The Coded Aperture Imaging

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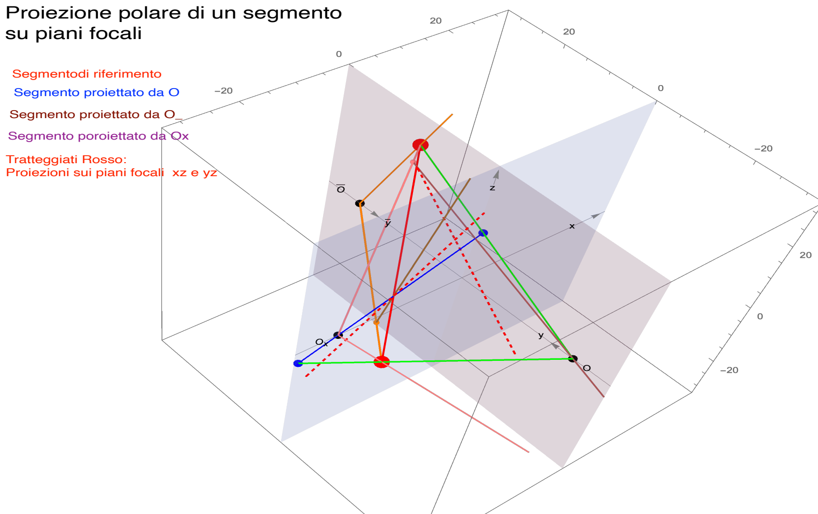
Outline

Proiezione


Proiezione polare di un segmento
su piani focali
Segmentodi riferimento
Segmento proiettato da O
Segmento proiettato da O_
Segmento poroiettato da Ox
Tratteggiati Rosso:
Proiezioni sui piani focali xz e yz
20
20
0
0
−20
−20
20
0
−20
O
y
z
x
y
Ox
O

$$T_{\mathcal{M}}P = \frac{T_M P}{p_d} = -\frac{b/\delta_P}{a}\frac{p_m}{p_d}(\mu_P, \nu_P) + \frac{a + b/\delta_P}{a}\frac{p_m}{p_d}\sum_{\sigma=0}^{\rho}\mathcal{M}_\sigma,$$

$$(\mathcal{M}_\sigma)_{i,j} = (\mathcal{M})_{i,j}(i,j) + \sigma_{i,j}, \quad \sigma_{i,j} = (s_1, s_2), \; |s_k| \leq \tfrac{1}{2}$$

$$T_{\mathcal{M}}P|_{dev} = [T_{\mathcal{M}}P]$$

$$Y = \Phi \; P + E$$

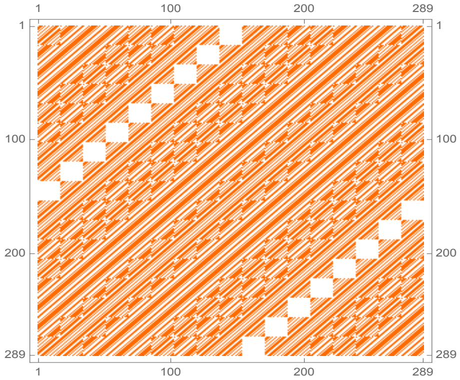


Figura: The transfer matrix Φ for a MURA 17×17 for sources on the focal plane, i.e. $\delta_P = 1.0$

The spectrum

$$\sigma(\Phi) = \left\{ \frac{q^2 - 1}{2} = K \ (deg = 1), \pm\frac{q+1}{2} \ (deg = K/2), \pm\frac{q-1}{2} \ (deg = K/2) \right\}$$

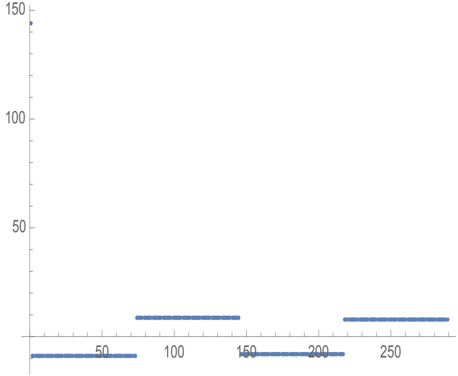


Figura: The spectrum of transfer matrix Φ for the MURA 17×17 at $\delta_P = 1.0$

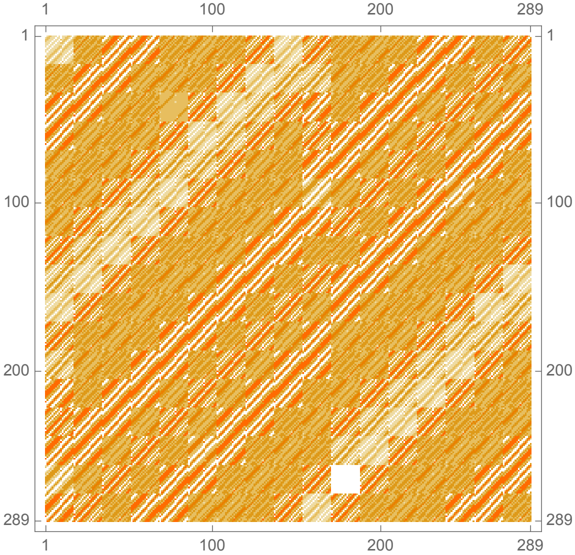


Figura: Transfer matrix for $\delta = 1.1$

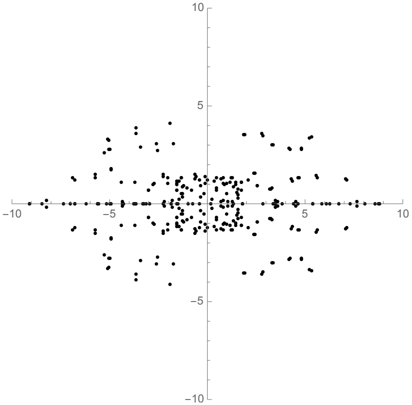


Figura: Transfer matrix spectrum for $\delta = 1.1$. Max 145.76

3D to 2D camera projection.

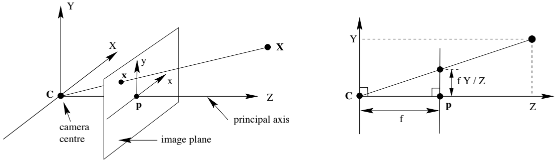


Figura: Model of camera. The three image points defined by $\mathbf{p_i}$ are the vanishing points of the directions of the world axes

$$\mathrm{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} = \begin{bmatrix} \mathbf{P}^{1\mathsf{T}} \\ \mathbf{P}^{2\mathsf{T}} \\ \mathbf{P}^{3\mathsf{T}} \end{bmatrix}$$

$$\mathbf{x} = P\,\mathbf{X} \qquad P = K\,[R\,|\,C] \quad K = \begin{pmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{pmatrix}$$

1. Given two images, and no other information, compute matches between the images, and the 3D position of the points that generate these matches and the cameras that generate the images.
2. Given three images, and no other information, similarly compute the matches between images of points and lines, and the position in 3D of these points and lines and the cameras.
3. Compute the epipolar geometry of a stereo rig, and trifocal geometry of a trinocular rig, without requiring a calibration object.
4. Compute the internal calibration of a camera from a sequence of images of natural scenes.

1. Given a set of 3D points $\mathbf{X_i}$ and a set of corresponding points $\mathbf{x_i}$ in an image. Find the "experimental "projective camera P and its properties.
2. Given a set of points $\mathbf{x_i}$ in one image, and corresponding points $\mathbf{x_i'}$ in another image, compute the fundamental matrix F: a singular 3×3 satisfying

$$\mathbf{x_i'}^{\,T}\, F\, \mathbf{x_i} = 0 \quad \forall i.$$

3. Given a set of point correspondences $\mathbf{x_i} \leftrightarrow \mathbf{x_i'} \leftrightarrow \mathbf{x''_i}$ across three images, compute the Trifocal Tensor T_i^{jk} relating points or lines in three views

$$\sum_{ijk} x^i\, \ell_j\, \ell_k\, T_i^{jk} = 0$$

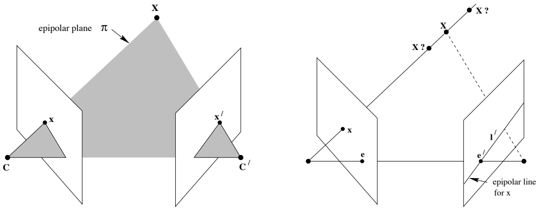


Figura: $\mathbf{x} \rightarrow \ell'$

$$F = K'^{-T} R K^T \left[K R^T \mathbf{t} \right]$$

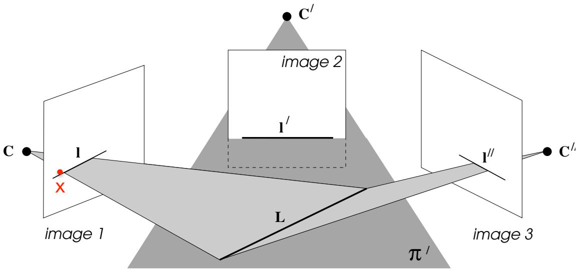
Projective reconstruction theorem

Suppose that $\mathbf{x_i} \leftrightarrow \mathbf{x_i'}$ is a set of correspondences between points in two images and that the fundamental matrix F is uniquely determined by the condition $\mathbf{x_i'}^T F \mathbf{x_i} = 0 \quad \forall i$

Let $(\mathbf{P_1}, \mathbf{P_1'}, \{\mathbf{X_{1\,i}}\})$ and $(\mathbf{P_2}, \mathbf{P_2'}, \{\mathbf{X_{2\,i}}\})$ be two reconstructions of the correspondences $\mathbf{x_i} \leftrightarrow \mathbf{x_i'}$. Then there exists a non-singular matrix H such that $\mathbf{P_2} = \mathbf{P_1} H^{-1}$ and $\mathbf{P_2'} = \mathbf{P_1'} H^{-1}$

Errors in the images, Transfer error and Reprojection errors

Trifocal Geometry



Figura

$$\sum_{ijk} x^i \, \ell_j \, \ell_k \, T_i^{jk} = 0$$

Estimating the trifocal tensor

1. Linear methods based on direct solution of a set of linear equations
2. Iterative methods, that minimizes the algebraic error, while satisfying all appropriate constraints on the tensor
3. Iterative method that minimizes geometric error

Application of the Random Matrix Theory : $A_{meas} = A_0 + W$, W has independent (complex) Gaussian entries: the measure of the state density converges to the Tracy-Widom distribution.