A Finite S-matrix

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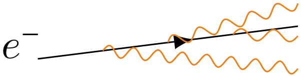
arXiv: 1810.10022 with C. Frye, N. Paul, M. Schwartz, and K. Yan

arXiv: 1906.03271 with M. Schwartz

arXiv: 1911.06821 with M. Schwartz

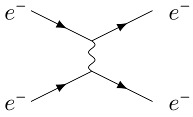


Outline

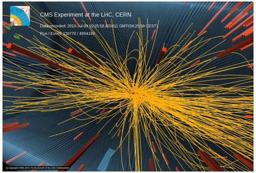
- Introduction: IR divergences
- Ideas for IR finiteness:
  - Cross section method
  - Finite S-matrix
  - Coherent states
- Conclusions & Future directions

The Scattering Matrix (S-matrix)

$\langle f|S|i\rangle$: **Probability amplitude** for measuring a final state $|f\rangle$ given an initial state $|i\rangle$

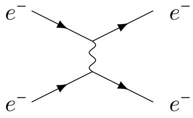


- Used in most **Quantum Field Theory** calculations.
  - Leads to predictions for **collider experiments**.
  - Standard Model observables computed to high precision.
  - Calculated using **Feynman diagrams**.



Problem with S-matrix: Divergences

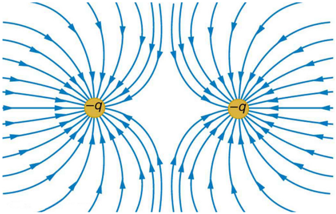
- Probability of e^-e^- scattering is naively $\propto \frac{1}{\epsilon} \to \infty$.
- **UV divergences** at high energies.
- **IR divergences** at low energies.



Problems provide an opportunity: Explore and gain new insight

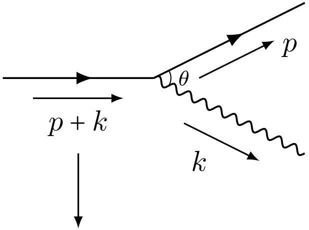
Problem with S-matrix: Divergences

Physical Reason: We are not including the electromagnetic field correctly in scattering calculations.



Lumen Learning

IR Divergences in QFT



Propagator: $\frac{1}{(p+k)^2} \sim \frac{1}{|k|(1-\cos\theta)}$

Singularities: $|k| \to 0$ *soft*
$\theta \to 0$ *collinear*
} IR divergences

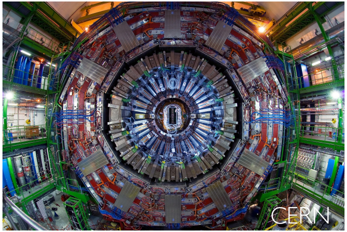
Ideas for IR Finiteness

1. **Finite cross sections** $\sigma \propto \int |\langle f| S |i\rangle|^2 d\Pi_f$
    - Bloch-Nordsieck theorem
    - KLN theorem
2. **Finite *S*-matrix**
3. **Finite scattering amplitudes** $S_{fi} = \langle f| S |i\rangle$

1. Finite Cross Sections

Cross Section Method - Introduction

Idea: Cross section is **measurable** and hence should be finite.



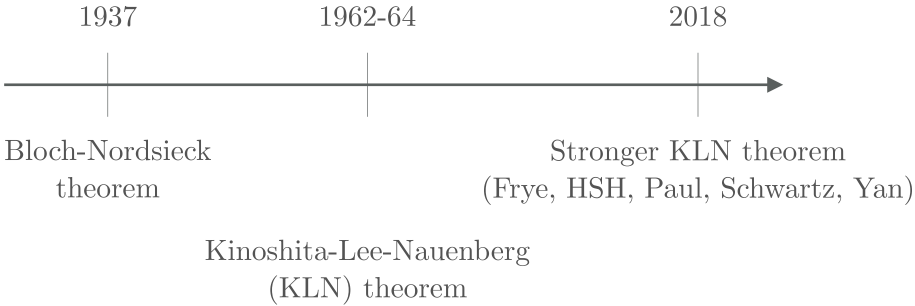
Need to calculate the same quantity as we measure.

Cross Section Method - Introduction

Physical Motivation: All physical observables are finite.

Theoretical Goal: Find the *minimal set* of Feynman diagrams needed for finiteness.

Previous Theorems on IR Divergences



Previous Theorems on IR Divergences

Bloch-Nordsieck (1937): Soft IR divergences cancel in QED when summing over final state photons with finite energy resolution.

Doria, Frenkel, Taylor (1980): Counterexample in QCD: $qq \to \mu\mu qq$ + final state gluons is soft IR divergent at 2-loops.

KLN Theorem (1962-64): S-matrix elements squared are IR finite when summing over **final states and initial states** within some energy window:

$$\sum_{f,i \in [E-E_0, E+E_0]} |\langle f | S | i \rangle|^2 < \infty$$

Stronger KLN Theorem

KLN Theorem (1962-64): S-matrix elements squared are IR finite when summing over **final states and initial states** within some energy window:

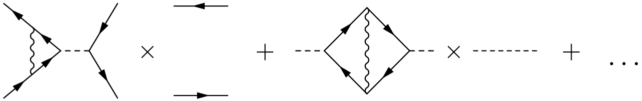
$$\sum_{f,i\in[E-E_0,E+E_0]} |\langle f|\,S\,|i\rangle|^2 < \infty$$

Stronger KLN Theorem (2018): S-matrix elements squared are IR finite when summing over **final states or initial states:**

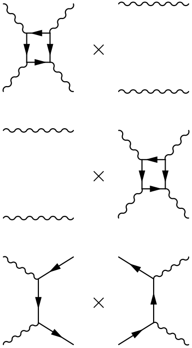
$$\sum_f |\langle f|\,S\,|i\rangle|^2 < \infty, \qquad \sum_i |\langle f|\,S\,|i\rangle|^2 < \infty$$

Forward Scattering

- KLN is a trivial consequence of **unitarity**:
  - Probability of $i \to$ anything is $1 < \infty$
  - Probability of anything $\to f$ is $1 < \infty$
- **KLN requires a term where $f = i$ → forward scattering**



$\gamma\gamma \to \gamma\gamma$ SCATTERING



$$\sigma \propto \frac{1}{2\,\epsilon}$$

$$\sigma \propto \frac{1}{2\,\epsilon}$$

$$\sigma \propto -\frac{1}{\epsilon}$$

Rate to produce no charged particles in photon collisions is not IR safe

$m_e = 0$, Dim reg, CM frame

Conclusion of Cross Section Method

$$\sum_f |\langle f| S |i\rangle|^2 \propto 1 < \infty$$

Conclusion: KLN theorem = unitarity.

If we sum over **all possible diagrams** we get 1 by unitarity, and 1 is IR finite.

Not closer to finding the **minimal set of diagrams** needed for IR finiteness.

Conclusion of Cross Section Method

$$\sum_f |\langle f|\, S\, |i\rangle|^2 \propto 1 < \infty$$

Conclusion: KLN theorem = unitarity.

If we sum over **all possible diagrams** we get 1 by unitarity, and 1 is IR finite.

Not closer to finding the **minimal set of diagrams** needed for IR finiteness.

Need new ideas beyond the cross section method.

2. A Finite S-matrix

The Scattering Matrix (S-matrix)

- Properties extensively studied.
  - How to **encode its content**? *Spinors, twistors, amplituhedron?*
  - What are its **symmetries**? *Lorentz invariance, Dual conformal invariance?*
  - What **constraints** can we impose? *Steinmann relations, limits?*
- Still, **the S-matrix does not exist** in theories with massless particles.
  - Divergent in perturbation theory.
  - Zero non-perturbatively.

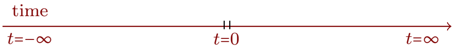
The Scattering Matrix (S-matrix)

Why are our previous calculations valuable?

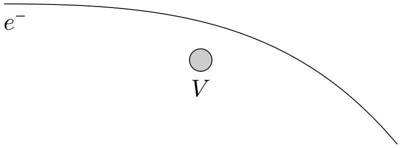
What is the **fundamental object** we should calculate?

What do we gain from a **firmer mathematical ground?**

What is Scattering?



$e^{-iH_0 t} \|i\rangle$	e^{-iHt}	$e^{-iH_0 t} \|f\rangle$
$\downarrow$	$\downarrow$	$\downarrow$
freely evolving state	scattering,	freely evolving state
in far past	evolution with H	in far future



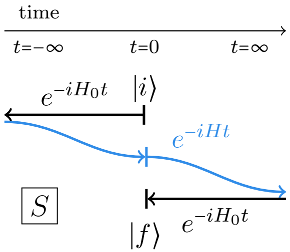
$\langle f | S | i \rangle$: **Probability amplitude**

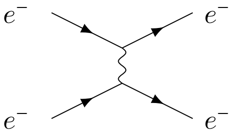
for measuring $|f\rangle$ given $|i\rangle$

What is Scattering?

S-matrix: **Probability amplitude** for measuring $|f\rangle$ given $|i\rangle$

$$S_{fi} = \lim_{t_\pm \to \pm\infty} \langle f| \, e^{iH_0 t_+} e^{-iHt_+} e^{iHt_-} e^{-iH_0 t_-} \, |i\rangle$$





Traditional Definition of S-matrix

$$S_{fi} = \lim_{t_\pm \to \pm\infty} \langle f|\, e^{iH_0 t_+} e^{-iHt_+} e^{iHt_-} e^{-iH_0 t_-} \,|i\rangle$$

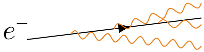
Free Theory: $S = \mathbb{1}$ $\quad S_{fi} = \langle f|i\rangle$ ✓

QM, short range potential: ✓

Const. potential $\quad H = H_0 + V_0$: $\quad S_{fi} = \langle f|i\rangle \lim_{T\to\infty} e^{-2iV_0 T}$?

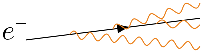
QED: $\quad S = \mathbb{1} - \frac{\alpha}{\epsilon^2} + \cdots = -\infty$?

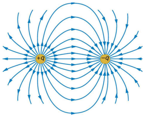
$$S = \exp\left\{-\frac{\alpha}{\epsilon^2}\right\} = 0?$$

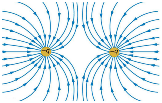


Modify S-matrix to S_H

Recall: Interactions do not vanish as $t \to \pm\infty$ in QED.





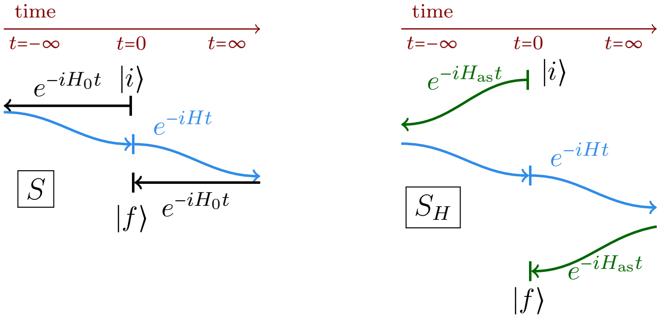


Redefine S-matrix in theories with long range interactions:

$$S_{fi} = \lim_{t_\pm \to \pm\infty} \langle f| \, e^{iH_0 t_+} e^{-iHt_+} e^{iHt_-} e^{-iH_0 t_-} \, |i\rangle$$

$$\to S^H_{fi} = \lim_{t_\pm \to \pm\infty} \langle f| \, e^{iH_{\rm as} t_+} e^{-iHt_+} e^{iHt_-} e^{-iH_{\rm as} t_-} \, |i\rangle$$

MODIFY S-MATRIX TO S_H



Questions

$$S^H_{fi} = \lim_{t_\pm \to \pm\infty} \langle f | e^{iH_{\mathrm{as}} t_+} e^{-iHt_+} e^{iHt_-} e^{-iH_{\mathrm{as}} t_-} | i \rangle$$

(i) How to pick H_{as}?

- Criteria: IR finite, easy to calculate, useful in practice, consistent with every measurement to date.

(ii) How to calculate matrix elements of S_H?

(iii) How to interpret S_H?

Choice of H_{AS}

(i) *How to pick H_{as}?*

- Use **factorization**, and techniques from Soft-Collinear Effective Theory (SCET):

$$H_{\text{as}} = H_{SCET}$$

- IR finite by construction due to **universality of IR divergences**.
- States **evolve independently of how they scatter**.
- New UV divergences dealt with using renormalization.
- No scales, most integrals are zero in dim reg.

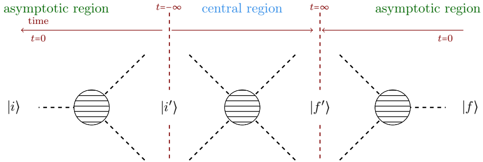
Three Part Calculation

(ii) *How to calculate matrix elements of S_H?*

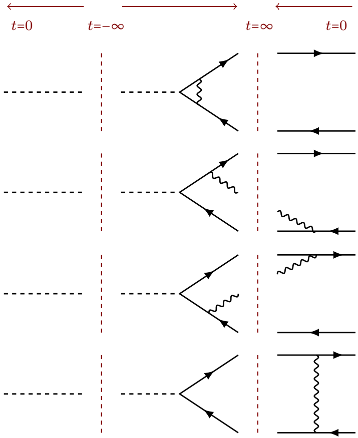
- Calculation trick in perturbation theory:

$$S^H_{fi} = \int d\Pi'_f \int d\Pi'_i \underbrace{\langle f|\,\Omega^{\rm as}_+\,|f'\rangle}_{\text{TOPT rules}} \underbrace{\langle f'|\,S\,|i'\rangle}_{\text{usual Feynman rules}} \underbrace{\langle i'|\,\Omega^{\rm as}_+\,|i\rangle}_{\text{TOPT rules}}$$

- Calculations split into three parts:



Example: $Z \to e^+ \, e^-$ for $H_{\text{AS}} = H_{\text{SCET}}$



$$= \mathcal{M}_0 \frac{\alpha}{4\pi}\left[\frac{1}{\epsilon_{\text{UV}}} - \frac{2}{\epsilon_{\text{IR}}^2} - \frac{4+2L}{\epsilon_{\text{IR}}} - 8 + \frac{\pi^2}{6} - L^2 + 3L\right]$$

$$= \mathcal{M}_0 \frac{\alpha}{4\pi}\left[\frac{2}{\epsilon_{\text{IR}}^2} + \frac{4+2L}{\epsilon_{\text{IR}}} - \frac{2}{\epsilon_{\text{UV}}^2} - \frac{4+2L}{\epsilon_{\text{UV}}}\right]$$

$$\langle e^+ e^- | \, S_H \, | Z \rangle^{\overline{\text{MS}}} = \mathcal{M}_0 + \mathcal{M}_0 \frac{\alpha}{4\pi}\left[-8 + \frac{\pi^2}{6} - L^2 + 3L\right]$$

$m_e{=}0$, $L{=}\ln \frac{-E_{CM}^2}{\mu^2}$, $\mathcal{M}_0$: LO matrix element, Dim reg, CM frame

Interpretation of S_H

(iii) *How to interpret S_H?*

a. Wilson coefficients in Soft-Collinear Effective Theory (SCET)

b. Remainder functions in $\mathcal{N} = 4$ Supersymmetric Yang-Mills theory (SYM)

c. Dressed states / Coherent states

a. Wilson Coefficients

$$\langle e^+e^- |\, S_H \,| Z\rangle^{\overline{\text{MS}}} = \mathcal{M}_0 + \mathcal{M}_0 \frac{\alpha}{4\pi}\left[-8 + \frac{\pi^2}{6} - L^2 + 3L\right]$$

Familiar expression:

S_H amplitudes = Wilson coefficients in SCET

- **Wilson coefficients:** Coupling constants in the effective field theory.
- Encode **hard dynamics** of a scattering process.

a. Wilson Coefficients

Advantages to alternative definition:

Properties of Wilson coefficients identical to those of $\boldsymbol{S_H}$.

- Usually: difference of matrix elements in different theories.
- Here: matrix elements of a **single operator**.
- *What are the **analytic** and **symmetry** properties of S_H?*

b. $\mathcal{N} = 4$ Remainder Functions

- $\mathcal{N} = 4$ supersymmetric Yang-Mills theory obeys dual conformal invariance (DCI).
  - Amplitudes **bootstrapped** to 6 and 7 loops (Caron-Huot et al. 2019)
  - DCI violated at 1-loop.
- Compute **remainder functions R** instead of amplitudes
  - **Ratio** of full amplitude and an exponentiation of 1-loop divergences.
  - Sometimes obey **Steinmann relations** and **DCI**.

IR divergences obscure simplicity of $\mathcal{N} = 4$ amplitudes.

4-Point Amplitude in $\mathcal{N} = 4$ is Complicated

$$M_4^{(1)}(\epsilon) = -\frac{2}{\epsilon^2} + \frac{1}{\epsilon} M_4^{(1)}(\epsilon^{-1}) + M_4^{(1)}(\epsilon^0) + \cdots$$

$$M_4^{(1)}(\epsilon^{-1}) = -\ln\frac{\mu^2}{-s} - \ln\frac{\mu^2}{-t}$$

$$M_4^{(1)}(\epsilon^{0}) = -\ln\frac{\mu^2}{-t}\ln\frac{\mu^2}{-s} + \frac{2\pi^2}{3}$$

$$\begin{aligned}M_4^{(1)}(\epsilon^{1}) = &-\frac{\pi^2}{2}\ln\frac{-s}{u} - \frac{1}{3}\ln^3\frac{-s}{u} + \frac{\pi^2}{12}\ln\frac{\mu^2}{-s} - \frac{1}{6}\ln^3\frac{\mu^2}{-s} + \frac{\pi^2}{4}\ln\frac{\mu^2}{u} + \frac{1}{2}\ln^2\frac{-s}{u}\ln\frac{\mu^2}{u} - \frac{1}{2}\ln\frac{-s}{u}\ln\frac{-t}{u}\ln\frac{\mu^2}{u}\\ &- \ln\frac{-s}{u}\mathrm{Li}_2\frac{-s}{u} + \mathrm{Li}_3\frac{-s}{u} + \frac{7}{3}\zeta_3 + (s \leftrightarrow t)\end{aligned}$$

$$\begin{aligned}M_4^{(1)}(\epsilon^{2}) = &\frac{5\pi^2}{24}\ln^2\frac{-s}{u} + \frac{1}{8}\ln^4\frac{-s}{u} + \frac{3}{8}\ln\frac{-s}{u}\ln\frac{-t}{u} + \frac{1}{6}\ln^3\frac{-s}{u}\ln\frac{-t}{u} - \frac{1}{4}\ln^2\frac{-s}{u}\ln^2\frac{-t}{u} + \frac{\pi^2}{24}\ln^2\frac{\mu^2}{-s} - \frac{1}{24}\ln^4\frac{\mu^2}{s} - \frac{\pi^2}{2}\ln\frac{-s}{u}\ln\frac{\mu^2}{u}\\ &- \frac{1}{3}\ln^3\frac{-s}{u}\ln\frac{\mu^2}{u} + \frac{\pi^2}{8}\ln^2\frac{\mu^2}{u} + \frac{1}{4}\ln^2\frac{-s}{u}\ln^2\frac{\mu^2}{u} - \frac{1}{4}\ln\frac{-s}{u}\ln\frac{-t}{u}\ln^2\frac{\mu^2}{u} + \frac{7}{3}\zeta_3\ln^2\frac{\mu^2}{-s} + \frac{1}{2}\ln^2\frac{-s}{u}\mathrm{Li}_2\frac{-s}{u} - \ln\frac{-s}{u}\ln\frac{\mu^2}{u}\mathrm{Li}_2\frac{-s}{u}\\ &+ \ln\frac{\mu^2}{u}\mathrm{Li}_3\frac{-s}{u} - \ln\frac{-s}{u}\mathrm{Li}_3\frac{-t}{u} - \mathrm{Li}_4\frac{-s}{u} + \frac{49\pi^4}{720} + (s \leftrightarrow t)\end{aligned}$$

Use universality of IR divergences to simplify.

b. $\mathcal{N} = 4$ Remainder Functions

S_H approach:

- **Subtract** divergent amplitudes instead of taking ratios.
- S_H amplitude for 4-points, 1-loop after renormalization:

$$\widehat{M}_4^{\mathrm{BDS},(1)} = -\ln\frac{\mu^2}{-t}\ln\frac{\mu^2}{-s} + \frac{5\pi^2}{6}$$

- S_H amplitude for 4-points, 2-loop after renormalization:

$$\widehat{M}_4^{\mathrm{BDS},(2)} = \frac{1}{2}\left[\widehat{M}_4^{\mathrm{BDS},(1)} - \frac{\pi^2}{6}\right]^2$$

b. $\mathcal{N}=4$ Remainder Functions

$$M_4^{(1)}(\epsilon) = -\frac{2}{\epsilon^2} + \frac{1}{\epsilon} M_4^{(1)}(\epsilon^{-1}) + M_4^{(1)}(\epsilon^0) + \cdots$$

$$\begin{aligned}
M_4^{(1)}(\epsilon^{-1}) &= -\ln\frac{\mu^2}{-s} - \ln\frac{\mu^2}{-t} \\
M_4^{(1)}(\epsilon^{0}) &= -\ln\frac{\mu^2}{-t}\ln\frac{\mu^2}{-s} + \frac{2\pi^2}{3} \\
M_4^{(1)}(\epsilon^{1}) &= -\frac{\pi^2}{2}\ln\frac{-s}{u} - \frac{1}{3}\ln^3\frac{-s}{u} + \frac{\pi^2}{12}\ln\frac{\mu^2}{-s} - \frac{1}{6}\ln^3\frac{\mu^2}{-s} + \frac{\pi^2}{4}\ln\frac{\mu^2}{u} + \frac{1}{2}\ln^2\frac{-s}{u}\ln\frac{\mu^2}{u} - \frac{1}{2}\ln\frac{-s}{u}\ln\frac{-t}{u}\ln\frac{\mu^2}{u} \\
&\quad - \ln\frac{-s}{u}\mathrm{Li}_2\frac{-s}{u} + \mathrm{Li}_3\frac{-s}{u} + \frac{7}{3}\zeta_3 + (s \leftrightarrow t) \\
M_4^{(1)}(\epsilon^{2}) &= \frac{5\pi^2}{24}\ln^2\frac{-s}{u} + \frac{1}{8}\ln^4\frac{-s}{u} + \frac{3}{8}\ln\frac{-s}{u}\ln\frac{-t}{u} + \frac{1}{6}\ln^3\frac{-s}{u}\ln\frac{-t}{u} - \frac{1}{4}\ln^2\frac{-s}{u}\ln^2\frac{-t}{u} + \frac{\pi^2}{24}\ln^2\frac{\mu^2}{-s} - \frac{1}{24}\ln^4\frac{\mu^2}{s} - \frac{\pi^2}{2}\ln\frac{-s}{u}\ln\frac{\mu^2}{u} \\
&\quad - \frac{1}{3}\ln^3\frac{-s}{u}\ln\frac{\mu^2}{u} + \frac{\pi^2}{8}\ln^2\frac{\mu^2}{u} + \frac{1}{4}\ln^2\frac{-s}{u}\ln^2\frac{\mu^2}{u} - \frac{1}{4}\ln\frac{-s}{u}\ln\frac{-t}{u}\ln^2\frac{\mu^2}{u} + \frac{7}{3}\zeta_3\ln^2\frac{\mu^2}{-s} + \frac{1}{2}\ln^2\frac{-s}{u}\mathrm{Li}_2\frac{-s}{u} - \ln\frac{-s}{u}\ln\frac{\mu^2}{u}\mathrm{Li}_2\frac{-s}{u} \\
&\quad + \ln\frac{\mu^2}{u}\mathrm{Li}_3\frac{-s}{u} - \ln\frac{-s}{u}\mathrm{Li}_3\frac{-t}{u} - \mathrm{Li}_4\frac{-s}{u} + \frac{49\pi^4}{720} + (s \leftrightarrow t)
\end{aligned}$$

b. $\mathcal{N} = 4$ Remainder Functions

$$M_4^{(1)}(\epsilon) = -\frac{2}{\epsilon^2} + \frac{1}{\epsilon} M_4^{(1)}(\epsilon^{-1}) + M_4^{(1)}(\epsilon^0) + \cdots$$

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$$M_4^{(1)}(\epsilon^0) = -\ln\frac{\mu^2}{-t}\ln\frac{\mu^2}{-s} + \frac{2\pi^2}{3}$$

$$\begin{aligned} M_4^{(1)}(\epsilon^1) = &-\frac{\pi^2}{2}\ln\frac{-s}{u} - \frac{1}{3}\ln^3\frac{-s}{u} + \frac{\pi^2}{12}\ln\frac{\mu^2}{-s} - \frac{1}{6}\ln^3\frac{\mu^2}{-s} + \frac{\pi^2}{4}\ln\frac{\mu^2}{u} + \frac{1}{2}\ln^2\frac{-s}{u}\ln\frac{\mu^2}{u} - \frac{1}{2}\ln\frac{-s}{u}\ln\frac{-t}{u}\ln\frac{\mu^2}{u} \\ &- \ln\frac{-s}{u}\mathrm{Li}_2\frac{-s}{u} + \mathrm{Li}_3\frac{-s}{u} + \frac{7}{3}\zeta_3 + (s \leftrightarrow t) \end{aligned}$$

$$\begin{aligned} M_4^{(1)}(\epsilon^2) = &\frac{5\pi^2}{24}\ln^2\frac{-s}{u} + \frac{1}{8}\ln^4\frac{-s}{u} + \frac{3}{8}\ln\frac{-s}{u}\ln\frac{-t}{u} + \frac{1}{6}\ln^3\frac{-s}{u}\ln\frac{-t}{u} - \frac{1}{4}\ln^2\frac{-s}{u}\ln^2\frac{-t}{u} + \frac{\pi^2}{24}\ln^2\frac{\mu^2}{-s} - \frac{1}{24}\ln^4\frac{\mu^2}{s} - \frac{\pi^2}{2}\ln\frac{-s}{u}\ln\frac{\mu^2}{u} \\ &- \frac{1}{3}\ln^3\frac{-s}{u}\ln\frac{\mu^2}{u} + \frac{\pi^2}{8}\ln^2\frac{\mu^2}{u} + \frac{1}{4}\ln^2\frac{-s}{u}\ln^2\frac{\mu^2}{u} - \frac{1}{4}\ln\frac{-s}{u}\ln\frac{-t}{u}\ln^2\frac{\mu^2}{u} + \frac{7}{3}\zeta_3\ln^2\frac{\mu^2}{-s} + \frac{1}{2}\ln^2\frac{-s}{u}\mathrm{Li}_2\frac{-s}{u} - \ln\frac{-s}{u}\ln\frac{\mu^2}{u}\mathrm{Li}_2\frac{-s}{u} \\ &+ \ln\frac{\mu^2}{u}\mathrm{Li}_3\frac{-s}{u} - \ln\frac{-s}{u}\mathrm{Li}_3\frac{-t}{u} - \mathrm{Li}_4\frac{-s}{u} + \frac{49\pi^4}{720} + (s \leftrightarrow t) \end{aligned}$$

$$\widehat{M}_4^{\mathrm{BDS},(1)} = -\ln\frac{\mu^2}{-t}\ln\frac{\mu^2}{-s} + \frac{5\pi^2}{6}$$

b. $\mathcal{N} = 4$ Remainder Functions

Factorization, and techniques from SCET,

explain why remainder functions have simple forms.

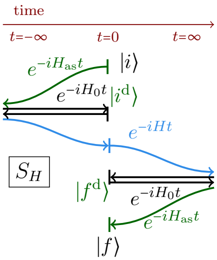
$$\widehat{M}_4^{\mathrm{BDS},(1)} = -\ln\frac{\mu^2}{-t}\ln\frac{\mu^2}{-s} + \frac{5\pi^2}{6}$$

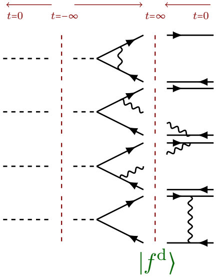
3. Finite Scattering Amplitudes

C. Coherent States

- Arise as intermediate steps in S_H calculations:

$$S^H_{fi} = \sum_{f'} \sum_{i'} \underbrace{\langle f|\,\Omega^{\mathrm{as}}_{+}\,|f'\rangle\,\langle f'|}_{\langle f^{\mathrm{d}}|}\,S\,\underbrace{|i'\rangle\,\langle i'|\,\Omega^{\mathrm{as}}_{+}\,|i\rangle}_{|i^{\mathrm{d}}\rangle}$$





c. Coherent States

- Arise as intermediate steps in S_H calculations:

$$S^H_{fi} = \sum_{f'} \sum_{i'} \underbrace{\langle f | \Omega^{\text{as}}_+ | f' \rangle \langle f' |}_{\langle f^{\text{d}} |} S \underbrace{| i' \rangle \langle i' | \Omega^{\text{as}}_+ | i \rangle}_{| i^{\text{d}} \rangle}$$

Mathematically the same as the finite *S*-matrix

Future Directions: Analytic Structure of S_H

We have explored:

S_H provides an alternative definition of familiar QFT objects.

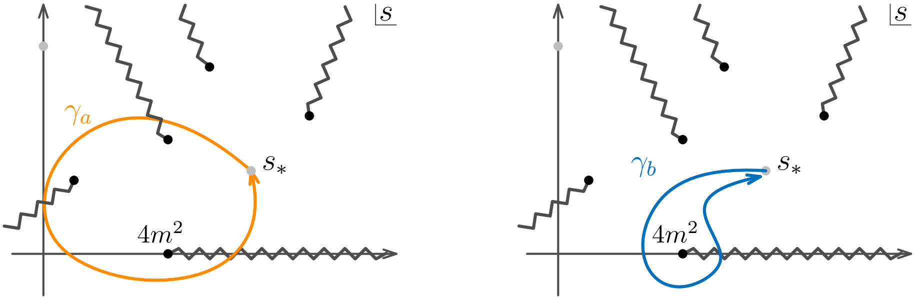
New goal:

Examine properties of S_H, e.g. using **bootstrapping** methods.

Tools needed:

Better handle on **analytic structure** of amplitudes.

Future Directions: Analytic Structure of S_H



Can we deduce branch-point structure of S_H?

Conclusions

- **IR divergences** remain a problem in QFT
- Explored **three solutions**:
  1. Finite cross sections: *Sum over all diagrams for finiteness.*
  2. Finite S-matrix: *Encodes hard dynamics of scattering processes.*
  3. Finite scattering amplitudes (Coherent states): *Same as Finite S-matrix.*
- Future directions: Explore **analytic structure**

Thanks!