NEUTRINO COSMOLOGY AND DARK MATTER

MASSIMILIANO LATTANZI

INFN, sezione di Ferrara

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From Theory to Observations

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OUTLINE

- Neutrino thermal history: Neutrino decoupling, electron-positron annihilation
- Present neutrino energy density
- Neutrino energy density during the radiation-dominated era; effective number of relativistic species
- Effect of neutrinos on cosmological observables

REFERENCES

- Lesgourgues J, Mangano G, Miele G, Pastor S. *Neutrino Cosmology*. Cambridge, UK: Cambridge University Press (2013).
- Gerbino M and Lattanzi M (2018) *Status of Neutrino Properties and Future Prospects—Cosmological and Astrophysical Constraints.* Front. Phys. 5:70. doi: 10.3389/fphy.2017.00070
- J. Lesgourgues and L. Verde, "25. *Neutrinos in Cosmology*", in Review of Particle Physics

Neutrino Thermal History

Let us apply what you learned so far to reconstruct the thermal history of neutrinos in the framework of the standard hot Big Bang model.

The early Universe was very hot and radiation-dominated (RD). Thermal equilibrium was established by particle interactions whose rates were very large when compared to the expansion rate of the Universe, so that all particle species had a common temperature.

The energy density at a given temperature is dominated by the species with T >> m. Then

$$\rho = \sum_i \rho_i = \left(\sum_{\rm Bosons} g_i + \frac{7}{8} \sum_{\rm Fermions} \right) \frac{\pi^2}{30} T^4 \equiv g_* \frac{\pi^2}{30} T^4$$

where the sum only runs on particle species with m << T. The Friedmann eq. in the RD era reads:

$$H = 1.66 g_*^{1/2} \frac{T^2}{m_{\rm pl}}$$

Neutrino Thermal History

As the Universe cools down, interactions slow down and possibly become ineffective in maintaining equilibrium. The condition for an interaction with rate Γ to be effective is:

$$\Gamma(T) > H(T)$$

Let us consider the Universe at temperatures between m_e=0.5 MeV and m_μ=100 MeV. The energy density is dominated by the species that are UR these temperatures, i.e. photons, electrons, positrons and neutrinos. Neutrinos are kept in equilibrium with the other particles by the weak interactions, through processes like

$$\nu + e^{\pm} \leftrightarrow \nu + e^{\pm} \qquad\qquad \nu + \bar{\nu} \leftrightarrow e^{+} + e^{-}$$

The cross sections for these processes, and the corresponding rates, are roughly

$$\langle \sigma_{\mathrm{weak}} v \rangle \simeq \langle G_F^2 E^2 \rangle \simeq G_F^2 T^2 \,; \qquad \Gamma_{\mathrm{weak}} = \langle \sigma_{\mathrm{weak}} v \rangle n \simeq G_F^2 T^5$$

NEUTRINO THERMAL HISTORY

Then the condition for equilibrium reads

$$G_F^2 T^5 > 1.66 g_*^{1/2} \frac{T^2}{m_{\rm pl}} \longrightarrow T > 1\,\mathrm{MeV}$$

Thus at temperatures below ~1 MeV, neutrinos ***decouple*** from the cosmological plasma, since weak interactions become so rare that they cannot maintain equilibrium anymore.

It can be shown that, since decoupling happens when neutrinos are ultrarelativistic, the phase space density of neutrinos evolves in such a way to keep the form of an ultrarelativistic fermion at equilibrium:

$$f_\nu(\vec{p}) = \frac{1}{e^{p/T_\nu} + 1}$$

The (effective) neutrino temperature T_ν goes down with the expansion like 1/a .

NEUTRINO THERMAL HISTORY

Shortly after neutrino decouple, the temperature drops below the electron rest mass and electron-positron pairs begin to annihilate without being replenished by the inverse reaction $\gamma\gamma \rightarrow e^+e^-$

The entropy of electrons and positrons goes into heating the photons. On the other hand neutrinos, being already decoupled, are not heated up. The result is that after e^+e^- annihilation, photons are hotter than neutrinos. The ratio between temperatures can be easily computed from entropy conservation:

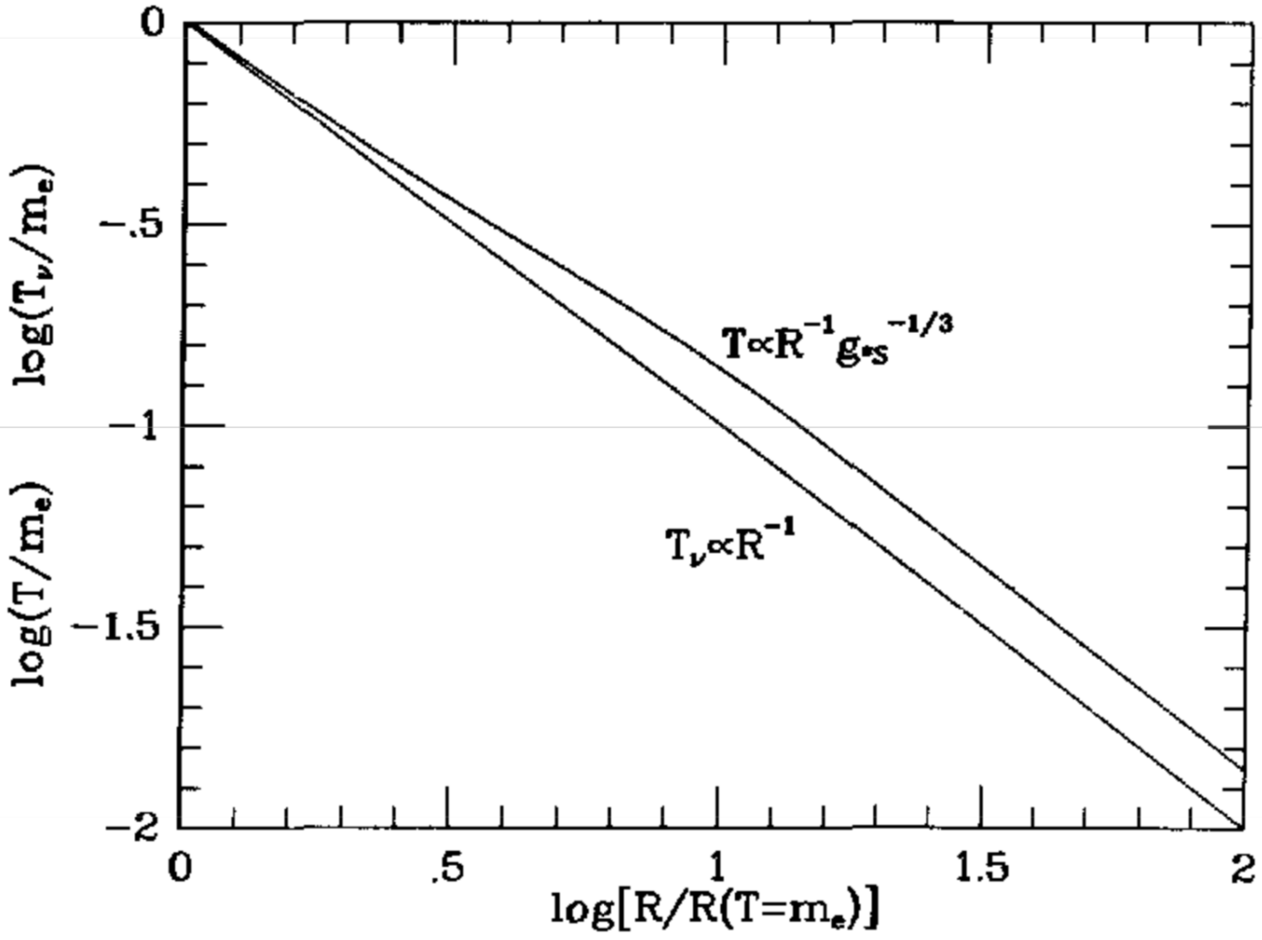
$$g_{*s} T_\gamma^3|_{\rm before} = g_{*s} T_\gamma^3|_{\rm after} \, ; \qquad T_\nu|_{\rm before} = T_\nu|_{\rm after}$$

yielding

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \quad \text{for } T_\gamma < m_e$$

(in reality, neutrinos share a small part of the entropy production, see below)

NEUTRINO THERMAL HISTORY



From
Kolb&Turner

Fig. 3.8: The evolution of T and T_ν through the epoch of $e^\pm$ annihilation.

Neutrino Thermal History

The present photon temperature is well measured: $T_{\gamma,0}$ = 2.725 K. Then:

$$T_{\nu,0} \simeq 1.9\mathrm{K} \simeq 1.6 \times 10^{-4}\mathrm{eV}$$

Remember:
1K = 0.86 x 10^{-4} eV

that corresponds to a present day density per neutrino species

$$n_{\nu,0} = \frac{3}{2}\frac{\zeta(3)}{\pi^2}T^3_{\nu,0} \simeq 113\mathrm{cm}^{-3}$$

(remember that for photons $n_0 \simeq$ 400 cm^{-3})

Thus we expect that the Universe is filled by a background of relic thermal neutrinos, the ***Cosmic Neutrino Background (CNuB)*** with temperature and density of the same order of magnitude as CMB photons.
Clearly, the former are much harder to detect than the latter!

Relic Neutrinos in the RD era

Let us focus for the moment on early times, when neutrinos are relativistic. After the e+e- annihilation we have

$$\rho_\nu = 3 \times 2 \times \frac{7}{8} \times \frac{\pi^2}{30} T_\nu^4 = 3 \times \frac{7}{8} \times \left(\frac{4}{11}\right)^{4/3} \rho_\gamma$$

and the total radiation density is

$$\rho_r = \rho_\nu + \rho_\gamma = \left[1 + 3 \times \frac{7}{8} \times \left(\frac{4}{11}\right)^{4/3}\right] \rho_\gamma$$

RELIC NEUTRINOS IN THE RD ERA

This is usually rewritten to allow for a contribution of additional exotic light relics (e.g. sterile neutrinos, thermal axions), or for a nonstandard contribution of neutrinos, to the radiation density (note that now this becomes a **definition!**)

$$\rho_r \equiv \left[1 + N_{\mathrm{eff}} \times \frac{7}{8} \times \left(\frac{4}{11}\right)^{4/3}\right] \rho_\gamma$$

N_{eff} is called the ”effective number of neutrino species”. It is in fact a measure of the energy density of light species, other than photons, in the early Universe, normalized to the energy density of a single massless neutrino with standard thermal history.

For active neutrinos, one would expect N_{eff} =3. In fact, due to non-instantaneous decoupling, the standard prediction is slightly larger: N_{eff} =3.045.

Relic Neutrinos in the RD era

Nnu=3 is true in the instantaneous neutrino decoupling approximation. However:

- Decoupling is not instantaneous -> high-momentum neutrinos still coupled to the photon bath -> distortions in the neutrino distribution function + photon/neutrino temperature < (11/4)^1/3
  One should solve the Boltzmann evolution equation (~ % correction to Nnu=3)

$$\left(\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right) f_{\nu_\alpha}(t,p) = C[f_{\nu_\alpha}; f_{\nu_\beta}; f_{e_\pm}]$$

- QED radiative corrections must be taken into account (finite-temperature corrections to the plasma equation of state)
  sub-percent correction to Nnu=3
- Flavour oscillations are active around neutrino decoupling and must be accounted for
  One should switch to the density matrix approach when solving the Boltzmann equation
  sub-dominant effects

$$\left(\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right) \rho_p(t) = -i\left[\left(\frac{1}{2p}M_F - \frac{8\sqrt{2}G_F p}{3m_w^2}E\right), \rho_p(t)\right] + I(\rho_p(t))$$

$$N_{eff} = 3.0440 \pm 0.0002$$

Dolgov; Mangano+ 2005; ….; Akita&Yamaguchi 2020; Bennett+,2020; Froustey+ 2020

RELIC NEUTRINOS – PRESENT ENERGY DENSITY

Neutrinos with a mass > 10^{-4} eV would be nonrelativistic today, with density

$$\rho_{\nu,0} = m_\nu n_{\nu,0} \quad \longrightarrow \quad \Omega_\nu h^2 = \frac{m_\nu}{93.14\,\mathrm{eV}}$$

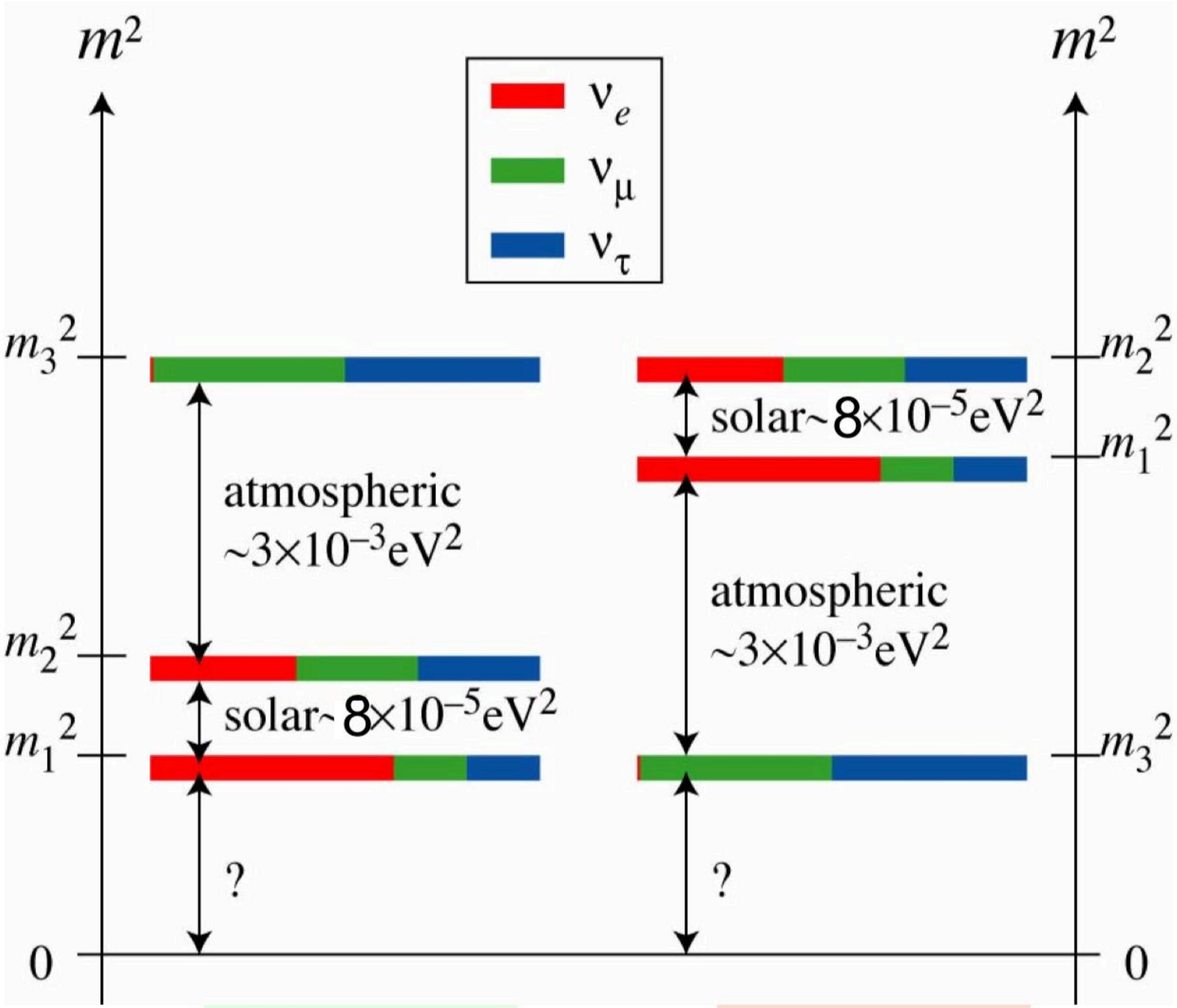
We know from flavour oscillation experiments that at least two of the three active neutrino eigenstates should be non relativistic today, and that the sum of neutrino masses

$$\sum m_i \geq 0.06\,\mathrm{eV}\ \text{(Normal ordering), or}\ \geq 0.10\,\mathrm{eV}\ \text{(Inverted ordering)}$$

which implies

$$\Omega_\nu h^2 \geq 6 \times 10^{-4}\ \text{(NO), or}\ \geq 10^{-3}\ \text{(IO)}$$

Relic Neutrinos – Present Energy Density



Normal Inverted

RELIC NEUTRINOS – PRESENT ENERGY DENSITY

The lower bound on neutrino masses from oscillation experiments implies, in the framework of the standard cosmological model (SCM), a lower limit on the present neutrino density.

On the other hand, if we can obtain from observations an upper limit on the present neutrino density, this can be translated on an upper limit on the neutrino mass, again in the framework of the SCM. This is exactly how cosmological limits on neutrino masses are obtained.

This was understood already in the 60's by **Gerstein and Zel'dovich** (and Cowsik+McClelland), who realized that the simple requirement that neutrinos do not overclose the Universe ($\Omega_\nu < 1$) implies that the sum of the masses should be < 100 eV.

This was far better than bounds on the sum of neutrino masses provided by laboratory experiments at the times!

Energy density of Relic Neutrinos

In general, after decoupling the energy density of a single neutrino species is given by

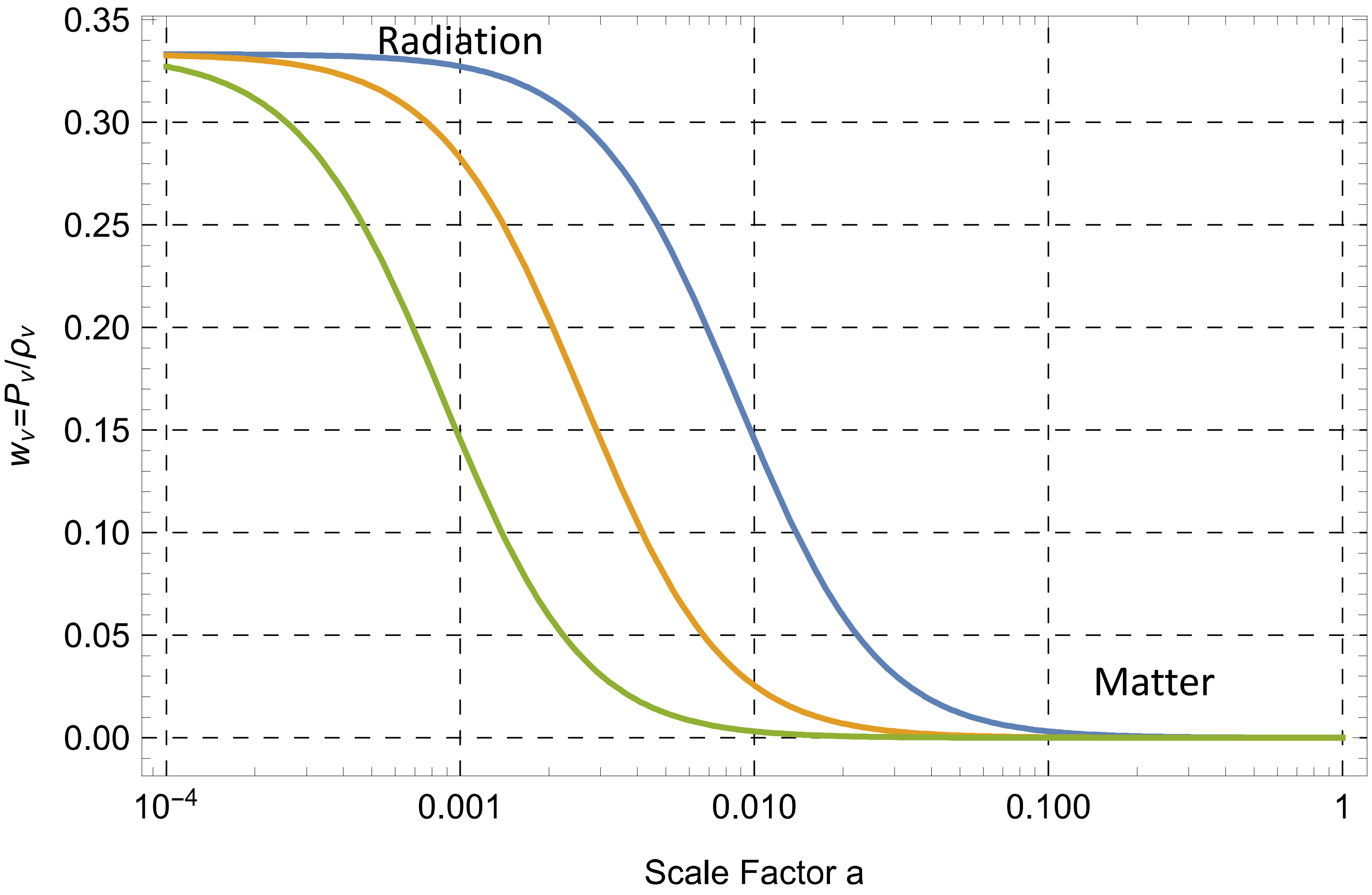
$$\rho_\nu = \frac{1}{\pi^2}\int \frac{\sqrt{p^2+m^2}}{e^{p/T_\nu}+1}p^2dp = \frac{T_\nu^4}{\pi^2}\int \frac{\sqrt{y^2+\frac{m^2}{T_\nu^2}}}{e^y+1}y^2dy$$

From this it is clear that neutrinos behave as "radiation" at early times, when T ≫ m, with their energy density scaling like $(1+z)^4$, while they behave as "matter" at late times, when T ≪ m , with their energy density scaling like $(1+z)^3$.

Transition between the two regimes happens when

$$\langle p\rangle \simeq m_\nu \longrightarrow 1+z_{\rm nr} \simeq 1900\left(\frac{m_\nu}{\rm eV}\right)$$

EQUATION OF STATE OF RELIC NEUTRINOS



m_ν = 0.06 eV

m_ν = 0.2 eV

m_ν = 0.6 eV

THE COSMIC NEUTRINO BACKGROUND

The presence of a background of relic neutrinos (***CνB***) is a basic prediction of the standard cosmological model

- Neutrinos are kept in thermal equilibrium with the cosmological plasma by weak interactions until T ~ 1 MeV (z ~ 10^{10});
- Below T ~ 1 MeV, neutrino free stream keeping an equilibrium spectrum:

$$f_\nu(p) = \frac{1}{e^{p/T} + 1}$$

- Today T_n = 1.9 K and n_n = 113 part/cm^3 per species
- Free parameters: the three masses (but cosmological evolution mostly depends on their sum)

The Cosmic Neutrino Background

Weak cross section: $\sigma \simeq G_F^2 T^2$

Weak interaction rate $\Gamma = n\langle \sigma v \rangle \sim G_F^2 T^5$

Expansion rate $H \simeq \frac{T^2}{m_{\rm p}}$

Interactions become ineffective when $T = T_d$ such that

$$1 \simeq \frac{\Gamma}{H} \sim G_F^2 T^3 m_{\rm p} \sim \left(\frac{T}{\rm MeV}\right)^3$$

Given this, we can use conservation laws to compute the temperature, density, etc… of neutrinos at a given time.

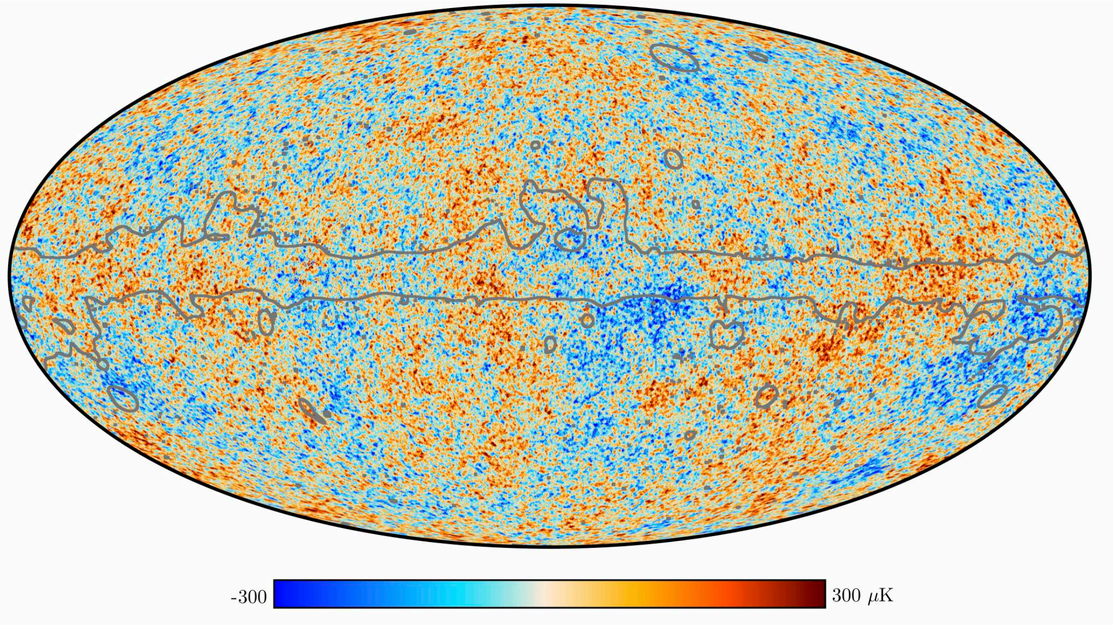
The Cosmic Neutrino Background

This picture relies on the following:

- only weak and gravitational interactions for ν's;
- no sterile neutrinos or other light relics;
- perfect lepton symmetry (zero chemical potential);
- no entropy generation after neutrino decoupling beyond e^+e^- annihilation;
- neutrinos are stable;
- in general, there are no interactions that could lead to neutrino scattering/annihilation/decay

Relic Neutrinos and Cosmological Observables

The Cosmic Microwave Background

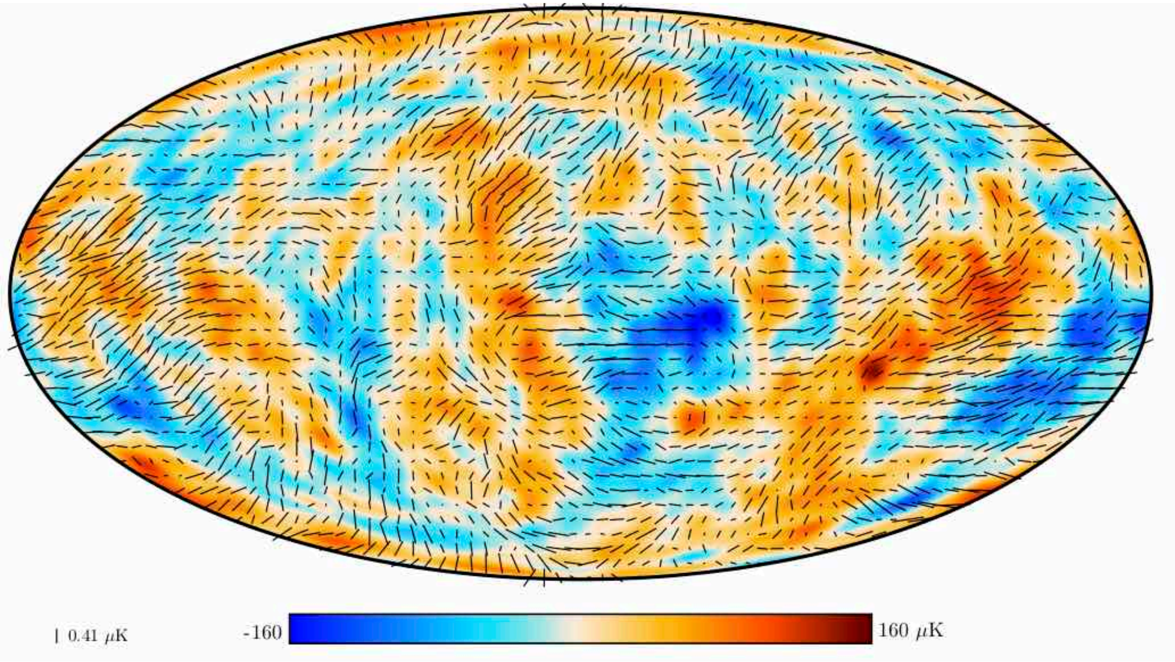


Spherical harmonic expansion:

$$T(\vec{n}) = \sum_{\ell,m} a^T_{\ell m} Y_{\ell m}(\vec{n})$$

$$\langle a^T_{\ell m} a^{T*}_{\ell' m'} \rangle = C^{TT}_\ell \delta_{\ell\ell'} \delta_{mm'}$$

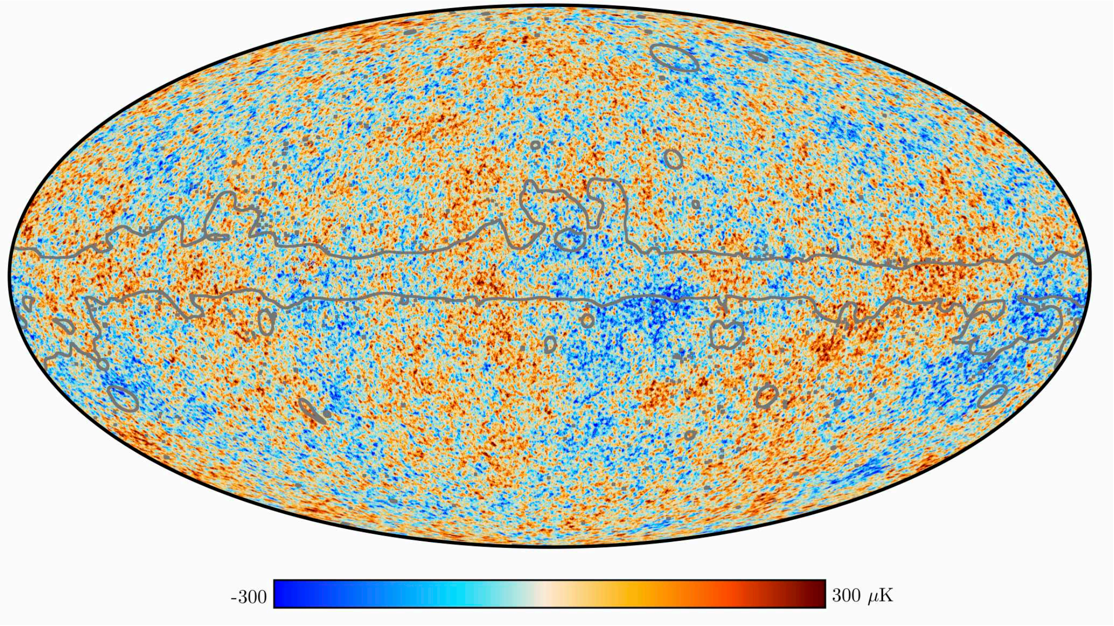
(similarly for E and B pol.)



Fluctuations are gaussian, so all information about the stochastic properties of the maps is contained in their (auto and cross) power spectra C_l

Temperature and polarization maps from Planck (2018)

The Cosmic Microwave Background



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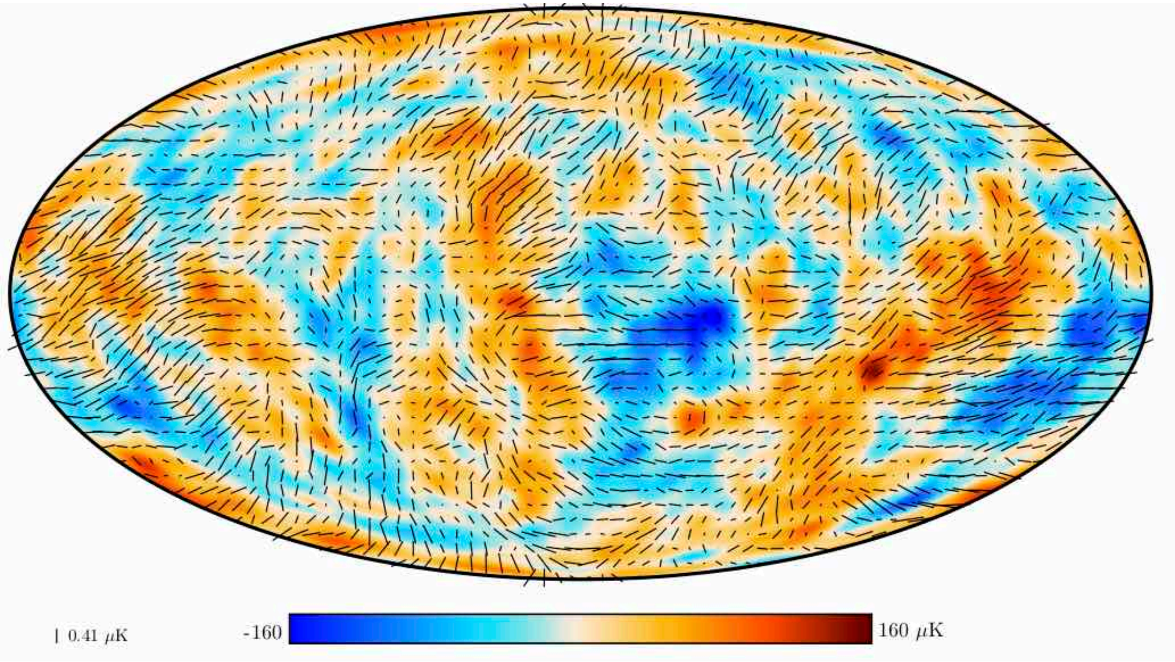
(similarly for E and B pol.)

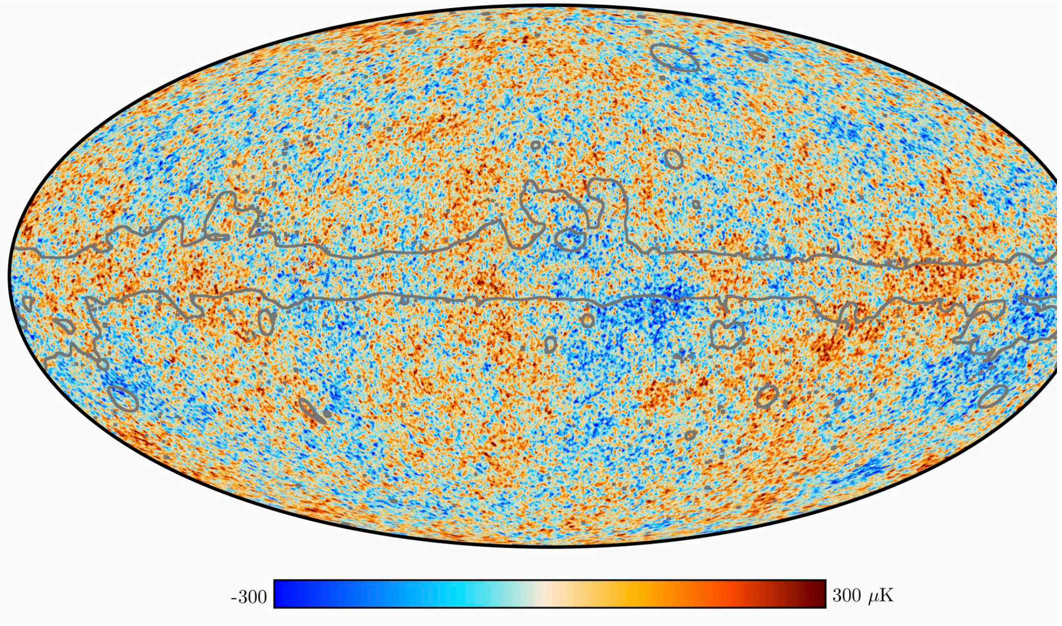
3 maps, 6 correlations: TT, EE, BB, TE, TB, EB

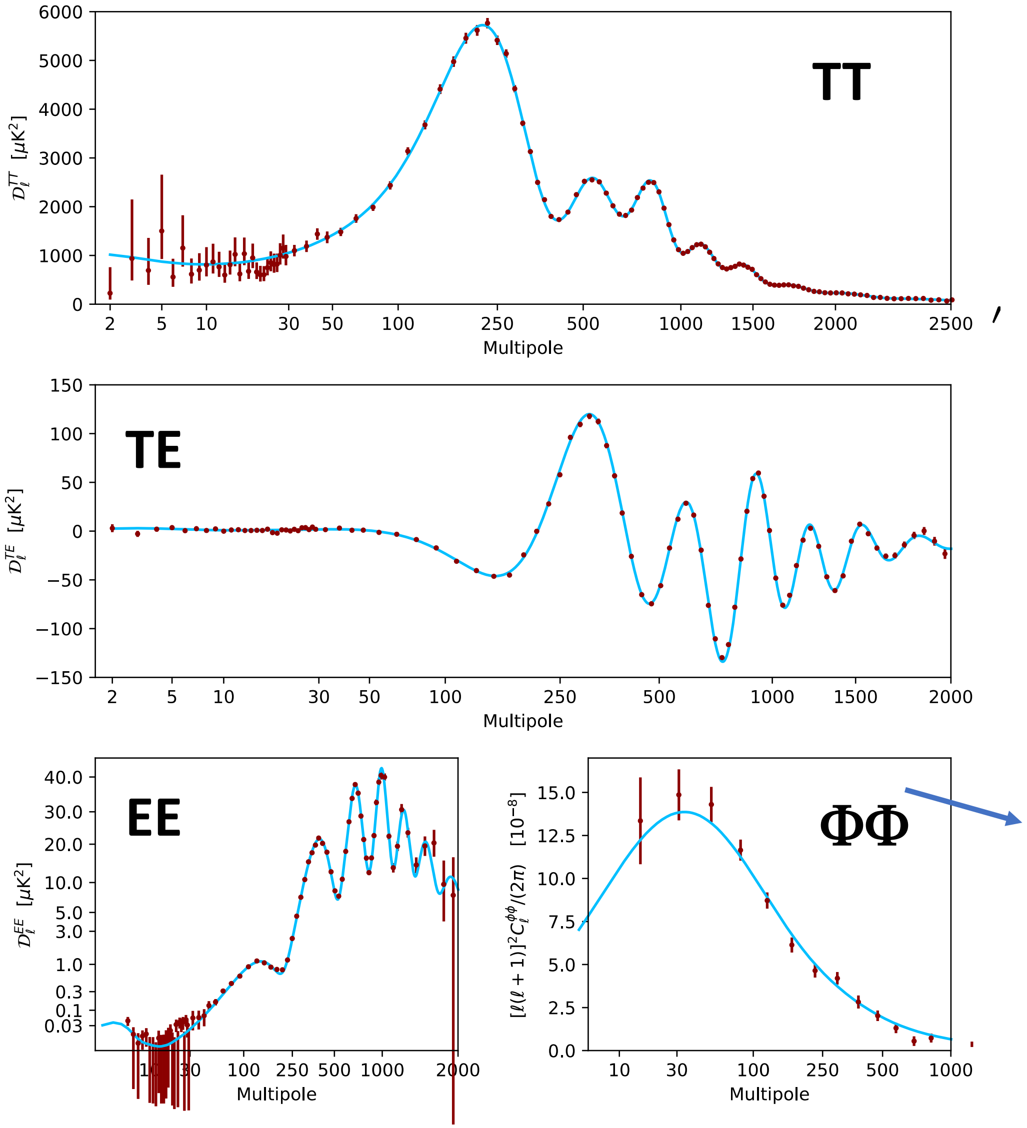
But TB,EB=0 (parity conservation)



Temperature and polarization maps from Planck (2018)

The Cosmic Microwave Background





3 maps, 6 correlations: TT, EE, BB, TE, TB, EB

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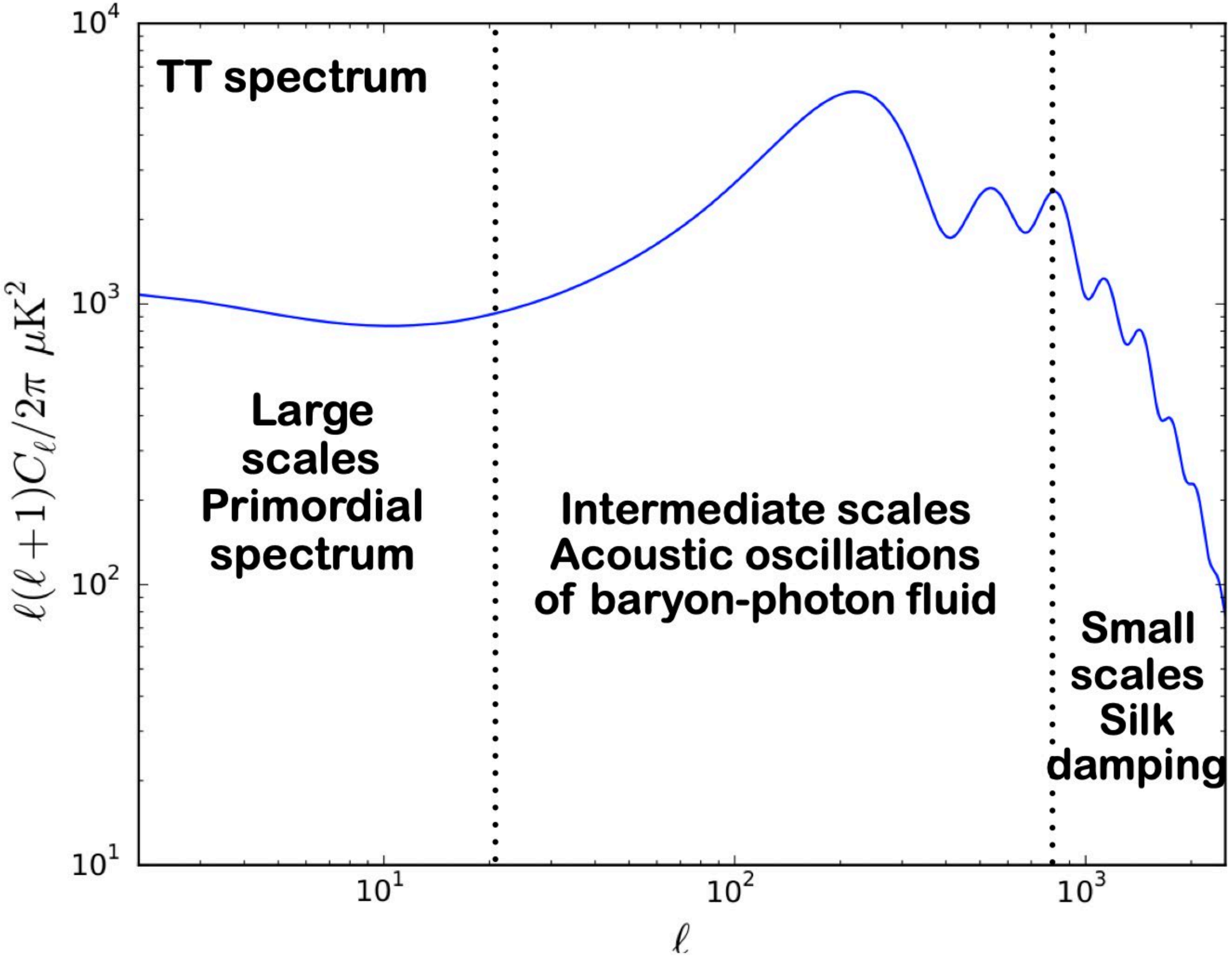
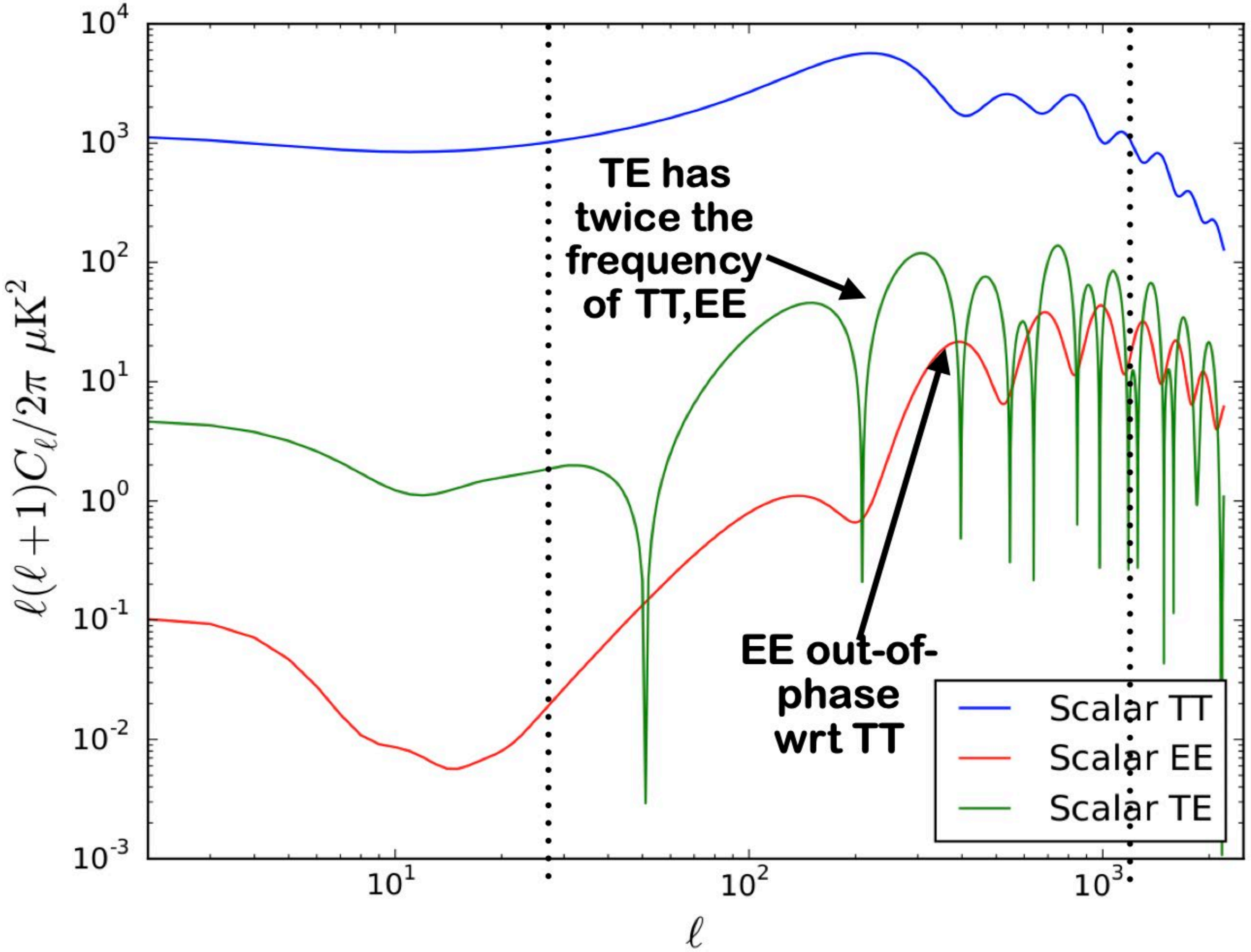
Lensing power spectrum (see below!)

Temperature and polarization maps from Planck (2018)

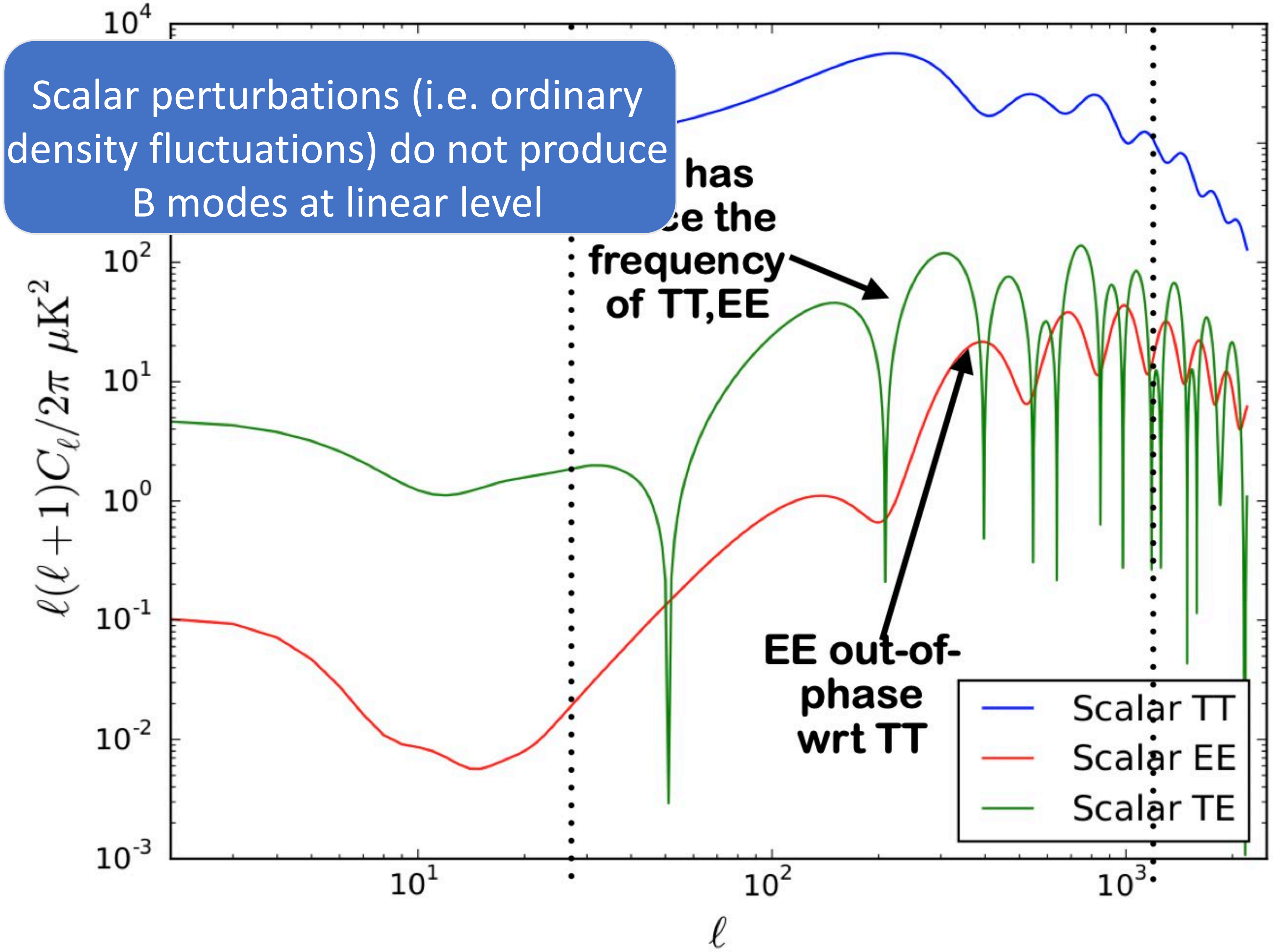
IMPRINT OF EARLY-UNIVERSE PHYSICS



Imprint of Early-Universe Physics



IMPRINT OF EARLY-UNIVERSE PHYSICS

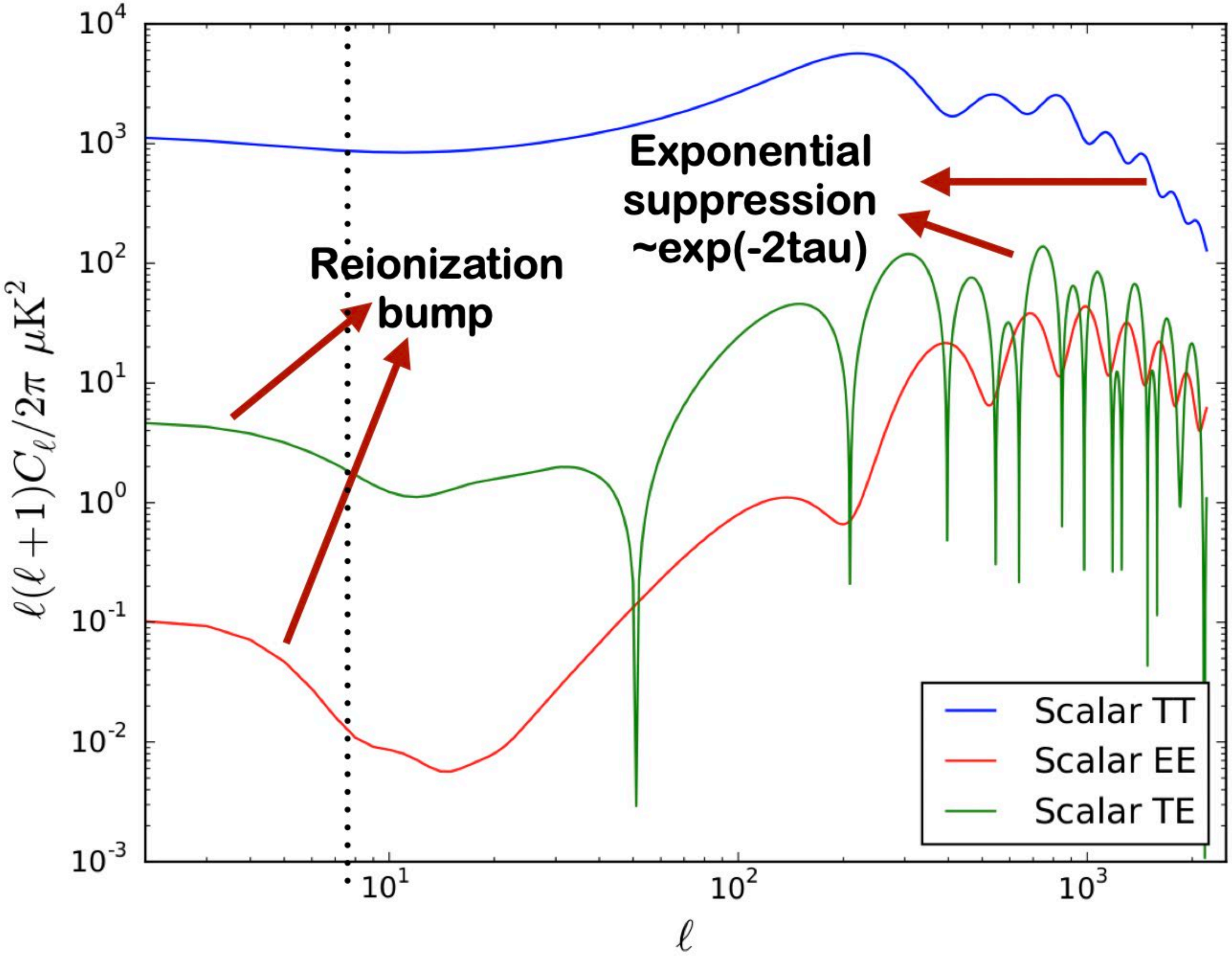


Imprint of Late-Universe Physics

In the late Universe, CMB photons are partly rescattered by a new population of free electrons from the ionization of neutral hydrogen caused by the lighf of the first stars, a process that goes by the name of "reionization".

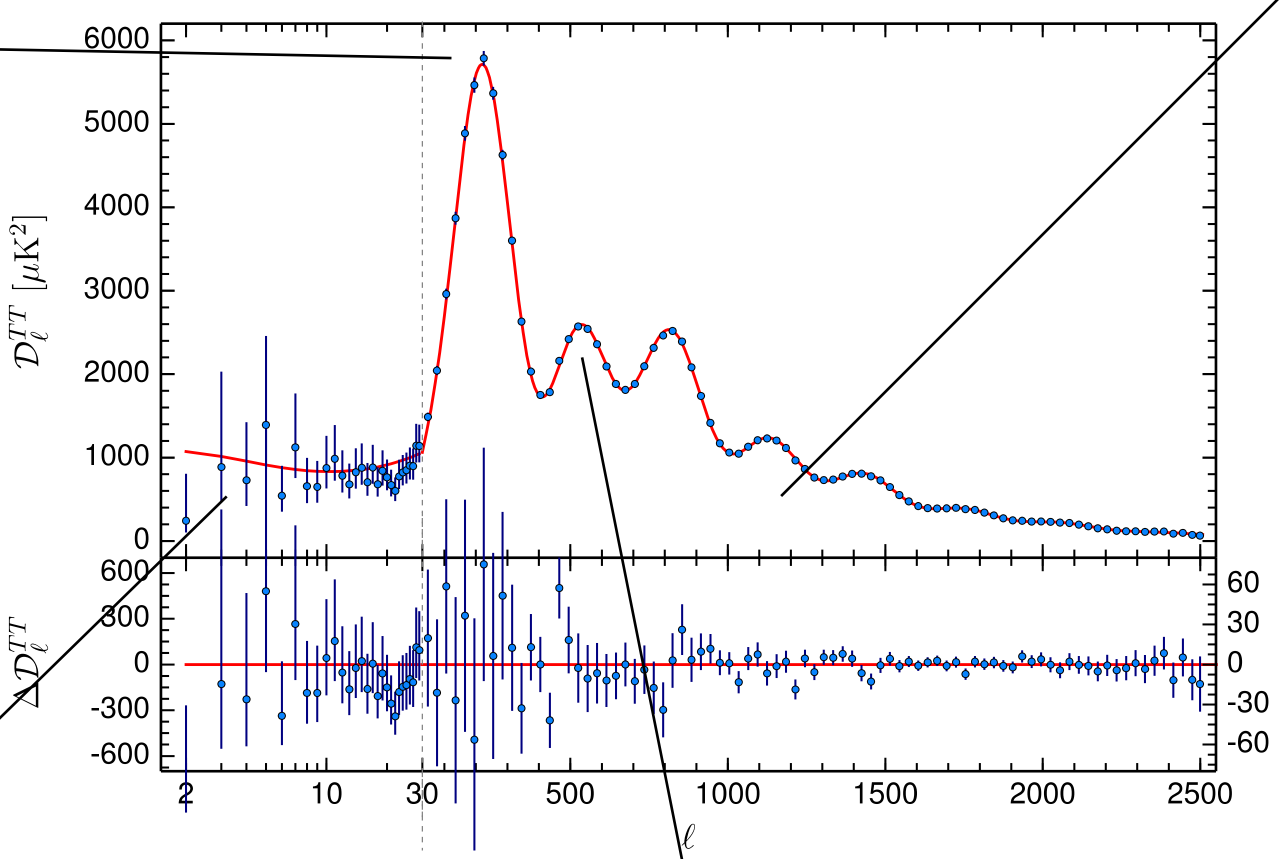
The optical depth to reionization τ gives the fraction of rescattered photons

This suppresses the anisotropies as exp($-\tau$), but also generates new polarization at large scales ("reionization bump")



CMB AND COSMOLOGICAL PARAMETERS

First peak:
spatial curvature;
relative abundance of matter and radiation;
distance to the last scattering surface;
H_0, Ω_m, Ω_k



Damping tail:
Photon diffusion length at recombination;
Slope of the primordial spectrum;
N_{eff}, Ω_b, Y_p, n_s

+ Overall power
$A_s e^{-2\tau}$

+ low-ell polarization (not shown)
Reionization history
τ

Large ell plateau: Primordial power spectrum; late time expansion (relative abundance of matter and dark energy);
A_s, Ω_Λ

Relative height of first and second peak: Baryon abundance
Ω_b

Characteristic scales in the CMB angular power spectrum

Characteristic length scales imprinted in the CMB power spectrum:

sound horizon
sets position of the first peak and spacing between peaks

$$r_s(z_*) = \int_{z_*}^{\infty} c_s \frac{dz}{H(z)}$$

damping length
controls exponential suppression of the power at small scales (large multipoles)
$\exp(-k^2 r_d^2)$

$$r_d(z_*) = \left[\int_{z_*}^{\infty} dz \frac{(1+z)F(R)}{\sigma_T n_e H} \right]^{1/2}$$

(why the square root? Because it's a random walk!)

We actually see length scales projected on the sky through the ***angular diameter distance***

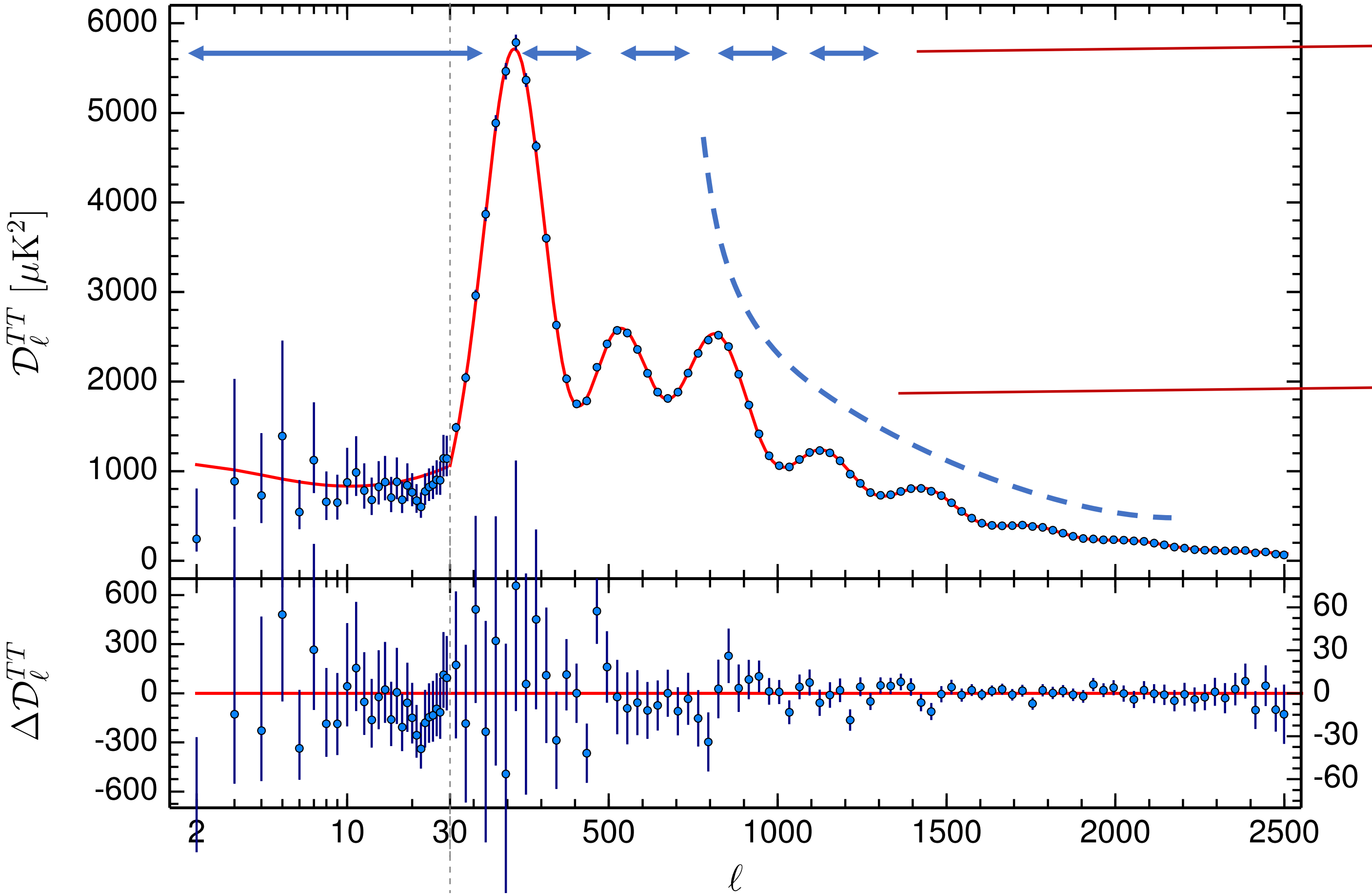
$$D_A(z_*) = \int_0^{z_*} c \frac{dz}{H(z)}$$

Characteristic ***angular*** scales imprinted in the CMB power spectrum:

$$\theta_{d*} = \frac{r_d(z_*)}{D_A(z_*)} \qquad \theta_{s*} = \frac{r_s(z_*)}{D_A(z_*)}$$

(all lengths in this slide are comoving)

Characteristic scales in the CMB angular power spectrum



The positions and spacing of the peaks are set by

$$\theta_{s*} = \frac{r_s(z_*)}{D_A(z_*)}$$

The damping envelope* $e^{-\ell^2\theta_d^2}$ is set by

$$\theta_{d*} = \frac{r_d(z_*)}{D_A(z_*)}$$

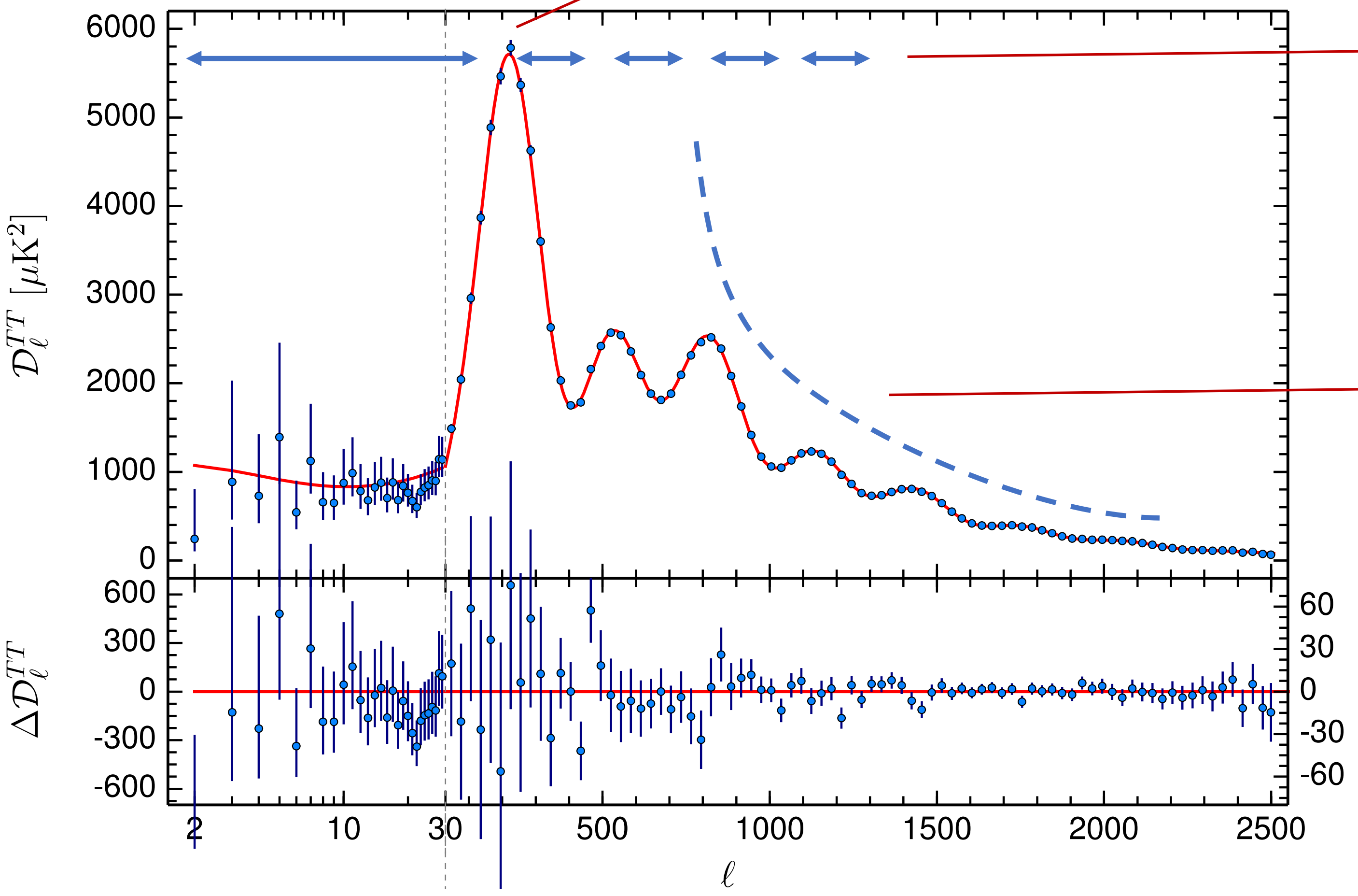
Note that the ratio θ_s/θ_d does **not** depend of D_A

*beware of factors of pi

Characteristic scales in the CMB angular power spectrum

Height of the first peak depends on the redshift of matter-radiation equality

$$1 + z_{\rm eq} = \frac{\Omega_m h^2}{\Omega_r h^2}$$



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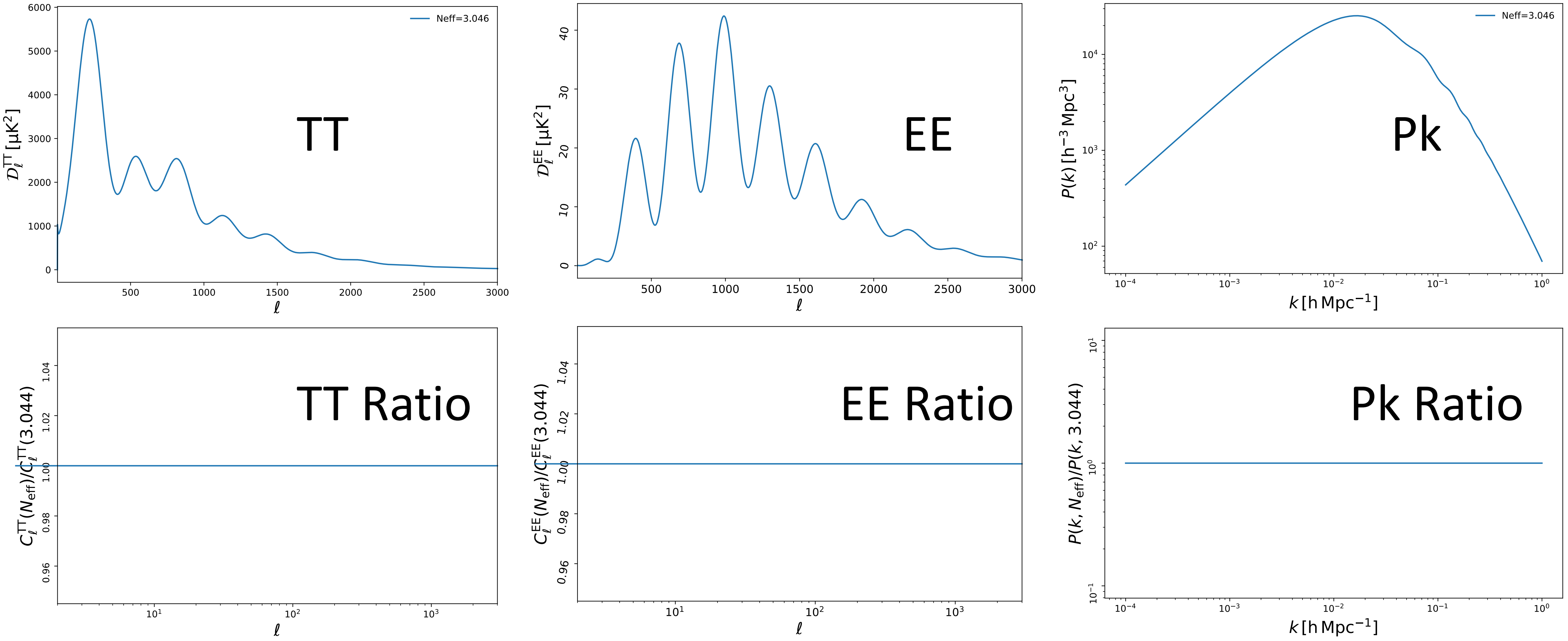
Characteristic scales in the CMB angular power spectrum

- In a flat Universe with NR matter, a cosmological constant, photons, and massless neutrinos, the Hubble rate before recombination is given by:

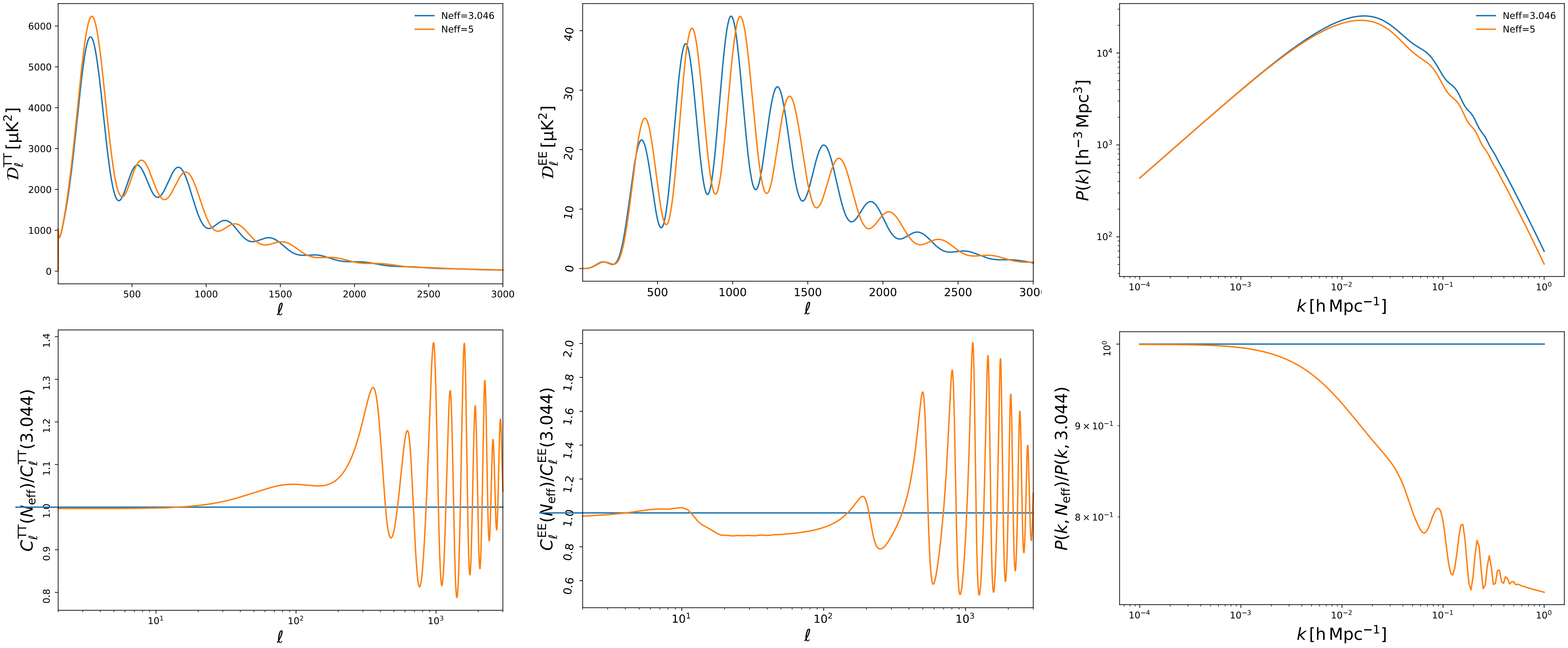
$$H(z) = H_0 \left[\Omega_\gamma \left(1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_{\mathrm{eff}} \right) (1+z)^4 + \Omega_m (1+z)^3 \right]^{1/2}$$

- Increasing the energy density of neutrinos (or of any effectively massless species), i.e. increasing N_{eff}, will increase H(z), making the Universe younger, and expanding faster, at a given *z.*
- Increasing N_{eff} will thus decrease both r_s and r_d; however, their ratio will decrease like $H^{-1/2}$
- In the end, *for fixed position of the first peak* (which is very well measured!), an increase in N_{eff} will increase θ_d and thus result in *more damping at small scales.*
- Note that $\omega_\gamma = \Omega_\gamma h^2$ is well constrained by the measurement of the CMB energy density.
- The matter density $\omega_m = \Omega_m h^2$ represents an additional degree of freedom that can however be constrained by the height of the first peak (that is sensitive to the ratio ω_m / ω_{rad})

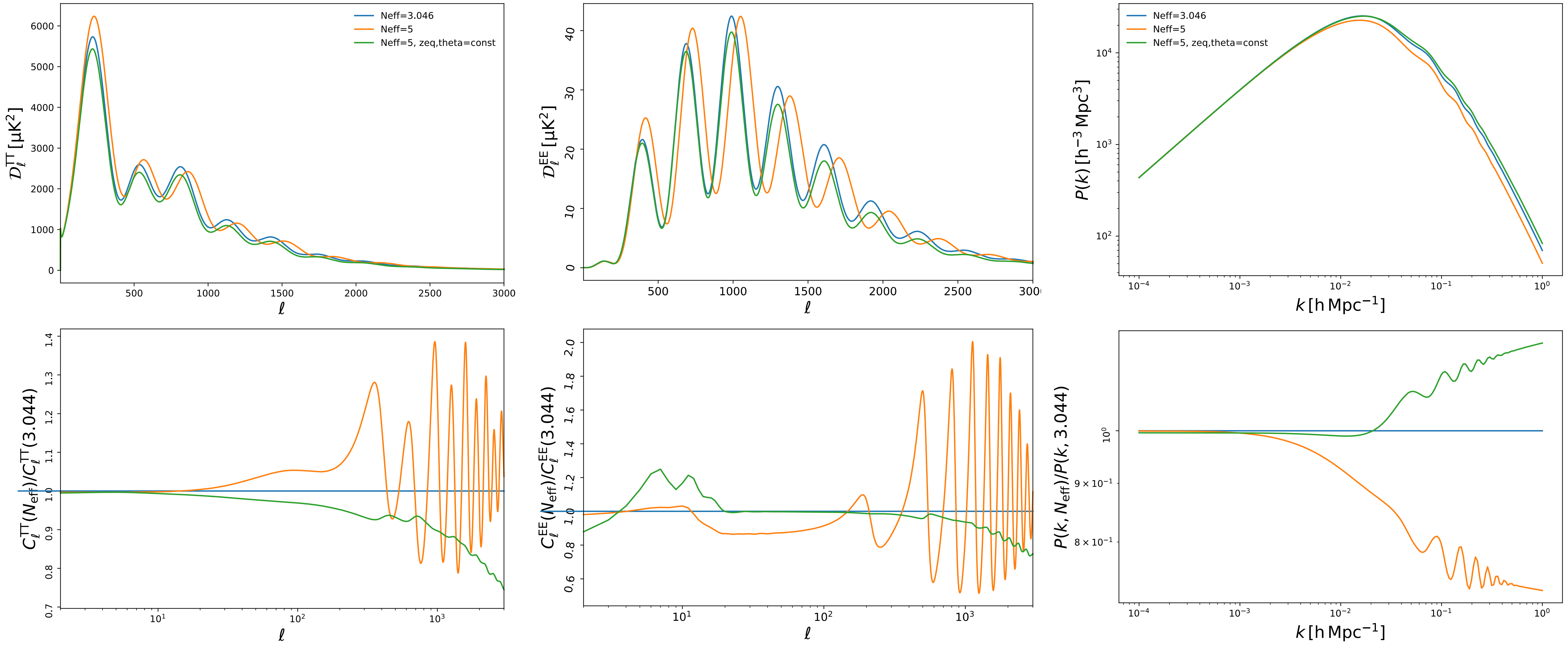
Neff - Reference



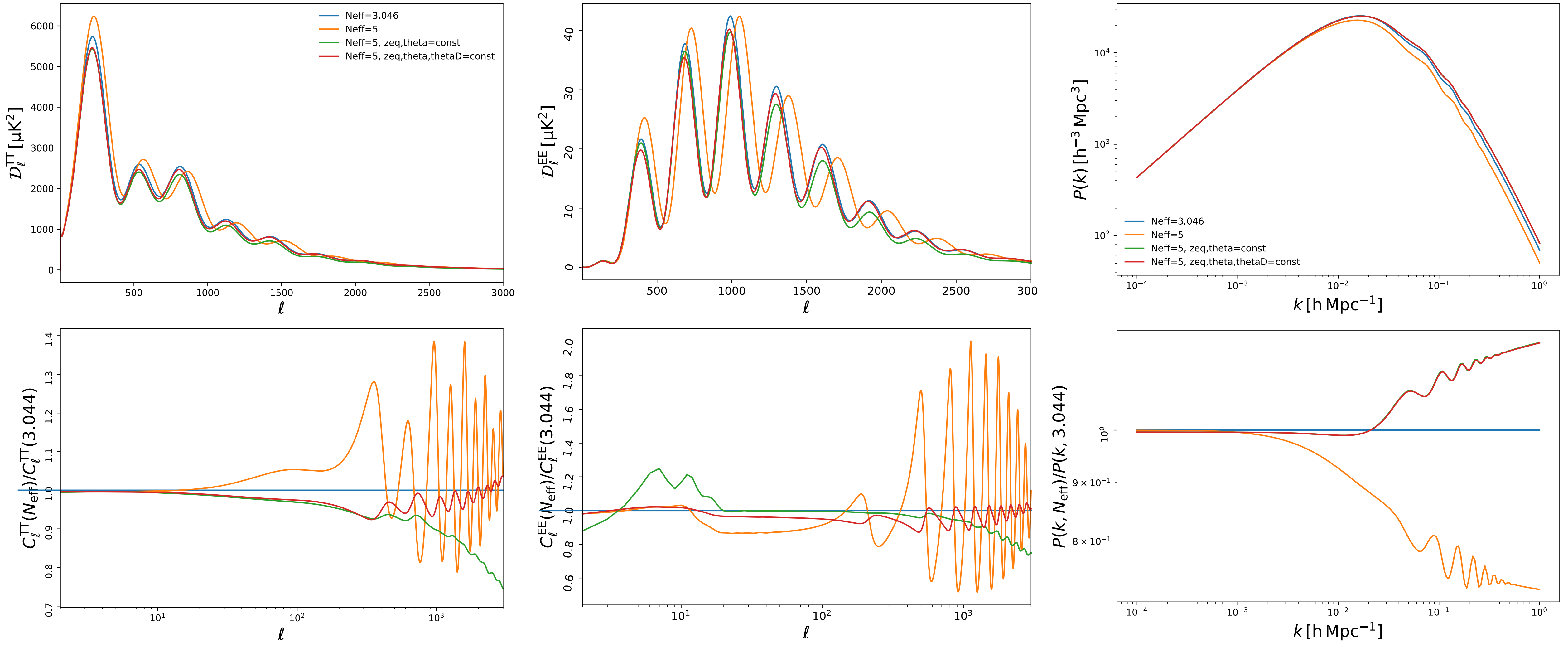
NEFF – INCREASING NEFF WRT REFERENCE



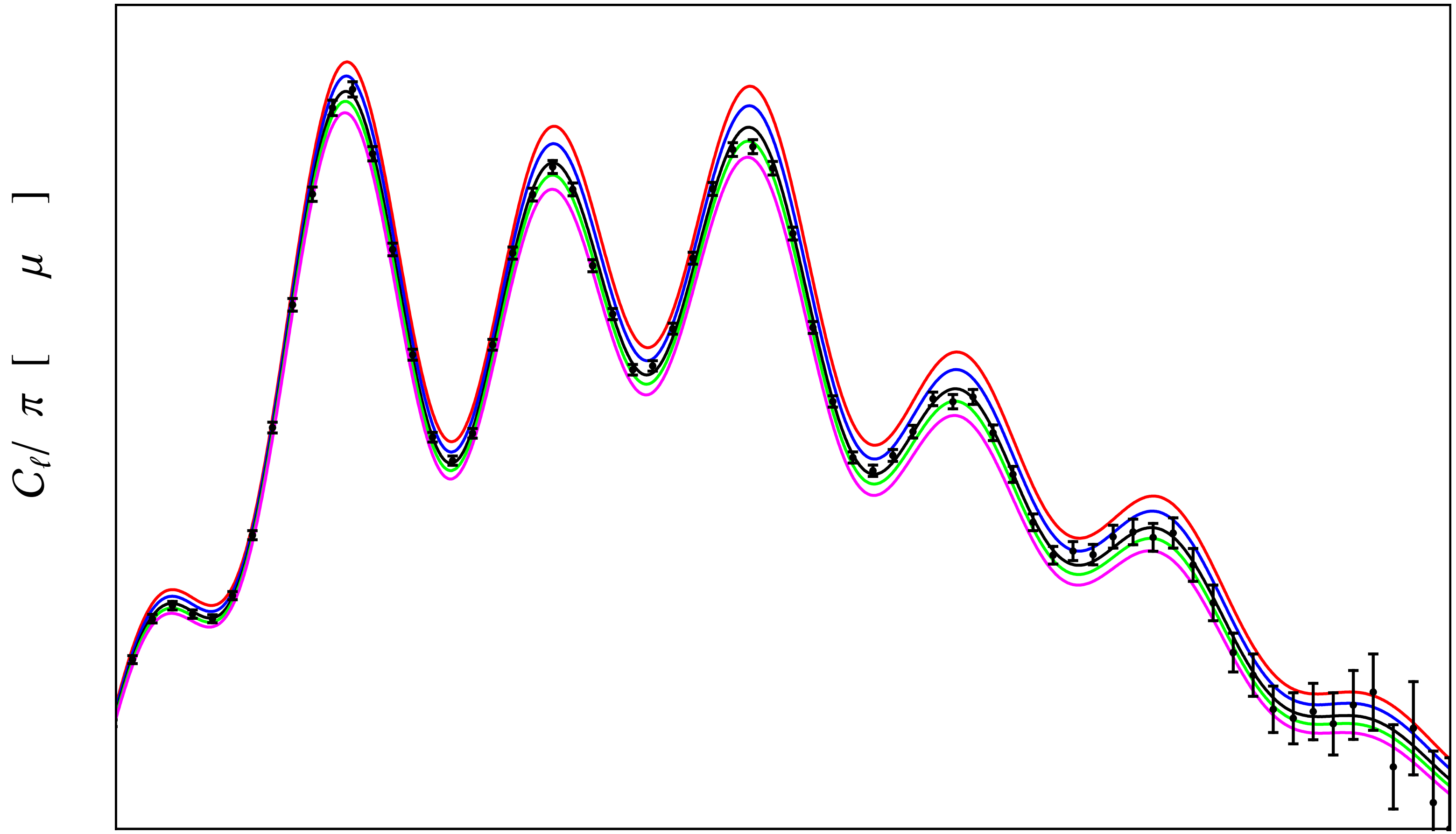
NEFF – KEEPING ZEQ, THETA* FIXED



NEFF – KEEPING ZEQ, THETA*, THETAD FIXED



OBSERVING THE $C\nu B$

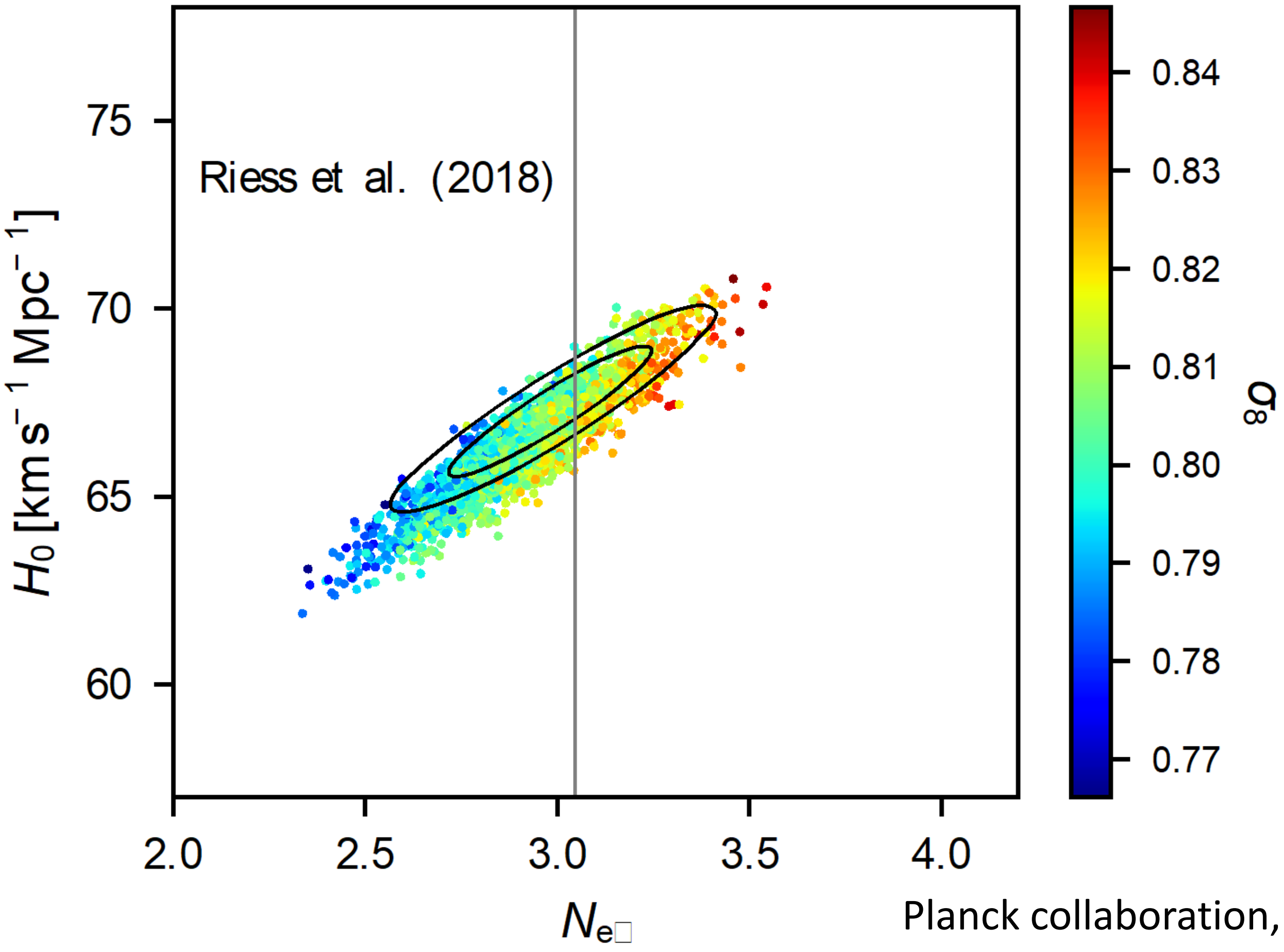


Planck 2018
Neff = 2.89 +/- 0.19

Planck 2018 + BAO:
Neff = 2.99+/- 0.17

(note I am showing ~ $l^4 C_l$, not $l^2 C_l$)

Effective number of neutrinos from Planck

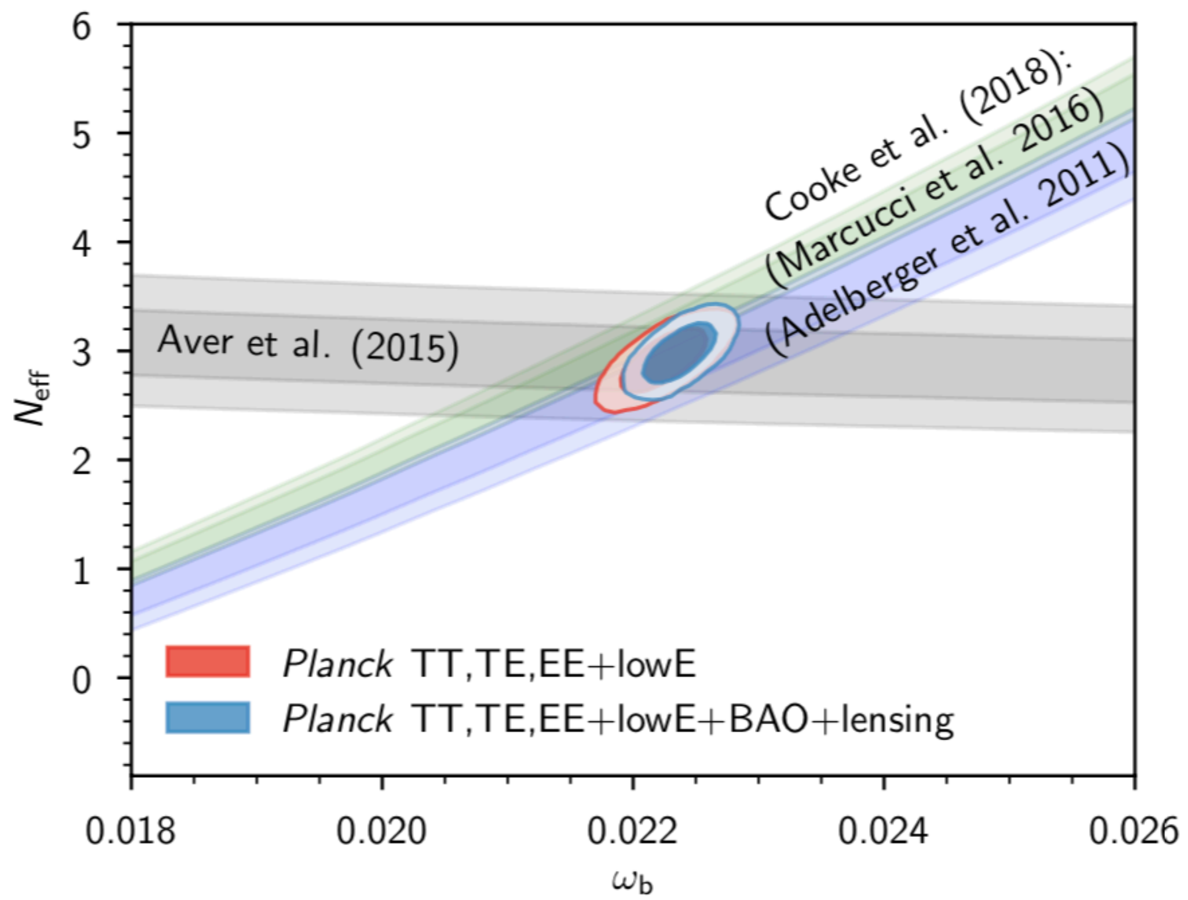


Planck collaboration, VI 2018

Planck 2018:
Neff = 2.89+/- 0.19

Excludes a fourth, very light, ***thermalized*** neutrino at more than 5σ

Effective number of neutrinos and Big Bang Nucleosynthesis



Planck collaboration VI, 2018

The synthesis of light elements is essentially a competition between the nuclear reactions and expansion.
As such, it is pretty sensitive to N_{eff}.

Constraints on the energy density of light species from the abundance of light elements and from CMB observations are in excellent agreement

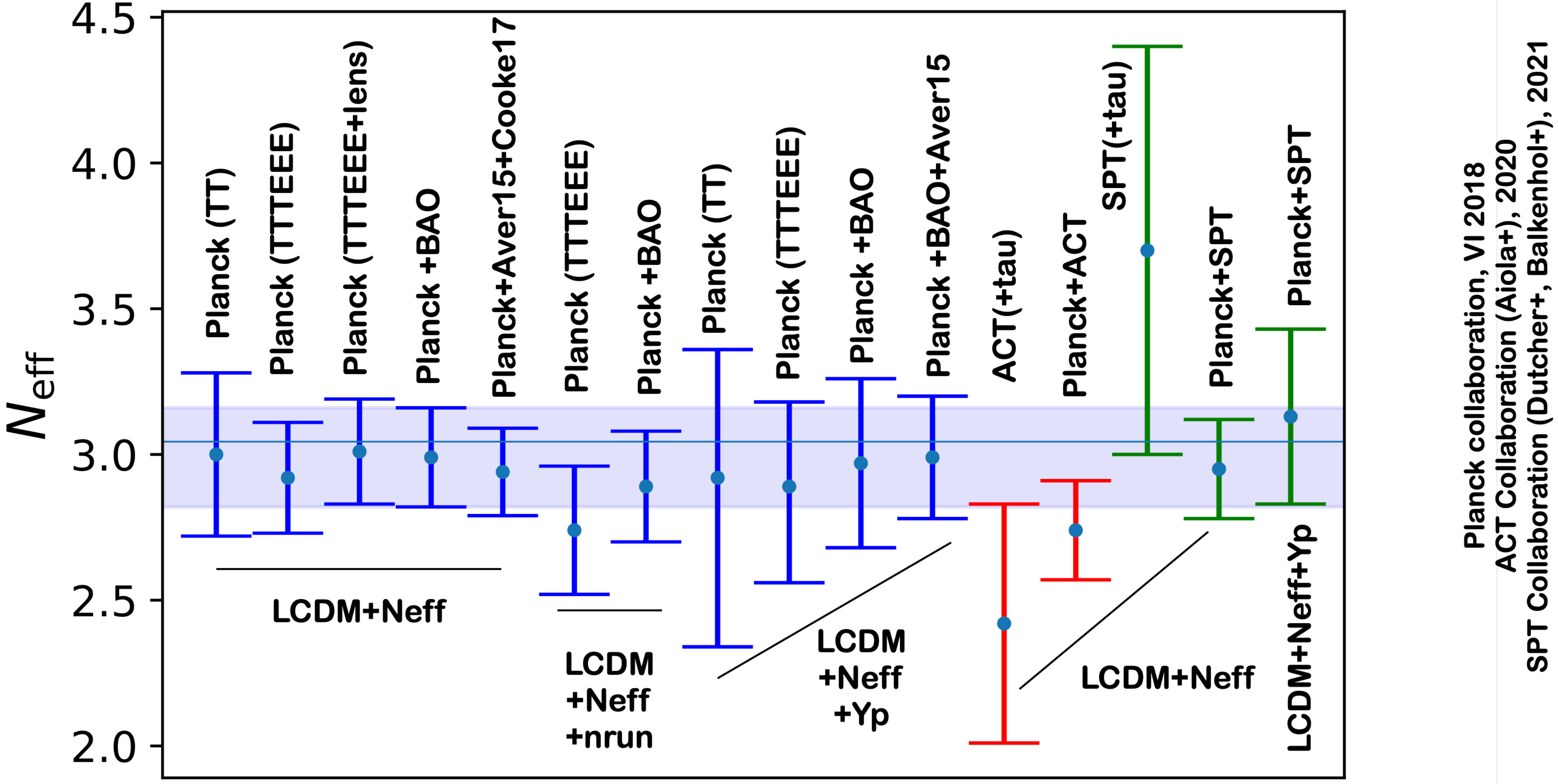
N_{eff} = 2.86 +/- 0.28 [Yp + D/H]

Combining BBN and CMB data:

N_{eff} = 2.88 +/- 0.15 [BBN + CMB]

Pisanti et al, JCAP 2021
Yeh et al., JCAP 2021

CURRENT LIMITS ON NEFF (68% CL)



Credit: M. Gerbino