Phase transitions in the early universe

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1. Electroweak phase transition beyond the Standard Model

Without supplemental particles the Standard Model (SM) has no first-order thermal phase transition. In fact it is a cross-over [1]. Beyond the SM extensions could, however, allow for a first-order phase transition. While supplementing the SM with additional scalars is one viable option to achieve this, also higher-dimensional operators can alter the phase transition. This exercise follows closely the calculations of [2].

Inspect the pure scalar sector of the Minkowskian SM Lagrangian

$$\mathcal{L}_{\mathrm{M}} = (D_\mu\phi)^\dagger(D_\mu\phi) - V(\phi) \;, \tag{1.1}$$

$$V(\phi) = \mu_h^2\phi^\dagger\phi + \lambda\big(\phi^\dagger\phi\big)^2 \;, \tag{1.2}$$

where D_μ is the covariant derivative acting on the SU(2) Higgs doublet ϕ. Using a constant background field $\bar{\phi}$, we parameterise

$$\phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(\bar{\phi} + h + iG^0) \end{pmatrix} \;, \tag{1.3}$$

where h is the physical Higgs field and G^+, G^0 are Goldstone bosons and $G^- = (G^+)^\dagger$.

Exercise 1.1. Construct the tree-level effective potential $V_{\text{eff}}^{(0)}(\bar{\phi})$ by employing the parameterisation (1.3) in the Higgs potential (1.1). For this analysis assume vanishing $h, G^\pm, G^0 \to 0$. From the resulting potential, relate the $\overline{\text{MS}}$ -renormalised parameters of the Lagrangian to physical observables using the vacuum expectation value of the Higgs $v = 246$ GeV and the physical Higgs mass $M_h = 125$ GeV.

Exercise 1.2. The effective potential receives loop corrections such that $V_{\text{eff}} = V_{\text{eff}}^{(0)} + V_{\text{eff}}^{(1)}$. The one-loop contribution to the effective potential then takes the finite-temperature form

$$V_{\text{eff}}^{(1)} = \sum_i n_i \,\SumInt_P \ln(P^2 + m_i^2) \;, \tag{1.4}$$

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where $D = d+1 = 4-2\epsilon$ and the Euclidean four-momenta $P = (\omega_n, \mathbf{p})$. The summation runs over all species that couple to ϕ and n_i is the number of degrees of freedom of the i-th field with mass m_i.

Show that the one-loop contribution to the effective potential (1.4) is of the form

$$V_{\text{eff}}^{(1)}(\bar{\phi}) = \sum_i n_i \Big[\int \frac{\mathrm{d}^D p}{(2\pi)^D} \ln\big(p^2 + m_i^2(\bar{\phi})\big) + J_{\mathrm{B,F}}\Big(\frac{m_i^2(\bar{\phi})}{T^2}\Big)\Big] \,, \tag{1.5}$$

where the first term is the zero-temperature Coleman-Weinberg potential [3] and temperature effects [4] are encoded in the thermal functions

$$J_{\mathrm{B,F}}(m_i^2) = -T \int_{\mathbf{p}} \ln\big(1 \pm n_{\mathrm{B,F}}(\varepsilon_p, T)\big) \,, \quad n_{\mathrm{B,F}}(\varepsilon_p^i, T) = \frac{1}{e^{\varepsilon_p^i/T} \mp 1} \,, \quad \varepsilon_p^i = \sqrt{p^2 + m_i^2} \,. \tag{1.6}$$

Here, $n_{\mathrm{B,F}}$ are the bosonic and fermionic distribution functions, respectively. Show that one can expand them at high temperature i.e. $z \ll 1$ with $z^2 = m^2(\bar{\phi})/T^2$

$$\frac{J_{\mathrm{B}}(z^2)}{T^4} = -\frac{\pi^2}{90} + \frac{1}{24} z^2 - \frac{1}{12\pi}(z^2)^{\frac{3}{2}} + \mathcal{O}(z^4) \,, \tag{1.7}$$

$$\frac{J_{\mathrm{F}}(z^2)}{T^4} = +\frac{7}{8}\frac{\pi^2}{90} - \frac{1}{48} z^2 + \mathcal{O}(z^4) \,. \tag{1.8}$$

Using the above expressions,

- derive the one-loop effective potential at leading order in the high-temperature expansion $z^2 = m^2(\bar{\phi})/T^2$.
- determine the functional form of the effective potential as a function of $\bar{\phi}$ through the corresponding leading-order terms at $\bar{\phi}^2$, $\bar{\phi}^4$ and $\bar{\phi}^6$.

Exercise 1.3. By adding the sextic interaction $|\phi|^6$ of dimension six, the Higgs potential (1.1) in the symmetric phase is augmented by the operator

$$\mathcal{O}_6 = M^{-2}\big(\phi^\dagger \phi\big)^3 \,, \tag{1.9}$$

this is the minimal SM effective theory (SMEFT). In relation to the SM in Exercise 1.1 and Exercise 1.2

- visualise the difference between the tree-level effective potentials of the pure SM and the SMEFT.
- include the M-dependence in μ_h^2 and λ. For the additional parameter M there will be now a barrier in the effective potential already at tree-level. Determine how the global minimum depends on M.
- derive the one-loop effective potential at leading order in the high-T expansion.

References

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