Effect of the N3LO three-nucleon contact interactions on p-d scattering observables

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December 19, 2022

Summary

Nuclear physics goal

- To arrive at a fundamental understanding of nuclear properties using a unified theoretical perspective with fundamental forces among nucleons
  - To achieve predictive power and control of uncertainties

- Two major obstacles:
  - Interactions among nucleons are not known precisely $\Rightarrow$ Chiral EFT
  - Many-body problem extremely hard to solve

Our purpose is to study nuclear systems with interactions derived from Chiral EFT and with an *ab initio* method, that is, by solving the Schroedinger equation

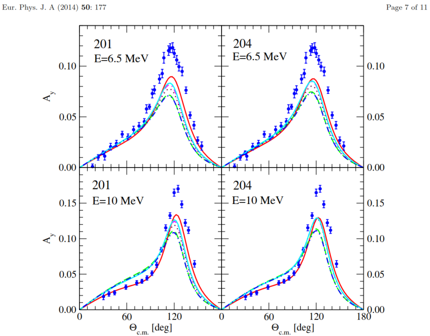
Motivations

- A more sophisticated version of the three-body force used so far is needed to
  - Equation of state for neutron matter ⇒ Neutron stars ⇒ Gravitational waves
  - Structure of nuclei
  - Calculations on the fusion between polarized nuclei (d-d, p-7Li, p-11B) [Lecce-Pisa group]

  - Long-standing discrepancies: The N-d Ay puzzle

$$A_y \equiv \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow}$$

where $\sigma_\uparrow$ and $\sigma_\downarrow$ denote the differential cross section with the spin of the incoming nucleon oriented normal to the scattering plane



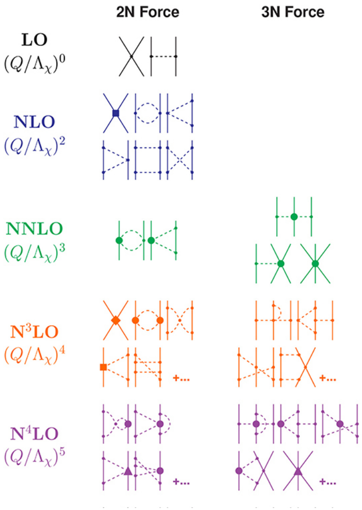
[J. Golak et al. Eur. Phys. J. A 50 (2014) 177]

Chiral effective field theory

EFT based on an approximate QCD symmetry: the Chiral symmetry [Weinberg, 1979]

- Explicit degrees of freedom: π, N
  - Nucleon and pion fields $N = \begin{pmatrix} p \\ n \end{pmatrix}$ $\quad \vec{\pi} = (\pi_x, \pi_y, \pi_z)$
- Bottom up approach: Symmetries require the operator structure
- Perturbative expansion in a power $(Q/\Lambda_\chi)^\nu$
  - ν =chiral order, $Q \sim m_\pi$ = typical value of nucleon momentum inside a nucleus, $\Lambda_\chi = 4\pi f_\pi \sim 1$ GeV
  - $\mathcal{L} = \sum_\nu \mathcal{L}^{(\nu)} = \mathcal{L}^{(0)}_{NN} + \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{NN} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(2)}_{\pi\pi} + \dots$
- Low-energy constants (LECs) absorb short-range physics
  - Now determined from experimental data

Chiral EFT potentials

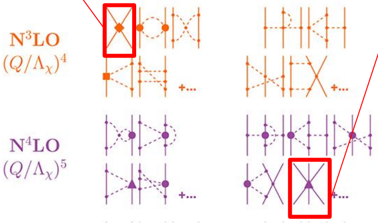


Chiral EFT potentials

$$V^{(4)} = D_1 k^4 + D_2 Q^4 + D_3 k^2 Q^2 + D_4 (\boldsymbol{k}\times\boldsymbol{Q})^2 + \left(D_5 k^4 + D_6 Q^4 + D_7 k^2 Q^2 + D_8 (\boldsymbol{k}\times\boldsymbol{Q})^2\right)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$$
$$+ \frac{i}{2}\left(D_9 k^2 + D_{10} Q^2\right)(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\boldsymbol{Q}\times\boldsymbol{k}) + \left(D_{11}k^2 + D_{12}Q^2\right)(\boldsymbol{\sigma}_1\cdot\boldsymbol{k})(\boldsymbol{\sigma}_2\cdot\boldsymbol{k}) + \left(D_{13}k^2 + D_{14}Q^2\right)(\boldsymbol{\sigma}_1\cdot\boldsymbol{Q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{Q})$$
$$+ D_{15}\boldsymbol{\sigma}_1\cdot(\boldsymbol{k}\times\boldsymbol{Q})\boldsymbol{\sigma}_2\cdot(\boldsymbol{k}\times\boldsymbol{Q}) + iD_{16}\boldsymbol{k}\cdot\boldsymbol{Q}\boldsymbol{Q}\times\boldsymbol{P}\cdot(\sigma_1-\sigma_2) + D_{17}\boldsymbol{k}\cdot\boldsymbol{Q}(\boldsymbol{k}\times\boldsymbol{P})\cdot(\boldsymbol{\sigma_1}\times\boldsymbol{\sigma_2})$$

$$V = \sum_{i\neq j\neq k}\Big[-E_1\mathbf{k}_i^2 - E_2\mathbf{k}_i^2\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j - E_3\mathbf{k}_i^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j - E_4\mathbf{k}_i^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j - E_5\left(3\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_i\cdot\boldsymbol{\sigma}_j - \mathbf{k}_i^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right)$$
$$- E_6\left(3\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_i\cdot\boldsymbol{\sigma}_j - \mathbf{k}_i^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right)\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j + \frac{i}{2}E_7\mathbf{k}_i\times(\mathbf{Q}_i-\mathbf{Q}_j)\cdot(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)$$
$$+ \frac{i}{2}E_8\mathbf{k}_i\times(\mathbf{Q}_i-\mathbf{Q}_j)\cdot(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k - E_9\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_j\cdot\boldsymbol{\sigma}_j - E_{10}\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_j\cdot\boldsymbol{\sigma}_j\tau_i\cdot\boldsymbol{\tau}_j$$
$$-E_{11}\mathbf{k}_i\cdot\boldsymbol{\sigma}_j\mathbf{k}_j\cdot\boldsymbol{\sigma}_i - E_{12}\mathbf{k}_i\cdot\boldsymbol{\sigma}_j\mathbf{k}_j\cdot\boldsymbol{\sigma}_i\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j - E_{13}\mathbf{k}_i\cdot\boldsymbol{\sigma}_j\mathbf{k}_j\cdot\boldsymbol{\sigma}_i\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k\Big]$$

with $\boldsymbol{k}_i = \boldsymbol{p}_i' - \boldsymbol{p}_i, \quad \boldsymbol{Q}_i = \frac{\boldsymbol{p}_i+\boldsymbol{p}_i'}{2}$



Unitarity Transformations

$$U = \mathrm{e}^{\alpha_i T_i}$$

$$T_1 = \int d^3x N^\dagger \overleftrightarrow{\nabla}^i N \nabla^i \left(N^\dagger N\right)$$

$$T_2 = \int d^3\mathbf{x} N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^i \left(N^\dagger \sigma^j N\right)$$

$$T_3 = \int d^3x \left[N^\dagger \overleftrightarrow{\nabla}^i \sigma^i N \nabla^j \left(N^\dagger \sigma^j N\right) + N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^j \left(N^\dagger \sigma^i N\right)\right]$$

$$T_4 = i\epsilon^{ijk} \int d^3x N^\dagger \overleftrightarrow{\nabla}^i N N^\dagger \overleftrightarrow{\nabla}^j \sigma^k N$$

$$T_5 = \int d^3x \left[N^\dagger \overleftrightarrow{\nabla}^i \sigma^i N \nabla^j \left(N^\dagger \sigma^j N\right) - N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^j \left(N^\dagger \sigma^i N\right)\right]$$

$$U^\dagger H_0 U = H_0 + \alpha_i [H_0, T_i] + \cdots \equiv H_0 + \alpha_i \delta_i H_0 + \cdots \Rightarrow$$ 

$$U^\dagger H_{C_S/C_T}\Big($$  $$\Big) U = H_{C_S/C_T} + \alpha_i [H_{C_S/C_T}, T_i] + \cdots \equiv H_{C_S/C_T} + \alpha_i \delta_i H_{C_S/C_T} + \cdots \Rightarrow$$ 

α_i are related with the D_i and E_i LECs

$$\delta E_1 = \alpha_1 (C_S + C_T) + \alpha_2 (C_S - 2C_T)$$
$$\delta E_2 = 3\alpha_2 C_T + 2\alpha_3 C_T - 8\alpha_4 C_T + 2\alpha_5 C_T$$
$$\delta E_3 = 2\alpha_1 C_T + \alpha_2 (2C_S - C_T) + \frac{2}{3}\alpha_3 (2C_S - C_T) + 8\alpha_4 C_T - 2\alpha_5 C_T,$$
$$\delta E_4 = \frac{2}{3}\alpha_1 C_T + \frac{1}{3}\alpha_2 (2C_S - 7C_T) - \frac{2}{3}\alpha_3 C_T + \frac{8}{3}\alpha_4 C_T - \frac{2}{3}\alpha_5 C_T,$$
$$\delta E_5 = 2\alpha_1 C_T + 2\alpha_2 (C_S - 2C_T) + \frac{2}{3}\alpha_3 (2C_S - C_T) + 8\alpha_4 C_T - 2\alpha_5 C_T,$$
$$\delta E_6 = \frac{2}{3}\alpha_1 C_T + \frac{2}{3}\alpha_2 (C_S - 2C_T) - \frac{2}{3}\alpha_3 C_T + \frac{8}{3}\alpha_4 C_T - \frac{2}{3}\alpha_5 C_T,$$
$$\delta E_7 = 24\alpha_4 C_T,$$
$$\delta E_8 = \frac{1}{3}\delta E_7,$$
$$\delta E_9 = 3\alpha_1 C_T + 3\alpha_2 (C_S - 2C_T) + 2\alpha_3 (C_S - 2C_T) - \alpha_4 (C_S - 11C_T) + 2\alpha_5 (C_S - 2C_T),$$
$$\delta E_{10} = \alpha_1 C_T + \alpha_2 (C_S - 2C_T) - \frac{1}{3}\alpha_4 (3C_S - 15C_T),$$
$$\delta E_{11} = 3\alpha_1 C_T + 3\alpha_2 (C_S - 2C_T) + 2\alpha_3 (C_S - 2C_T) + \alpha_4 (C_S - 11C_T) - 2\alpha_5 (C_S - 2C_T),$$
$$\delta E_{12} = \alpha_1 C_T + \alpha_2 (C_S - 2C_T) + \frac{1}{3}\alpha_4 (3C_S - 15C_T)$$
$$\delta E_{13} = -16\alpha_4 C_T + 4\alpha_5 C_T$$

$$\delta D_3 = -\frac{4}{m}\alpha_1$$
$$\delta D_4 = \frac{4}{m}\alpha_1$$
$$\delta D_7 = -\frac{4}{m}\alpha_2$$
$$\delta D_8 = \frac{4}{m}\alpha_2 + \frac{2}{m}\alpha_3$$
$$\delta D_{15} = -\frac{4}{m}\alpha_3$$
$$\delta D_{12} = -\frac{4}{m}\alpha_3$$
$$\delta D_{13} = -\frac{4}{m}\alpha_3$$
$$\delta D_{17} = -\frac{4}{m}\alpha_3 - \frac{2}{m}\alpha_5$$
$$\delta D_{16} = -\frac{2}{m}\alpha_4$$

Therefore:

- It is possible to remove 5 D_i LECs from the NN contact potential as long as we consider the shifts δE_i as an effect at N3LO
- The five LECs parametrizing the N3LO NN off-shell interaction can be fitted to observables of the 3N system and interpreted as a 3N interaction

$$\Downarrow$$

Using the relations between the δE_i and α_i, we tried to fit the α parameters to the p-D scattering observables

As a first step, we used and hybrid model with a phenomenological two body potential (the Av18) and the δE_i N3LO terms

Bound and scattering wave functions

The ^3H wave function is written as an expansion over spin-isospin-Hyperspherical Harmonics (HH) states times hyperradial functions, which are themselves expanded on a basis of Laguerre polynomials,

$$\Psi = \sum_{\mu} c_\mu \phi_\mu \tag{1}$$

where μ denotes collectively the quantum numbers specifying the combination ϕ_μ of spin-isospin-HH states.

The Rayleigh-Ritz variational principle,

$$\delta \langle \Psi | H - E | \Psi \rangle = 0$$

is used to determine the expansion coefficients c_μ and bound state energy E

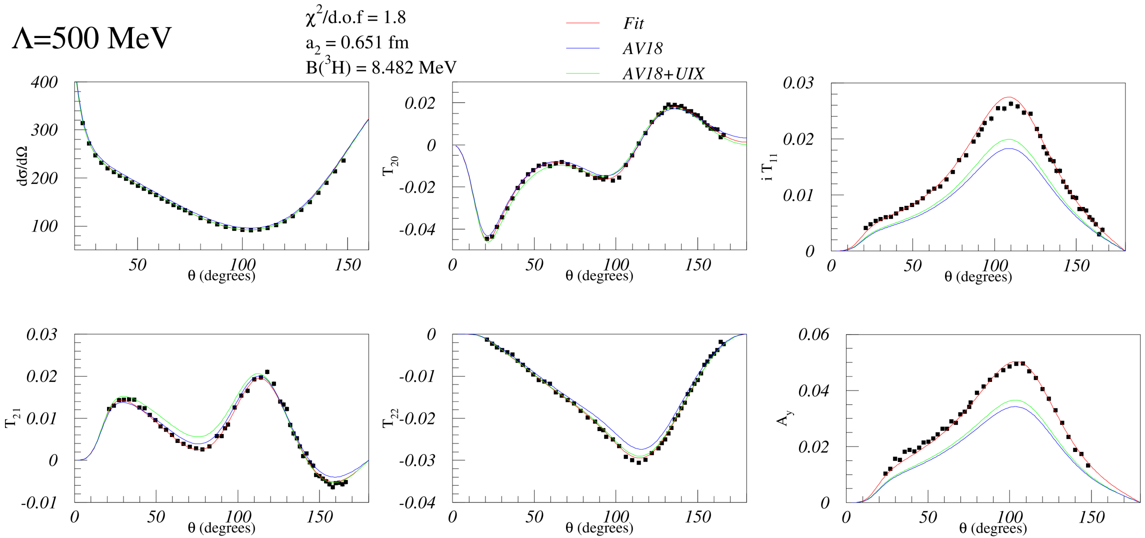
To describe $\mathrm{N-d}$ scattering states below the deuteron breakup threshold the w.f. is taken as

$$\Psi = \Psi_C + \Omega^R + \sum_{\mu} \mathcal{R}_\mu \Omega^I_\mu$$

Ψ_C describes configurations in which all the particles of the system are close to each other and is decomposed as Eq (1), $\Omega^{\lambda=I,R}$ are functions describing the asymptotic region. $\mathcal{R}_\mu$ are the $\mathcal{R}$-matrix elements.

The Ψ_C coefficients c_μ and $\mathcal{R}_\mu$ are determined by using the Kohn variational principle which guarantees that the $\mathcal{R}$-matrix elements, considered as functionals of the w.f., are stationary with respect to variations of all the trial parameters

Hybrid Fit on p-d observables



Conclusions

- There are 5 free LECs in the 3N force at N3LO to improve the description of scattering data
- Preliminary investigations show that the N-d Ay problem could be solved in this way
- Tests with this three-body force for $A > 3$ are running on Cineca, we had in 2022:
  - 660,000 hours for the INFN-Cineca agreement
  - 1,300,000 hours for the IscraB "ddfusion" project

.. and Outlooks

- Use of unitary transformations in the N3LO Chiral potential
  - Add the δE_i N3LO terms
  - Remove the redundant D_i terms
- Calculation of scattering observables and quantitative error estimation