

Status of Kalman Filter in Heavy Neutral Lepton study

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BEYOND STANDARD MODEL PHYSICS

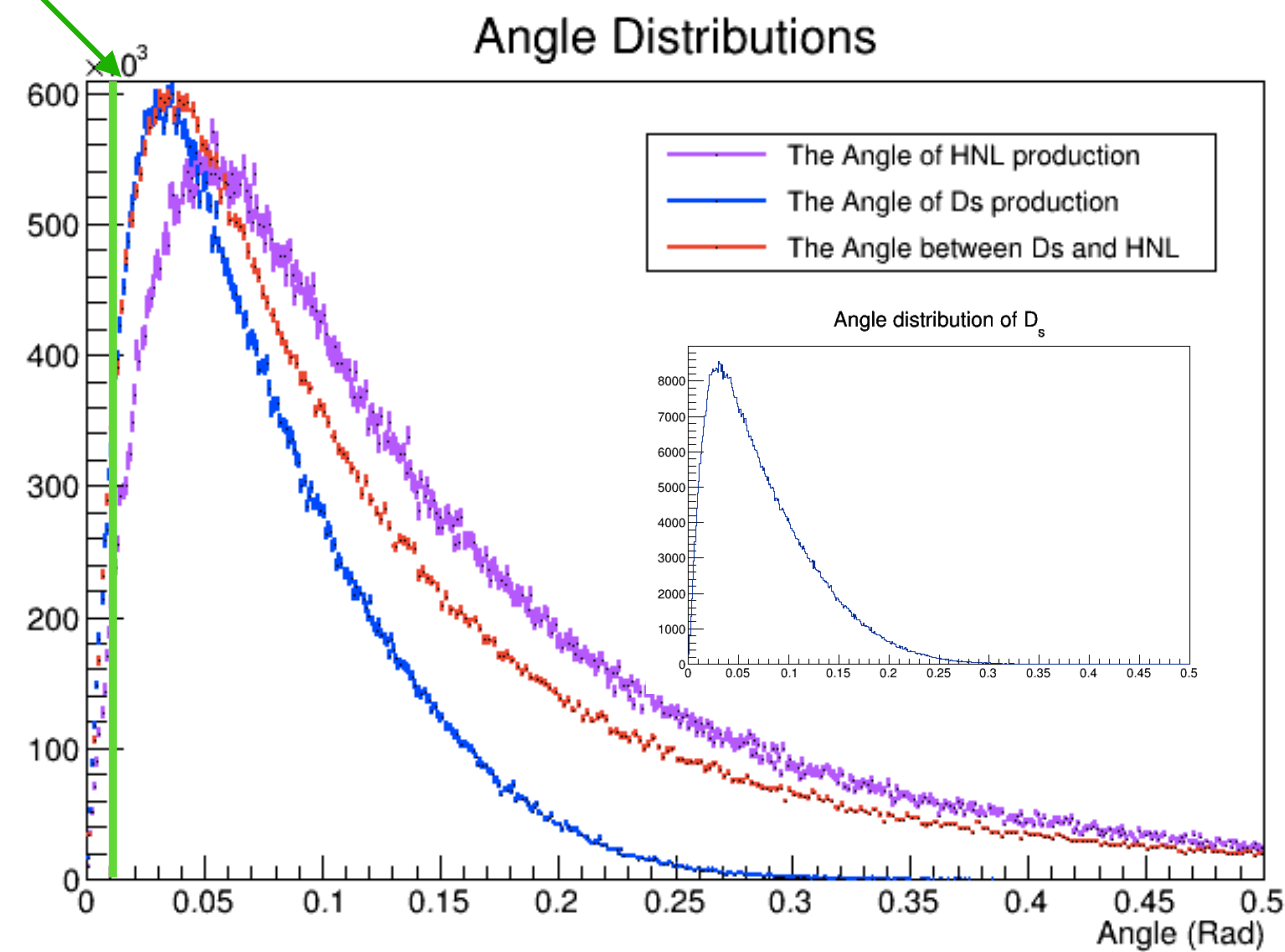
$$\mathcal{L}_{\nu\text{MSM}} = \mathcal{L}_{\text{MSM}} + \bar{\tilde{N}}_I i \partial_\mu \gamma^\mu \tilde{N}_I - F_{\alpha I} \bar{L}_\alpha \tilde{N}_I \tilde{\Phi} - M \tilde{N}_2^c \tilde{N}_3 - \frac{\Delta M_{IJ}}{2} \bar{\tilde{N}}_I^c \tilde{N}_J + \text{h.c.},$$

model I : $f_e^2 : f_\mu^2 : f_\tau^2 \approx 52 : 1 : 1, \quad \kappa = 1,$

model II : $f_e^2 : f_\mu^2 : f_\tau^2 \approx 1 : 16 : 3.8, \quad \kappa = 2,$

model III : $f_e^2 : f_\mu^2 : f_\tau^2 \approx 0.061 : 1 : 4.3, \quad \kappa = 2.$

Acceptance



Mesons:
- Primary
- Secondary

*Steps

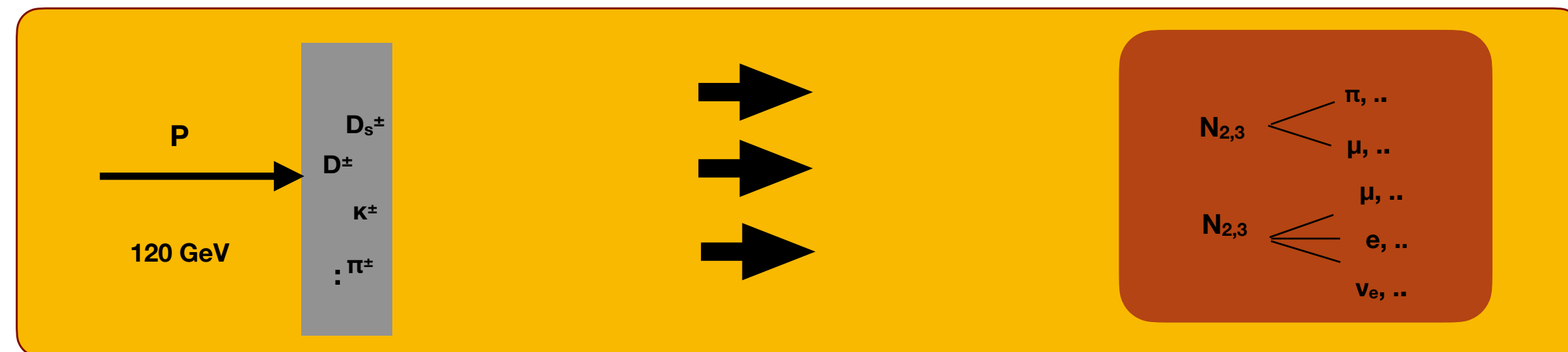
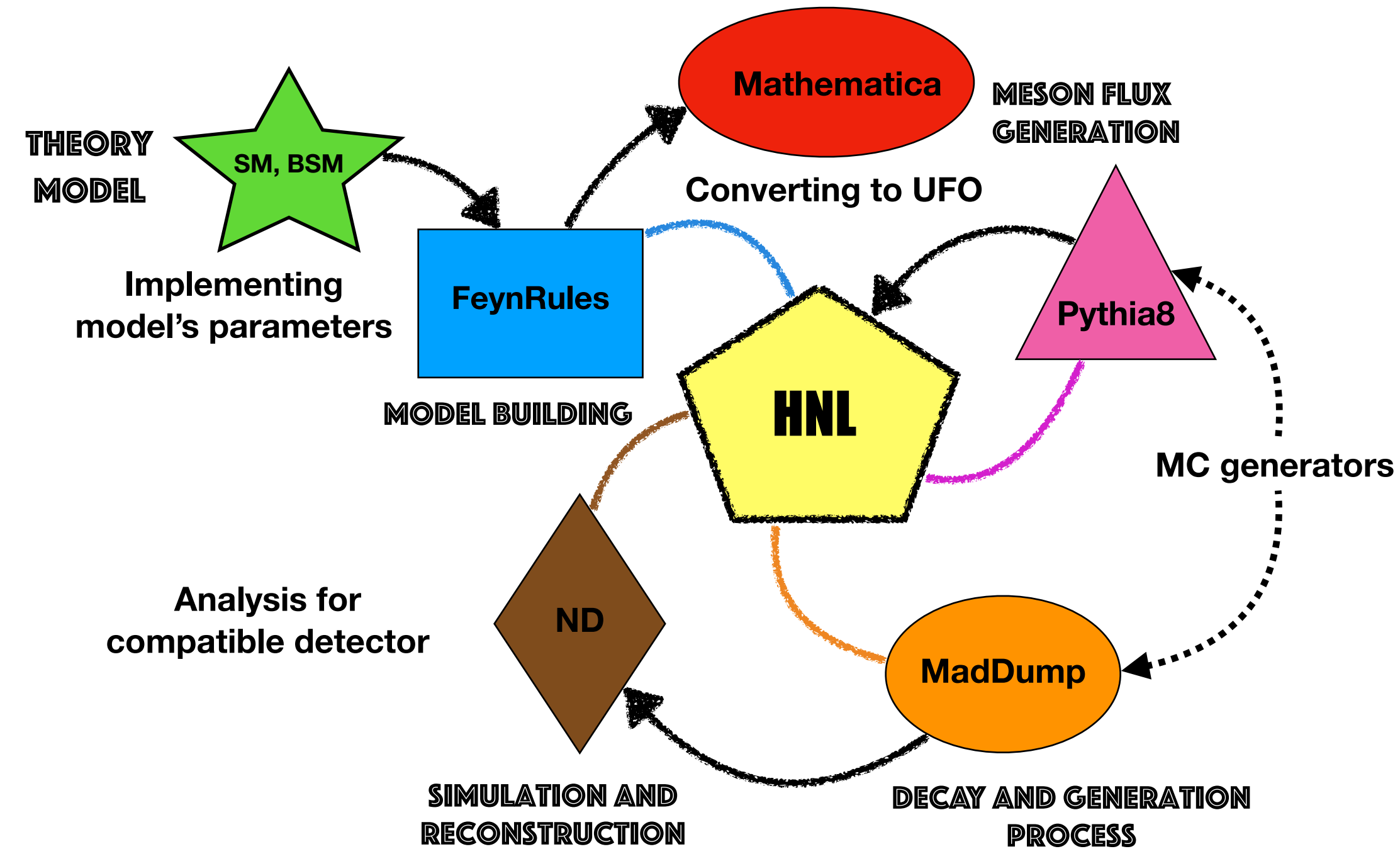
- Building the model, choosing the right parameters ->(Channels, Coupling, Mass, Mixing,...)
- Generating external meson(s) flux -> Heavy Meson (Ds)
- Final check of the parameters
- Launching Mad-dump
- Analyzing the results

Heavy Fermions:
- Long lived.

	Channel	BR (%)	m_N/MeV
$D_s^+ \rightarrow$	$\tau^+ \nu_\tau$	5.48	191.42
	$\mu^+ \nu_\mu$	0.55	1862.63
	$e^+ \nu_e$	0.008	1967.78
$\tau^+ \rightarrow$	$\pi^+ \pi^0 \bar{\nu}_\tau$	25.49	1502.31
	$\bar{\nu}_\tau e^+ \nu_e$	17.82	1776.35
	$\bar{\nu}_\tau \mu^+ \nu_\mu$	17.39	1671.20
	$\pi^+ \bar{\nu}_\tau$	10.82	1637.29

$$N \rightarrow \pi^\mp \mu^\pm$$

MODEL BUILDING



500 m

<https://arxiv.org/abs/0705.1729>

<https://arxiv.org/abs/1812.06771>

BEYOND STANDARD MODEL PHYSICS

$$\mathcal{L}_{\nu\text{MSM}} = \mathcal{L}_{\text{MSM}} + \bar{\tilde{N}}_I i \partial_\mu \gamma^\mu \tilde{N}_I - F_{\alpha I} \bar{L}_\alpha \tilde{N}_I \tilde{\Phi} - M \bar{\tilde{N}}_2^c \tilde{N}_3 - \frac{\Delta M_{IJ}}{2} \bar{\tilde{N}}_I^c \tilde{N}_J + \text{h.c.},$$

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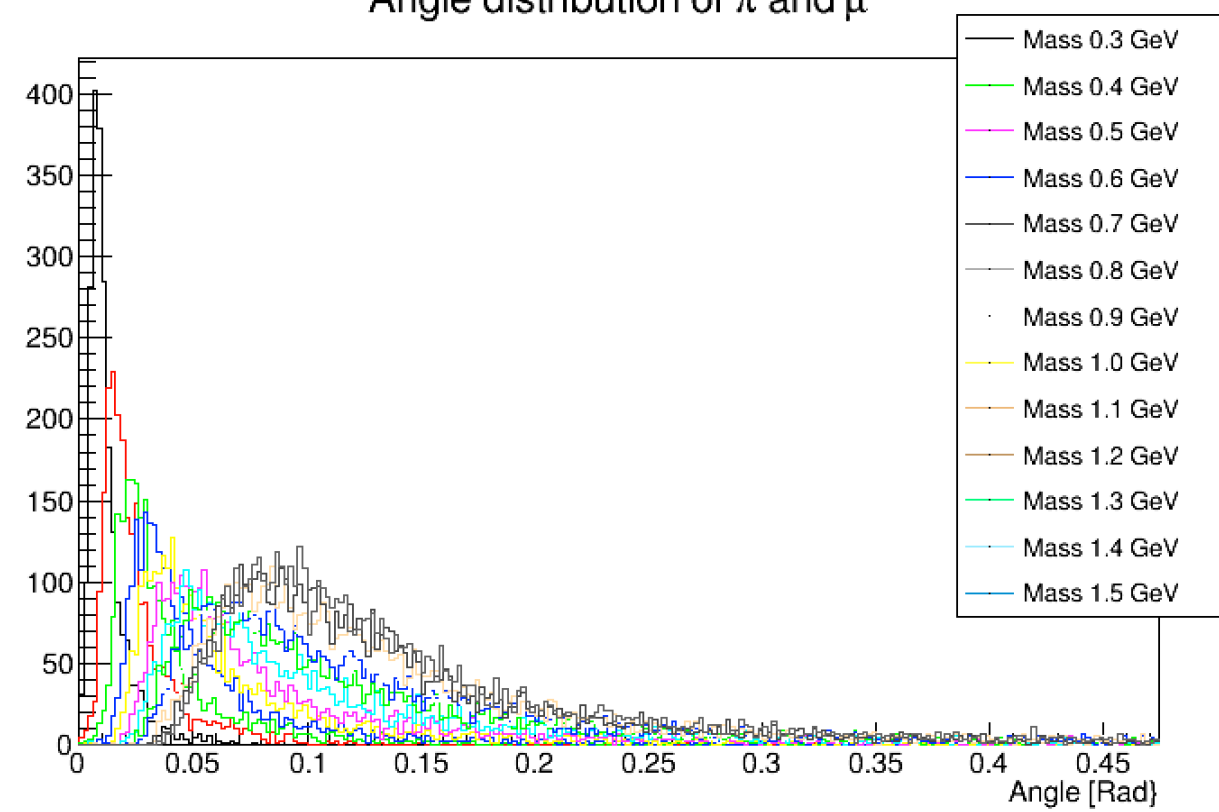
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$$D_s^\pm \rightarrow N \mu^\pm$$

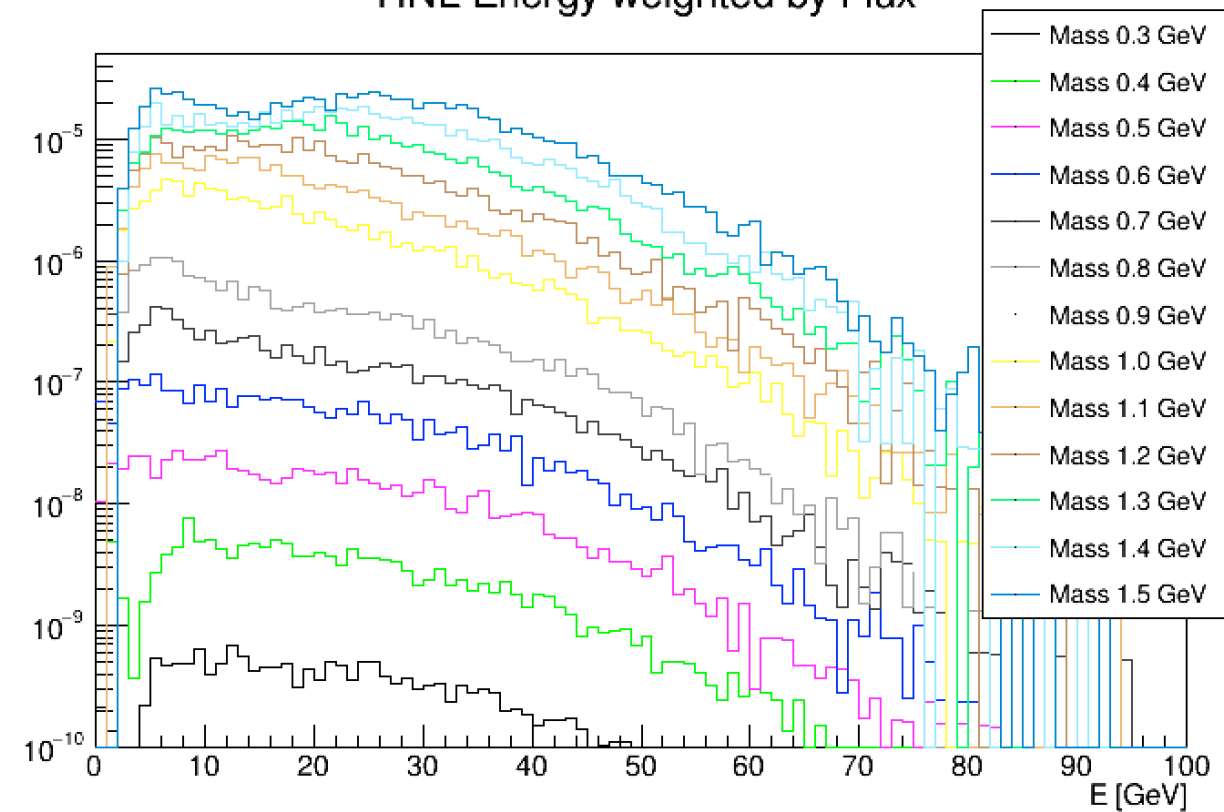
$$N \rightarrow \pi^\mp \mu^\pm$$

Angle distribution of π and μ

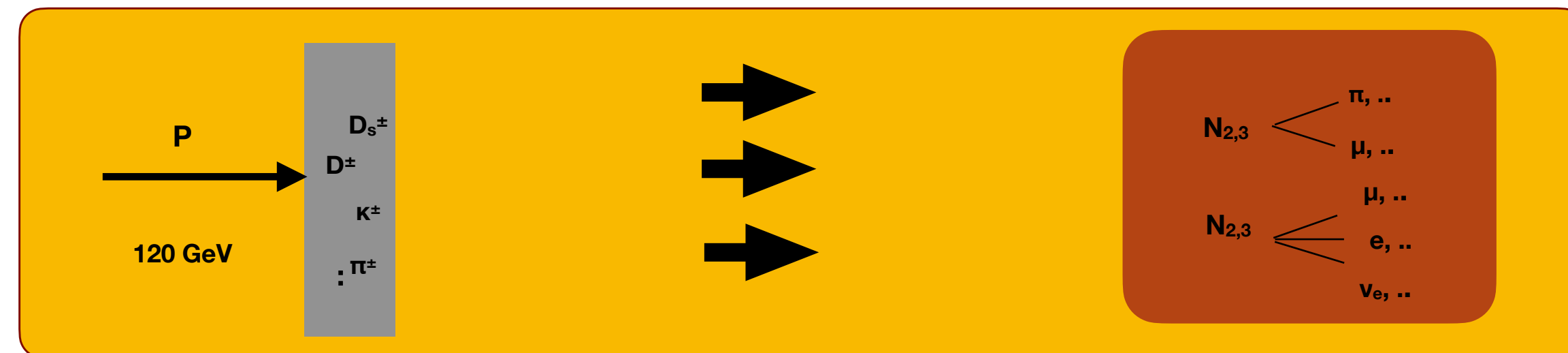


- # Mesons:
 - Primary
 - Secondary

HNL Energy weighted by Flux



- # Heavy Fermions:
 - Long lived.



<https://arxiv.org/abs/0705.1729>

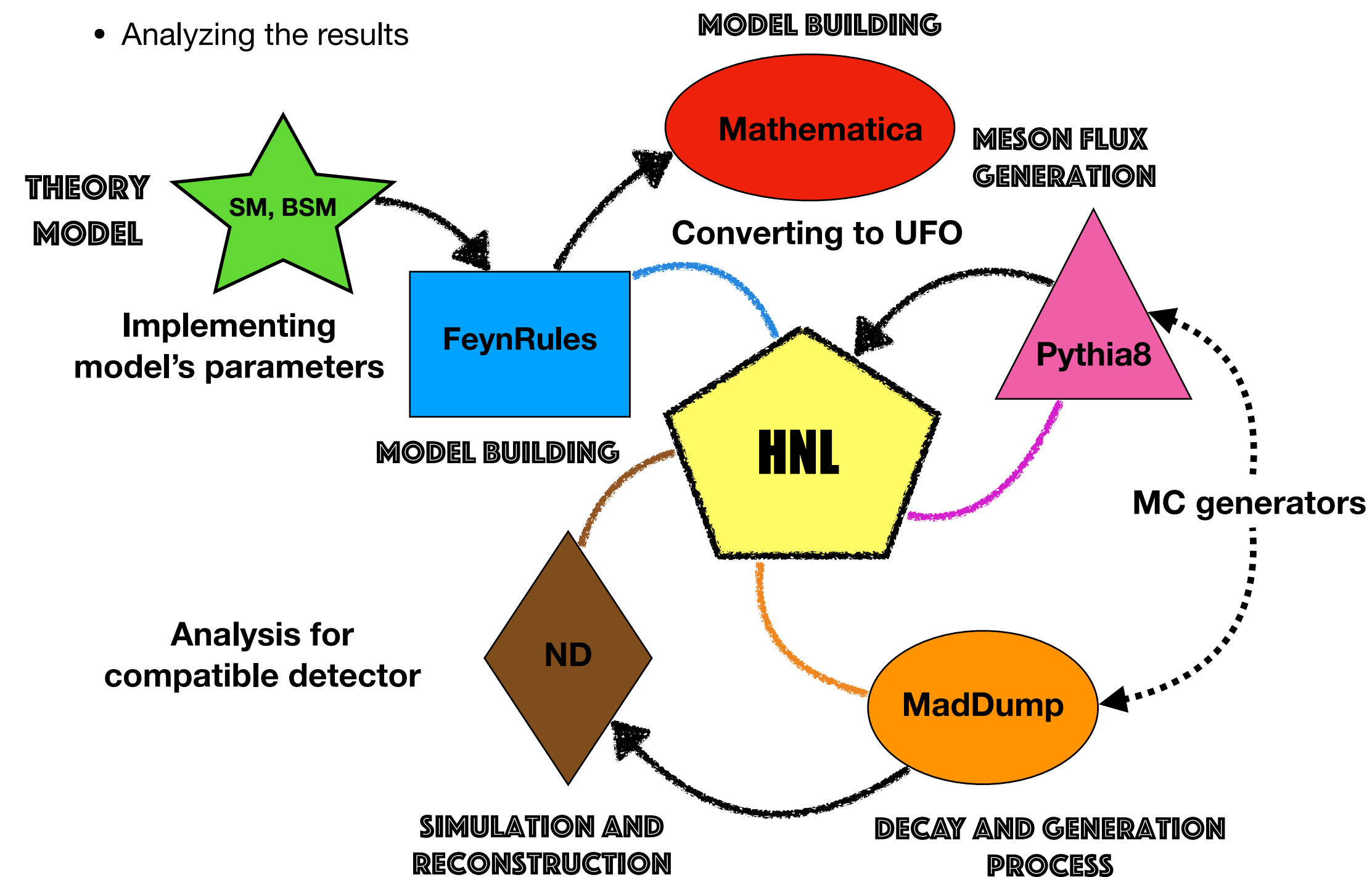
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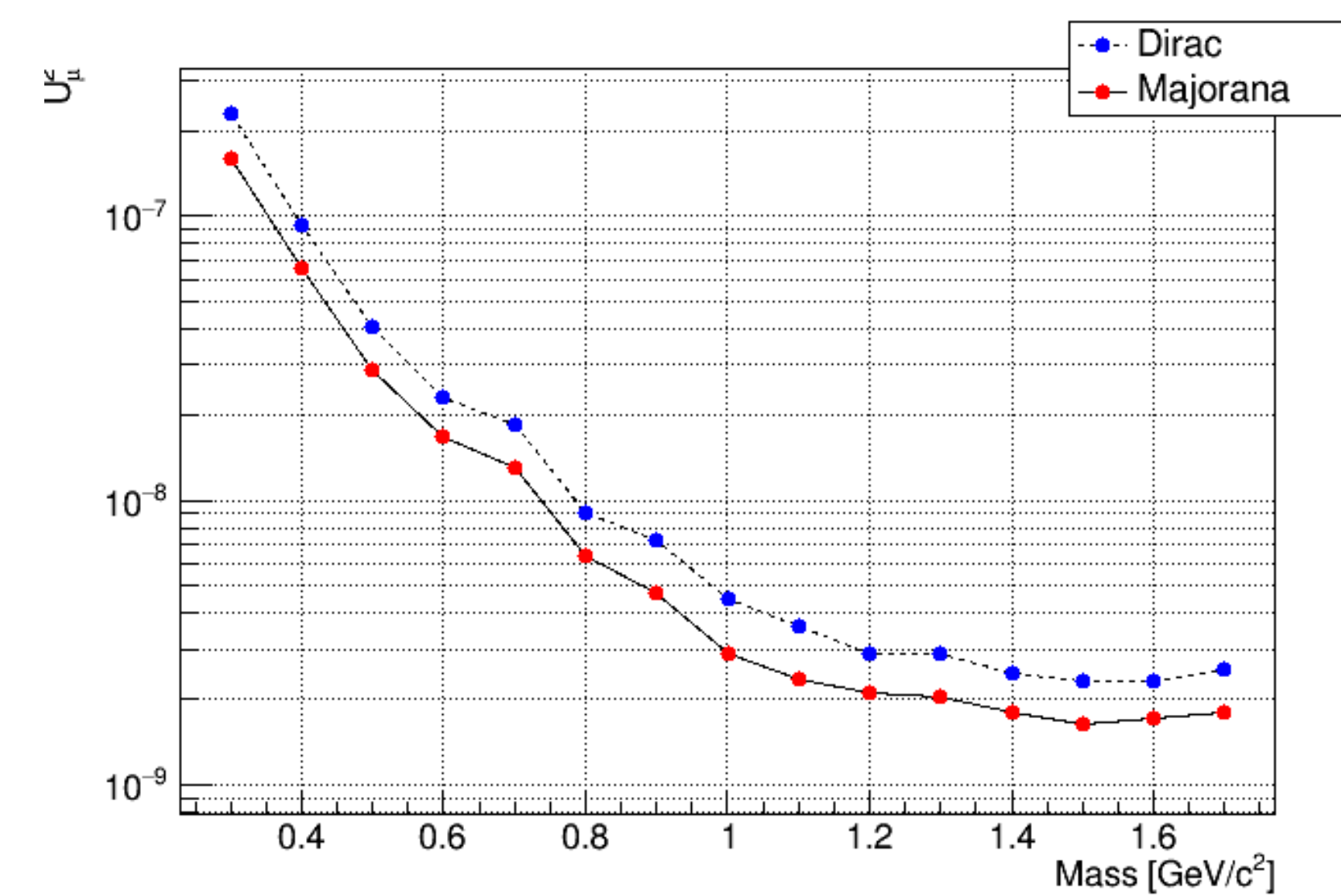
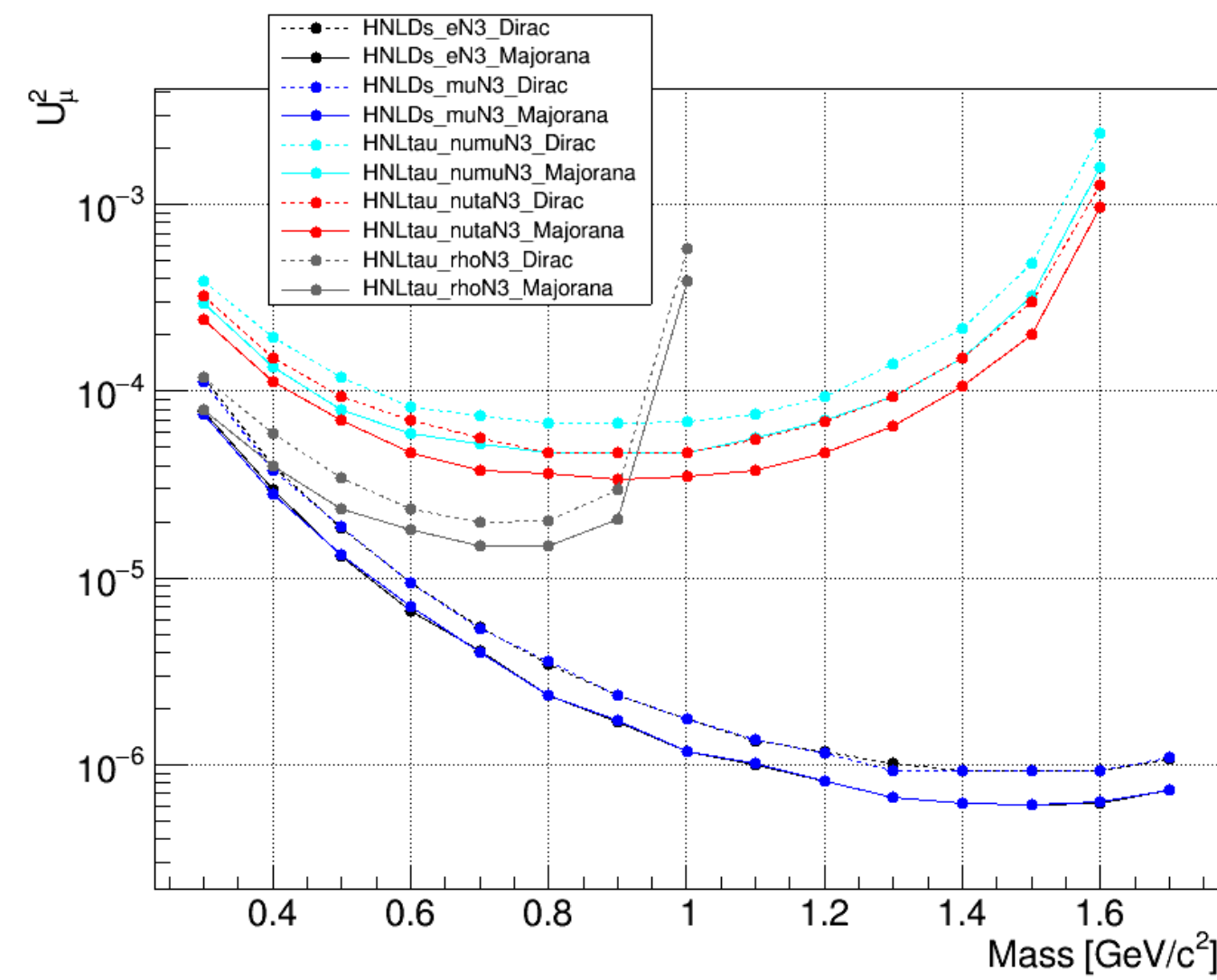
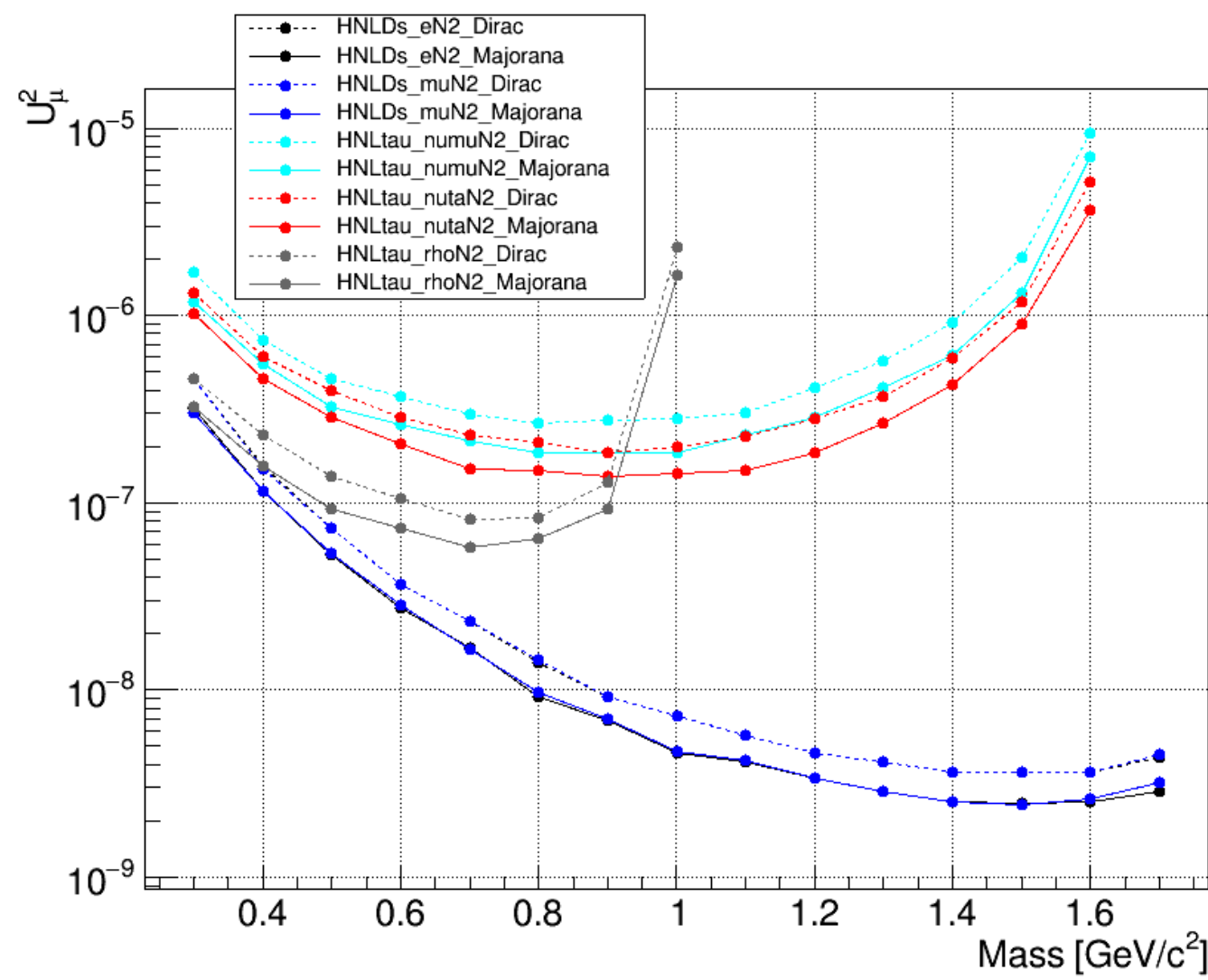
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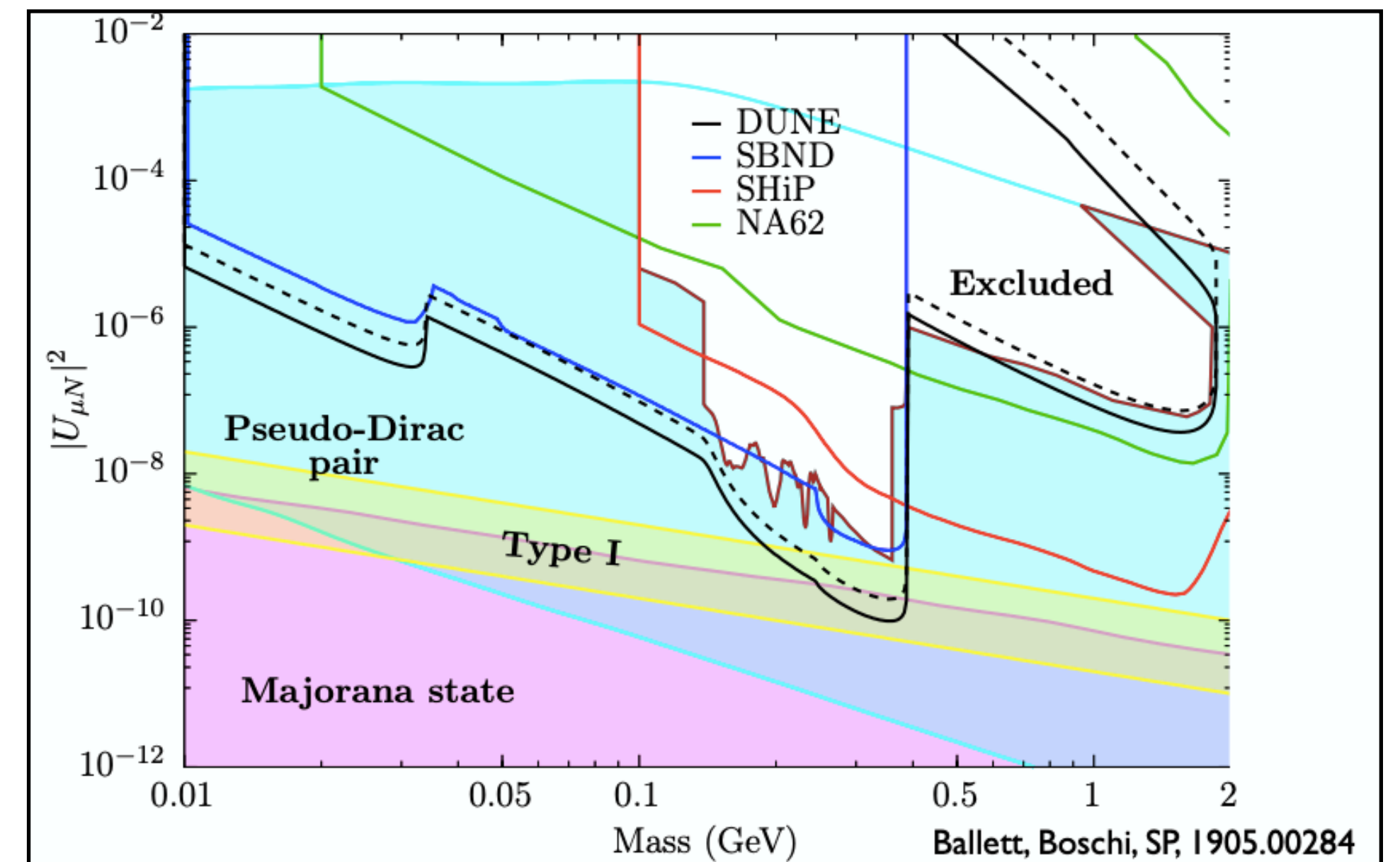




- 6 years exposure, 1.1×10^{21} NPOT/1yr
- Coupling follows Benchmark II, mass range is extended to 1.7 GeV
- Very small mixing between N2 and N3 due BAU lower bound
- Detector's Geometry has been modified to fit with SAND transversally
- All correction has been checked
- All Ds channels has been covered

✱ **Reconstruction**

- Mad-Dump output file gets transformed to Genie-like output
- Genie-like output processed with Edepsim : edepsim output
- Digitization
- Existing Reco didn't work for me -> Motivation for implementing Kalman Filter for my work

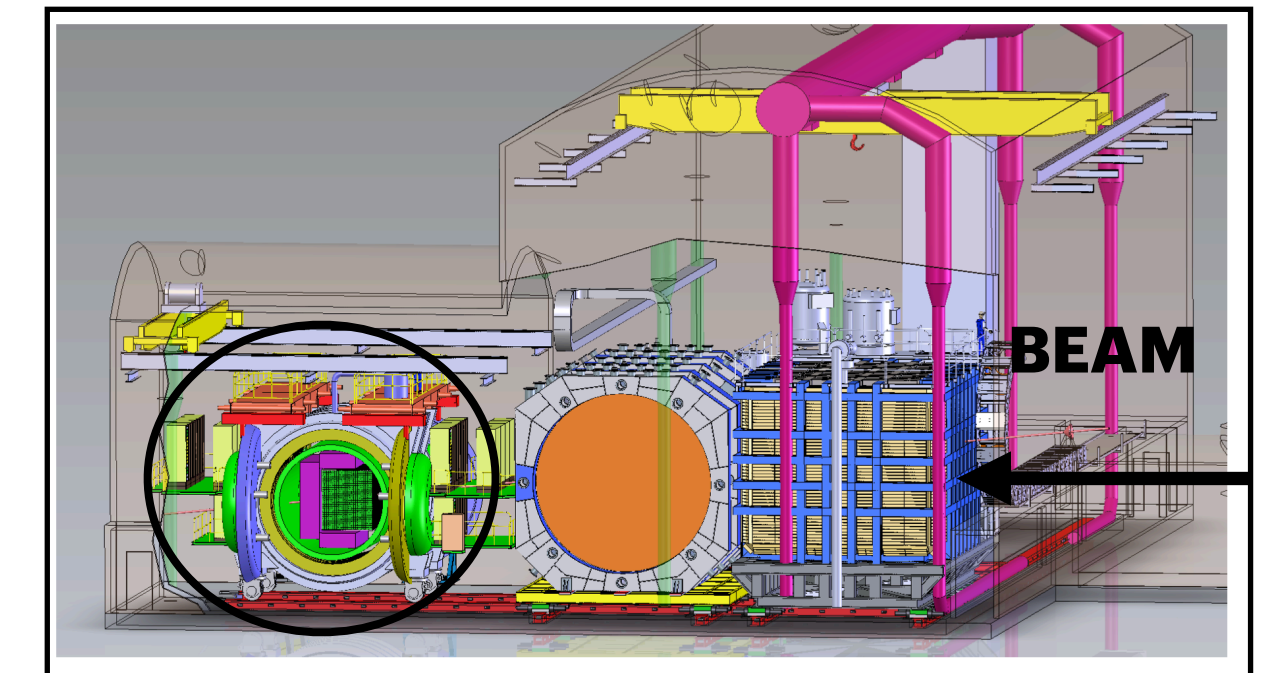
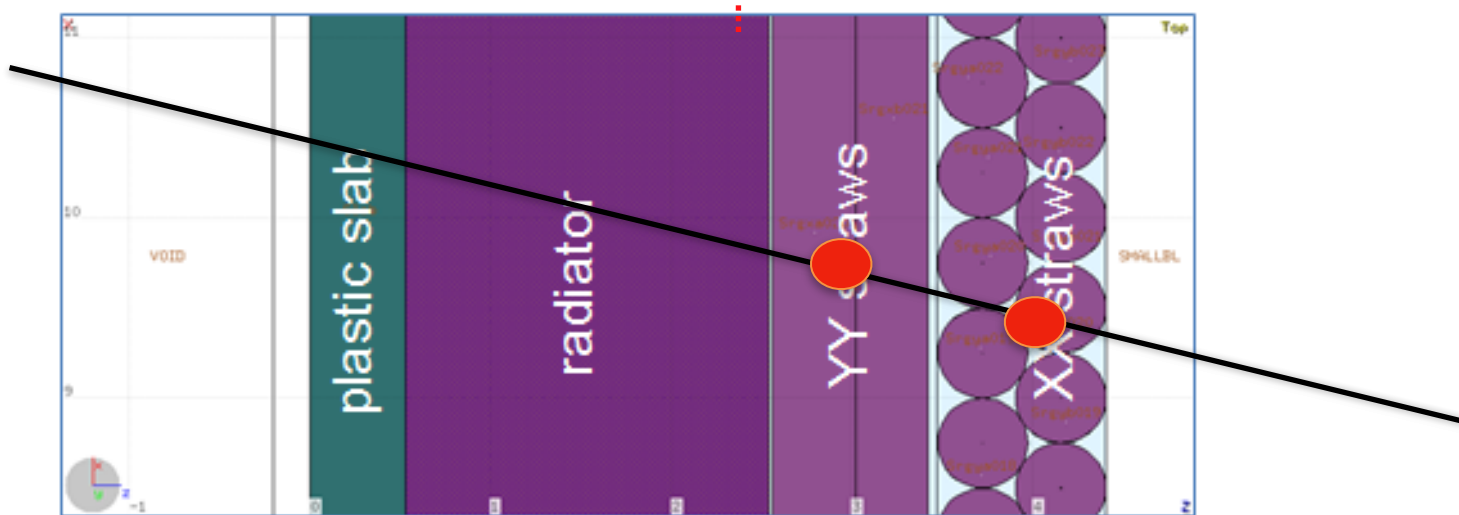
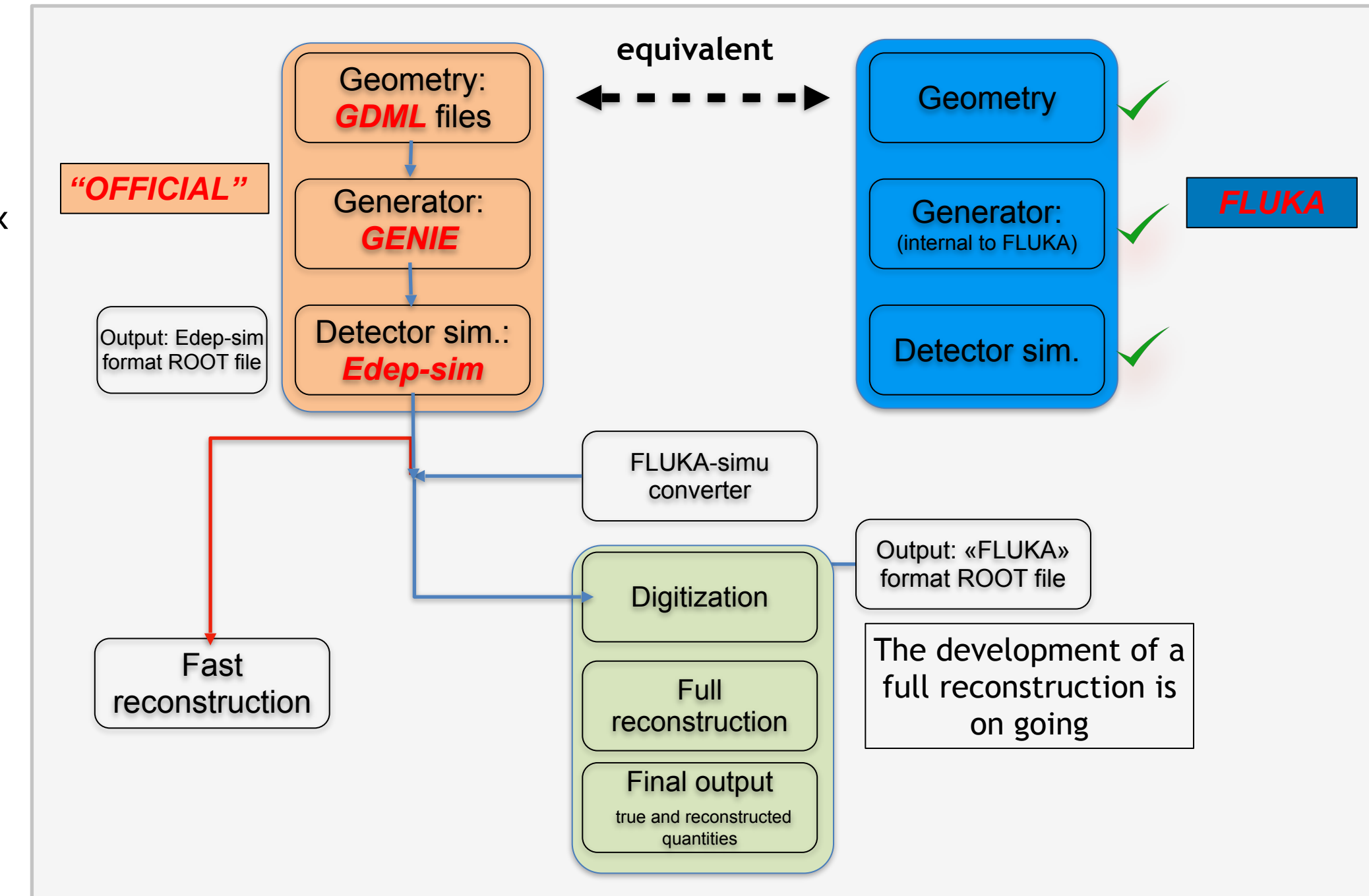
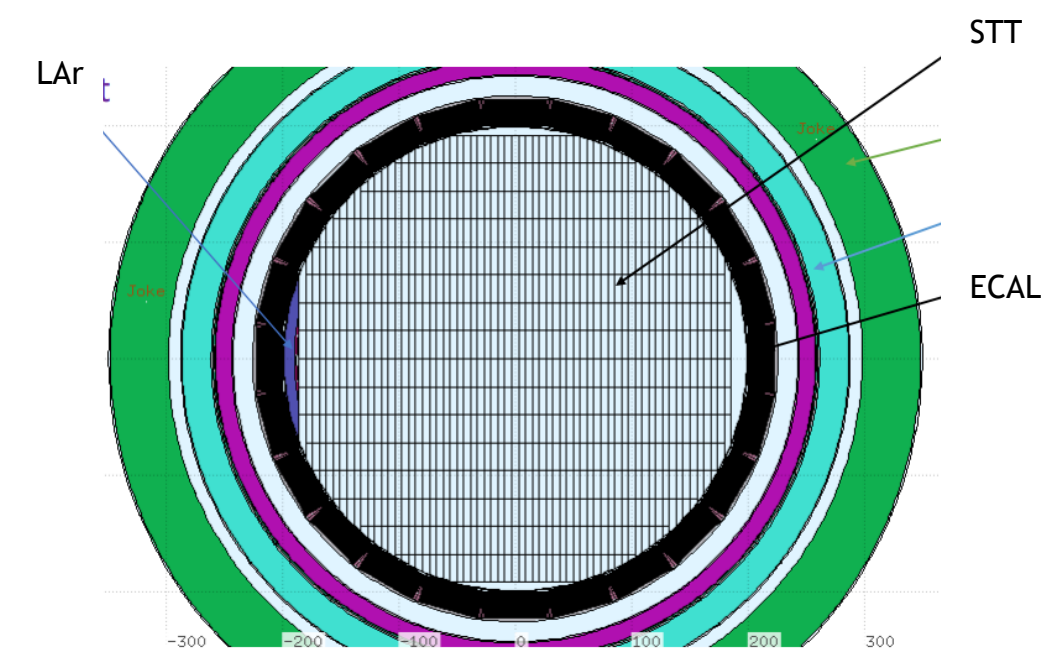
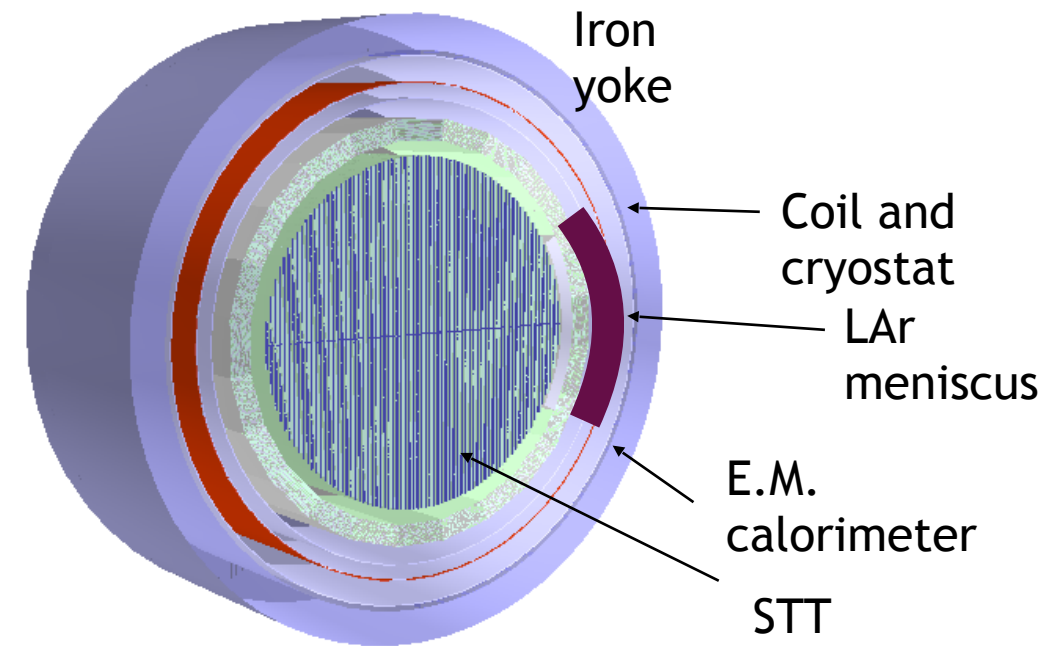
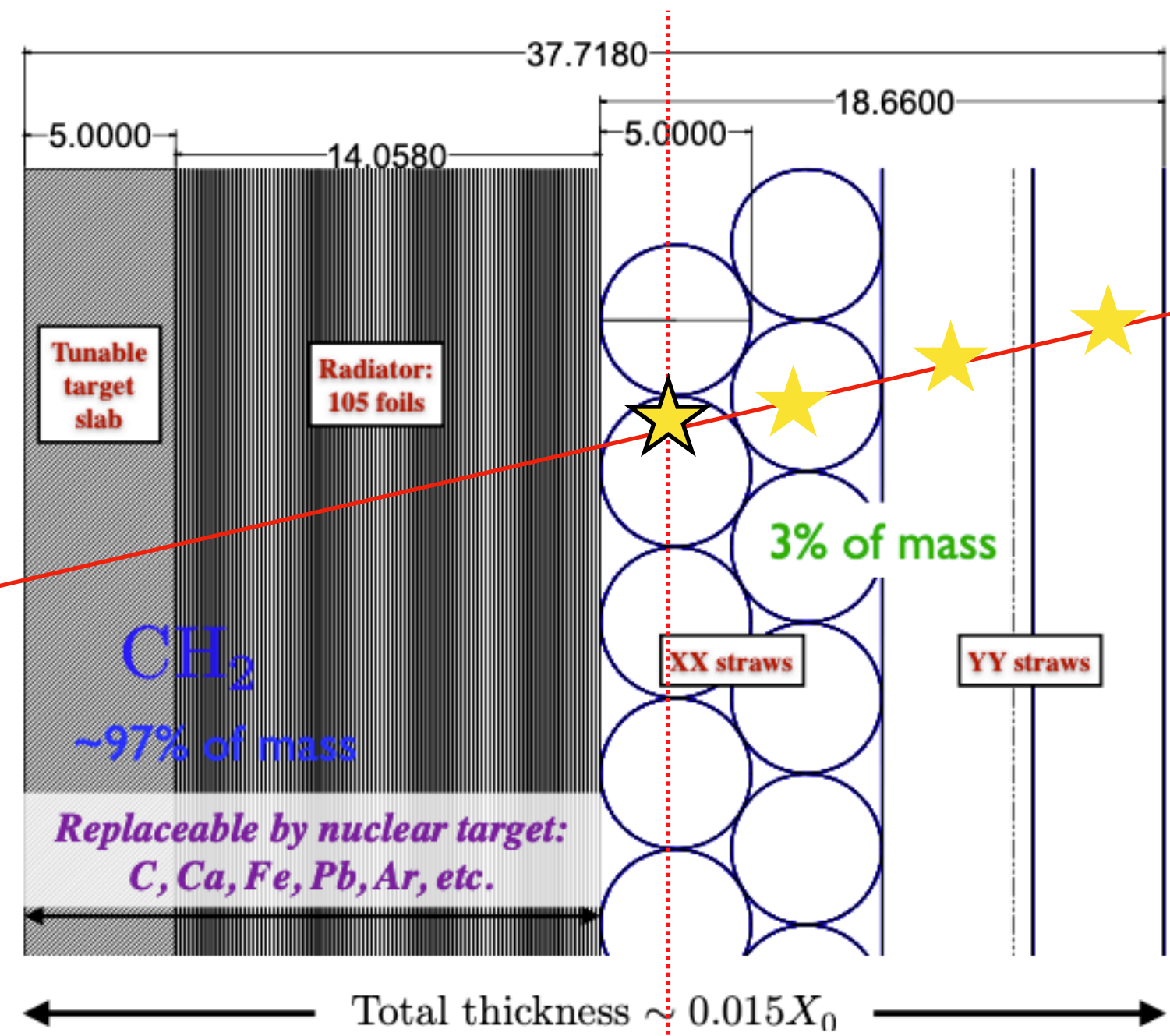


DUNE, Near detector, SAND, Designs

Geometry of the Tracker

ECAL + STT + Grain (LAr meniscus) → GEANT4 and FLUKA

- FLUKA
 - HeaderTree → -TG4PrimaryVertex
 - HitsTree → -TG4Trajectories
 - CellTree → -TG4HitSegment
 - STTTree → -TG4HitSegment
- EDEPSIM
 - TG4PrimaryVertex
 - TG4Trajectories
 - TG4HitSegment



Reconstruction, Kalman Filter

- Kalman Filter is an algorithm that determines the trajectory of a state vector of a dynamical system from a set of measurements taken at different times, taking into account gaussian fluctuations
- It proceeds progressively from one measurement to the next, improving the knowledge about the trajectory with each new measurement.
- There are three steps for Kalman Filter
 - **Predicting** : an estimate is made for the next measurement from current knowledge of the state vector
 - **Filtering/Updating** : Kalman Filter in Theory updates the state vector using the measurement
 - **Smoothing** : recursive operation, step by step in the direction opposite to that of filter

Residuals

Predict

$$\tilde{x}_k^{k-1} = F_k \tilde{x}_{k-1}$$

$$\tilde{r}_k^{k-1} = m_k - H_k \tilde{x}_{k-1}^k$$

$$\tilde{C}_k^{k-1} = F_k C_{k-1} F_k^T + Q_k \quad (=MCS)$$

$$\tilde{R}_k^{k-1} = V_k + H_k C_{k-1}^k H_k^T$$

Update

$$K_k = C_{k-1}^k H^T (V_k + H_k C_{k-1}^k H_k^T)^{-1}$$

$$r_k = (1 - K_k H_k) r_{k-1}^k$$

$$\tilde{x}_k = \tilde{x}_k^{k-1} + K_k (m_k - H_k \tilde{x}_{k-1}^k)$$

$$R_k = (1 - K_k H_k) V_k$$

$$C_k = (1 - K_k H_k) C_{k-1}^k$$

$$\chi_{k,F}^2 = r_k^T R_k^{-1} r_k$$

Smooth

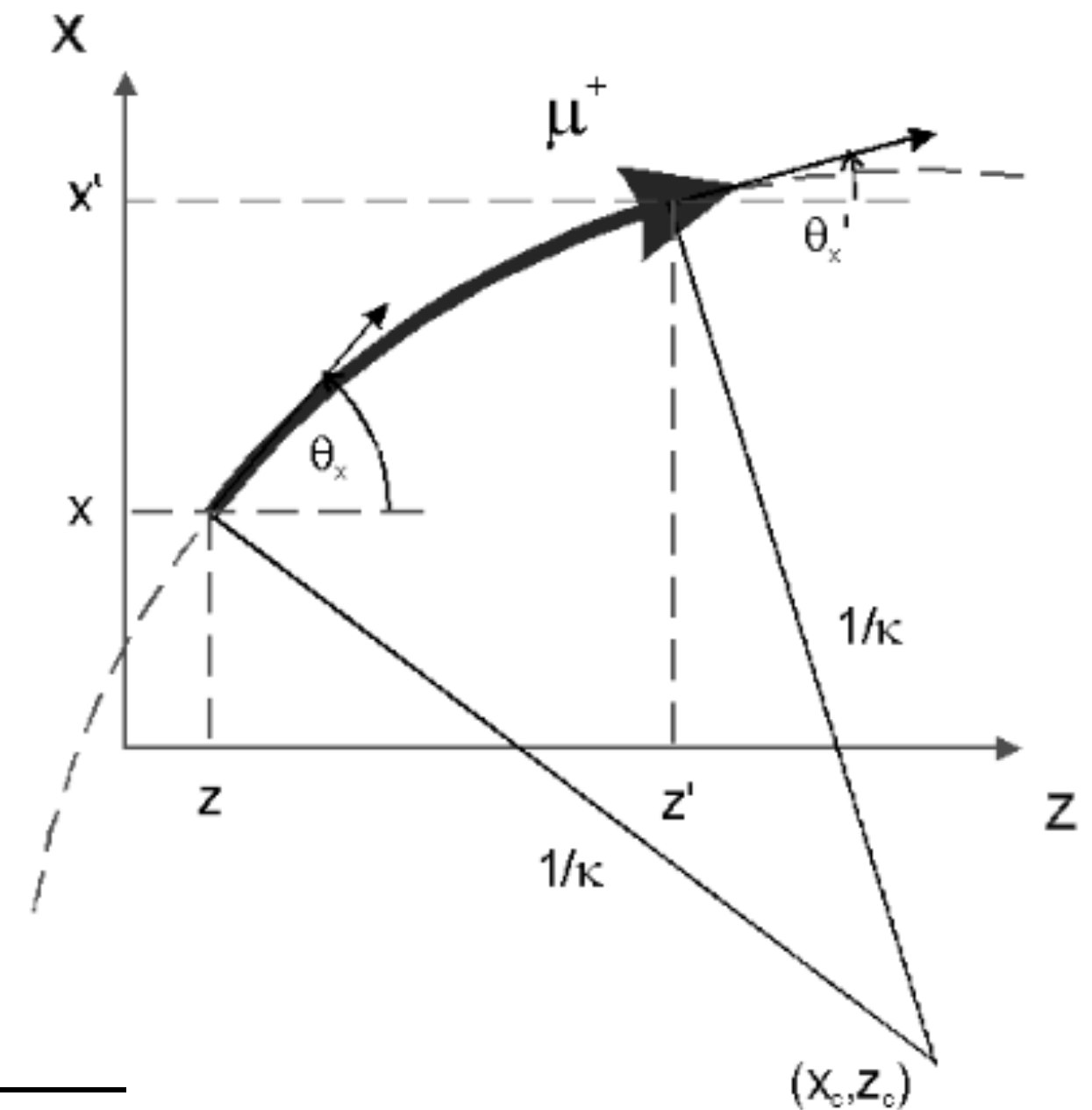
$$A_k = C_k F_{k+1}^T (C_{k+1}^k)^{-1}$$

$$r_k^n = m_k - H_k \tilde{x}_k^n$$

$$\tilde{x}_k^n = \tilde{x}_k + A_k (\tilde{x}_{k+1}^n - \tilde{x}_{k+1}^k)$$

$$R_k^n = R_k - H_k A_k (C_{k+1}^n - C_{k+1}^k) A_k^T H_k^T$$

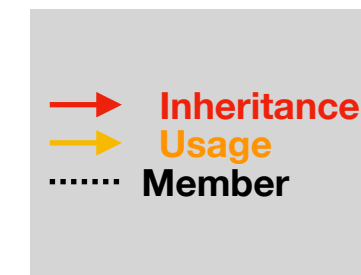
$$C_k^n = C_k + A_k (C_{k+1}^n - C_{k+1}^k) A_k^T$$



$$X = \left(x, y, t_x, t_y, \frac{q}{P_T} \right)$$

$$\frac{q}{P_T} \left[\frac{e}{GeV} \right] = \frac{1}{R \times 0.3 \times B}$$

Kalman Filter, Obj Oriented Coding

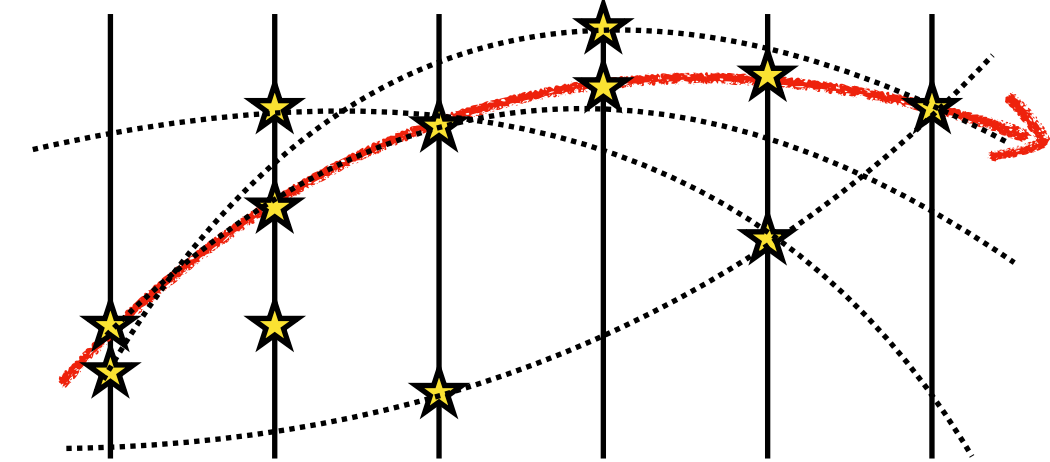
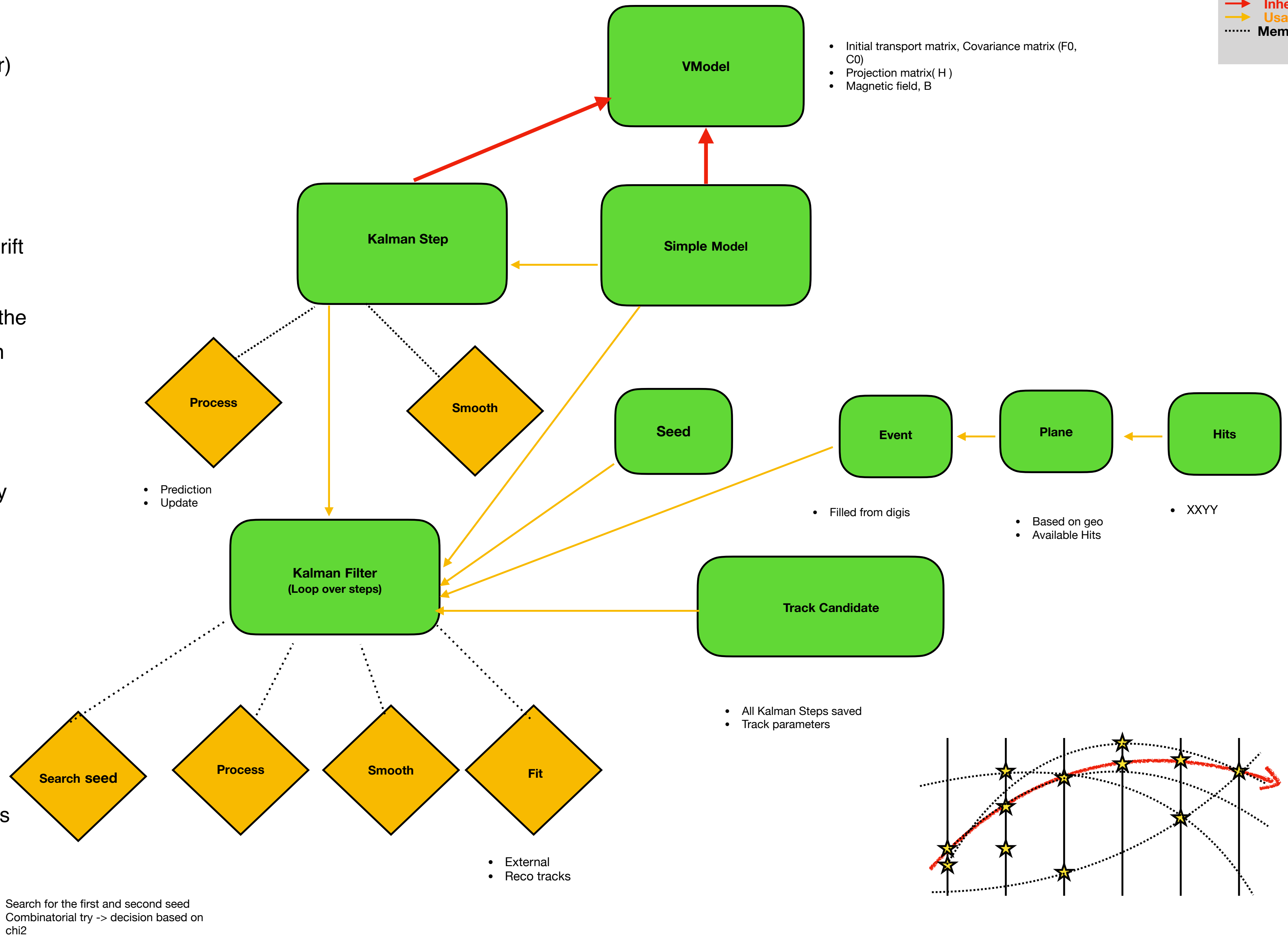


* Coding structure

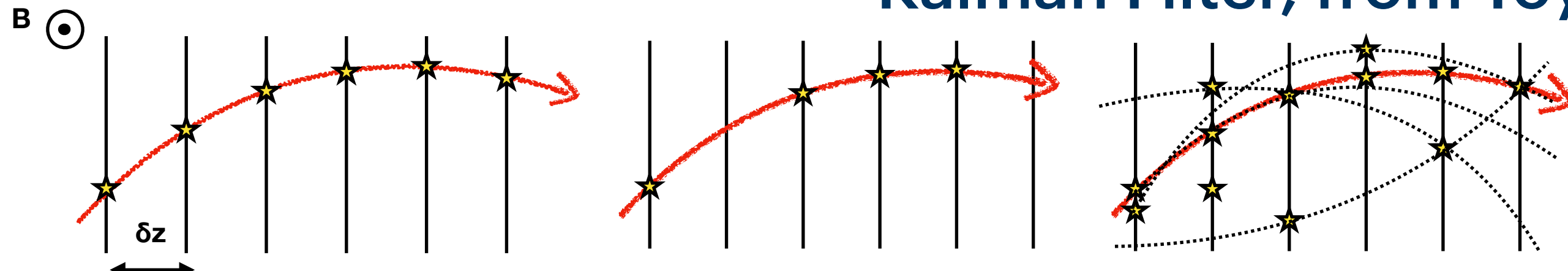
- Started with simple code (it cannot go far)
- Progressed to Object Oriented

* Procedure

- Hits
 - Digitization provides 1-D hits (X,Y), drift radius is not stored
 - Having the drift radius instead of the coordinate directly, leaves with an ambiguity on left and right (to be solved using the other hits in the same coordinate)
 - Not very realistic but it's still ok for my study
- Kalman Step
 - In each step Kalman Process and Smooth acts
- Kalman Filter
 - Search Seed
 - Two hits combinations
 - Build potential tracks
 - Discard candidates with < 6 points
 - Fit
 - Save the track



Kalman Filter, from Toy MC to Geant4 MC



* Toy MC

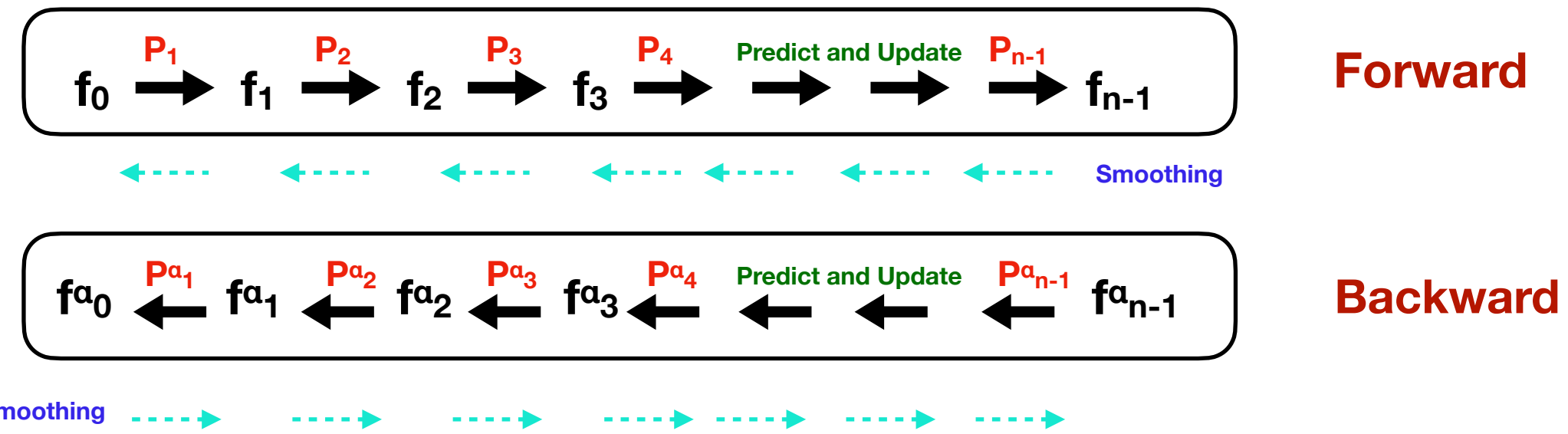
- Toy MC starting with one single track, moved to multiple tracks, noise hits added
- Forward (Backward) Kalman
- **Measurements**
 - Assuming uniform B field, 0.6 T, constant δz for the planes (ideal, zero thickness)
 - RN generation with uniform distribution for initial position and initial momentum
 - For each plane x , y according to analytic extrapolation, with 95% efficiency.
 - Smearing 0.1mm for x and y

* Preparation for Geant4 MC

- Mad-Dump -> Genie-like output
- EdepSim (nd_hall_kloe_sttonly.gdml) -> Edep-Sim output
- Digitization (200 μm smearing)-> wire position added (to meet with Kalman Filter discrete process that goes in steps, e.g. zero uncertainty on z coordinate of the plane)
- XY hits are combined into an extrapolated measurement at the z of the wire of the upstream plane of the module

* Kalman Filter Geant4 MC

- HNL sample 1 GeV mass
- Forward/Backward
- **My Kalman Filter Assumptions**
 - Straw modules -> XXYY or XXYYXX (present in this geometry)
 - Uniform B field
 - Processing hits
 - Separate measurement for X and Y are recombined to (X, Y) referring to the Z of the first straw layer of each module

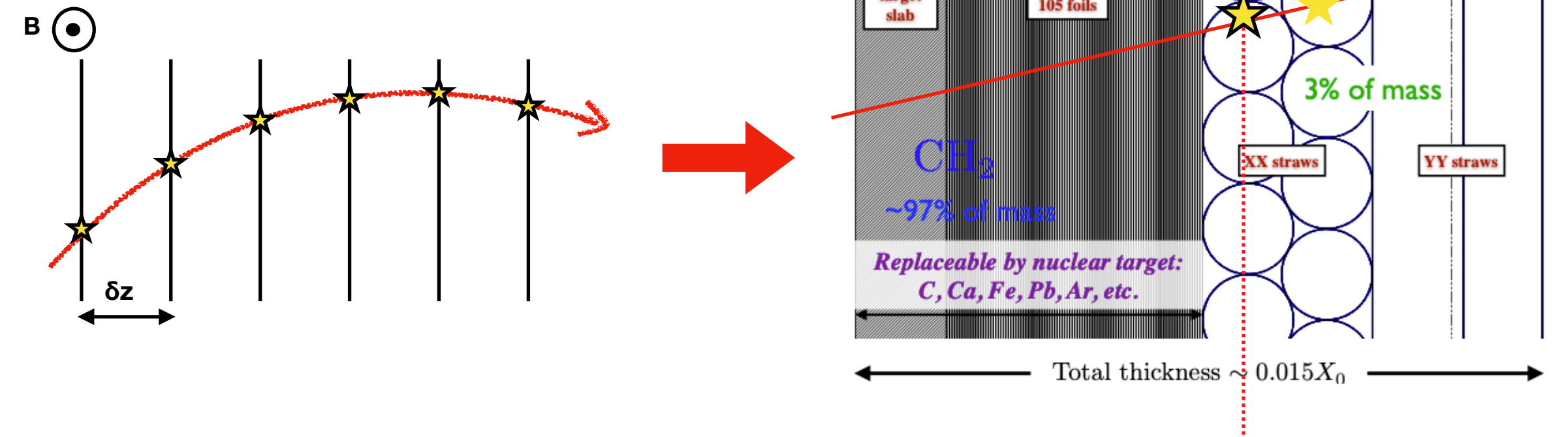


* Kalman Filter Procedure:

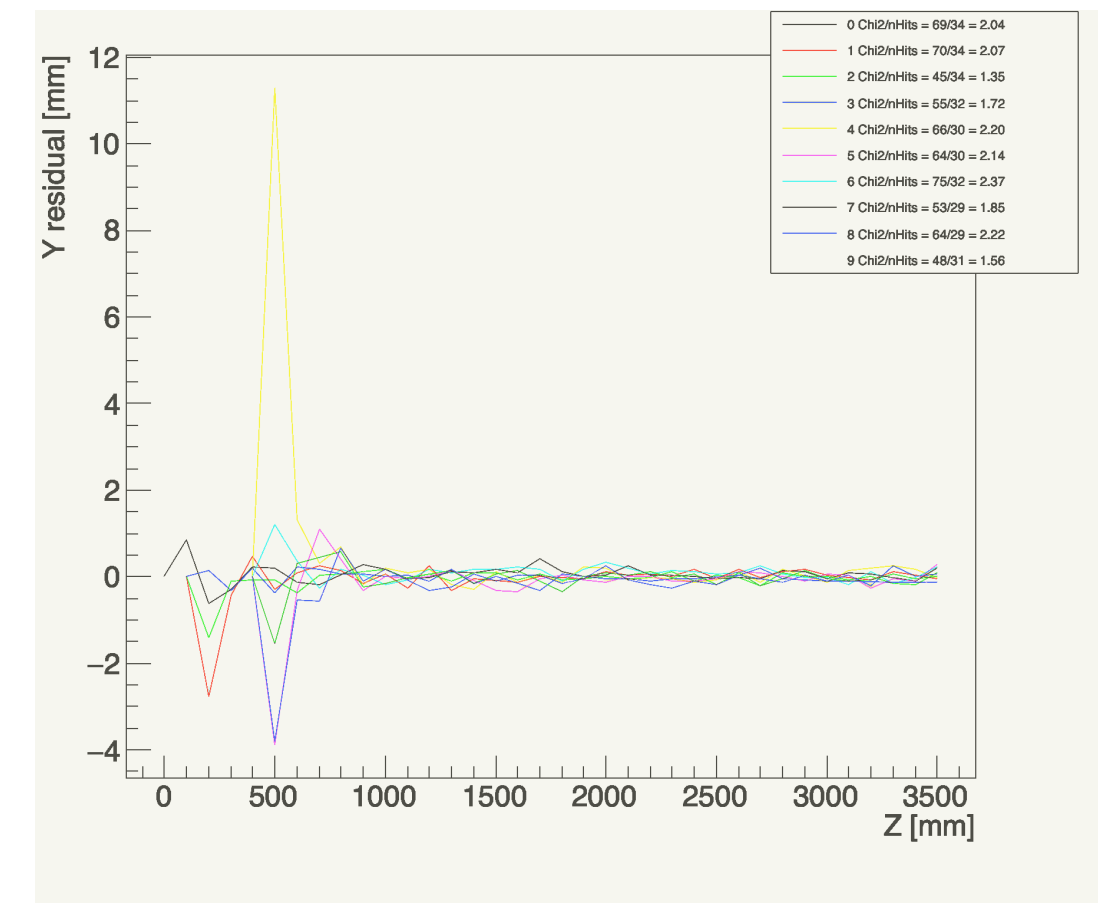
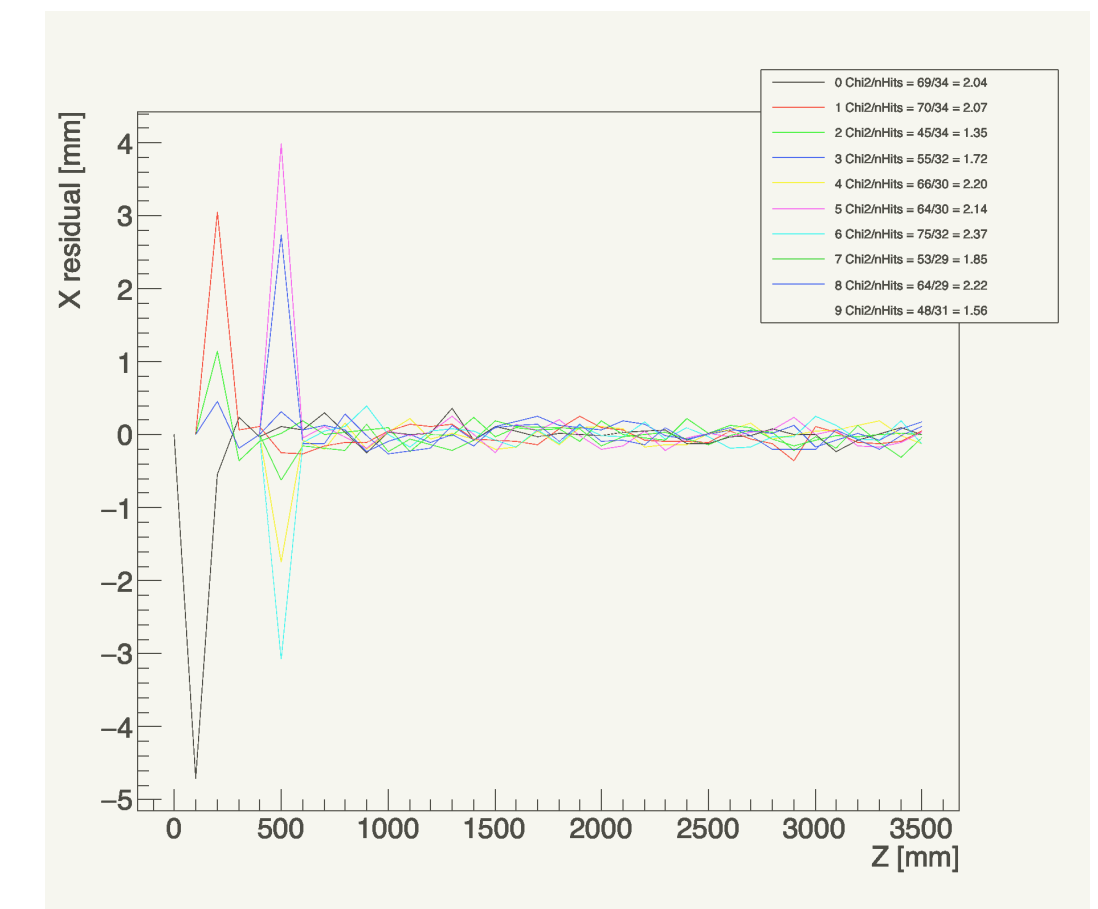
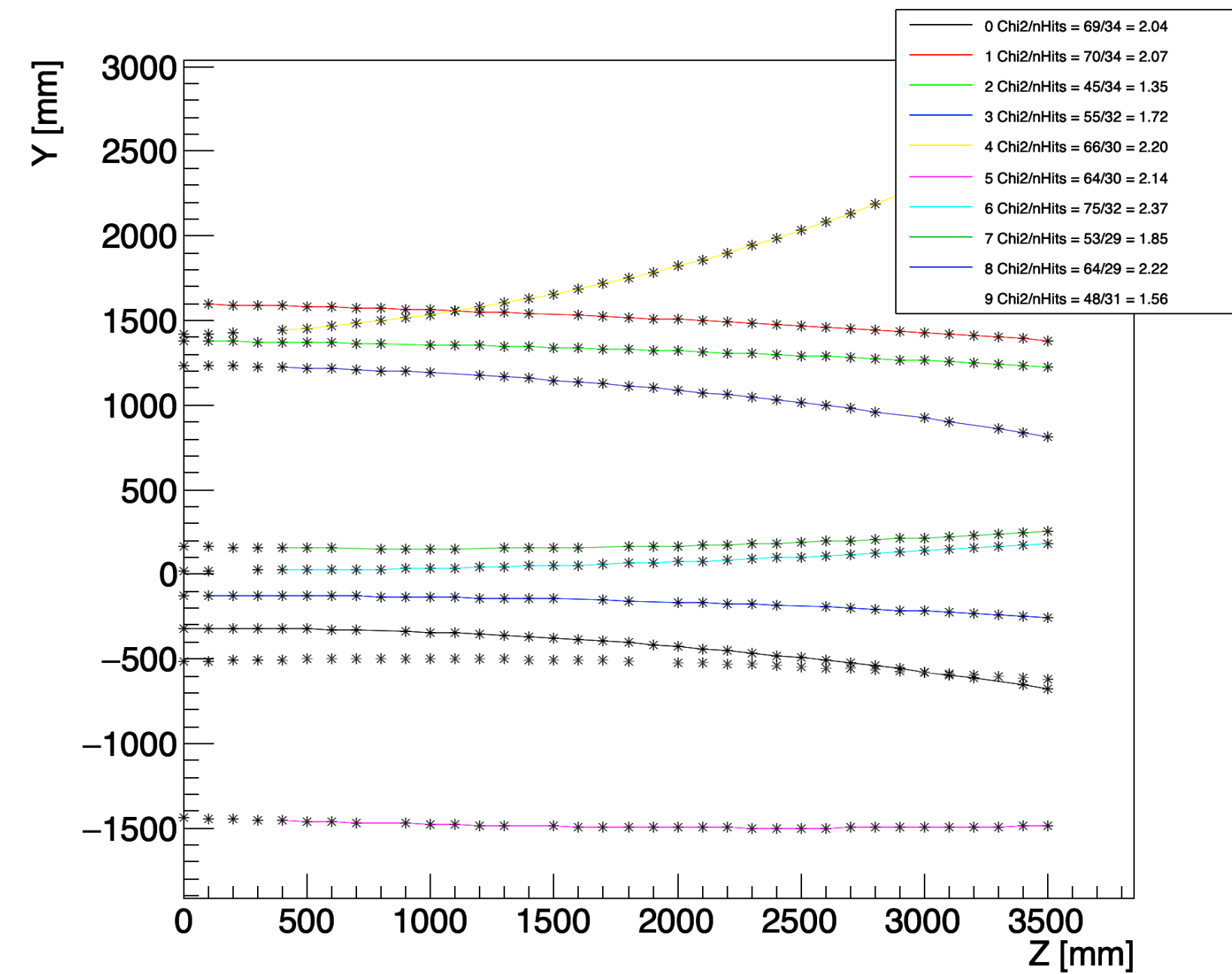
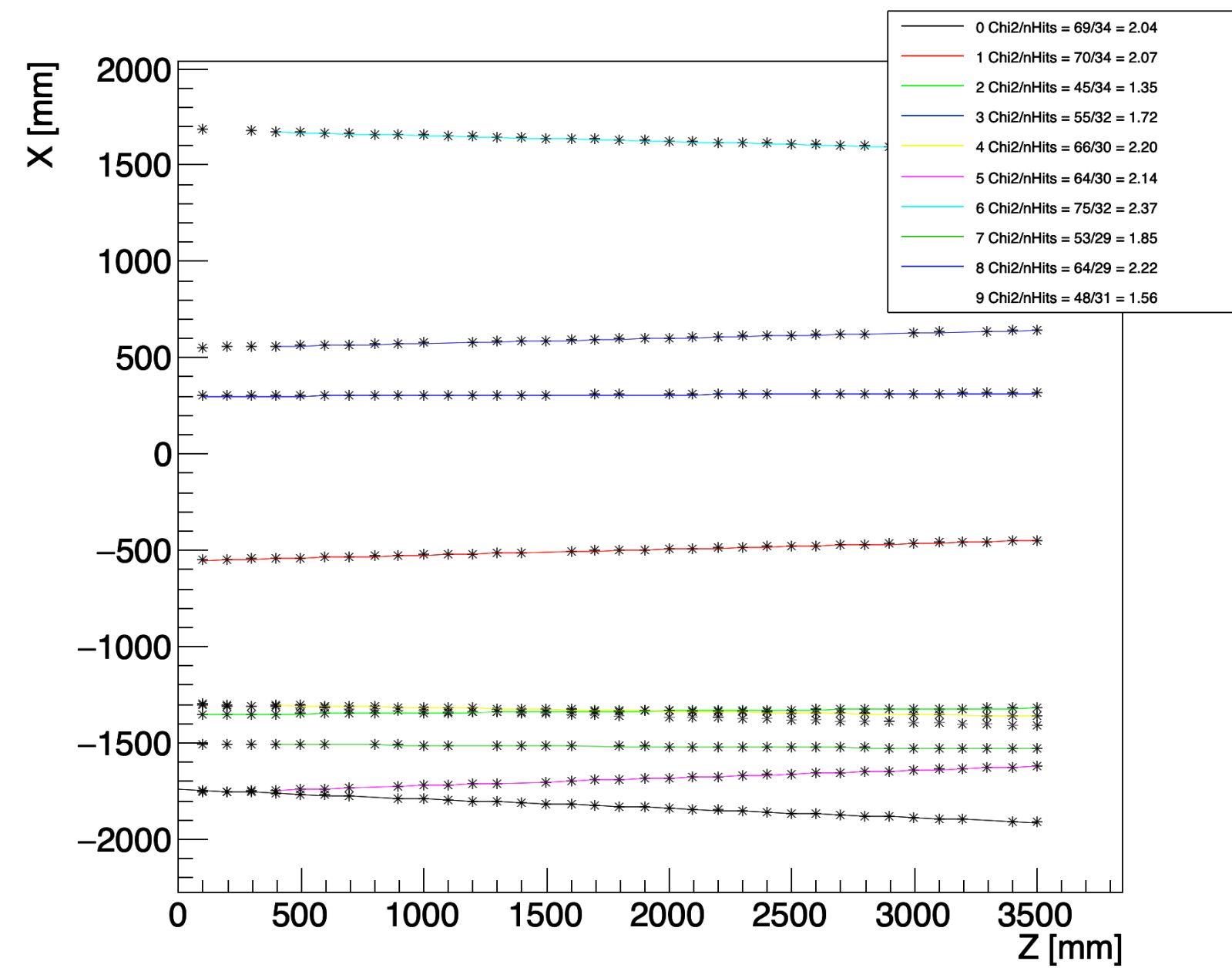
- Sequentially adding new information on each hits to get an optimal track
- **Strategy:**
 - Prediction and Update (Filtering, Residual , χ^2) -> forward and backward, Smoothing

* Kalman Filter Assumptions

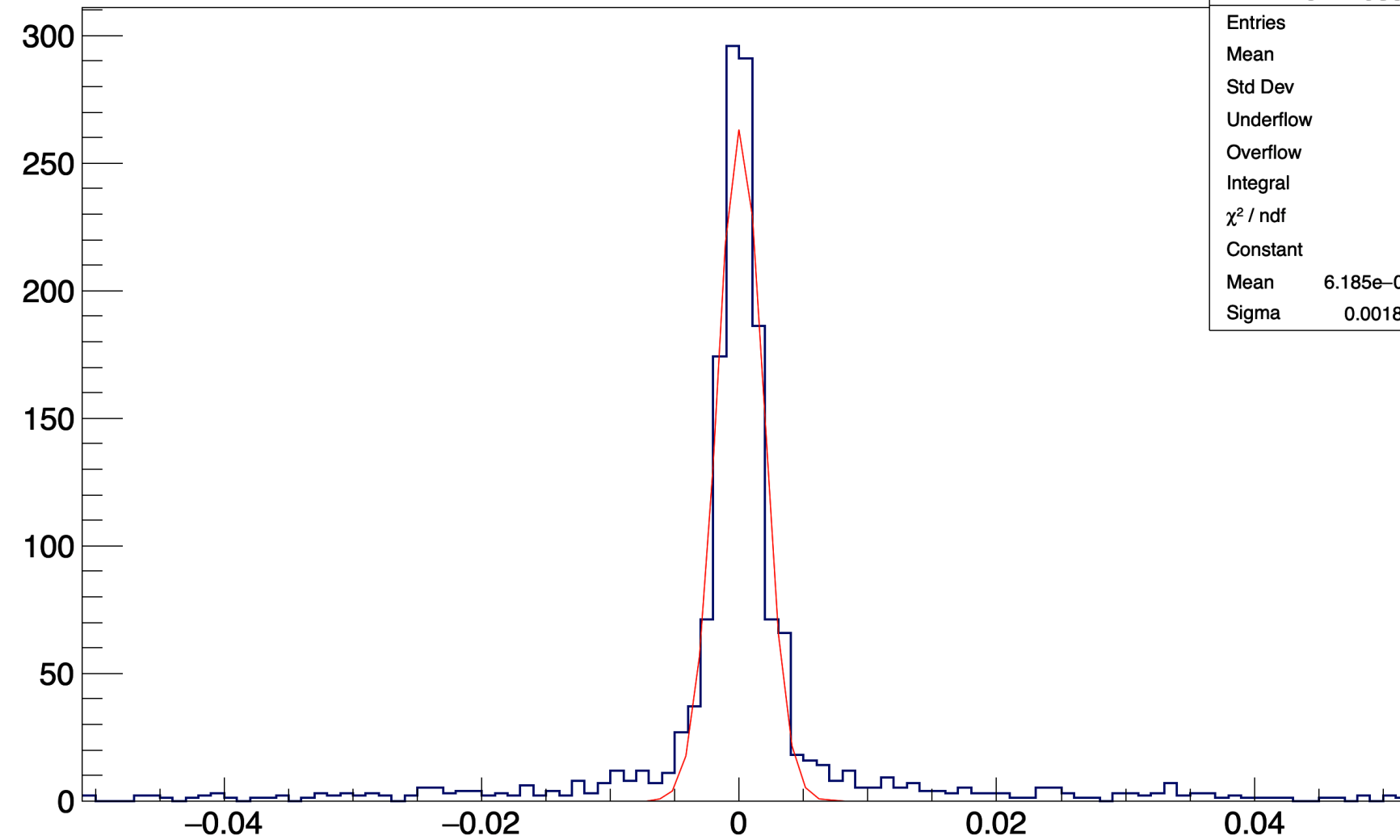
- Uniform B field
- Prediction step is an analytical extrapolation



Kalman Filter, Toy MC



momentum resolution



MomResolution	
Entries	2011
Mean	0.00026
Std Dev	0.01002
Underflow	37
Overflow	0
Integral	1546
χ^2 / ndf	354.8 / 88
Constant	263.5 \pm 12.1
Mean	6.185e-05 \pm 5.248e-05
Sigma	0.001804 \pm 0.000064

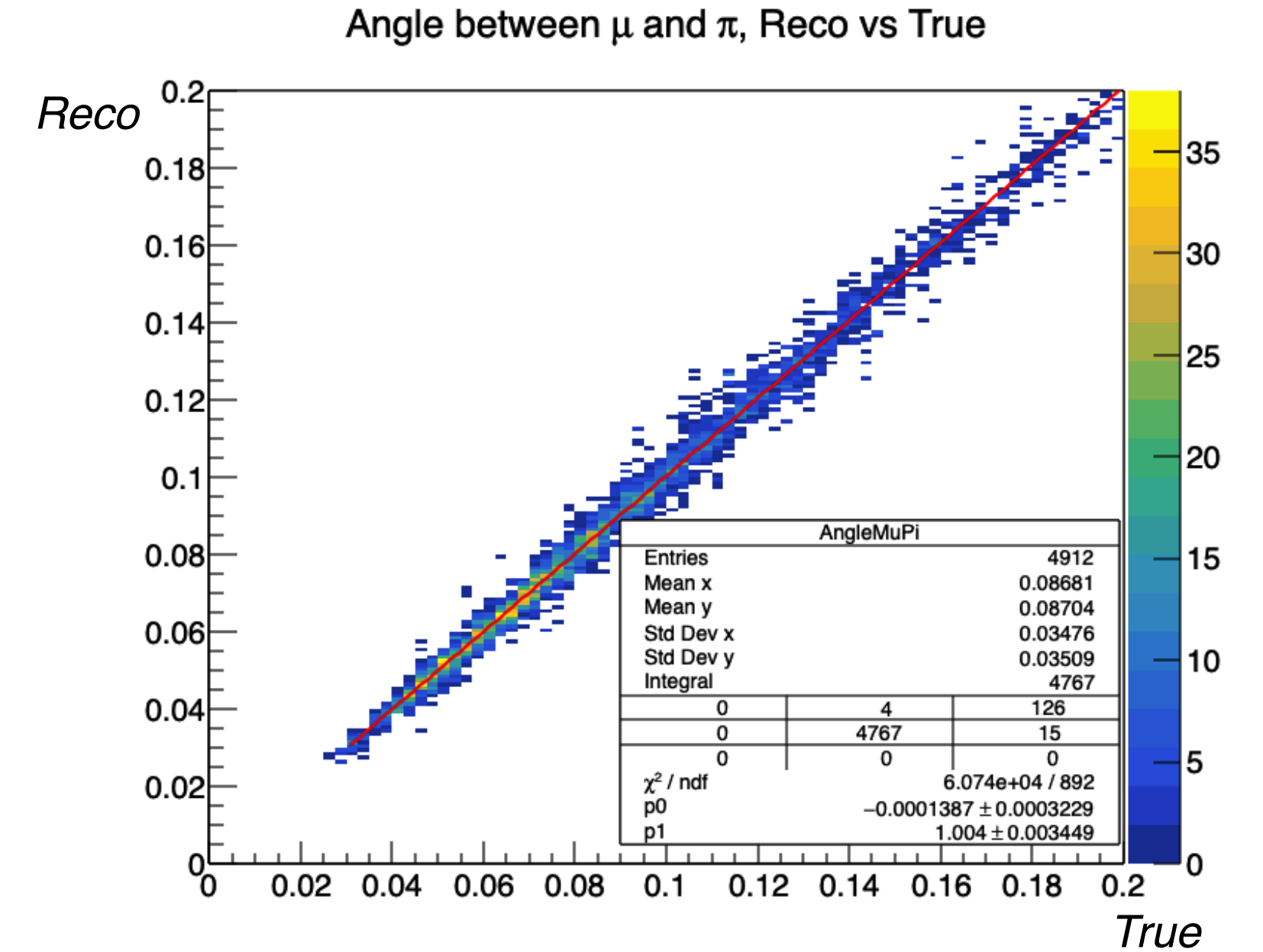
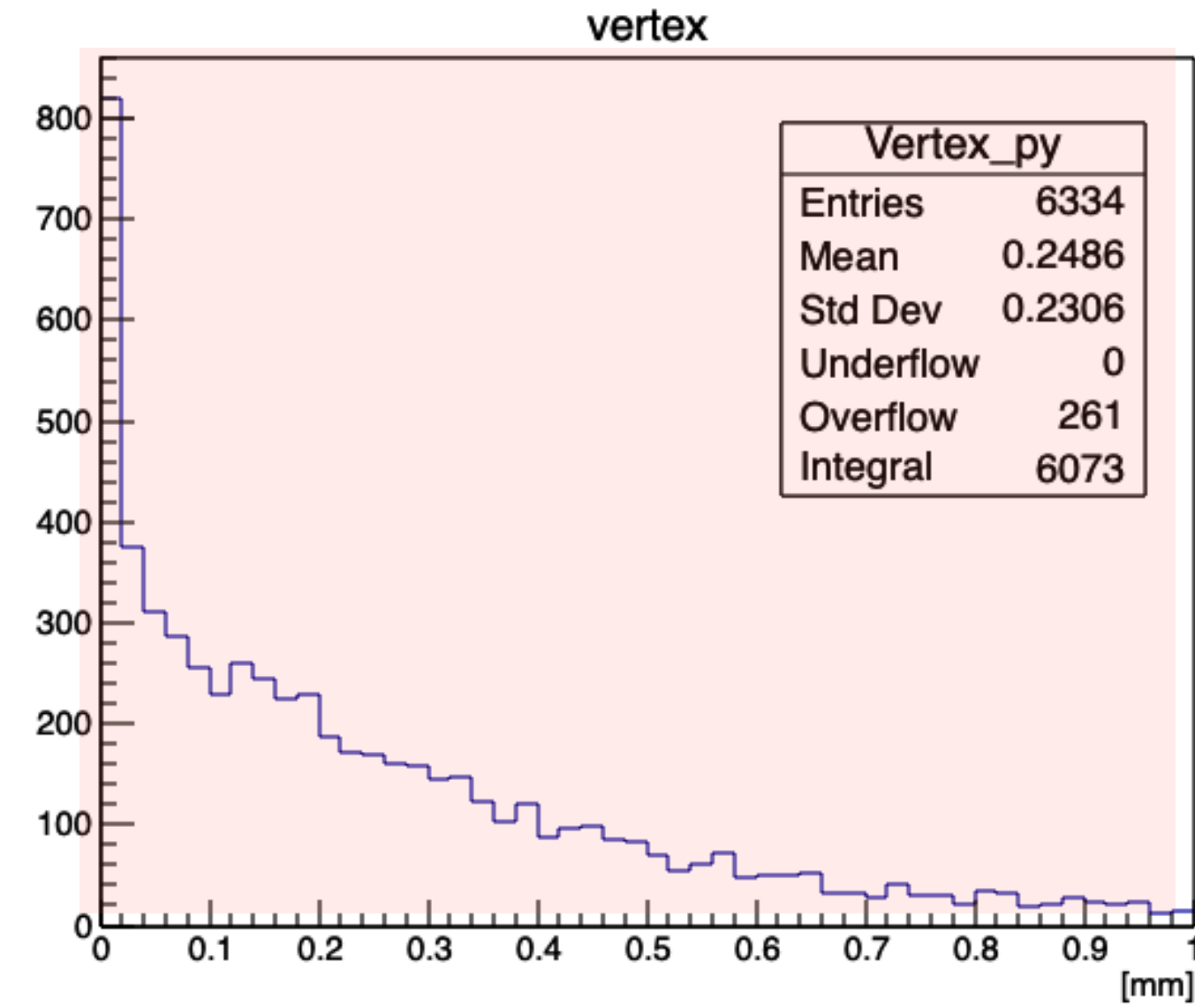
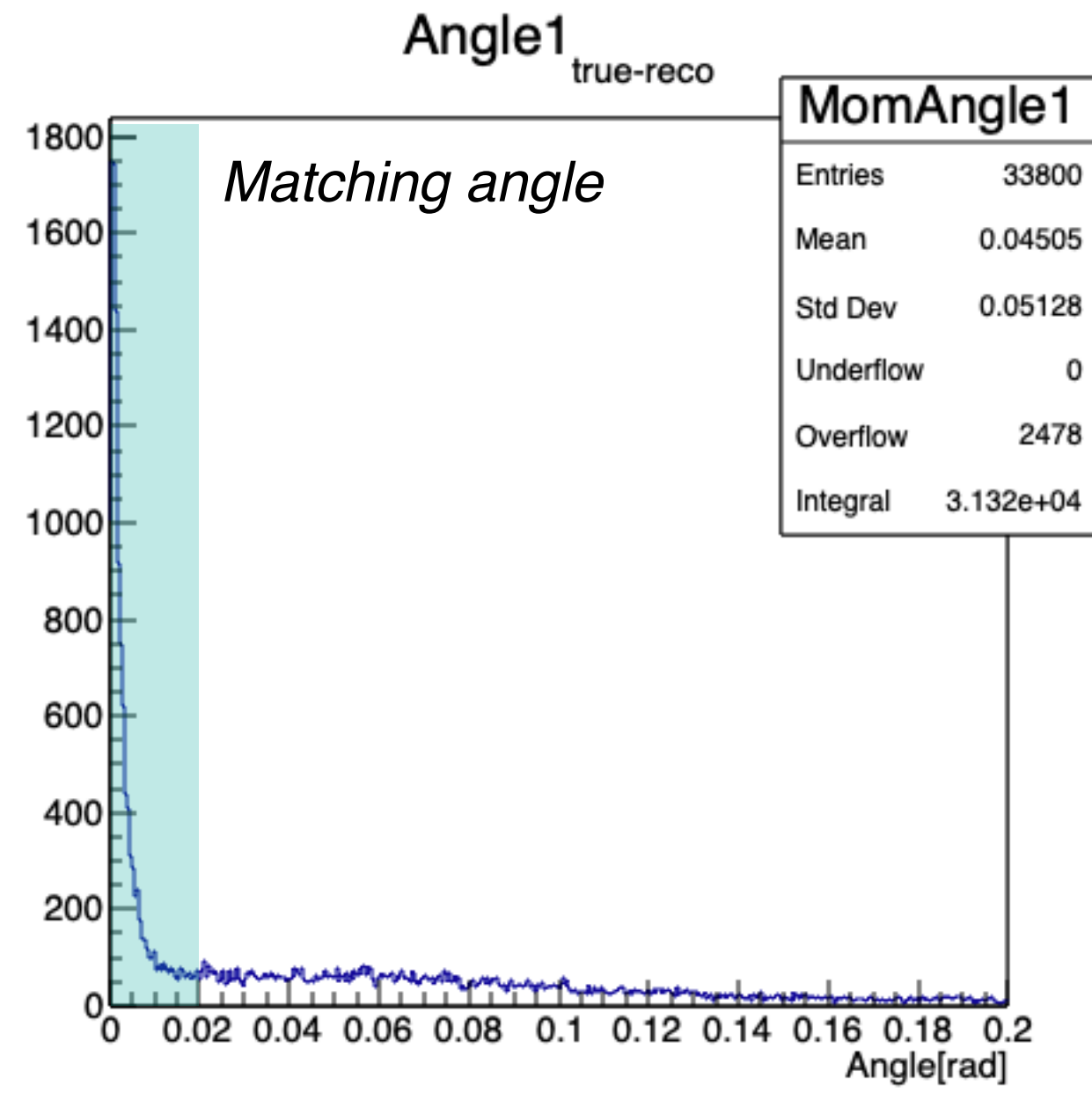
* Momentum Resolution

- Sigma 0.2% -> contribution of the Kalman Filter process to momentum resolution
 - Detector's contribution under estimated by the simple simulation approach
- Toy MC could be improved:
 - mimic a closer geometry to the real one, e.g. the separate measurements for X and Y
 - Individual effects like MS can be studied

* Efficiency

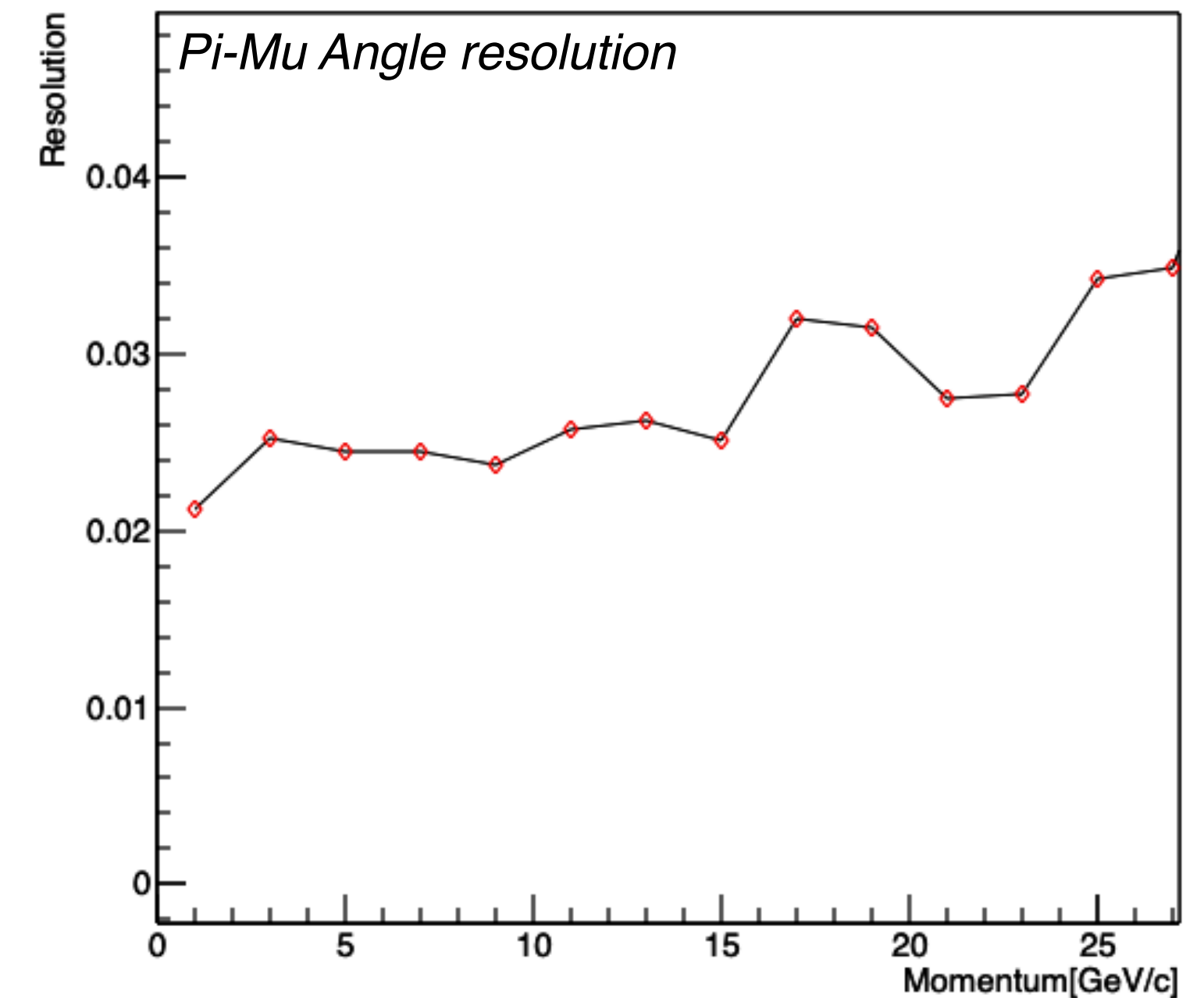
- Seems to find all tracks up to 10 tracks but a realistic estimate on efficiency is not worth for this toy MC

Kalman Filter, Angles and Vertex



* Dominant contributions to Invariant Mass (besides Momentum)

- **Quality of the Vertex:**
 - cut is $< 1\text{mm}$ (most of statistics)
- **Quality of the Reco final product angle:**
 - The final product angle resolution is around 30 [mrad] up to 25 [GeV]
- **MC truth Matching angle (preliminary):**
 - Significant tail on the single particle angle resolution (currently cut by MC truth matching ~ 20 [mrad])



Kalman Filter, GEANT4 MC

* Improvements

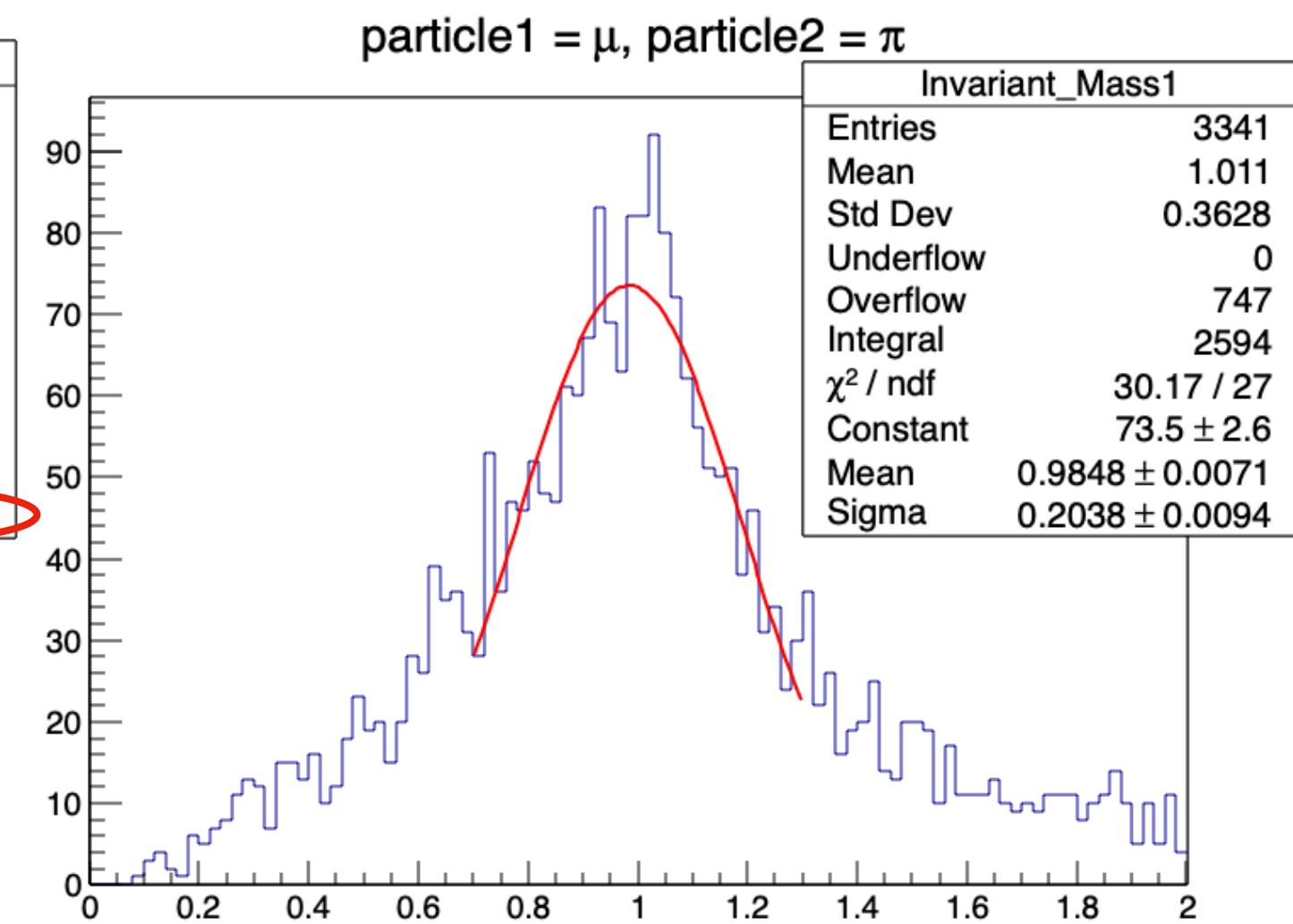
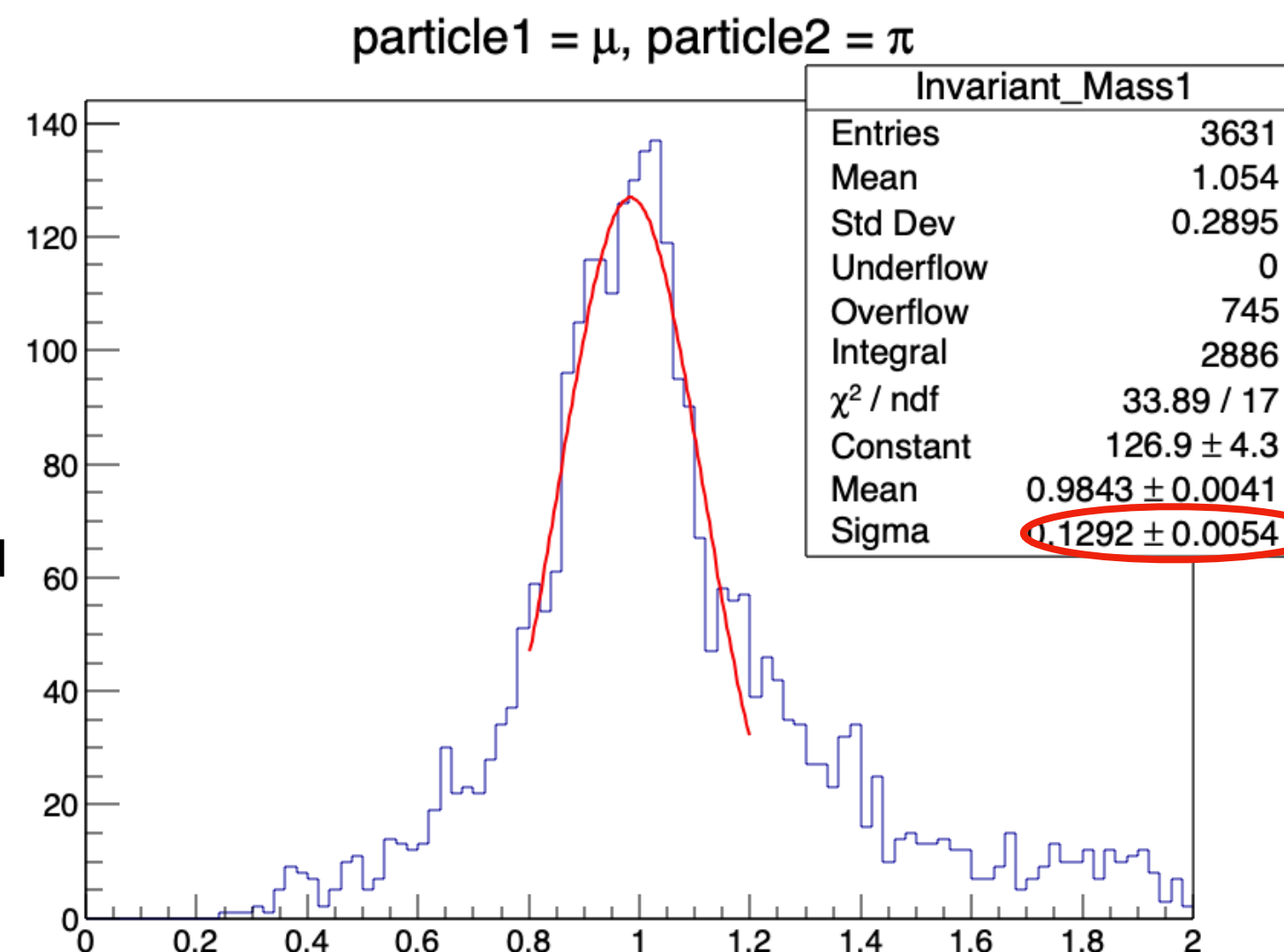
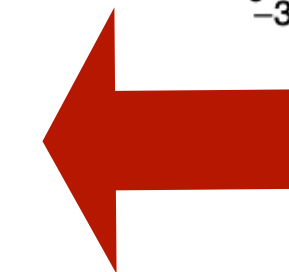
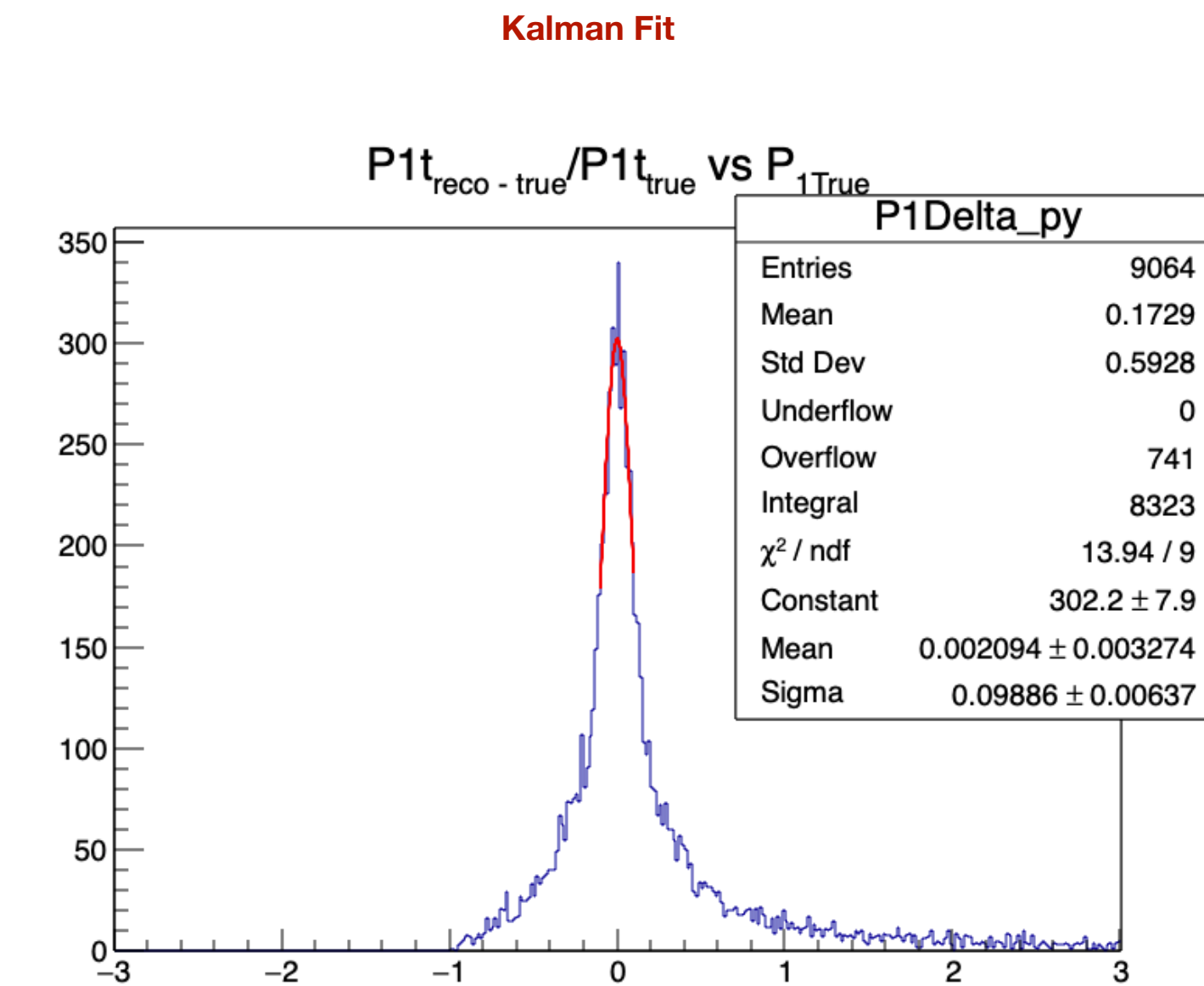
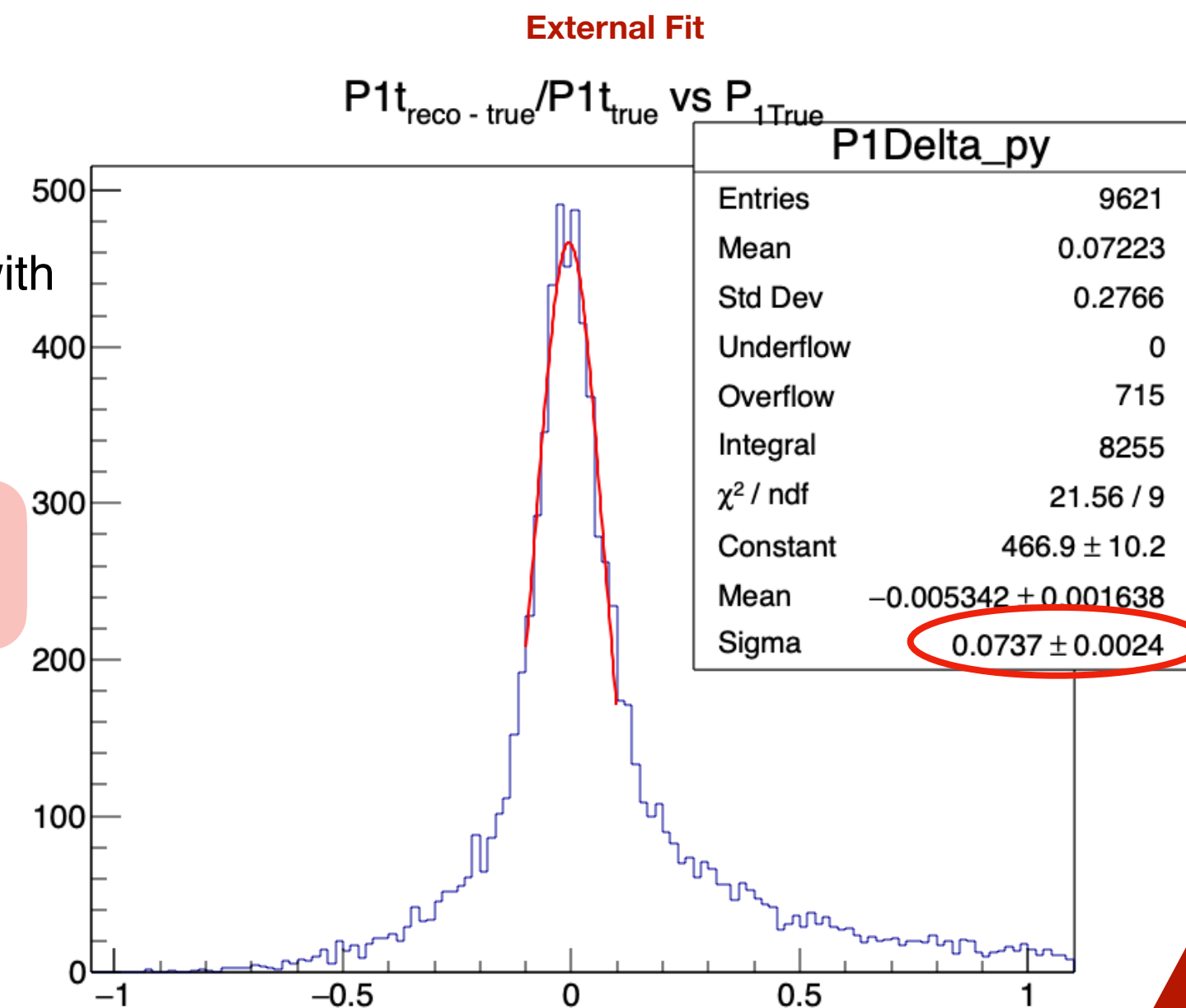
- Backward Kalman direction implemented, in this case backward is more efficient: the initial hit is found easier and more precise (MCS not messing with the hits much)
- Multiple scattering has been added (changing the resolution by 0.1%)
- For better precision, external helical fit has been used (hits are coming from Kalman Filter, the used fit is the external one)

* Items to have an eye on:

- Invariant Mass resolution
- Momentum resolution
- Kalman Filter parameters (Pull plots)
- Goodness of the fit (χ^2)

* Procedure:

- Kalman Filter
 - Forward/Backward Kalman and smoothing.
 - External helical fit.
 - Reco tracks:
 - A. Choosing either forward or backward as Reco tracks.
 - B. Matching the forward/backward Reco tracks ($\geq 90\%$ shared hits), choosing the right combo for the final Reco track collection.
- Matching the Reco and the True.
- Momentum resolution, Invariant mass resolution, Pull plots



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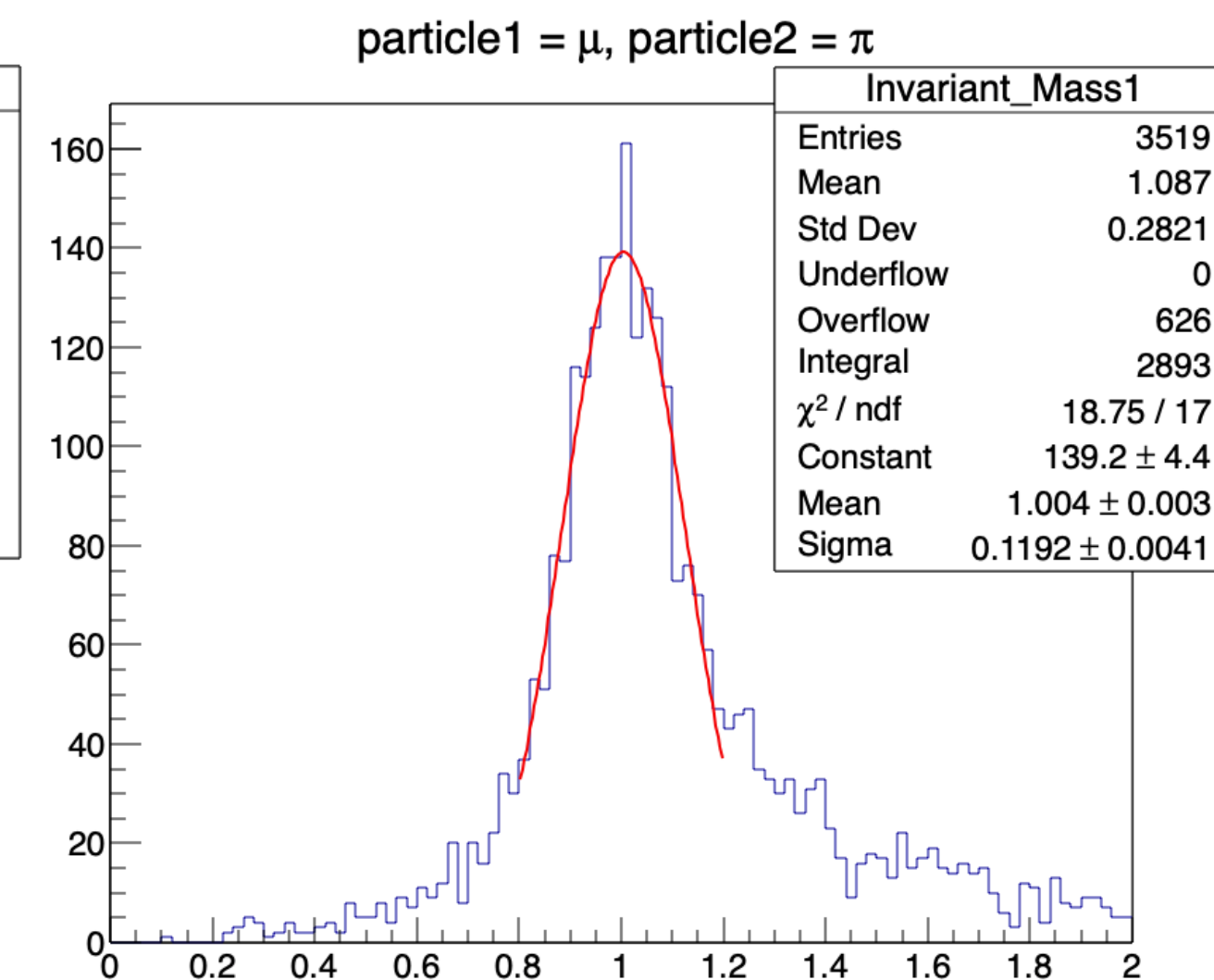
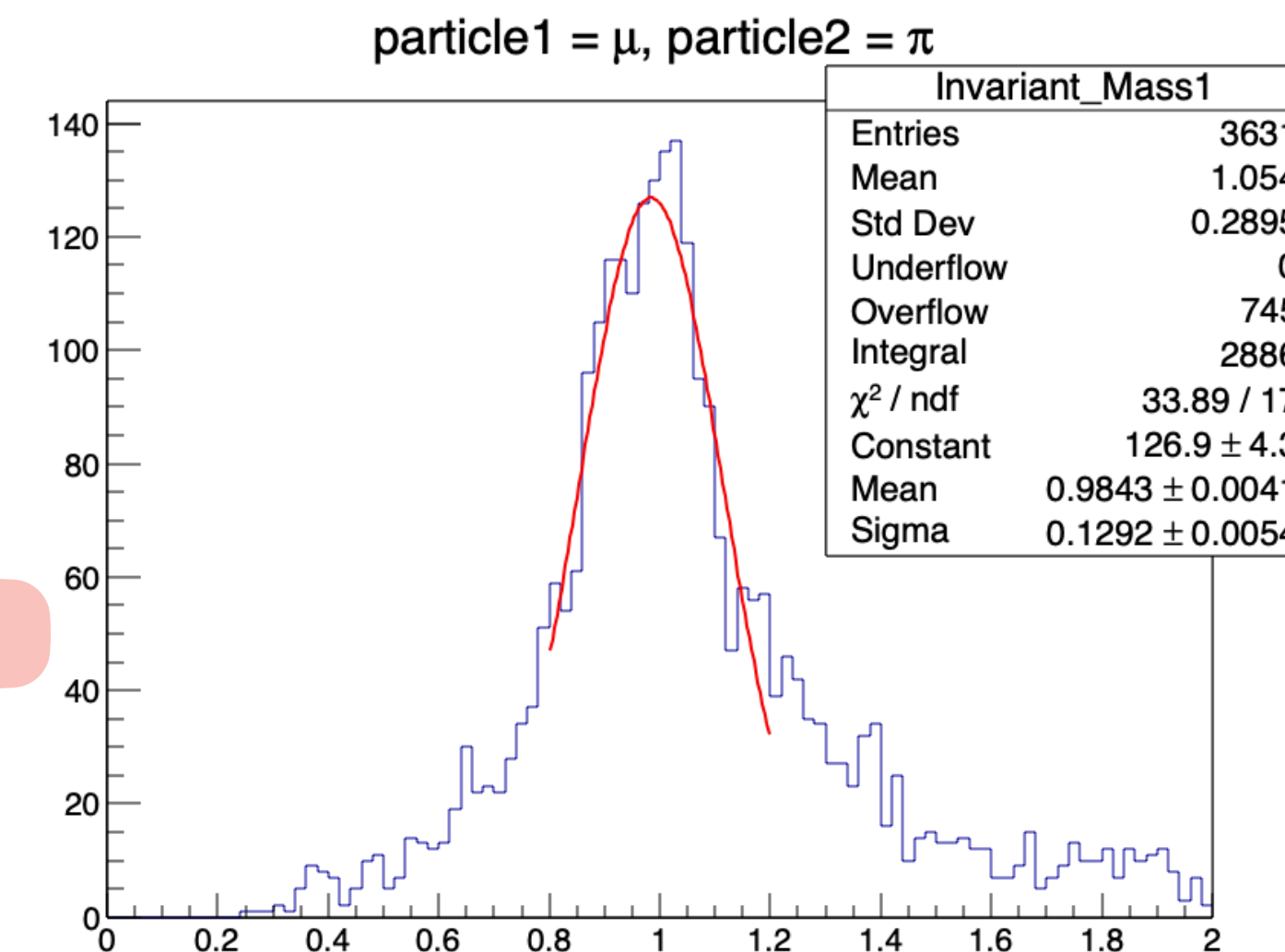
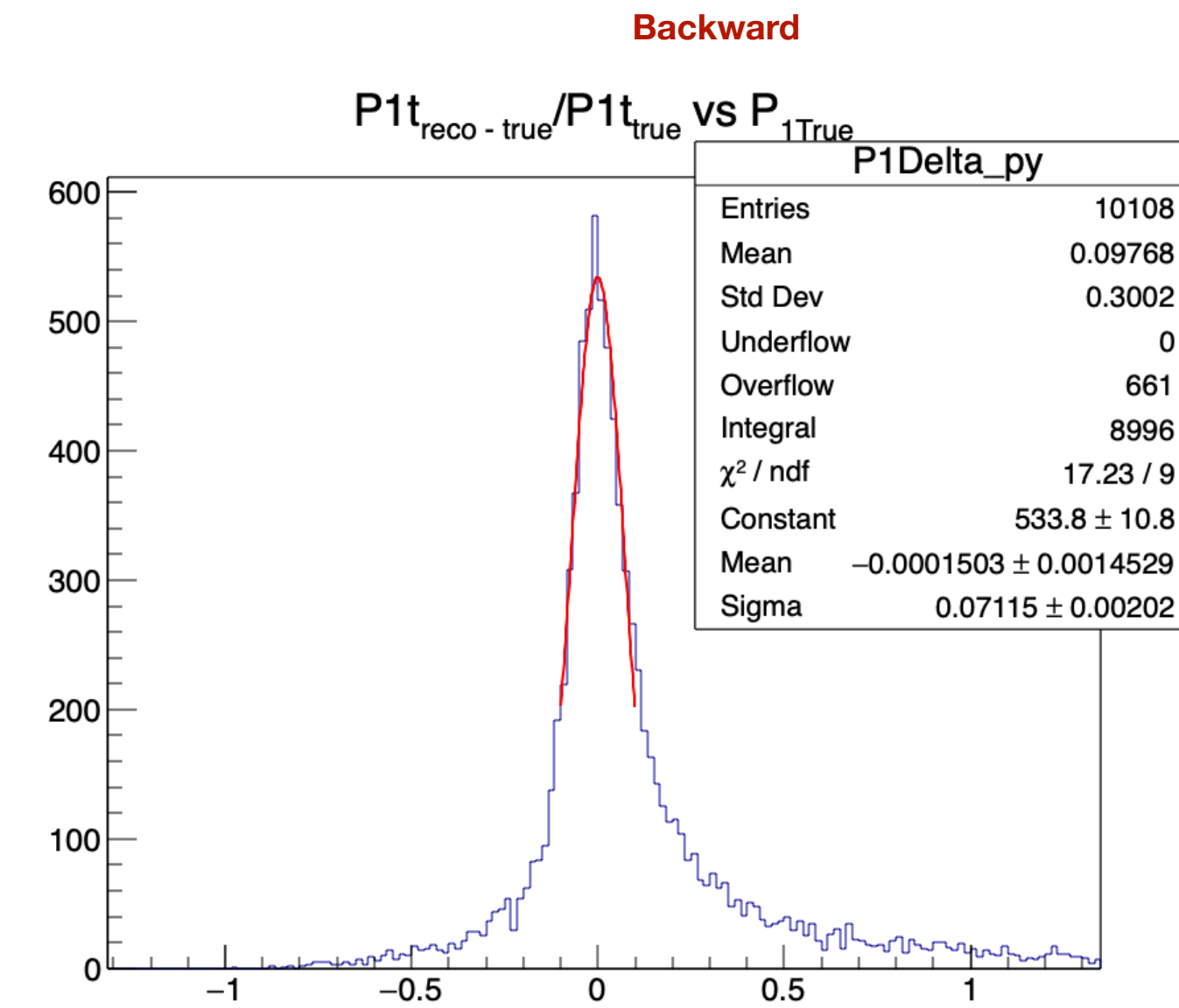
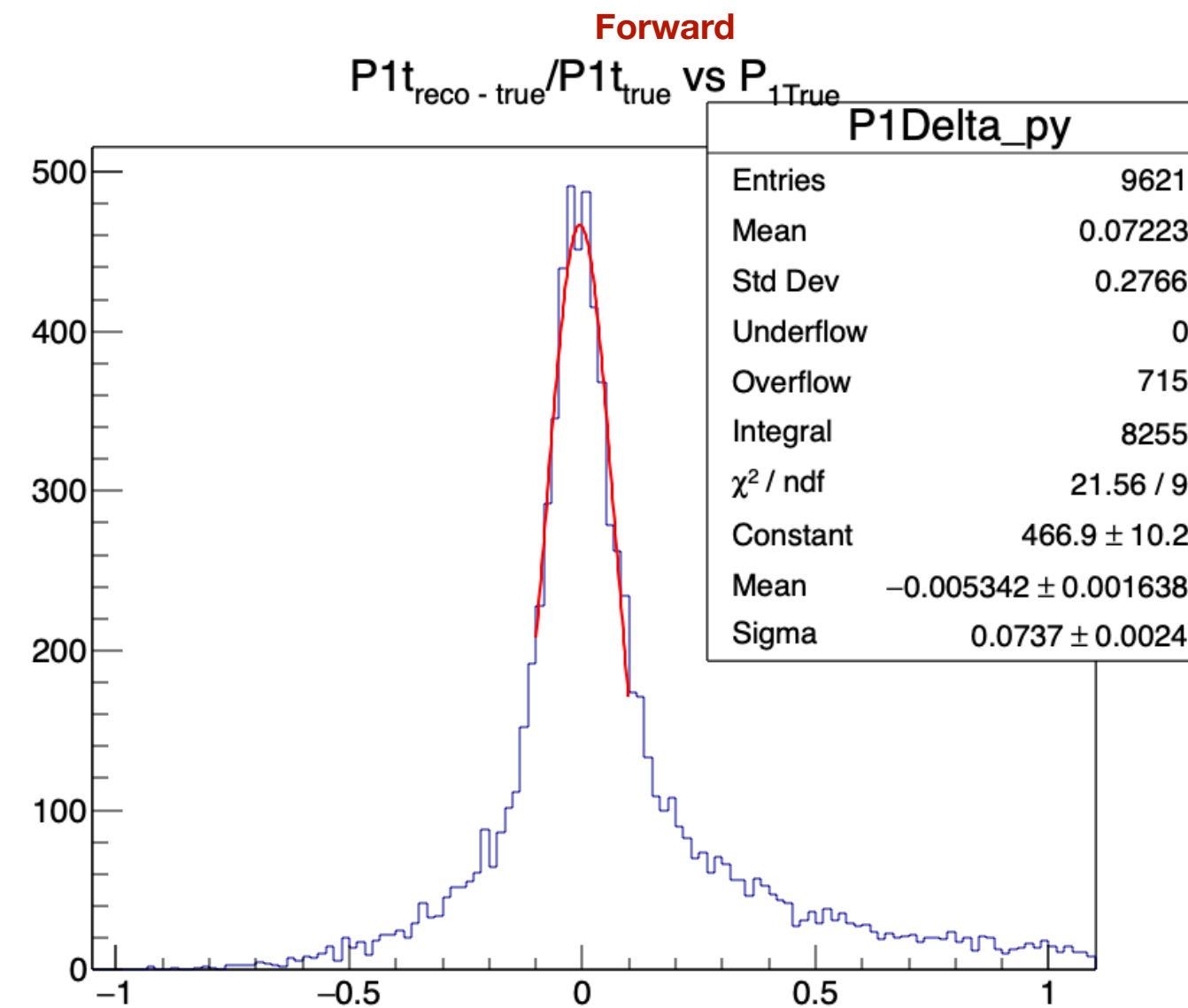
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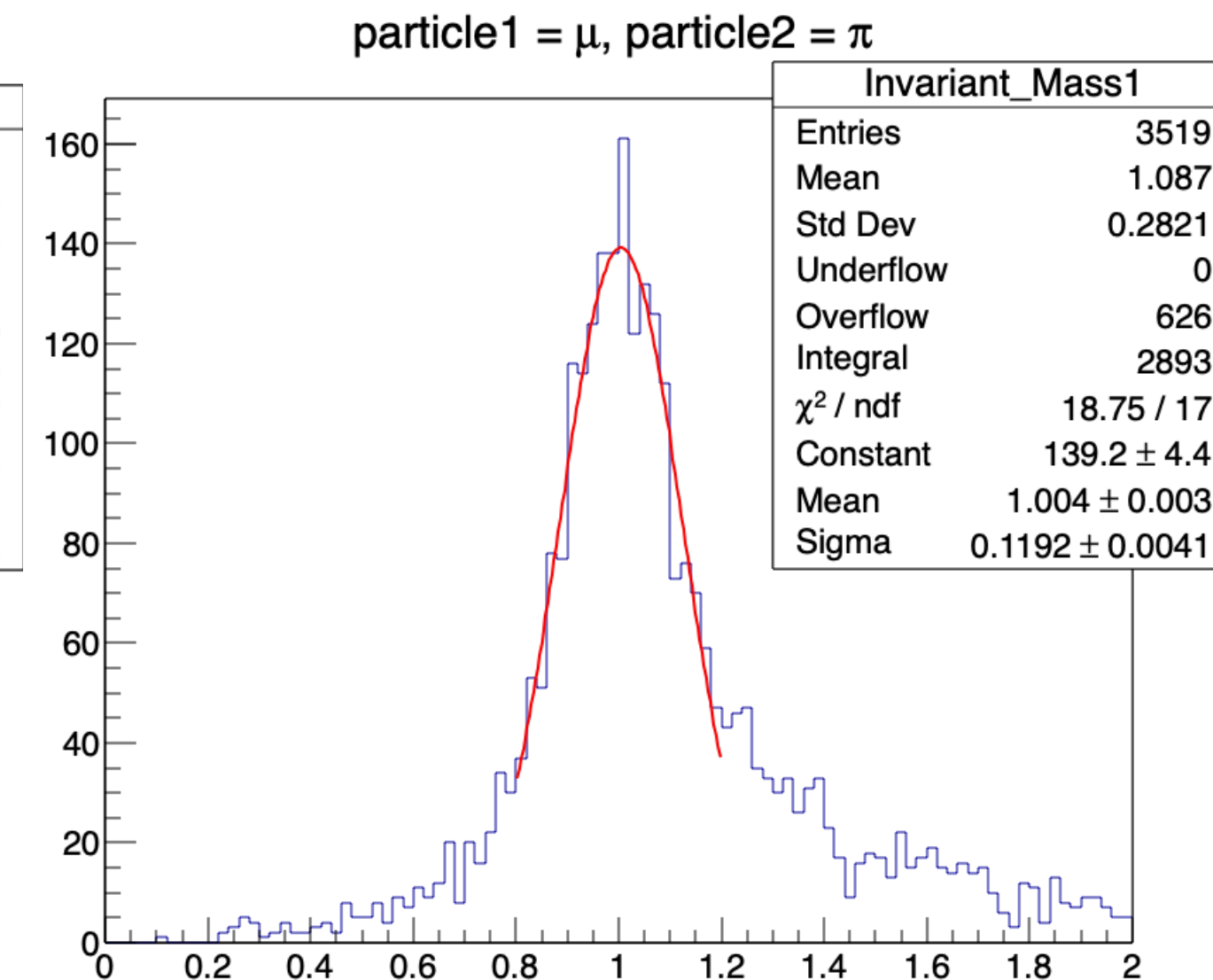
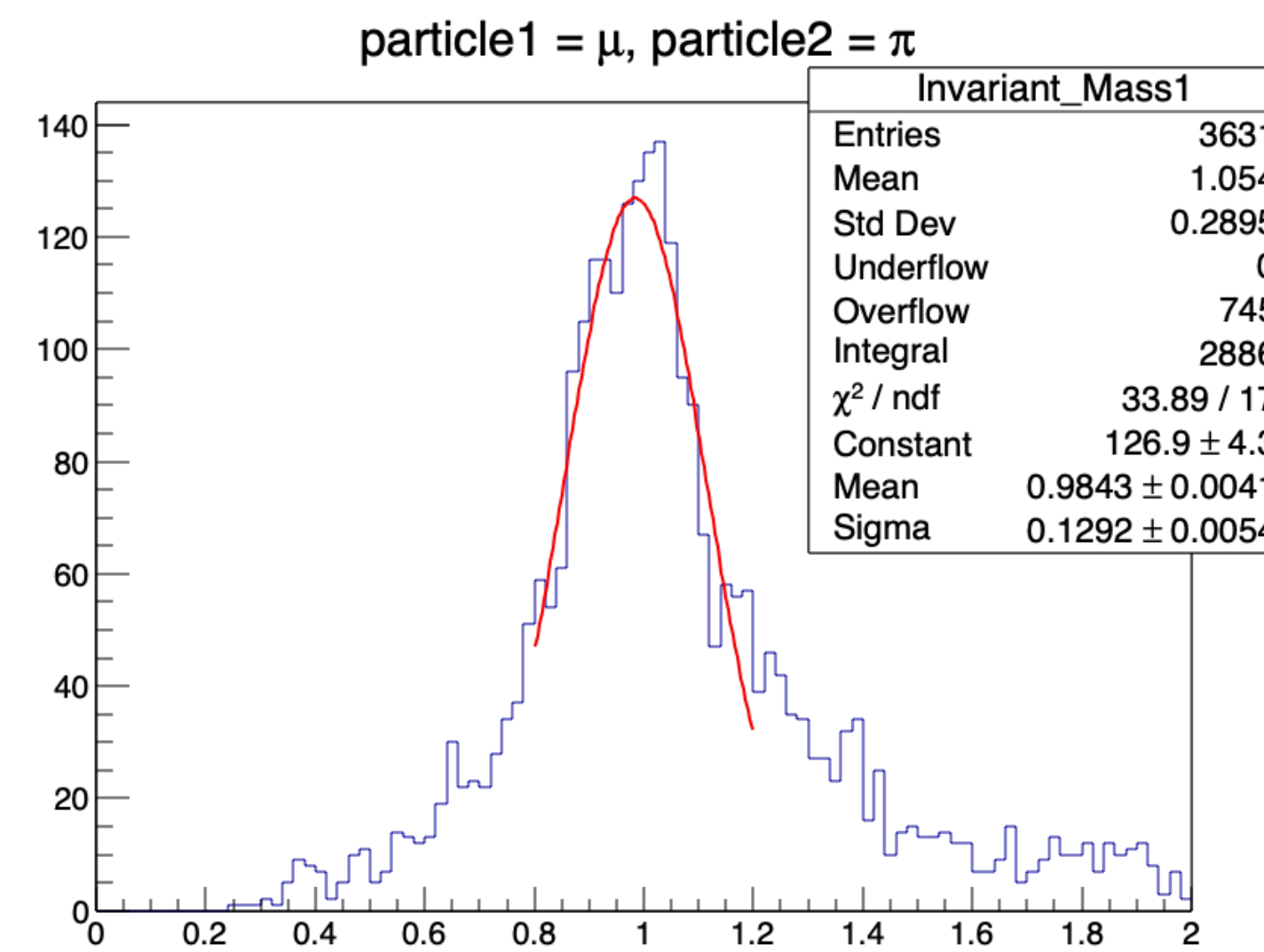
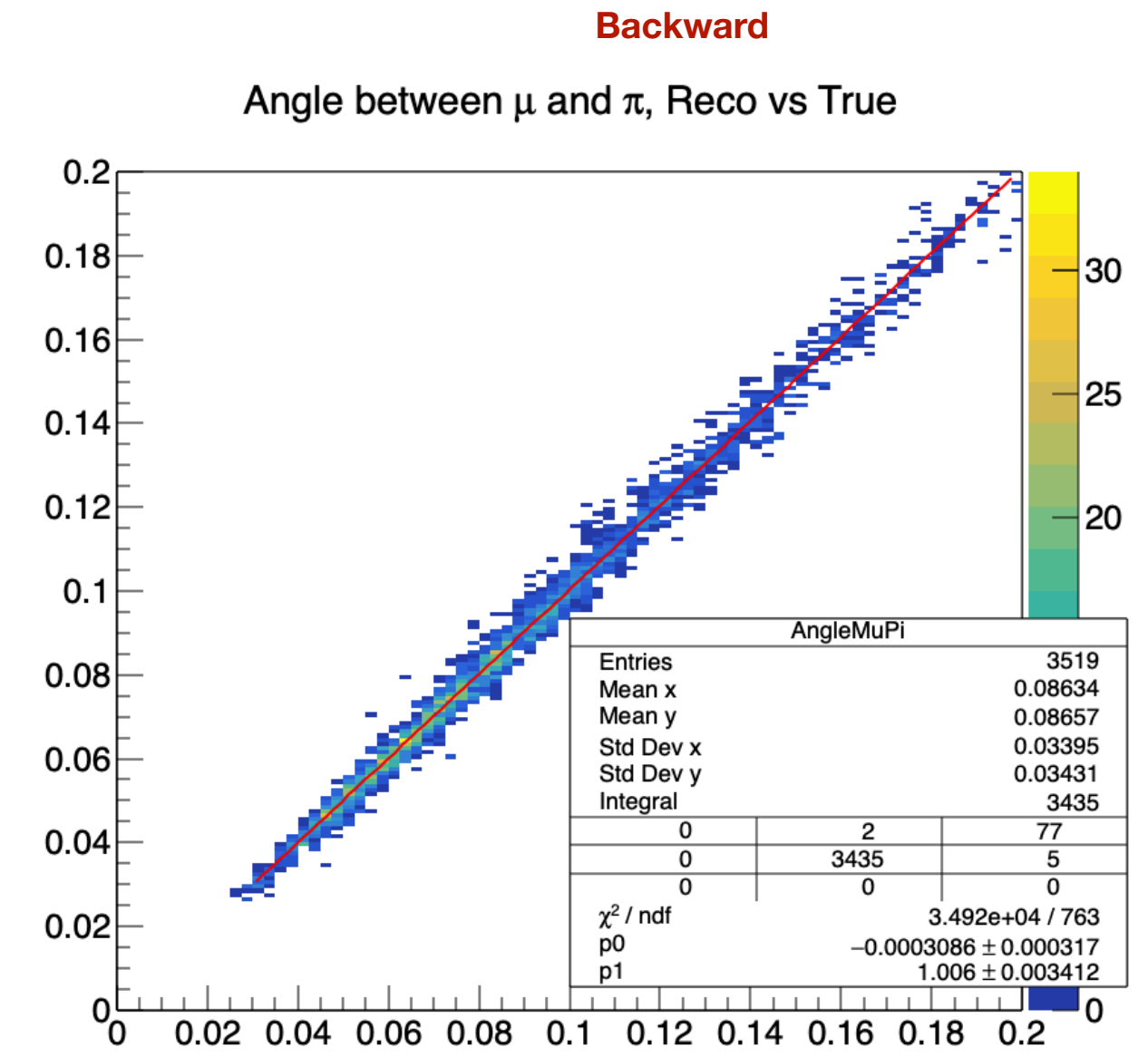
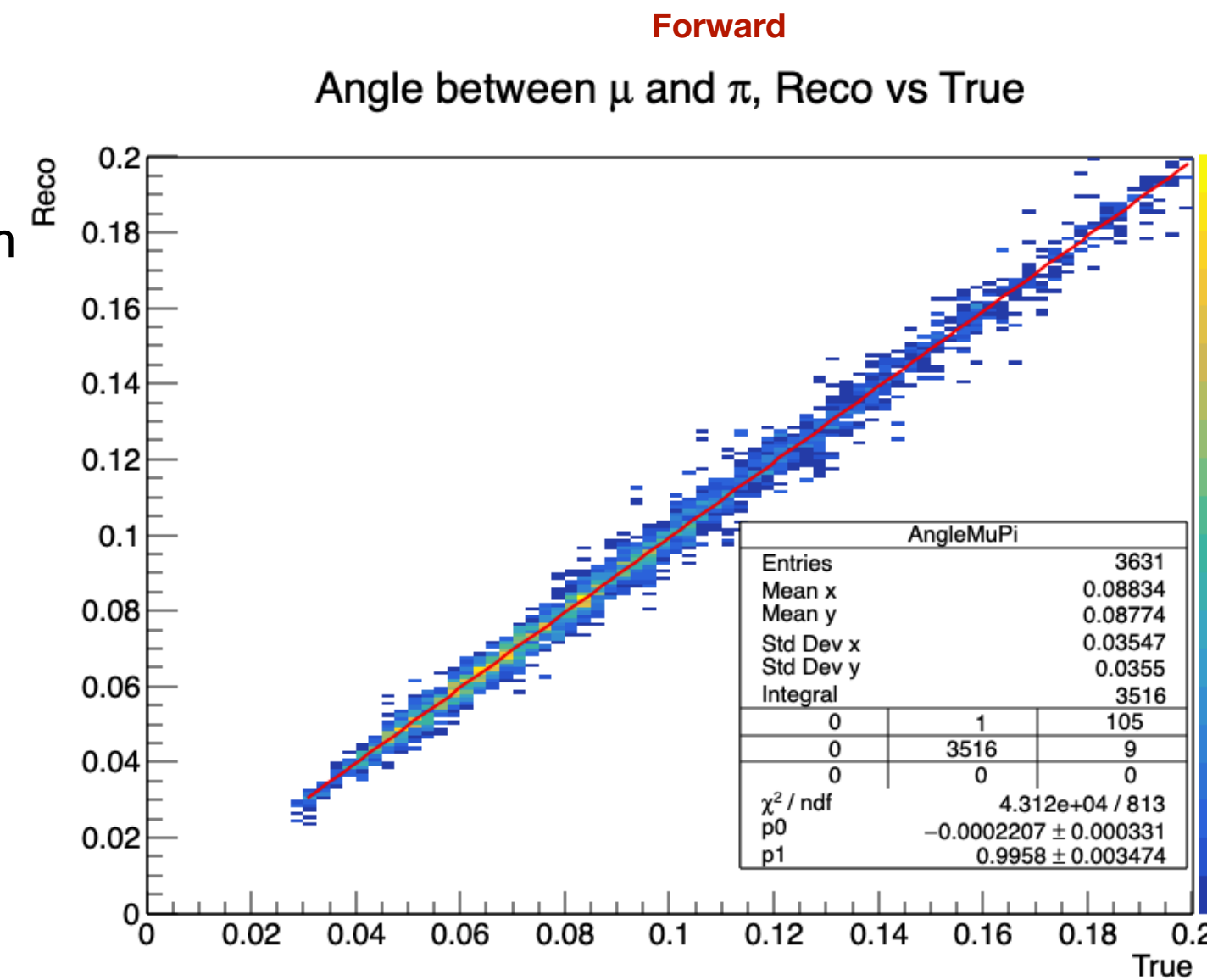
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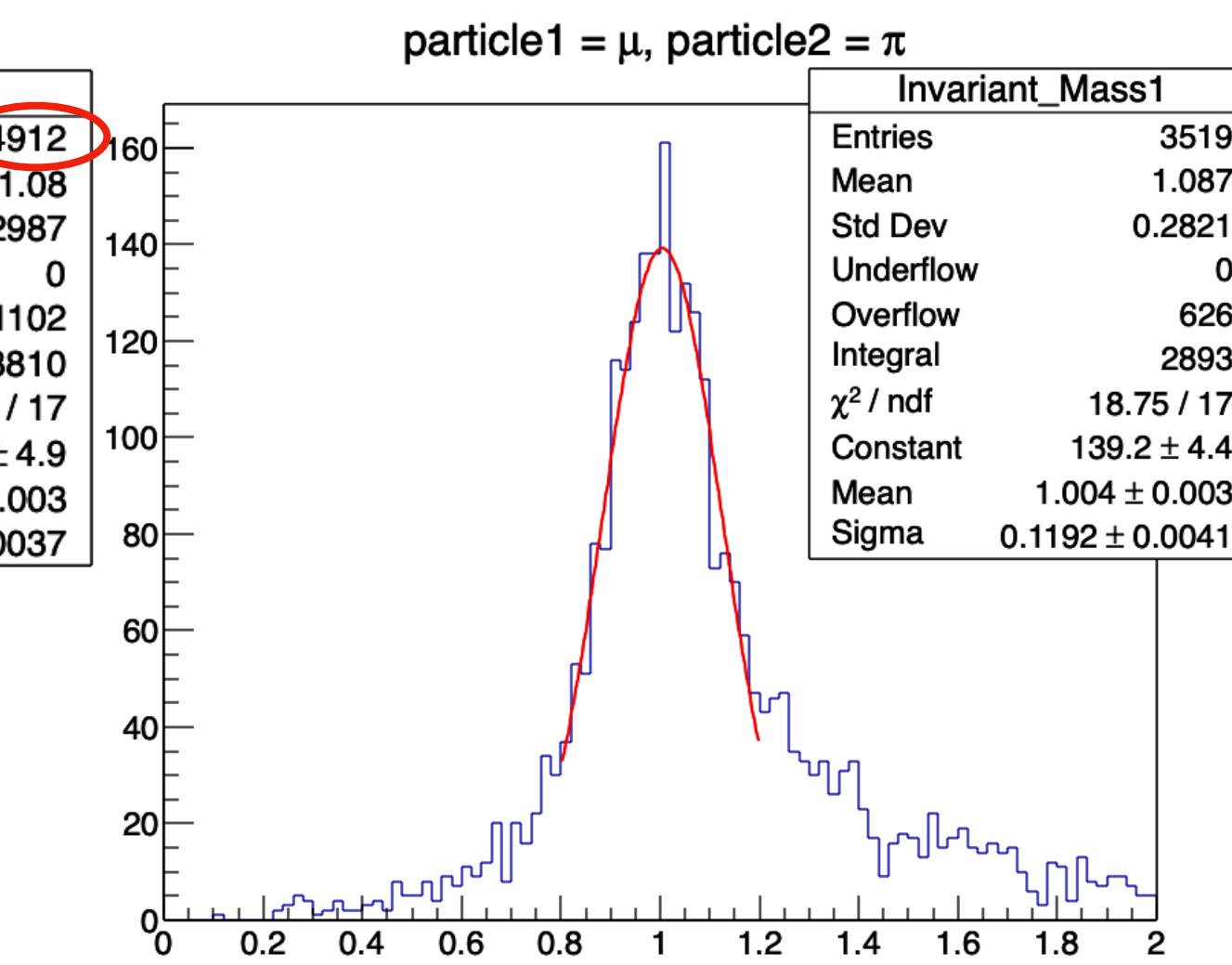
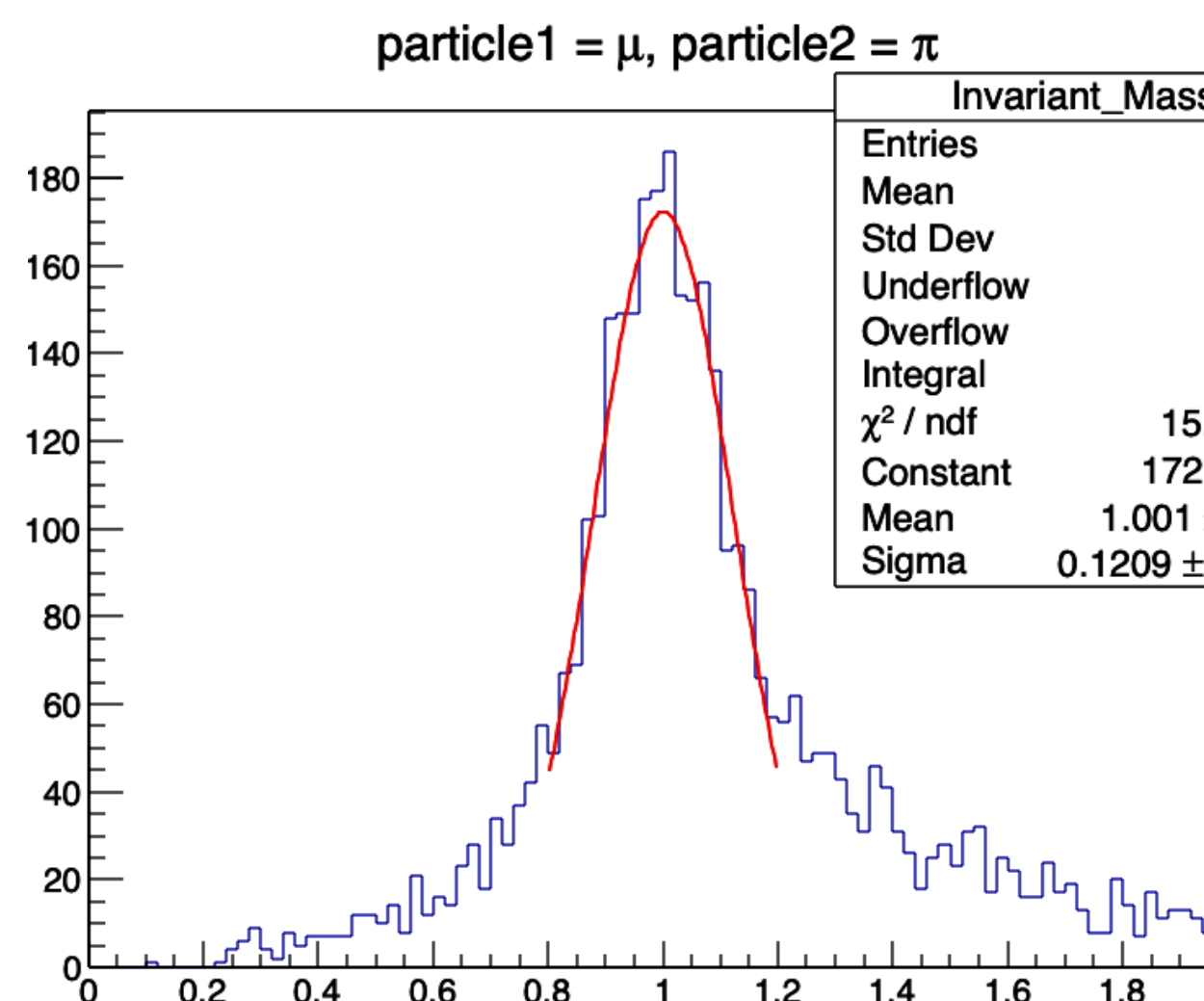
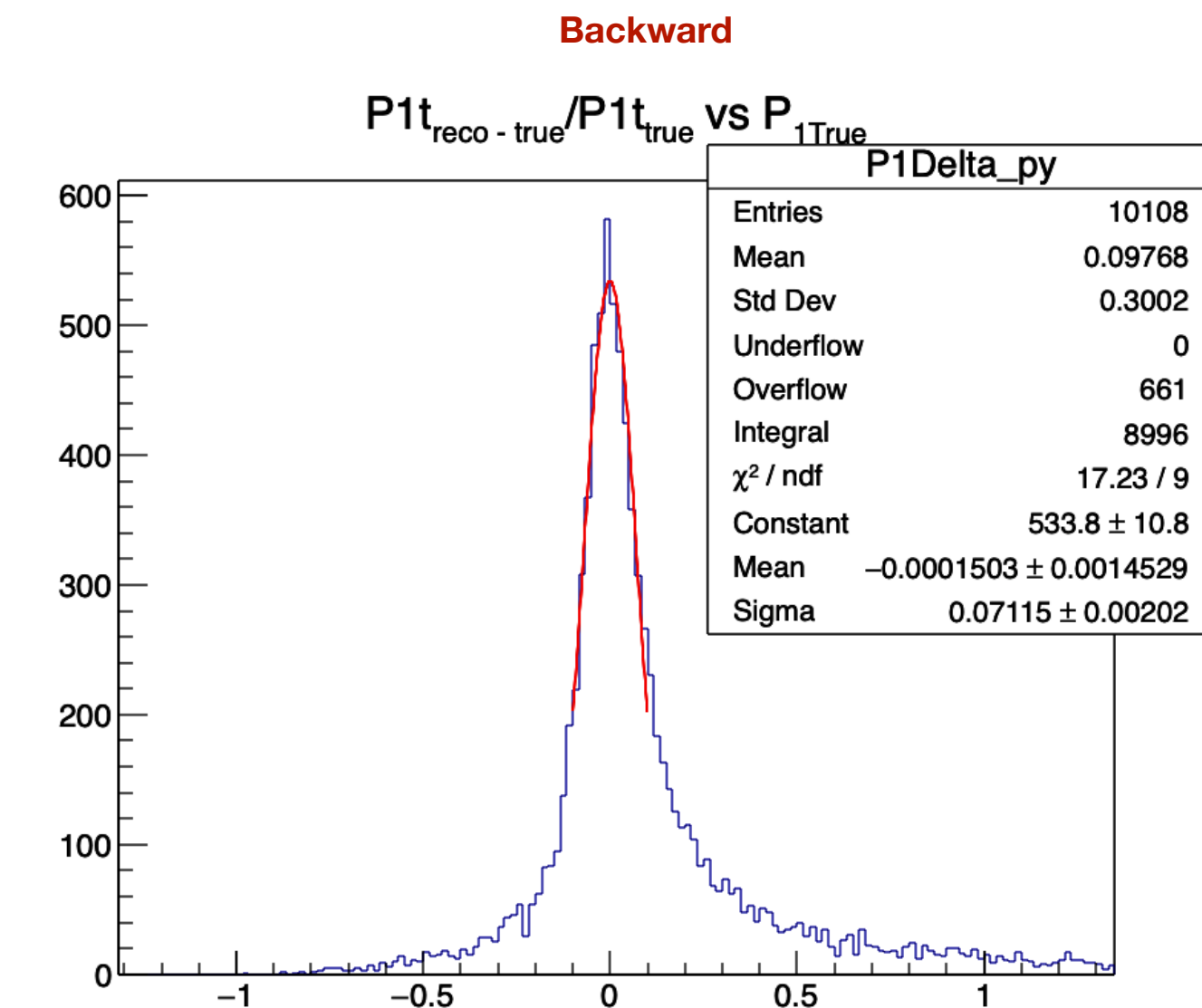
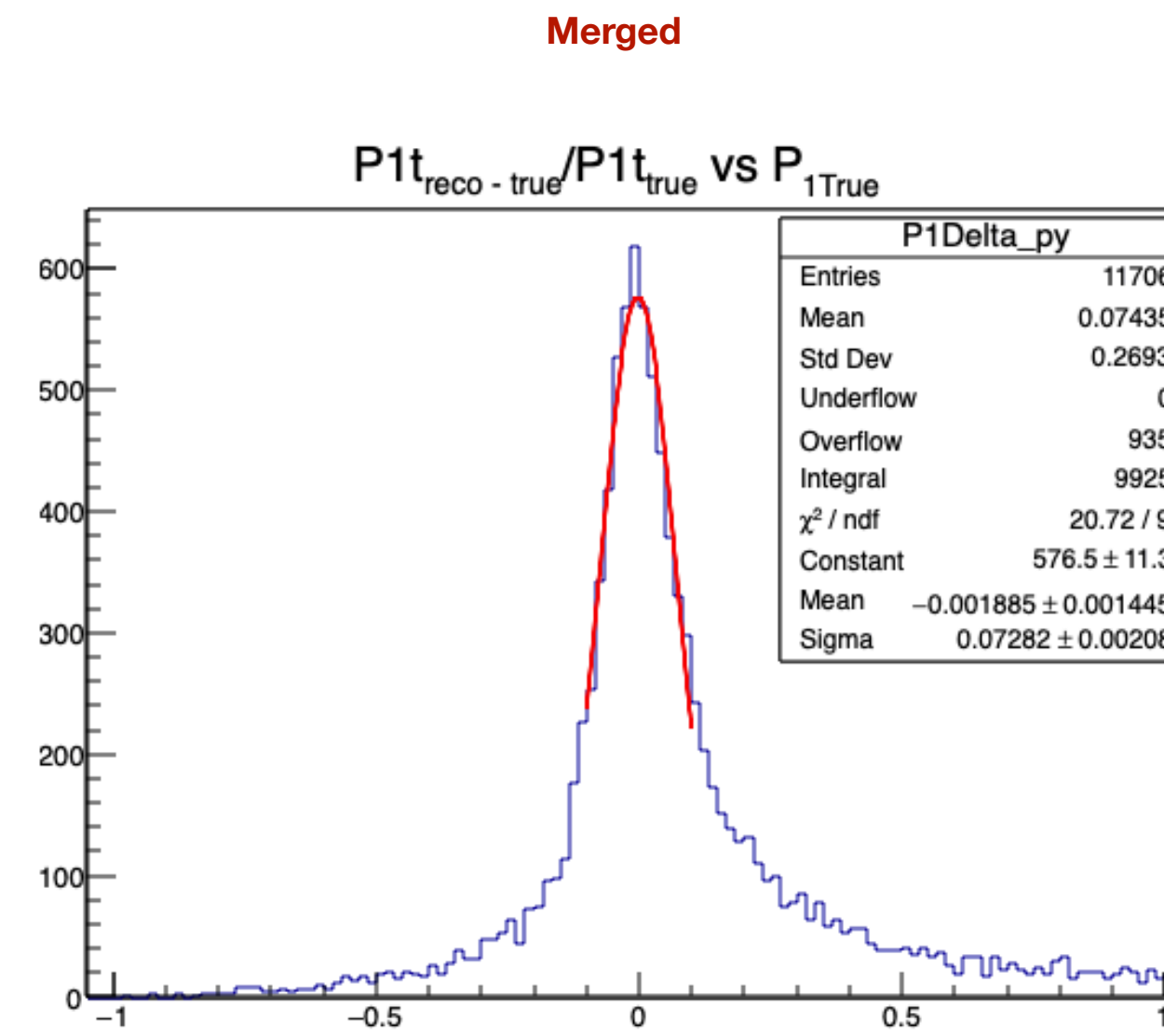
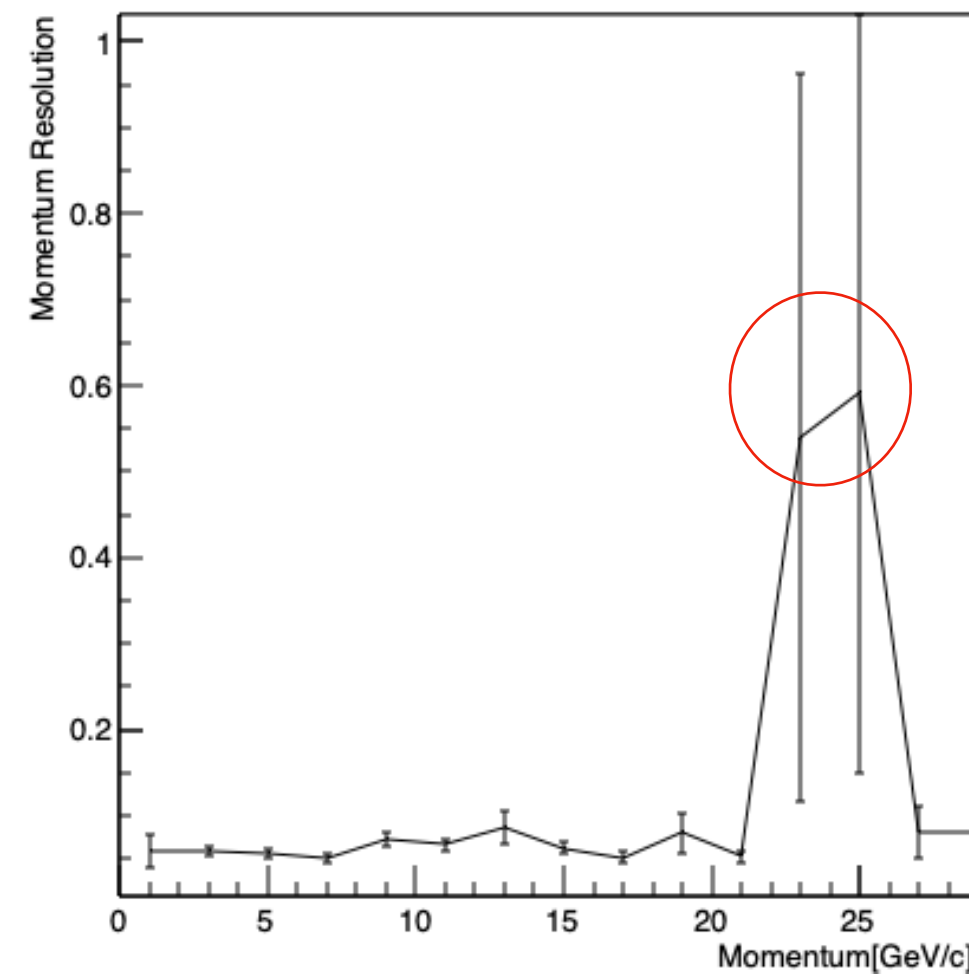
- Backward Kalman direction implemented, in this case backward is more efficient: the initial hit is found easier and more precise (MCS not messing with the hits much)
- Multiple scattering has been added (changing the resolution by 0.1%)
- For better precision, external helical fit has been used (hits are coming from Kalman Filter, the used fit is the external one)

* Items to have an eye on:

- Invariant Mass resolution
- Momentum resolution
- Kalman Filter parameters (Pull plots)
- Goodness of the fit (χ^2)

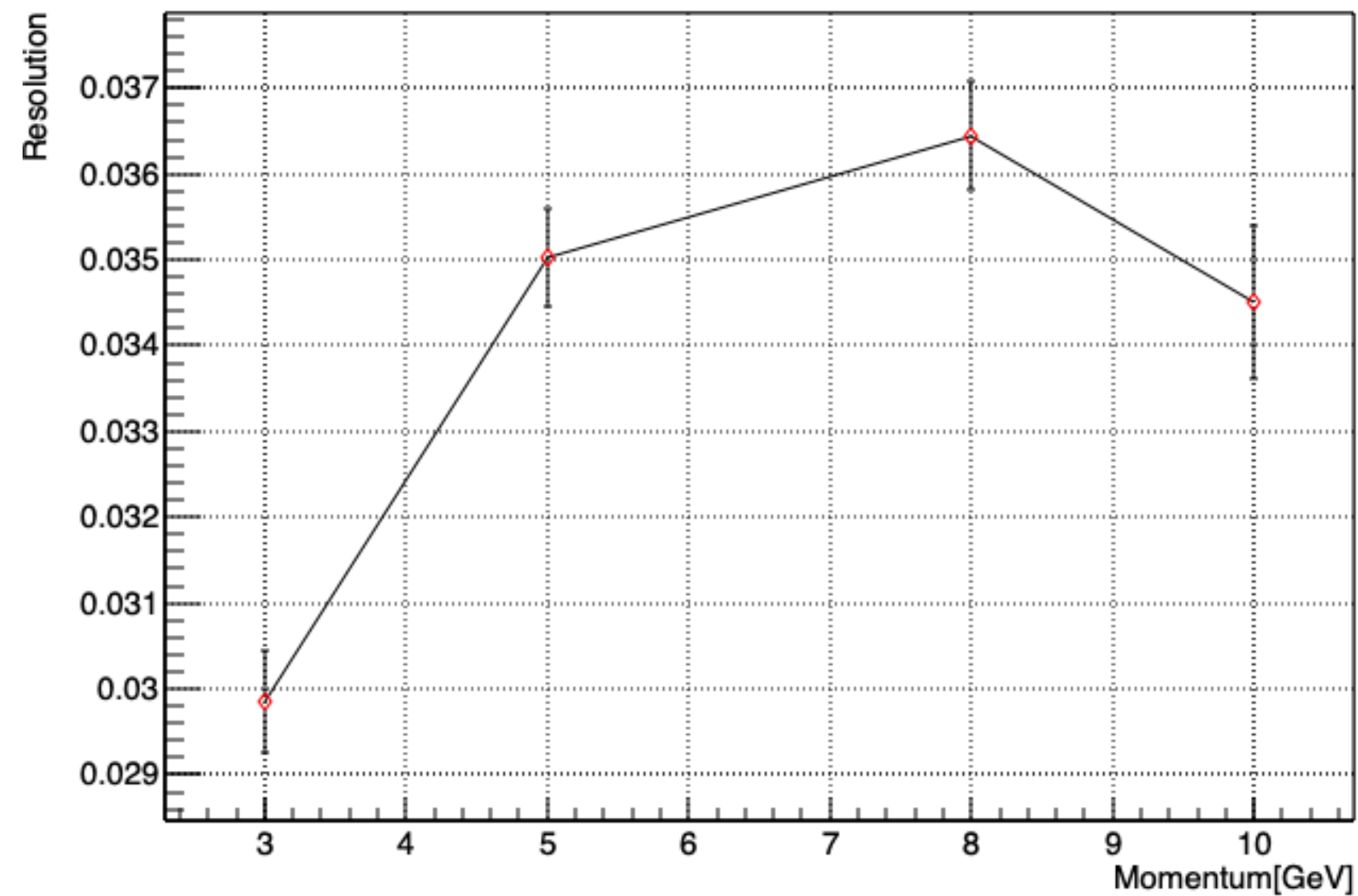
* Procedure:

- Kalman Filter
 - Forward/Backward Kalman and smoothing.
 - External helical fit.
 - Reco tracks:
 - A. Choosing either forward or backward as Reco tracks.
 - B. Matching the forward/backward Reco tracks ($\geq 90\%$ shared hits), choosing the right combo for the final Reco track collection: Matched \rightarrow minChi2/Backward, Non-Matched \rightarrow Both
- Matching the Reco and the True.
- Momentum resolution, Invariant mass resolution, Pull plots

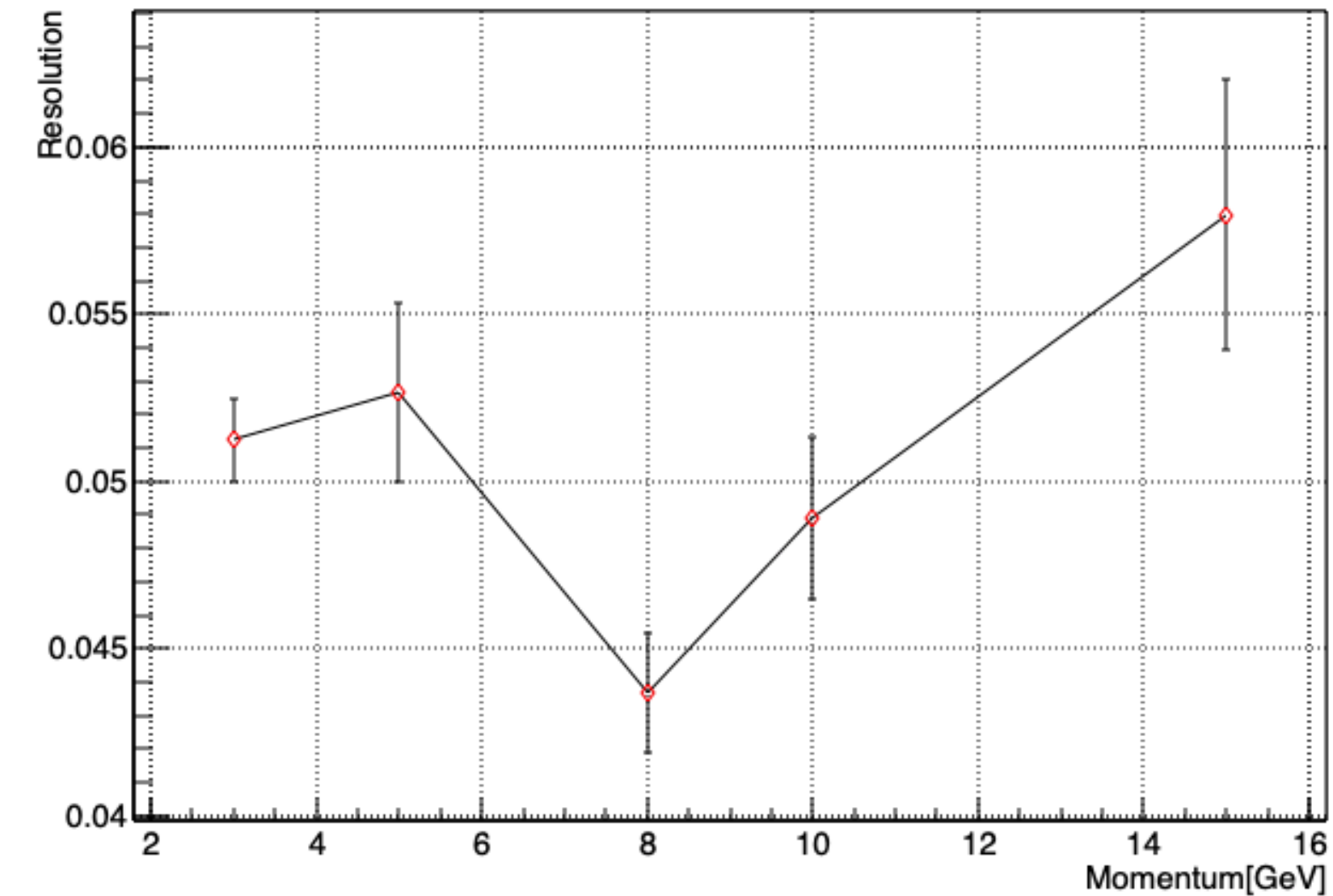


Kalman Filter, Resolution Comparison

Momentum Resolution, A



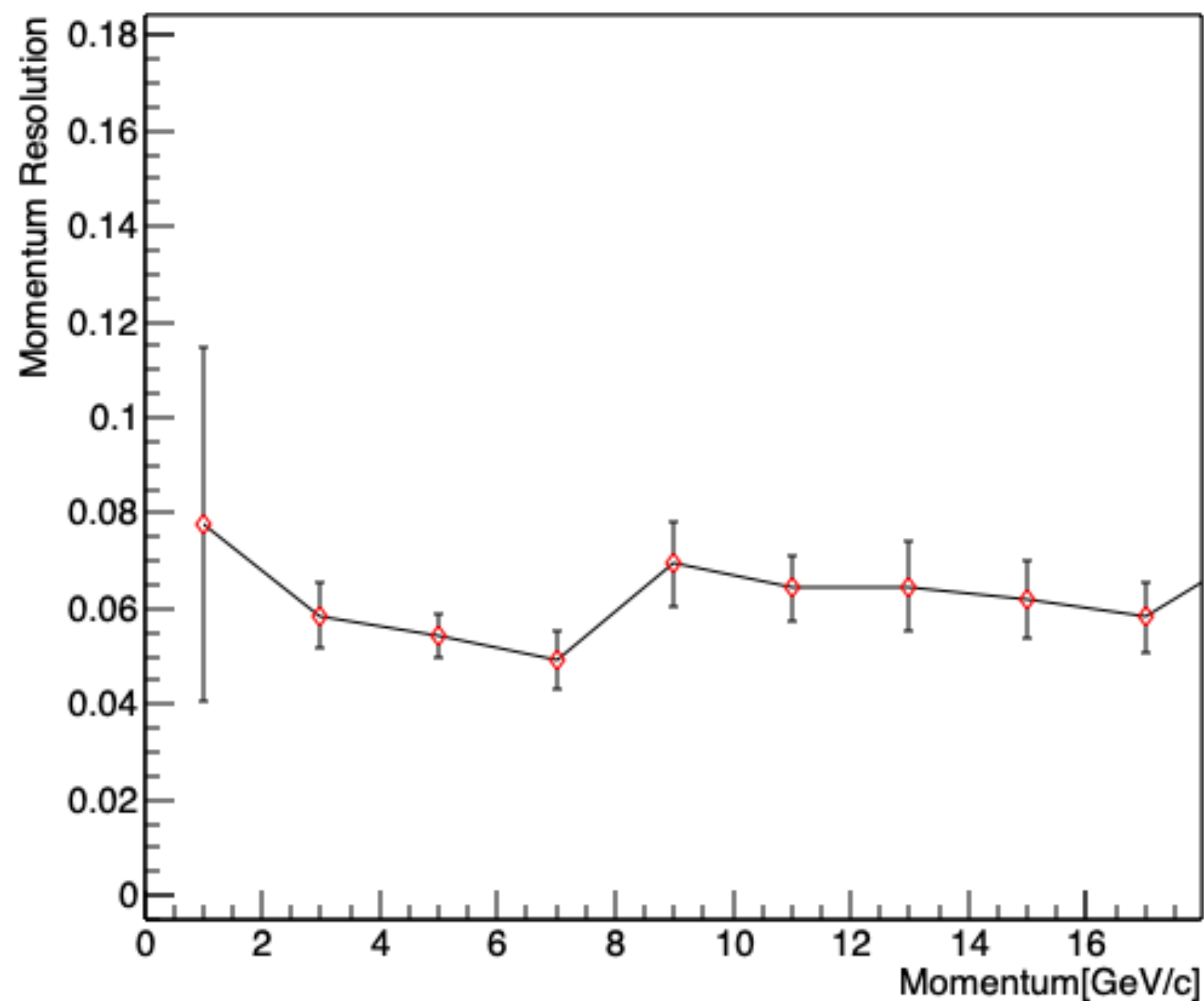
Momentum Resolution, B



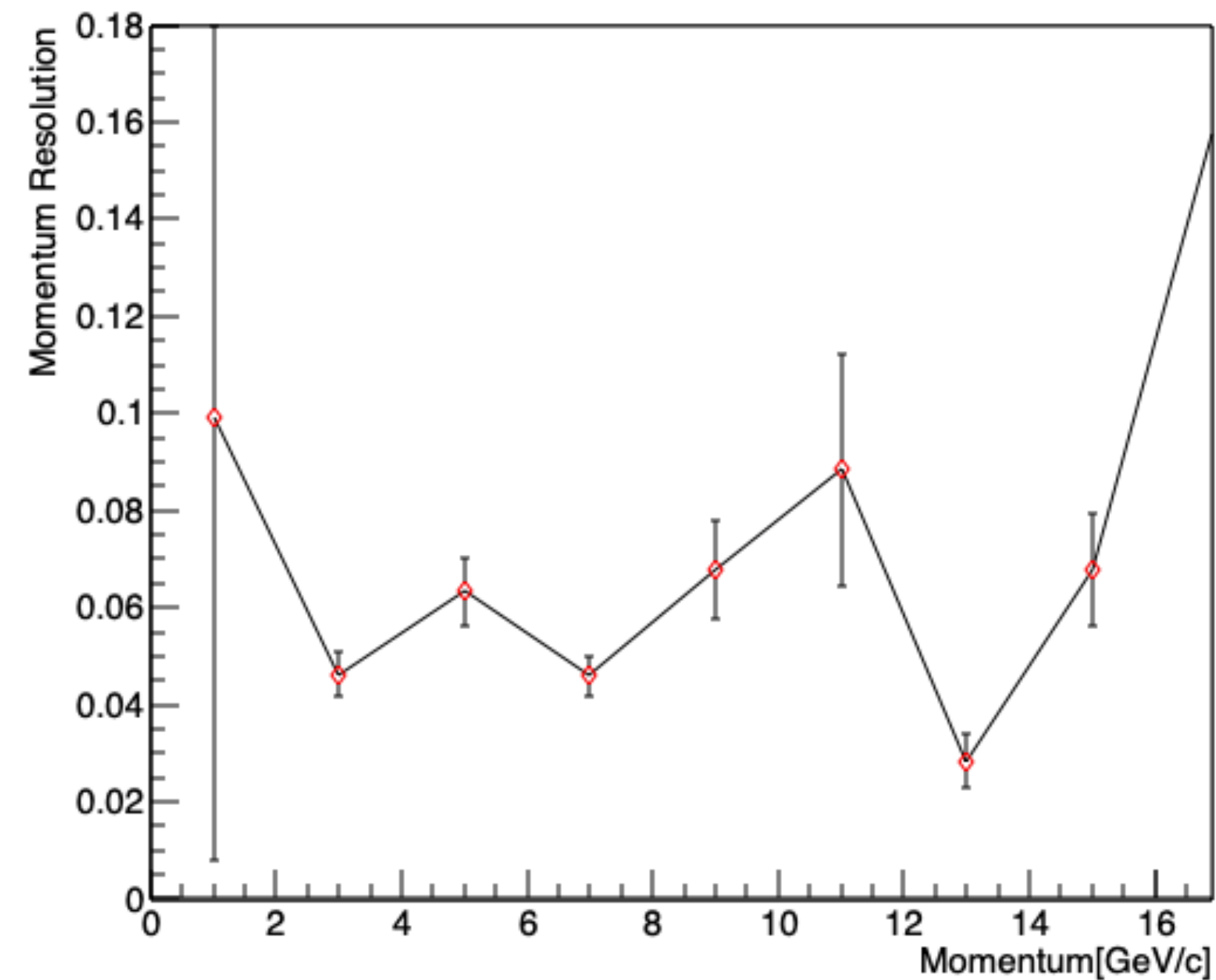
* Samples

- A. Monochromatic, simple single muon: fixed point, fixed direction (horizontal)
- B. My event-like muons (Cylindrical distribution, comparable angle to my sample)
- C. HNL sample

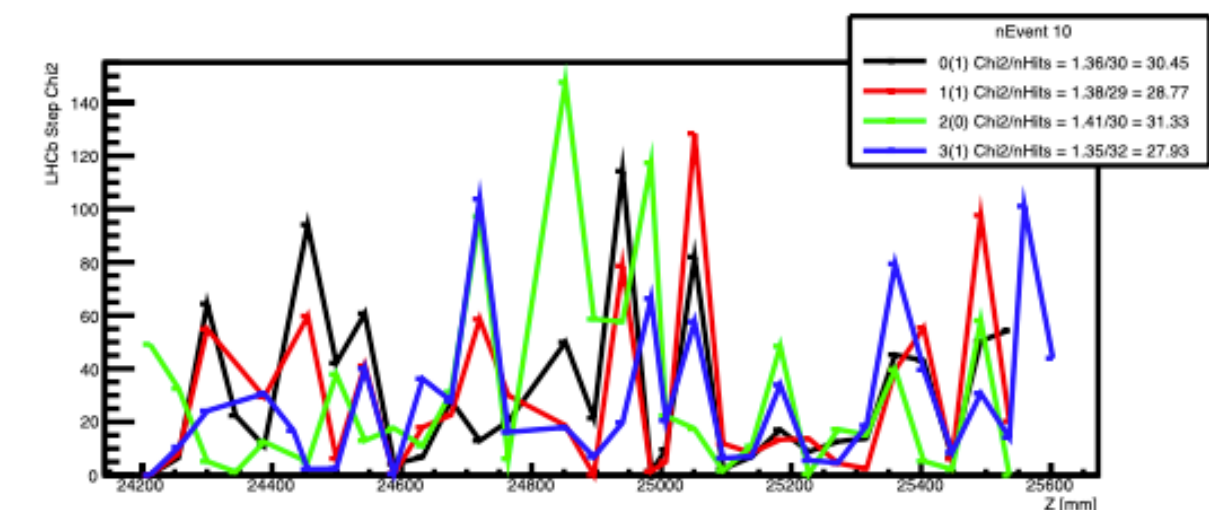
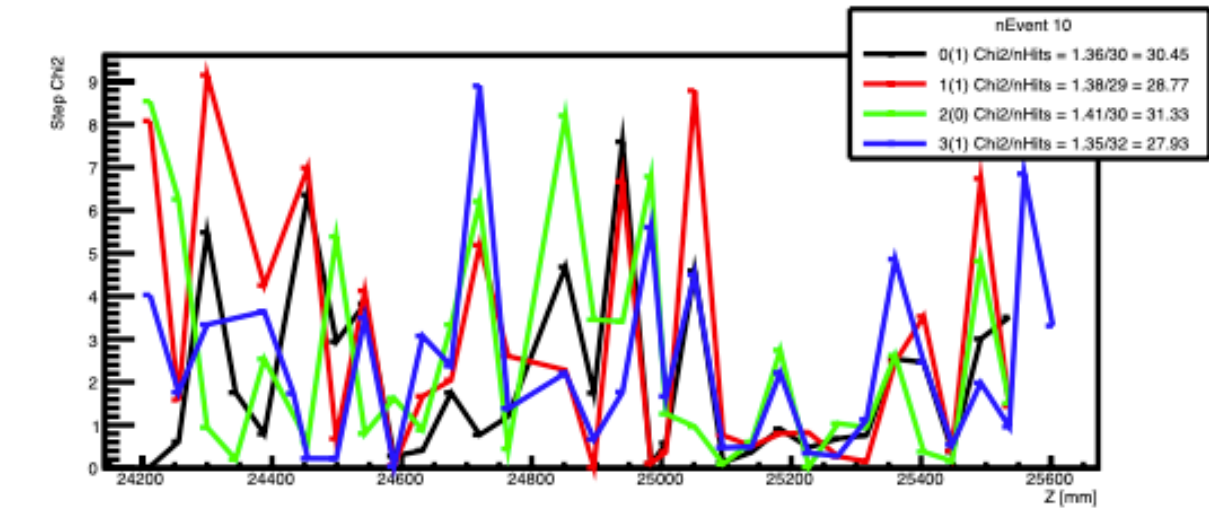
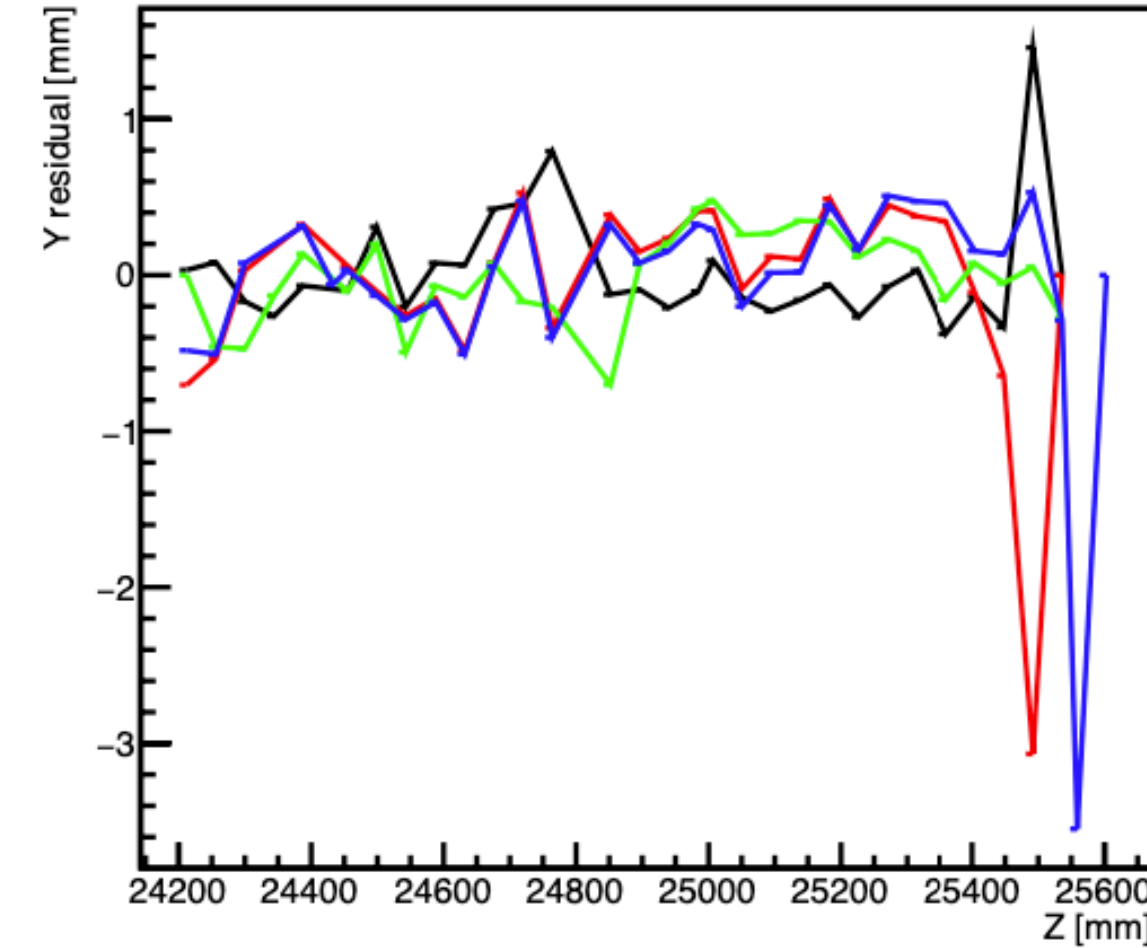
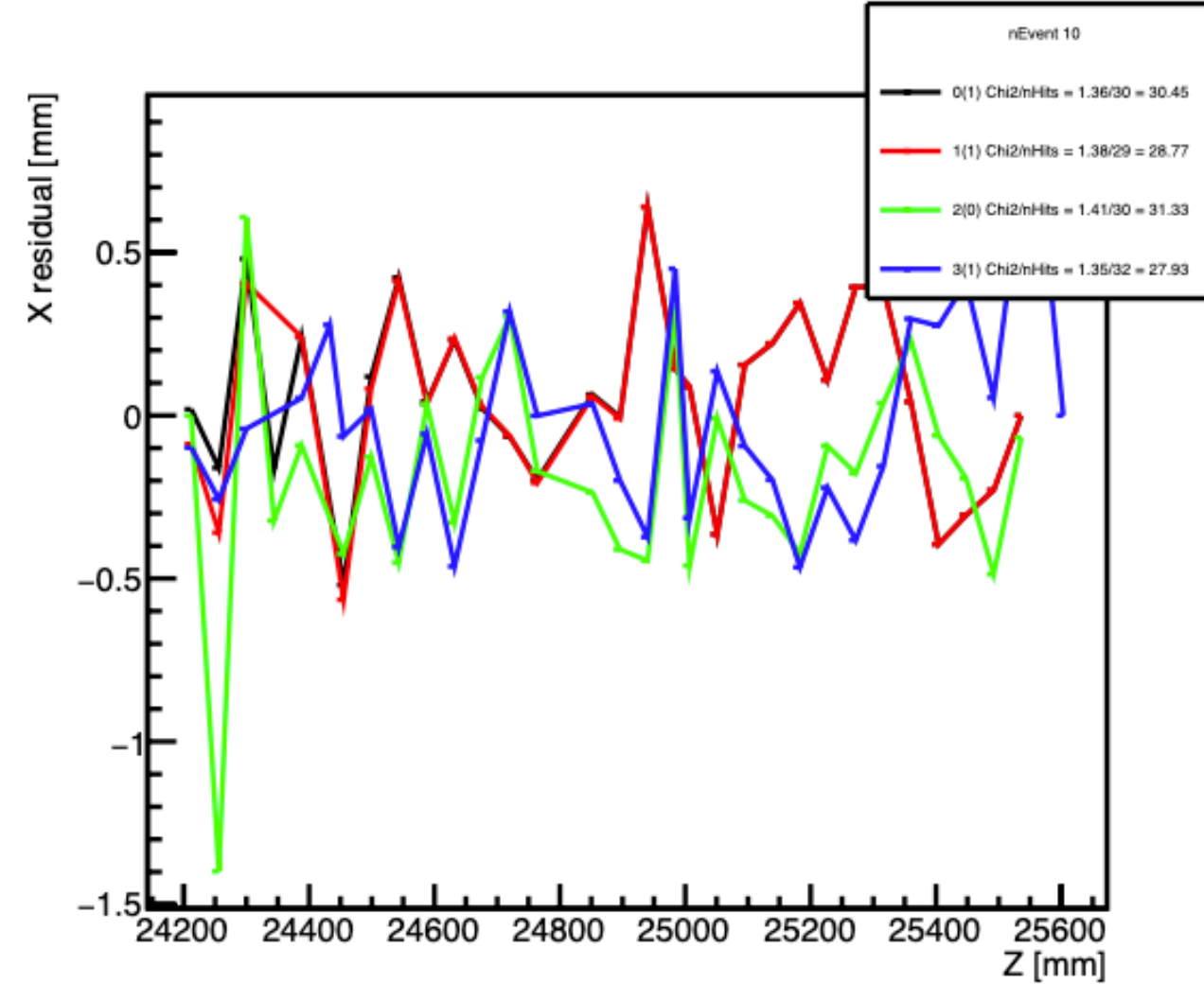
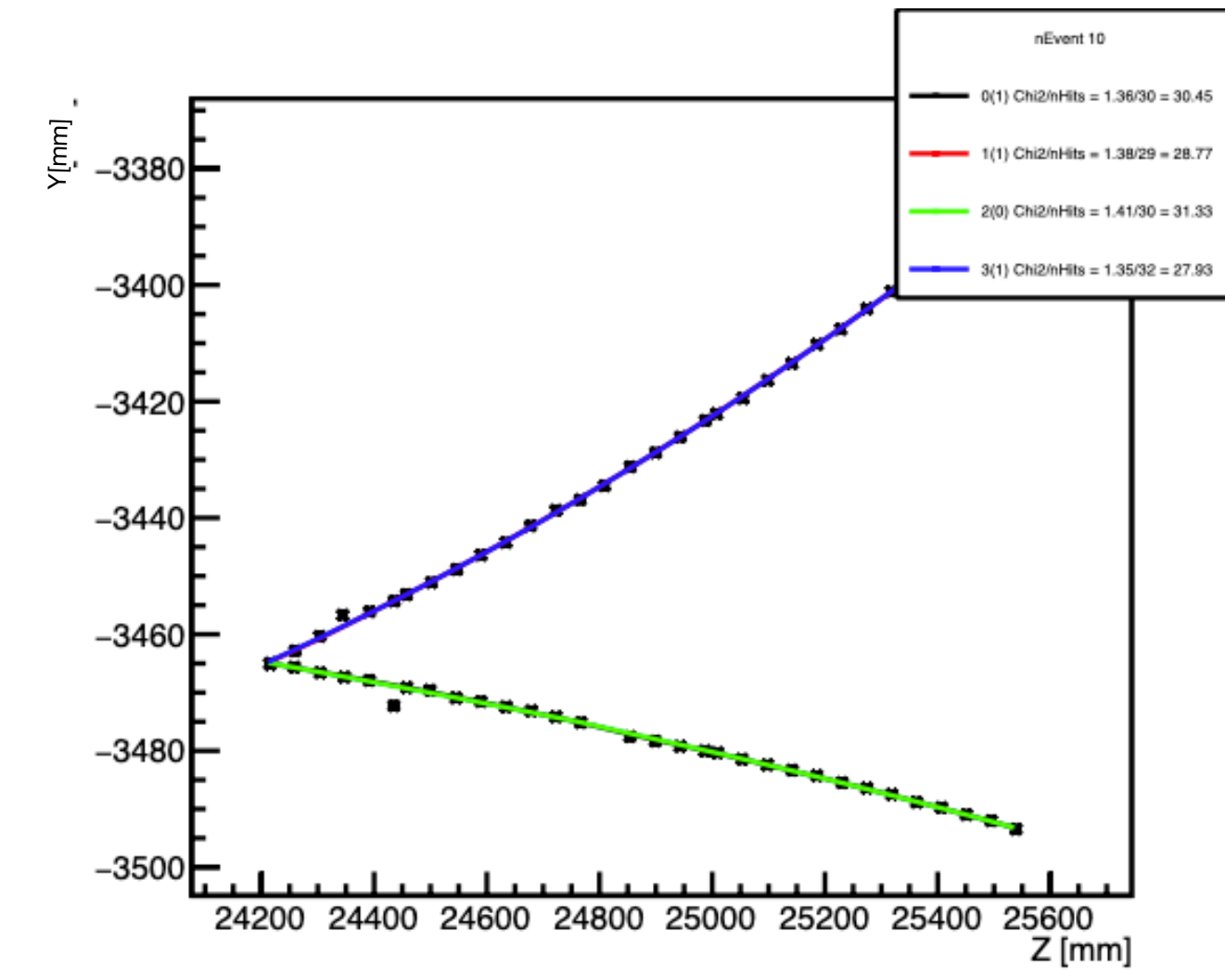
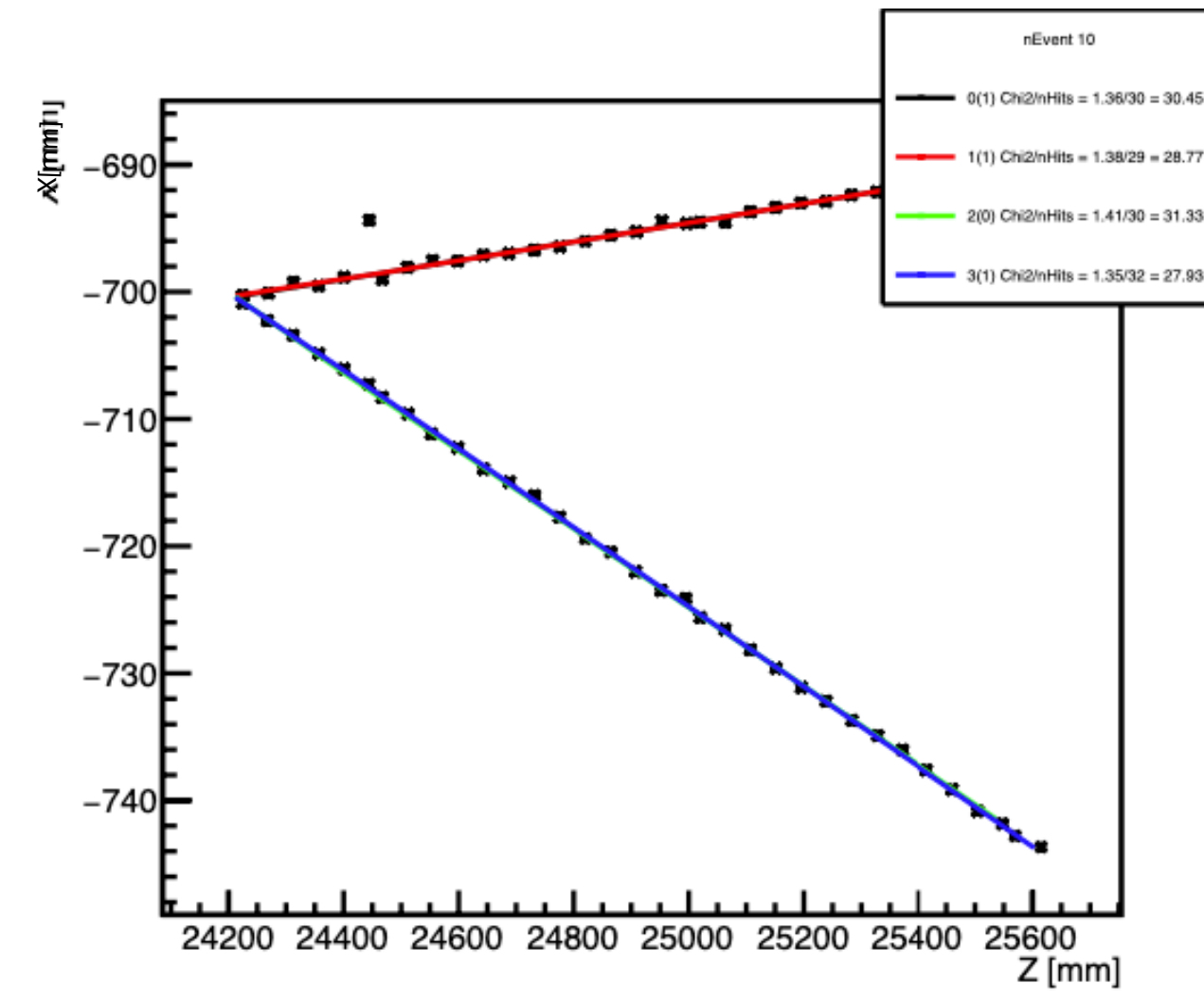
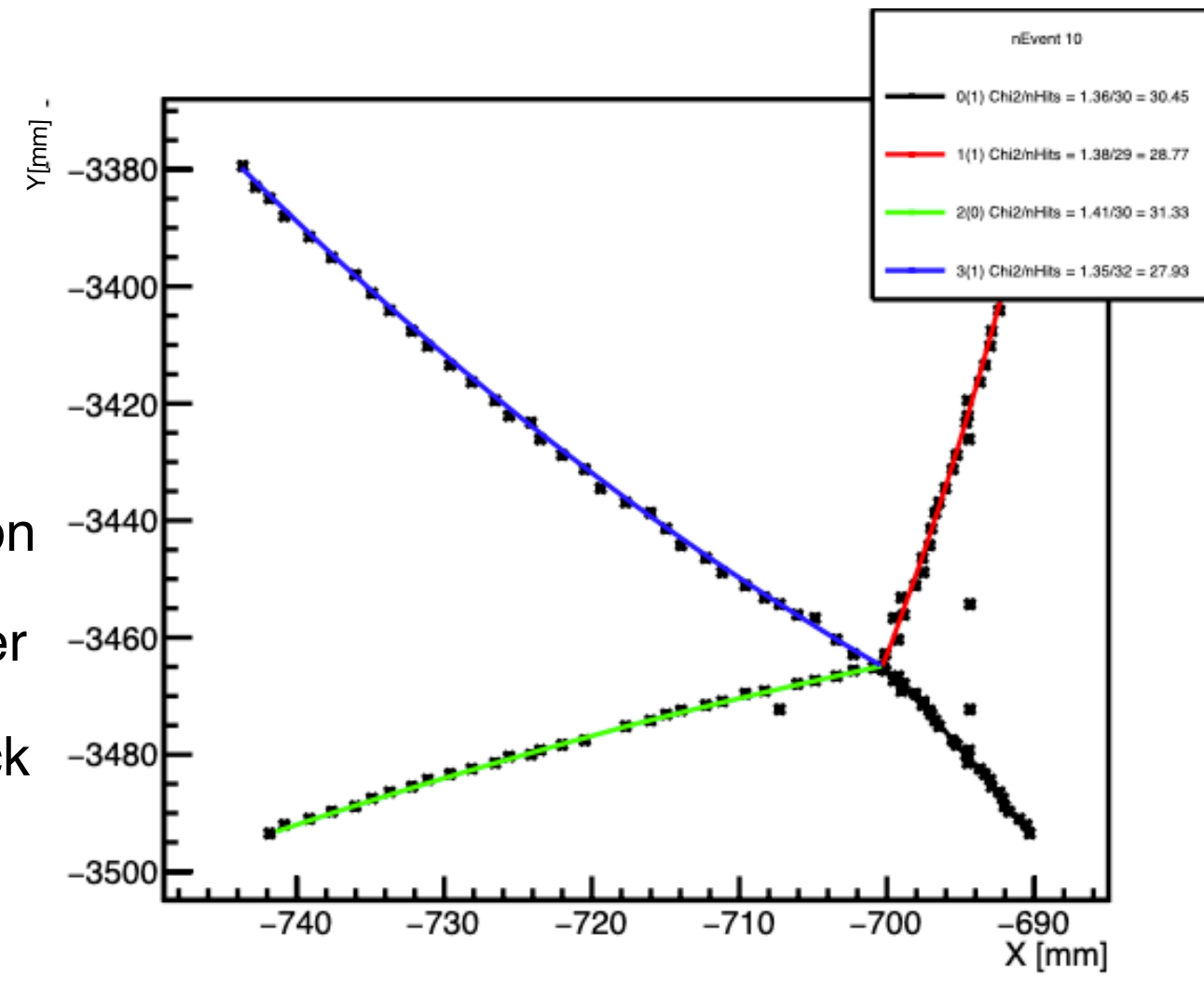
Momentum Resolution, Muon (Merged Kalman)



Momentum Resolution, Pion (Merged Kalman)



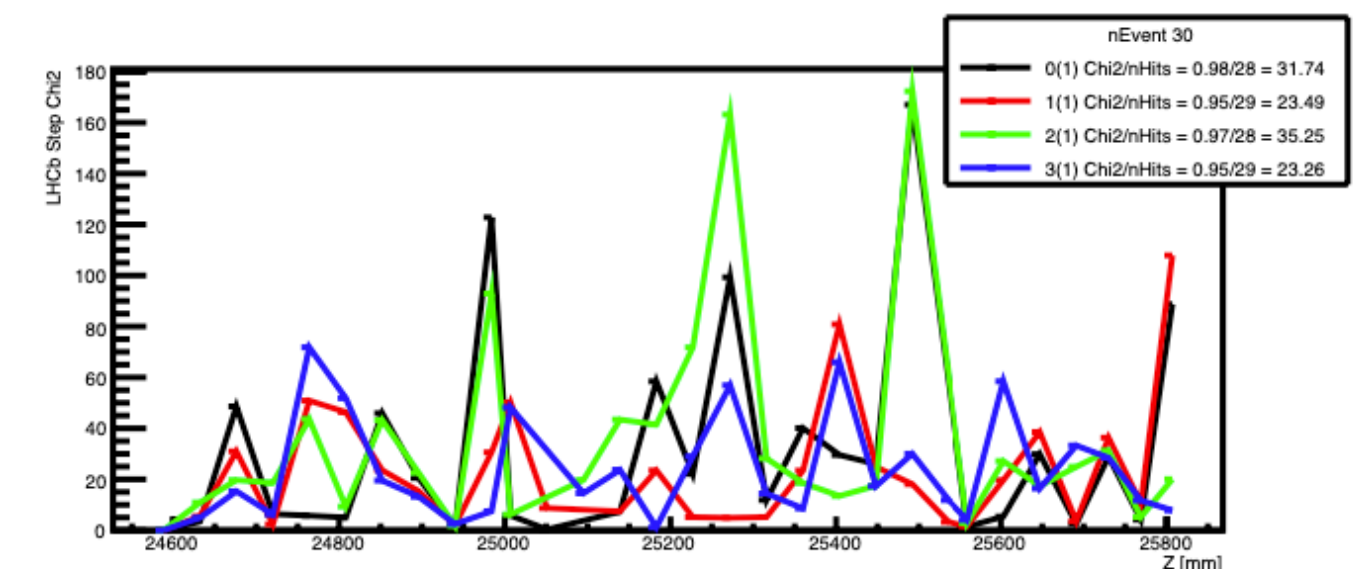
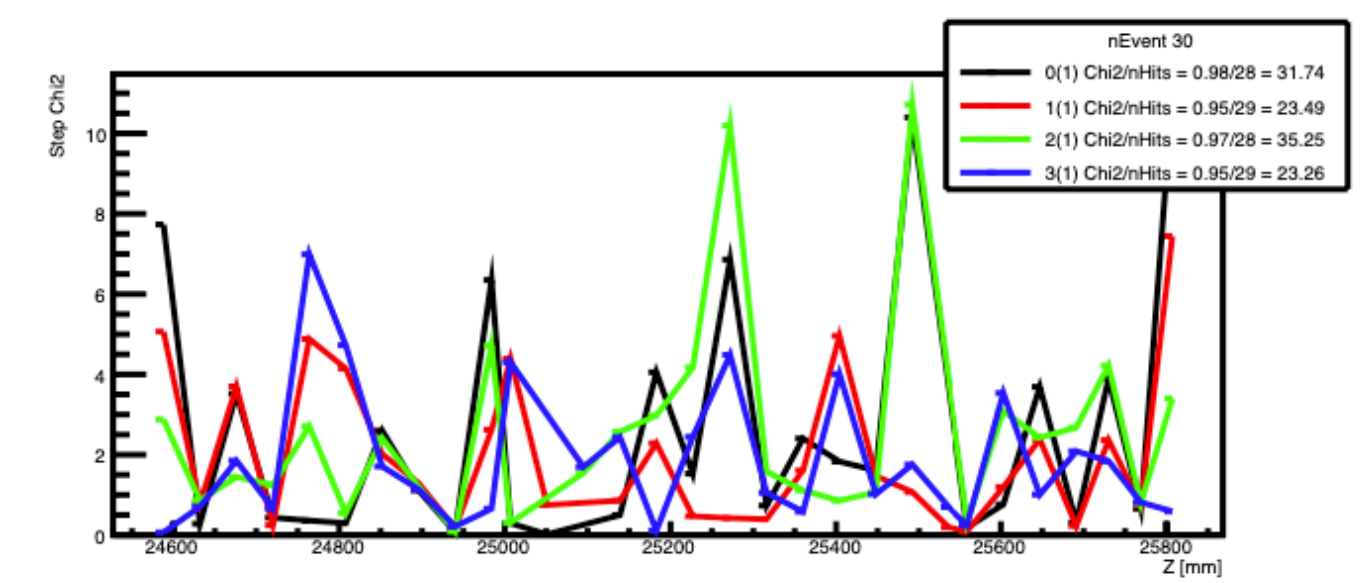
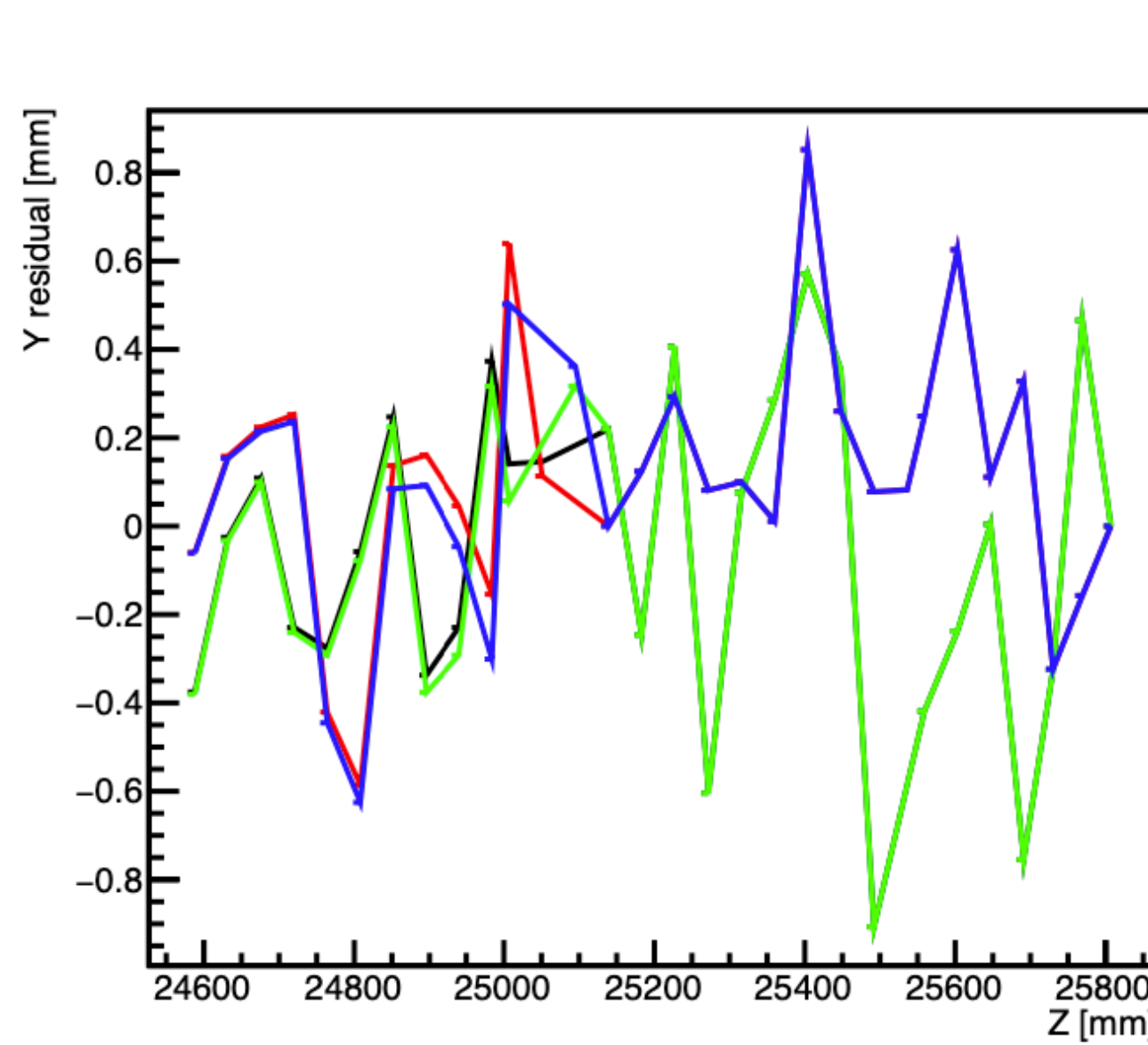
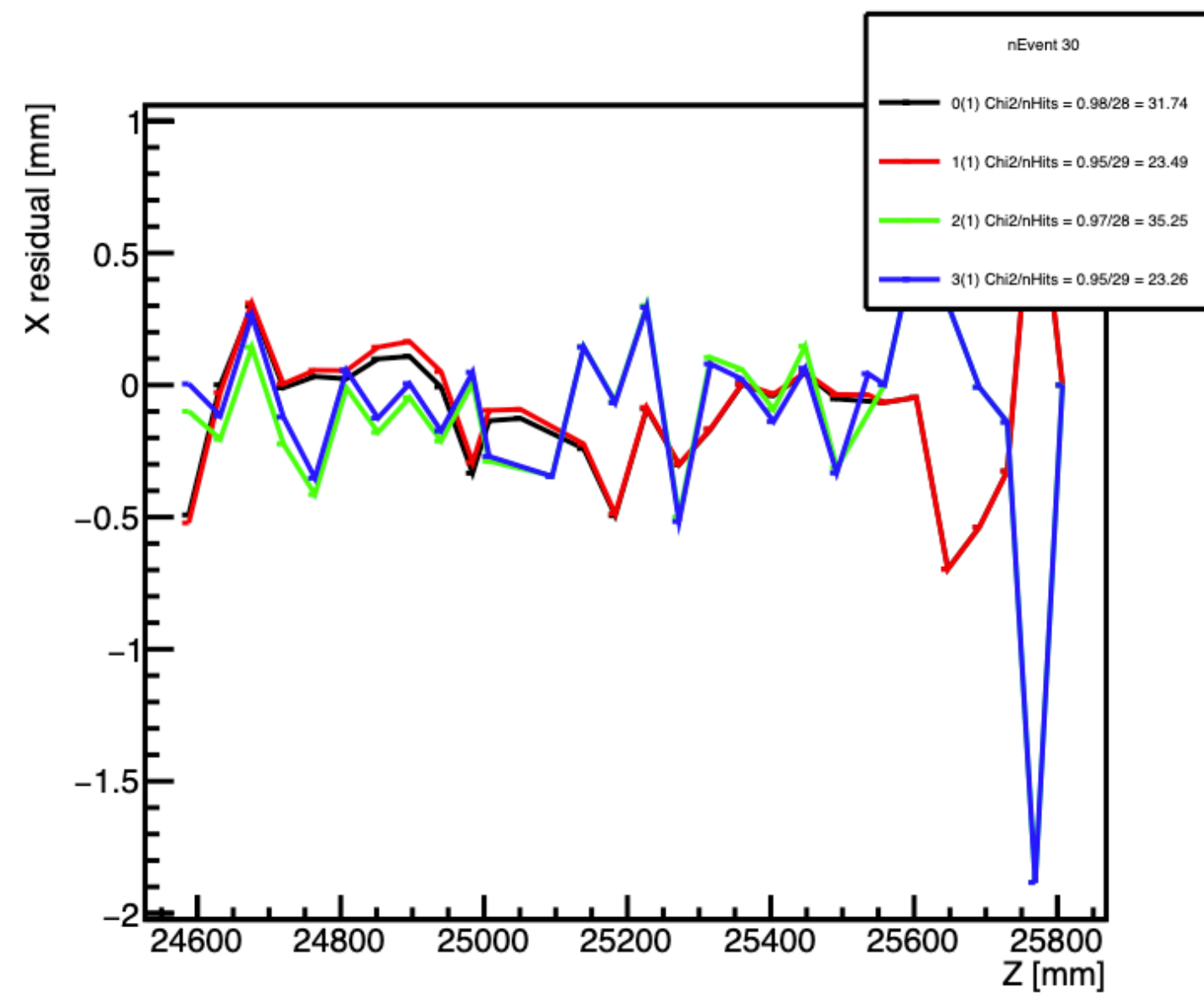
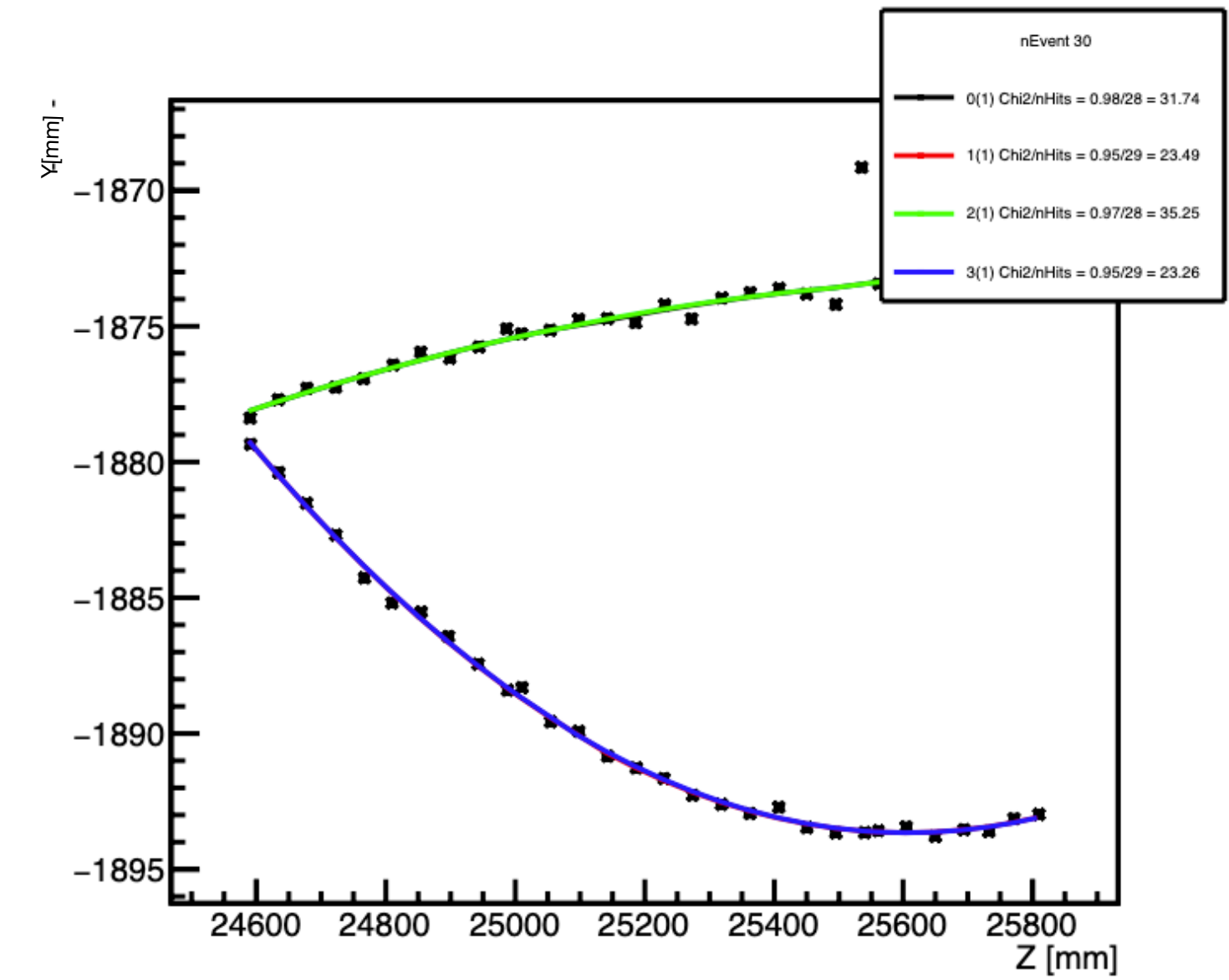
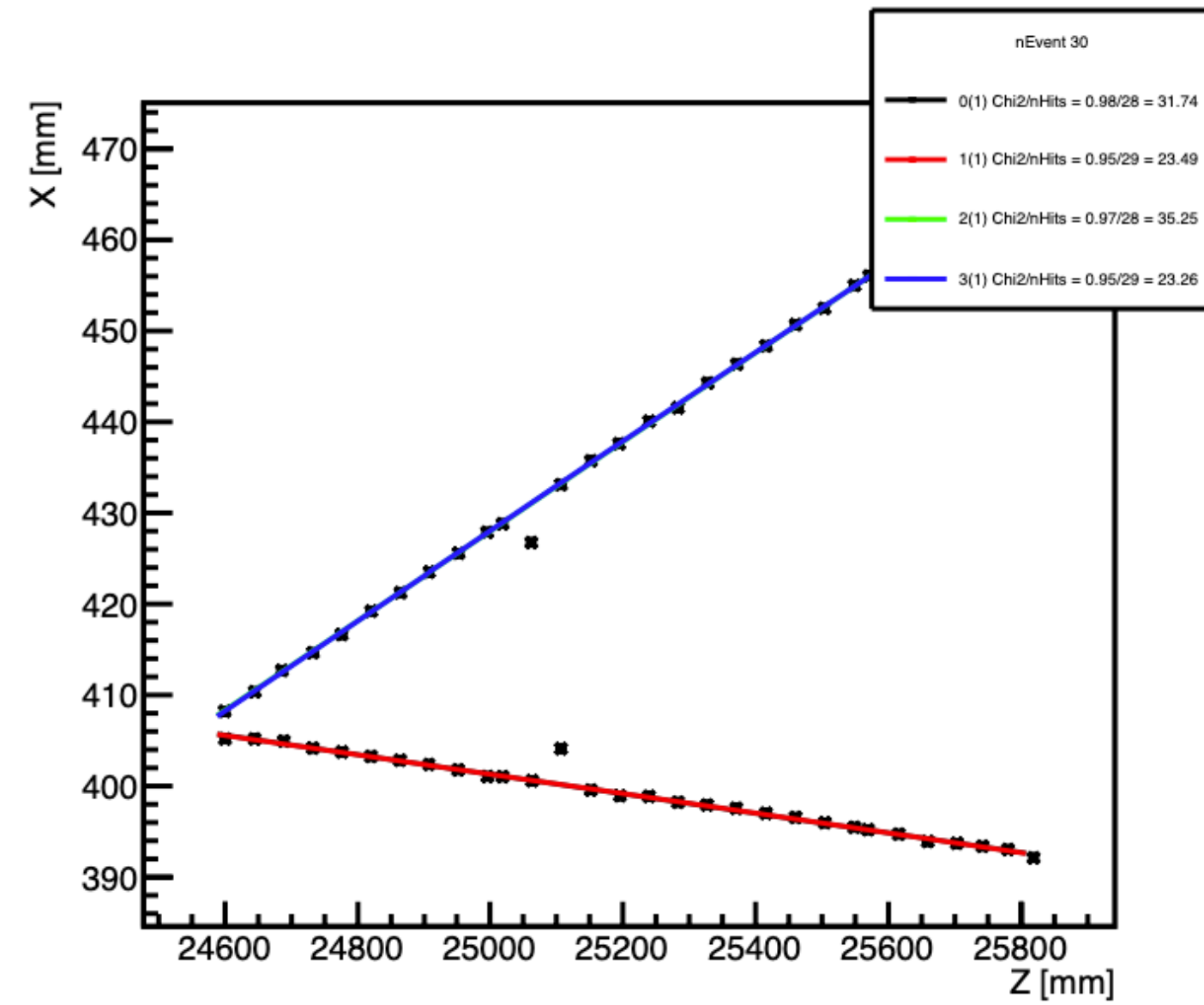
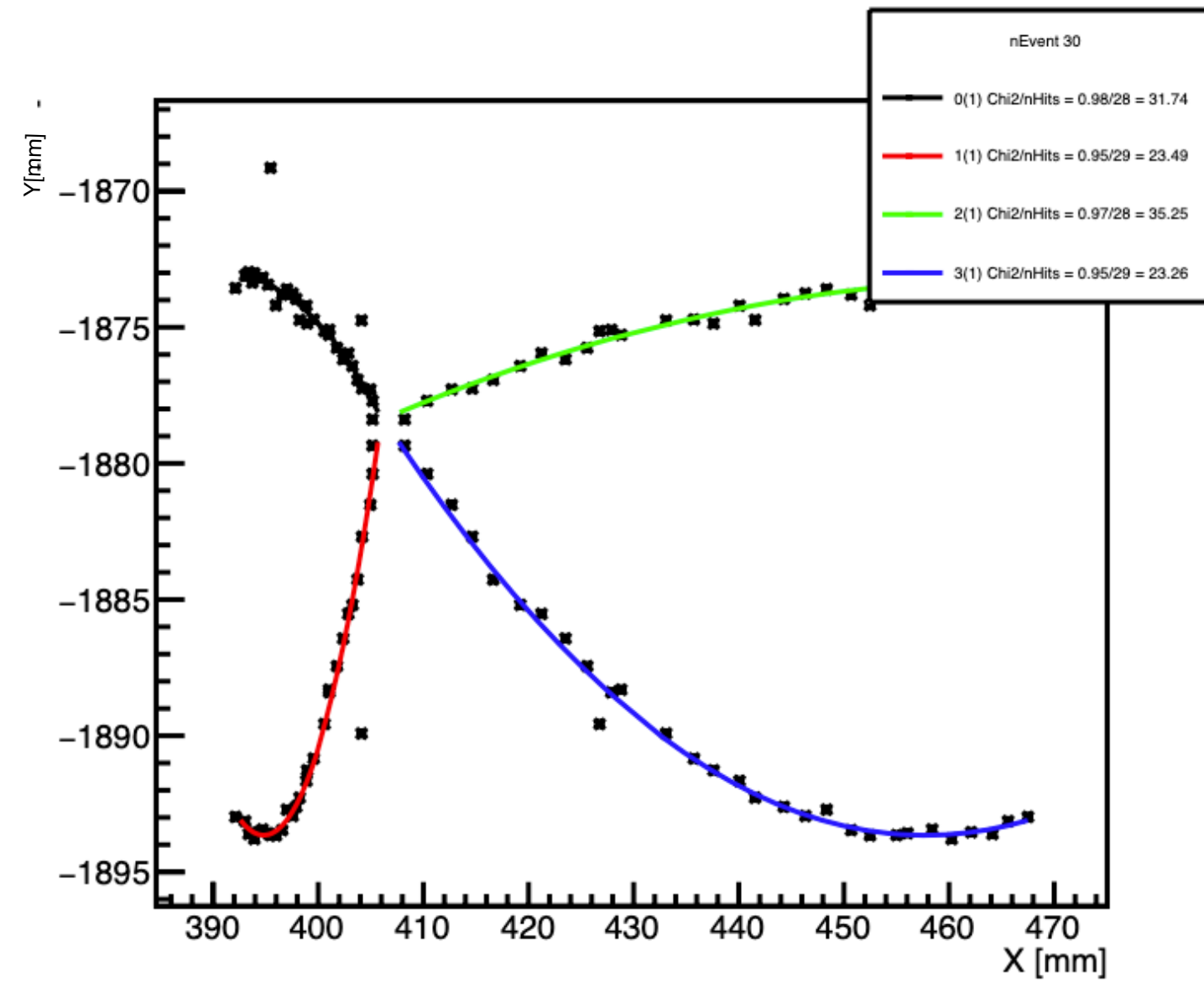
Kalman Filter, GEANT4 MC, Event Display



↑
Point rejection χ^2

- Ignoring MC truth matching
- The hits are coming from the digitization and the solid lines are the Kalman Filter
- The ghosts usually are not back to back (potential discriminant for this channel)
- The χ^2 :
 - Point rejection chi2 (Update stage)
~ threshold 300
 - The external fit chi2 for the tracks
- The bottom graphs show the spacial residual fluctuations

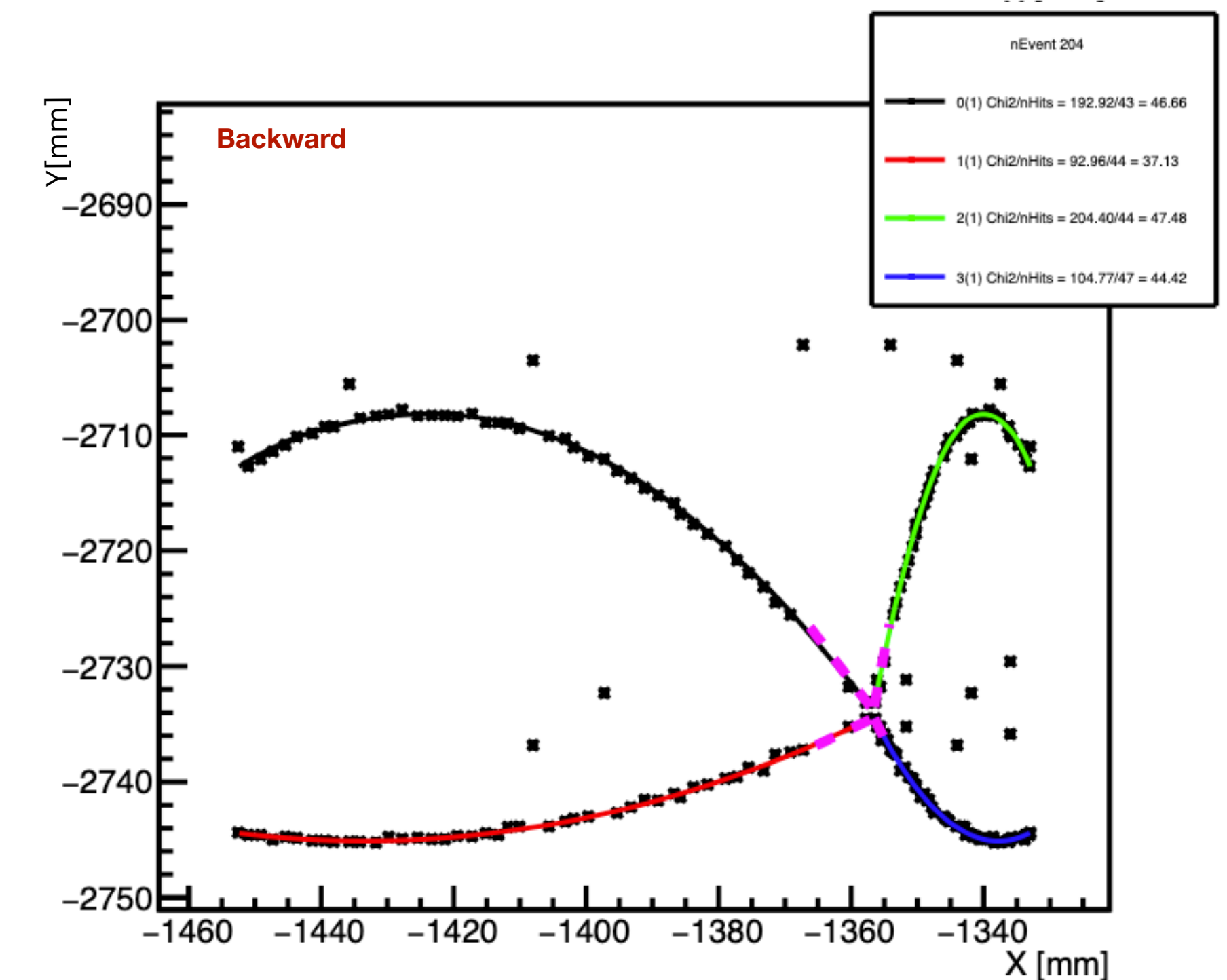
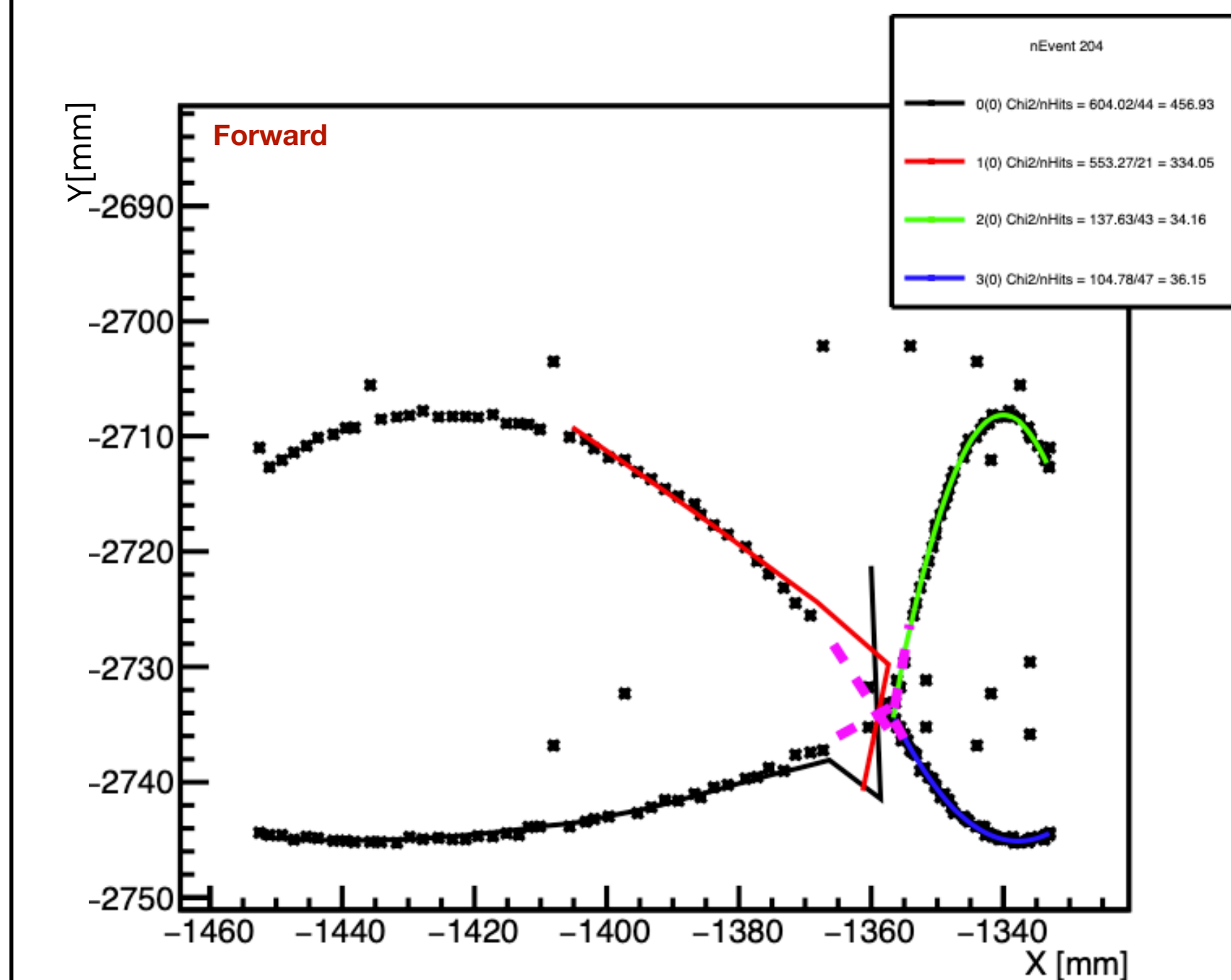
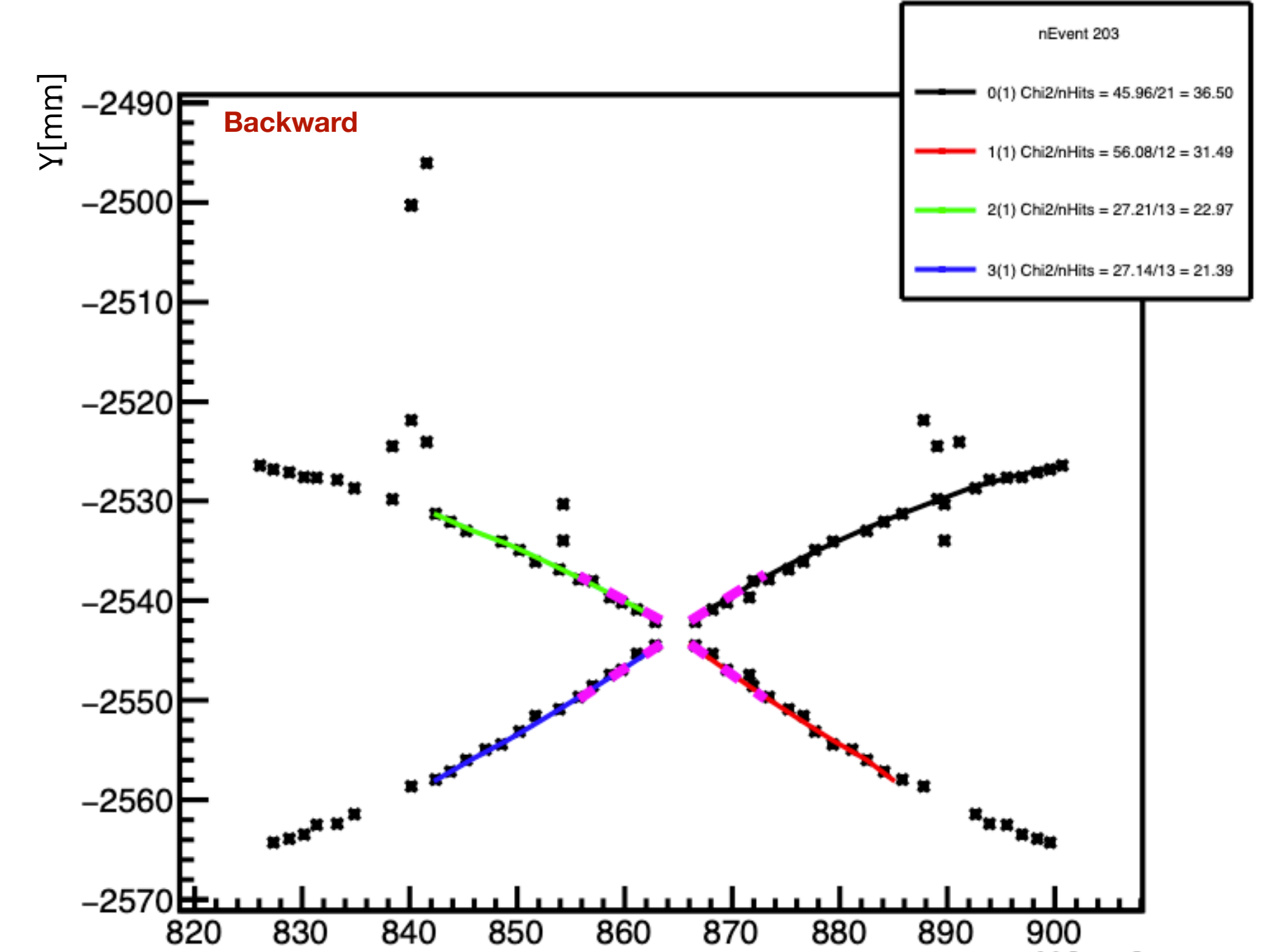
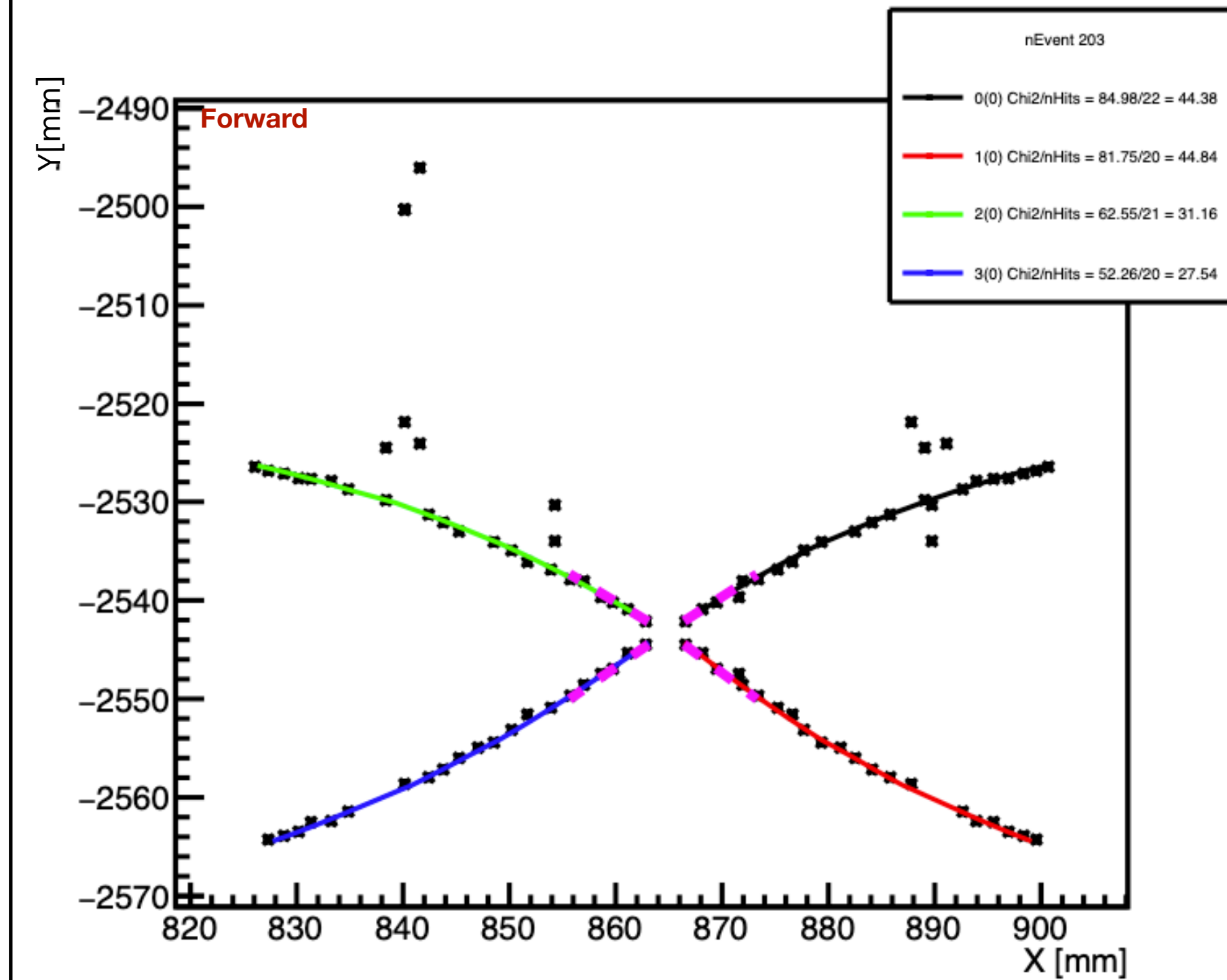
Kalman Filter, GEANT4 MC, Event Display



Kalman Events

* Kalman Event Selection

- **For/Backward tracks**
 - The more apart the hits the better the track recognition
 - Statistically, the backward Kalman is more efficient
- **Merged tracks**
 - Enhancing the InvarMass resolution
 - Recovering the events failed in either of For/Backward process
 - Saving the better reconstructed event
 - Shared hits are checked to make sure of no double counting
- **Preliminary Efficiency estimate**
 - FidCut is done:
 - Extrapolation of the track up to the exiting point, count the # plane
 - Number of planes = 6
 - Single track efficiency ~ 70%
 - Event (pair of tracks with a vertex < 1mm) efficiency ~ 80%
 - Still to investigate the apparent statistical correlation



Events Selection

- **Production cut**

- Cubic FidVol cut to mad-Dump output

- **Events Kinematic**

- Heavy Neutrino: High P, mostly with low theta \rightarrow back to back (XY) 2-body decay

- **Treatment for Ghosts**

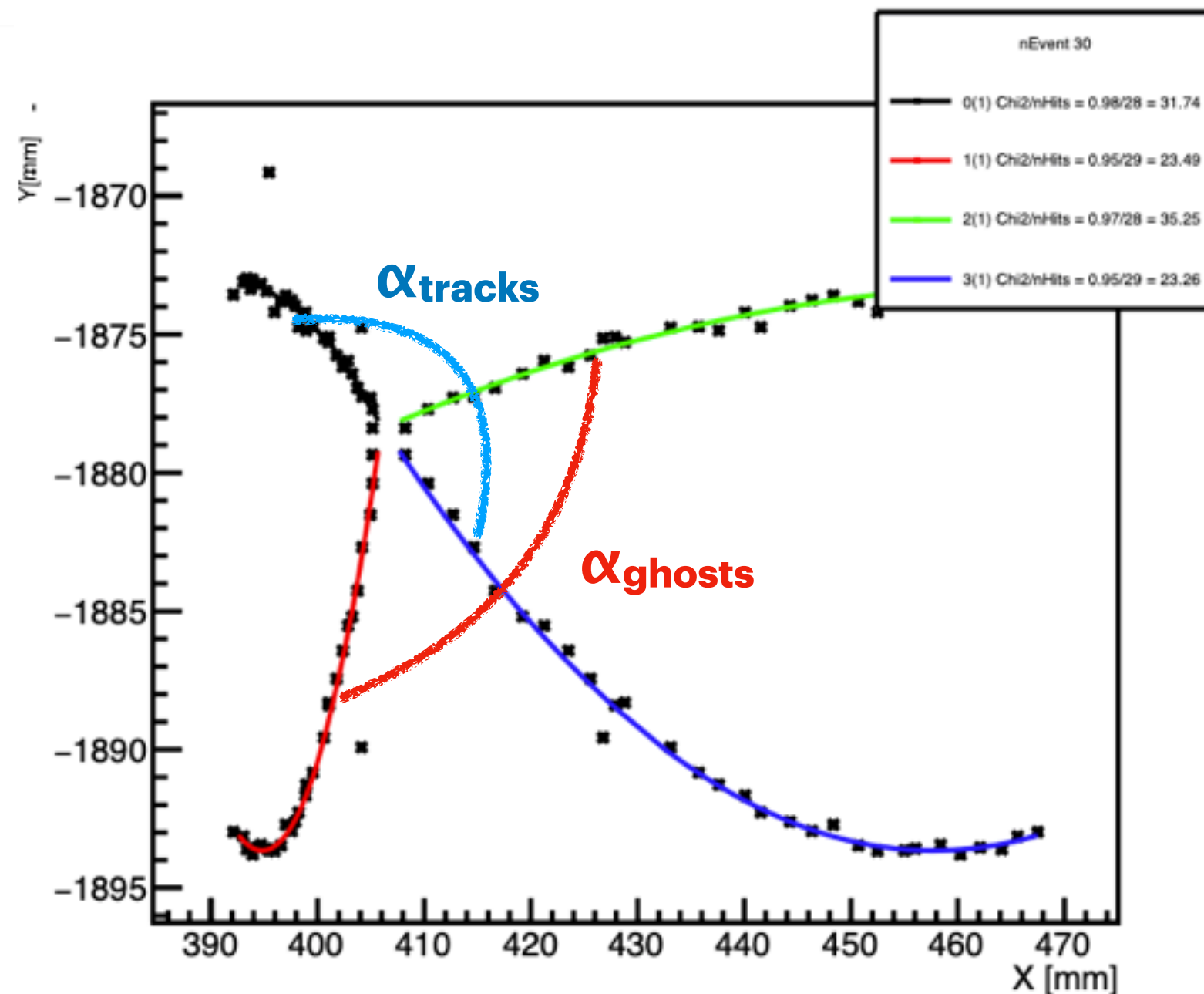
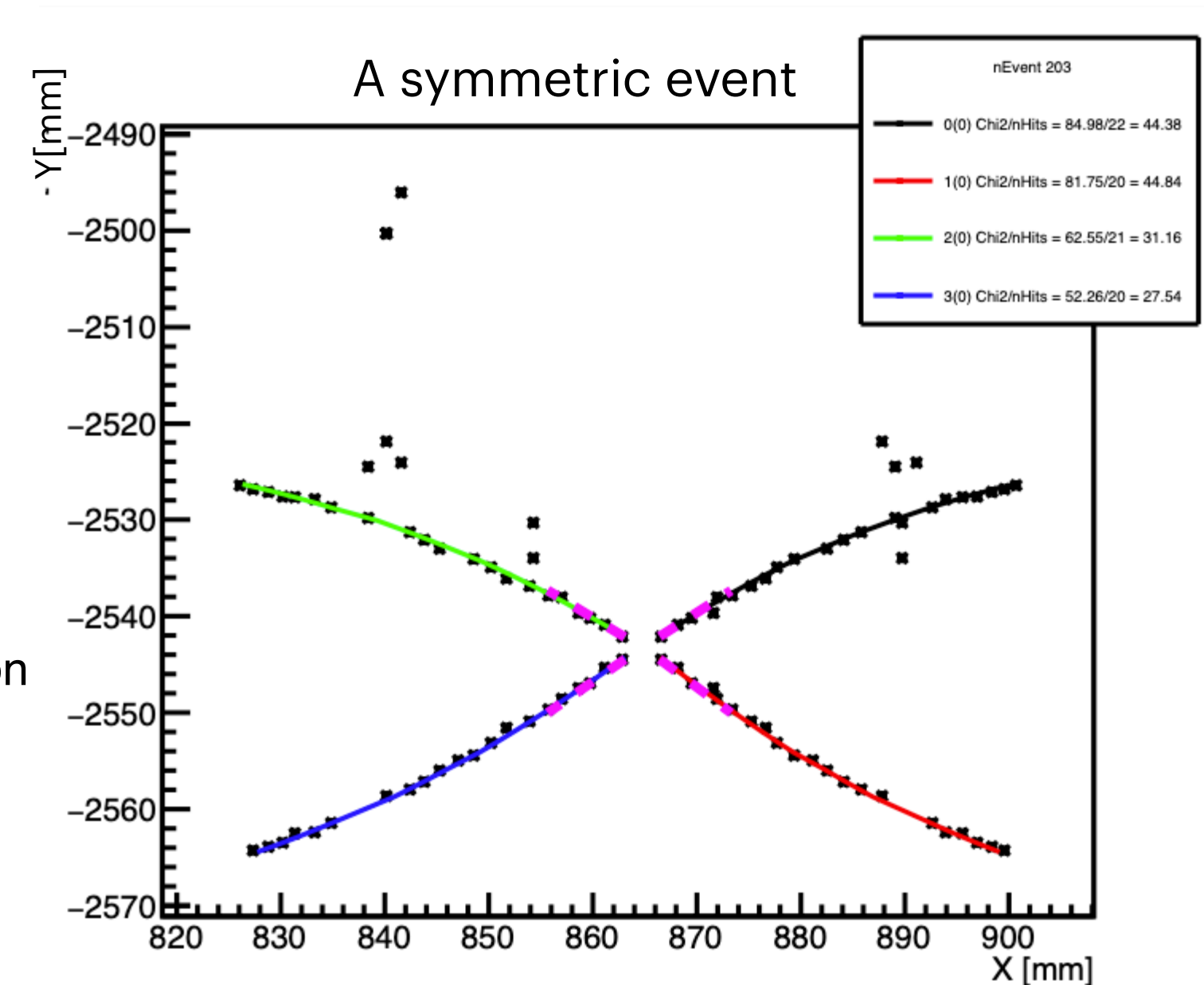
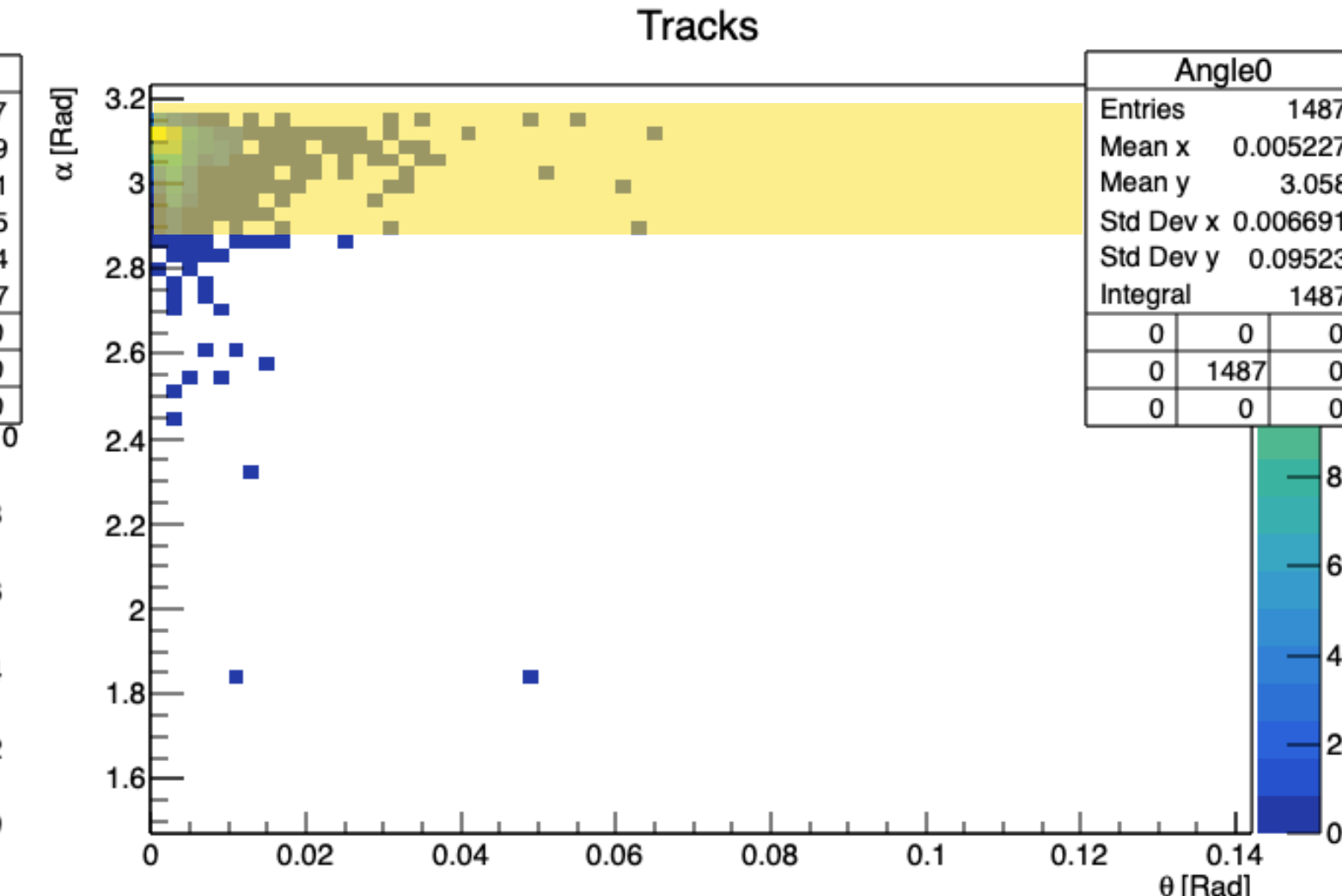
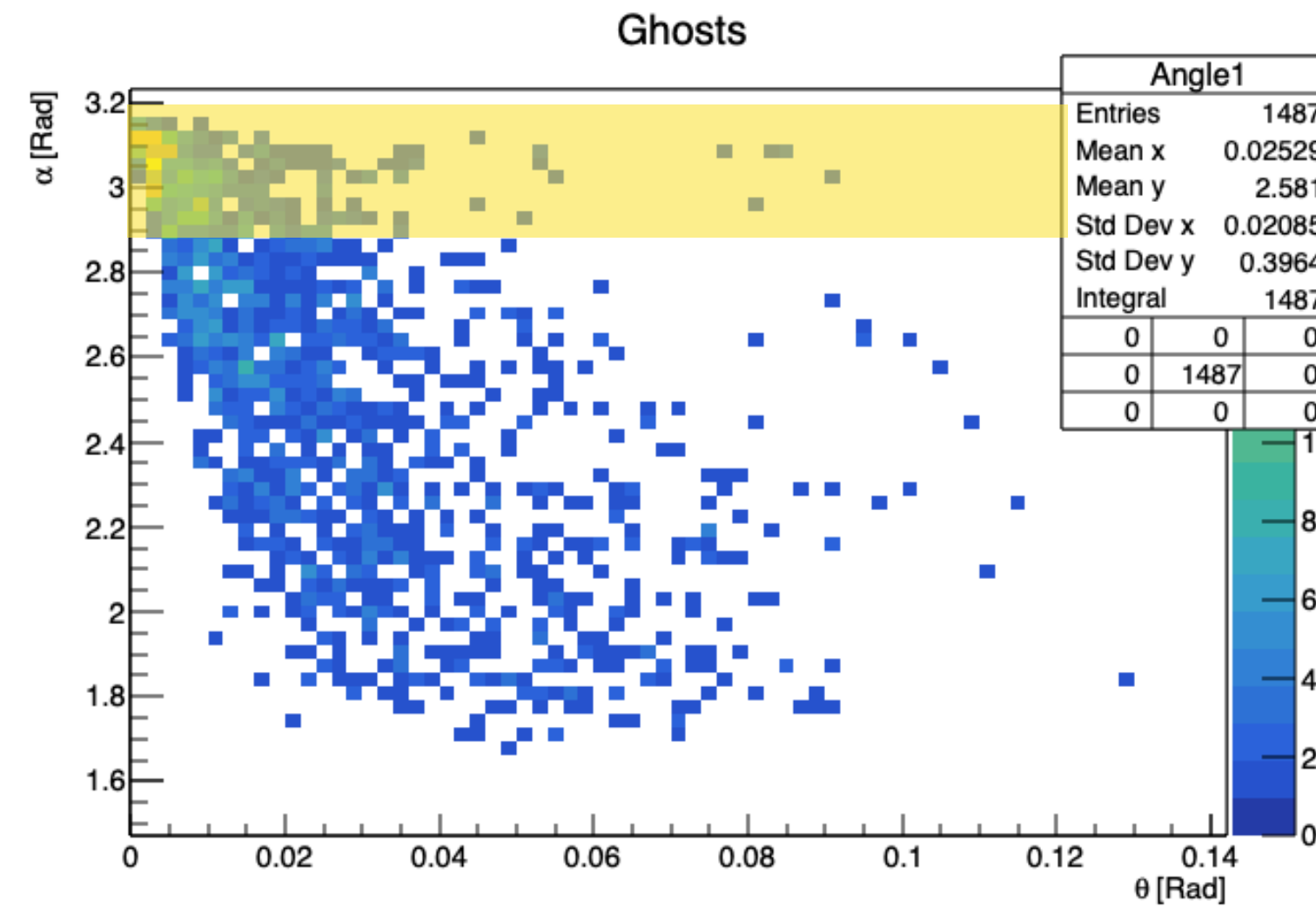
- Opposite charges and tracks in opposite quadrants XY.
- Alpha angle in XY between the ghosts or the tracks
- Theta is the angle of HNL with respect to the z-axis
- Alpha is geometrically correlated with theta
- A cut can be made for selecting the tracks from ghosts \rightarrow theta < 0.02, alpha > 2.9
 - Removes most of the ghosts contaminating the signal
 - To be optimized based on Signal/Background(regular neutrino interactions)
 - The remnant ghosts:
 - No effect on the resolution (very symmetric events)
 - Compensated by a correction factor

- **Particle ID**

- Not necessary at this stage
 - Swapping pi-mu has negligible effect on Invariant mass resolution

- **Background**

- Working on generating a neutrino beam-like sample with the tracker geometry i'm using



Summary and Outlook

- **Kalman Filter Implementation**

- Toy (Uniform B, Const δz , (x,y) measurement based on analytical function, 95% off, 0.1 mm smearing)
- Geant4 (Uniform B , Geometry “nd_hall_kloe_sttonly.gdml”, XXYY, XXYYXX for δz , separate X, Y measurement, 200 μm smearing)

- **Kalman Selection**

- Heavy Neutrino: High P, mostly with low theta \rightarrow back to back (XY) 2-body decay
- **Ghosts treatment**
 - Angle cut in XY plane combined with the angle cut on theta
 - Removes most of the ghosts contaminating the signal
 - To be optimized based on s/b
 - The remnant ghosts:
 - No effect on the resolution (very symmetric events)
 - Compensated by a correction factor

- **Event Selection and Efficiency**

- FidCut based on track extrapolation to the detectors walls (passing ≥ 6 planes)
- Efficiency (preliminary) of single track \sim of an event (pair with a vertex $< 1\text{mm}$)
- **Particle ID**
 - Not needed at this stage \rightarrow highly boosted event

- **Background**

- Working on generating a neutrino beam-like sample with the tracker geometry i'm using
- More accurate efficiency can be estimated after the bg sample is ready

- **To be done**

- Further Investigation on the efficiency of single and paired tracks
- Run background and signal to assess “final” sensitivity
- Kalman Filter optimization



DUNE DEEP UNDERGROUND
NEUTRINO EXPERIMENT

THANKS!

Meeting Annuale della Collaborazione Nazionale di DUNE
11-12 Nov Bologna