Non-perturbative thermal QCD at very high temperatures: mesonic screening masses

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Based on:

Dalla Brida, LG, Harris, Laudicina, Pepe, JHEP 04 (2022) 034 [arXiv:2112.05427]

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Outline

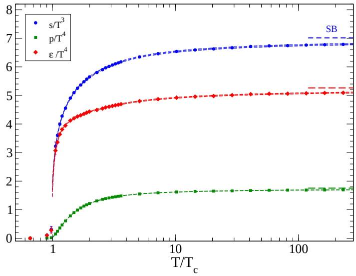
- Introduction and motivations
- Effective field theories at large T
- Lattice setup and renormalization
- Results for mesonic screening masses
- Discussion and interpretation
- Conclusions and Outlook

Introduction and motivations

Study thermal QCD up to electroweak scale non perturbatively (NP)

- Early evolution of Universe
- Properties of Quark gluon plasma
- Intrinsic theoretical interest
- Non-perturbative renormalization
- ......

[LG, Pepe 17]

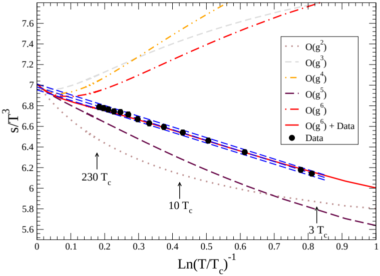


Introduction and motivations

Why NP up to the electroweak scale?

Because the various (analytic) approximations are heavily limited:

- Matching coefficients of EFTs computable in PT only up to finite order
- Perturbative expansion has a very poor convergence rate
- EFTs must be finally solved and matched NP



$$\frac{s(T)}{T^3} = \frac{32\pi^2}{45}\left\{1 + s_2 g^2 + s_3 g^3 + s_4 g^4 + s_5 g^5 + s_6 g^6 + \mathrm{NP}\right\}$$

For the $SU(3)$ Yang–Mills theory, the NP contribution is still $\sim 50\%$ of the sum of all other interacting terms at $T \sim 68$ GeV

Effective field theories at large T: EQCD

- Physics at energies $E \ll \pi T$ is described by a 3-dimensional effective gauge theory dubbed Electrostatic QCD (EQCD)

$$S_{\mathrm{EQCD}}=\frac{1}{g_{\mathrm{E}}^2}\int d^3x\left\{\frac{1}{2}\,\mathrm{Tr}\,[F_{ij}F_{ij}]+\mathrm{Tr}\,[(D_jA_0)(D_jA_0)]+m_{\mathrm{E}}^2\,\mathrm{Tr}\left[A_0^2\right]\right\}+\ldots$$

where the fields are the Matsubara zero-modes of 4D gauge field

- The 4D temporal component A_0 behaves as a 3D scalar field of mass m_{E} in the adjoint representation of the gauge group

- When the QCD coupling g^2 is small, perturbative matching gives

$$m_{\mathrm{E}}^2=\frac{3}{2}g^2T^2+\ldots \qquad \text{and} \qquad g_{\mathrm{E}}^2=g^2T+\ldots$$

and at asymptotically hight T, three energy scales develop

$$\frac{g_{\mathrm{E}}^2}{\pi}\ll m_{\mathrm{E}}\ll \pi T$$

Effective field theories at large T: MQCD

- For Physics at energies $E = O(g_{\mathrm{E}}^2)$, the scalar field can be integrated out, and one is left with Magnetostatic QCD (MQCD)

$$S_{\mathrm{MQCD}} = \frac{1}{g_{\mathrm{E}}^2} \int d^3x \left\{ \frac{1}{2} \mathrm{Tr}\left[F_{ij}F_{ij}\right] \right\} + \ldots$$

- This is a 3D Yang–Mills theory which needs to be solved NP. All dimensionful quantities proportional to appropriate power of g_{E}^2

- As a result, at asymptotically high T the mass gap developed by thermal QCD is proportional to $g_{\mathrm{E}}^2 = g^2 T + \ldots$

- Quarks have very heavy masses $M = \pi T(1 + \frac{g^2}{6\pi^2} + \ldots)$, and can be considered, in first approximation, as static fields

Renormalization

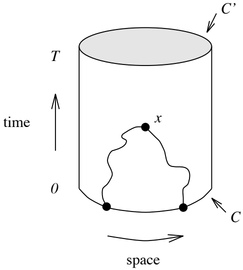
Hadronic renormalization scheme is not a viable option because

$$M_{\mathrm{hadron}} \ll T$$

Accommodating 2 very different scales on a lattice is too expensive

Way to go is the NP renormalization of the coupling:

[Lüscher et al 91]



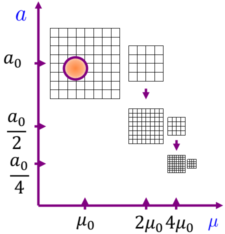
- Define the renormalized g^2 NP, e.g. SF or GF couplings
- Define quark masses NP by WIs
- Avoid zero-temperature subtraction in renormalization of fields by adopting shifted boundary conditions, e.g. Equation of State

Renormalization

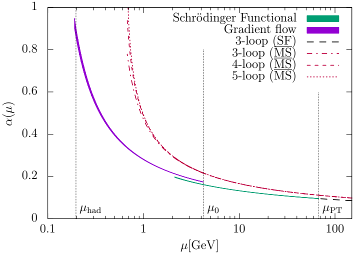
- Relate $g^2(\mu_{\rm hadron})$ to $M_{\rm hadron}$ NP
- Determine running of $g^2(\mu)$ NP
- Compute $g^2(\mu)$ for μ up to electroweak scale
- For each value of T, renormalize thermal QCD by requiring

$$\bar{g}^2_{\rm SF}(g_0^2, a\mu) = \bar{g}^2_{\rm SF}(\mu)$$

with $a\mu \ll 1$ and $T \sim \mu$



[Bruno et al. 17]



- Last condition fixes the dependence of the bare g_0^2 on a, for values of a at which μ and T can be easily accommodated

Lattice setup

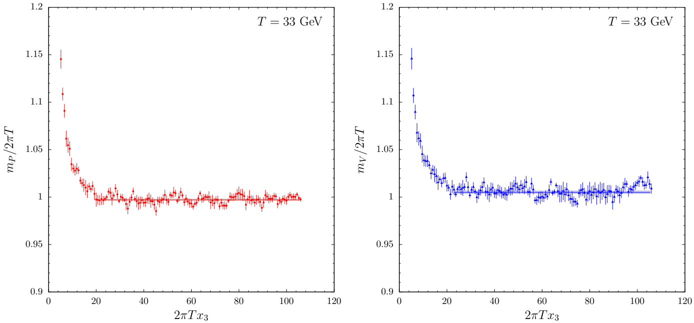
- Wilson (T_0–T_8) and Lüscher–Weisz (T_9–T_{11}) actions for gluons
- NP $O(a)$-improved Wilson quarks
- Four lattice spacings for each T, $L_0/a = 4, 6, 8$ and 10
- Shifted boundary conditions
- Restriction to zero topology thanks to the high temperature

T	$\bar{g}^2_{\rm SF}(\mu = T\sqrt{2})$	T (GeV)
T_0	–	164.6(5.6)
T_1	1.11000	82.3(2.8)
T_2	1.18446	51.4(1.7)
T_3	1.26569	32.8(1.0)
T_4	1.3627	20.63(63)
T_5	1.4808	12.77(37)
T_6	1.6173	8.03(22)
T_7	1.7943	4.91(13)
T_8	2.0120	3.040(78)

T	$\bar{g}^2_{\rm GF}(\mu = T/\sqrt{2})$	T (GeV)
T_9	2.7359	2.833(68)
T_{10}	3.2029	1.821(39)
T_{11}	3.8643	1.167(23)

- The linear extension of spatial directions is $L/a = 288$, i.e. $20 < LT < 50$. Finite volume effects negligible given the mass gap. Explicitly checked at the highest and lowest temperature

Screening mass definition



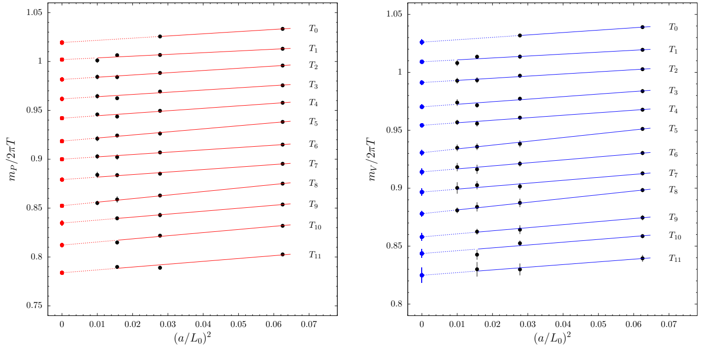
From the two-point correlators $[\mathcal{O} = \{S, P, V_\mu, A_\mu\}]$

$$C_{\mathcal{O}}(x_3) = a^3 \sum_{x_0, x_1, x_2} \langle \mathcal{O}^a(x) \mathcal{O}^a(0) \rangle$$

screening masses are defined as

$$a m_{\mathcal{O}}(x_3) = \mathrm{arcosh}\left[\frac{C_{\mathcal{O}}(x_3 + a) + C_{\mathcal{O}}(x_3 - a)}{2\, C_{\mathcal{O}}(x_3)}\right]$$

Continuum limit



The tree-level improved definitions

$$m_{\mathcal{O}} \to m_{\mathcal{O}} - \left[m_{\mathcal{O}}^{\text{free}} - 2\pi T \right]$$

have been extrapolated to the continuum linearly in $(a/L_0)^2$

Results for mesonic screening masses

Effective theory + NLO matching predict

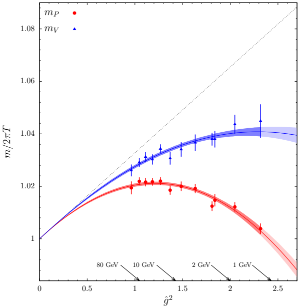
$$m_{\mathcal{O}}^{\rm PT} = 2\pi T \left(1 + p_2^{\rm PT} g^2\right)$$

where $p_2^{\rm PT} = 0.03274$. In particular m_P and m_v are degenerate

NP Results can be fitted by a quartic polynomial in

$$\frac{1}{\hat{g}^2(T)} \equiv \frac{9}{8\pi^2} \ln \frac{2\pi T}{\Lambda_{\overline{\rm MS}}} + \frac{4}{9\pi^2} \ln \left(2 \ln \frac{2\pi T}{\Lambda_{\overline{\rm MS}}} \right)$$

where for our purpose this is a function of T designed to coincide with the NLO inverse coupling in the $\overline{\rm MS}$ scheme



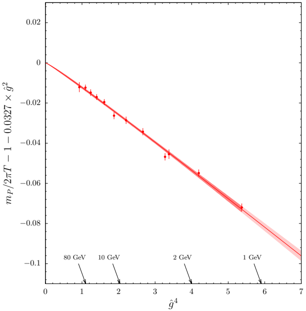
Masses non-degenerate even at electroweak scale!

Discussion and interpretation

Pseudoscalar mass:

$$\frac{m_P}{2\pi T} = 1 + p_2^{\mathrm{PT}}\,\hat{g}^2 + p_3\,\hat{g}^3 + p_4\,\hat{g}^4$$

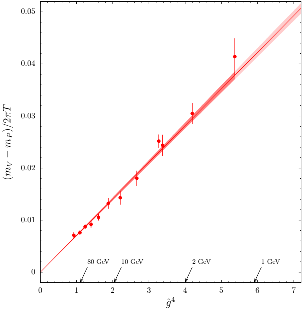
p_3=0.0038(22) and p_4=−0.0161(17)



Pseudoscalar-vector mass difference:

$$\frac{(m_V - m_P)}{2\pi T} = s_4\,\hat{g}^4$$

$s_4 = 0.00704(14)$

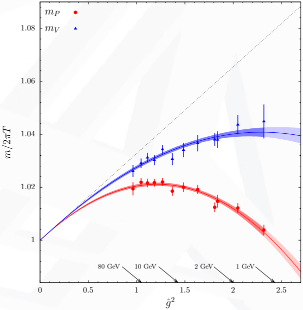


An effective $\hat{g}^4$ term explain the difference with PT in both cases over 2 orders of magnitude in T!

Conclusions and Outlook

Possible to simulate thermal QCD for T up to the electroweak scale

First NP results compatible with effective field theory expectations but not with NLO matching



The strategy proposed here opens the way to study many other properties of thermal QCD in the high temperature regime:

- Equation of State
- Baryon masses
- Transport coefficients
- .....