Non-thermal messengers from the Universe - High energy physics processes in astrophysics

Exercises to be discussed in class, Part II (Roughly lectures III and IV)

Exercise 1.

Starting from Newton's law in presence of electromagnetic fields, show that the rigidity of a particle obeys the equation

$$\frac{\mathrm{d}\vec{\mathcal{R}}\cdot\vec{\mathcal{R}}}{\mathrm{d}t} = 2\vec{\mathcal{R}}\cdot\mathbf{E} \tag{1}$$

i.e. any electromagnetic acceleration process orders particle distribution functions with respect to their rigidity. Use this to justify why, in the relativistic limit ($E_N \gg m_N$), we must expect that electrons are subleading to the hadronic (approximate, protonic) CR component, and estimate by how much, expressing them in terms of ratio of fluxes with respect to kinetic energy, $J(T)$. Consider both species accelerated from a non-relativist bath with typical kinetic energy $T_0 \ll m_e$, for which charge neutrality applies (so that equal numbers of particles are accelerated above T_0 for both species). For momentum power-spectra of index s, you should find

$$\frac{J_e(T)}{J_p(T)} \to \left(\frac{m_e}{m_p}\right)^{(s-1)/2} \tag{2}$$

in the relativistic limit.

Exercise 2.

Knowing that SN are estimated to happen 2-3 times per century in the Galaxy, each releasing a few times 10^{51} erg in kinetic energy, what it their "kinetic" luminosity? Compare that with the power needed to sustain a steady-state population of CRs, with integrated energy density of about $0.5\mathrm{eV}/\mathrm{cm}^3$ filling a confinement volume of the Milky Way assumed to be a cylinder with radius 15 kpc and height 4 kpc, and typical "lifetime" of $\tau_{CR} \simeq$10 Myr. Is it enough for the SNRs to power CRs? What is the ratio of the two, or if you wish the efficiency of macrocopic kinetic energy conversion into CR acceleration?

Exercise 3.

Calculate the minimum energy in the lab for a CR proton, hitting another one at rest, to produce one antiproton. *Hint:* remembering that baryon number is conserved, what is the lightest final state containing an antiproton, i.e. what is the lightest X in $pp \to \bar{p}\,X$?

Exercise 4.

Compute the energy of background photons at threshold for pair-production for incoming photons of 1 TeV or 1 PeV energy. What type of "light" are these bands corresponding to? In the case of CMB, described by a blackbody spectrum at 2.7 K, compute the mean-free path of a photon of typical PeV energies.

Exercise 5.

Consider a (globally neutral) plasma of electrons and protons (remember: the two species are in tight e.m. coupling!), spherically symmetric of mass M and radius R, kept in equilibrium by the balance of gravity and radiation pressure. What is the luminosity (called *Eddington luminosity*) supporting such a "star" on the verge of disruption? Does our Sun satisfy that limit? If yes, it is close to it? If not, can you explain why?

Exercise 6.

The Crab nebula, associated to the explosion of a SN in AD 1054, presents a roughly broken power-law spectrum, with a steepening around 5 eV, interpreted as synchrotron radiation, see Fig. 1. If we attribute this phenomenon to a "cooling break" (i.e. the electrons producing the photons below the break have a cooling lifetime longer than its age, and the ones

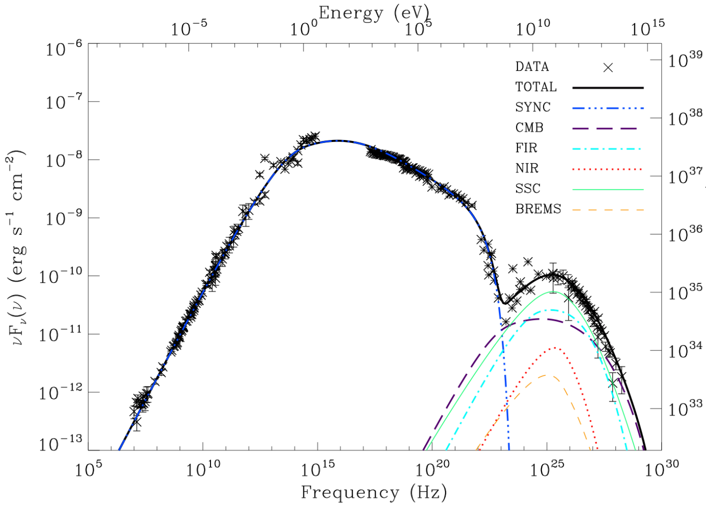

FIG. 1: The multiwavelength photon spectrum of the Crab nebula, also compared to models, see notes for refs..

producing photons above it have a shorter cooling timescale), determine the magnetic field in the source, and the energy of the electrons associated to this "break point". What is the energy of IC photons produced when those electrons hit the very synchrotron photons at the break point?

Exercise 7.
Modify the following transport equation

$$-\frac{\partial}{\partial z}\left(K\frac{\partial \phi}{\partial z}\right) = 2\,q_0(p)\,h\delta(z)\;. \tag{3}$$

to account for a catastrophic loss term, of the type $-2h\,\Gamma_\sigma\,\phi_P\,\delta(z)$. Consider the same equation written for secondaries, i.e. without primary source term q, but sourced by primary spallation, i.e. $2h\,\Gamma_{P\to S}\,\phi_P\,\delta(z)$. Prove that

$$\frac{\phi_S(p)}{\phi_P(p)} \simeq \Gamma_{P\to S}\tau_{\mathrm{eff},P} \simeq \Gamma_{P\to S}\frac{H\,h}{K(p)}. \tag{4}$$

Exercise 8.
Consider the following Eq as a toy model for *primary* cosmic ray antiprotons:

$$-\frac{\partial}{\partial z}\left(K\frac{\partial \phi_X}{\partial z}\right) = q_X(p) - 2h\,\Gamma_\sigma\,\phi_X\delta(z)\,. \tag{5}$$

where the production of antiprotons q_X is considered uniform over the vertical (diffusive) extent $\pm H$, as a proxy for a dark matter halo origin. Prove that Eq. (5) implies

$$\frac{1}{\tau_{\mathrm{eff}}(p)}\phi_X^0 = \frac{H}{h}\,q_X(p)\,, \tag{6}$$

with a suitable $\tau_{\mathrm{eff}}(p)$. Note that now the expected antiproton signal from DM depends on one extra parameter, the ratio H/h as opposed to the product $H\,h$ entering τ_d. This extra dependence makes physical sense, because it is linear in the size of the volume from which one collects injected particles.