5. Baryogenesis and magnetic fields

What is baryon asymmetry?

The visible universe is made of particles (antiparticles can be produced when cosmic rays collide with atmosphere, but only in tiny amounts). So we have a matter asymmetry, "particles minus antiparticles". Since most of the mass is in protons, we frequently refer to baryon asymmetry.

Sakharov conditions

The criteria for generating a baryon asymmetry dynamically are:

(i) there must be baryon number violation
[otherwise any reaction generates as many antibaryons as baryons]

(ii) there must be C and CP violation
[otherwise the number densities of particles and antiparticles coincide]

(iii) the system must depart from thermal equilibrium
[in equilibrium any reaction and inverse reaction proceed equally fast]

In principle these could be satisfied by extensions of the Standard Model:

(i) baryon asymmetry is violated by the chiral anomaly:

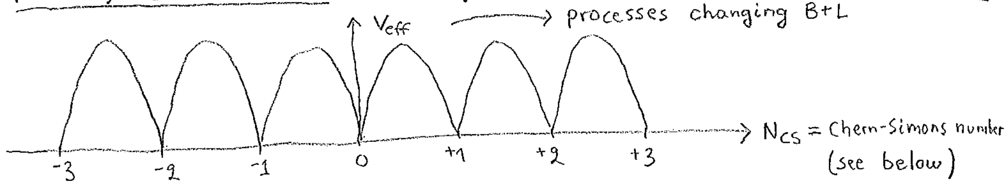
$$\partial_\mu \underbrace{\sum_k}_{\text{generations}} \Big\{ \underbrace{\bar{q}_k \gamma^\mu q_k}_{\text{"B"}} + \underbrace{\bar{\ell}_k \gamma^\mu \ell_k}_{\text{"L"}} \Big\} = \frac{(\sum_k)}{32\pi^2}\, \varepsilon^{\mu\nu\rho\sigma} \Big\{ \underbrace{g_w^2 G^a_{\mu\nu} G^a_{\rho\sigma}}_{SU_L(2)} - \underbrace{g_1^2 F_{\mu\nu} F_{\rho\sigma}}_{U_Y(1)} \Big\} .$$

(ii) C is violated by weak interactions, CP by a phase in the CKM matrix.

(iii) bubble dynamics could provide for deviation from equilibrium.

Sphaleron rate

The rate of B+L violation is closely related to the nucleation rate on p. 10.



The rate can be estimated in various limits:

(i) $T=0 \Rightarrow$ quantum tunnelling $\Rightarrow \Gamma_{B+L} \propto e^{-\frac{8\pi^2}{g_w^2}} \sim$ very small.

(ii) $0<T<T_c \Rightarrow$ thermal fluctuation, through a τ-independent "sphaleron" configuration $\Rightarrow \Gamma_{B+L} \propto e^{-\frac{8\pi m_W}{g_w^2 T}\phi\left(\frac{m_h}{m_W}\right)}$,
where $\phi \sim 1$ is a slowly varying function.

(iii) $T>T_c \Rightarrow v \simeq 0 \Rightarrow m_W \simeq 0 \Rightarrow$ no barrier!
In this case Γ_{B+L} becomes fast, $\Gamma_{B+L} \sim \alpha_w^5 T$, $\alpha_w \equiv \frac{g_w^2}{4\pi}$.
It is still called the sphaleron rate, even though no semi-classical configuration exists and the result must be determined numerically.
Since $\Gamma_{B+L} \gg H \sim \frac{T^2}{m_{Pl}}$ at $T \lesssim 10^{10}$ GeV, B+L averages to zero.

A nice way to see that baryon and lepton asymmetries are connected

Let us boldly assign chemical potentials to all fermions! What could go wrong?

$$L_E \supset \bar\psi_k \gamma_\mu D_\mu \psi_k = \bar\psi_{kL}\gamma_\mu D_\mu \psi_{kL} + \bar\psi_{kR}\gamma_\mu D_\mu \psi_{kR}$$

$\mathbb{1} = a_L + a_R$

$$\rightarrow \bar\psi_{kL}\left[\gamma_\mu D_\mu - \gamma_0 \mu_{kL}\right]\psi_{kL} + \bar\psi_{kR}\left[\gamma_\mu D_\mu - \gamma_0\mu_{kR}\right]\psi_{kR}\,.$$

In Feynman rules, this means that $\gamma_0 i\omega_n \to \gamma_0(i\omega_n - \mu)$, i.e. $\omega_n \to \omega_n + i\mu$, where ω_n is a fermionic Matsubara frequency.

We can now derive the dimensionally reduced effective theory for this setup. Because the chemical potentials break discrete symmetries (charge conjugation C), additional operators appear, compared with p. 7.

Diagrams to compute:

...

Result: (we assign separate chemical potentials to left and right-handed modes only for leptons, because for quarks so-called strong sphalerons guarantee that there is chiral equilibrium)

$$L_{eff} = \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{2}m^2_{E2}A^a_0 A^a_0 + \frac{1}{2}m^2_{E1}B_0 B_0 + i j_{E1} B_0$$
$$+ \Big(3N_g\mu_q + \sum_{k=1}^{N_g}\mu_{kL}\Big)(n_{cs2} - n_{cs1}) + 2\sum_{k=1}^{N_g}(\mu_{kL}-\mu_{kR})\, n_{cs1} + \ldots\,,$$

$N_g = 3$ denotes the number of generations

quark chemical potential

lepton chemical potentials

$$n_{cs2} \equiv \frac{g_w^2}{32\pi^2}\epsilon_{ijk}\Big(A^a_i G^a_{jk} - \frac{g_w}{3}f^{abc}A^a_i A^b_j A^c_k\Big)\,,$$

$$n_{cs1} \equiv \frac{g_1^2}{32\pi^2}\epsilon_{ijk} B_i F_{jk}\,.$$

$N_{CS} = \int_x n_{cs}$ is called the Chern-Simons number.

Note that $n_{cs2} - n_{cs1}$ is similar to the right-hand side of the anomaly equation on p. 17.

Interpretations:

$N_s = 1$ denotes the number of scalar doublets

* $$m^2_{E2} = g_w^2\left[\Big(\frac{2}{3} + \frac{N_g}{3} + \frac{N_s}{6}\Big)T^2 + 3N_g\frac{\mu_q^2}{4\pi^2} + \sum_{k=1}^{N_g}\frac{\mu_{kL}^2}{4\pi^2}\right] + O(g_w^4)$$

  corresponds to Debye screening. A non-zero mass for these components implies that A^a_0, B_0 might be integrated out (cf. p. 7).

* $$j_{E1} = g_1\left[\frac{N_g\mu_q}{3}\Big(T^2 + \frac{\mu_q^2}{\pi^2}\Big) - \sum_{k=1}^{N_g}\Big(\frac{\mu_{kL}+\mu_{kR}}{6}T^2 + \frac{\mu_{kL}^3+\mu_{kR}^3}{6\pi^2}\Big)\right] + O(g_1^3)$$

  implies that B_0 develops an (imaginary) expectation value in the perturbative vacuum. This is OK, nothing to worry about.

* n_{cs2} is not gauge invariant in "large" gauge transformations
  $\Rightarrow$ it should not appear $\Rightarrow$ the coefficient should be zero
  $\Rightarrow$ thus there is a relation between quark and lepton chemical potentials. [if sphalerons are in equilibrium].

* n_{cs1} is gauge invariant (up to a total derivative):

$$\epsilon_{ijk}B_i F_{jk} \xrightarrow[B_i \to B_i - \partial_i\theta]{} \epsilon_{ijk}B_i F_{jk} \underbrace{- \partial_i\{\epsilon_{ijk}\theta F_{jk}\}}_{\text{total derivative}} + \theta\,\underbrace{\epsilon_{ijk}\partial_i F_{jk}}_{\frac{1}{3}\epsilon_{ijk}(\partial_i F_{jk} + \partial_j F_{ki} + \partial_k F_{ij}) = 0}\,.$$

"Tachyonic instability" from an Abelian Chern-Simons term (n_{CS1})

Let us go to Minkowskian spacetime and consider an Abelian Chern-Simons term,

$$\mathcal{L}_M = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - c\, \varepsilon_{ijk} B_i F_{jk} = -\frac{1}{2} \partial^\mu B^\nu \left(\partial_\mu B_\nu - \partial_\nu B_\mu \right) - 2c\, \varepsilon_{ijk} B^i \partial_j B^k$$

after partial integration

$$= +\frac{1}{2} B^\alpha \left(\partial^\mu \partial_\mu \eta_{\alpha\beta} - \partial_\alpha \partial_\beta \right) B^\beta - 2c\, \varepsilon_{ijk} B^i \partial_j B^k$$

Choose gauge $B^0 = 0$

$$= \frac{1}{2} B^i \Big(\partial^\mu \partial_\mu \underbrace{\eta_{ij}}_{-\delta_{ij}} - \partial_i \partial_j \Big) B^j - 2c\, \varepsilon_{ijk} B^i \partial_j B^k .$$

From here we obtain an equation of motion:

$$\left[(\partial_0^2 - \nabla^2)(-\delta_{ij}) - \partial_i \partial_j \right] B^j = 4c\, \varepsilon_{ijk} \partial_j B^k \quad \forall\, i .$$

Momentum space: $B^j(t,\vec{x}) = \int_{\vec{k}} B^j(t,\vec{k})\, e^{-i\vec{k}\cdot\vec{x}}$, with $\vec{k} \equiv (0,0,k)$.

$$\Rightarrow \begin{pmatrix} -\partial_0^2 - k^2 & & \\ & -\partial_0^2 - k^2 & \\ & & -\partial_0^2 - \cancel{k^2} + \cancel{k^2} \end{pmatrix} \begin{pmatrix} B^1 \\ B^2 \\ B^3 \end{pmatrix} = -4cik \begin{pmatrix} -B^2 \\ B^1 \\ 0 \end{pmatrix} . \quad \begin{array}{l} \leftarrow \{i=1, j=3\} \\ \leftarrow \{i=2, j=3\} \\ \leftarrow \{i=3, j=3\} \end{array}$$

For B^1 and B^2, this gives

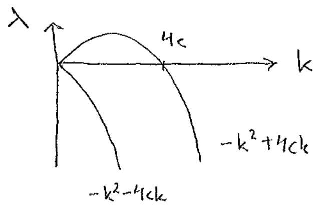
$$\begin{pmatrix} \partial_0^2 & \\ & \partial_0^2 \end{pmatrix} \begin{pmatrix} B^1 \\ B^2 \end{pmatrix} = \begin{pmatrix} -k^2 & -4cik \\ 4cik & -k^2 \end{pmatrix} \begin{pmatrix} B^1 \\ B^2 \end{pmatrix} .$$

Eigenvalues:

$$(-k^2 - \lambda)^2 - 16 c^2 k^2 = 0$$

$$\lambda^2 + 2\lambda k^2 + k^4 - 16 c^2 k^2 = 0$$

$$\lambda = -k^2 \pm \sqrt{k^4 - k^4 + 16 c^2 k^2} = -k^2 \pm 4ck .$$



So for $k < 4c$, there is a positive eigenvalue!

Diagonalizing the system, we thus find

$$\partial_0^2 B_{pos.} = +|\lambda|\, B_{pos.} \quad \Rightarrow \quad B_{pos.}(t) = \alpha\, e^{+\sqrt{|\lambda|}\,t} + \beta\, e^{-\sqrt{|\lambda|}\,t} .$$

exponential growth!

Summary: Returning to p. 18, we must have $\mu_{KL} = \mu_{KR}$, in order to have a stable (static) system*. Applying the same logic to n_{CS2} and assuming that the non-linear terms do not change the situation qualitatively, we find another derivation of the sphaleron equilibrium relation between μ_q, μ_{KL}.

* i.e. without any exponential growth

— o — o —

A bit about magnetic fields

Summary for $\vec{B}$:

* observation : ??
* theory:
  - inflation ?
  - reheating ?
  - phase transitions?
  - ⋮
  - galactic dynamics?

Apart from baryon asymmetry, another important issue in cosmology is whether large-scale magnetic fields could be generated primordially, for instance in phase transitions.

As the empirical evidence for large-scale magnetic fields is not easy to interpret, and theoretical computations are hard, the field is not as well developed as for gravitational waves and baryon asymmetry. We just illustrate one mechanism, which has also been used in the context of inflation.

Let us assume that after integrating out some fields, the Lagrangian contains an axion-like (i.e. pseudoscalar) field and coupling:

$$\mathcal{L}_M \supset -\frac{\varphi}{f} \cdot \frac{g_1^2}{64\pi^2} \varepsilon^{\mu\nu\rho\lambda} F_{\mu\nu} F_{\rho\lambda} .$$

Recall : $K^\mu \equiv c\, \varepsilon^{\mu\nu\rho\lambda} B_\nu F_{\rho\lambda} = 2c\, \varepsilon^{\mu\nu\rho\lambda} B_\nu \partial_\rho B_\lambda$

$$\Rightarrow \partial_\mu K^\mu = 2c\, \varepsilon^{\mu\nu\rho\lambda} \{ \partial_\mu B_\nu \partial_\rho B_\lambda + \underbrace{B_\nu \partial_\mu \partial_\rho B_\lambda}_{\text{vanishes by antisymmetry of } \varepsilon^{\mu\nu\rho\lambda}} \}$$

$$= \frac{c}{2} \varepsilon^{\mu\nu\rho\lambda} F_{\mu\nu} F_{\rho\lambda} .$$

Choosing now $c = \frac{g_1^2}{32\pi^2}$, we find

$$\mathcal{L}_M \supset -\frac{\varphi}{f} \partial_\mu K^\mu , \quad \text{with } K^0 = n_{cs1} .$$

Assume that φ is constant but depends on time, and integrate over spacetime

$$\Rightarrow S_M \supset -\int_X \frac{\varphi(t)}{f} \{ \partial_0 K^0 + \underbrace{\partial_i K^i}_{\text{total derivative} \to 0} \}$$

$$\overset{\text{partial integration}}{=} \int_X \frac{\dot\varphi(t)}{f} n_{cs1} .$$

So we find the same term as on p. 19, and the same exponential production of large-scale (small-k) magnetic fields!

Literature

* thermal field theory : ML & A. Vuorinen, 1701.01554
* phase transitions & gravitational waves : M.B. Hindmarsh et al, 2008.09136
* baryon asymmetry : D. Bödeker & W. Buchmüller, 2009.07294
* notes for these lectures : www.laine.itp.unibe.ch/ggi22.pdf

---