# Non-thermal messengers from the Universe - High energy physics processes in astrophysics

**Exercises to be discussed in class, Part I (roughly lectures I and II)**

## Exercise 1.

Argue that cosmic rays impinging on the Earth along the magnetic field (fraction of a G) direction travel unimpeded, while CR arriving along the magnetic equator suffer a deflection. Compute the radius of curvature and determine the momentum/charge (known as rigidity, measured in GV) below which this radius is smaller than the Earth radius. Discuss how this depends on the latitude. A charged CR nature clearly predicts that an isotropic flux far away from Earth would be observed in a "magnetic-latitude" modulated way. The critical rigidity is known as *rigidity (or magnetic) cutoff*. For estimates, you can use the following dipole model of the Earth:

$$B_r = -2B_0\left(\frac{R_E}{r}\right)^3\cos\theta \quad (1)$$

$$B_\theta = -B_0\left(\frac{R_E}{r}\right)^3\sin\theta \quad (2)$$

$$|B| = B_0\left(\frac{R_E}{r}\right)^3\sqrt{1+3\cos^2\theta} \quad (3)$$

where $B_0 = 3.12\times10^{-5}$ T$=0.321$ G, $R_E$ is the mean radius of the Earth (approximately 6370 km), $r$ is the radial distance from the center of the Earth, and $\theta$ is the geomagnetic co-latitude measured from the north magnetic pole (if $\lambda$ is the magnetic latitutde, $\theta = \pi/2-\lambda$). For more professional modeling and links with geographical coordinate systems, you can have a look at https://www.ngdc.noaa.gov/IAGA/vmod/igrf.html Be aware that the magnetic dipole is tilted (by something like 11°, but depends on time!) with respect to the rotation axis of the Earth. For rough calculations you can confound the two, otherwise please use dedicated software like at http://www.geomag.bgs.ac.uk/data_service/models_compass/coord_calc.html for conversions.

## Exercise 2.

Consider an isothermal (exponential) model for the (upper) atmosphere,

$$\rho(h) \simeq \rho_0\exp(-h/h_0) \quad h_0\simeq 6.4\text{ km} \quad \rho_0 h_0 \simeq 1300\text{ g/cm}^2 \quad (4)$$

$$X(\ell,\theta) = \int_\ell^\infty \mathrm{d}l\,\rho(h(l,\theta)) \quad (5)$$

$$h(l,\theta) = \sqrt{R_\oplus^2 + 2lR_\oplus\cos\theta + l^2} \approx l\cos\theta + \frac{l^2}{2R_\oplus}\sin^2\theta\,. \quad (6)$$

- Estimate what is the height in the above model of atmosphere, Eqs. (4)(5),(6) at which the first interaction of a downgoing energetic photon takes place.
- Based on the Heitler model, how many particles are expected in a 1 TeV (100 GeV) gamma-ray induced shower? In the above model, what is the typical height of this maximum for a downgoing photon?

## Exercise 3.

Consider two CR species: primaries with number density $n_p$ and secondaries (initially not produced!) with number density $n_s$. If the two are coupled by the spallation process $p\to s+\ldots$, then

$$\frac{dn_p}{dX} = -\frac{n_p}{\lambda_p} \quad (7)$$

$$\frac{dn_s}{dX} = -\frac{n_s}{\lambda_s} + \frac{p_{\mathrm{p}\to\mathrm{s}}n_p}{\lambda_p} \quad (8)$$

where $X \equiv \int d\ell\rho(\ell)$ is the *grammage*, the density integrated along the actual path followed by the particle, measured in g/cm$^2$; $\lambda_i = \rho_{ISM}/(n_{ISM}\sigma_i) = m_{ISM}/\sigma_i$ is the *interaction length* of the species $i$ in the ISM medium, in terms of the effective ISM mass $m_{ISM} \simeq m_p$, and total inelastic cross-section $\sigma_i$ (appropriately weighted by the medium composition); it is measured in g/cm$^2$, as $X$; $p_{\rm p\to s}$ is the probability that, in an inelastic interaction of species $p$, the species $s$ is produced (the relevant b.r.), and is dimensionless. Note that $\lambda_i$ are dependent from laboratory measurements and the ISM composition, not really from the CR path. For instance, one has $\sigma_{\rm CNO} \simeq 6.7$ g/cm$^2$, $\sigma_{\rm LiBeB} \simeq 10$ g/cm$^2$, and $p \simeq 0.35$ between these two groups. From the measured value $n_s/n_p \simeq 0.25$, deduce $X$, and compare with the predictions for $X$ for straight lines crossing the Milky Way disk with an angle between $30°$ and 60°, assuming a density of 1 hydrogen atom/cm$^3$ and a Galactic half-thickness $h \simeq$100 pc. Based on this, estimate the typical timescales the CRs spent in the gaseous disk.

### Exercise 4.

Compute the energy at which a proton and a iron nucleus gyroradius exceeds the kpc scale, if $B_0 = 3\,\mu$G. If I told you that the skymap of CRs at $E \simeq 10^{19}$ eV looks roughly isotropic, what would you infer about the source locations?

### Exercise 5.

A CR source is at a distance $d$. For $d \simeq H \simeq 3$kpc (a typical Galactic distance), assuming the value of the average Galactic diffusion coefficient is $K = 0.3(\mathcal{R}/10{\rm GV})^{0.5}$kpc$^2$/Myr, a) compute the time it takes for the CR of rigidity 10 GV to reach us; compare it with the estimated age of a few famous supernova remnants that you can search on the web: Are you surprised? b) Reverse the previous exercise: For a few SNRs that you found by browsing some catalogue online, take their distance and age, and estimate the minimum energy of CRs that could have reached us. c) The rigidity at which the diffusion approximation breaks down, since the diffusion speed nominally exceeds the speed of light.

### Exercise 6.: "Leaky box" from slab model

Let us assume that sources are confined to an infinitesimal disk. The steady state transport equation simplifies into

$$-\frac{\partial}{\partial z}\left(K\frac{\partial \phi}{\partial z}\right) = 2\,q_0(p)\,h\delta(z)\ . \tag{9}$$

Find the CR flux $\phi(z,p)$. Compare it with the solution of a "leaky box" type of model, where injection at rate $Q$ happens in a homogeneous and isotropic medium from which cosmic rays take a (momentum dependent) time $\tau_{\rm esc}(p)$ to escape, leading to the steady state solution $\phi(p) = Q(p)\tau_{\rm esc}(p)$.