National Technical University of Athens

Modified Gravity Cosmology III

Emmanuel N. Saridakis

National Observatory of Athens

CANTATA

Cosmology and Astrophysics Network for Theoretical Advances and Training Actions

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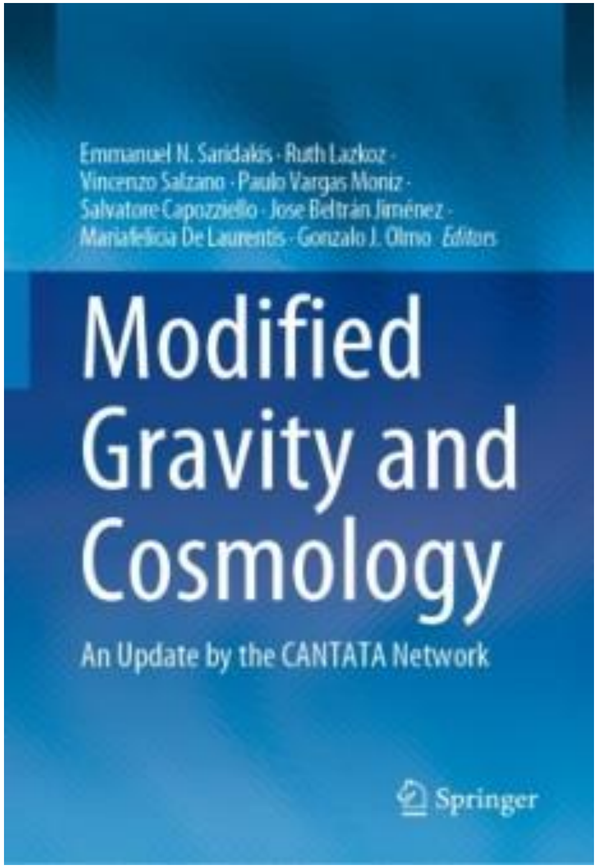
Bibliography



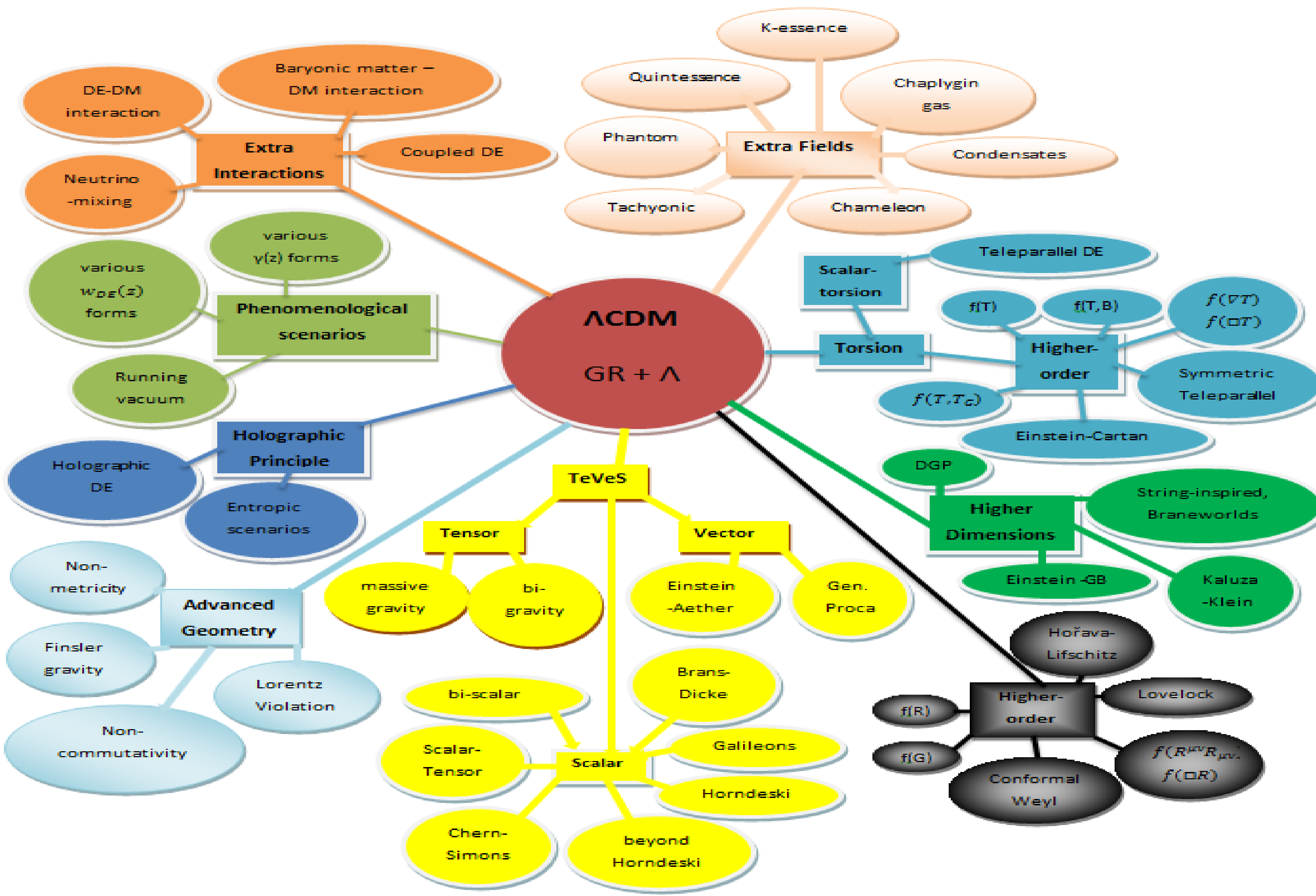
- Modified Gravity and Cosmology: An Update by the CANTATA Network
  E. N. Saridakis et al. 2105.12582 [gr-qc]

- Modified Gravity and Cosmology,
  T. Clifton, P. G. Ferreira, A. Padilla, C. Skordis, Phys.Rept. 513 (2012) 1-189, 1106.2476 [astro-ph.CO]

- Extended Theories of Gravity
  S. Capozziello, M. De Laurentis, Phys.Rept. 509 (2011) 167-321, 1108.6266 [gr-qc]

- f(T) teleparallel gravity and cosmology
  YF. Cai, S. Capozziello, M. De Laurentis, E. N. Saridakis, Rept.Prog.Phys. 79 (2016) 10, 106901, 1511.07586 [gr-qc]

Modified Gravity



TORSIONAL GRAVITY

"Those that do not know geometry are not allowed to enter".
Front Door of Plato's Academy

Introduction

- Einstein 1916: **General Relativity**:

  energy-momentum source of spacetime Curvature

  Levi-Civita connection: Zero Torsion

- Einstein 1928: **Teleparallel Equivalent of GR:**

  Weitzenbock connection: Zero Curvature

Curvature and Torsion

- Vierbeins e_A^{μ} : four linearly independent fields in the tangent space

  $g_{\mu\nu}(x) = \eta_{AB}\, e_{\mu}^{A}(x)\; e_{\nu}^{B}(x)$

- Connection: ω_{ABC}

- Curvature tensor: $R^{A}_{B\mu\nu} = \omega^{A}_{B\nu,\mu} - \omega^{A}_{B\mu,\nu} + \omega^{A}_{C\mu}\omega^{C}_{B\nu} - \omega^{A}_{C\nu}\omega^{C}_{B\mu}$

- Torsion tensor: $T^{A}_{\mu\nu} = e^{A}_{\nu,\mu} - e^{A}_{\mu,\nu} + \omega^{A}_{B\mu}e^{B}_{\nu} - \omega^{A}_{B\nu}e^{B}_{\mu}$

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- Torsion tensor: $T^A_{\mu\nu} = e^A_{\nu,\mu} - e^A_{\mu,\nu} + \omega^A_{B\mu}e^B_\nu - \omega^A_{B\nu}e^B_\mu$
- Levi-Civita connection and Contortion tensor: $\omega_{ABC} = \Gamma_{ABC} + K_{ABC}$

$$K_{ABC} = \frac{1}{2}\left(T_{CAB} - T_{BCA} - T_{ABC}\right) = -K_{BAC}$$

- Curvature and Torsion Scalars: $\boxed{R = \overline{R} + T - 2\left(T_\nu^{\ \nu\mu}\right)_{;\mu}}$

$$R = g^{\mu\nu}R_{\mu\nu} = g^{\mu\nu}R^\rho_{\mu\rho\nu}$$

$$T = \frac{1}{4}T^{\rho\mu\nu}T_{\rho\mu\nu} + \frac{1}{2}T^{\rho\mu\nu}T_{\nu\mu\rho} - T^\rho_{\ \rho\mu}T_\nu^{\ \nu\mu}$$

Introduction

- **Gauge Principle**: global symmetries replaced by local ones:

  The group generators give rise to the compensating fields

  It works perfect for the standard model of strong, weak and E/M interactions

  $$SU(3) \times SU(2) \times U(1)$$

- Can we apply this to gravity?

Introduction

- Formulating the gauge theory of gravity (mainly after 1960):
- Start from Special Relativity

$\Longrightarrow$ Apply (Weyl-Yang-Mills) gauge principle to its Poincaré-group symmetries

$\Longrightarrow$ Get Poinaré gauge theory:

Both curvature and torsion appear as field strengths

- Torsion is the field strength of the translational group

(Teleparallel and Einstein-Cartan theories are subcases of Poincaré theory)

Introduction

- One could extend the gravity gauge group (SUSY, conformal, scale, metric affine transformations) obtaining SUGRA, conformal, Weyl, metric affine gauge theories of gravity

- In all of them torsion is always related to the gauge structure.
- Thus, a possible way towards gravity quantization would need to bring torsion into gravity description.

Introduction

- 1998: Universe acceleration

$\Rightarrow$Thousands of work in Modified Gravity

(f(R), Gauss-Bonnet, Lovelock, nonminimal scalar coupling, nonminimal derivative coupling, Galileons, Hordenski, massive etc)

- Almost all in the curvature-based formulation of gravity

Introduction

- 1998: Universe acceleration

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- Almost all in the curvature-based formulation of gravity

- So question: Can we modify gravity starting from its torsion-based formulation?

torsion $\Rightarrow$ gauge $? \Rightarrow$ quantization

modification $\Rightarrow$ full theory $? \Rightarrow$ quantization

Teleparallel Equivalent of General Relativity (TEGR)

- Let’s start from the simplest tosion-based gravity formulation, namely TEGR:
- Vierbeins e_A^{μ} : four linearly independent fields in the tangent space

$$g_{\mu\nu}(x) = \eta_{AB}\, e_{\mu}^{A}(x)\, e_{\nu}^{B}(x)$$

- Use curvature-less Weitzenböck connection instead of torsion-less Levi-Civita one: $\Gamma_{\nu\mu}^{\lambda\{W\}} = e_A^{\lambda}\partial_{\mu} e_{\nu}^{A}$
- Torsion tensor:

$$T_{\mu\nu}^{\lambda} = \Gamma_{\nu\mu}^{\lambda\{W\}} - \Gamma_{\mu\nu}^{\lambda\{W\}} = e_A^{\lambda}\left(\partial_{\mu} e_{\nu}^{A} - \partial_{\nu} e_{\mu}^{A}\right)$$

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- Torsion tensor:

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- Lagrangian (imposing coordinate, Lorentz, parity invariance, and up to 2nd order in torsion tensor)

$$\boxed{L \equiv T = \frac{1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\mu\rho} - T_{\rho\mu}^{\rho} T_{\nu}^{\nu\mu}}$$

- **Completely equivalent** with GR at the level of equations

f(T) Gravity and f(T) Cosmology

- f(T) Gravity: Simplest torsion-based modified gravity
- Generalize T to f(T) (inspired by f(R))

$$S = \frac{1}{16\pi G}\int d^4x\, e\,\left[T + f(T)\right] + S_m$$

- Equations of motion:

$$e^{-1}\partial_\mu\left(e e_A^\rho S_\rho^{\mu\nu}\right)\left(1+f_T\right) - e_A^\lambda T_{\mu\lambda}^\rho S_\rho^{\nu\mu} + e_A^\rho S_\rho^{\mu\nu}\partial_\mu(T)\, f_{TT} - \frac{1}{4}e_A^\nu[T+f(T)] = 4\pi G e_A^\rho \mathrm{T}_\rho^{\nu\{\mathrm{EM}\}}$$

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- Equations of motion:

$$e^{-1}\partial_\mu\left(e e_A^\rho S_\rho^{\mu\nu}\right)(1+f_T) - e_A^\lambda T^\rho_{\mu\lambda} S_\rho^{\nu\mu} + e_A^\rho S_\rho^{\mu\nu}\partial_\mu(T) f_{TT} - \frac{1}{4} e_A^\nu [T + f(T)] = 4\pi G e_A^\rho \mathrm{T}_\rho^{\nu\{\mathrm{EM}\}}$$

- f(T) Cosmology: Apply in FRW geometry:

$$e_\mu^A = diag(1,a,a,a) \;\Rightarrow\; ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j$$ (not unique choice)

- Friedmann equations:

$$H^2 = \frac{8\pi G}{3}\rho_m - \frac{f(T)}{6} - 2 f_T H^2$$

$$\dot{H} = -\frac{4\pi G(\rho_m + p_m)}{1 + f_T - 12H^2 f_{TT}}$$

- Find easily

$$T = -6H^2$$

f(T) Cosmology: Background

- Effective Dark Energy sector:

$$\rho_{DE} \equiv \frac{3}{8\pi G_N}\left[-\frac{f}{6}+\frac{Tf_T}{3}\right],$$

$$P_{DE} \equiv \frac{1}{16\pi G_N}\left[\frac{f-f_T T+2T^2 f_{TT}}{1+f_T+2Tf_{TT}}\right]$$

$$w_{DE} = -\frac{f - Tf_T + 2T^2 f_{TT}}{[1+f_T+2Tf_{TT}][f-2Tf_T]}$$

[Linder PRD 82]

- Interesting cosmological behavior: Late-time acceleration, Inflation etc

[Cai, Capozziello, De Laurentis, Saridakis, Rept.Prog.Phys. 79]

f(T) Cosmology: Background

- Re-write Firedmann Equation as:

$$E^2(z,\mathbf{r}) = \Omega_{m0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \Omega_{F0}\,y(z,\mathbf{r})$$

with $E^2(z) \equiv \frac{H^2(z)}{H_0^2} = \frac{T(z)}{T_0}$ and $\Omega_{F0} = 1 - \Omega_{m0} - \Omega_{r0}$;

- $y(z,\mathbf{r}) = \frac{1}{T_0\Omega_{F0}}\left[f - 2Tf_T\right]$ quantifies the deviation from ΛCDM (for f=const. we obtain ΛCDM)

f(T) Cosmology: Perturbations

- For scalar perturbations:

$$e^0_\mu = \delta^0_\mu (1+\psi) \ , \ e^\alpha_\mu = \delta^\alpha_\mu a(1-\phi)$$

$$\Rightarrow \ ds^2 = (1+2\psi)dt^2 - a^2(1-2\phi)\delta_{ij} dx^i dx^j$$

- Obtain Perturbation Equations.

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\text{eff}}\, \rho\, \delta \approx 0$$

$$Q(a) = \frac{G_{\text{eff}}(a)}{G_N} = \frac{1}{1+f_T}$$

[Chen, Dent, Dutta, Saridakis PRD 83], [Dent, Dutta, Saridakis JCAP 1101]

$$\begin{aligned}
\delta \overset{\mathbf{em}}{T}{}_0{}^0 &= -\delta\rho_m \\
\delta \overset{\mathbf{em}}{T}{}_0{}^i &= a^{-2}(\rho_m + p_m)(-\partial_i \delta u) \\
\delta \overset{\mathbf{em}}{T}{}_i{}^0 &= (\rho_m + p_m)(\partial_i \delta u) \\
\delta \overset{\mathbf{em}}{T}{}_i{}^j &= \delta_{ij}\delta p_m + \partial_i\partial_j \pi^S.
\end{aligned}$$

$$\begin{aligned}
E^0_0 \equiv\ & (1+f'_0)(\nabla^2\phi) + 6(1+f'_0)H\dot{\phi} \\
& +6(1+f'_0)H^2\psi - 3f'_1H^2 \\
& -\frac{T_1+f_1}{4} = -4\pi G\delta\rho_m, \\
E^i_0 \equiv\ & (1+f'_0)\partial_i\dot{\phi} + (1+f'_0)H\partial_i\psi \\
& -12H\dot{H}f''_0\partial_i\phi = -4\pi G(\rho_m+p_m)\partial_i\delta u, \\
E^0_a \equiv\ & 12H^2\partial_i\delta^i_a(\dot{\phi}+H\psi)f''_0 - (1+f'_0)\partial_i\delta^i_a(\dot{\phi}+H\psi) \\
& = 4\pi G(\rho_m+p_m)\partial_i\delta^i_a\delta u,
\end{aligned}$$

$$\begin{aligned}
E^i_a\delta^a_i \equiv\ & \frac{f'_1}{a}\left(-3H^2-\dot{H}\right) + \frac{f''_1}{a}\left(12H^2\dot{H}\right) \\
& -\frac{(1+f'_0)}{2a}\sum_{b\neq a}\partial^j\delta^b_j\partial_i\delta^i_b(\psi-\phi) \\
& -\frac{\phi(T_0+f_0)}{4a} - \frac{T_1+f_1}{4a} \\
& +\frac{(1+f'_0)}{a}[6H\dot{\phi} + 6H^2\psi - 3H^2\phi \\
& \qquad +\ddot{\phi} + \dot{H}(2\psi-\phi) + H\dot{\psi}] \\
& +\frac{f''_0}{a}(-24H\dot{H}\dot{\phi} - 48\psi H^2\dot{H} - 12H^2\ddot{\phi} \\
& \qquad -12H^3\dot{\psi} + 12H^2\dot{H}\phi) \\
& \qquad = \frac{4\pi G}{a}(p_m\phi + \delta p_m), \\
E^i_{b;\,b\neq a}\delta^a_i \equiv\ & \frac{(1+f'_0)}{2}\partial_j\delta^j_b\partial^i\delta^a_i(\phi-\psi) \\
& = 4\pi G a^2\partial_j\delta^j_b\partial^i\delta^a_i\pi^S
\end{aligned}$$

Viable f(T) models

- 1) Power-law model (f1CDM)

$$f(T) = \alpha(-T)^b \qquad \alpha = (6H_0^2)^{1-b}\frac{\Omega_{F0}}{2b-1}$$

$$y(z,b) = E^{2b}(z,b) \qquad G_{\rm eff}(z) = \frac{G_N}{1+\frac{b\Omega_{F0}}{(1-2b)E^{2(1-b)}}}$$

- 2) The Linder model (f2CDM)

$$f(T) = \alpha T_0(1-e^{-p\sqrt{T/T_0}}) \qquad \alpha = \frac{\Omega_{F0}}{1-(1+p)e^{-p}}$$

$$y(z,p) = \frac{1-(1+pE)e^{-pE}}{1-(1+p)e^{-p}} \qquad G_{\rm eff}(z) = \frac{G_N}{1+\frac{\Omega_{F0}p\ e^{-pE}}{2E[1-(1+p)e^{-p}]}}$$

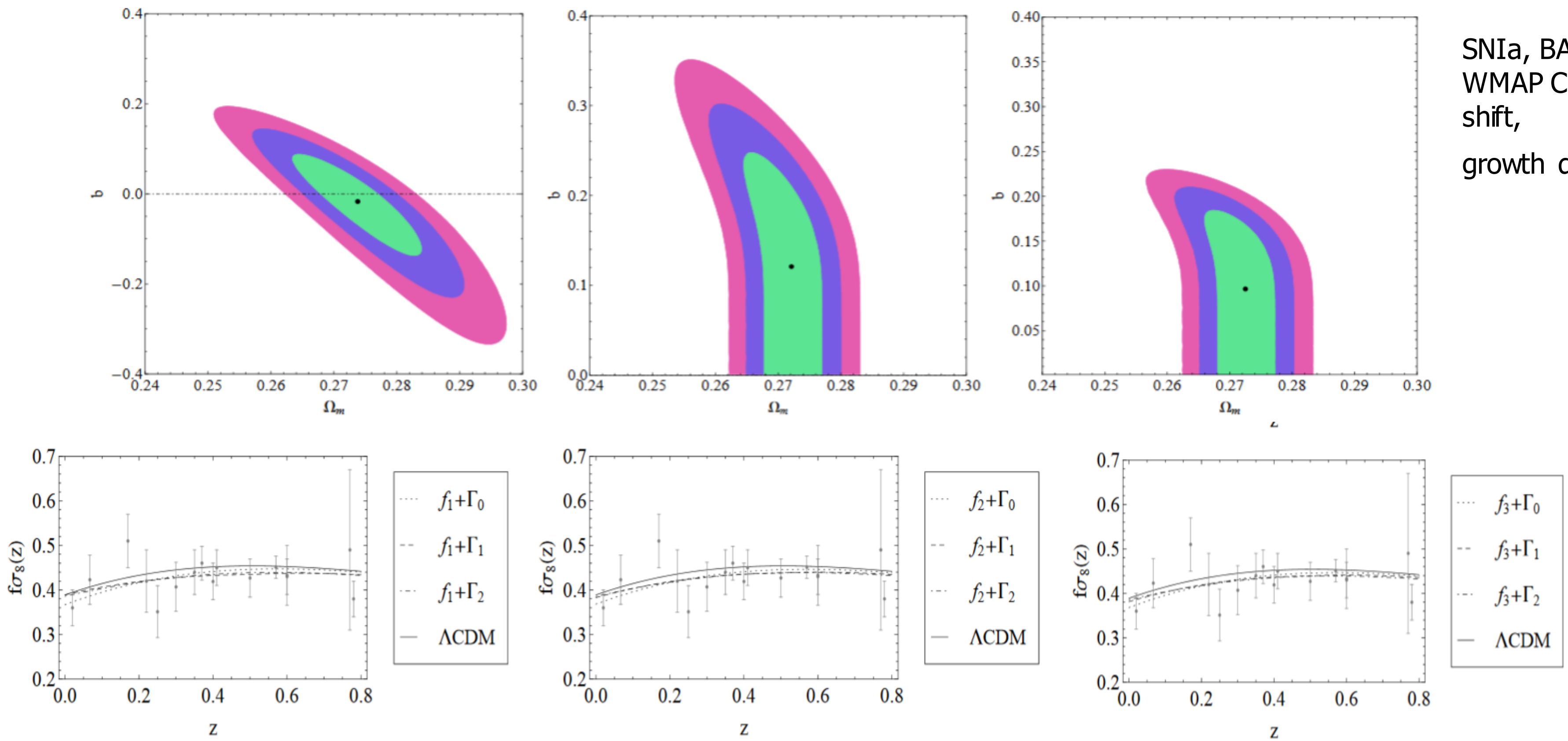
- 3) The exponential model (f3CDM)

$$f(T) = \alpha T_0(1-e^{-pT/T_0}) \qquad \alpha = \frac{\Omega_{F0}}{1-(1+2p)e^{-p}}$$

$$y(z,p) = \frac{1-(1+2pE^2)e^{-pE^2}}{1-(1+2p)e^{-p}} \qquad G_{\rm eff}(z) = \frac{G_N}{1+\frac{\Omega_{F0}p\ e^{-pE^2}}{1-(1+2p)e^{-p}}}$$

[Nesseris, Basilakos, Saridakis, Perivolaropoulos, PRD 88]

Viable f(T) models



SNIa, BAO, WMAP CMB shift,

growth data

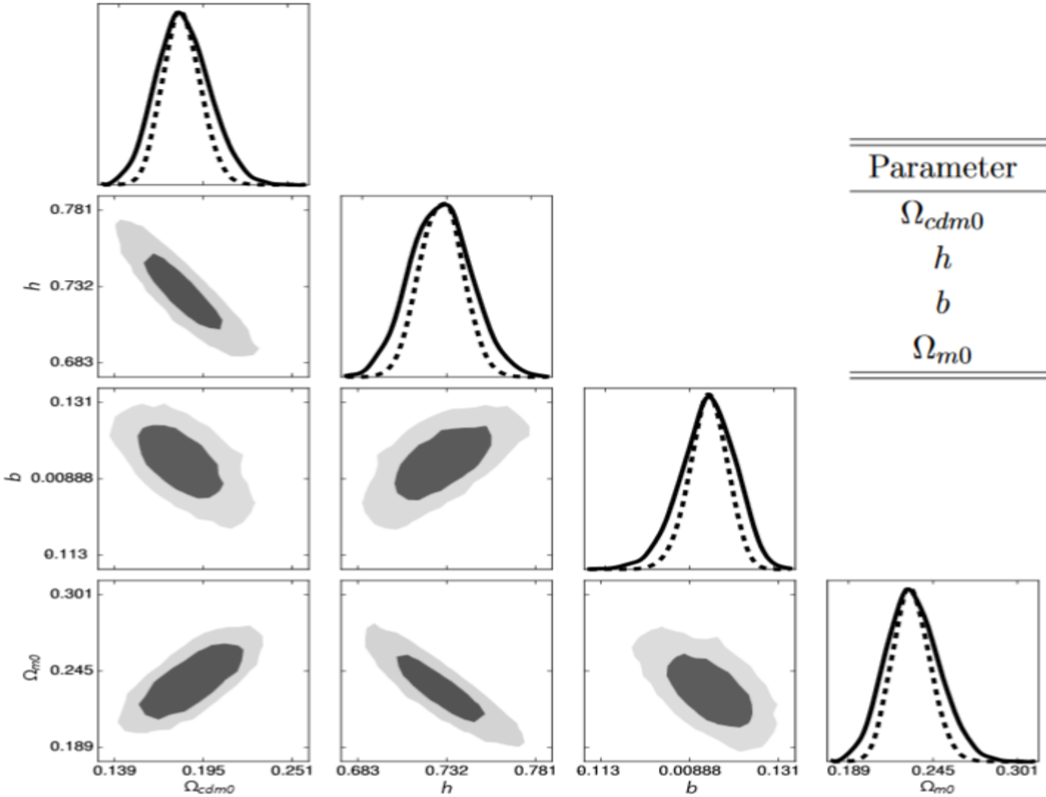
$$\gamma(z) = \begin{cases} \gamma_0, & \Gamma_0 \text{ model} \\ \gamma_0 + \gamma_1 z, & \Gamma_1 \text{ model} \\ \gamma_0 + \gamma_1 (1-a), & \Gamma_2 \text{ model.} \end{cases}$$

[Nesseris, Basilakos, Saridakis, Perivolaropoulos, PRD 88]

[Nunes, Pan, Saridakis, JCAP 1608]

Viable f(T) models

- Power-law (f1CDM): $f(T) = \alpha(-T)^b$ $\alpha = (6H_0^2)^{1-b} \dfrac{\Omega_{F0}}{2b-1}$



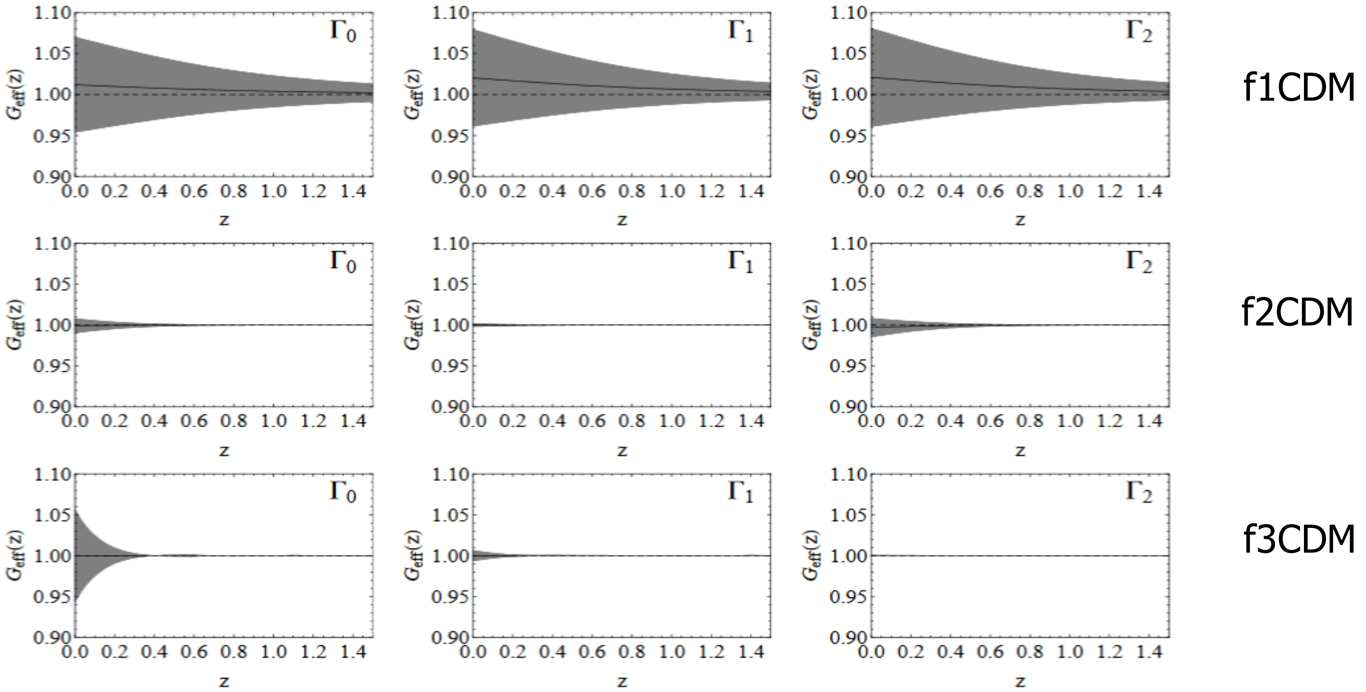
Parameter	best-fit	mean$\pm$ 1σ	95% lower	95% upper
Ω_{cdm0}	0.1806	$0.1835^{+0.016}_{-0.019}$	0.1503	0.2179
h	0.7292	$0.7275^{+0.017}_{-0.018}$	0.6945	0.7616
b	0.05536	$0.05128^{+0.025}_{-0.019}$	0.00622	0.09329
Ω_{m0}	0.2306	$0.2335^{+0.016}_{-0.019}$	0.2003	0.2679

SNIa, BAO, CC

[Nunes, Pan, Saridakis, JCAP 1608]

[Basilakos, Nesseris, Anagnostopoulos, Saridakis, 1803.09278]

Viable f(T) models



- In f(T) gravity we can indeed obtain $G_{\rm eff}/G_{\rm N}$ <1 for z<2, without affecting the background evolution.
- fσ8 tension may be alleviated. [Nesseris, Basilakos, Saridakis, Perivolaropoulos, PRD 88]

In other modified gravities: Not possible

- This behavior is not possible in other modified gravities. e.g.:

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}f(R,\phi,X) + \mathcal{L}_m\right) \qquad X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi.$$

$$G_{\rm eff}(a,k)/G_{\rm N} = \frac{1}{F}\frac{f_{,X} + 4\left(f_{,X}\frac{k^2}{a^2}\frac{F_{,R}}{F} + \frac{F_{,\phi}^2}{F}\right)}{f_{,X} + 3\left(f_{,X}\frac{k^2}{a^2}\frac{F_{,R}}{F} + \frac{F_{,\phi}^2}{F}\right)} \qquad F = F(R,\phi,X) = \partial_R f(R,\phi,X)$$

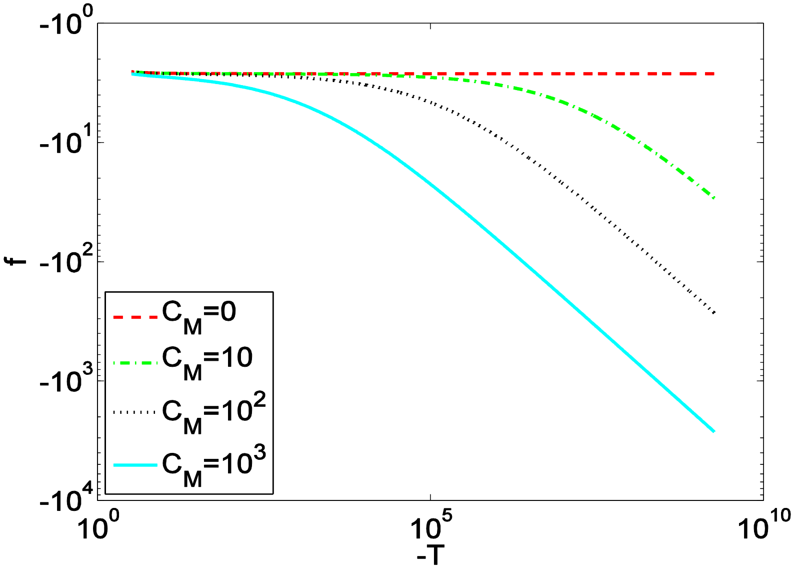
- $G_{\rm eff}/G_{\rm N}$ >1 for all models that do not have ghosts (i.e. with fR,fRR>0).

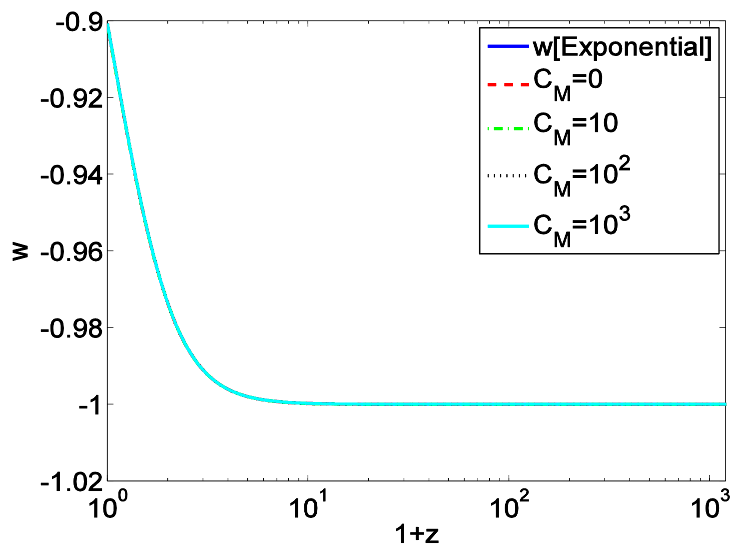
- On the contrary, f(T) gravity has second-order field equations and moreover perturbations are stable in a large part of the parameter phase.

f(T) Cosmology: Perturbations

- Application: Distinguish f(T) from quintessence
- 1) Reconstruct f(T) to coincide with a given **quintessence** scenario:

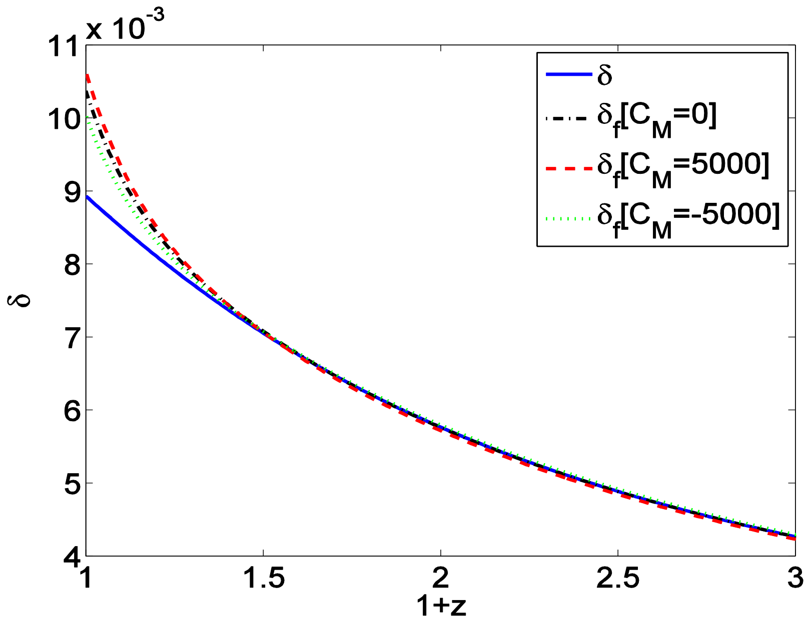
$$f(H) = 16\pi G H \int \frac{\rho_Q}{H^2} dH \;\; + CH \quad \text{with} \quad \rho_Q = \dot{\phi}^2/2 + V(\phi) \quad \text{and} \quad \mathrm{H} = \sqrt{-\mathrm{T}/6}$$

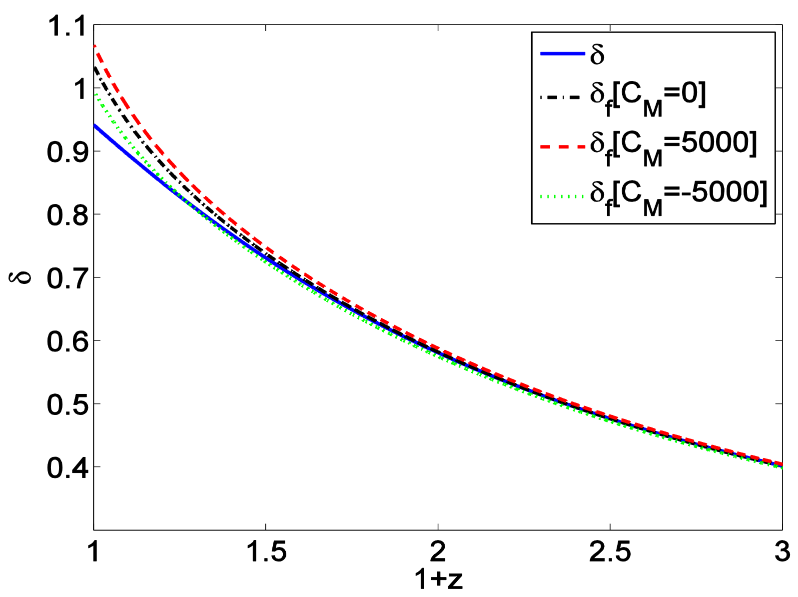




f(T) Cosmology: Perturbations

- Application: Distinguish f(T) from quintessence
- 2) Examine evolution of matter overdensity $\delta \equiv \frac{\delta\rho_m}{\rho_m}$





Non-minimally coupled scalar-torsion theory

- In curvature-based gravity, apart from $R+f(R)$ one can use $R+\xi R\varphi$^2
- Let's do the same in torsion-based gravity:

$$S=\int d^4x\, e\left[\frac{T}{2\kappa^2}+\frac{1}{2}\left(\partial_\mu\varphi\partial^\mu\varphi+\xi T\varphi^2\right)-V(\varphi)+L_m\right]$$

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- Friedmann equations in FRW universe:

$$H^2=\frac{\kappa^2}{3}(\rho_m+\rho_{DE})$$

$$\dot{H}=-\frac{\kappa^2}{2}(\rho_m+p_m+\rho_{DE}+p_{DE})$$

with effective Dark Energy sector: $\rho_{DE}=\frac{\dot{\varphi}^2}{2}+V(\varphi)-3\xi H^2\varphi^2$

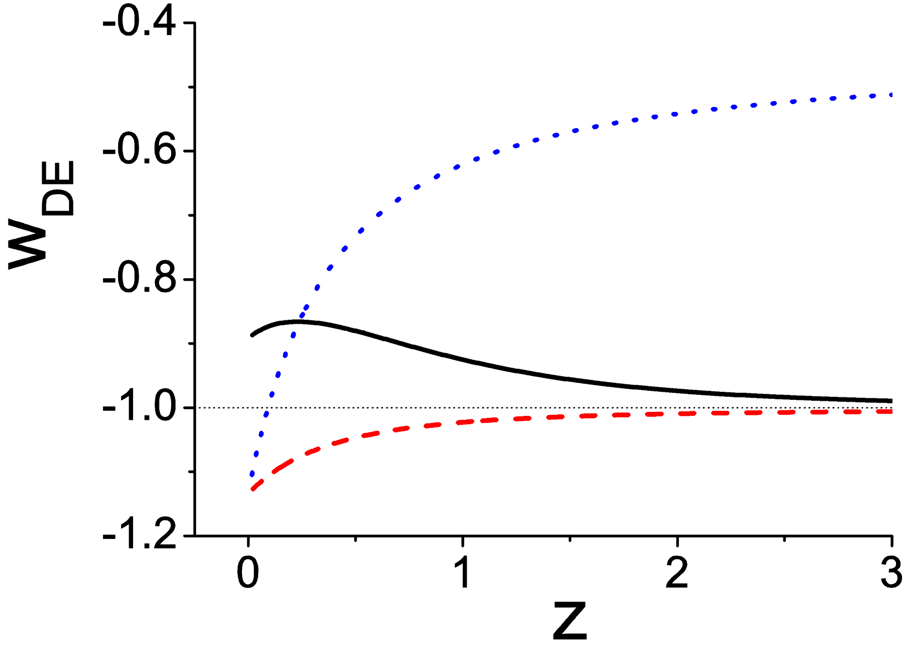
$$p_{DE}=\frac{\dot{\varphi}^2}{2}-V(\varphi)+4\xi H\varphi\dot{\varphi}+\xi\left(3H^2+2\dot{H}\right)\varphi^2$$

- **Different** than non-minimal quintessence!

(no conformal transformation in the present case)

Non-minimally coupled scalar-torsion theory

- Main advantage: Dark Energy may lie in the phantom regime or/and experience the phantom-divide crossing
- Teleparallel Dark Energy:



Scalar-torsion theories – Teleparallel Horndeski

$$a_\mu = \frac{1}{6}\epsilon_{\mu\nu\sigma\rho}T^{\nu\sigma\rho}\,,$$
$$v_\mu = T^\sigma{}_{\sigma\mu}\,,$$
$$t_{\sigma\mu\nu} = \frac{1}{2}\left(T_{\sigma\mu\nu} + T_{\mu\sigma\nu}\right) + \frac{1}{6}\left(g_{\nu\sigma}v_\mu + g_{\nu\mu}v_\sigma\right) - \frac{1}{3}g_{\sigma\mu}v_\nu\,,$$

$$T_{\rm ax} = a_\mu a^\mu = \frac{1}{18}\left(T_{\sigma\mu\nu}T^{\sigma\mu\nu} - 2T_{\sigma\mu\nu}T^{\mu\sigma\nu}\right)\,,$$
$$T_{\rm vec} = v_\mu v^\mu = T^\sigma{}_{\sigma\mu}T_\rho{}^{\rho\mu}\,,$$
$$T_{\rm ten} = t_{\sigma\mu\nu}t^{\sigma\mu\nu} = \frac{1}{2}\left(T_{\sigma\mu\nu}T^{\sigma\mu\nu} + T_{\sigma\mu\nu}T^{\mu\sigma\nu}\right) - \frac{1}{2}T^\sigma{}_{\sigma\mu}T_\rho{}^{\rho\mu}\,,$$

$$T = \frac{3}{2}T_{\rm ax} + \frac{2}{3}T_{\rm ten} - \frac{2}{3}T_{\rm vec}\,.$$

$$I_1 = t^{\mu\nu\sigma}\phi_{;\mu}\phi_{;\nu}\phi_{;\sigma}\,,$$
$$I_2 = v^\mu\phi_{;\mu}\,,$$
$$I_3 = a^\mu\phi_{;\mu}\,.$$

$$J_1 = a^\mu a^\nu\phi_{;\mu}\phi_{;\nu}\,,$$
$$J_2 = v^\mu v^\nu\phi_{;\mu}\phi_{;\nu}\,,$$
$$J_3 = v_\sigma t^{\sigma\mu\nu}\phi_{;\mu}\phi_{;\nu}\,,$$
$$J_4 = v_\mu t^{\sigma\mu\nu}\phi_{;\sigma}\phi_{;\nu}\,,$$
$$J_5 = t^{\sigma\mu\nu}t_\sigma{}^{\bar\mu}{}_\nu\phi_{;\mu}\phi_{;\bar\mu}\,,$$
$$J_6 = t^{\sigma\mu\nu}t_\sigma{}^{\bar\mu\bar\nu}\phi_{;\mu}\phi_{;\nu}\phi_{;\bar\mu}\phi_{;\bar\nu}\,,$$
$$J_7 = t^{\sigma\mu\nu}t^{\bar\sigma\bar\mu}{}_\sigma\phi_{;\mu}\phi_{;\nu}\phi_{;\bar\sigma}\phi_{;\bar\mu}\,,$$
$$J_8 = t^{\sigma\mu\nu}t_{\sigma\mu}{}^{\bar\nu}\phi_{;\nu}\phi_{;\bar\nu}\,,$$
$$J_9 = t^{\sigma\mu\nu}t^{\bar\sigma\bar\mu\bar\nu}\phi_{;\sigma}\phi_{;\mu}\phi_{;\nu}\phi_{;\bar\sigma}\phi_{;\bar\mu}\phi_{;\bar\nu}\,,$$
$$J_{10} = \epsilon^\mu{}_{\nu\rho\sigma}a^\nu t^{\alpha\rho\sigma}\phi_{;\mu}\phi_{;\alpha}\,.$$

$$\mathcal{L}_{\rm Tele} = G_{\rm Tele}(\phi, X, T, T_{\rm ax}, T_{\rm vec}, I_2, J_1, J_3, J_5, J_6, J_8, J_{10}),$$

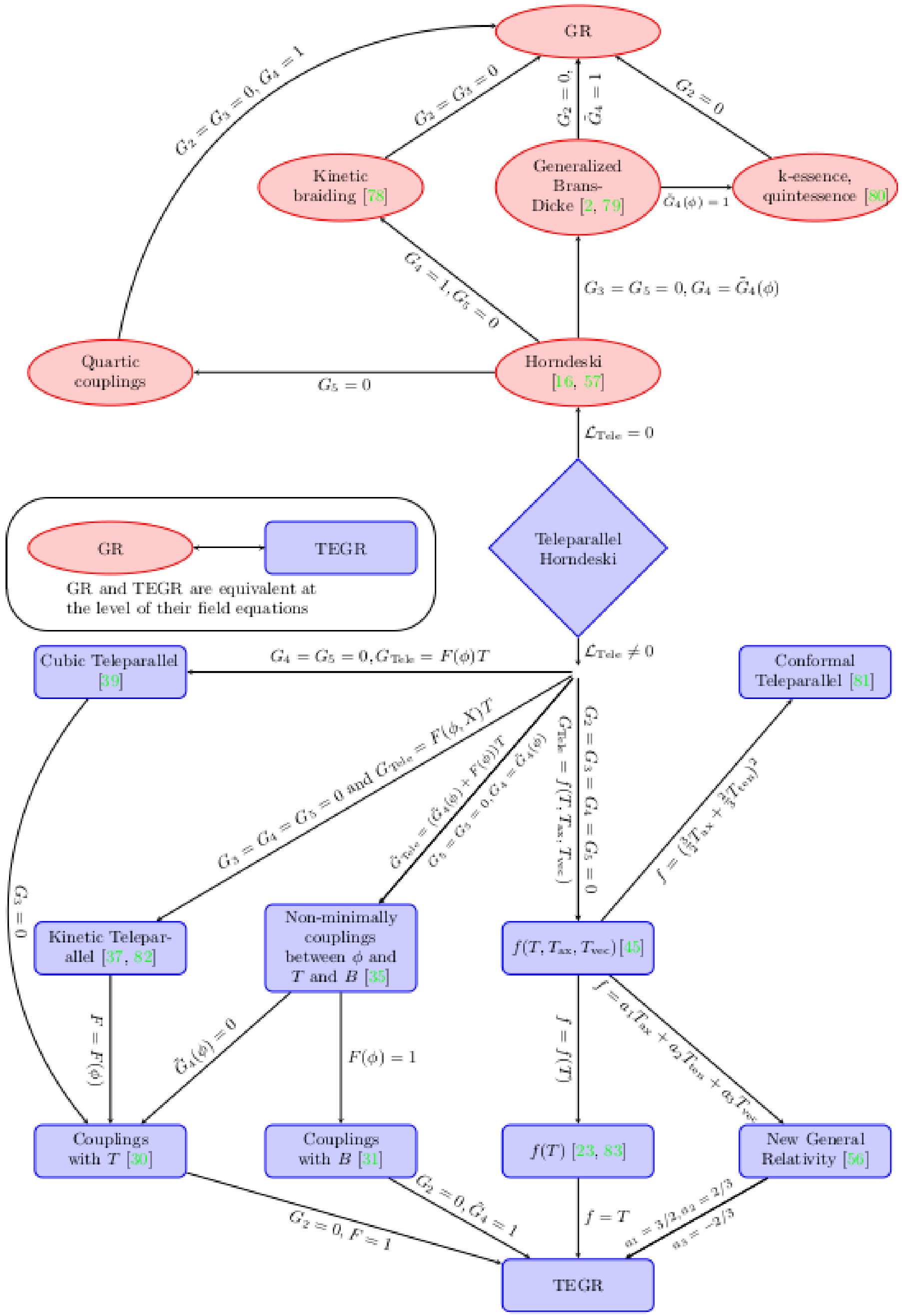


FIG. 1: Relationship between Teleparallel Horndeski and various theories.

Non-minimally matter-torsion coupled theory

- In curvature-based gravity, one can use $f(R)L_m$ coupling
- Let's do the same in torsion-based gravity:

$$S = \frac{1}{2\kappa^2}\int d^4x\, e\, \left\{T + f_1(T) + \left[1 + \lambda\, f_2(T)\right] L_m\right\}$$

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- Friedmann equations in FRW universe:

$$H^2=\frac{\kappa^2}{3}(\rho_m+\rho_{DE})$$

$$\dot{H}=-\frac{\kappa^2}{2}(\rho_m+p_m+\rho_{DE}+p_{DE})$$

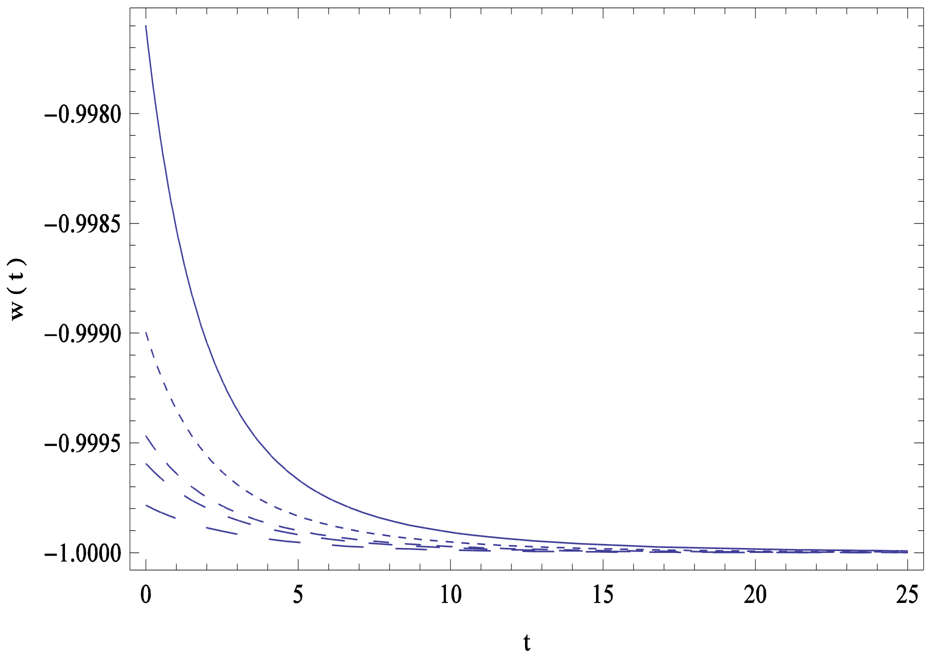
with effective Dark Energy sector: $\rho_{DE}=-\frac{1}{2\kappa^2}\left(f_1+12H^2f_1'\right)+\lambda\rho_m\left(f_2+12H^2f_2'\right)$

$$p_{DE}=(\rho_m+p_m)\left[\frac{1+\lambda\left(f_2+12H^2f_2'\right)}{1+f_1'-12H^2f_1''-2\kappa^2\lambda\rho_m\left(f_2'-12H^2f_2''\right)}\right]+\frac{\lambda\left(f_1+12H^2f_1'\right)}{2\kappa^2}-\lambda\rho_m\left(f_2+12H^2f_2'\right)$$

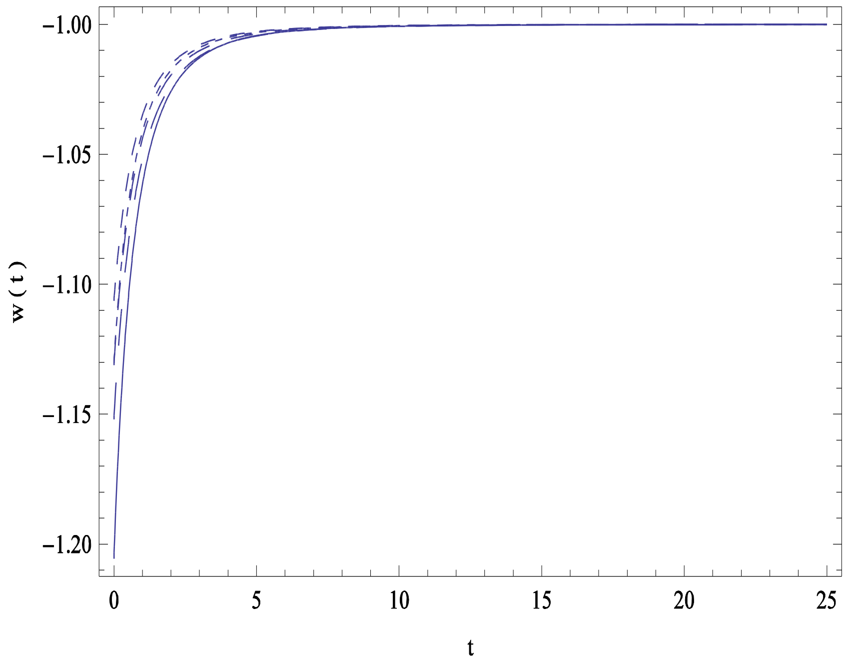
- **Different** than non-minimal matter-curvature coupled theory

Non-minimally matter-torsion coupled theory

- Interesting phenomenology



$$f_1(T) = -\Lambda + \alpha_1 T^2, \quad f_2(T) = \beta_1 T^2$$



$$f_1(T) = -\Lambda, \quad f_2(T) = \alpha_1 T + \beta_1 T^2$$

Teleparallel Equivalent of Gauss-Bonnet and f(T,T_G) gravity

- In curvature-based gravity, one can use higher-order invariants like the Gauss-Bonnet one $G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\kappa\lambda}R^{\mu\nu\kappa\lambda}$
- Let's do the same in torsion-based gravity:
- Similar to $e\overline{R} = -eT + 2\left(eT_{\nu}^{\nu\mu}\right)_{,\mu}$ we construct $e\overline{G} = eT_G + tot.diverg$ with

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$$T_G = \left(K^{a_1}_{\ \ ea_2}K^{ea_2}_{\ \ \ b}K^{a_3}_{\ \ fc}K^{fa_4}_{\ \ \ d} - 2K^{a_1a_2}_{\ \ \ \ a}K^{a_3}_{\ \ eb}K^{e}_{\ fc}K^{fa_4}_{\ \ \ d} + 2K^{a_1a_2}_{\ \ \ \ a}K^{a_3}_{\ \ eb}K^{ea_4}_{\ \ \ f}K^{f}_{\ cd} + +2K^{a_1a_2}_{\ \ \ \ a}K^{a_3}_{\ \ eb}K^{ea_4}_{\ \ \ f}K^{f}_{\ c,d}\right)\delta^{abcd}_{a_1a_2a_3a_4}$$

- $f(\mathrm{T}, T_G)$ gravity:

$$S = \frac{1}{2\kappa^2}\int d^4x\, e\,\{T + f(T, T_G)\} + S_m$$

- **Different** from $f(R,G)$ and $f(T)$ gravities

Teleparallel Equivalent of Gauss-Bonnet and f(T,T_G) gravity

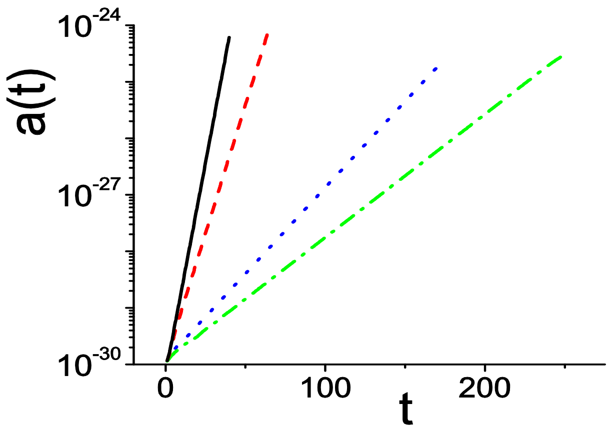
- Cosmological application:

$$\rho_{DE} = -\frac{1}{2\kappa^2}\left[f - 12H^2 f_T - T_G f_{T_G} + 24H^3 \dot{f}_{T_G}\right]$$

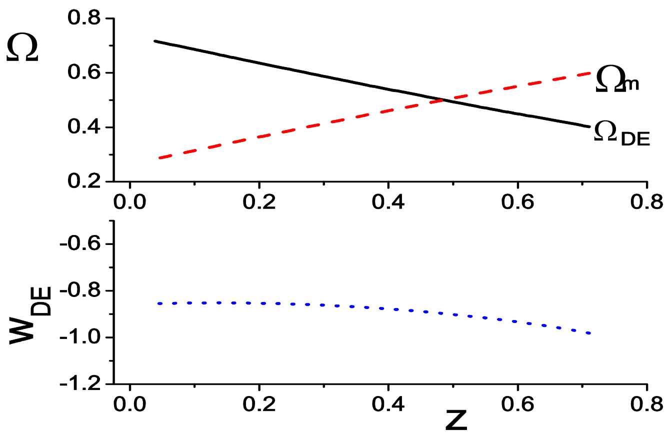
$$p_{DE} = \frac{1}{2\kappa^2}\left[f - 4\left(\dot{H} + 3H^2\right)f_T - 4H\dot{f}_T - T_G f_{T_G} + \frac{2}{3H}T_G \dot{f}_{T_G} + 8H^2 \ddot{f}_{T_G}\right]$$

$$T = 6H^2$$

$$T_G = 24H^2\left(\dot{H} + H^2\right)$$



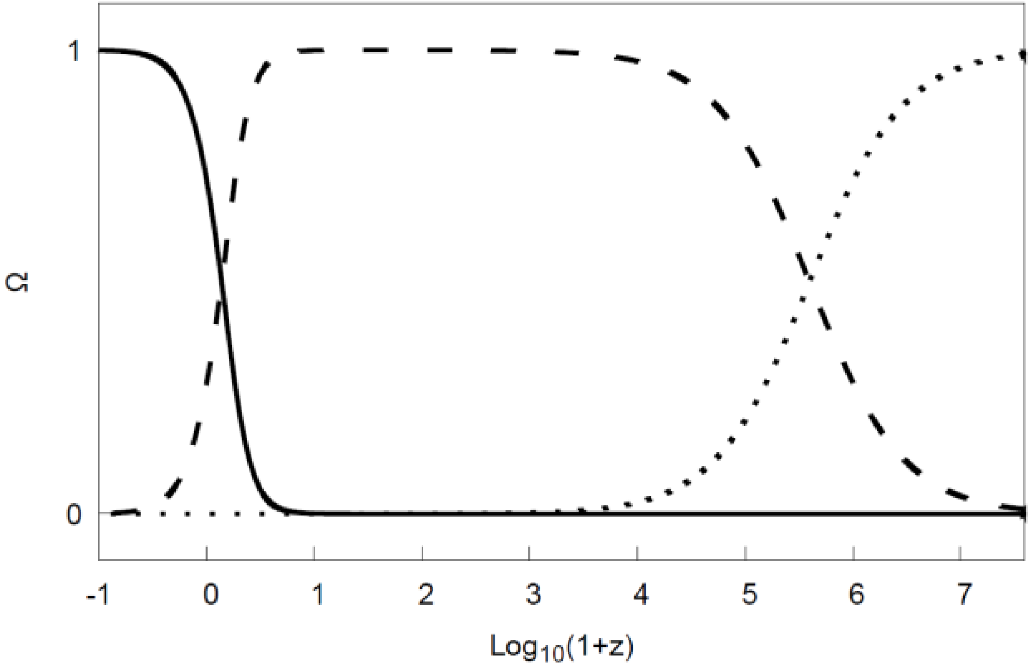
$f(T,T_G) = \alpha_1 T^2 + \alpha_2 T\sqrt{|T_G|}$

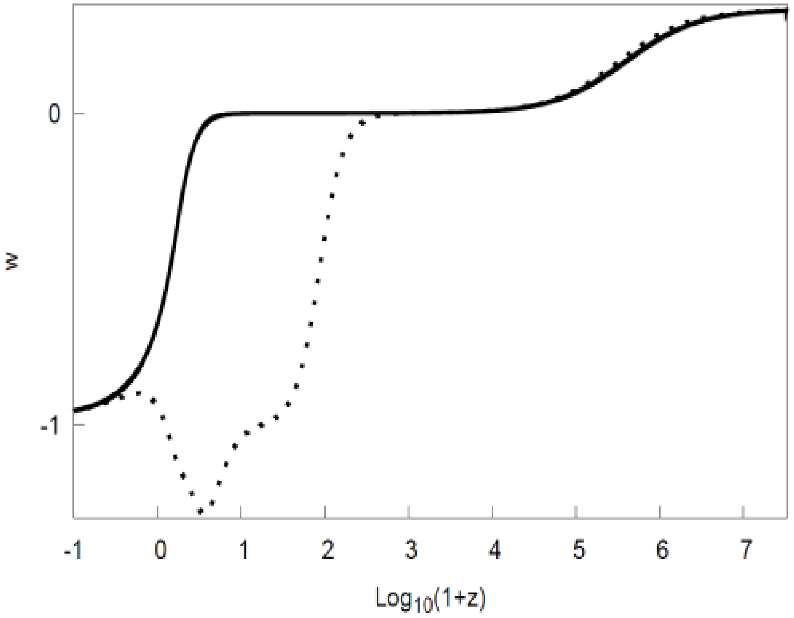


$f(T,T_G) = \beta_1\sqrt{T^2 + \beta_2 T_G}$

Torsional Gravity with higher derivatives

$$S = \frac{1}{2\kappa^2}\int d^4x\, e\, F\left(T, (\nabla T)^2, \Diamond T\right) + S_m(e^A_\mu, \Psi_m)$$

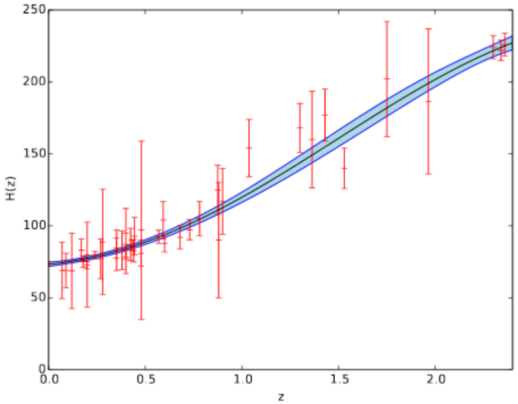


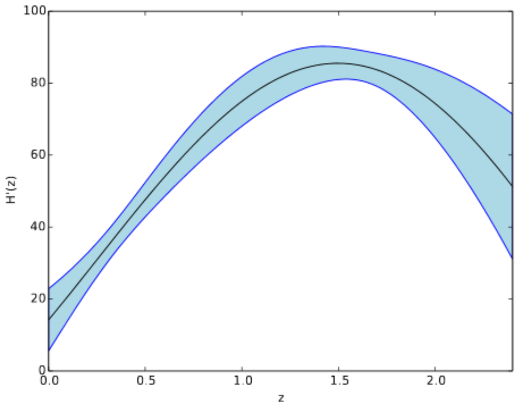


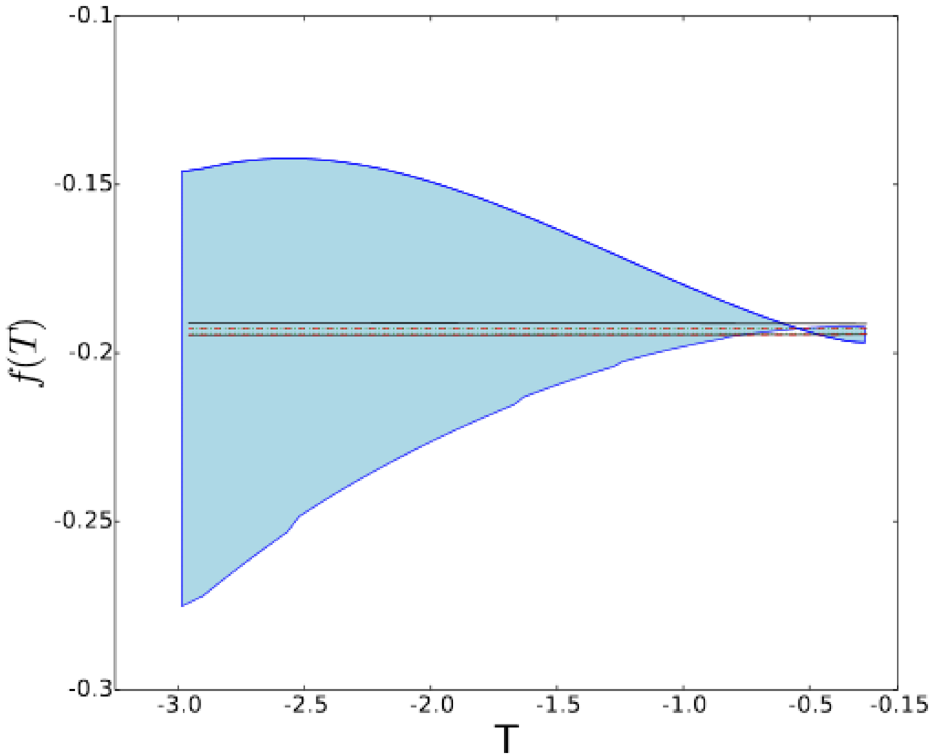
Growth-index constraints on f(T) gravity

- Perturbations: $\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G_{eff}\ \rho_m \delta_m$, clustering growth rate: $\frac{d\ \ln \delta_m}{d\ \ln a} = \Omega_m^{\gamma}(a)$
- γ(z): Growth index. $G_{eff} = \frac{1}{1 + f'(T)}$

Gaussian Process constraints on f(T)

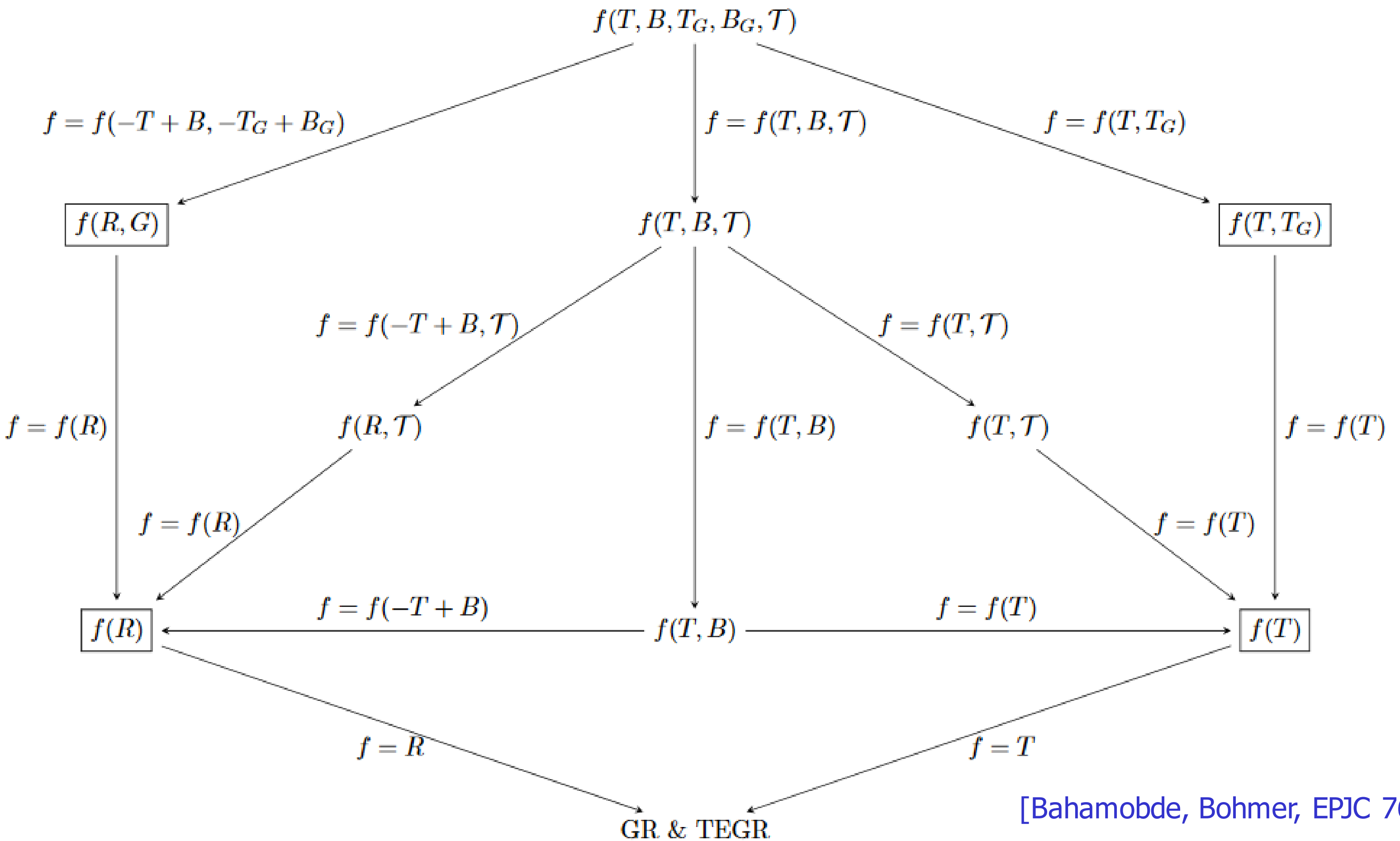






[Cai, Khurshudyan, Saridakis, Astroph. J 888]

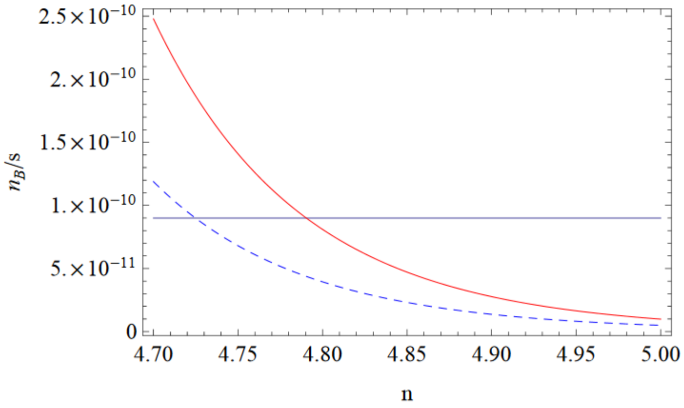
Torsional Modified Gravity



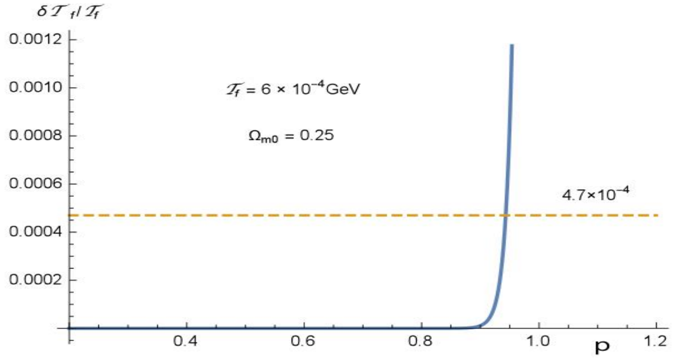
[Bahamobde, Bohmer, EPJC 76]

Baryogenesis and BBN constraints on f(T) gravity

- Baryon-anti-baryon asymmetry through CP violating term: $\frac{1}{M_*^2}\int d^4x\, e\left[\partial_\mu f(T)\right]J^\mu$



- BBN constraints: $\frac{\delta T_f}{T_f} \approx \frac{\rho_T}{\rho}\frac{H_{GR}}{10qT_f^5}$



Solar System constraints on f(T) gravity

- Apply the black hole solutions in Solar System:
- Assume corrections to TEGR of the form $f(T)=\alpha T^2+O(T^3)$

$$\Rightarrow F(r)^2=1-\frac{2GM}{c^2r}-\frac{\Lambda}{3}r^2+\alpha\left[-6\Lambda-\frac{6}{r^2}-\frac{4GM\Lambda}{c^2r}\right]$$

$$\Rightarrow G(r)^2=1-\frac{2GM}{c^2r}-\frac{\Lambda}{3}r^2+\alpha\left[\frac{8\Lambda}{3}-\frac{24}{r^2}-2\Lambda^2r^2-\frac{2GM}{c^2r}\left(8\Lambda-\frac{8}{r^2}\right)\right]$$

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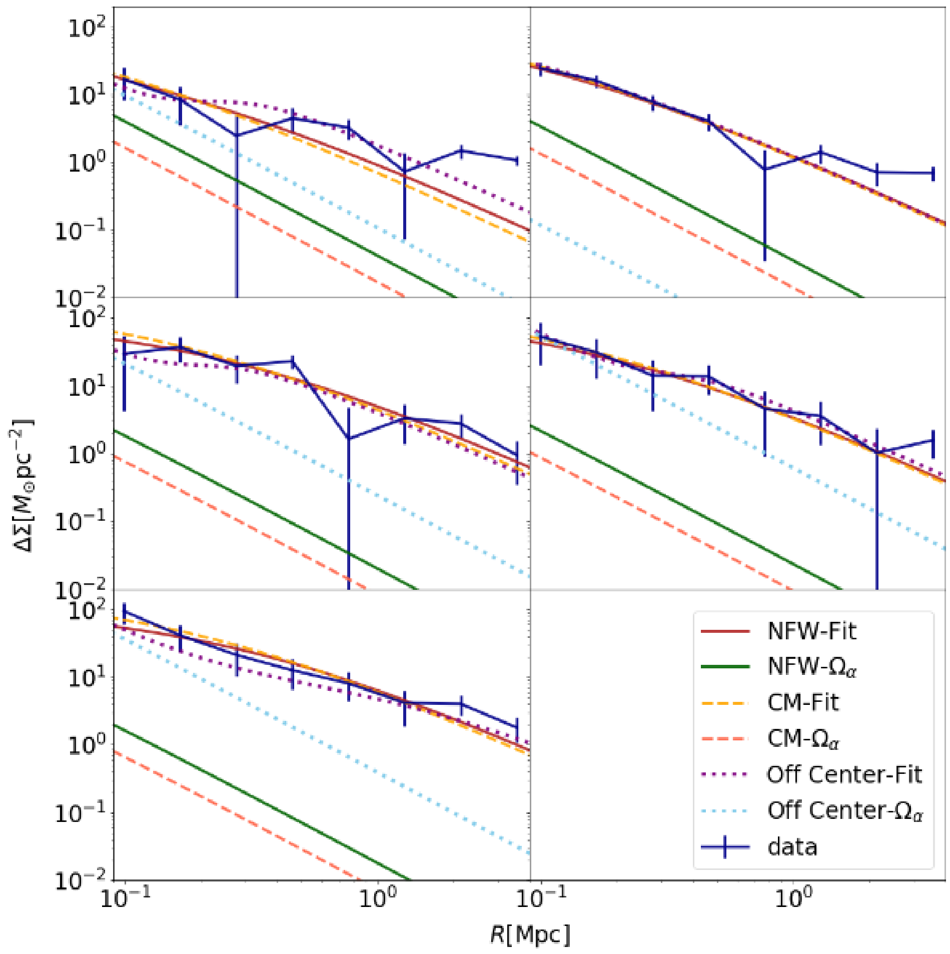
- Use data from Solar System orbital motions:

$$\Delta U_{f(T)}\leq 6.2\times10^{-10}$$

T<<1 so consistent

- f(T) divergence from TEGR is very small
- This was already known from cosmological observation constraints up to $O(10^{-1}-10^{-2})$
- With Solar System constraints, much more stringent bound.

Galaxy-Galaxy lensing constraints on f(T) gravity

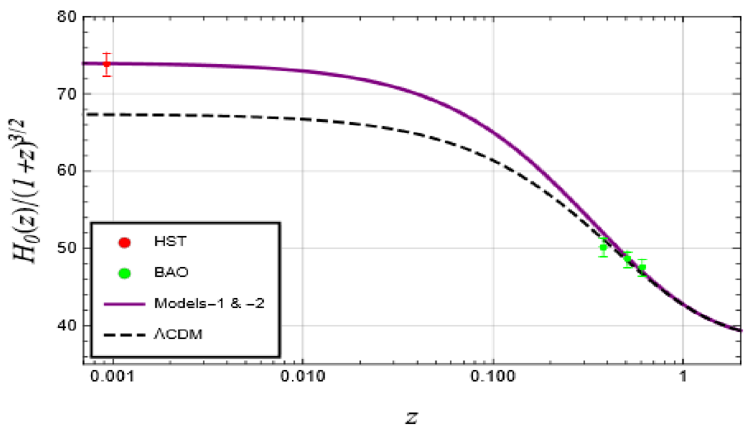


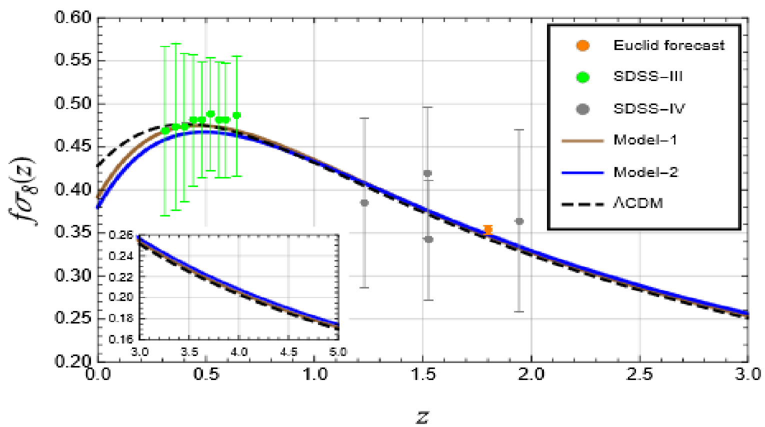
$$f(T) = -T - 2\Lambda/M_P^2 + \alpha T^\beta$$

[Chen. Luo, Cai, Saridakis, 1907.12225, PRD 102]

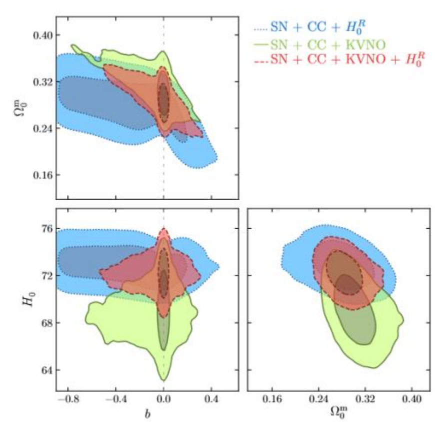
H0 and σ8 tension can be alleviated

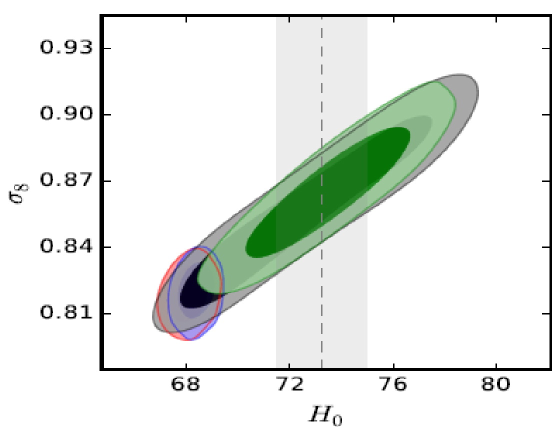
Power-law (f1CDM): $f(T) = T + \alpha(-T)^n$ $\alpha = (6H_0^2)^{1-n}\frac{\Omega_{F0}}{2n-1}$





Parameter	CMB + BAO	CMB + BAO + H_0
$10^2\omega_b$	$2.235^{+0.013}_{-0.013}$	$2.235^{+0.013}_{-0.013}$
ω_{cdm}	$0.1181^{+0.001}_{-0.001}$	$0.118^{+0.001}_{-0.001}$
$100\theta_s$	$1.041^{+0.00027}_{-0.00027}$	$1.041^{+0.00030}_{-0.00027}$
$\ln 10^{10}A_s$	$3.078^{+0.023}_{-0.023}$	$3.08^{+0.022}_{-0.022}$
n_s	$0.9678^{+0.0039}_{-0.0039}$	$0.9684^{+0.0039}_{-0.0044}$
τ_{reio}	$0.073^{+0.012}_{-0.013}$	$0.075^{+0.012}_{-0.012}$
n	$0.0043^{+0.0033}_{-0.0039}$	$0.0054^{+0.0020}_{-0.0020}$
$\log\alpha$	$10.00^{+0.081}_{-0.12}$	$10.03^{+0.06}_{-0.06}$
Ω_{F0}	$0.73^{+0.021}_{-0.028}$	$0.738^{+0.015}_{-0.015}$
H_0	$72.4^{+3.3}_{-4.1}$	$73.5^{+2.1}_{-2.1}$
σ_8	$0.855^{+0.023}_{-0.033}$	$0.866^{+0.02}_{-0.02}$
$\chi^2_{min}/2$	6480.48	6482.27



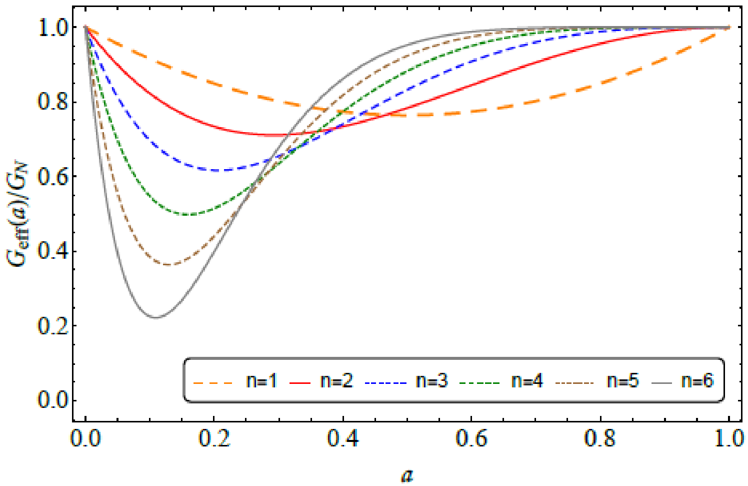


Tension2 – fσ8

- A possible wav to lower $f\sigma_8(z)$ is by considering modifications of gravity leading to: $G_{\rm eff}/G_{\rm N}$ <1
- Solar system constraints: $\left|\frac{1}{G_N}\frac{dG_{\rm eff}(z)}{dz}\Big|_{z=0}\right| < 10^{-3}h^{-1}$, $\left|\frac{1}{G_N}\frac{d^2G_{\rm eff}(z)}{dz^2}\Big|_{z=0}\right| < 10^{5}h^{-2}$
- BBN constraints: $|G_{\rm eff}/G_N - 1| \leq 0.2$
- A consistent parametrization that respects these constraints is

$$\frac{G_{\rm eff}(a,g_a,n)}{G_{\rm N}} = 1 + g_a(1-a)^n - g_a(1-a)^{n+m}$$
$$= 1 + g_a\left(\frac{z}{1+z}\right)^n - g_a\left(\frac{z}{1+z}\right)^{n+m}$$

[Kazantzidis, Perivolaropoulos, PRD97]



Tension2 – fσ8

- We consider the following ansatz:

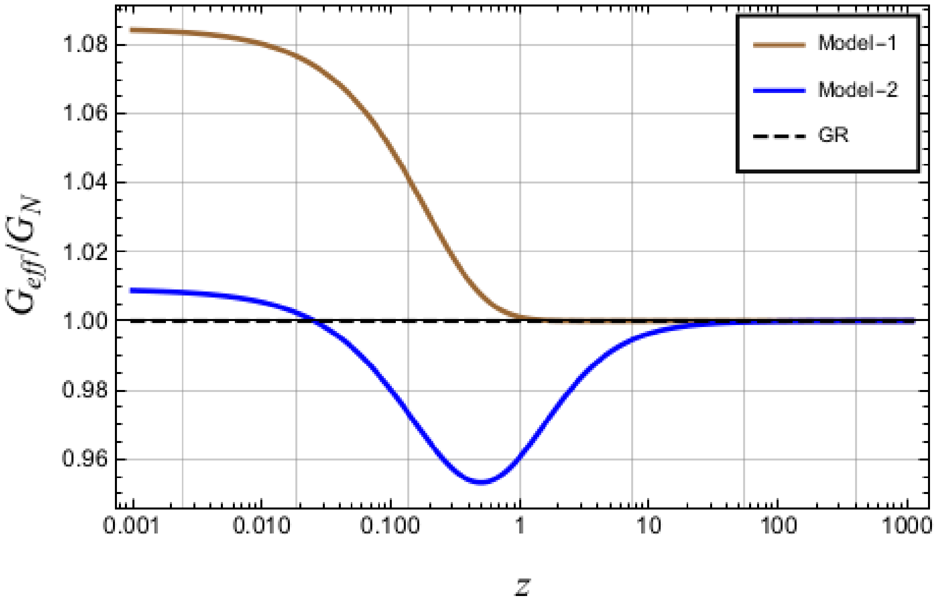
$$f(T) = -[T + 6H_0^2(1 - \Omega_{m0}) + F(T)],$$

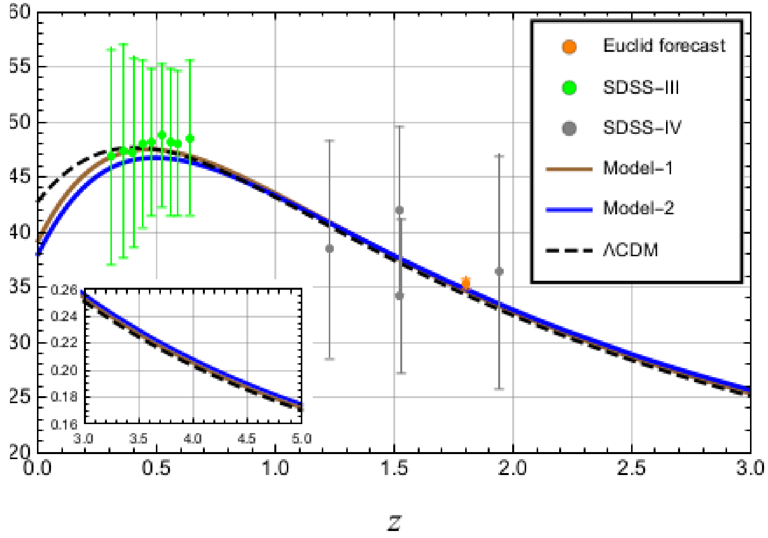
  where $F(T)$ describes the deviation from GR
  The first Friedmann equation becomes

$$T(z) + 2\frac{F'(z)}{T'(z)}T(z) - F(z) = 6H^2_{\Lambda CDM}(z)\ .$$

- In order to solve the H_0 tension, we need
  $T(0) = 6H_0^2 \simeq 6(H_0^{CC})^2$, with $H_0^{CC} = 74.03\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$,
  while in the early era of $z \gtrsim 1100$ we require the Universe expansion to evolve as in ΛCDM, namely
  $H(z \gtrsim 1100) \simeq H_{\Lambda CDM}(z \gtrsim 1100)$
  This implies $F(z)|_{z \gtrsim 1100} \simeq cT^{1/2}(z)$ (the value $c = 0$ corresponds to standard GR, while for $c \neq 0$ we obtain ΛCDM too).

Tension2 – fσ8





Metric-Affine Modified Gravity

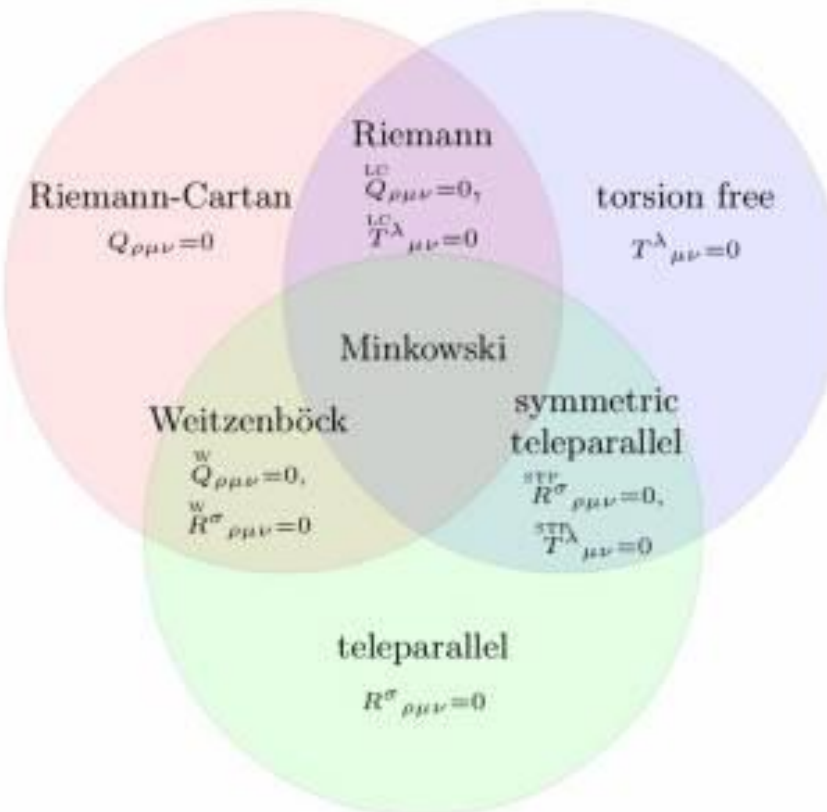
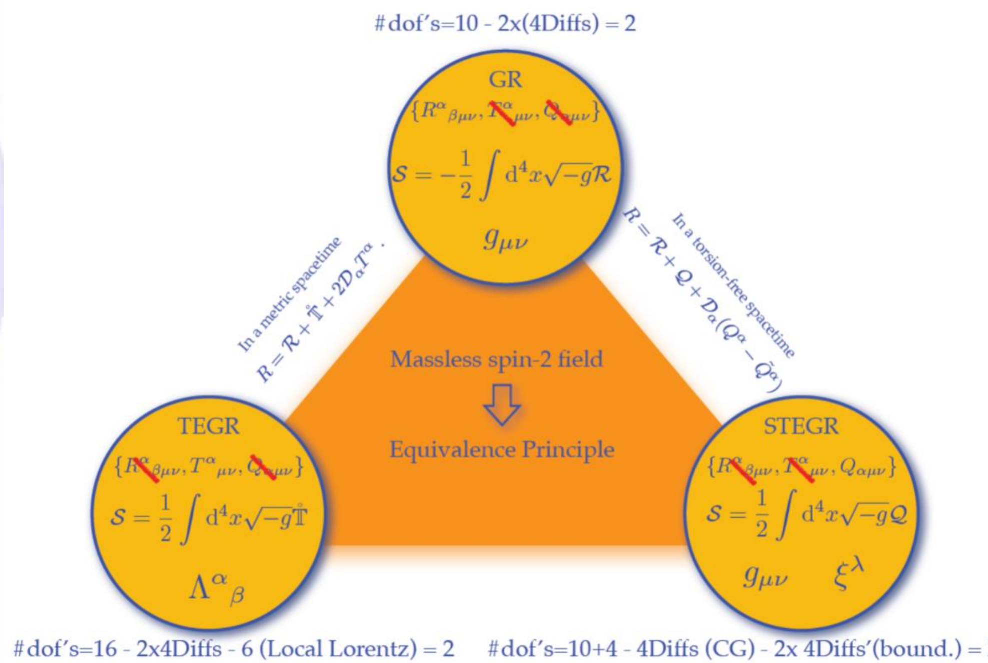
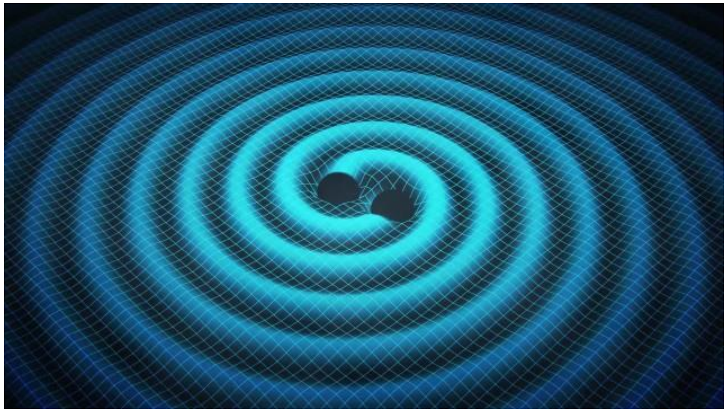


FIG. 1. Subclasses of metric-affine geometry, depending on the properties of connection.



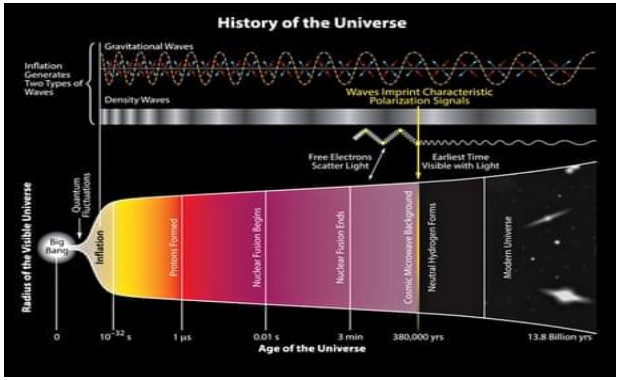
Gravitational waves

- The GWs are the tensor perturbations of the metric. Predicted in 1915, first observed in 2015. First astronomical observation ever, not related to E/M (or neutrinos).
- GWs from mergers:



[Abbott et al, LIGO Virgo PRL 116]

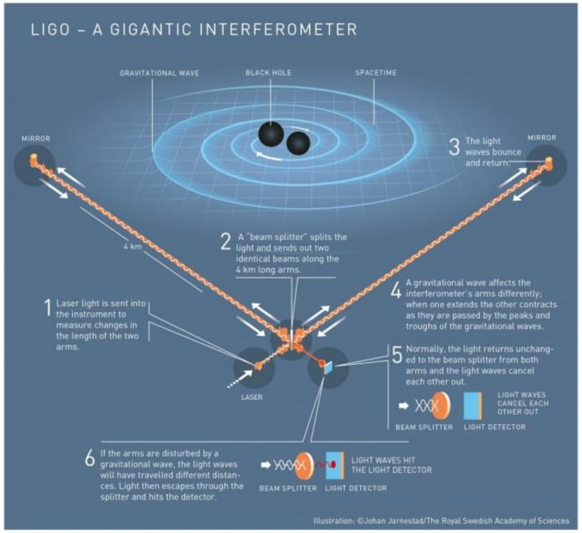
- Primordial GWs:

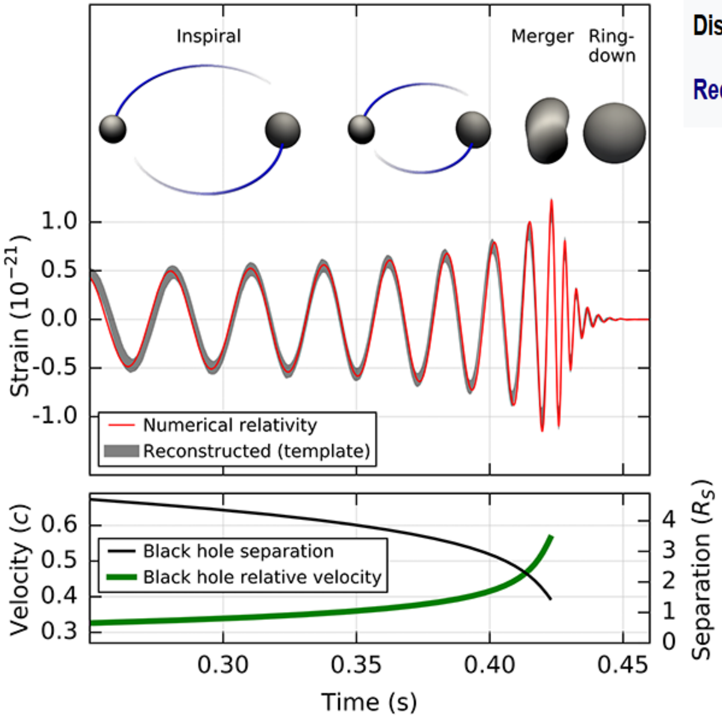


Gravitational waves

- GW150914: Two black holes with 36^{+5}_{-4} M⊙ and 29^{+4}_{-4} M⊙, resulting in a 62^{+4}_{-4} M⊙ black hole

Louisiana.
Washington
4km
10^{-18}m





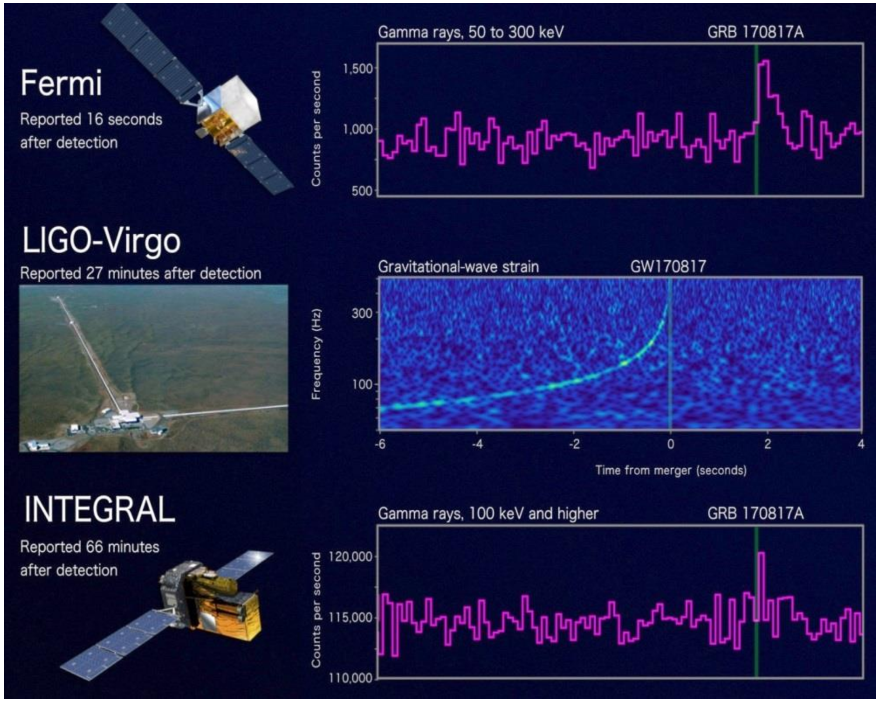
Distance	410^{+160}_{-180} Mpc
Redshift	$0.093^{+0.030}_{-0.036}$

[Abbott et al, LIGO Virgo PRL 116]

2017 Nobel Price in Physics

Gravitational waves

- GW170817: Two neutron stars, distance 40 Mpc, redshift 0.0099
- GRB170817A: The Electromagnetic counterpart.



- The era of multi-messenger astronomy begins!

Gravitational waves

- In case of GWs from black hole mergers we know their properties at the moment of detection, and their direction (in case of three detectors). Assuming GR and ΛCDM we can extract their speed, distance, and properties at the moment of emission.

Gravitational waves

- In case of GWs from black hole mergers we know their properties at the moment of detection, and their direction (in case of three detectors). Assuming GR and ΛCDM we can extract their speed, distance, and properties at the moment of emission.

- In case of GWs from neutron star mergers, and their E/M counterpart, we know their properties at the moment of detection and their direction, but using the implied physics from the E/M information we can extract their speed, distance and properties at the moment of emission, independently of the underlying gravitational theory and cosmological scenario.

- Great tool for testing General Relativity and cosmological scenarios!

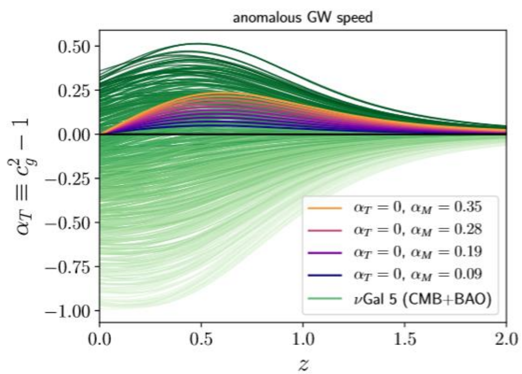
Gravitational waves

- An immediate result: The speed of GWs is equal to the speed of light!

  GW170817 time delay $1.74 \pm 0.05\text{s}$ constrains: $-3 \cdot 10^{-15} \leq c_g/c - 1 \leq 7 \cdot 10^{-16}$

- Excludes a large number of theories that were consistent with other data!

	$c_g = c$	$c_g \neq c$
Horndeski	General Relativity	quartic/quintic Galileons [13, 14]
	quintessence/k-essence [46]	Fab Four [15]
	Brans-Dicke/$f(R)$ [47, 48]	de Sitter Horndeski [49]
	Kinetic Gravity Braiding [50]	$G_{\mu\nu}\phi^\mu\phi^\nu$ [51], $f(\phi)$·Gauss-Bonnet [52]
beyond H.	Derivative Conformal (19) [17]	quartic/quintic GLPV [18]
	Disformal Tuning (21)	quadratic DHOST [20] with $A_1 \neq 0$
	quadratic DHOST with $A_1 = 0$	cubic DHOST [23]
	Viable after GW170817	Non-viable after GW170817



[Ezquiaga, Zumalacarregui PRL 119]

Gravitational waves

- For tensor perturbations:
$$g_{00} = -1\,, \quad g_{0i} = 0\,,$$
$$g_{ij} = a^2\left(\delta_{ij} + h_{ij} + \frac{1}{2}h_{ik}h_{kj}\right)$$

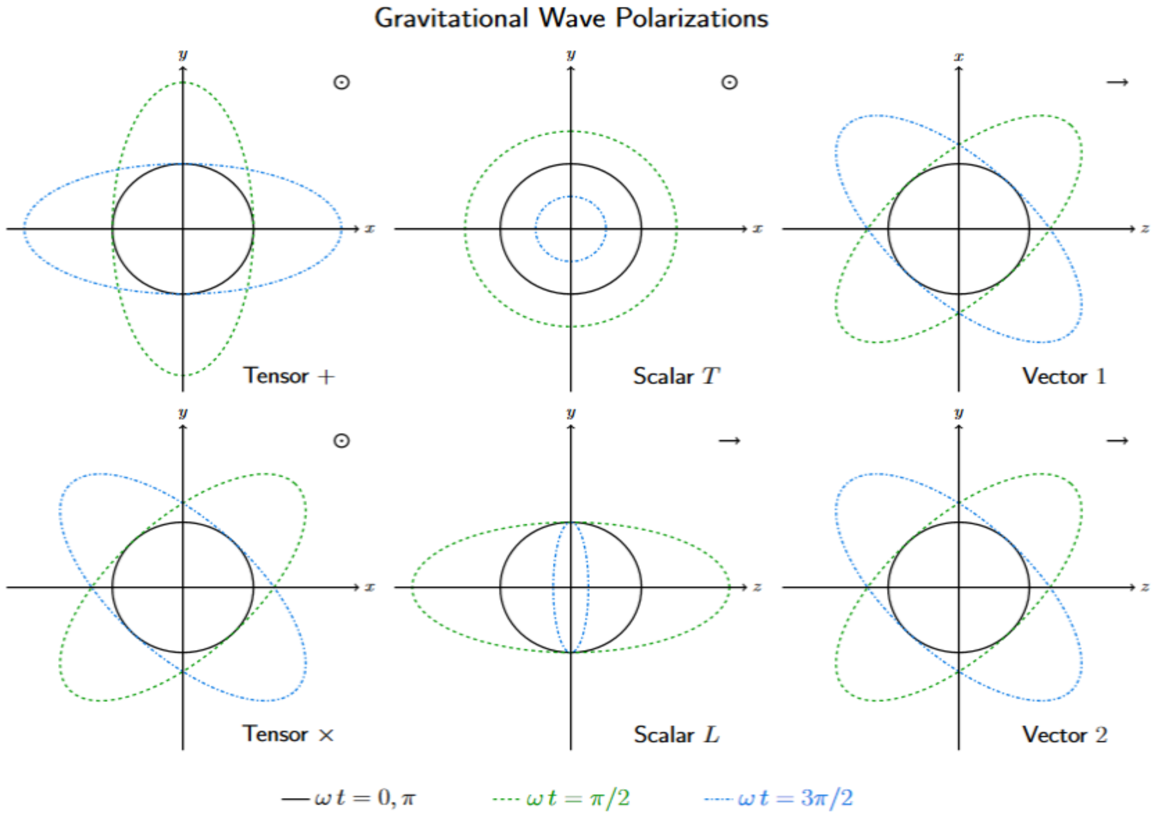
$$\boxed{\ddot{h}_{ij} + (3+\alpha_M)\dot{h}_{ij} + (1+\alpha_T)\frac{k^2}{a^2}h_{ij} = 0}$$

$$\alpha_M = \frac{d\log(M_*^2)}{d\log a} \qquad\qquad c_g^2 = (1+\alpha_T)$$

- $$h_{\rm GW} \sim h_{\rm GR} \quad \underbrace{e^{-\frac{1}{2}\int \nu\mathcal{H}d\eta}}_{\text{Affects amplitude}} \quad \underbrace{e^{ik\int(\alpha_T + a^2m^2/k^2)^{1/2}d\eta}}_{\text{Affects phase}}$$

Gravitational waves

- Polarizations:



Gravitational waves in modified gravity

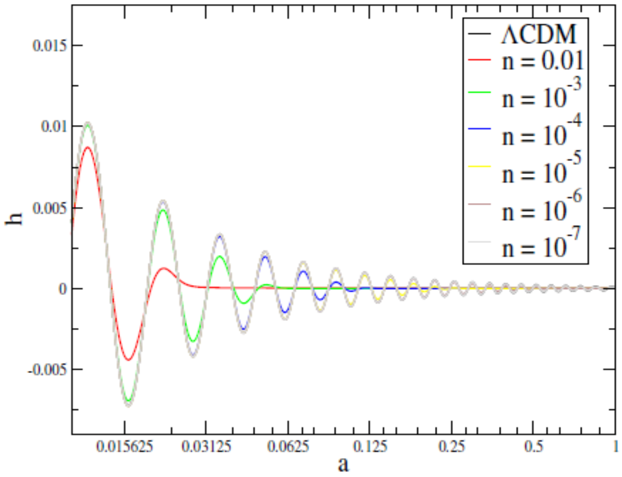
- Gw's propagation at cosmological scales: $h = e^{-\mathcal{D}} e^{-ik\Delta T} h_{GR}$

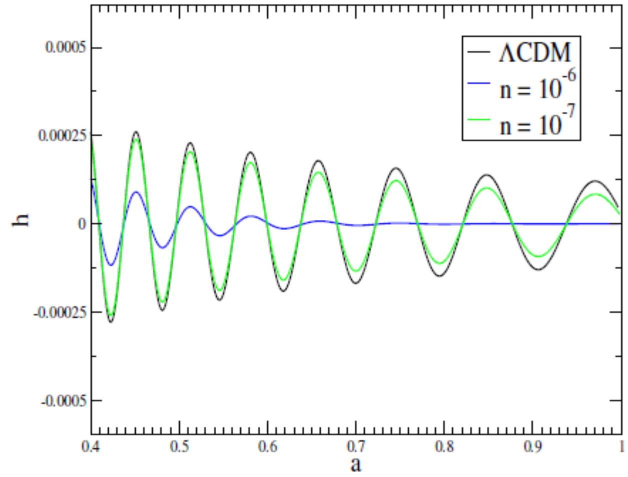
$\mathcal{D} = \frac{1}{2}\int \nu\mathcal{H}d\tau'$ (affects amplitude) $\Delta T = \int \left(1 - c_T - \frac{a^2\mu^2}{2k^2}\right) d\tau'$ (affects phase)

- In f(T) gravity:

$$\ddot{h}_{ij} + 3H(1-\beta_T)\dot{h}_{ij} + \frac{k^2}{a^2}h_{ij} = 0$$

$$\beta_T \equiv -\frac{\dot{f}_T}{3Hf_T}$$





Gravitational Waves in Modified Teleparallel Theories

- $$S=\frac{1}{16\pi G}\int d^4x\, e\, f(T,B)+\int d^4x\, e\,\mathcal{L}_m$$ $$R=-T-2\nabla^\mu T^\nu{}_{\mu\nu}$$

$$-f_T G_{\mu\nu}+\left(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu\right)f_B$$
$$+\frac{1}{2}g_{\mu\nu}\left(f_B B+f_T T-f\right)$$
$$+2S_\nu{}^\alpha{}_\mu\partial_\alpha\left(f_T+f_B\right)=8\pi G\,\Theta_{\mu\nu}$$

- $$g_{\mu\nu}=\eta_{\mu\nu}+h^{(1)}_{\mu\nu}+\mathcal{O}\left(h^{(2)}_{\mu\nu}\right)$$

$$h^{(1)}_{\mu\nu}=\begin{pmatrix}-2A\exp(ik_\mu x^\mu)-\frac{f^{(0)}_{BB}B^{(1)}}{f^{(0)}_T} & B_1\exp(ik_\mu x^\mu) & B_2\exp(ik_\mu x^\mu) & -2A\exp(ik_\mu x^\mu)\\ B_1\exp(ik_\mu x^\mu) & h_+ +\frac{f^{(0)}_{BB}B^{(1)}}{f^{(0)}_T} & h_\times & B_1\exp(ik_\mu x^\mu)\\ B_2\exp(ik_\mu x^\mu) & h_\times & -h_+ +\frac{f^{(0)}_{BB}B^{(1)}}{f^{(0)}_T} & B_2\exp(ik_\mu x^\mu)\\ -2A\exp(ik_\mu x^\mu) & B_1\exp(ik_\mu x^\mu) & B_2\exp(ik_\mu x^\mu) & -2A\exp(ik_\mu x^\mu)+\frac{f^{(0)}_{BB}B^{(1)}}{f^{(0)}_T}\end{pmatrix}$$

[Farrugia, Said, Gakis, Saridakis, PRD 97]

The Effective Field Theory (EFT) approach

- The EFT approach allows to ignore the details of the underlying theory and write an action for the perturbations around a time-dependent background solution.
- One can systematically analyze the perturbations separately from the background evolution. [Arkani-Hamed, Cheng JHEP0405 (2004)]

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$$S = \int d^4x \Big\{ \sqrt{-g} \Big[\frac{M_P^2}{2} \Psi(t) R - \Lambda(t) - b(t) g^{00}$$
<- background

$$+ M_2^4 (\delta g^{00})^2 - \bar{m}_1^3 \delta g^{00} \delta K - \bar{M}_2^2 \delta K^2 - \bar{M}_3^2 \delta K_\mu^\nu \delta K_\nu^\mu$$
<- linear evolution of perturbations

$$+ m_2^2 h^{\mu\nu} \partial_\mu g^{00} \partial_\nu g^{00} + \lambda_1 \delta R^2 + \lambda_2 \delta R_{\mu\nu} \delta R^{\mu\nu} + \mu_1^2 \delta g^{00} \delta R \Big]$$
<- linear evolution of perturbations

$$+ \gamma_1 C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma} + \gamma_2 \epsilon^{\mu\nu\rho\sigma} C_{\mu\nu}{}^{\kappa\lambda} C_{\rho\sigma\kappa\lambda}$$
<- linear evolution of perturbations

$$+ \sqrt{-g} \Big[\frac{M_3^4}{3} (\delta g^{00})^3 - \bar{m}_2^3 (\delta g^{00})^2 \delta K + \ldots \Big] \Big\} ,$$
<- 2nd-order evolution of perturbations

The functions Ψ(t), Λ(t), b(t), are determined by the background solution

[Gubitosi, Piazza, Vernizzi, JCAP1302]

The (EFT) approach to torsional gravity

- Application of the EFT approach to torsional gravity leads to include terms:
- i) Invariant under 4D diffeomorphisms: e.g. R,T multiplied by functions of time.
- ii) Invariant under spatial diffeomorphisms: e.g. g^{00}, R^{00} and T^{0}
- ii) Invariant under spatial diffeomorphisms: e.g. ${}^{(3)}R_{\mu\nu\rho\sigma}$, ${}^{(3)}T^{\rho}{}_{\mu\nu}$, $K_{\mu\nu}$, $\hat{K}_{\mu\nu}$

  the extrinsic torsion is defined as

$$\hat{K}_{\mu\nu} \equiv h^{\sigma}_{\mu}\hat{\nabla}_{\sigma}n_{\nu} = K_{\mu\nu} - \mathcal{K}^{\lambda}{}_{\nu\mu}n_{\lambda} + n_{\mu}\frac{1}{g^{00}}T^{00}{}_{\nu}$$

  with n_{μ} the orthogonal to t=cont. surfaces unitary vector $n_{\mu} = \frac{\delta^{0}_{\mu}}{\sqrt{-g^{00}}}$

[Cai, Li, Saridakis, Xue, PRD 97], [Li, Cai, Cai, Saridakis, JCAP18]

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Using the projection operator h^{μ}_{ν} we can express ${}^{(3)}R_{\mu\nu\rho\sigma} = h^{\alpha}_{\mu}h^{\beta}_{\nu}h^{\gamma}_{\rho}h^{\delta}_{\sigma}R_{\alpha\beta\gamma\delta} - K_{\mu\rho}K_{\nu\sigma} + K_{\nu\rho}K_{\mu\sigma}$

$$h^{d}_{a}h^{c}_{b}h^{f}_{e}T^{e}{}_{dc} = {}^{(3)}T^{f}{}_{ab}$$

[Cai, Li, Saridakis, Xue, PRD 97], [Li, Cai, Cai, Saridakis, JCAP18]

The (EFT) approach to torsional gravity

- We perturb the previous tensors, and we finally obtain:

$$R^{(0)}_{\mu\nu\rho\sigma} = f_1(t) g_{\mu\rho} g_{\nu\sigma} + f_2(t) g_{\mu\rho} n_\nu n_\sigma + f_3(t) g_{\mu\sigma} g_{\nu\rho} + f_4(t) g_{\mu\sigma} n_\nu n_\rho + f_5(t) g_{\nu\sigma} n_\mu n_\rho + f_6(t) g_{\nu\rho} n_\mu n_\sigma ,$$

$$T^{(0)}_{\rho\mu\nu} = g_1(t) g_{\rho\nu} n_\mu + g_2(t) g_{\rho\mu} n_\nu ,$$

$$K^{(0)}_{\mu\nu} = f_7(t) g_{\mu\nu} + f_8(t) n_\mu n_\nu ,$$

$$\hat{K}^{(0)}_{\mu\nu} = 0 \, .$$

where the time-dependent functions are determined by the background solution.

[Cai, Li, Saridakis, Xue, PRD 97], [Li, Cai, Cai, Saridakis, JCAP18]

The (EFT) approach to torsional gravity

- Finally, the EFT action of torsional gravity becomes:

$$S = \int d^4x\sqrt{-g}\Big[\frac{M_P^2}{2}\Psi(t)R - \Lambda(t) - b(t)g^{00} + \frac{M_P^2}{2}d(t)T^0\Big] + S^{(2)} ,$$

- The perturbation part contains:

  i) Terms present in curvature EFT action

  ii) Pure torsion terms such as δT^2 , $\delta T^0\delta T^0$ and $\delta T^{\rho\mu\nu}\delta T_{\rho\mu\nu}$

  iii) Terms that mix curvature and torsion, such as $\delta T\delta R$, $\delta g^{00}\delta T$, $\delta g^{00}\delta T^0$ and $\delta K\delta T^0$

[Cai, Li, Saridakis, Xue, PRD 97], [Li, Cai, Cai, Saridakis, JCAP18]

The (EFT) approach to f(T) gravity: Background

- For the case of f(T) gravity, at the background level, we have:

$$S = \frac{M_P^2}{2}\int d^4x\sqrt{-g}\big[-f_T(T^{(0)})R + 2\dot{f}_T(T^{(0)})T^{(0)} - T^{(0)}f_T(T^{(0)}) + f(T^{(0)})\big]$$

where by comparison:

$$\Psi(t) = -f_T(T^{(0)})\ ,$$
$$\Lambda(t) = \frac{M_P^2}{2}\left[T^{(0)}f_T(T^{(0)}) - f(T^{(0)})\right]\ ,$$
$$d(t) = -2\dot{f}_T(T^{(0)})\ ,$$
$$b(t) = 0\ .$$

- Performing variation we obtain the background equations of motion (Friedmann Eqs):

$$b(t) = M_P^2\Psi\left(-\dot{H} - \frac{\ddot{\Psi}}{2\Psi} + \frac{H\dot{\Psi}}{2\Psi} - \frac{\dot{d}}{4\Psi} + \frac{3Hd}{4\Psi}\right) - \frac{1}{2}(\rho_m + p_m),$$

$$\Lambda(t) = M_P^2\Psi\left(3H^2 + \frac{5H\dot{\Psi}}{2\Psi} + \dot{H} + \frac{\ddot{\Psi}}{2\Psi} + \frac{\dot{d}}{4\Psi} + \frac{3Hd}{4\Psi}\right) - \frac{1}{2}(\rho_m - p_m),$$

[Li, Cai, Cai, Saridakis, JCAP18]

The (EFT) approach to f(T) gravity: Tensor Perturbations

- For tensor perturbations: $g_{00} = -1\ ,\quad g_{0i} = 0\ ,$

$$g_{ij} = a^2\big(\delta_{ij} + h_{ij} + \frac{1}{2}h_{ik}h_{kj}\big)$$

i.e.

$$\bar{e}^0_\mu = \delta^0_\mu\ ,$$
$$\bar{e}^a_\mu = a\delta^a_\mu + \frac{a}{2}\delta^i_\mu\delta^{aj}h_{ij} + \frac{a}{8}\delta^i_\mu\delta^{ja}h_{ik}h_{kj}\ ,$$
$$\bar{e}^\mu_0 = \delta^\mu_0\ ,$$
$$\bar{e}^\mu_a = \frac{1}{a}\delta^\mu_a - \frac{1}{2a}\delta^{\mu i}\delta^j_a h_{ij} + \frac{1}{8a}\delta^{i\mu}\delta^j_a h_{ik}h_{kj}$$

- We obtain:

$$^{(3)}R \approx -\frac{1}{4}a^{-2}(\partial_i h_{kl}\partial_i h_{kl})\ ,$$
$$K^{ij}K_{ij} \approx 3H^2 + \frac{1}{4}\dot{h}_{ij}\dot{h}_{ij}\ ,$$
$$K \approx 3H\ ,$$

$$\boxed{T = T^{(0)} + O(h^2) = 6H^2 + O(h^2)}$$

- And finally:

$$S = \frac{M_P^2}{2}\int d^4x\sqrt{-g}\Big[\frac{f_T}{4}\big(a^{-2}\vec{\nabla}h_{ij}\cdot\vec{\nabla}h_{ij} - \dot{h}_{ij}\dot{h}_{ij}\big) + 6H^2 f_T - 12H\dot{f}_T - T^{(0)}f_T + f(T^{(0)})\Big]$$

[Cai, Li, Saridakis, Xue, PRD 97]

The (EFT) approach to f(T) gravity: Scalar Perturbations

- For scalar perturbations:

$$g_{00} = -1 - 2\phi \ ,$$
$$g_{0i} = 0 \ ,$$
$$g_{ij} = a^2[(1-2\psi)\delta_{ij} + \partial_i\partial_j F]$$

i.e

$$e^0_\mu = \delta^0_\mu + \delta^0_\mu \phi + a\delta^i_\mu \partial_i \chi \ ,$$
$$e^a_\mu = a\delta^i_\mu \delta^a_i + \delta^0_\mu \delta^a_i \partial^i \mathcal{E} + a\delta^i_\mu \delta^a_j \left[\epsilon_{ijk}\partial_k \sigma - \psi\delta_{ij} + \frac{1}{2}\partial_i\partial_j F\right]$$

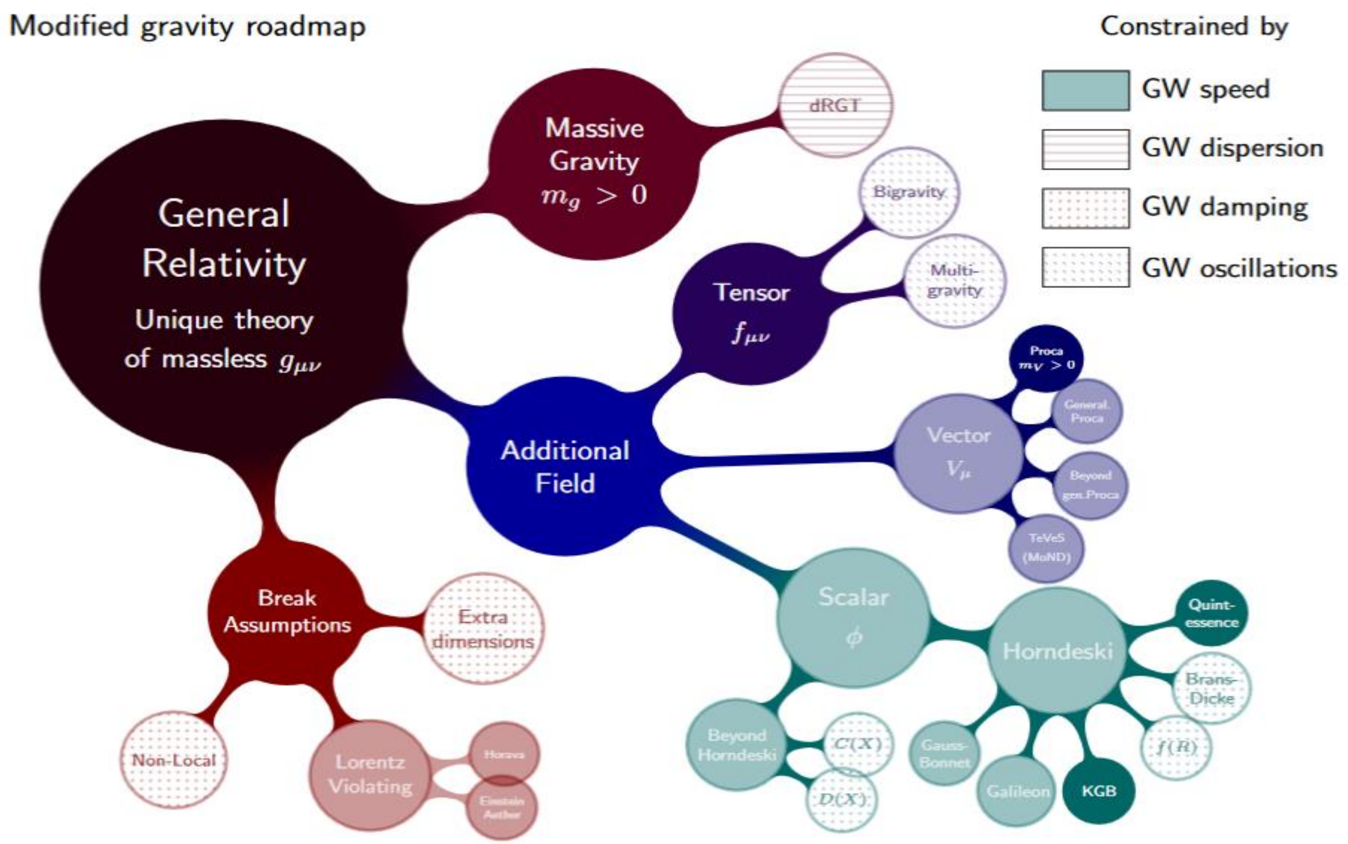
- So $$T^0 = g^{0\mu}T^\nu{}_{\mu\nu} = -3H + 6H\phi + 3\dot{\psi} - 6H\phi^2 - 6\dot{\psi}\phi + \frac{1}{a}\partial_i\partial_i\chi - \frac{1}{2a}\partial_i\phi\partial_i\chi - \frac{3}{2a}\phi\partial_i\partial_i\chi - \frac{1}{2a}\partial_i\psi\partial_i\chi + \frac{1}{2a}\psi\partial_i\partial_i\chi$$

- Thus:

$$S = \int d^4x \Big[\frac{M_P^2}{2}\Big(-2af_T\partial_i\psi\partial_i\psi + 4af_T\partial_i\phi\partial_i\psi + 4a^2\dot{f}_T\partial_i\psi\partial_i\chi + 4\dot{f}_T a^2 H\partial_i\pi\partial_i\chi\Big) + a^3M^2\pi^2 - a^3\phi\delta\rho_m\Big]$$

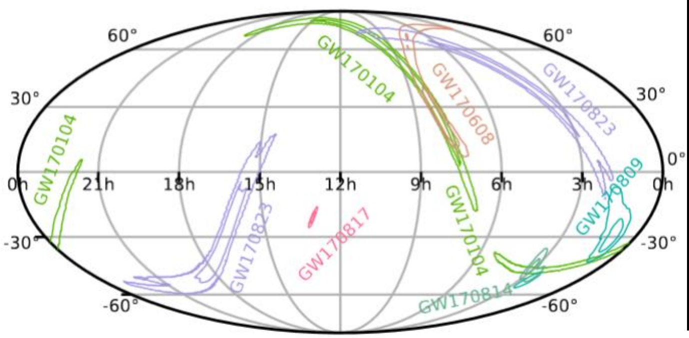
[Li, Cai, Cai, Saridakis, JCAP18]

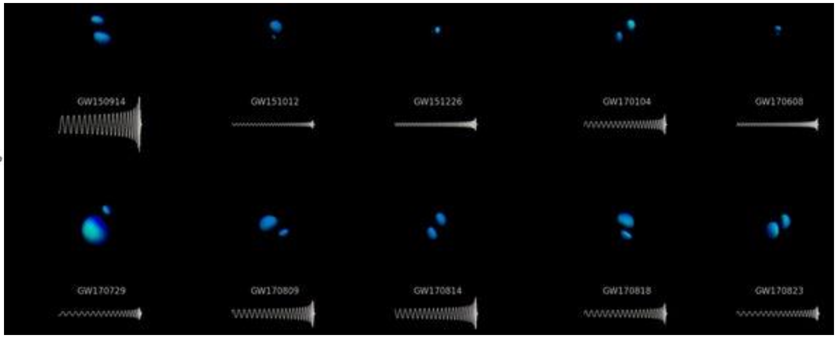
Gravitational waves and Modified Gravity

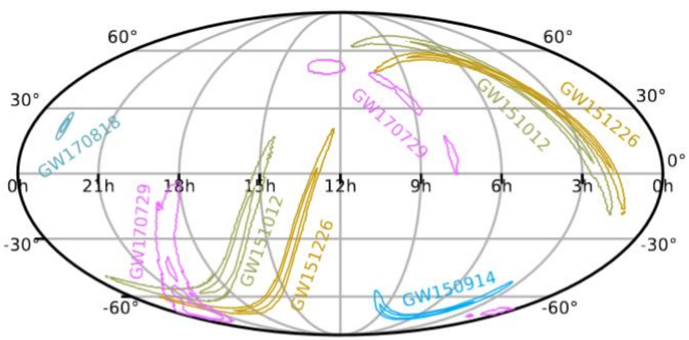


Gravitational waves - Observations

- Observations: 53 up to now (43 BH-BH, 3 NS-NS, 2 NS-BH, 5 uncertain, 19-85 Msun, 320-2800 MPc)







Event	$m_1/M_\odot$	$m_2/M_\odot$	$\mathcal{M}/M_\odot$
GW150914	$35.6^{+4.8}_{-3.0}$	$30.6^{+3.0}_{-4.4}$	$28.6^{+1.6}_{-1.5}$
GW151012	$23.3^{+14.0}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	$15.2^{+2.0}_{-1.1}$
GW151226	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.6}$	$8.9^{+0.3}_{-0.3}$
GW170104	$31.0^{+7.2}_{-5.6}$	$20.1^{+4.9}_{-4.5}$	$21.5^{+2.1}_{-1.7}$
GW170608	$10.9^{+5.3}_{-1.7}$	$7.6^{+1.3}_{-2.1}$	$7.9^{+0.2}_{-0.2}$
GW170729	$50.6^{+16.6}_{-10.2}$	$34.3^{+9.1}_{-10.1}$	$35.7^{+6.5}_{-4.7}$
GW170809	$35.2^{+8.3}_{-6.0}$	$23.8^{+5.2}_{-5.1}$	$25.0^{+2.1}_{-1.6}$
GW170814	$30.7^{+5.7}_{-3.0}$	$25.3^{+2.9}_{-4.1}$	$24.2^{+1.4}_{-1.1}$
GW170817	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	$1.186^{+0.001}_{-0.001}$
GW170818	$35.5^{+7.5}_{-4.7}$	$26.8^{+4.3}_{-5.2}$	$26.7^{+2.1}_{-1.7}$
GW170823	$39.6^{+10.0}_{-6.6}$	$29.4^{+6.3}_{-7.1}$	$29.3^{+4.2}_{-3.2}$

[LIGO-Virgo Collaborations 1811.12907]

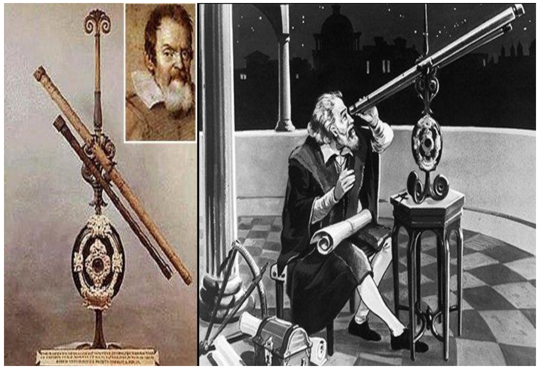
Designation
150914+09:50:45UTC
151226+03:38:53UTC
151012+09:54:43UTC
151019+00:23:16UTC
150928+10:49:00UTC
151218+18:30:58UTC
160103+05:48:36UTC
151202+01:18:13UTC
160104+03:51:51UTC
151213+00:12:20UTC
150923+07:10:59UTC
151029+13:34:39UTC
151206+14:19:29UTC
151202+15:32:09UTC
151012+06:30:45UTC
151116+22:41:48UTC
151121+03:34:09UTC
150922+05:41:08UTC
151008+14:09:17UTC
151127+02:00:30UTC

- Expectations: Many thousands in the next years

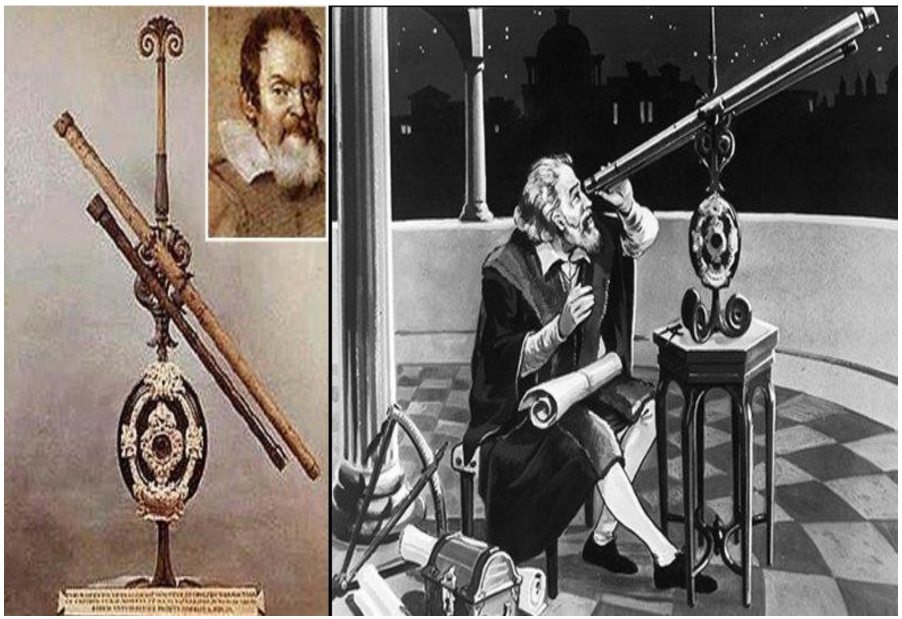
5000 years of observations
500 years of organized observations







Multi-messenger Astronomy Era!



EM observations: 400 years

GW observations: 6 years

- "There are the ones that invent occult fluids to understand the Laws of Nature. They come to conclusions, but they now run out into dreams and chimeras neglecting the true constitutions of the things...
  However there are those that from the simplest observation of Nature, they reproduce New Forces"...

From the Preface of PRINCIPIA (II edition) 1687 by Isaac Newton, written by Mr. Roger Cotes.

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THANK YOU!