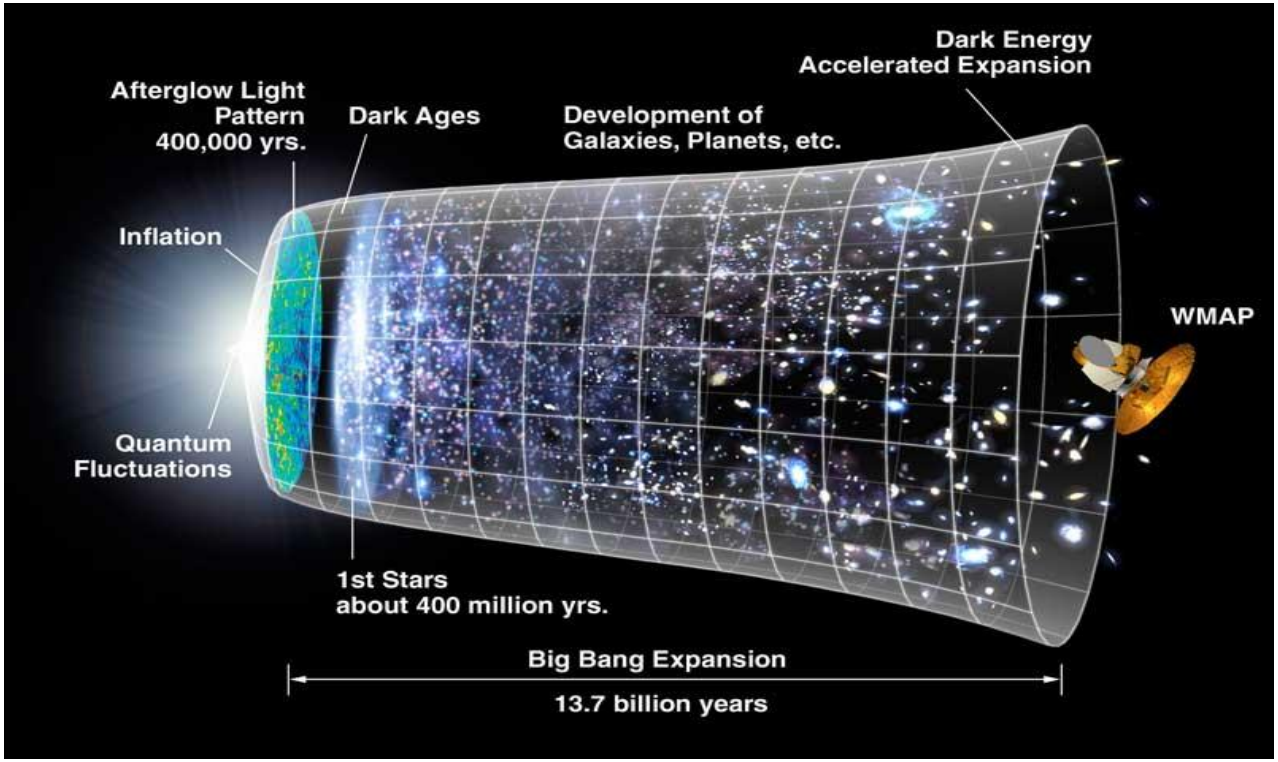
# Inflation can also explain CMB and seeds of LSS

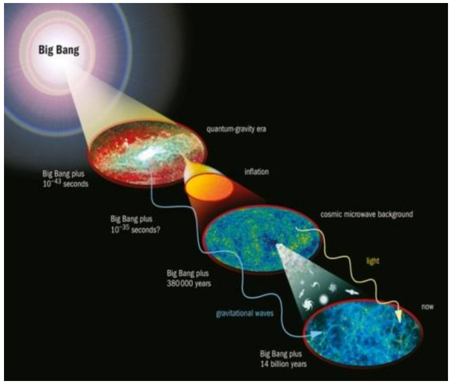
- Additional success: Inflation provides the necessary primordial fluctuations, which letter gave the Large Scale Structure of matter:

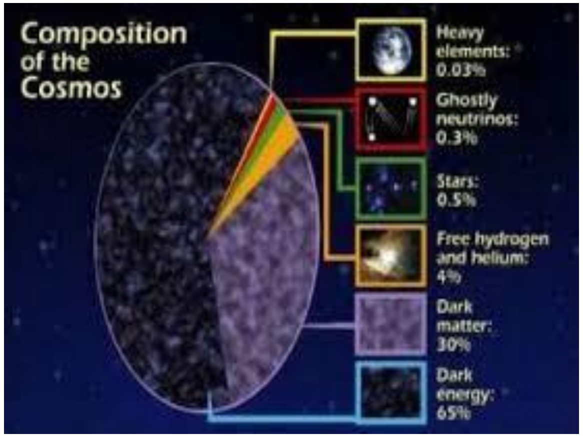


# Summary of Observations

The Universe history:

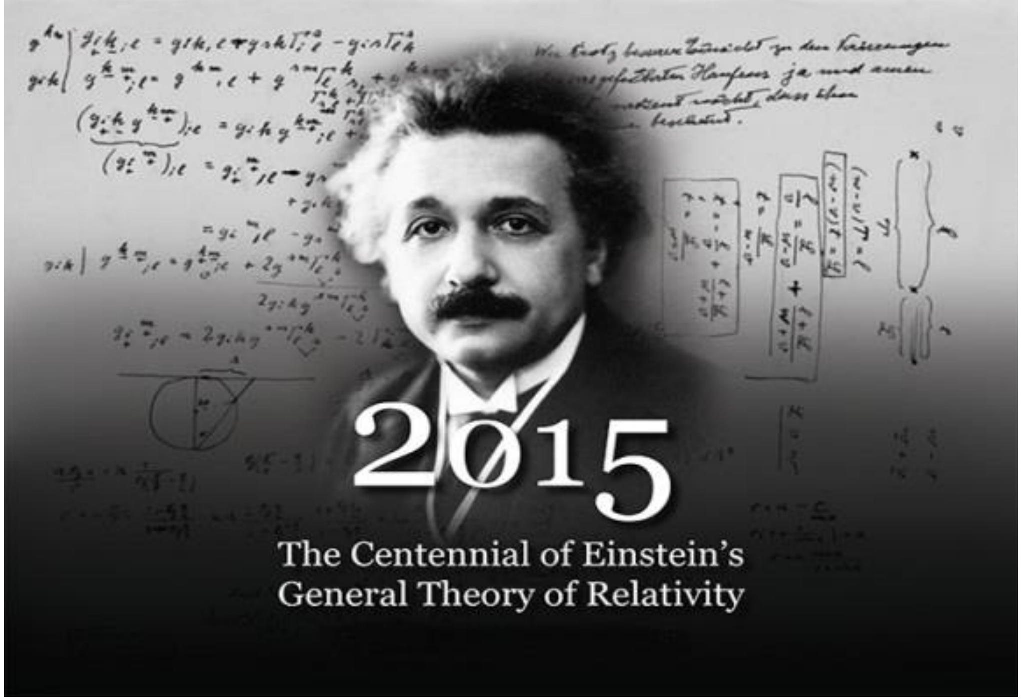






# How to describe the Expanding Universe?

- General Relativity: The evolution of the 4-dimensional spacetime is determined by the distribution of matter



# G.R. model of the physical world

| Physics | Geometry |
|---|---|
| • The **when** and the **where** of any physical physical phenomenon constitute an **event.**<br>• The set of all events is a continuous space, named **space-time**<br>• **Gravitational phenomena** are manifestations of the **geometry** of space—time<br>• **Point-like particles** move in space—time following special world-lines that are **“straight”**<br>• The **laws of physics** are the same for **all observers** | • An **event** is a **point** in a topological space<br>• **Space-time** is a **differentiable manifold** $M$<br>• **The gravitational field** is a **metric** **g** on $M$<br>• **Straight lines** are **geodesics**<br>• **Field equations** are **generally covariant** under diffeomorphisms |

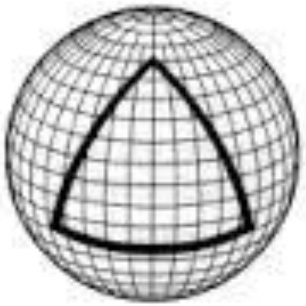
# Describing Expanding Universe

- Homogeneous and Isotropic (Friedman-Robertson-Walker metric):

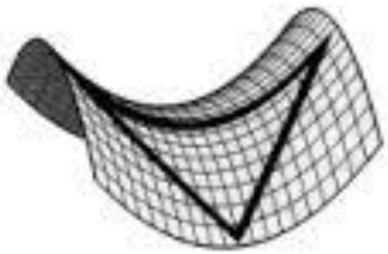
$$\boxed{ds^2 = -c^2 dt^2 + a(t)^2 \left( \frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2 \right)}$$

$a(t)$ : scale factor,

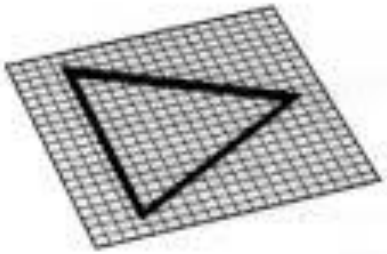
k=0,-1,+1 flat, closed open 3D spatial geometry



Positive Curvature



Negative Curvature



Flat Curvature

# Describing Expanding Universe

$$S = \int d^4x \sqrt{-g}\left[\frac{R}{16\pi G}\right] + S_m$$

- Field equations in FRW geometry (Friedmann Equations):

$$\boxed{H(t)^2 + \frac{k}{a(t)^2} = \frac{8\pi G}{3}\rho(t)}$$

$$\boxed{\dot{H}(t) - \frac{k}{a(t)^2} = -4\pi G\left[\rho(t) + p(t)\right]}$$

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)}$$

- Conservation Equation of matter perfect fluid

$$\boxed{\dot{\rho}(t) + 3H(t)\left[\rho(t) + p(t)\right] = 0}$$

# Describing Expanding Universe

- Equation of State: $\boxed{w \equiv \frac{p}{\rho}}$
- Evolution of the universe for a fluid with constant w, in flat space (k=0):

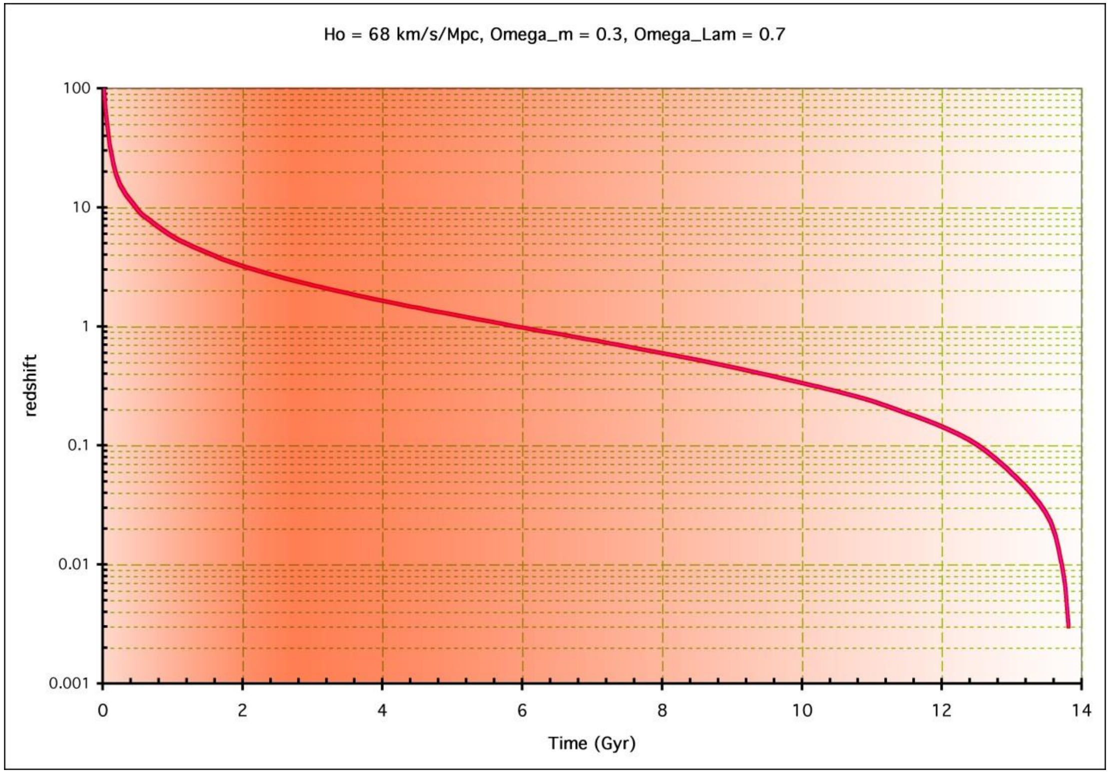
$$\dot{\rho}(t) + 3H(t)\left[\rho(t) + p(t)\right] = 0 \quad \boxed{\Rightarrow \ \rho(t) = \rho_0 a^{-3(1+w)}}$$

$$H(t)^2 = \frac{8\pi G}{3}\rho(t) \quad \boxed{\Rightarrow \ a(t) = \left[\frac{3(1+w)}{2}\sqrt{\frac{8\pi G\rho_0}{3}}\right]^{\frac{2}{3(1+w)}} t^{\frac{2}{3(1+w)}}}$$

- Matter Universe ($w_m = 0$): $\boxed{a(t) \propto t^{2/3}}$
- Radiation Universe ($w_r = 1/3$): $\boxed{a(t) \propto t^{1/2}}$

# Describing Expanding Universe

- We can use the redshift z as parameter of time evolution, up to recombination epoch z=1100:



$$\boxed{1+z \equiv \frac{a_0}{a}}$$

# Standard Model of Cosmology

# ΛCDM Paradigm + Inflation

$$\boxed{H(t)^2 + \frac{k}{a(t)^2} = \frac{8\pi G}{3}\left[\rho_{dm}(t) + \rho_b(t) + \rho_r(t)\right] + \frac{\Lambda}{3}}$$

$$\boxed{w_\Lambda \equiv \frac{p_\Lambda}{\rho_\Lambda} = -1}$$

$$\boxed{\dot{H}(t) - \frac{k}{a(t)^2} = -4\pi G\left[\rho_{dm}(t) + p_{dm}(t) + \rho_b(t) + p_b(t) + \rho_r(t) + p_r(t)\right]}$$

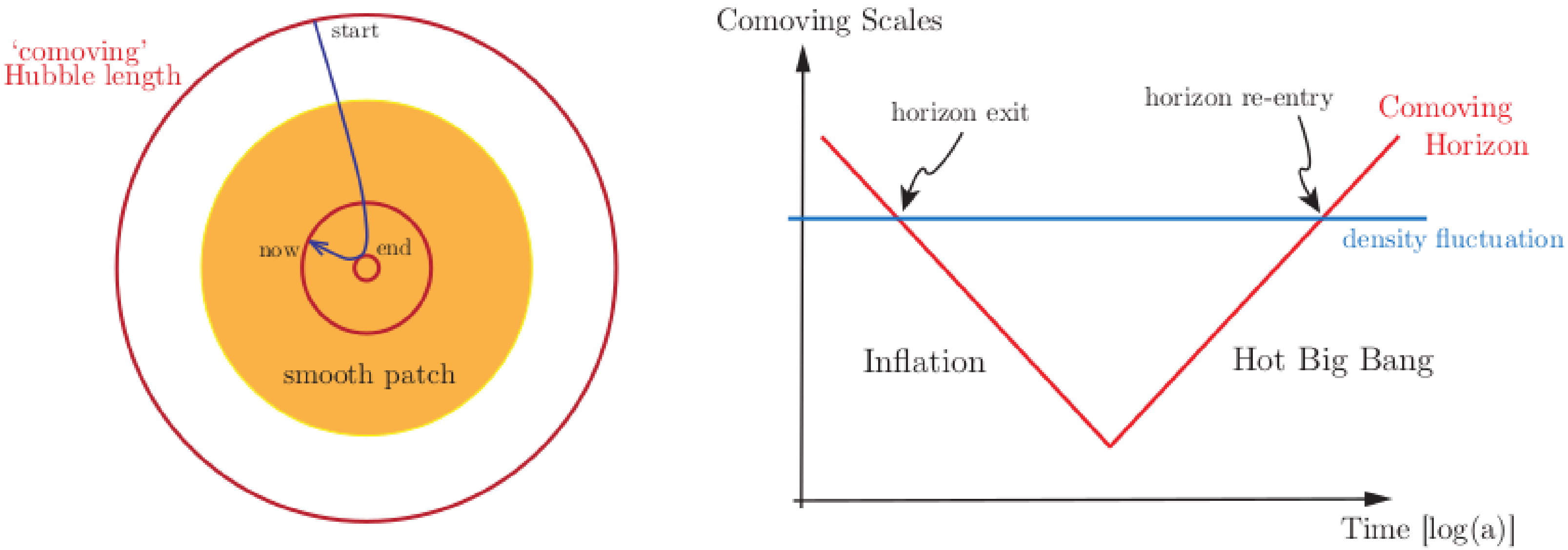
- Describes the thermal history of the Universe at the background level
- Epochs of inflation, radiation, matter, late-time acceleration

# ⇨ Synthesis

| | | | z | Age | T(K) | kT |
|---|---|---|---|---|---|---|
| Planck's Era | | | | | | |
| Radiation Era | GUT Epoch | Quantum-Gravity SB | $10^{32}$ | $5\times10^{-44}$s | $10^{32}$ | $10^{19}$ GeV |
| | | *gravitons* decoupling | | | | |
| | | Inflation | | | | |
| | Electroweak Epoch | GUT SB | $10^{26}$ | $10^{-34}$s | $10^{27}$ | $10^{15}$ GeV |
| | | Baryogenesis | | | | |
| | Quarks Epoch | Electroweak SB | $10^{14}$ | $10^{-10}$s | $10^{15}$ | 100 GeV |
| | | *quarks → hadrons* | $10^{12}$ | $10^{-5}$s | $10^{13}$ | 1 GeV |
| | Leptons Epoch | | | | | 150 MeV |
| | | $\nu$ decoupling | $10^{9}$ | 1s | $10^{11}$ | 1 MeV |
| | | $e^-/e^+$ annihilation | | | $10^{10}$ | 500 keV |
| | Plasma Epoch | P. Nucleosynthesis | $10^{8}$-$10^{9}$ | 100s | $10^{8}$-$10^{9}$ | 300 keV |
| | Matter-radiation equality | | 4000 | 10.000a | 62000 | 5.4 eV |
| Matter Era | Recombination | | 1400 | | 3800 | 0.33 eV |
| | $\gamma$ decoupling (CMBR) | | 1100 | 380.000a | 3000 | 0.26 eV |
| | Star and galaxy formation | | 10 | | | |
| | Reionization Epoch | | 6-15 | | | |
| $\Lambda$ Era | Accelerated Expansion Epoch | | 0.3 | | 3.6 | |
| | Now | | 0 | 13.7Ga | 2.725 | $2.35\times10^{-3}$eV |

## Horizon Problem Revisited

A decreasing comoving horizon means that large scales entering the present universe were inside the horizon before inflation Causal physics before inflation therefore established spatial homogeneity. With a period of inflation, the uniformity of the CMB is not a mystery.

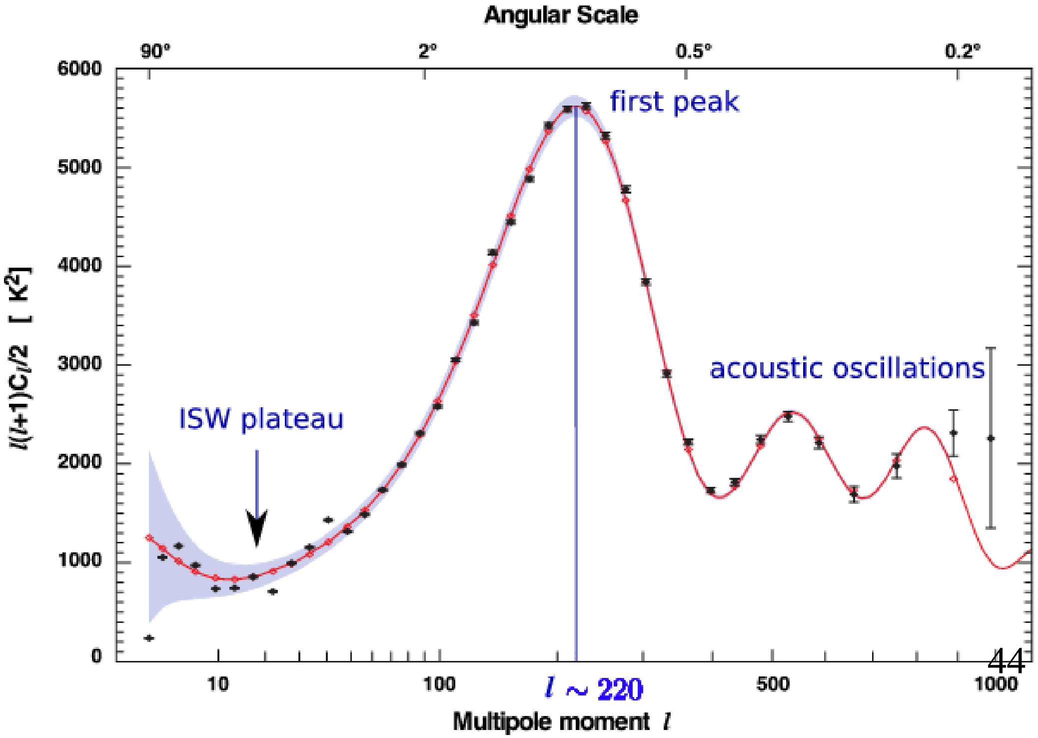


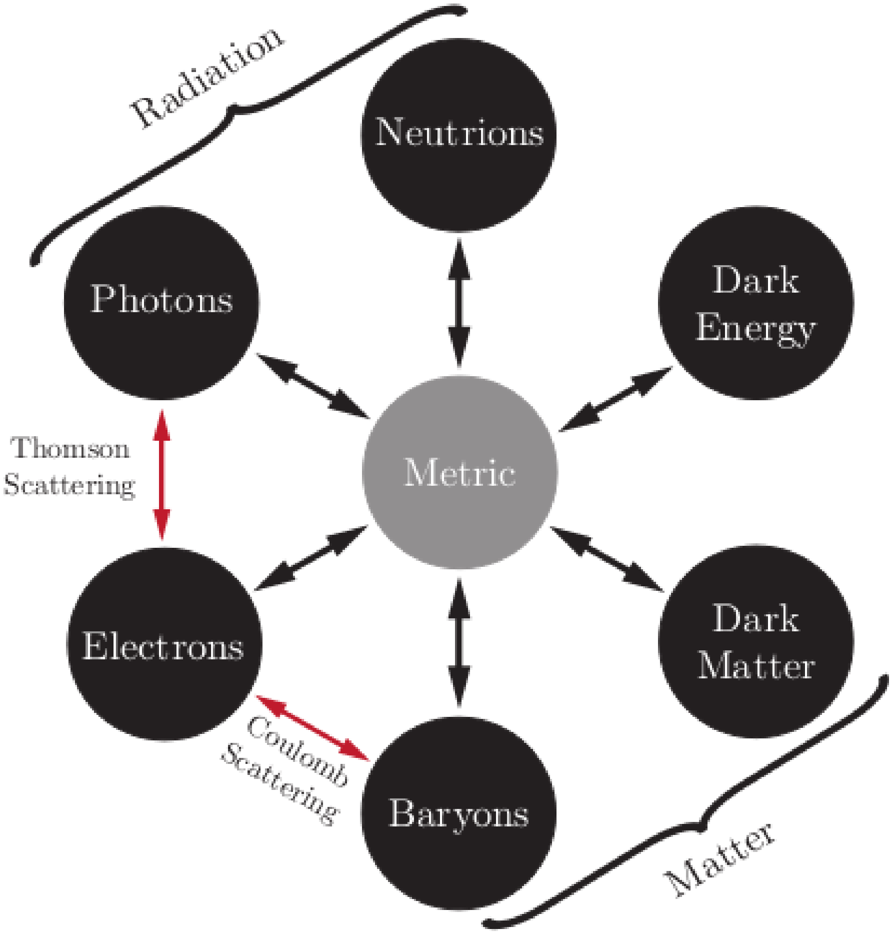
*Left:* Evolution of the comoving Hubble radius, $(aH)^{-1}$, in the inflationary universe. The comoving Hubble sphere shrinks during inflation and expands after inflation. Inflation is therefore a mechanism to 'zoom-in' on a smooth sub-horizon patch. *Right:* Solution of the horizon problem. All scales that are relevant to cosmological observations today were larger than the Hubble radius until $a \sim 10^{-5}$. However, at sufficiently early times, these scales were smaller than the Hubble radius and therefore causally connected. Similarly, the scales of cosmological interest came back within the Hubble radius at relatively recent times.

# COSMIC MICROWAVE BACKGROUND (CMB)

# Cosmic Microwave Background radiation

- From the fluctuation spectrum we extract information: The first peak provides the spatial curvature (it results to flat universe), the second peak the baryon energy density parameter, the third peak the dark matter energy density parameter, etc.



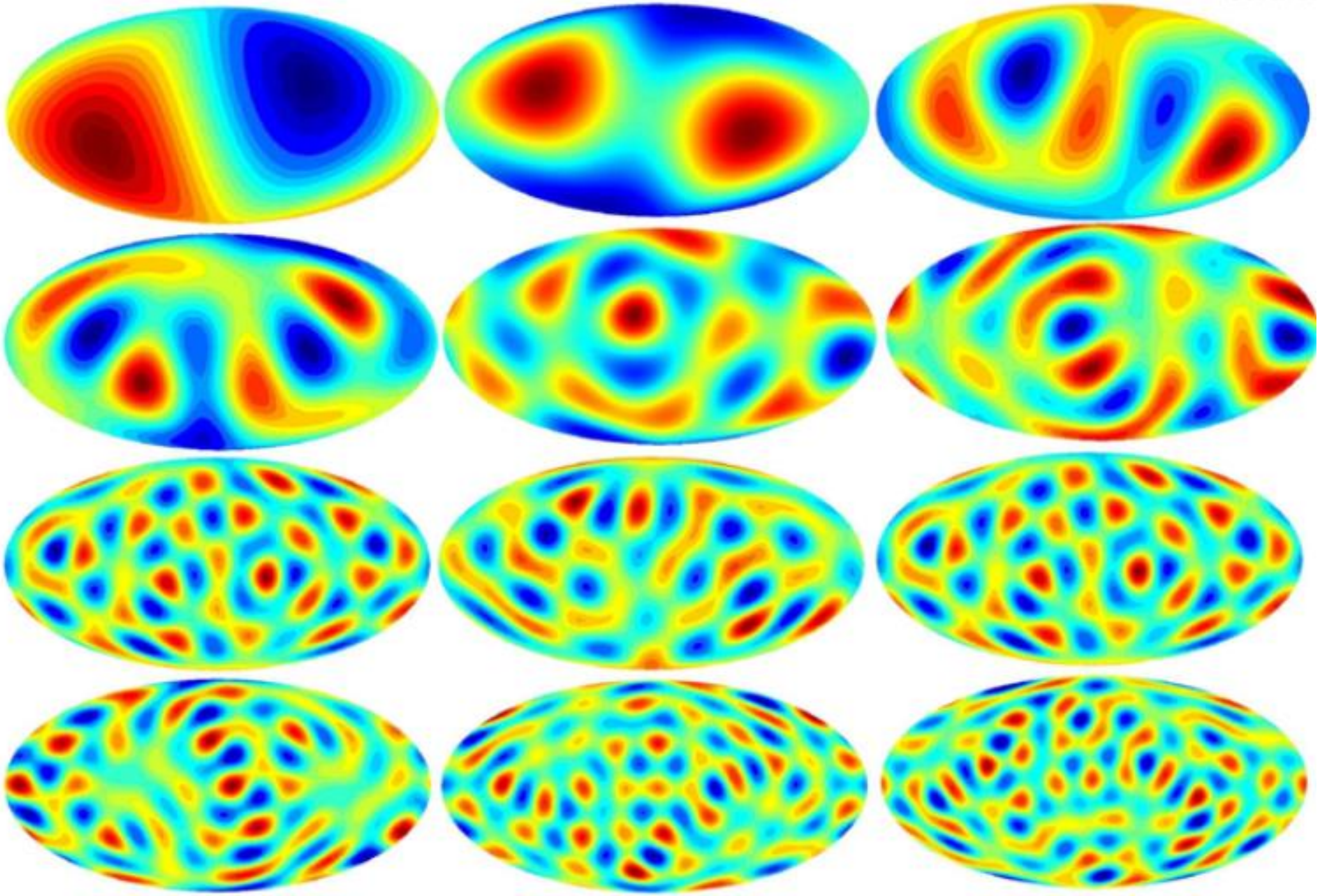


$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{d\mathbf{x}}{dt} \cdot \nabla_{\mathbf{x}} f + \frac{d\mathbf{p}}{dt} \cdot \nabla_{\mathbf{p}} f = \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + \mathbf{F} \cdot \nabla_{\mathbf{p}} f \equiv \hat{L}(f) \; , \quad (3.121)$$

where $\mathbf{v}$ is the particle velocity and $\mathbf{F}$ is the force acting on the particle. The operator $\hat{L}$ acting on $f$ is similar to the convective derivative used in fluid dynamics and is also called **Liouville operator**.

# ➢Multipole analysis

$l = 1{\sim}12.$



### Anisotropies from Inhomogeneities

We are interested in the temperature anisotropies observed today ($\eta_0$) at our location ($\mathbf{x}_0 \equiv \mathbf{0}$) as a function of the direction $\hat{\mathbf{n}}$ on the sky. Since a photon observed in the direction $\hat{\mathbf{n}}$ had to be travelling in the direction $\hat{\mathbf{p}} = -\hat{\mathbf{n}}$, we have

$$\tilde{\Theta}(\hat{\mathbf{n}}) \equiv \frac{\delta T}{T}(\hat{\mathbf{n}}) = \Theta(\eta_0, \mathbf{x}_0, \hat{\mathbf{p}} = -\hat{\mathbf{n}})\,. \tag{5.2}$$

This may also be written as

$$\begin{aligned}
\tilde{\Theta}(\hat{\mathbf{n}}) &= \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^{3/2}}\, e^{i\mathbf{k}\cdot\mathbf{x}_0}\, \Theta(\eta_0, \mathbf{k}, \hat{\mathbf{n}}) \\
&= \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^{3/2}}\, e^{i\mathbf{k}\cdot\mathbf{x}_0} \sum_l (-i)^l \Theta_l(\eta_0, \mathbf{k}) P_l(\hat{\mathbf{k}}\cdot\hat{\mathbf{n}}) \\
&= \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^{3/2}}\, e^{i\mathbf{k}\cdot\mathbf{x}_0} \sum_l (-i)^l \Theta_l(k) \mathcal{R}(\mathbf{k}) P_l(\hat{\mathbf{k}}\cdot\hat{\mathbf{n}})\,, \qquad (5.3)
\end{aligned}$$

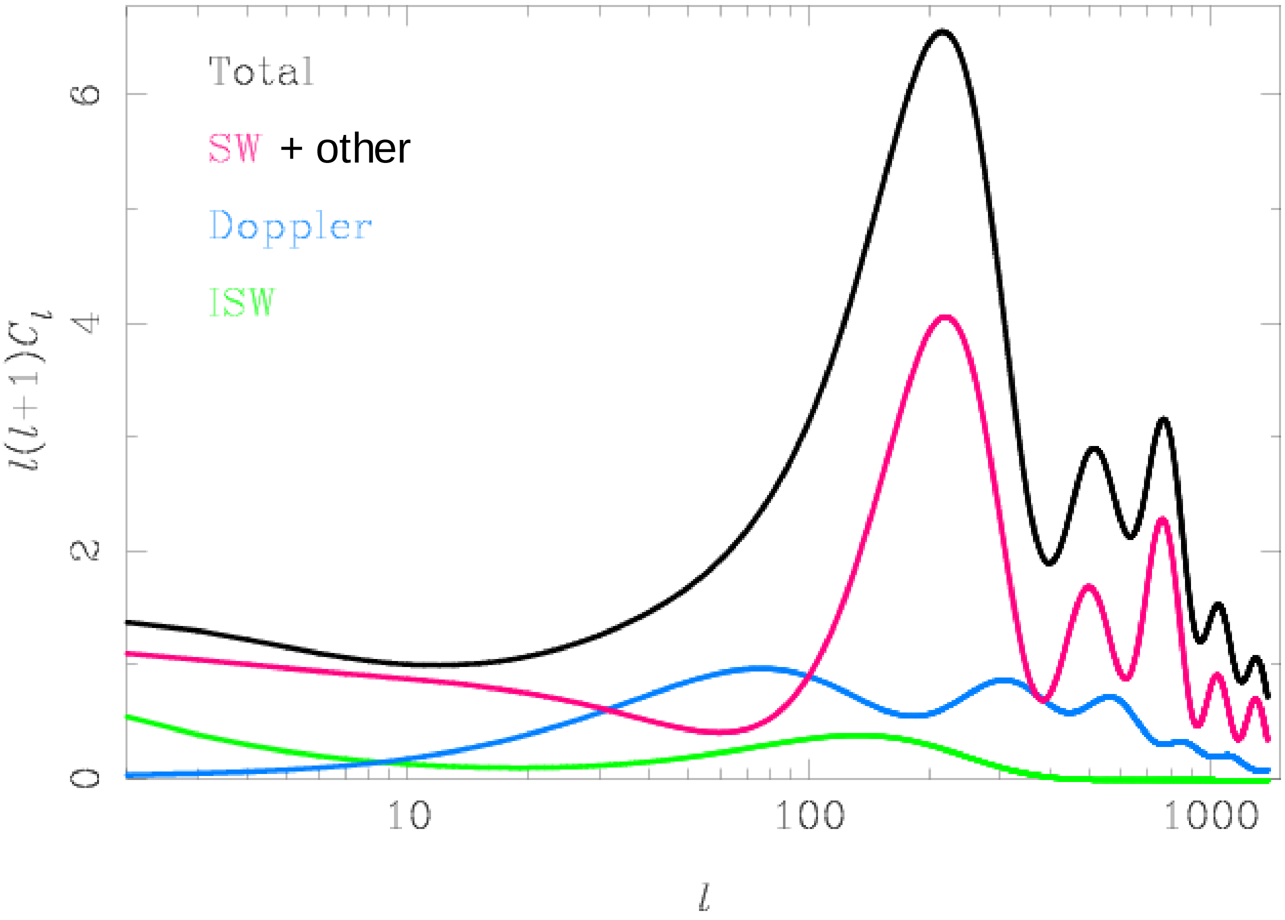
where we introduced the Fourier components of the inhomogeneous temperature field in the first line, expanded them into multipole moments in the second line, and introduced a *transfer function* for the linear evolution in the third line:

$$\boxed{\Theta_l(k) \equiv \frac{\Theta_l(\eta_0, \mathbf{k})}{\mathcal{R}(\mathbf{k})}}\,. \qquad (5.4)$$

The transfer function $\Theta_l(k)$ provides the map from the initial power spectrum of curvature perturbations to the angular power spectrum of temperature anisotropies:

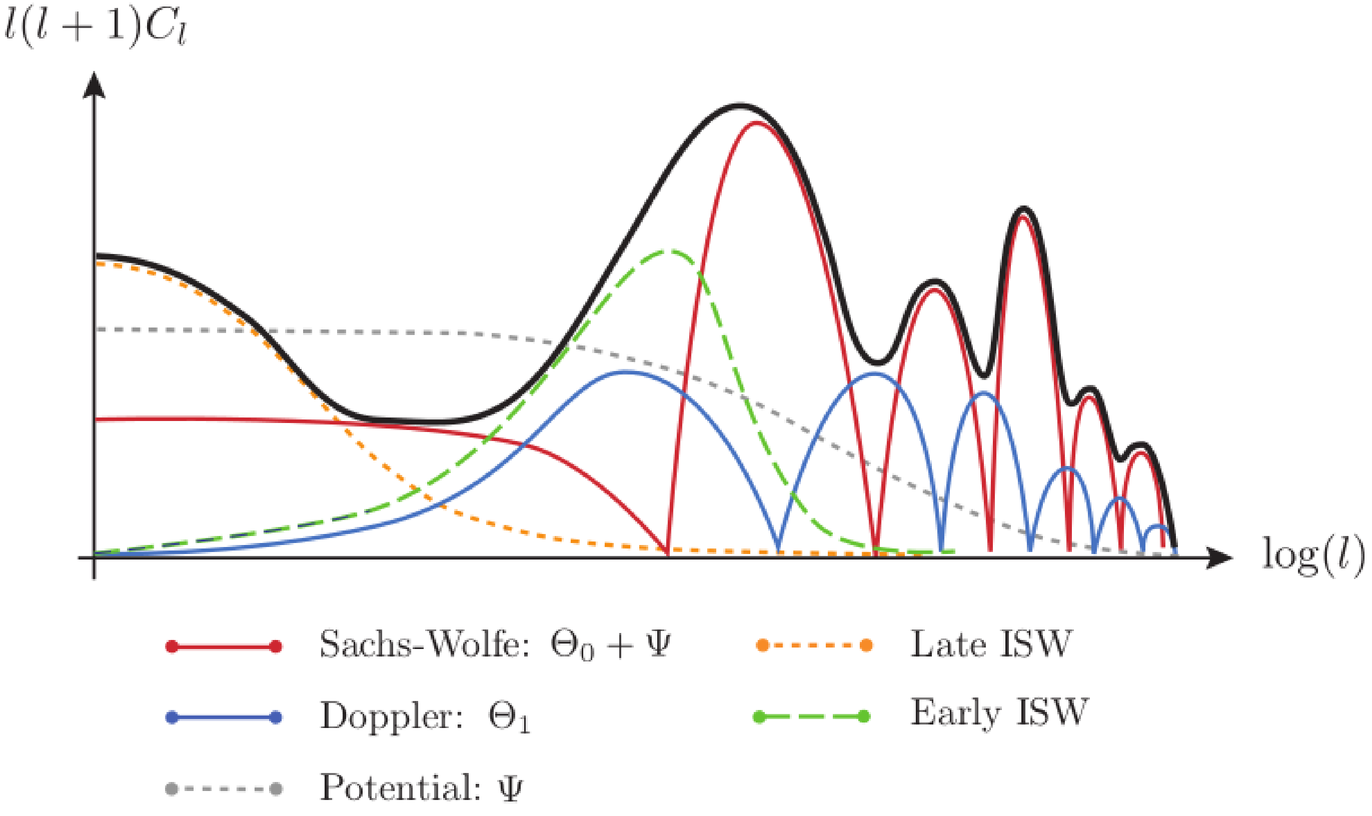
$$\boxed{C_l = \frac{4\pi}{(2l+1)^2} \int \mathrm{d}\ln k\ \Theta_l^2(k)\, \Delta_{\mathcal{R}}^2(k)}\,. \qquad (5.5)$$

## Contributions to temperature $C_l$



### Power Spectrum

Figure 25 shows a sketch of the different contributions to the CMB power spectrum. The shape of the Sachs-Wolfe and Doppler contributions can be understood from the analytical treatment of the previous section. Note the the velocity $v_\gamma \approx v_b$ vanishes outside the sound horizon and that the Doppler effect is therefore suppressed on large scales. The Sachs-Wolfe transfer function is a constant on large scales and the plateau in $l(l+1)C_l$ for small $l$ is therefore a direct reflection of the scale-invariant initial conditions. The late ISW effect leads to a small rise of the plateau. This is a measure of dark energy. The early ISW adds extra power near the first peak. Finally, diffusion damping suppresses all contributions to the power spectrum at large $l$.



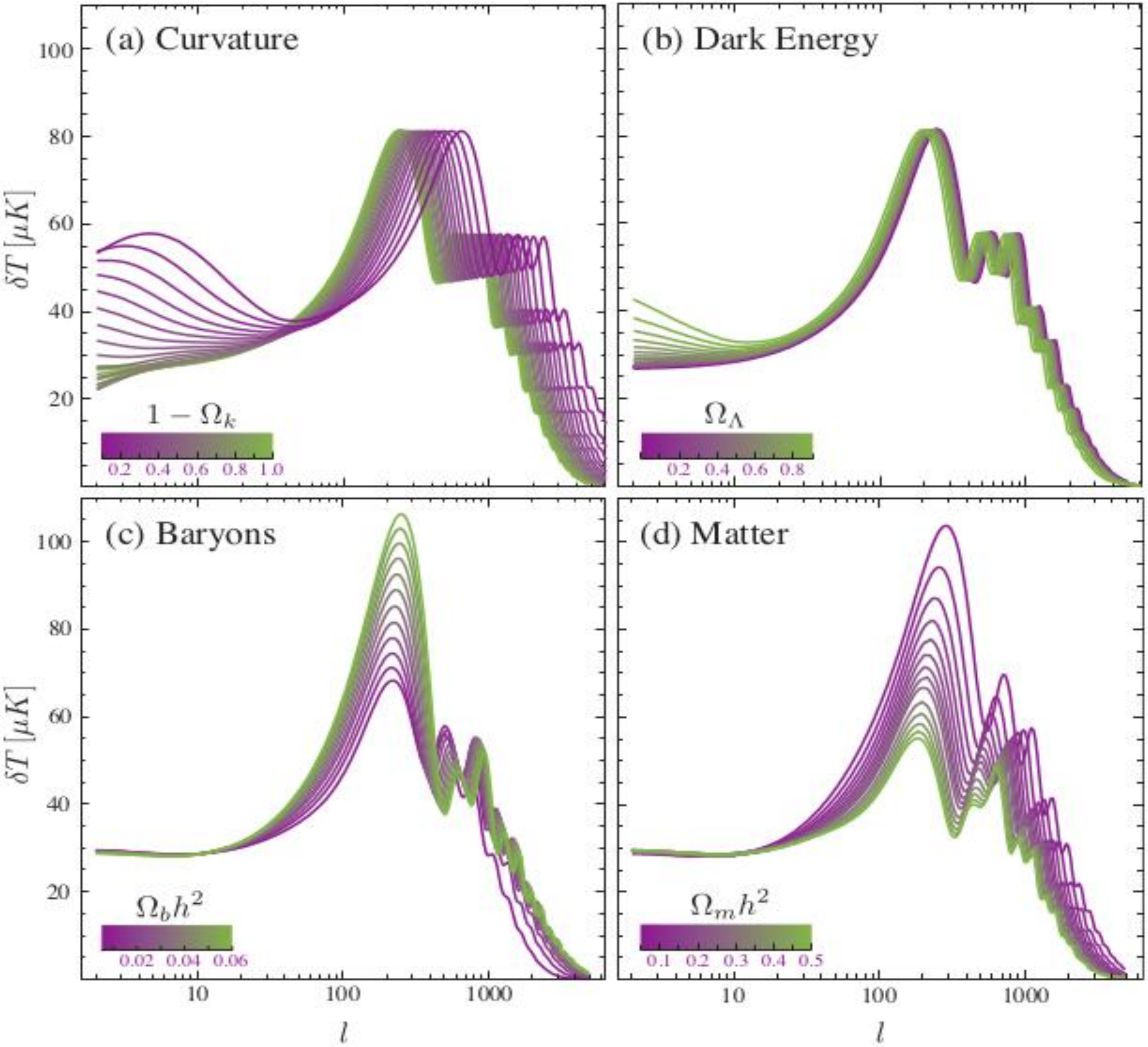


**Figure 26**. The CMB power spectrum as a function of cosmological parameters

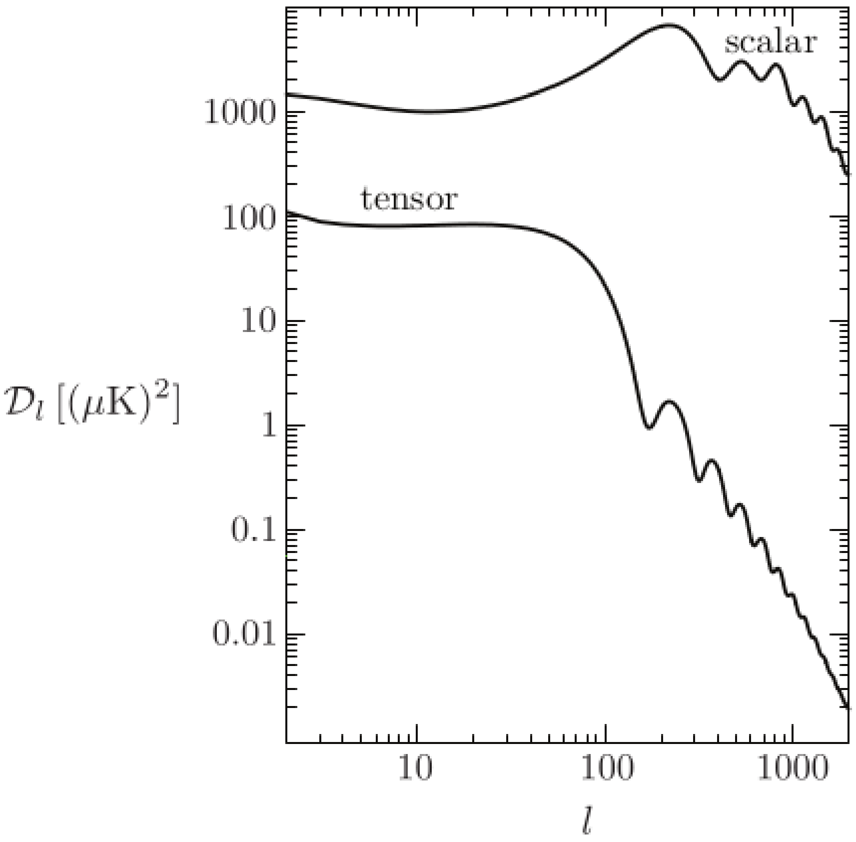


**Figure 27**. Comparison of the power spectra of CMB temperature anisotropies created by scalar and tensor perturbations.

The scalar quantities $E$ and $B$ completely specify the linear polarization field. $E$-mode polarization is often also characterized as a *curl-free* mode with polarization vectors that are radial around cold spots and tangential around hot spots on the sky. In contrast, $B$-mode polarization is *divergence-free* but has a *curl*: its polarization vectors have vorticity around any given point on the sky. Fig. 21 gives examples of $E$- and $B$-mode patterns. Although $E$ and $B$ are both invariant under rotations, they behave differently under parity transformations. Note that when reflected about a line going through the center, the $E$-mode patterns remain unchanged, while the $B$-moe patterns change sign.

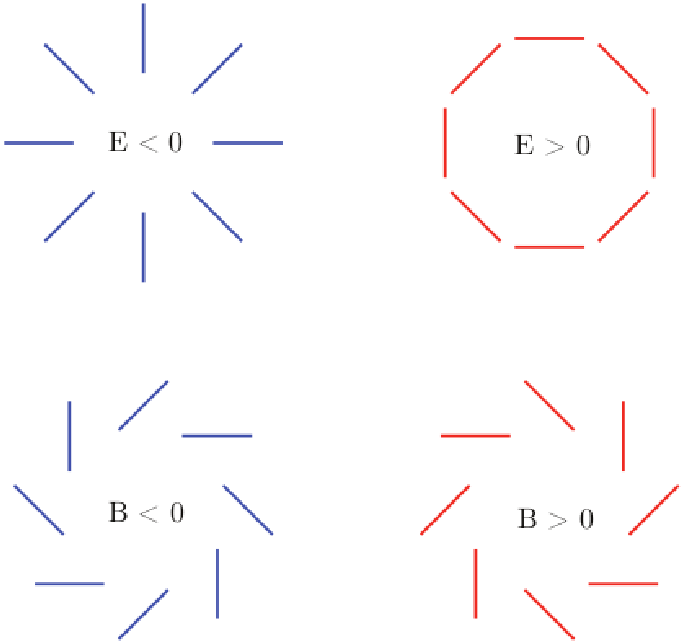


**Figure 21:** Examples of $E$-mode and $B$-mode patterns of polarization. Note that if reflected across a line going through the center the $E$-mode patterns are unchanged, while the positive and negative $B$-mode patterns get interchanged.

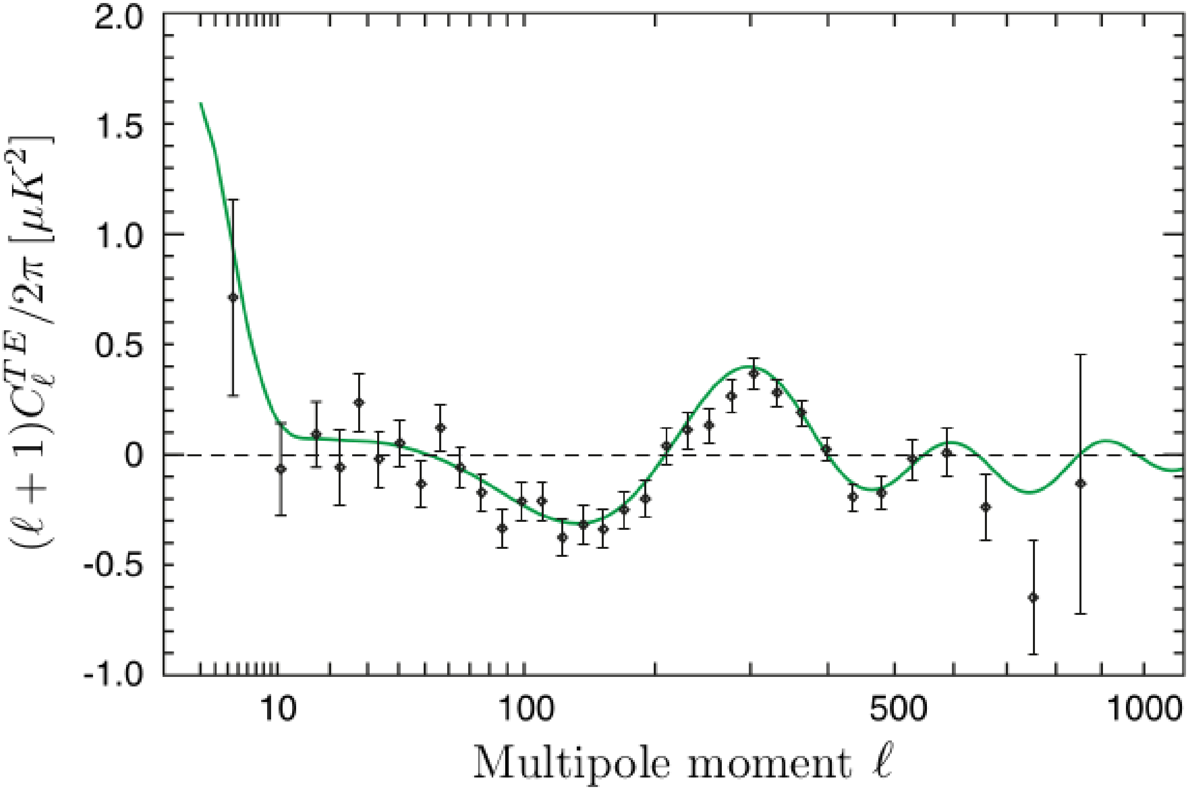
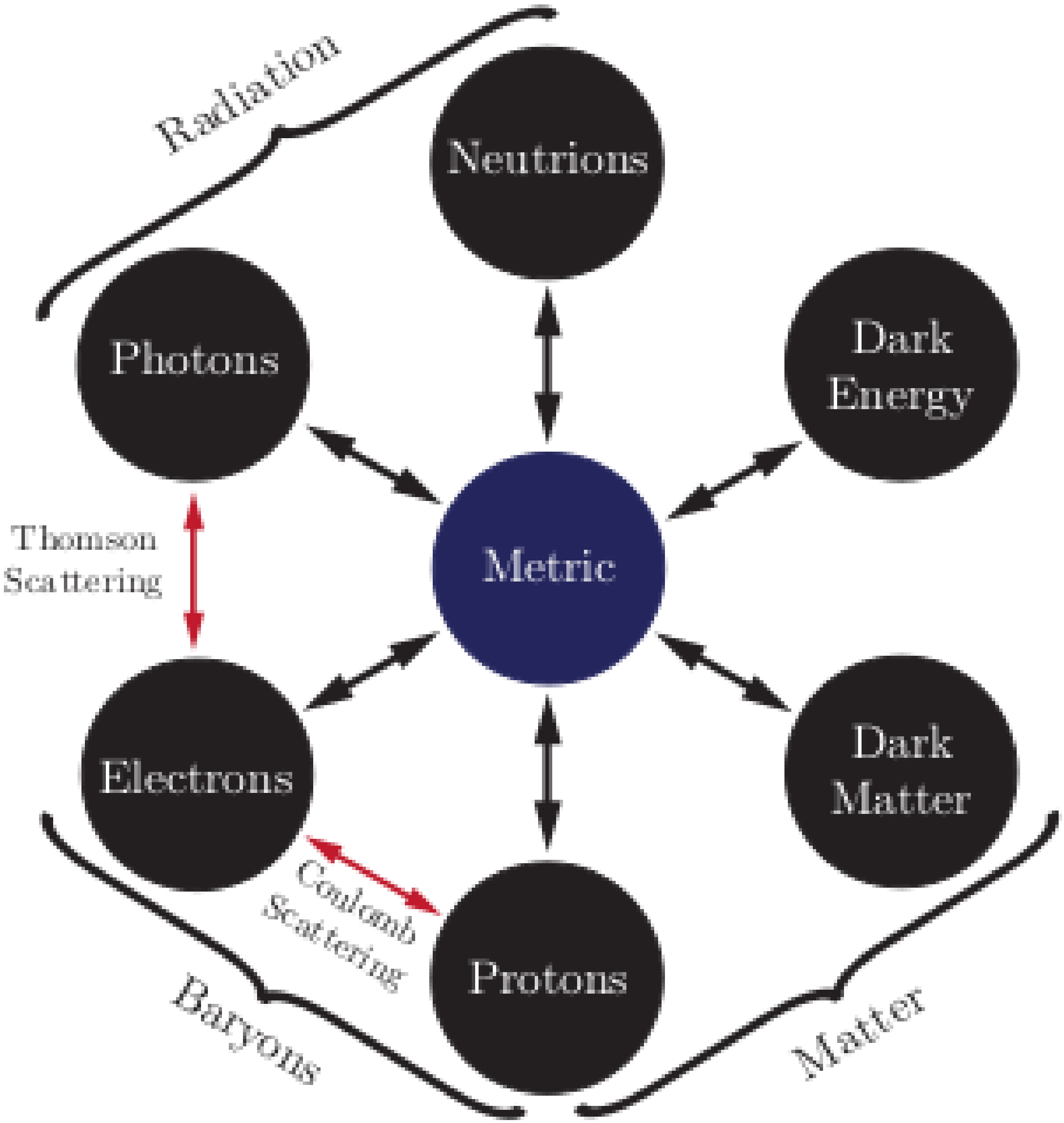


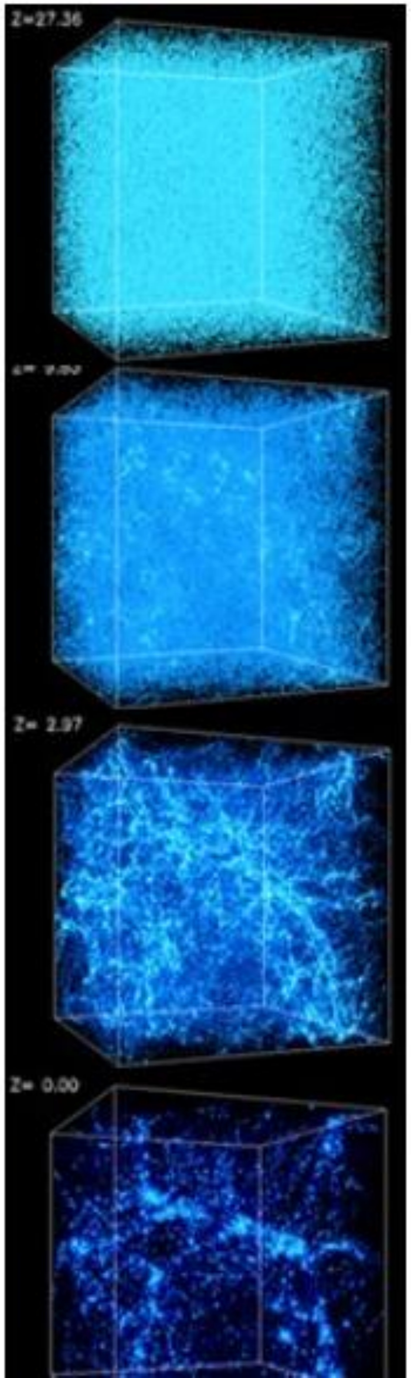
**Figure 22:** Power spectrum of the cross-correlation between temperature and $E$-mode polarization anisotropies . The anti-correlation for $\ell = 50 - 200$ (corresponding to angular separations $5^\circ > \theta > 1^\circ$) is a distinctive signature of adiabatic fluctuations on superhorizon scales at the epoch of decoupling , confirming a fundamental prediction of the inflationary paradigm.

In Fig. 22 we show the latest measurement of the $TE$ cross-correlation . The $EE$ spectrum has now begun to be measured, but the errors are still large. So far there are only upper limits on the $BB$ spectrum, but no detection.

# LARGE SCALE STRUCTURE


Radiation
Neutrinos
Photons
Dark Energy
Metric
Thomson Scattering
Electrons
Dark Matter
Coulomb Scattering
Protons
Baryons
Matter

# Evolution of the LSS – a brief history



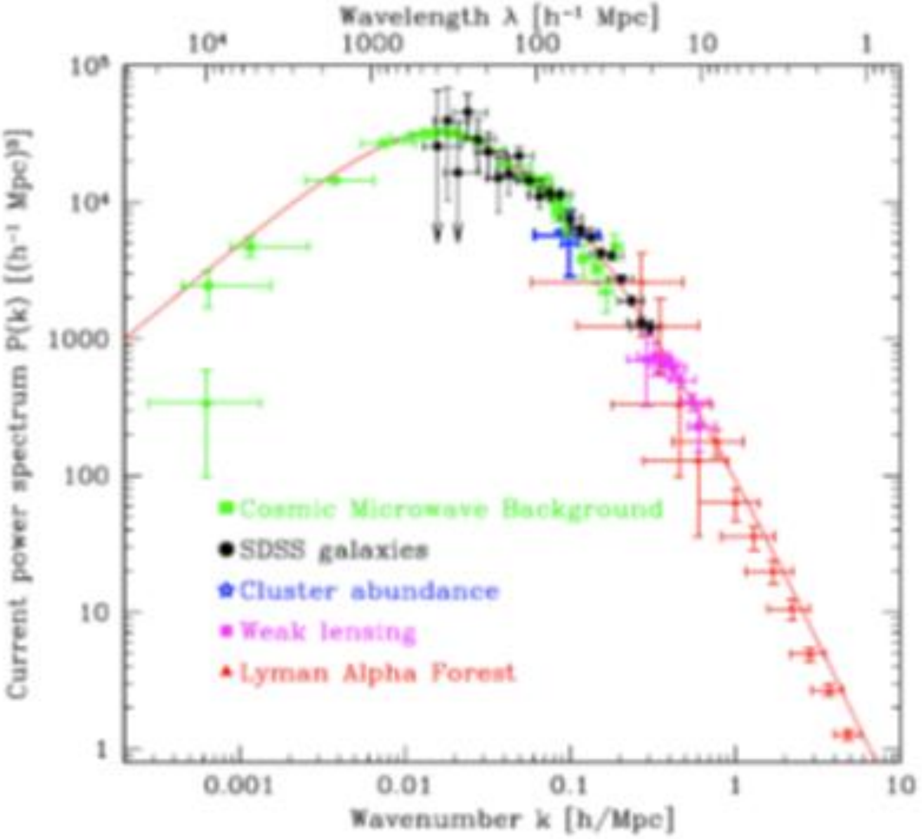
Somewhat after recombination --
density perturbations are small on nearly all spatial scales.

Dark Ages, prior to z=10 --
density perturbations in dark matter and baryons grow;
on smaller scales perturbations have gone non-linear, $\delta >> 1$;
small (low mass) dark matter halos form; massive stars
form in their potential wells and reionize the Universe.

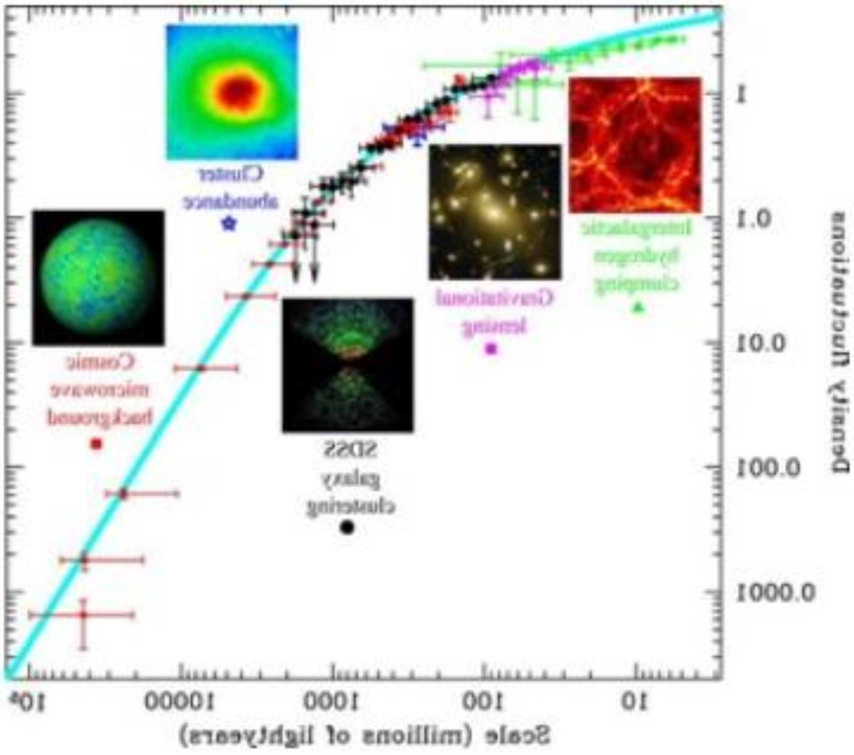
z=2 --
Most galaxies have formed; they are bright with stars;
this is also the epoch of highest quasar activity;
galaxy clusters are forming. In LCDM growth of structure
on large (linear) scales has nearly stopped, but smaller
non-linear scales continue to evolve.

z=0 --
Small galaxies continue to get merged to form larger ones;
in an open and lambda universes large scale (>10-100Mpc)
potential wells/hill are decaying, giving rise to late ISW.

# Matter Density Fluctuation Power Spectrum



A different convention:
plot $P(k)k^3$



- From the conservation of the stress-tensor, we derived the relativistic generalisations of the continuity equation and the Euler equation

$$\begin{aligned}\delta' + 3\mathcal{H}\left(\frac{\delta P}{\delta\rho} - \frac{\bar{P}}{\bar{\rho}}\right)\delta &= -\left(1+\frac{\bar{P}}{\bar{\rho}}\right)\left(\boldsymbol{\nabla}\cdot\boldsymbol{v} - 3\Phi'\right)\ , \quad (4.4.173)\\ \boldsymbol{v}' + 3\mathcal{H}\left(\frac{1}{3} - \frac{\bar{P}'}{\bar{\rho}'}\right)\boldsymbol{v} &= -\frac{\boldsymbol{\nabla}\delta P}{\bar{\rho}+\bar{P}} - \boldsymbol{\nabla}\Phi\ . \quad (4.4.174)\end{aligned}$$

These equations apply for the total matter and velocity, and also separately for any non-interacting components so that the individual stress-energy tensors are separately conserved.

- A very important quantity is the comoving curvature perturbation

$$\mathcal{R} = -\Phi - \frac{\mathcal{H}(\Phi' + \mathcal{H}\Phi)}{4\pi G a^2(\bar{\rho}+\bar{P})}\ . \quad (4.4.175)$$

We have shown that $\mathcal{R}$ doesn't evolve on super-Hubble scales, $k \ll \mathcal{H}$, unless non-adiabatic pressure is significant.

# SVT decomposition

A very important result of General Relativity is the *Scalar, Vector and Tensor decomposition theorem*: each type of metric perturbation evolves *independently* at *linear order*.

Putting each type of perturbation together we get:

$$\begin{aligned} S : ds^2 &= -(1+2\Phi)dt^2 + 2\alpha(t)B_{,i}dx^i dt + \alpha^2(t)[(1-2\Psi)\delta_{ij} + 2E_{,ij}]dx^i dx^j \\ V : ds^2 &= -dt^2 + 2\alpha(t)B_i dx^i dt + \alpha^2(t)[\delta_{ij} + 2V_{(i,j)}]dx^i dx^j \\ T : ds^2 &= -dt^2 + \alpha^2(t)[\delta_{ij} + h_{ij}^{TT}]dx^i dx^j \qquad (15) \end{aligned}$$

Note: We can perform a similar decomposition for $T_{\mu\nu}$ , which is also a symmetric and rank 2 tensor.

# Coordinate systems- Gauges

- General Relativity has diffeomorphism invariance → free choice of coordinate system or *gauge*.
- In certain systems some of the functions we introduced *gauge away*.
- We will first consider the *scalar* perturbations:

$$\begin{aligned} t \rightarrow \hat{t} &= t + \zeta(t, \vec{x}) \\ x^i \rightarrow \hat{x}^i &= x^i + \xi^{,i}(t, \vec{x}) \end{aligned} \tag{16}$$

- From (15):

$$\begin{aligned} g_{00} &= -(1 + 2\Phi) \\ g_{0i} &= -\alpha(t) B_{,i} \\ g_{ij} &= \alpha^2(t) \left[\delta_{ij}(1 - 2\Psi) + 2E_{ij}\right] \end{aligned}$$

# Scalar Perturbations

- The metric is rank 2 tensor, thus it transforms as follows:

$$g_{\mu\nu}(x) = \frac{\partial \widehat{x^{\alpha}}}{\partial x^{\mu}} \frac{\partial \widehat{x^{\beta}}}{\partial x^{\nu}} g_{\hat{\alpha}\beta}(\hat{x}) \tag{17}$$

- The time- time component of (17) is:

$$-(1+2\Phi) = -(1+2\hat{\Phi})\left(\frac{\partial \hat{t}}{\partial t}\right)^{2} \rightarrow -(1+2\Phi) = -(1+2\hat{\Phi})(1+\dot{\zeta})^{2}$$

$$\approx -1 - 2\hat{\Phi} - 2\dot{\zeta} \rightarrow \quad \hat{\Phi} = \Phi - \dot{\zeta}$$

# Scalar Perturbations

- Similarly we can work out the transformation of the other functions and find:

$$\begin{aligned} \hat{\Phi} &= \Phi - \dot{\zeta} \\ \hat{B} &= B - \zeta/\alpha + \alpha\dot{\xi} \\ \hat{E} &= E - \xi \\ \hat{\Psi} &= \Psi + H\zeta \end{aligned} \tag{18}$$

- Are all these perturbation functions necessary ⇔ are these perturbations *fictitious* or real ? → Need to construct gauge invariants:

$$\mathit{Bardeen} \quad \begin{cases} \Phi_B & \equiv \Phi - \frac{d}{dt}\left[\alpha^2(\dot{E} - B/\alpha)\right] \\ \Psi_B & \equiv \Psi + \alpha^2 H(\dot{E} - B/\alpha) \end{cases} \tag{19}$$

# Matter Perturbations

- From (23) we deduce that there are exactly *two* (4 - 2 = 2) such independent gauge invariant combinations.
- To actually describe the *structure* in the universe → need *matter* perturbations as well.
- We will consider perturbations around the homogeneous energy momentum tensor:

$$\bar{T}^{\mu}_{\nu} = (\bar{\rho} + \bar{p})\bar{u}^{\mu}\bar{u}_{\nu} + \bar{p}\delta^{\mu}_{\nu} \tag{20}$$

and write them as follows:

# Matter Perturbations

$$
\begin{aligned}
T^0_0 &= -(\bar{\rho} + \delta\rho) \\
T^0_i &= (\bar{\rho} + \bar{p})\alpha\upsilon_i \\
T^i_0 &= -(\bar{\rho} + \bar{p})(\upsilon^i - B^i)/\alpha \\
T^i_j &= \delta^i_j(\bar{p} + \delta p) + \Sigma^i_j
\end{aligned}
\tag{21}
$$

where $\upsilon^i \equiv dx^i/d\tau$ and $\Sigma^i_j$ is the anisotropic stress.
For each different universe constituent we have:

$$
\begin{aligned}
\delta\rho &= \sum_I \delta\rho_I & \delta p &= \sum_I \delta p_I \\
(\bar{\rho} + \bar{p})\upsilon^i &= \sum_I (\bar{\rho}_I + \bar{p}_I)\upsilon^i_I & \Sigma^{ij} &= \sum_I \Sigma^{ij}_I
\end{aligned}
\tag{22}
$$

# Matter Perturbations

- Velocities do not simply add → define $\delta q^i \equiv (\bar\rho + \bar p)\alpha\upsilon^i$ which is the 3- momentum density and $\delta q^i = \sum_I \delta q^i_I$
- A scalar function $\Phi(x)$ under (16) becomes:

$$\begin{aligned}\hat\Phi(\hat x) &= \Phi(\hat t - \zeta, \hat x^i - \xi^{,i}) = \bar\Phi(\hat t - \zeta) + \delta\Phi(\hat t - \zeta, \hat x^i - \xi^{,i}) \\ &\approx \bar\Phi(\hat t) - \zeta\frac{d\bar\Phi}{d\hat t} + \delta\Phi(\hat t, \hat x^i) \Rightarrow \delta\hat\Phi = \delta\Phi - \zeta\frac{d\bar\Phi}{dt}\end{aligned} \tag{23}$$

- From (23) and the tensor transformation law we get:

$$\begin{aligned}\delta\hat\rho &= \delta\rho - \bar\rho\zeta \\ \delta\hat p &= \delta p - \bar p\zeta \\ \delta\hat q &= \delta q + (\bar\rho + \bar p)\zeta\end{aligned} \tag{24}$$

where $q$ is the scalar part of the momentum density.

# Matter Perturbations

- Given (24) we can construct *gauge invariant* quantities like:

  i) The *comoving density* perturbation:

$$\delta\rho_m \equiv \delta\rho - 3H\delta q \tag{25}$$

  ii) The *curvature perturbation on uniform density hypersurfaces*:

$$-\zeta \equiv \Psi + H\delta\rho/\dot{\bar{\rho}} \tag{26}$$

  iii) The *comoving curvature perturbation* :

$$\mathcal{R} = \Psi - \frac{H}{\bar{\rho} + \bar{p}}\delta q \tag{27}$$

- The Fourier transformation is: $f(\vec{x}) = \int \frac{d^3x}{(2\pi)^3} \tilde{f}_{\vec{k}} \, e^{i\vec{k}\cdot\vec{x}}$

# Einstein Equations

- Can relate the metric and stress-energy perturbations via the *perturbed* Einstein equations:

$$\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu} \tag{28}$$

which to linear order and for *scalar* perturbations in Fourier space yields:

- *Energy and momentum constraint equations:*

$$\begin{aligned} 3H(\dot{\Psi} + H\Phi) + \frac{k^2}{\alpha^2}\left[\Psi + H(\alpha^2 \dot{E} - \alpha B)\right] &= -4\pi G \delta\rho \\ \dot{\Psi} + H\Phi &= -4\pi G \delta q \end{aligned} \tag{29}$$

- From (19), (25), (29) → gauge-invariant Poisson Equations:

$$\frac{k^2}{\alpha^2}\Psi_B = -4\pi G \delta\rho_m \tag{30}$$

# Einstein Equations

- From (29) we can also get the *evolution equations*:

$$\ddot{\Psi} + 3H\dot{\Psi} + H\dot{\Phi} + (3H^2 + 2\dot{H})\Phi = -4\pi G\big(\,\delta p - 2k^2\delta\Sigma/3\big)$$

$$(\partial_t + 3H)(\dot{E} - B/\alpha) + \frac{\Psi - \Phi}{\alpha^2} = 8\pi G\,\delta\Sigma \qquad (31)$$

- From (19) and (49) we can write:

$$\Psi_B - \Phi_B = 8\pi G\alpha^2\,\delta\Sigma \qquad (32)$$

Note: From (32) if $\delta\Sigma \approx 0 \;\Rightarrow\; \Psi_B \approx \Phi_B$

# Conservation Equations

- $\nabla_\mu T^{\mu\nu} = 0 \rightarrow$ *continuity equation* and *Euler equation* :

$$\delta\dot{\rho} + 3H(\delta\rho + \delta p) = \frac{k^2}{\alpha^2}\delta q + (\bar{\rho} + \bar{p})\left[3\dot{\Psi} + k^2(\dot{E} - B/\alpha)\right] \qquad (33)$$

$$\dot{\delta q} + 3H\delta q = -\delta p - (\bar{\rho} + \bar{p})\Phi + 2k^2\delta\Sigma/3 \qquad (34)$$

Note: It would be useful to note the 0th version of (33):

$$\dot{\bar{\rho}} = -3H(\bar{\rho} + \bar{p}) \qquad (35)$$

We are almost ready to pick a gauge and do explicit calculations

→ need *initial conditions* for our perturbations.

# Initial Conditions

- It is standard to assume that these perturbations are generated from *inflation*, which predicts that they are *isentropic or adiabatic* - also preferred by the data.

- Adiabatic perturbations are induced by a common, local shift in time of all background quantities so:

$$\delta\eta = \frac{\delta\rho_a}{\bar{\rho}'_a} = \frac{\delta\rho_b}{\bar{\rho}'_b} \quad \text{for each species a and b} \tag{36}$$

where conformal time is $dn \equiv dt/\alpha$ and from (35) :

$$\frac{\delta_a}{1 + w_a} = \frac{\delta_b}{1 + w_b} \tag{37}$$

where we defined $\delta \equiv \delta\rho/\rho$ the *fractional overdensity* and $w_a \equiv P_a/\rho_a$ the equation of state parameter.

# Newtonian Gauge

We can now start describing the evolution of structure. We shall pick the Newtonian gauge for it simplifies the analytic calculations greatly.

***Newtonian gauge:*** *Definition:* $B = E = 0$

The equations we have presented become:

$$ds^2 = -(1+2\Phi)dt^2 + \alpha^2(t)[(1-2\Psi)\delta_{ij}]dx^i dx^j \tag{38}$$

i) The Einstein equations:

$$\begin{aligned} 3H(\dot{\Psi} + H\Phi) + \frac{k^2}{\alpha^2}\Psi &= -4\pi G\,\delta\rho \\ \dot{\Psi} + H\Phi &= -4\pi G\,\delta q \\ \ddot{\Psi} + 3H\dot{\Psi} + H\dot{\Phi} + (3H^2 + 2\dot{H})\Phi &= -4\pi G\big(\,\delta p - 2k^2\delta\Sigma/3\big) \\ \frac{\Psi - \Phi}{\alpha^2} &= 8\pi G\,\delta\Sigma \end{aligned} \tag{39}$$

# Newtonian Gauge

The gauge invariant Poisson equation (30) becomes:

$$\frac{k^2}{\alpha^2}\Psi = -4\pi G\delta\rho \tag{40}$$

ii) The conservation equations:

$$\delta\dot\rho + 3H(\delta\rho + \delta p) = \frac{k^2}{\alpha^2}\delta q + 3(\bar\rho + \bar p)\dot\Psi \tag{41}$$

$$\dot{\delta q} + 3H\delta q = -\delta p - (\bar\rho + \bar p)\Phi + 2k^2\delta\Sigma/3 \tag{42}$$

It is useful to make (41) and (42) more explicit:

$$\begin{aligned} \dot\delta + 3H\Big(\frac{\delta\bar P}{\bar\rho} - \frac{\delta\bar P}{\bar\rho}\Big)\delta &= -\Big(1 + \frac{\bar P}{\bar\rho}\Big)\big(\frac{k_i u^i}{\alpha} - 3\dot\Phi\big) \\ \dot u^i + 3H\Big(\frac{1}{3} - \frac{\dot{\bar P}}{\dot{\bar\rho}}\Big)u^i &= -\frac{1}{\bar\rho + \bar P}\Big(\frac{k^i\delta P}{\alpha} + k_j\Sigma^{ij}\Big) - k^i\Psi \end{aligned} \tag{43}$$

# Growth Equations

We will now consider inhomogeneities in a fluid with $w = \bar{P}/\bar{\rho}, \Sigma_{ij} = 0$, $c_s^2 \equiv \delta P/\delta\rho$ and *adiabatic* perturbations → $c_s^2 \approx w$

Under these assumptions equations (43) become:

$$\dot{\delta} = -(1+w)\Big(\frac{k_i}{\alpha}u^i - 3\dot{\Psi}\Big) \tag{44}$$

$$\dot{u}^i = -H(1-3w)u^i - \frac{c_s^2}{1+w}\frac{k^i}{\alpha}\delta - \frac{k^i}{\alpha}\Psi \tag{45}$$

On subhorizon scales, that is $k >> \alpha H$, $\dot{\Psi} \approx 0$ and by taking the divergence of (45) and the Poisson equation (30) we get:

$$\boxed{\ddot{\delta} + (2-3w)H\dot{\delta} + c_s^2\frac{k^2}{\alpha^2} = (1+w)4\pi G\bar{\rho}\delta} \tag{46}$$

Which is called the *Jeans equation*.

# Growth Equations

If we now consider the *late time* evolution of perturbations ($\Rightarrow c_s \approx 0$) in the *matter domination era* ($w \approx 0$). Then (46) yields:

$$\boxed{\ddot{\delta} + 2H\dot{\delta} = 4\pi G\rho_M\,\delta} \tag{47}$$

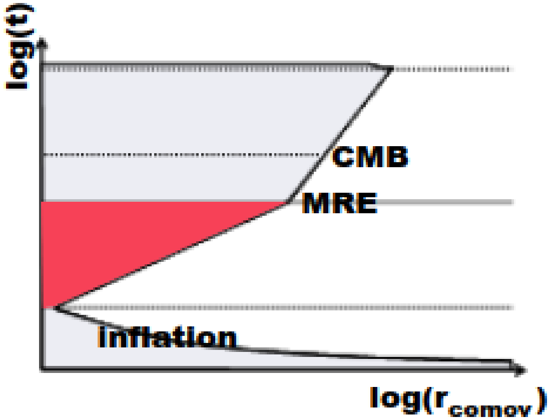
- By using this *linear* equation we can study the distribution of matter in the universe
- An analogous equation we can get in *modified gravity* theories, where the Newtonian $G$ in (47) is replaced by $G_{eff}$ .

# Linear growth of density perturbations:

## Sub-horizon, radiation dominated, pre recombination

Jeans linear perturbation analysis applies:

$$\ddot{\delta}_k + 2H\dot{\delta}_k + [\frac{c_s{}^2k^2}{a^2} - 4\pi G\rho_0]\,\delta_k = 0$$



$$a \propto t^{1/2}$$

$$\dot{a} \propto \tfrac{1}{2}t^{-1/2}$$

$$\frac{\dot{a}}{a} = H = \frac{1}{2t}$$

$$Ht = \tfrac{1}{2}$$

radiation dominates, and because radiation does not cluster → al$\delta_k$=0…

dark matter has no pressure of its own it is not coupled to photons, so there no restoring pressure force.

$$\ddot{\delta}_k + 2H\dot{\delta}_k + [\cancelto{\text{zero}}{\frac{c_s{}^2k^2}{a^2}} - 4\pi G\rho_0]\,\delta_k = 0$$

$$\ddot{\delta}_k + \tfrac{1}{t}\dot{\delta}_k = 0$$

$$\delta_k = \underbrace{A\ln(t)}_{\text{growing mode}} + \underbrace{B}_{\text{“decaying” mode}}$$

…but the rate of chang of $\delta_k$'s can be non-zer

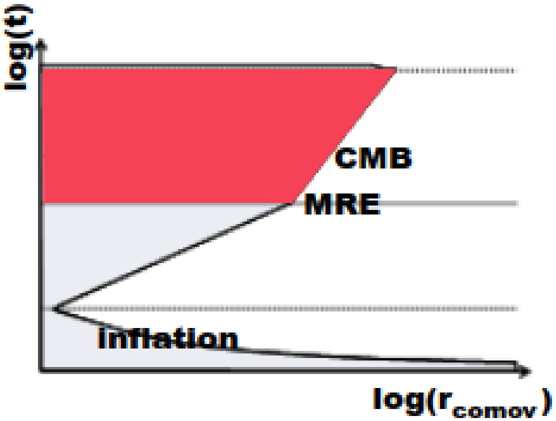
$$\boxed{\text{DM growing mode solution} \quad \delta_k \propto 2\ln(a)}$$

# Linear growth of density perturbations:

## Sub-horizon, matter dominated, pre & post recomb.

Jeans linear perturbation analysis applies:

$$\ddot{\delta}_k + 2H\dot{\delta}_k + [\frac{c_s{}^2 k^2}{a^2} - 4\pi G\rho_0]\,\delta_k = 0$$



$$a \propto t^{2/3}$$

$$\dot{a} \propto \tfrac{2}{3} t^{-1/3}$$

$$\frac{\dot{a}}{a} = H = \frac{2}{3t}$$

$$Ht = \tfrac{2}{3}$$

dark matter has no pressure of its ow
it is not coupled to photons, so there
no restoring pressure force.

$$\ddot{\delta}_k + 2H\dot{\delta}_k + [\frac{\cancel{c_s{}^2 k^2}}{a^2} - 4\pi G\rho_0]\,\delta_k = 0$$

zero

also, can assume that total density is the critical density at that epoch:

$$\rho = \rho_0 = \tfrac{3H^2}{8\pi G}$$

$$4\pi G\rho_0 = \tfrac{2}{3t^2}$$

$$\ddot{\delta}_k + \tfrac{4}{3t}\dot{\delta}_k - \tfrac{2}{3t^2}\delta_k = 0$$

$$\delta_k = \underbrace{At^{2/3}}_{\text{growing mode}} + \underbrace{Bt^{-1}}_{\text{decaying mode}}$$

Two linearly indep. solutions: **growing** mode always comes to dominate; ignore **decaying** mode soln.

$$\boxed{\text{DM growing mode solution} \quad \delta_k \propto a}$$

# Linear growth of density perturbations:

## Sub-horizon, lambda dominated, pre & post recomb.

Jeans linear perturbation analysis applies:

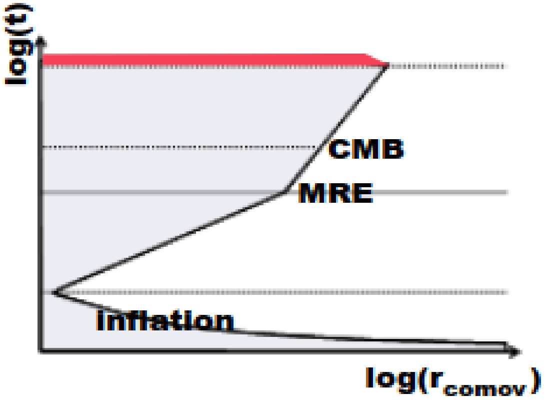
$$\ddot{\delta}_k + 2H\dot{\delta}_k + [\frac{c_s^2 k^2}{a^2} - 4\pi G\rho_0]\,\delta_k = 0$$

$$H^2 = H_0^2[\Omega_\Lambda]$$
$$H = \text{const}$$
$$a \propto e^{Ht}$$

dark matter has no pressure of its owr it is not coupled to photons, so there no restoring pressure force.

$$\ddot{\delta}_k + 2H\dot{\delta}_k + [\frac{\cancel{c_s^2 k^2}}{a^2} - 4\pi G\rho_0]\,\delta_k = 0$$

zero



can assume the amplitude of perturbations is zero, because lambda, which dominates, does not cluster:

$$\delta_k = 0$$

$$\ddot{\delta}_k + 2H\dot{\delta}_k = 0$$
$$\delta_k = \underbrace{A}_{\text{"growing" mode}} + \underbrace{Be^{-2Ht}}_{\text{decaying mode}}$$

Two linearly indep. solutions: **growing** mode always comes to dominate; ignore **decaying** mode soln.

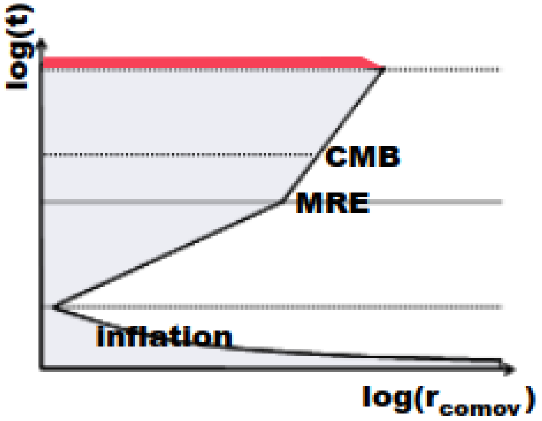
$$\boxed{\text{DM "growing" mode soln} \quad \delta_k \propto \text{const}}$$

# Linear growth of density perturbations:

## Sub-horizon, curvature dominated, pre & post recomb.

Jeans linear perturbation analysis applies:

$$\ddot{\delta}_k + 2H\dot{\delta}_k + [\frac{c_s{}^2k^2}{a^2} - 4\pi G\rho_o]\,\delta_k = 0$$



$$H^2 = H_0{}^2[(1-\Sigma\Omega_w)a^{-2}]$$
$$H \propto a^{-1}$$
$$\dot{a} = \mathrm{const}$$
$$a \propto t$$
$$Ht = 1$$

can assume the amplitude of perturbations is zero, because curvature, which dominates, does not cluster:

$$\delta_k = 0$$

dark matter has no pressure of its own it is not coupled to photons, so there no restoring pressure force.

$$\ddot{\delta}_k + 2H\dot{\delta}_k + [\underset{\text{zero}}{\cancel{\frac{c_s{}^2k^2}{a^2}}} - 4\pi G\rho_o]\,\delta_k = 0$$

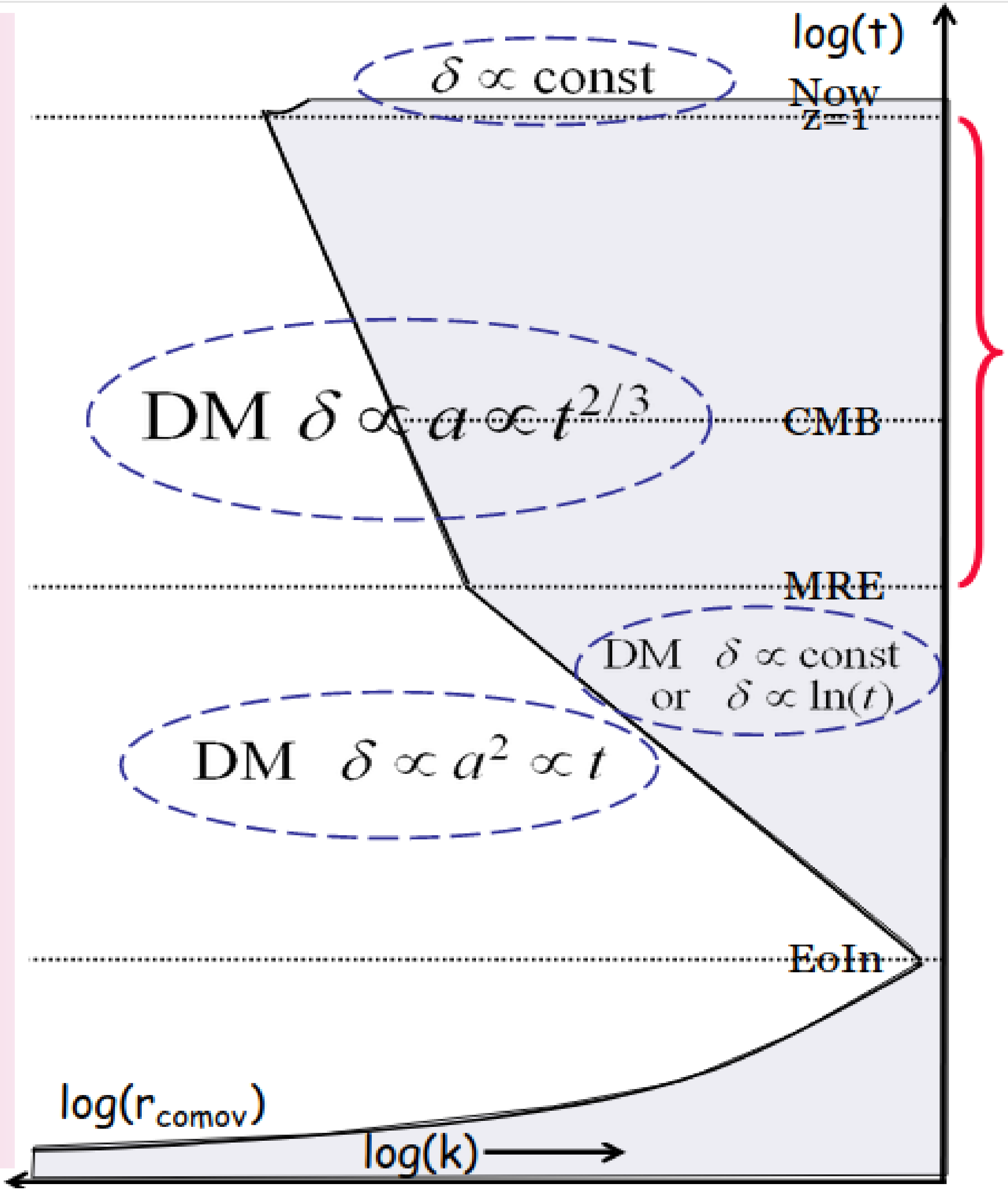
$$\ddot{\delta}_k + \tfrac{2}{t}\dot{\delta}_k = 0$$

$$\delta_k = \underbrace{A}_{\text{“growing” mode}} + \underbrace{Bt^{-1}}_{\text{decaying mode}}$$

Two linearly indep. solutions: **growing** mode always comes to dominate; ignore **decaying** mode soln.

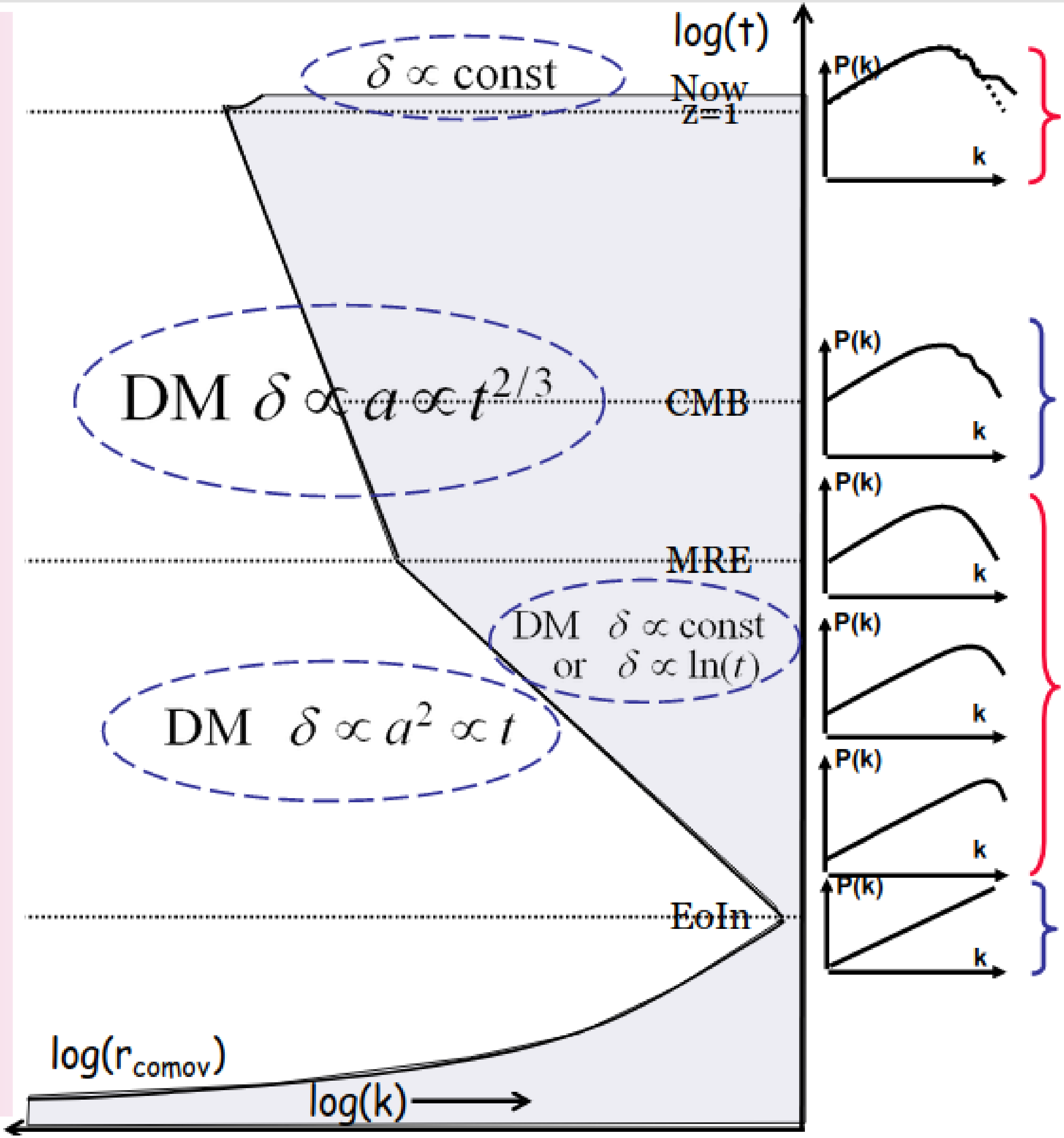
$$\boxed{\text{DM "growing" mode soln} \quad \delta_k \propto \mathrm{const}}$$

# Evolution of matter power spectrum



On sub-horizon scales **growth of structure begins and ends with matter domination**

# Evolution of matter power spectrum



high-k small scale perturbations grow fast, non-linearly

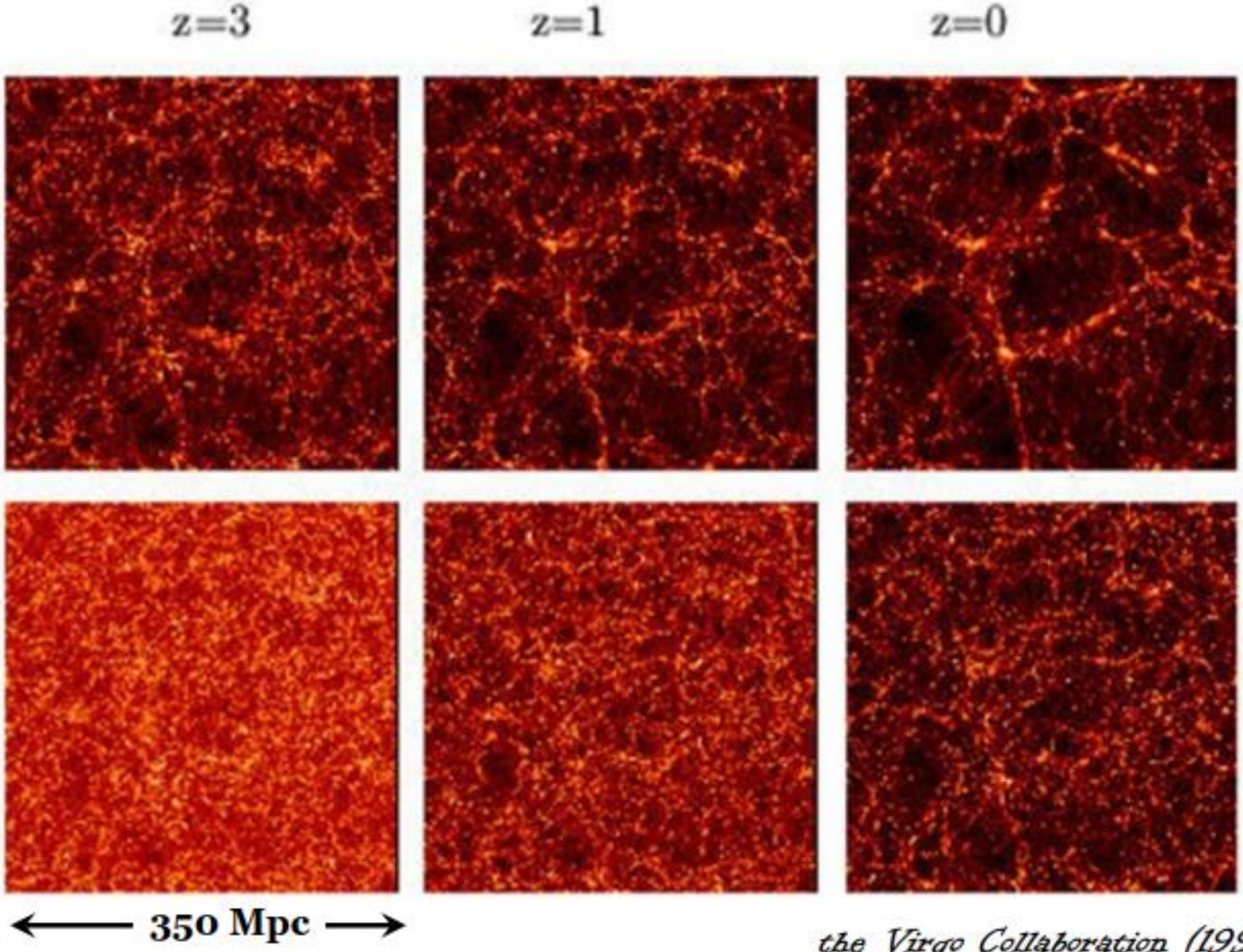
baryonic oscillations appear – the P(k) equivalent of CMB T power spectrum

sub-horizon perturb. do not grow during radiation dominated epoch

Harrison-Zeldovich spectrum P(k)~k from inflation

# Growth of large scale structure

Dark Matter density maps from N-body simulations



Lambda (DE)
spatially flat
$\Omega_{matter}$=0.3

**fractional overdensity ~const**

Standard
spatially flat
$\Omega_{matter}$=1.0

**fractional overdensity ~1/(1+z)**

*the Virgo Collaboration (1996,*

## Summary

Fig. 5.3 shows the evolution of the matter density contrast $\delta_m$ for the same modes as in fig. 5.2. Fluctuations are frozen until they enter the horizon. Subhorizon matter fluctuations in the radiation era only grow logarithmically, $\delta_m \propto \ln a$. This changes to power-law growth, $\delta_m \propto a$ when the universe becomes matter dominated. When the universe becomes dominated by dark energy, perturbations stop growing.

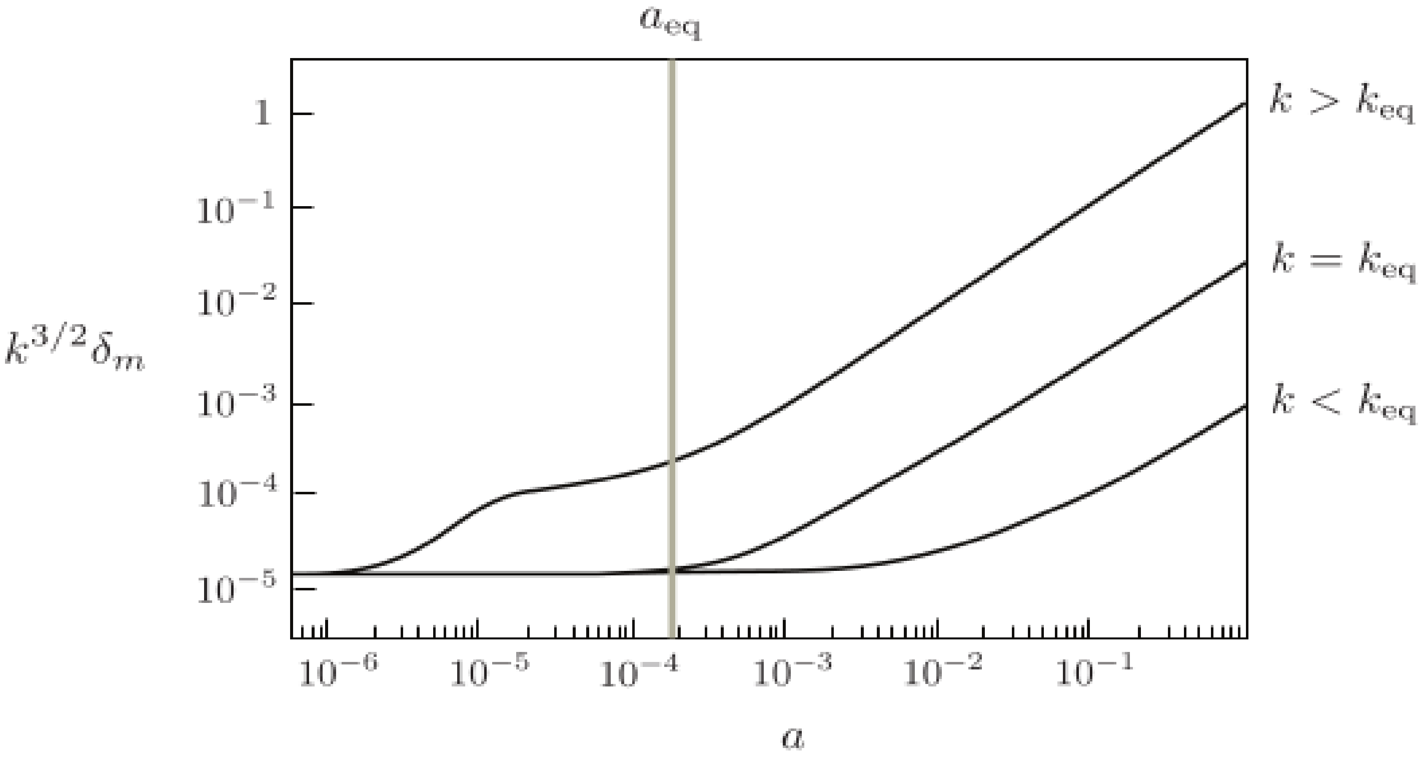
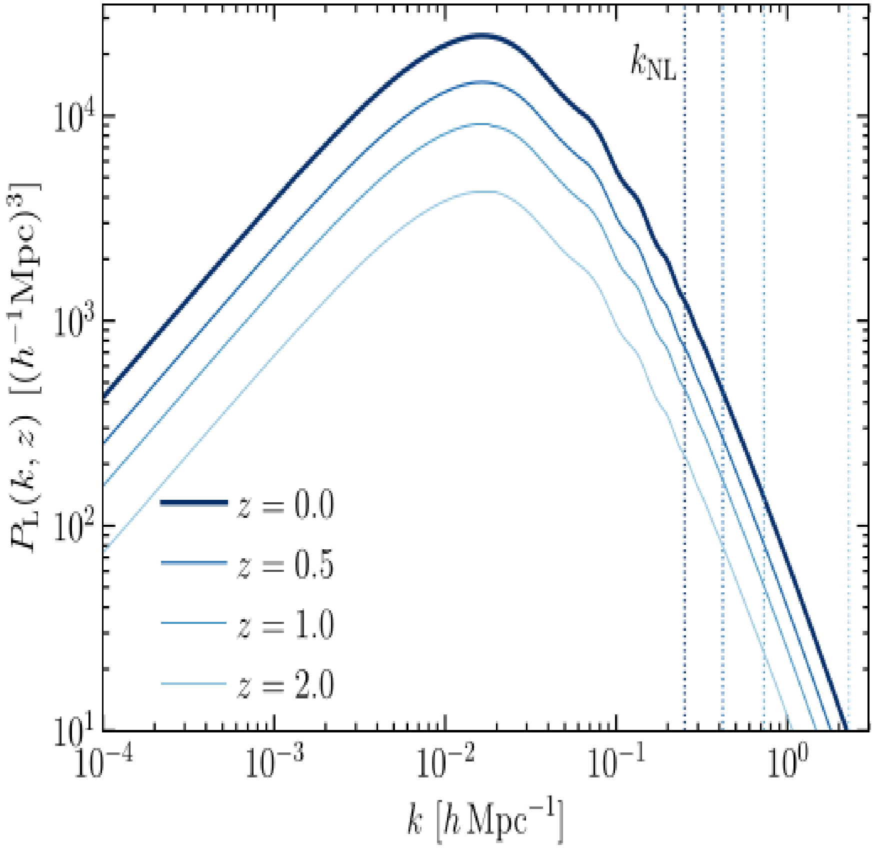


**Figure 5.3:** *Evolution of the matter density contrast for the same modes as in fig. 5.2.*



The linear matter power spectrum in the fiducial $\Lambda$CDM cosmology at different redshifts. Scales to the left of the vertical lines, which indicate $k_{\rm NL}(z)$ for each of the redshifts shown, are still evolving approximately linearly at each redshift.

# Summary:

## ΛCDM concordance model is almost perfect!

$$\boxed{H(t)^2 + \frac{k}{a(t)^2} = \frac{8\pi G}{3}\left[\rho_{dm}(t) + \rho_b(t) + \rho_r(t)\right] + \frac{\Lambda}{3}}$$

$$\boxed{\dot{H}(t) - \frac{k}{a(t)^2} = -4\pi G\left[\rho_{dm}(t) + p_{dm}(t) + \rho_b(t) + p_b(t) + \rho_r(t) + p_r(t)\right]}$$

$$\boxed{w_\Lambda \equiv \frac{p_\Lambda}{\rho_\Lambda} = -1}$$

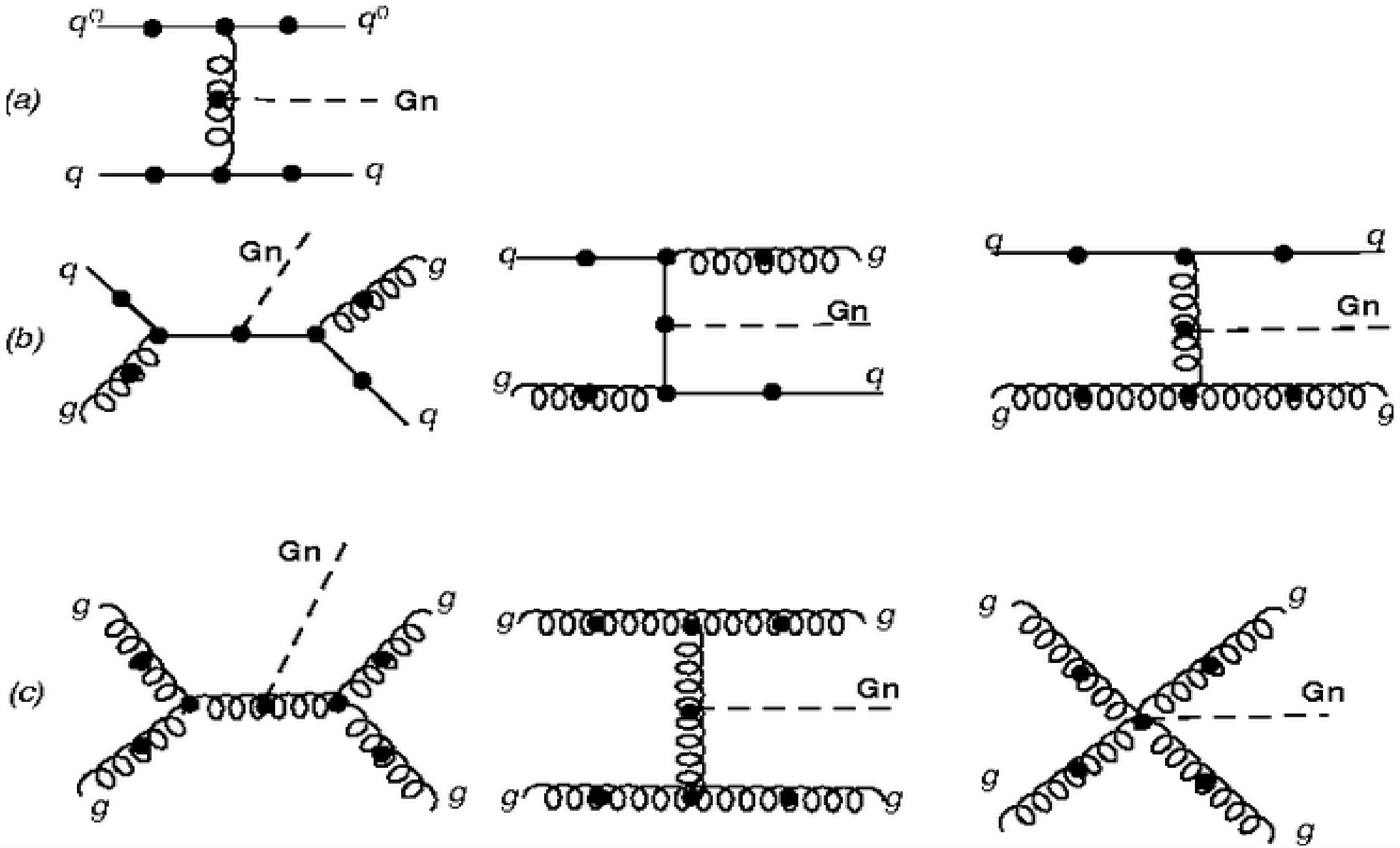
# Issues of ΛCDM Paradigm

- ΛCDM is a successful cosmological model:

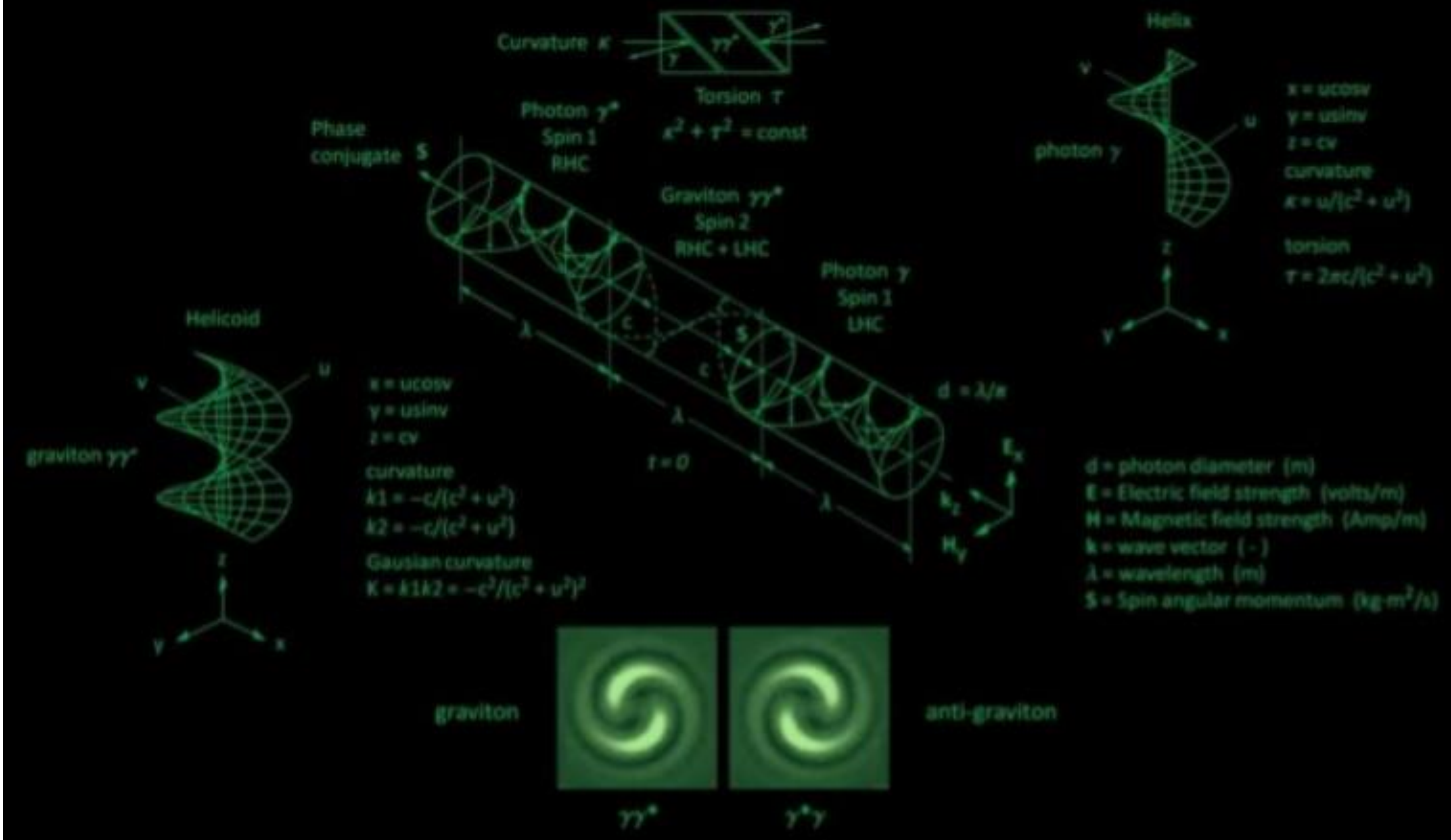
1) Describes the evolution of the universe at the background level
2) Describes the evolution of the universe at the perturbation level

- However there are open issues:
  1) General Relativity is non-renormalizable. It cannot get quantized.
  2) The cosmological-constant problem. Calculation of Λ gives a number 120 orders of magnitude larger than observed. Worst error in the ~~history of physics~~, ~~history of science~~, history
  3) How to describe primordial universe (inflation)
  4) Tensions with some data sets, e.g. H0, fσ8, AL data
  5) Missing galaxy satellites, curspy-core problems.

# Can General Relativity be quantized?



# Graviton



x = ucosv
y = usinv
z = cv
curvature
$\kappa = u/(c^2 + u^2)$
torsion
$\tau = 2\pi c/(c^2 + u^2)$

x = ucosv
y = usinv
z = cv
curvature
$k1 = -c/(c^2 + u^2)$
$k2 = -c/(c^2 + u^2)$
Gausian curvature
$K = k1k2 = -c^2/(c^2 + u^2)^2$

d = photon diameter (m)
E = Electric field strength (volts/m)
H = Magnetic field strength (Amp/m)
k = wave vector ( - )
$\lambda$ = wavelength (m)
S = Spin angular momentum (kg·m$^2$/s)

# COSMOLOGICAL CONSTANT PROBLEM

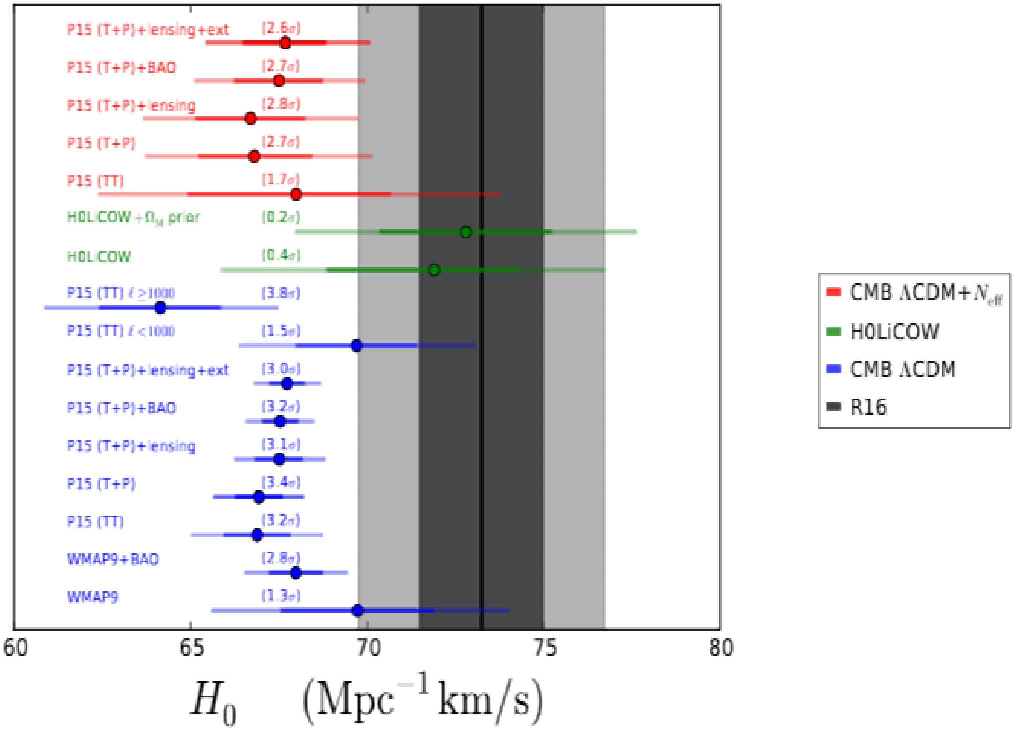
$$E_n \sim (n + 1/2) h\omega(k)$$

$$\rho_\Lambda(th) \sim M_p^4$$

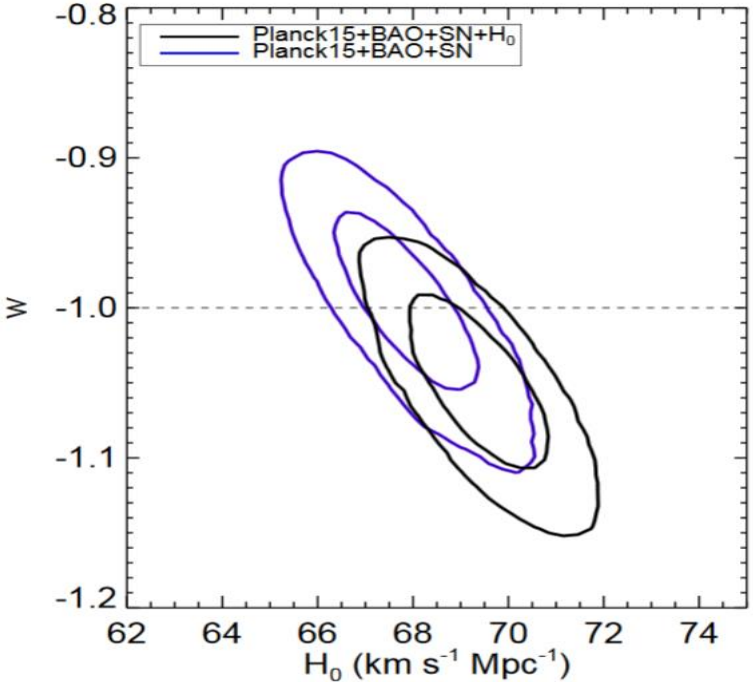
$$\rho_\Lambda^0 \sim 10^{-120} \rho_\Lambda^{th}$$

# Tension1 – H0

- Tension between the data (direct measurements) and Planck/ΛCDM (indirect measurements). The data indicate a lack of “gravitational power”.



[Bernal, Verde, Riess, JCAP1610]

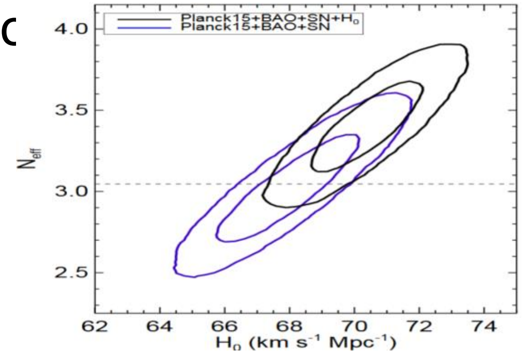


[Riess et al, Astrophys.J 826]

# Tension1 – H0

- Tension between the data (direct measurements) and Planck/ΛCDM (indirect measurements). The data indicate a lack of “gravitational power”.
- This tension could be due to systematics.

- If not systematics then we may need changes in ΛCDM in early or late time behavior.
- Higher number of effective relativistic species, dynamical dark energy, non-zero curvature, etc

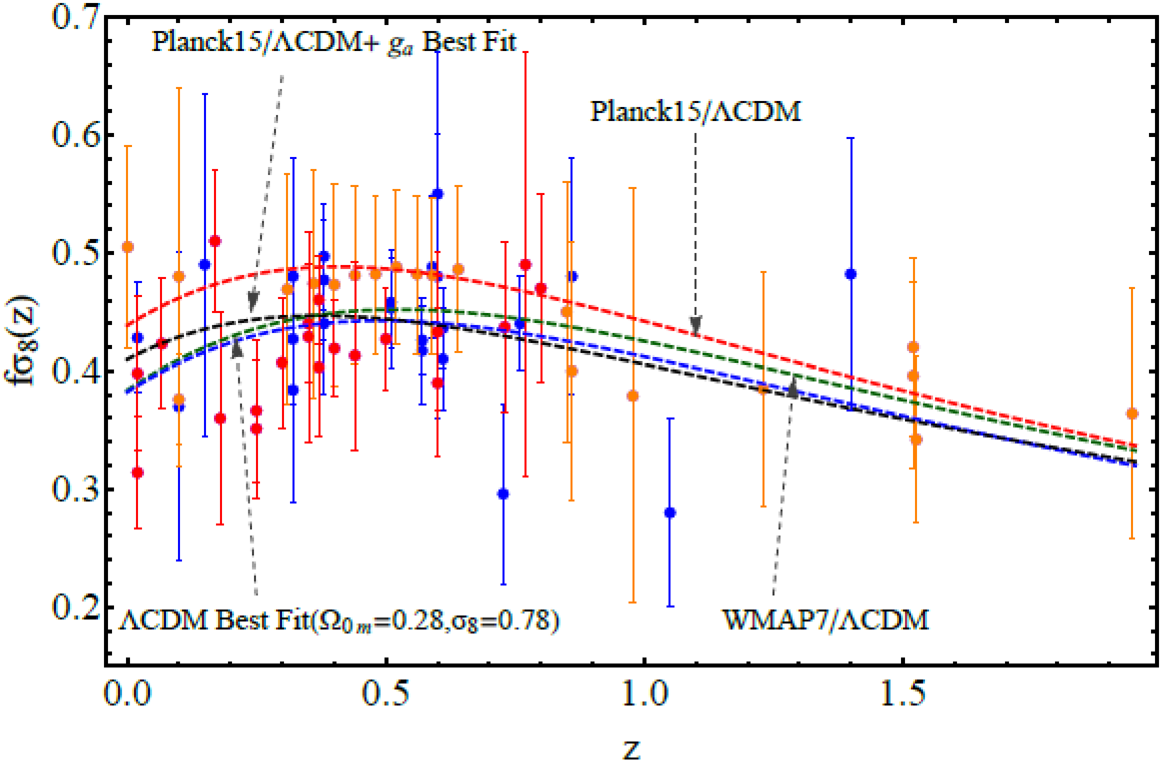


- Or Modified Gravity.

# Tension2 – fσ8

- Tension between the data and Planck/ΛCDM. The data indicate a lack of "gravitational power" in structures on intermediate-small cosmological scales.

| Parameter | Planck15/ΛCDM [12] | WMAP7/ΛCDM [45] |
|---|---|---|
| $\Omega_b h^2$ | $0.02225 \pm 0.00016$ | $0.02258 \pm 0.00057$ |
| $\Omega_c h^2$ | $0.1198 \pm 0.0015$ | $0.1109 \pm 0.0056$ |
| $n_s$ | $0.9645 \pm 0.0049$ | $0.963 \pm 0.014$ |
| $H_0$ | $67.27 \pm 0.66$ | $71.0 \pm 2.5$ |
| $\Omega_{0m}$ | $0.3156 \pm 0.0091$ | $0.266 \pm 0.025$ |
| $w$ | $-1$ | $-1$ |
| $\sigma_8$ | $0.831 \pm 0.013$ | $0.801 \pm 0.030$ |



# Tension2 – fσ8

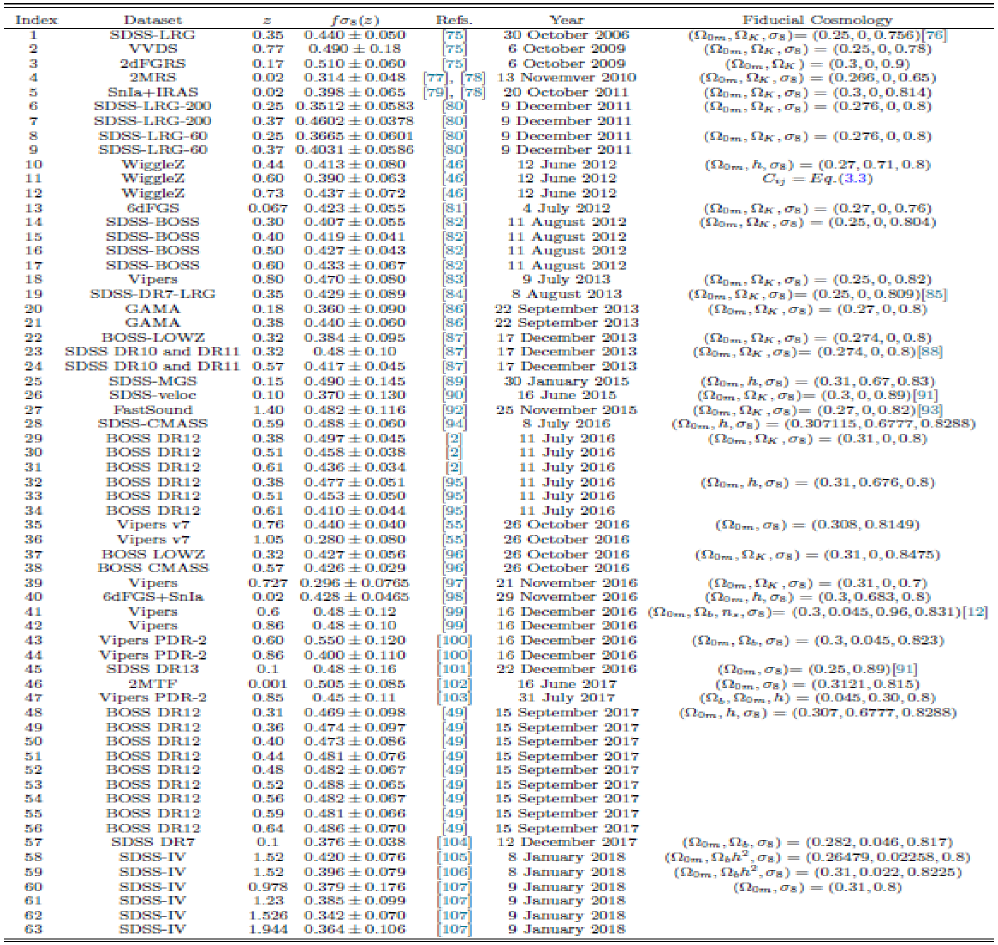


TABLE II: A compilation of RSD data that we found published from 2006 since 2018

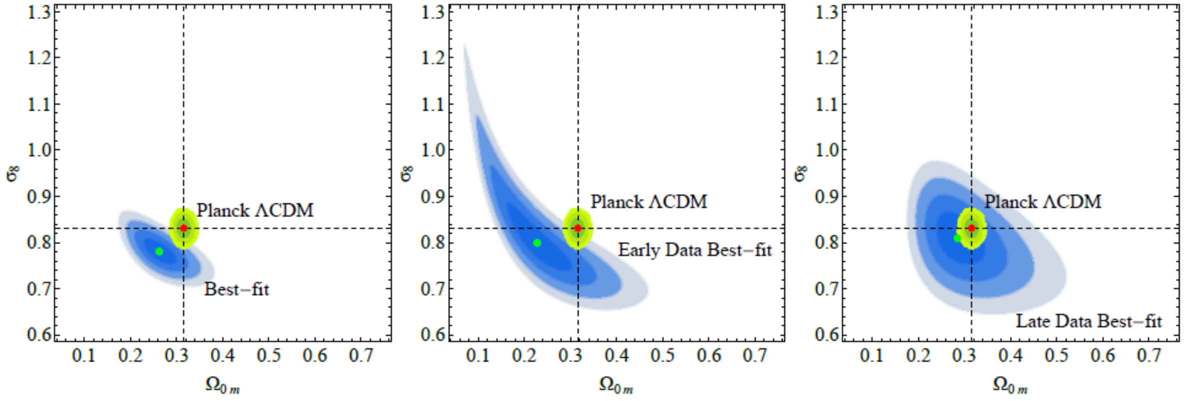
| Index | Dataset | $z$ | $f\sigma_8(z)$ | Refs. | Year | Fiducial Cosmology |
|---|---|---|---|---|---|---|
| 1 | SDSS-LRG | 0.35 | $0.440 \pm 0.050$ | [75] | 30 October 2006 | $(\Omega_{0m}, \Omega_K, \sigma_8) = (0.25, 0, 0.756)$[76] |
| 2 | VVDS | 0.77 | $0.490 \pm 0.18$ | [75] | 6 October 2009 | $(\Omega_{0m}, \Omega_K, \sigma_8) = (0.25, 0, 0.78)$ |
| 3 | 2dFGRS | 0.17 | $0.510 \pm 0.060$ | [75] | 6 October 2009 | $(\Omega_{0m}, \Omega_K) = (0.3, 0, 0.9)$ |
| 4 | 2MRS | 0.02 | $0.314 \pm 0.048$ | [77], [78] | 13 Novemver 2010 | $(\Omega_{0m}, \Omega_K, \sigma_8) = (0.266, 0, 0.65)$ |
| 5 | SnIa+IRAS | 0.02 | $0.398 \pm 0.065$ | [79], [78] | 20 October 2011 | $(\Omega_{0m}, \Omega_K, \sigma_8) = (0.3, 0, 0.814)$ |
| 6 | SDSS-LRG-200 | 0.25 | $0.3512 \pm 0.0583$ | [80] | 9 December 2011 | $(\Omega_{0m}, \Omega_K, \sigma_8) = (0.276, 0, 0.8)$ |
| 7 | SDSS-LRG-200 | 0.37 | $0.4602 \pm 0.0378$ | [80] | 9 December 2011 | |
| 8 | SDSS-LRG-60 | 0.25 | $0.3665 \pm 0.0601$ | [80] | 9 December 2011 | $(\Omega_{0m}, \Omega_K, \sigma_8) = (0.276, 0, 0.8)$ |
| 9 | SDSS-LRG-60 | 0.37 | $0.4031 \pm 0.0586$ | [80] | 9 December 2011 | |
| 10 | WiggleZ | 0.44 | $0.413 \pm 0.080$ | [46] | 12 June 2012 | $(\Omega_{0m}, h, \sigma_8) = (0.27, 0.71, 0.8)$ |
| 11 | WiggleZ | 0.60 | $0.390 \pm 0.063$ | [46] | 12 June 2012 | $C_{ij} = Eq.(3.3)$ |
| 12 | WiggleZ | 0.73 | $0.437 \pm 0.072$ | [46] | 12 June 2012 | |
| 13 | 6dFGS | 0.067 | $0.423 \pm 0.055$ | [81] | 4 July 2012 | $(\Omega_{0m}, \Omega_K, \sigma_8) = (0.27, 0, 0.76)$ |
| 14 | SDSS-BOSS | 0.30 | $0.407 \pm 0.055$ | [82] | 11 August 2012 | $(\Omega_{0m}, \Omega_K, \sigma_8) = (0.25, 0, 0.804)$ |
| 15 | SDSS-BOSS | 0.40 | $0.419 \pm 0.041$ | [82] | 11 August 2012 | |
| 16 | SDSS-BOSS | 0.50 | $0.427 \pm 0.043$ | [82] | 11 August 2012 | |
| 17 | SDSS-BOSS | 0.60 | $0.433 \pm 0.067$ | [82] | 11 August 2012 | |
| 18 | Vipers | 0.80 | $0.470 \pm 0.080$ | [83] | 9 July 2013 | $(\Omega_{0m}, \Omega_K, \sigma_8) = (0.25, 0, 0.82)$ |
| 19 | SDSS-DR7-LRG | 0.35 | $0.429 \pm 0.089$ | [84] | 8 August 2013 | $(\Omega_{0m}, \Omega_K, \sigma_8) = (0.25, 0, 0.809)$[85] |
| 20 | GAMA | 0.18 | $0.360 \pm 0.090$ | [86] | 22 September 2013 | $(\Omega_{0m}, \Omega_K, \sigma_8) = (0.27, 0, 0.8)$ |
| 21 | GAMA | 0.38 | $0.440 \pm 0.060$ | [86] | 22 September 2013 | |
| 22 | BOSS-LOWZ | 0.32 | $0.384 \pm 0.095$ | [87] | 17 December 2013 | $(\Omega_{0m}, \Omega_K, \sigma_8) = (0.274, 0, 0.8)$ |
| 23 | SDSS DR10 and DR11 | 0.32 | $0.48 \pm 0.10$ | [87] | 17 December 2013 | $(\Omega_{0m}, \Omega_K, \sigma_8) = (0.274, 0, 0.8)$[88] |
| 24 | SDSS DR10 and DR11 | 0.57 | $0.417 \pm 0.045$ | [87] | 17 December 2013 | |
| 25 | SDSS-MGS | 0.15 | $0.490 \pm 0.145$ | [89] | 30 January 2015 | $(\Omega_{0m}, h, \sigma_8) = (0.31, 0.67, 0.83)$ |
| 26 | SDSS-veloc | 0.10 | $0.370 \pm 0.130$ | [90] | 16 June 2015 | $(\Omega_{0m}, \Omega_K, \sigma_8) = (0.3, 0, 0.89)$[91] |
| 27 | FastSound | 1.40 | $0.482 \pm 0.116$ | [92] | 25 November 2015 | $(\Omega_{0m}, \Omega_K, \sigma_8) = (0.27, 0, 0.82)$[93] |
| 28 | SDSS-CMASS | 0.59 | $0.488 \pm 0.060$ | [94] | 8 July 2016 | $(\Omega_{0m}, h, \sigma_8) = (0.307115, 0.6777, 0.8288)$ |
| 29 | BOSS DR12 | 0.38 | $0.497 \pm 0.045$ | [2] | 11 July 2016 | $(\Omega_{0m}, \Omega_K, \sigma_8) = (0.31, 0, 0.8)$ |
| 30 | BOSS DR12 | 0.51 | $0.458 \pm 0.038$ | [2] | 11 July 2016 | |
| 31 | BOSS DR12 | 0.61 | $0.436 \pm 0.034$ | [2] | 11 July 2016 | |
| 32 | BOSS DR12 | 0.38 | $0.477 \pm 0.051$ | [95] | 11 July 2016 | $(\Omega_{0m}, h, \sigma_8) = (0.31, 0.676, 0.8)$ |
| 33 | BOSS DR12 | 0.51 | $0.453 \pm 0.050$ | [95] | 11 July 2016 | |
| 34 | BOSS DR12 | 0.61 | $0.410 \pm 0.044$ | [95] | 11 July 2016 | |
| 35 | Vipers v7 | 0.76 | $0.440 \pm 0.040$ | [55] | 26 October 2016 | $(\Omega_{0m}, \sigma_8) = (0.308, 0.8149)$ |
| 36 | Vipers v7 | 1.05 | $0.280 \pm 0.080$ | [55] | 26 October 2016 | |
| 37 | BOSS LOWZ | 0.32 | $0.427 \pm 0.056$ | [96] | 26 October 2016 | $(\Omega_{0m}, \Omega_K, \sigma_8) = (0.31, 0, 0.8475)$ |
| 38 | BOSS CMASS | 0.57 | $0.426 \pm 0.029$ | [96] | 26 October 2016 | |
| 39 | Vipers | 0.727 | $0.296 \pm 0.0765$ | [97] | 21 November 2016 | $(\Omega_{0m}, \Omega_K, \sigma_8) = (0.31, 0, 0.7)$ |
| 40 | 6dFGS+SnIa | 0.02 | $0.428 \pm 0.0465$ | [98] | 29 November 2016 | $(\Omega_{0m}, h, \sigma_8) = (0.3, 0.683, 0.8)$ |
| 41 | Vipers | 0.6 | $0.48 \pm 0.12$ | [99] | 16 December 2016 | $(\Omega_{0m}, \Omega_b, n_s, \sigma_8) = (0.3, 0.045, 0.96, 0.831)$[12] |
| 42 | Vipers | 0.86 | $0.48 \pm 0.10$ | [99] | 16 December 2016 | |
| 43 | Vipers PDR-2 | 0.60 | $0.550 \pm 0.120$ | [100] | 16 December 2016 | $(\Omega_{0m}, \Omega_b, \sigma_8) = (0.3, 0.045, 0.823)$ |
| 44 | Vipers PDR-2 | 0.86 | $0.400 \pm 0.110$ | [100] | 16 December 2016 | |
| 45 | SDSS DR13 | 0.1 | $0.48 \pm 0.16$ | [101] | 22 December 2016 | $(\Omega_{0m}, \sigma_8) = (0.25, 0.89)$[91] |
| 46 | 2MTF | 0.001 | $0.505 \pm 0.085$ | [102] | 16 June 2017 | $(\Omega_{0m}, \sigma_8) = (0.3121, 0.815)$ |
| 47 | Vipers PDR-2 | 0.85 | $0.45 \pm 0.11$ | [103] | 31 July 2017 | $(\Omega_b, \Omega_{0m}, h) = (0.045, 0.30, 0.8)$ |
| 48 | BOSS DR12 | 0.31 | $0.469 \pm 0.098$ | [49] | 15 September 2017 | $(\Omega_{0m}, h, \sigma_8) = (0.307, 0.6777, 0.8288)$ |
| 49 | BOSS DR12 | 0.36 | $0.474 \pm 0.097$ | [49] | 15 September 2017 | |
| 50 | BOSS DR12 | 0.40 | $0.473 \pm 0.086$ | [49] | 15 September 2017 | |
| 51 | BOSS DR12 | 0.44 | $0.481 \pm 0.076$ | [49] | 15 September 2017 | |
| 52 | BOSS DR12 | 0.48 | $0.482 \pm 0.067$ | [49] | 15 September 2017 | |
| 53 | BOSS DR12 | 0.52 | $0.488 \pm 0.065$ | [49] | 15 September 2017 | |
| 54 | BOSS DR12 | 0.56 | $0.482 \pm 0.067$ | [49] | 15 September 2017 | |
| 55 | BOSS DR12 | 0.59 | $0.481 \pm 0.066$ | [49] | 15 September 2017 | |
| 56 | BOSS DR12 | 0.64 | $0.486 \pm 0.070$ | [49] | 15 September 2017 | |
| 57 | SDSS DR7 | 0.1 | $0.376 \pm 0.038$ | [104] | 12 December 2017 | $(\Omega_{0m}, \Omega_b, \sigma_8) = (0.282, 0.046, 0.817)$ |
| 58 | SDSS-IV | 1.52 | $0.420 \pm 0.076$ | [105] | 8 January 2018 | $(\Omega_{0m}, \Omega_b h^2, \sigma_8) = (0.26479, 0.02258, 0.8)$ |
| 59 | SDSS-IV | 1.52 | $0.396 \pm 0.079$ | [106] | 8 January 2018 | $(\Omega_{0m}, \Omega_b h^2, \sigma_8) = (0.31, 0.022, 0.8225)$ |
| 60 | SDSS-IV | 0.978 | $0.379 \pm 0.176$ | [107] | 9 January 2018 | $(\Omega_{0m}, \sigma_8) = (0.31, 0.8)$ |
| 61 | SDSS-IV | 1.23 | $0.385 \pm 0.099$ | [107] | 9 January 2018 | |
| 62 | SDSS-IV | 1.526 | $0.342 \pm 0.070$ | [107] | 9 January 2018 | |
| 63 | SDSS-IV | 1.944 | $0.364 \pm 0.106$ | [107] | 9 January 2018 | |

- Model Dependence: Distance to galaxies is not measured directly, so a cosmological model is assumed in order to infer distances (ΛCDM with different parameters).
- Double counting: Some data points correspond to the same sample of galaxies analyzed by different groups/methods etc.

[Kazantzidis, Perivolaropoulos, PRD97]

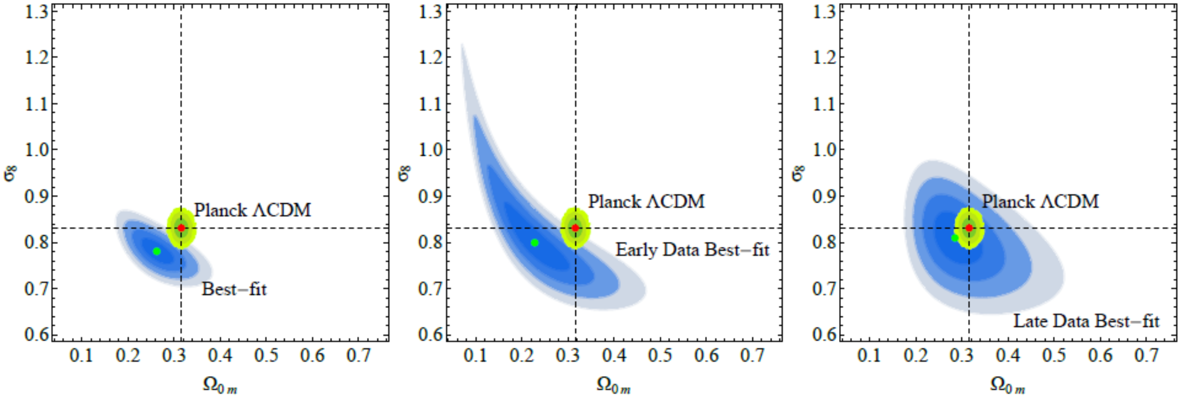
# Tension1 – fσ8

- Tension between the data and Planck/ΛCDM.
- This tension could be due to systematics. E.g:



# Tension2 – fσ8

- Tension between the data and Planck/ΛCDM.
- This tension could be due to systematics. E.g:

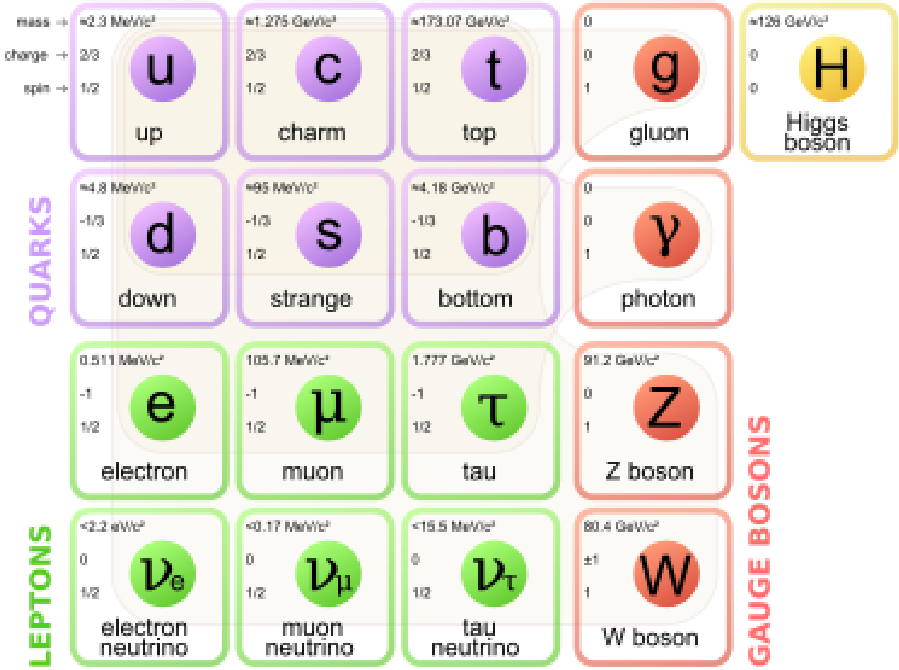


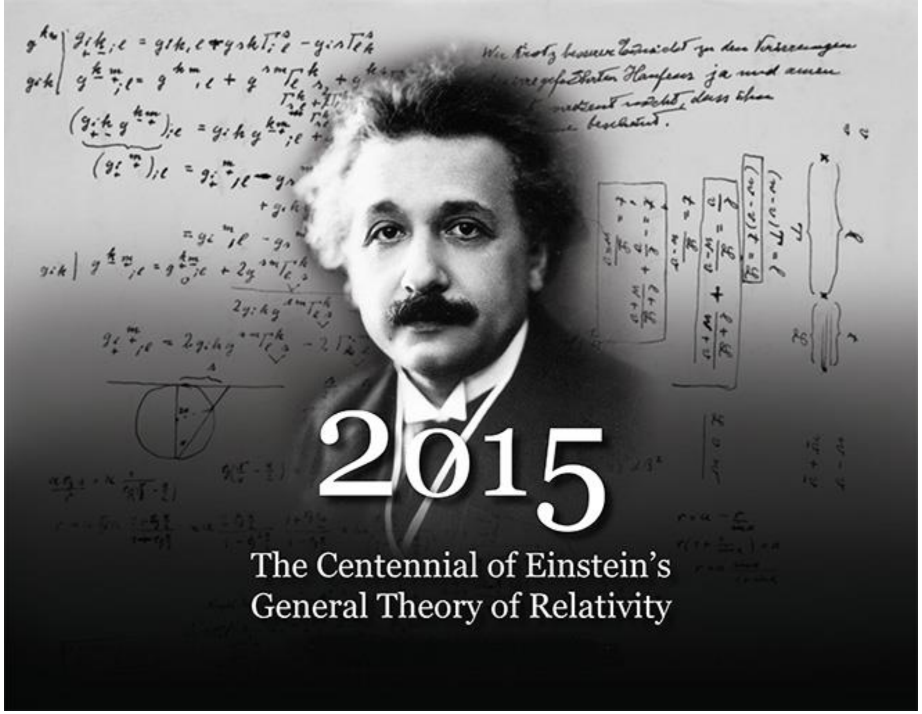
[Kazantzidis, Perivolaropoulos, PRD97]

- If not systematics, the data indicate a lack of "gravitational power" in structures on intermediate-small cosmological scales (expressed as smaller Ωm at z<0.6, or smaller σ8, or wDE<-1).
- It could be reconciled by a mechanism that reduces the rate of clustering between recombination and today: Hot Dark Matter, Dark Matter that clusters differently at small scales, or Modified Gravity.

# Knowledge of Physics

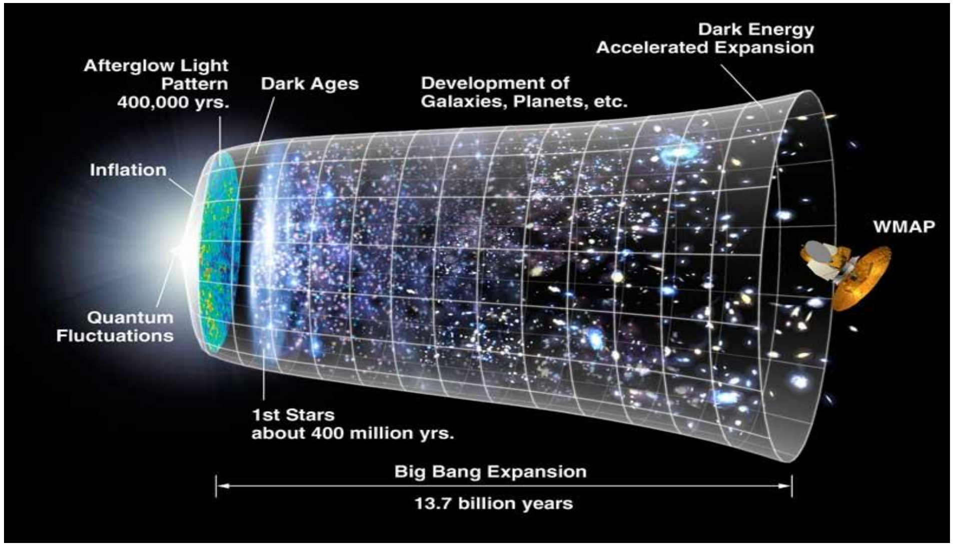
Knowledge of Physics: Standard Model + General Relativity

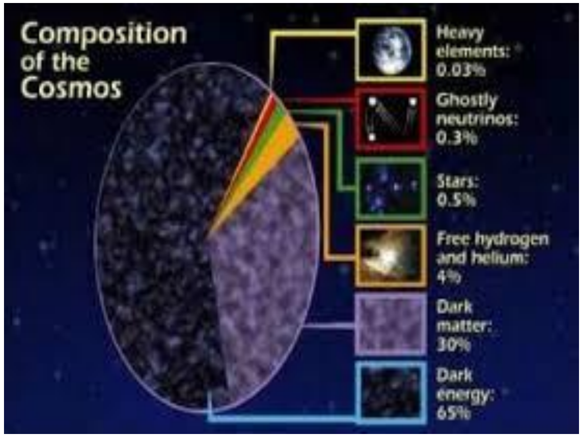




# Modified/new knowledge of physics

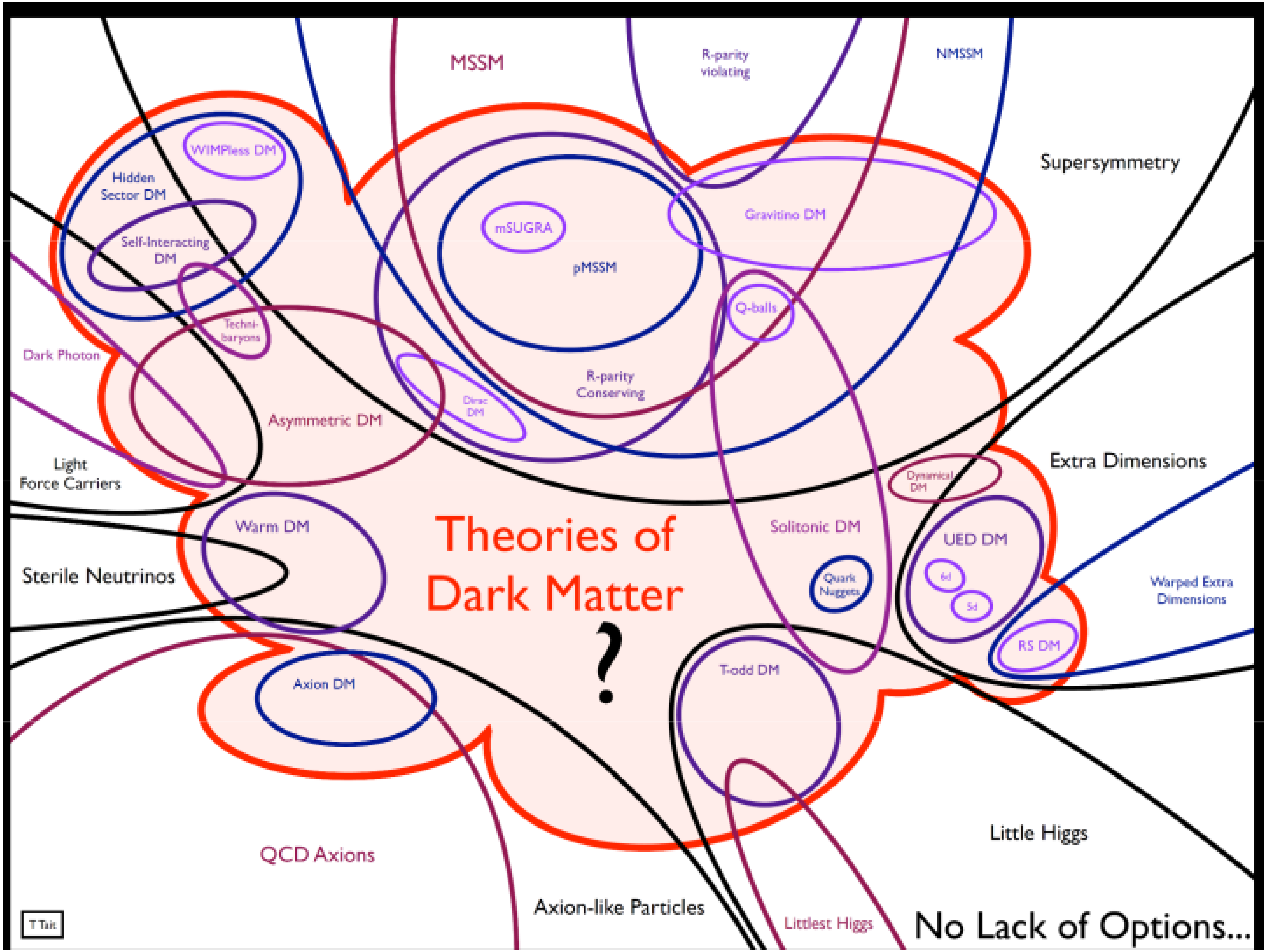
So can our knowledge of Physics describes all these?





Most probably, no!

We definitely need new physics for Inflation and Dark matter. Maybe for dark energy.



# Theories of Dark Matter

?

MSSM

R-parity violating

NMSSM

Supersymmetry

WIMPless DM

Hidden Sector DM

Self-Interacting DM

Techni-baryons

mSUGRA

pMSSM

Gravitino DM

Q-balls

Dark Photon

Dirac DM

R-parity Conserving

Asymmetric DM

Light Force Carriers

Extra Dimensions

Dynamical DM

Warm DM

Solitonic DM

UED DM

6d

5d

Sterile Neutrinos

Quark Nuggets

Warped Extra Dimensions

RS DM

Axion DM

T-odd DM

Little Higgs

QCD Axions

Axion-like Particles

Littlest Higgs

T Tait

## No Lack of Options...

# Why Modified Gravity?

We need to modify something:

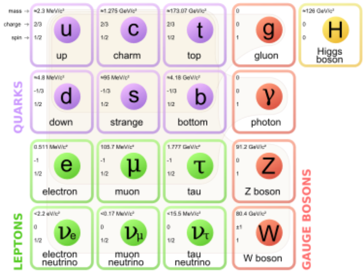
The universe content

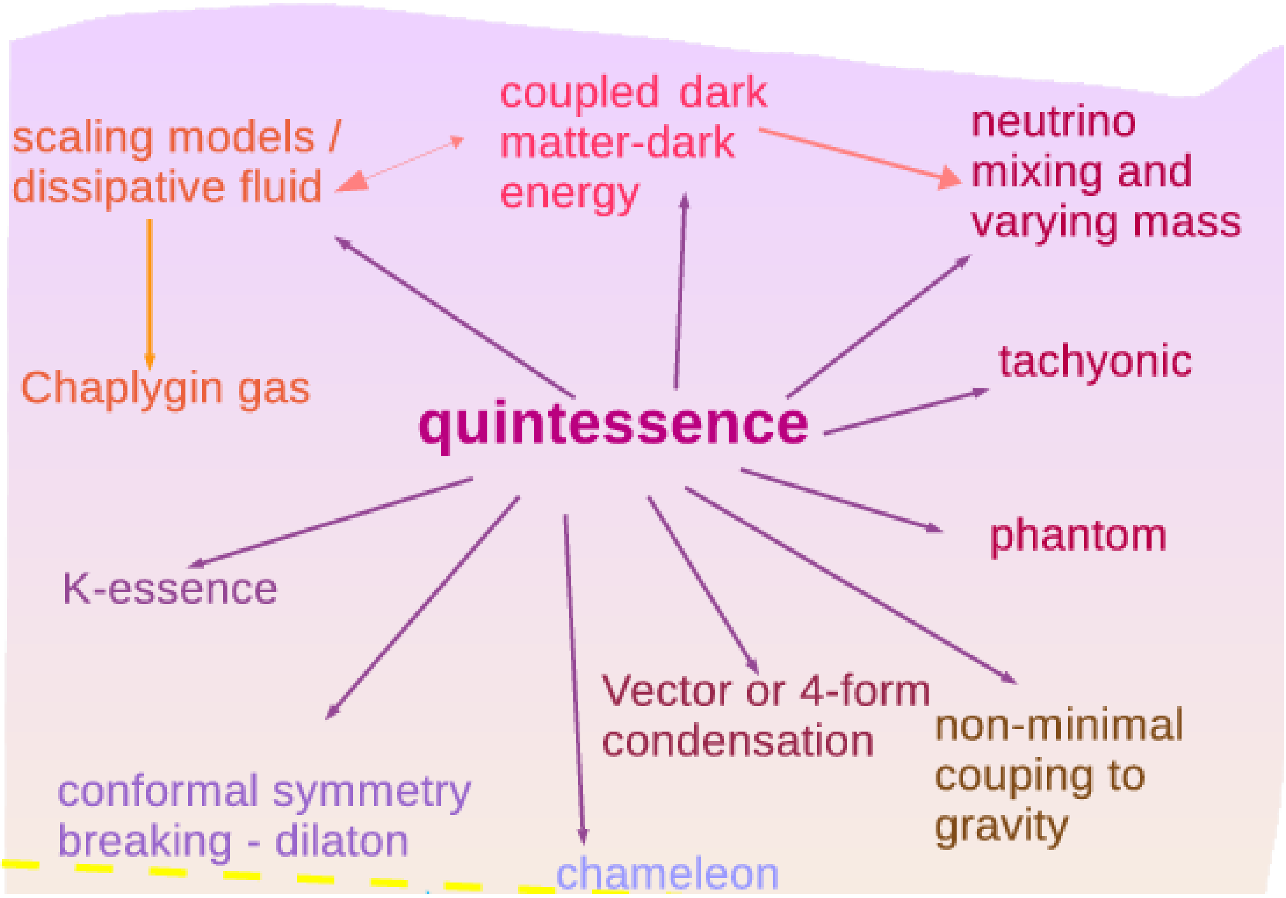
or

The theory of Gravity

# Dark Energy-Inflation

- Add a scalar field φ in the Universe content





scaling models / dissipative fluid

coupled dark matter-dark energy

neutrino mixing and varying mass

Chaplygin gas

tachyonic

**quintessence**

phantom

K-essence

Vector or 4-form condensation

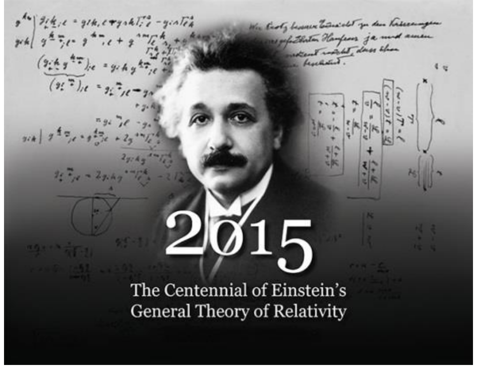
non-minimal couping to gravity

conformal symmetry breaking - dilaton

chameleon

# General Relativity

- Einstein 1915: **General Relativity**:


2015

The Centennial of Einstein's General Theory of Relativity

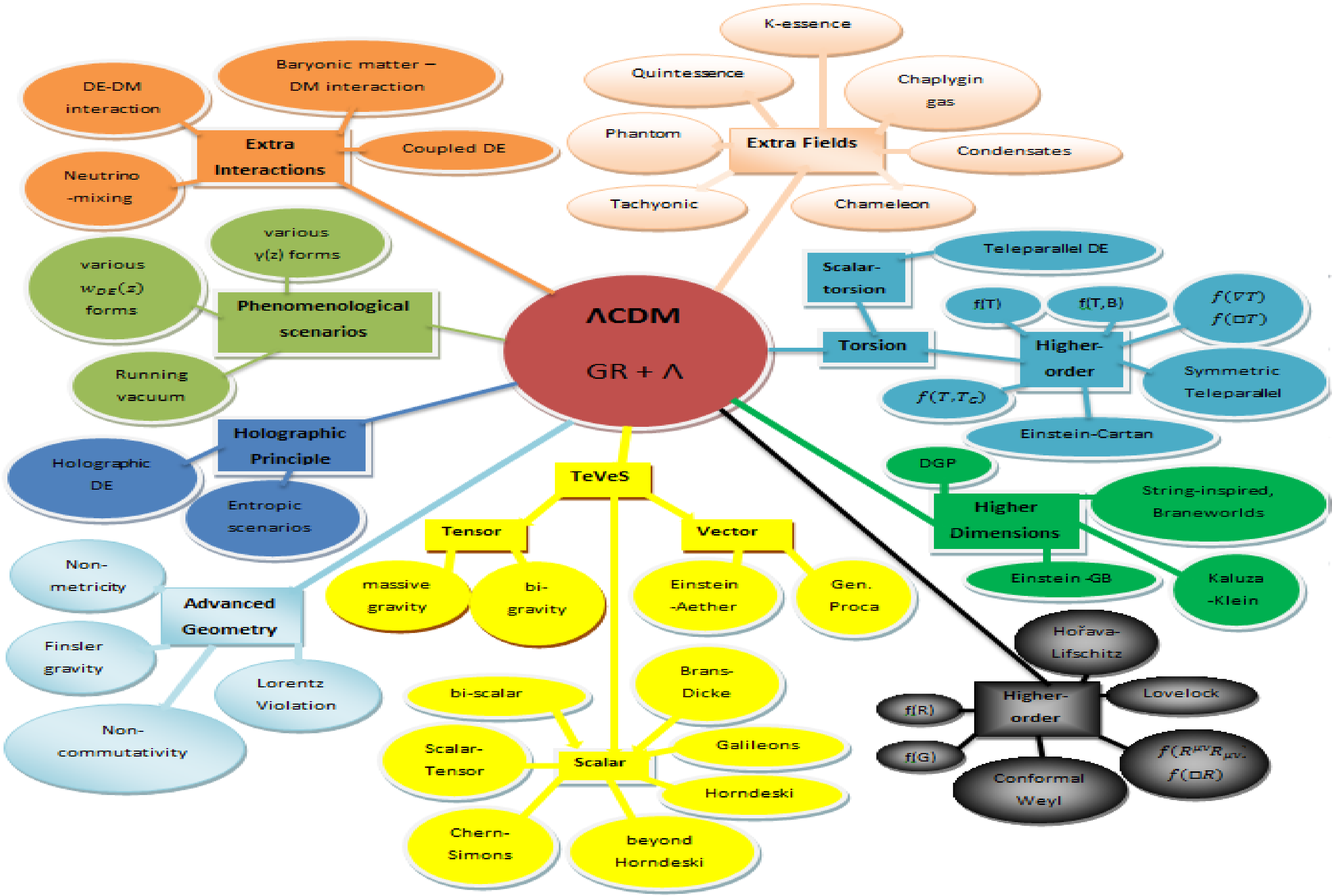
energy-momentum source of spacetime Curvature

$$\boxed{S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,[R-2\Lambda]+\int d^4x\, L_m(g_{\mu\nu},\psi)}$$

$$\Rightarrow \quad R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+g_{\mu\nu}\Lambda=8\pi G\, T_{\mu\nu}$$

with $$T^{\mu\nu}\equiv\frac{2}{\sqrt{-g}}\frac{\delta L_m}{\delta g_{\mu\nu}}$$

# Modified Gravity



**ΛCDM**

GR + Λ

- **Extra Interactions**
  - DE-DM interaction
  - Baryonic matter – DM interaction
  - Coupled DE
  - Neutrino -mixing
- **Extra Fields**
  - K-essence
  - Quintessence
  - Chaplygin gas
  - Phantom
  - Condensates
  - Tachyonic
  - Chameleon
- **Phenomenological scenarios**
  - various γ(z) forms
  - various $w_{DE}(z)$ forms
  - Running vacuum
- **Torsion**
  - Scalar-torsion
    - Teleparallel DE
  - Higher-order
    - f(T)
    - f(T,B)
    - $f(\nabla T)$ $f(\Box T)$
    - Symmetric Teleparallel
    - $f(T,T_G)$
    - Einstein-Cartan
- **Holographic Principle**
  - Holographic DE
  - Entropic scenarios
- **TeVeS**
  - Tensor
    - massive gravity
    - bi-gravity
  - Vector
    - Einstein -Aether
    - Gen. Proca
  - Scalar
    - bi-scalar
    - Brans-Dicke
    - Galileons
    - Scalar-Tensor
    - Horndeski
    - Chern-Simons
    - beyond Horndeski
- **Higher Dimensions**
  - DGP
  - String-inspired, Braneworlds
  - Einstein -GB
  - Kaluza -Klein
- **Advanced Geometry**
  - Non-metricity
  - Finsler gravity
  - Lorentz Violation
  - Non-commutativity
- **Higher-order**
  - Hořava-Lifschitz
  - Lovelock
  - f(R)
  - f(G)
  - $f(R^{\mu\nu}R_{\mu\nu})$ $f(\Box R)$
  - Conformal Weyl

# Cosmology-background

- Homogeneity and isotropy: $ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$

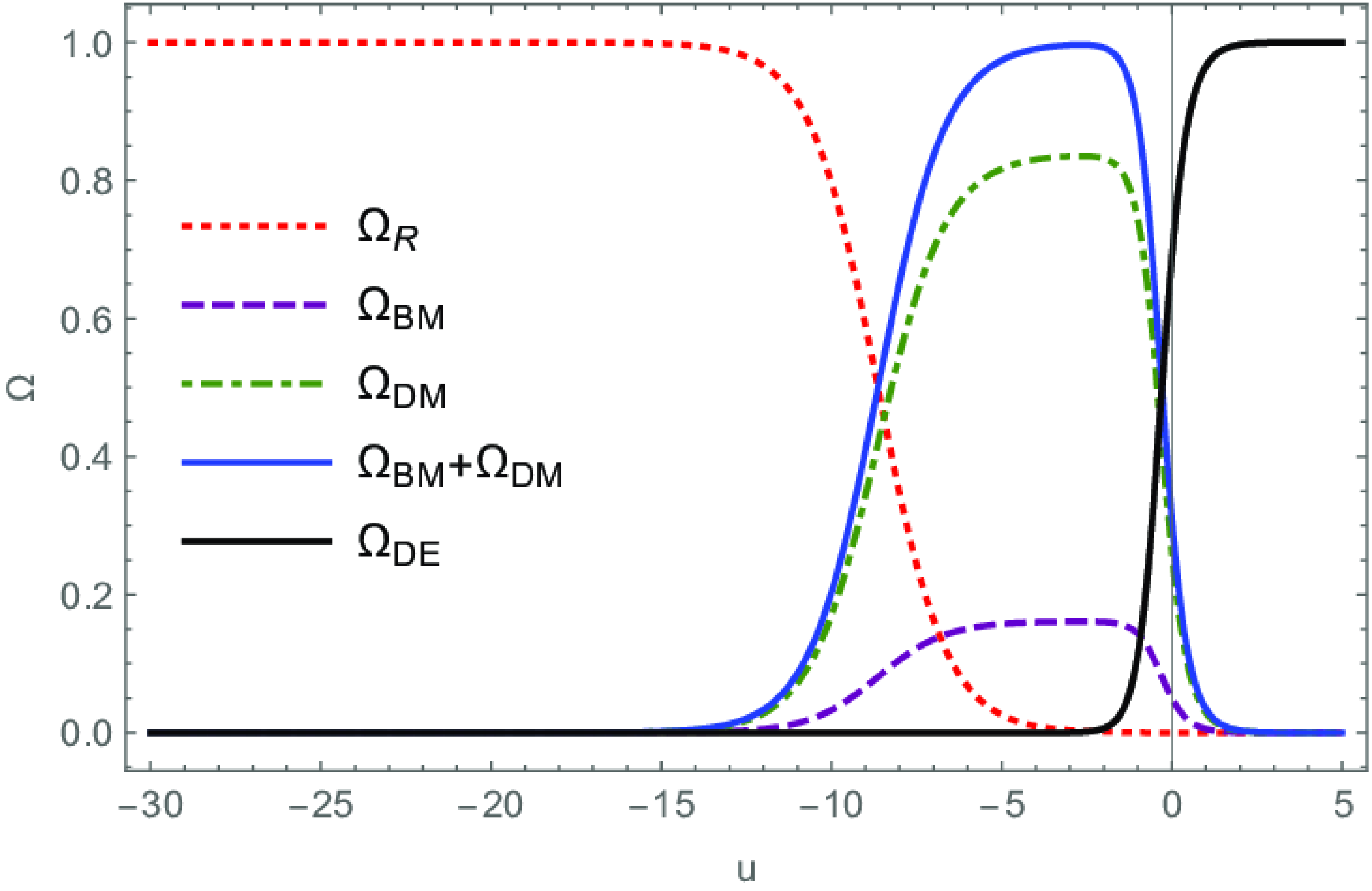
- Background evolution (Friedmann equations) in flat space

$$H^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_{DE}\right)$$
$$\dot{H} = -4\pi G\left(\rho_m + p_m + \rho_{DE} + p_{DE}\right),$$

(the effective DE sector can be either Λ or any possible modification)

- One must obtain a H(z) and Ωm(z) and wDE(z) in agreement with observations (SNIa, BAO, CMB shift parameter, H(z) etc)

# Cosmology-background



# Cosmology-perturbations

- Perturbation evolution: $\ddot{\delta}+2H\dot{\delta}-4\pi G_{\rm eff}\,\rho\,\delta\approx 0$ where $\delta\equiv\delta\rho/\rho$

  where $G_{\rm eff}(z,k)$ is the effective Newton's constant, given by

  $$\nabla^2\phi\approx 4\pi G_{\rm eff}\,\rho\,\delta,$$

  under the scalar metric perturbation $ds^2=-(1+2\phi)dt^2+a^2(1-2\psi)d\vec{x}^{\,2}$

- Hence: $\delta''+\left(\frac{(H^2)'}{2\,H^2}-\frac{1}{1+z}\right)\delta'\approx\frac{3}{2}(1+z)\frac{H_0^2}{H^2}\frac{G_{\rm eff}(z,k)}{G_N}\,\Omega_{0m}\delta$

  with $f(a)=\frac{dln\delta}{dlna}$ the growth rate, with $f(a)=\Omega_{\rm m}(a)^{\gamma(a)}$ and $\Omega_{\rm m}(a)\equiv\frac{\Omega_{0m}\,a^{-3}}{H(a)^2/H_0^2}$

- One can define the observable: $\boxed{f\sigma_8(a)\equiv f(a)\cdot\sigma(a)=\frac{\sigma_8}{\delta(1)}\,a\,\delta'(a)}$

  with $\sigma(a)=\sigma_8\frac{\delta(a)}{\delta 1}$ the z-dependent rms fluctuations of the linear density field within spheres of radius $R=8h^{-1}{\rm Mpc}$, and σ8 its value today.

# Cosmology-perturbations