Topology in Lattice QCD

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It is possible to add to the QCD Lagrangian a CP-violating term

q(x)

$$\mathcal{L} = \mathcal{L}_{QCD} + \theta \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a$$

$$\frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a = q(x)$$

$$Q = \sum q(x)$$

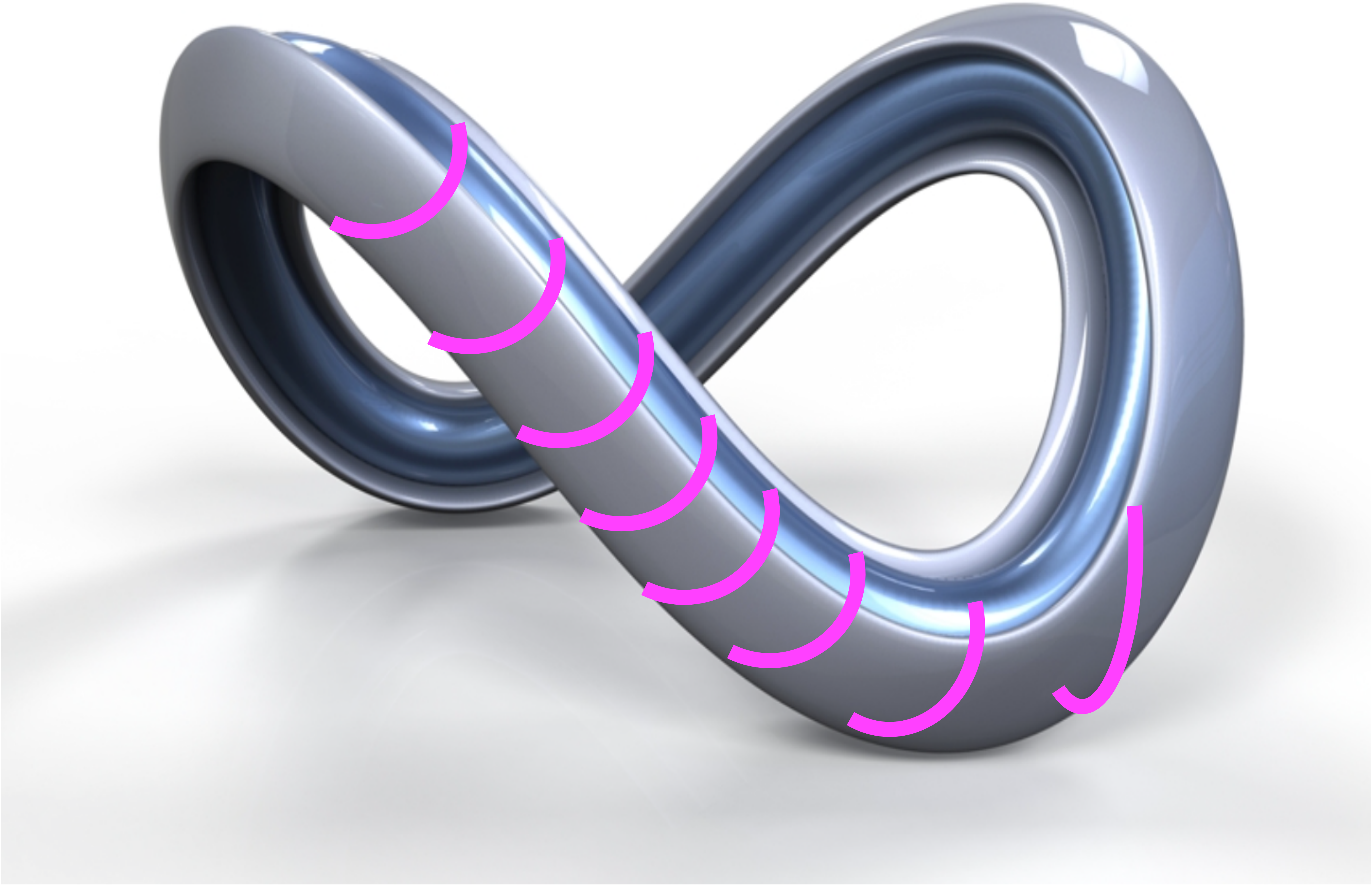
Defines a topological sector

Q — topological charge

CP-violating term

Lattice discretization — see talk by Jana Guenther —

is particularly delicate..
..the very concept of ‘topology’ seems lost



The talks by:

Claudio Bonanno
Francesco D’Angelo

discuss technical aspects and strategies

Outline

Topology in QCD

1. From low to high temperatures — a threshold in the plasma?

2. From low to high densities - experimental results and first theoretical analysis

3. Topology and the QCD axion

Topology, symmetries and spectrum

$$\mathcal{L} = \sum_{a=1}^{n} \bar{q}_{La} \partial\!\!\!/ q_{La} + \bar{q}_{Ra} \partial\!\!\!/ q_{Ra} - m(\bar{q}_{La} q_{La} + \bar{q}_{Ra} q_{Ra}) + \theta \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a + \mathcal{L}_{gauge}$$

With $m = 0$, invariant under
$q_L \to V_L q_L q_R \to V_R q_R$, with $V \in U(n)$

Global symmetry:

$$U(n)_L \times U(n)_R \cong SU(n) \times SU(n) \times U(1)_V \times U(1)_A$$

.

baryon number

Spontaneously Broken

Explicitly broken

Topology, symmetries and spectrum

$$\mathcal{L} = \sum_{a=1}^{n} \bar{q}_{La}\not{\partial} q_{La} + \bar{q}_{Ra}\not{\partial} q_{Ra} - m(\bar{q}_{La}q_{La} + \bar{q}_{Ra}q_{Ra}) + \theta\,\frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a + \mathcal{L}_{gauge}$$

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baryon number

Spontaneously Broken

Explicitly broken

Experimental evidence

$(n^2 - 1)$ pseudoGB

Heavy η'

Topology, η' and the $U_A(1)$ problem:

It can be proven that

$$\frac{1}{32\pi^2}\int d^4x F\tilde{F} = Q$$ Gluonic definition

and

$$Q = n_+ - n_-$$ Fermionic definition

The η' mass may now be computed from the decay of the correlation

Q(x) Q(y)

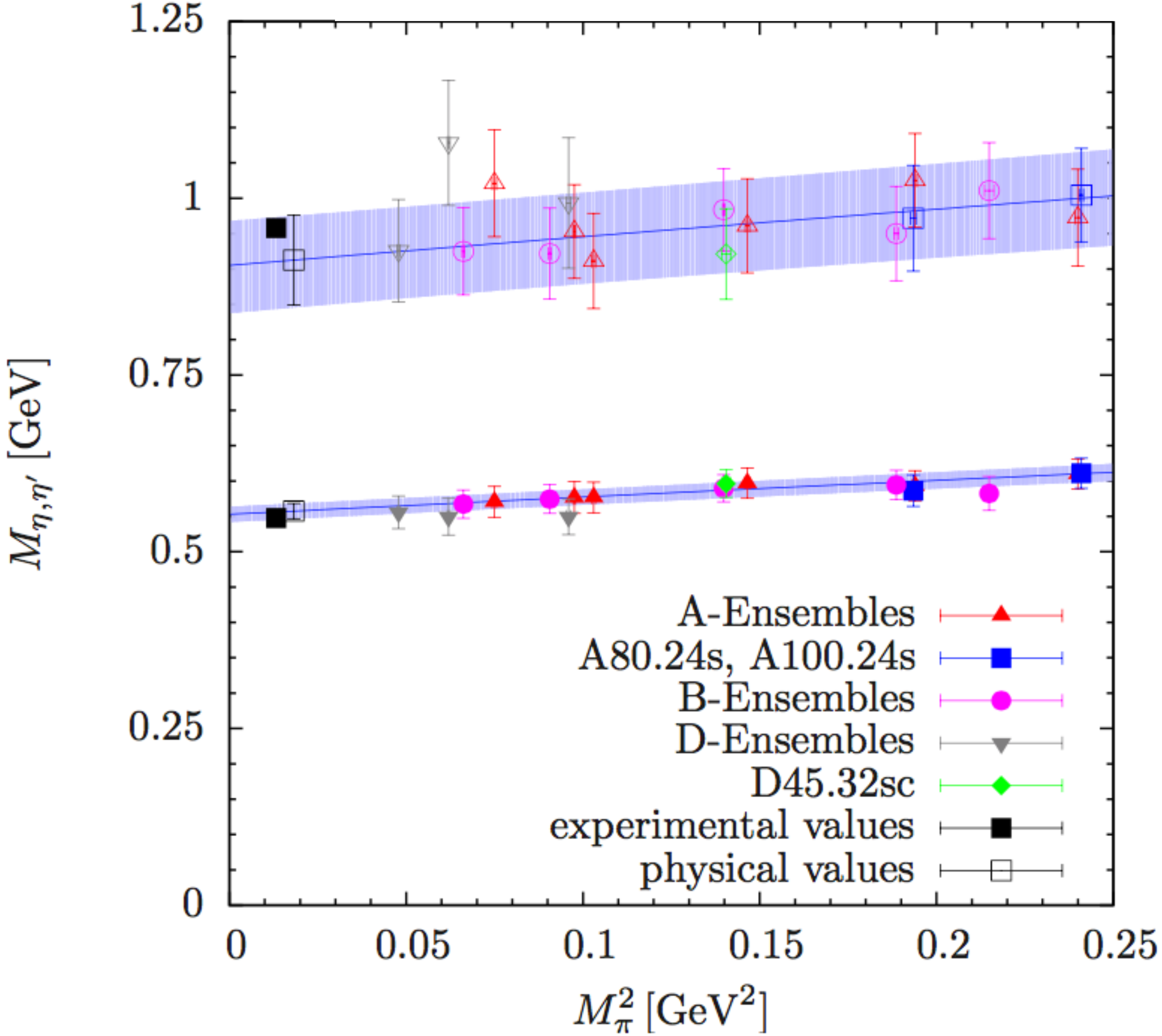
$$\langle \partial_\mu j_5^\mu(x) \partial_\mu j_5^\mu(y) \rangle \propto \frac{1}{N^2} \langle F(x)\tilde{F}(x) F(y)\tilde{F}(y) \rangle$$

which at leading order gives the Witten-Veneziano formula

$$m_{\eta'}^2 = \frac{2N_f}{F_\pi^2}\chi_t^{\rm qu}$$

Successful at T=0

Topology observable effects: EVIDENCES OF THE EXPLICIT $U(1)_A$ BREAKING.

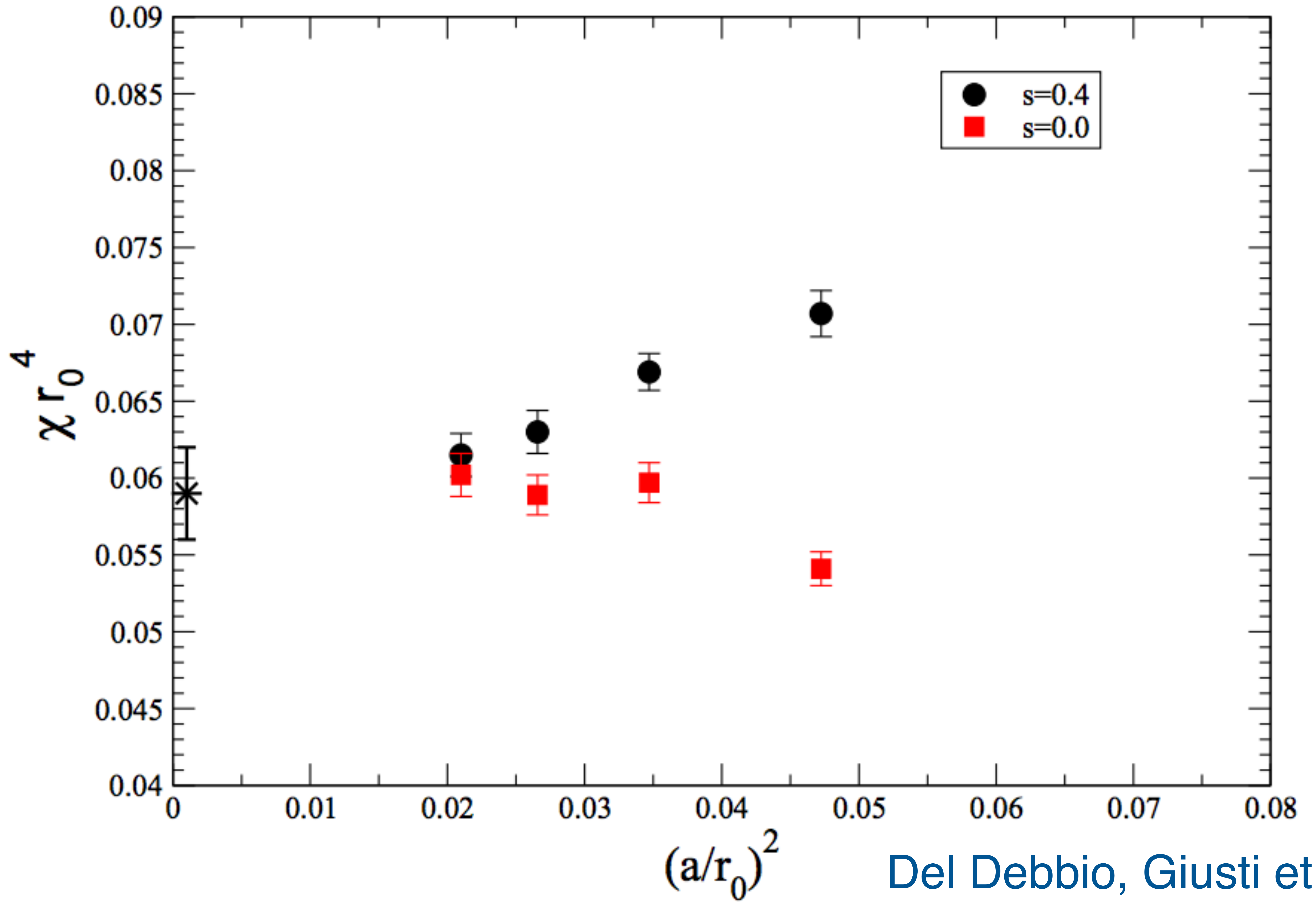


ETMC 2017

Topology observable effects:

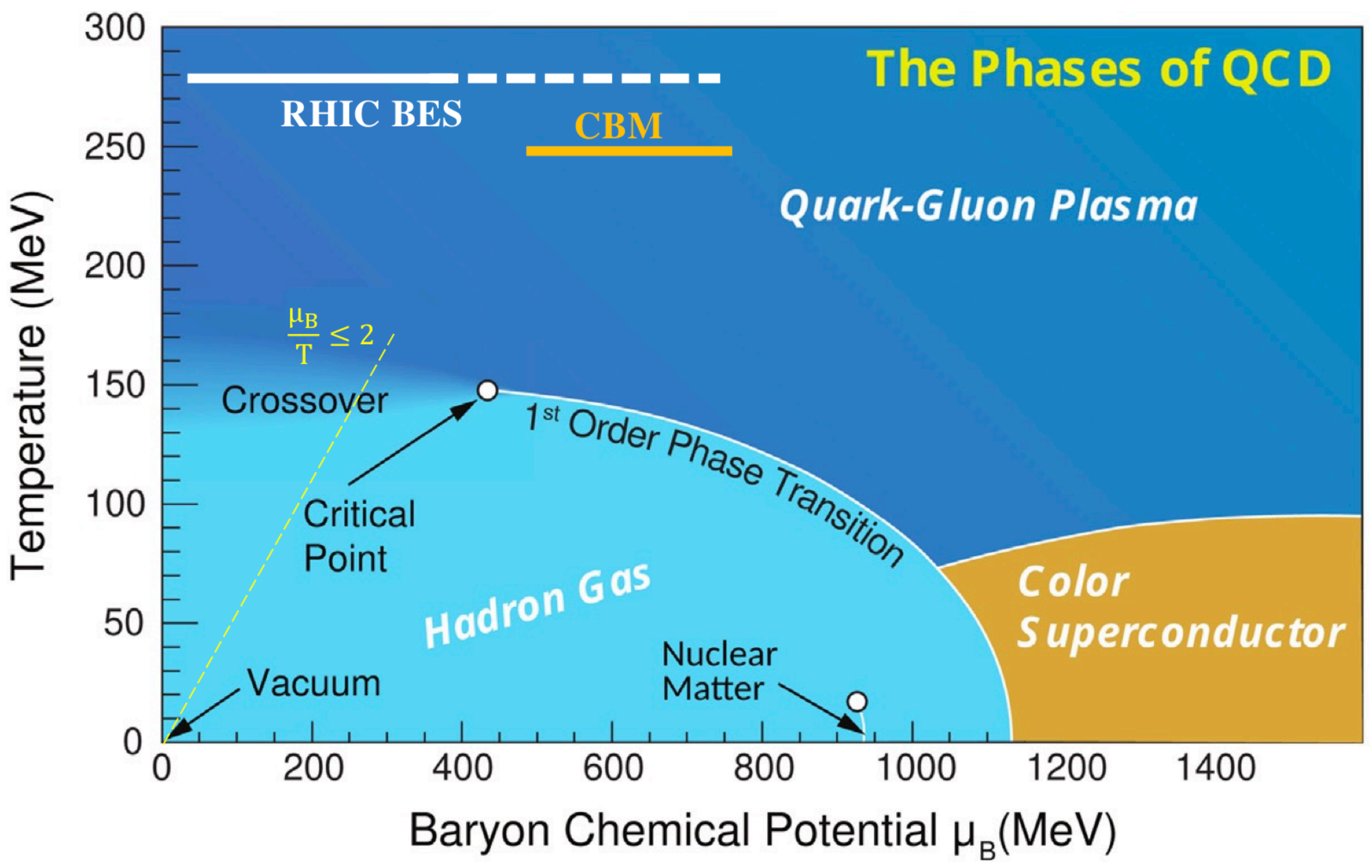
EVIDENCES OF THE EXPLICIT $U(1)_A$ BREAKING.

$\chi = (191 \pm 5\,\mathrm{MeV})^4$, Yang-Mills Topological Susceptibility



Del Debbio, Giusti et al. (2015)

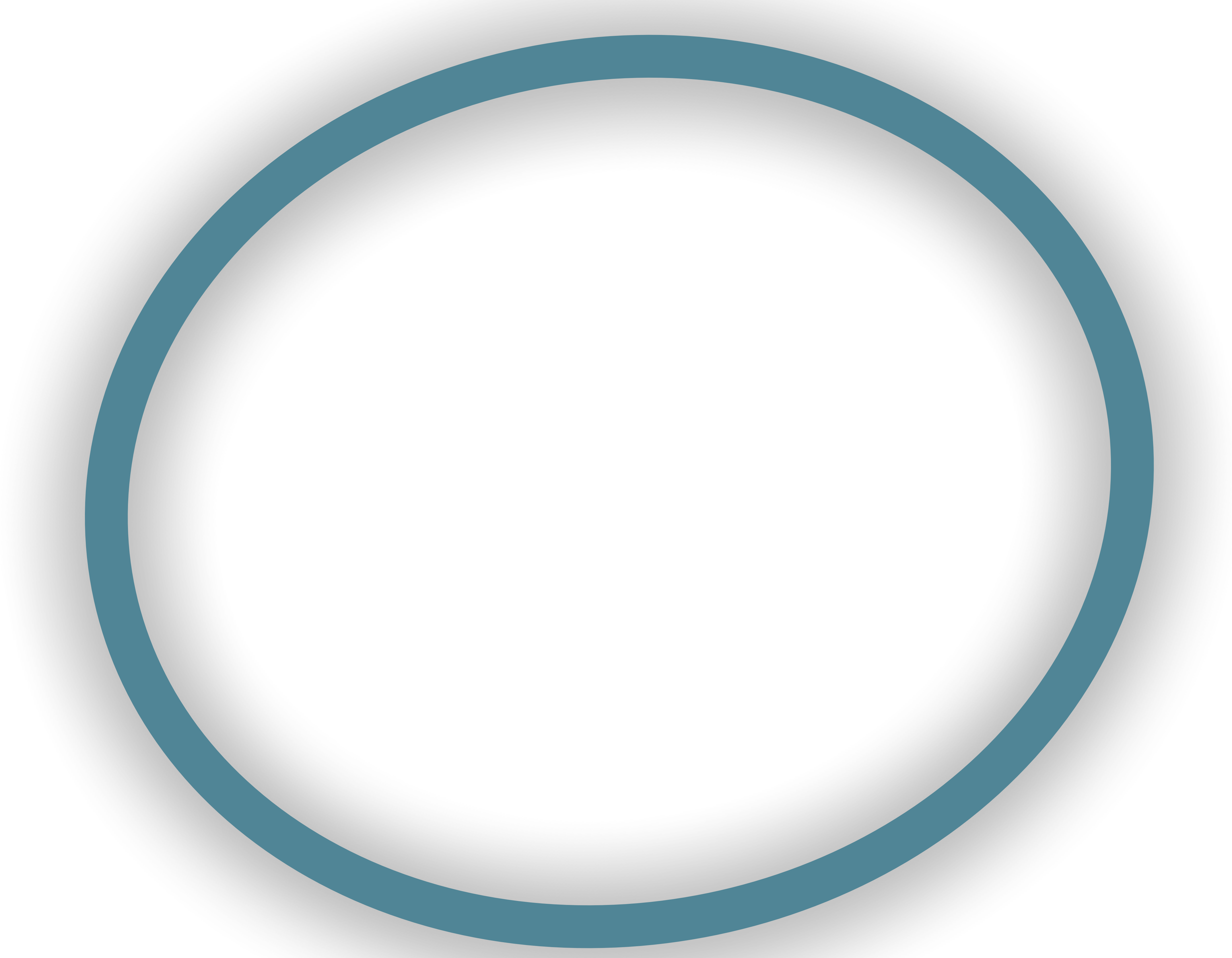
1. From low to high temperatures - A threshold in the plasma ?



Almanool et al. 2022

Topology from low to high Temperature

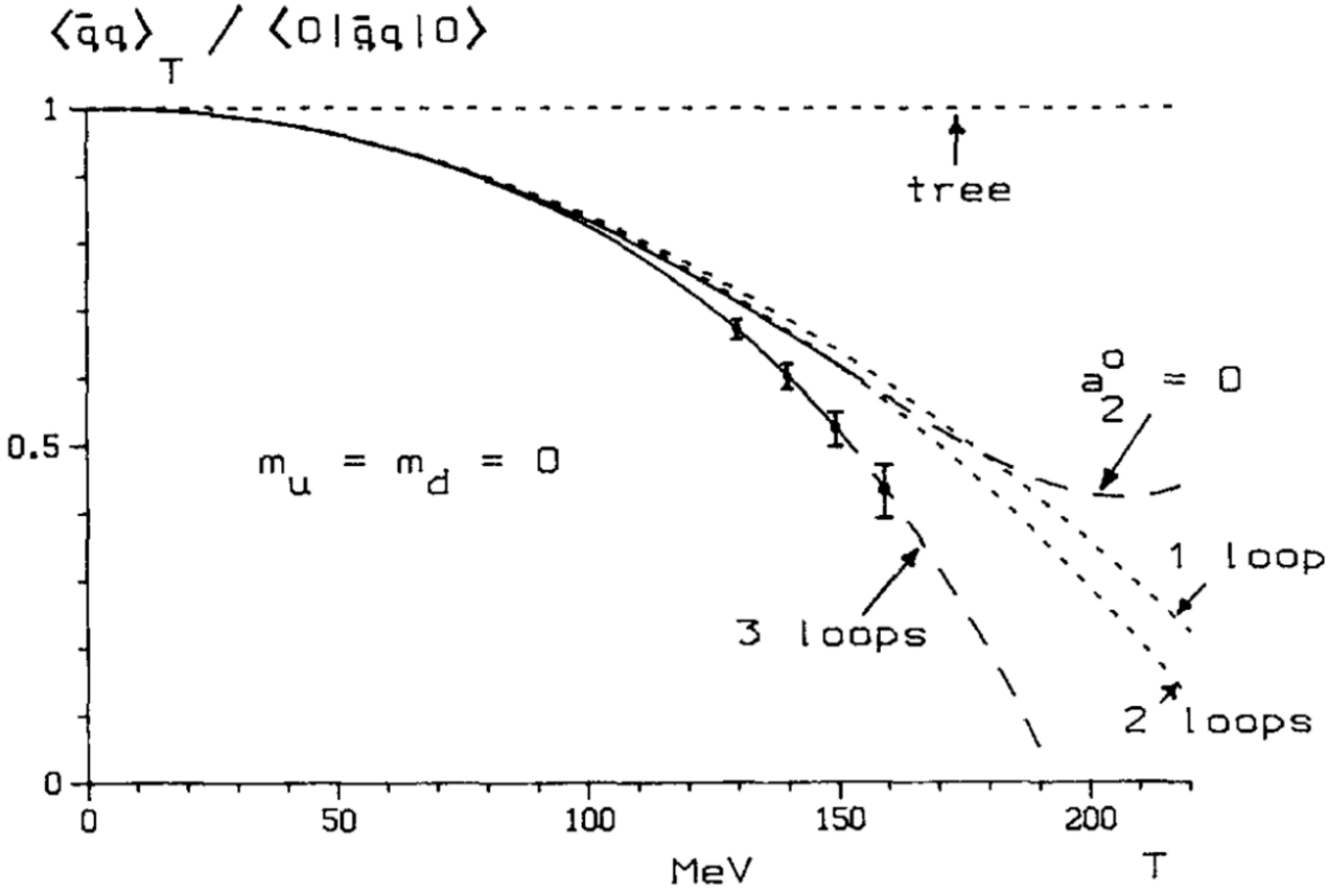
What happens to topology in the Quark Gluon Plasma?



Finite temperature

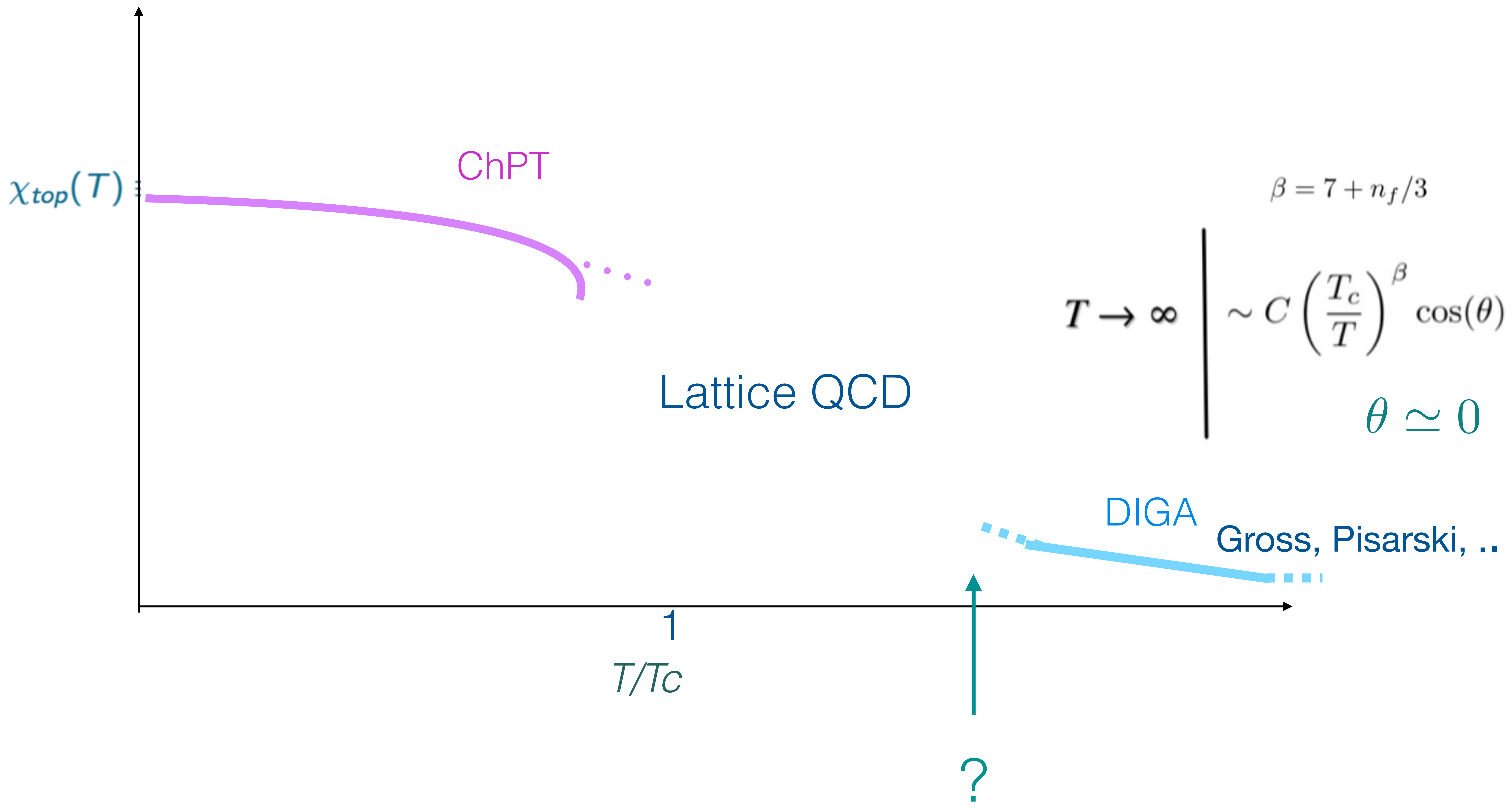
$$\frac{\chi_{top}(T)}{\chi_{top}} \overset{\text{NLO}}{=} \frac{m_\pi^2(T) f_\pi^2(T)}{m_\pi^2 f_\pi^2} = \frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle}$$

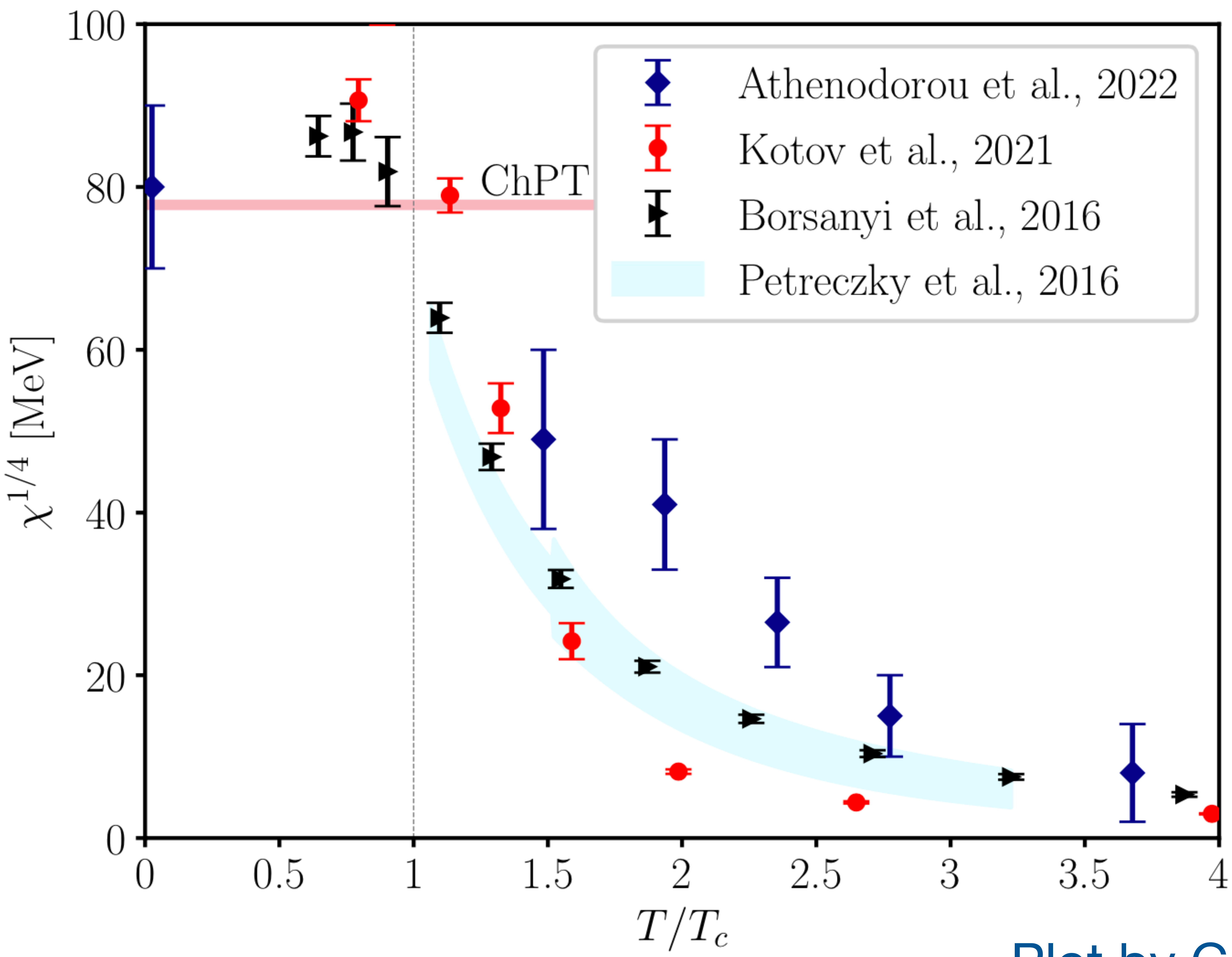
$$\langle \bar{q}q \rangle \overset{m \to 0}{=} \langle 0|\bar{q}q|0\rangle \left(1 - \frac{T^2}{8F^2} - \frac{T^4}{384F^4} - \frac{T^6}{288F^6} \ln\frac{\Lambda_q}{T} + O(T^8)\right)$$



Gasser-Leutwyler 1987-1989

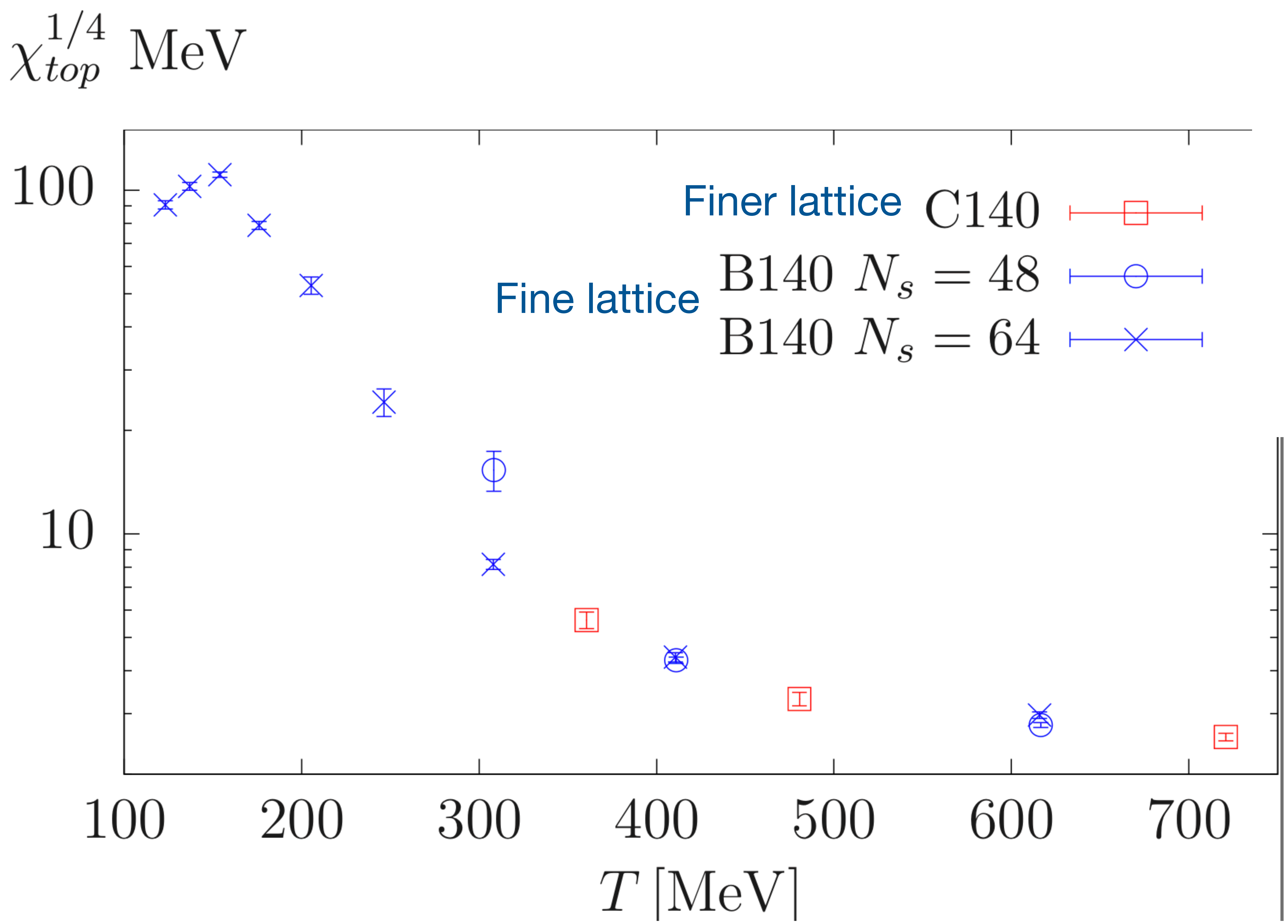
What do we know about $\chi_{top}(T) \equiv \left.\frac{\partial^2 F(\theta, T)}{\partial \theta^2}\right|_{\theta=0}$

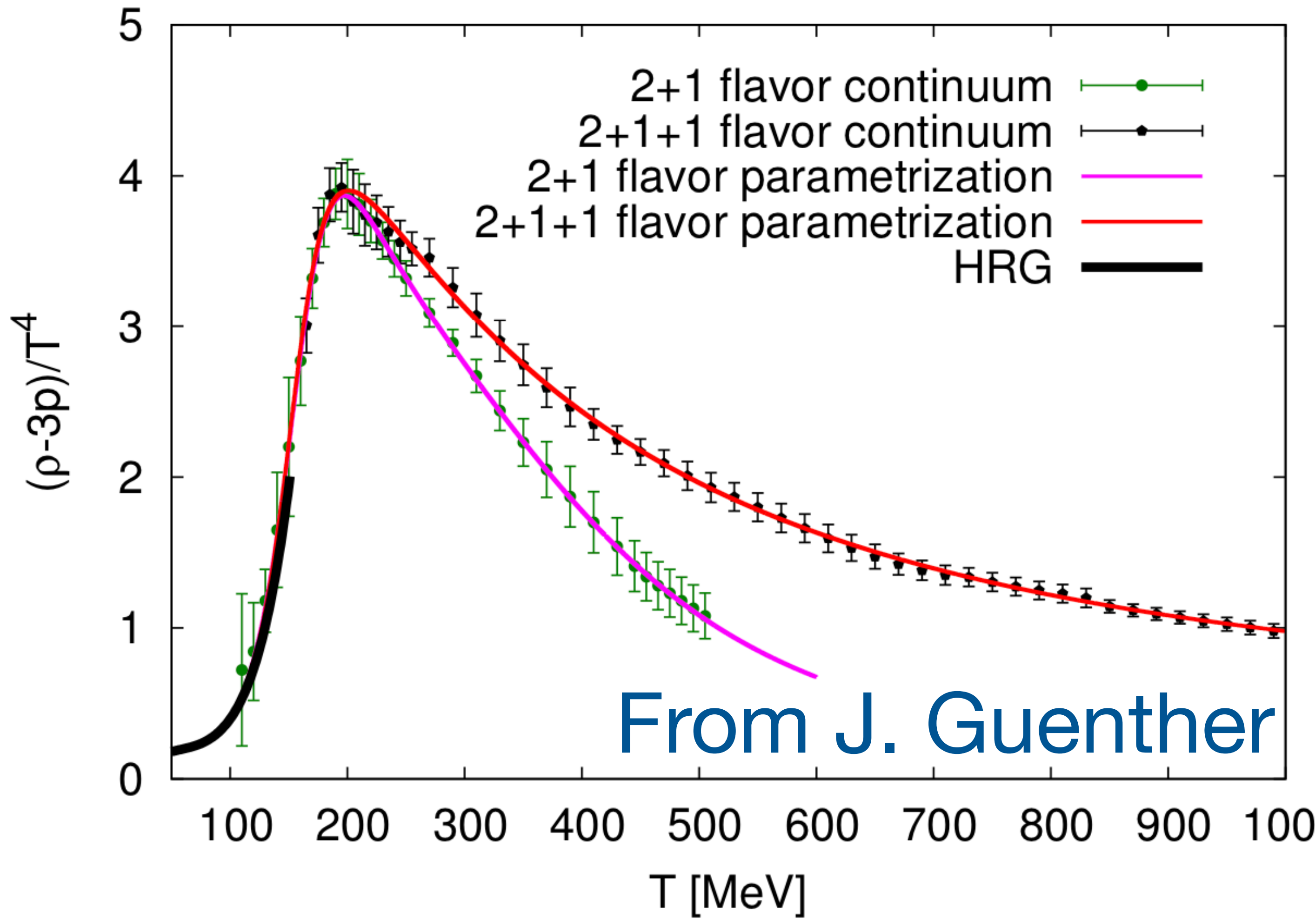




Plot by Claudio Bonanno

Systematics from twisted mass Wilson fermions 2+1+1 flavours

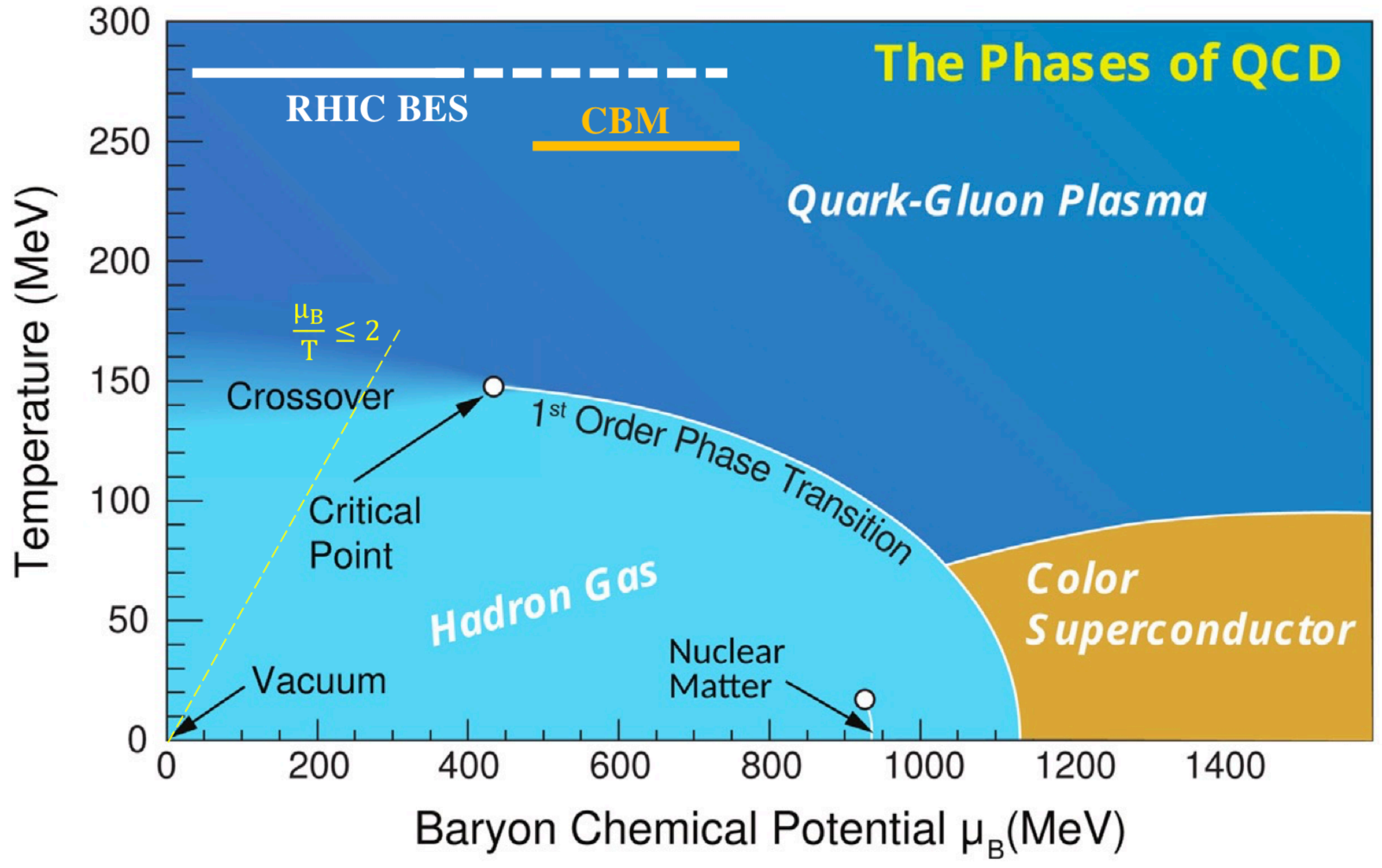




Kotov MpL Trunin (in progress)

A threshold in the plasma ?

Alexandru and Horvath (2019-2021);
Glozman et al; Glozman, Philipsen, Pisarski (2016-2022);
Burger, Kotov, MpL, Trunin (2018-2022..)



Almanool et al., 2022

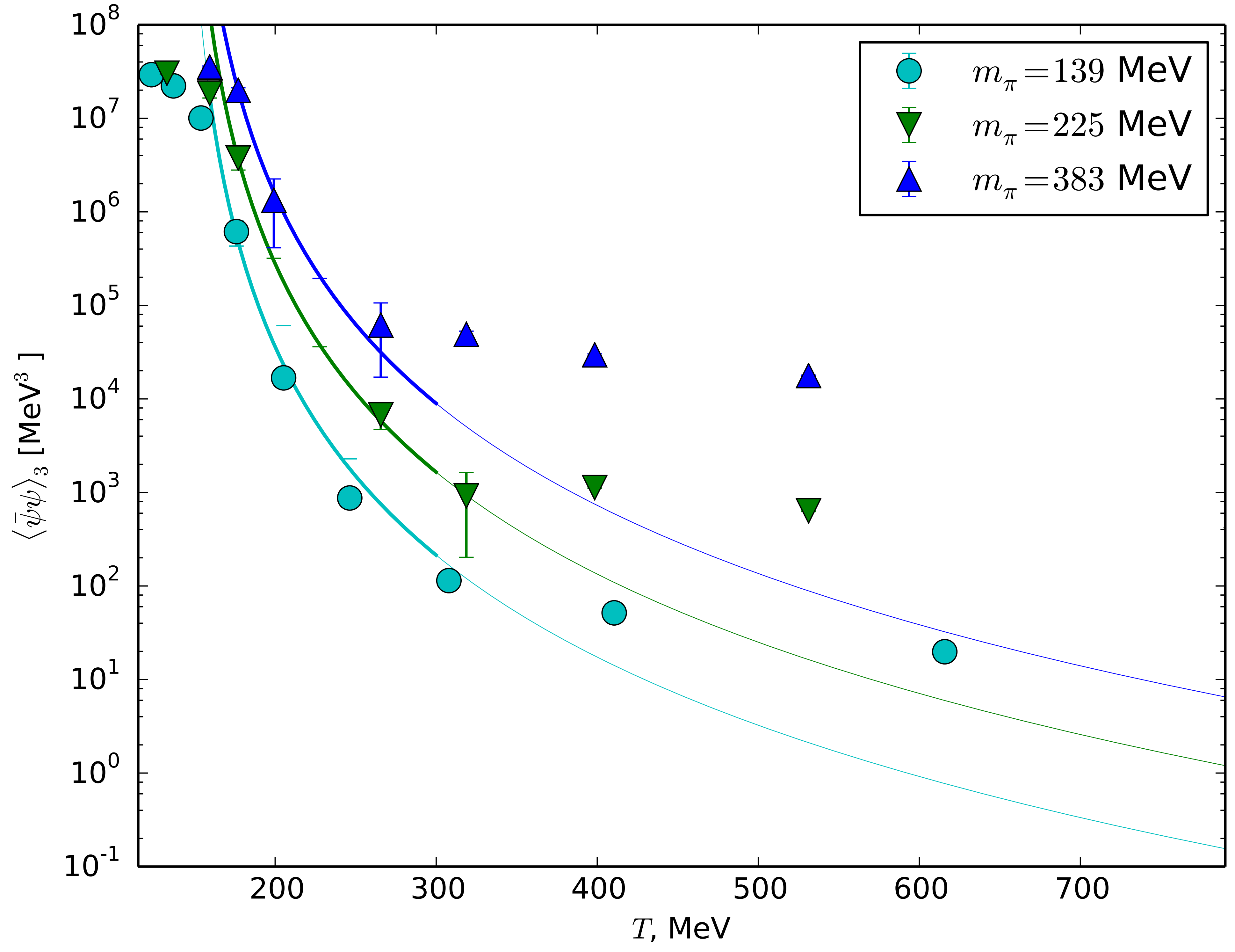
Generic features of a critical region

. Is the ‘threshold’ related with the crossover from the Ginzburg region to mean field ?
. Is the threshold related with topology?

Searching for the scaling window in temperature

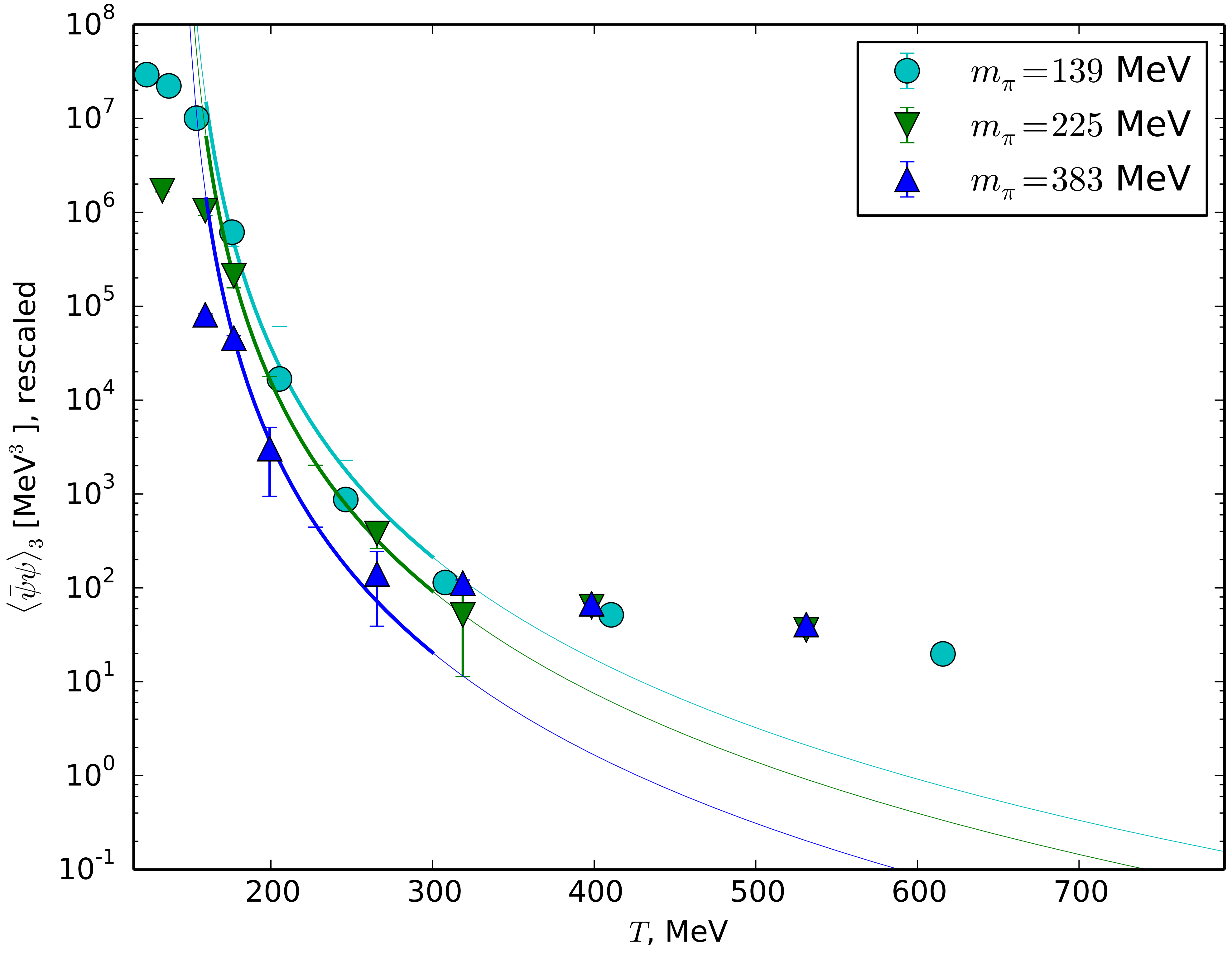
Kotov, MpL, Trunin 2021

Ginzburg region $T < 300$ MeV



$$\Delta_3 \propto t^{-\gamma-2\beta\delta}$$

Simple analytic behaviour $T > 300$ MeV



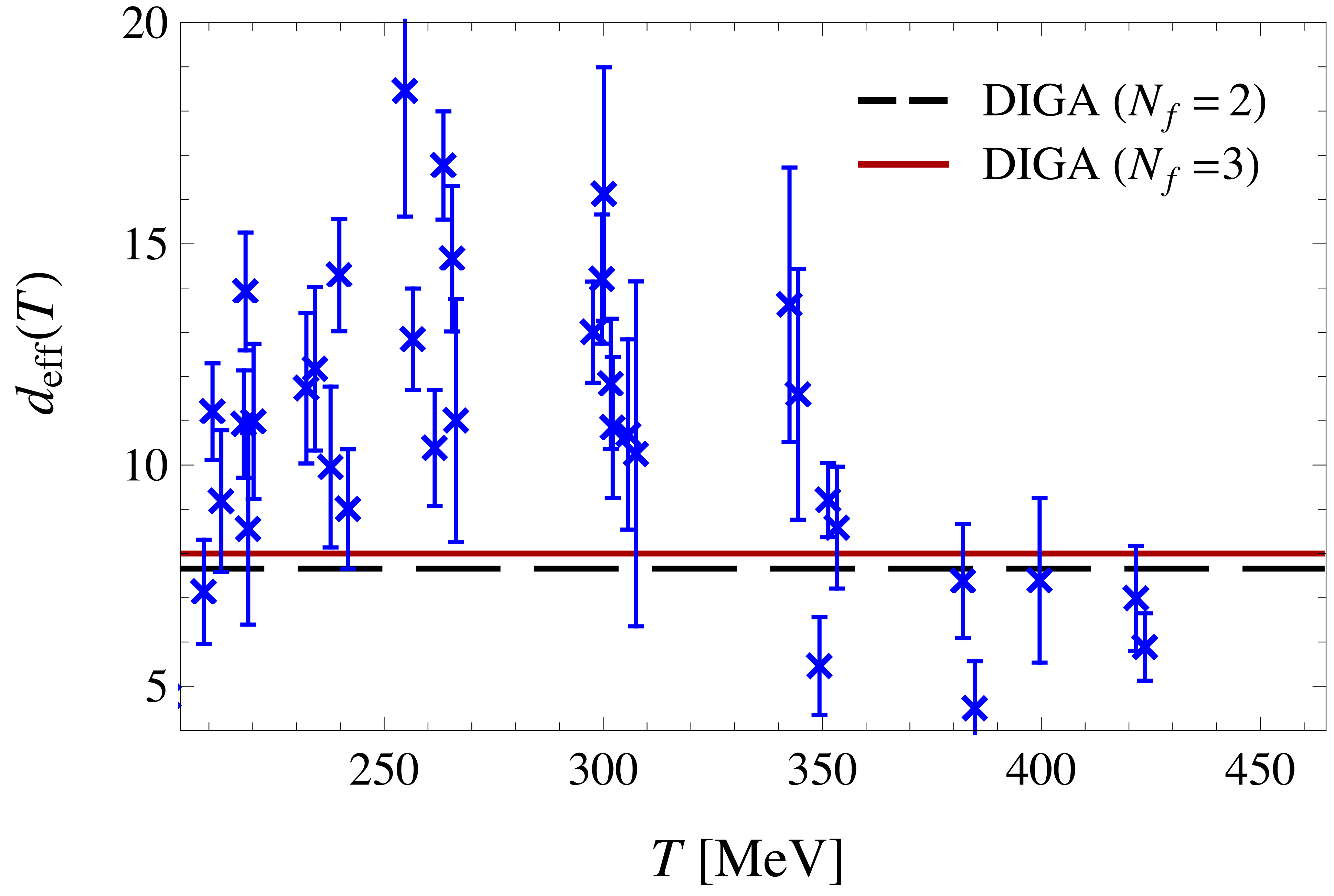
$$\Delta_3 \propto m_\pi^6$$

Power-law decay for T > 300 MeV

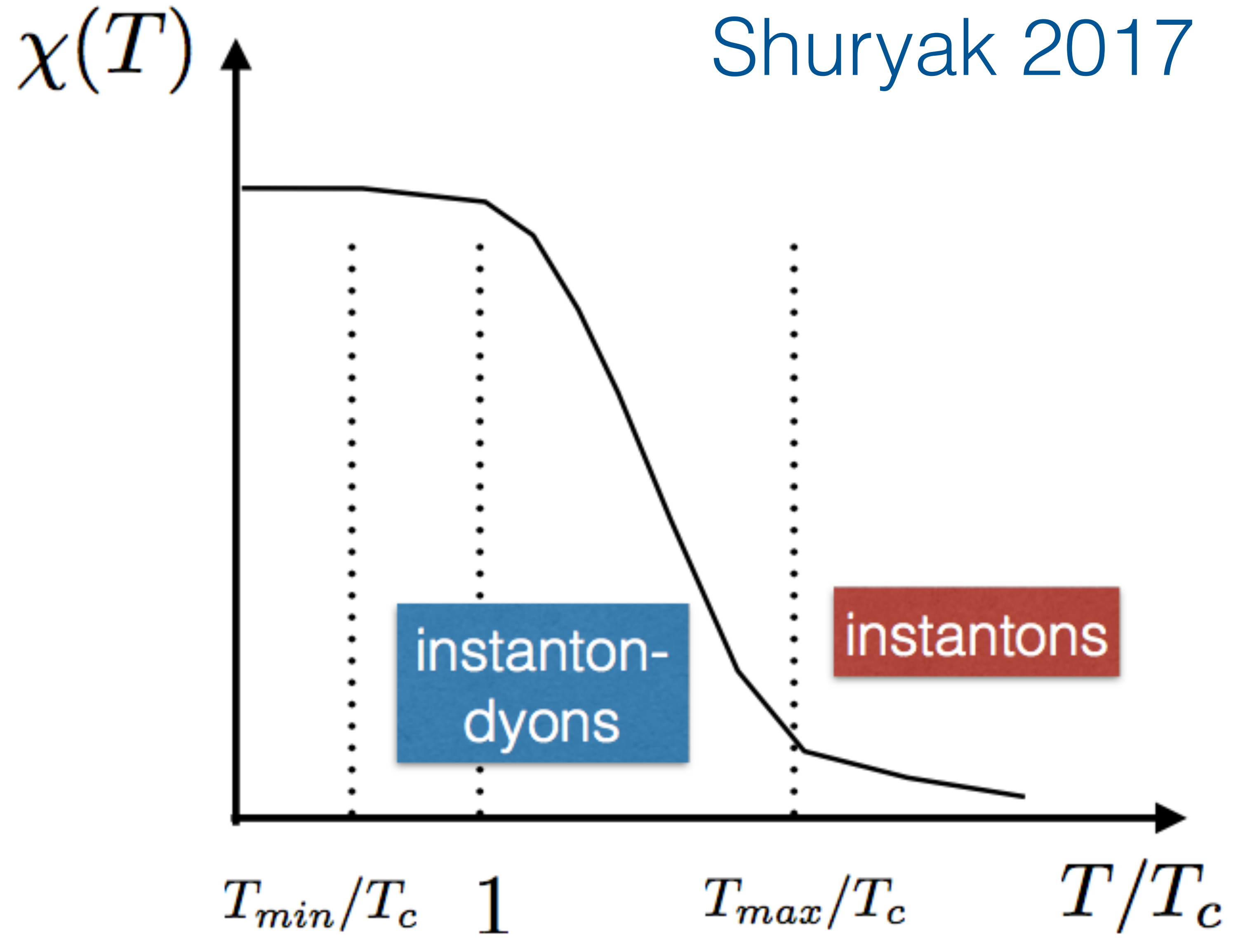
$$\chi^{0.25}(T) = aT^{-d(T)}$$

For instanton gas $\quad d(T) \equiv const \simeq (7 + \frac{N_f}{3})$

$$d(T) = -T\frac{d}{dT}\ln\chi^{0.25}(T)$$



Possibly consistent
with instant -dyon?
Shuryak 2017



Faster decrease before DIGA sets in

Burger, MpL , .. 2018

QCD - Summary of b parameter

$$\chi(T) = A\,T^{b}$$

Y. Taniguchi, K. Kanaya, H. Suzuki and T. Umeda (2017) (d),

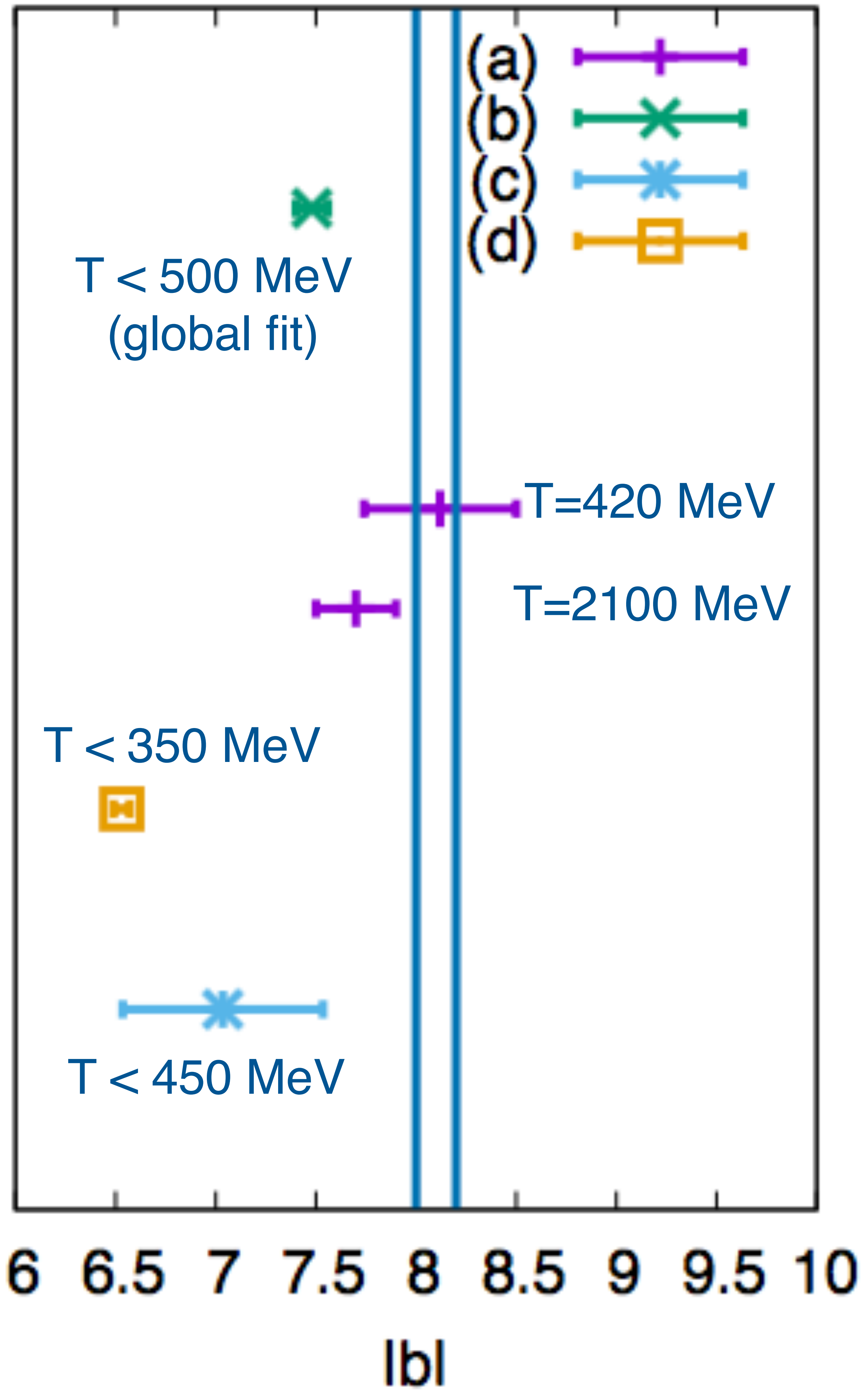
Borsanyi et al. (2016) (a)

Petreczky, Schlaeder, Scharma (2016) (b)

Burger et al. (2018) (c)

DIGA, Nf = 3

DIGA, Nf=4



For T > 300 MeV the DIGA exp is approached from below

Tc < T < 250 — 300 MeV ??

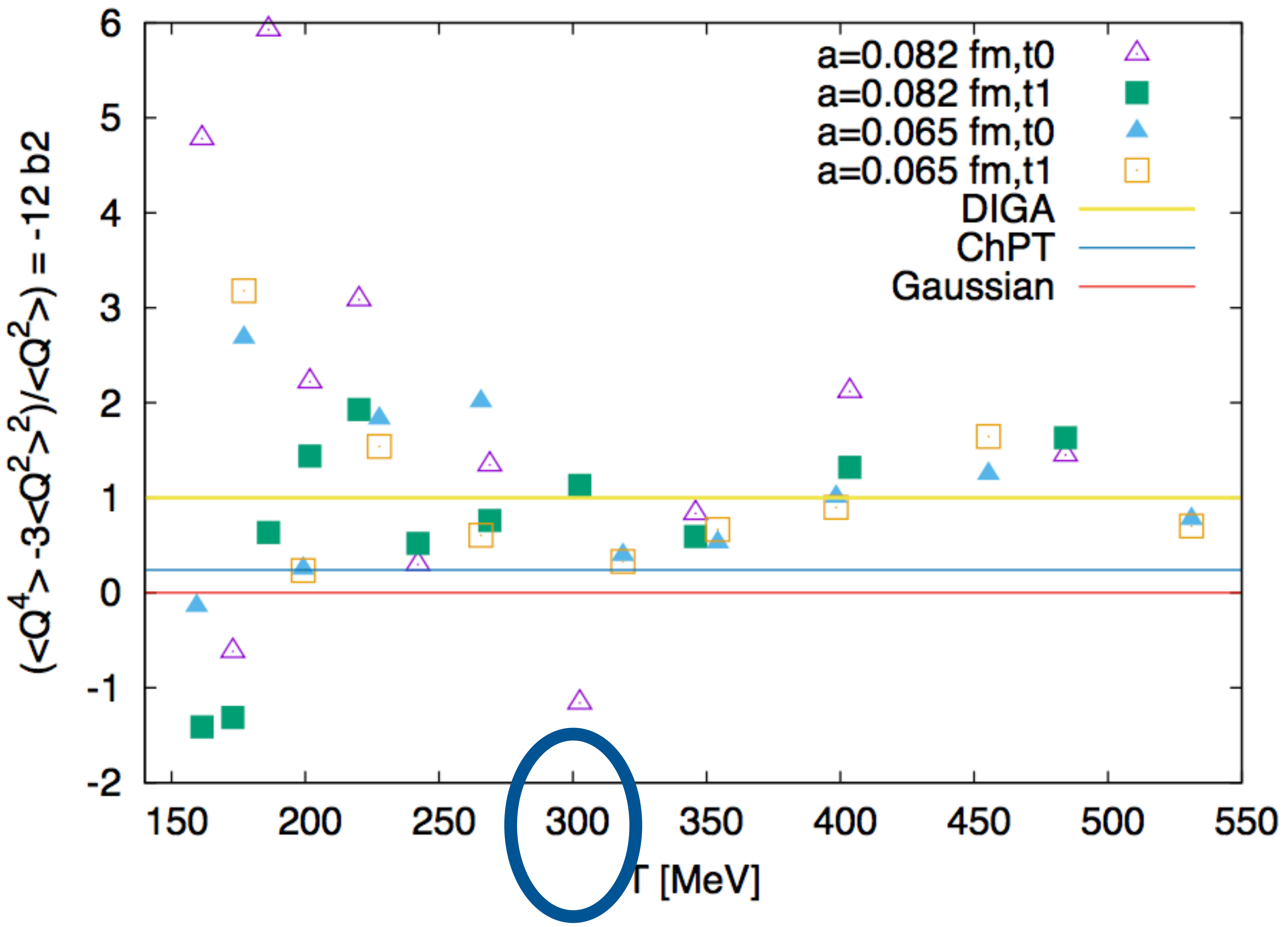
Further evidence of a fast crossover in topology

Beyond topological susceptibility

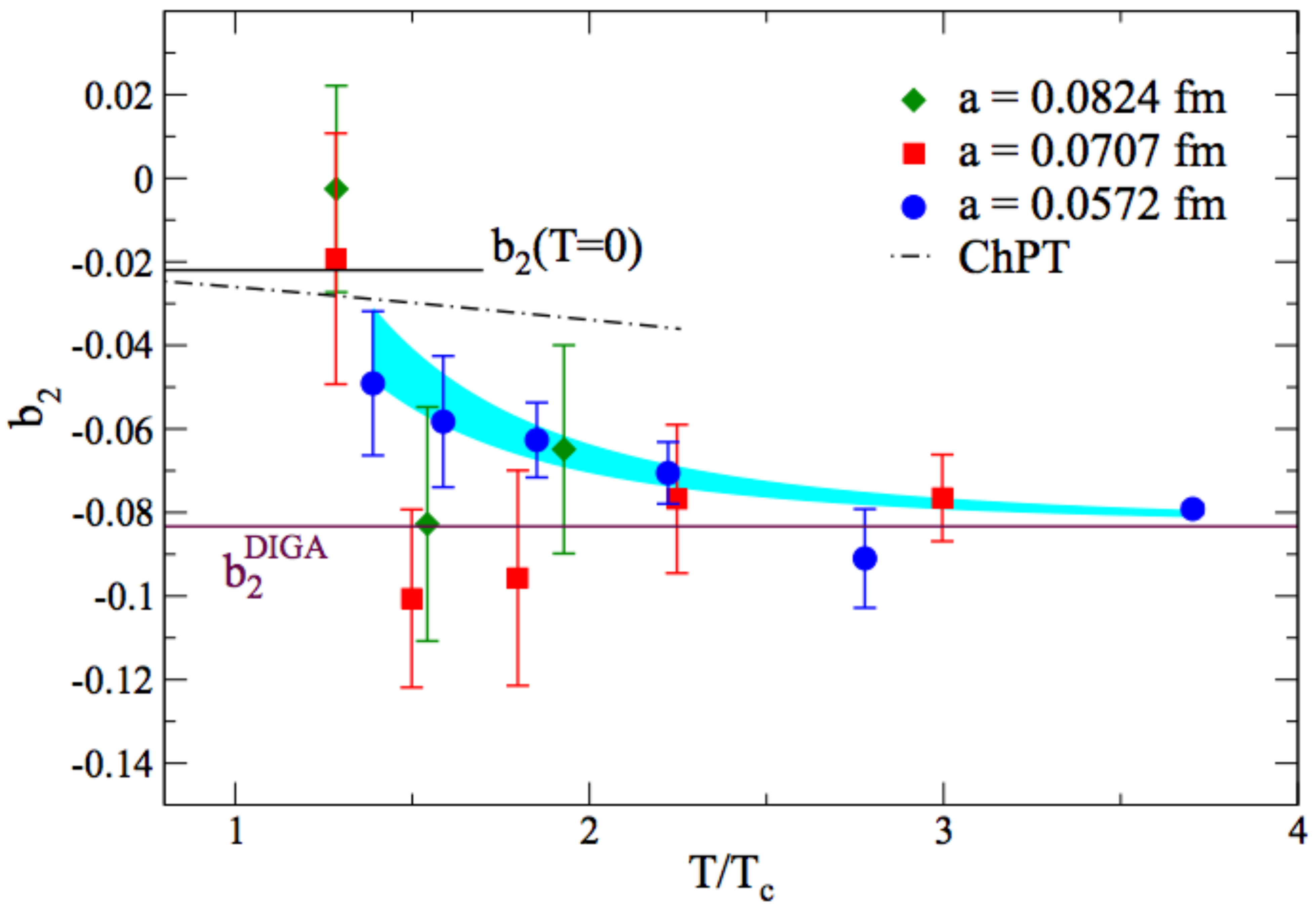
T > 250-300 MeV

$$C_n = (-1)^{n+1} \frac{d^{2n}}{d\theta^{2n}} F(\theta, T)\Big|_{\theta=0} = \langle Q^{2n} \rangle_{conn}.$$

d'Elia, Vicari (2013)



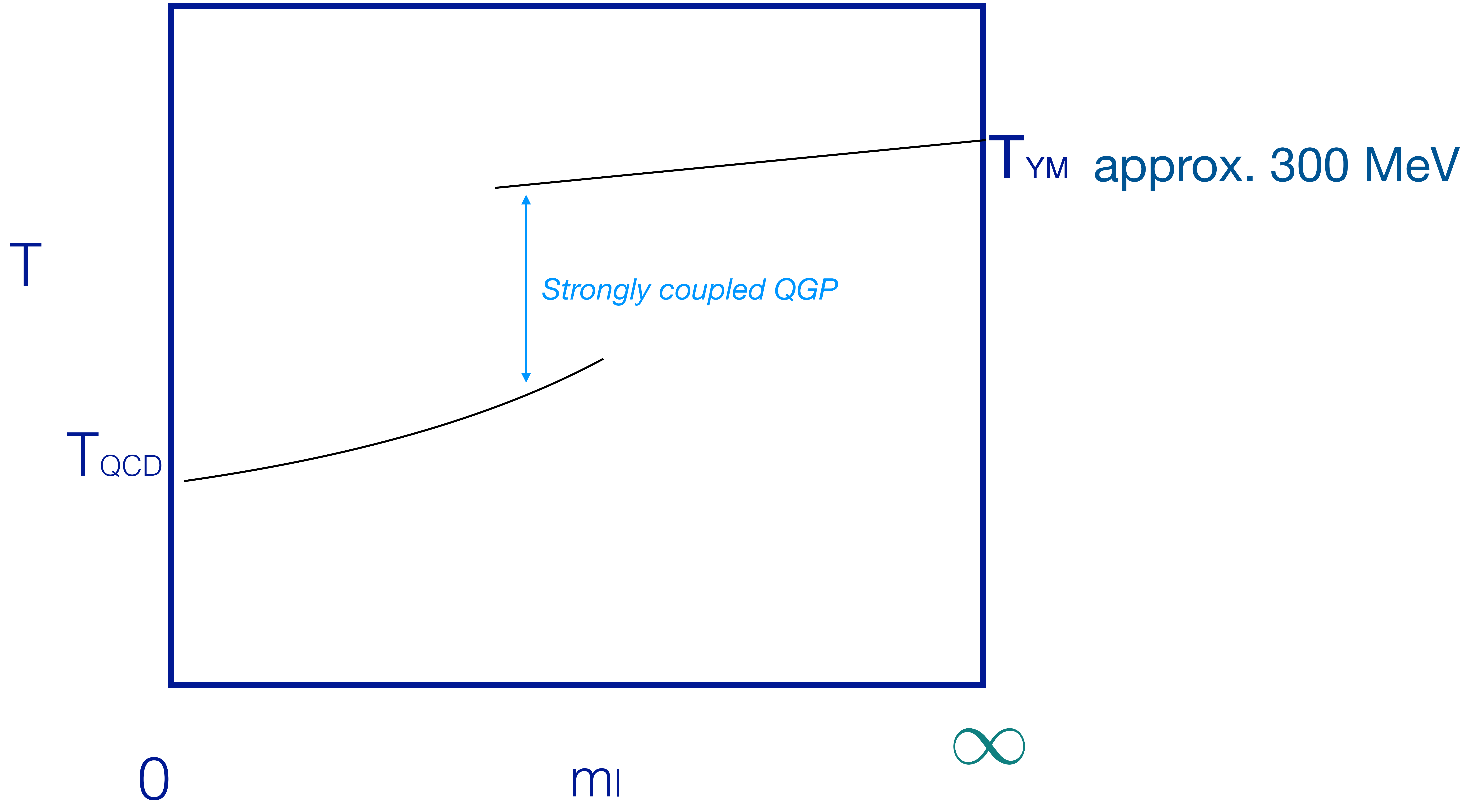
Trunin et al (2018)



Bonati et al. (2016)

.. a speculation…

Is a resilient YM topology producing a threshold in full QCD?



Topology from low to high Temperature - effects on the spectrum

In the hadronic phase topology solves the η' puzzle by explicit breaking $U(1)_A$

What happens to η' in the Quark Gluon Plasma?

PHYSICAL REVIEW D VOLUME 53, NUMBER 9 1 MAY 1996

Return of the prodigal Goldstone boson

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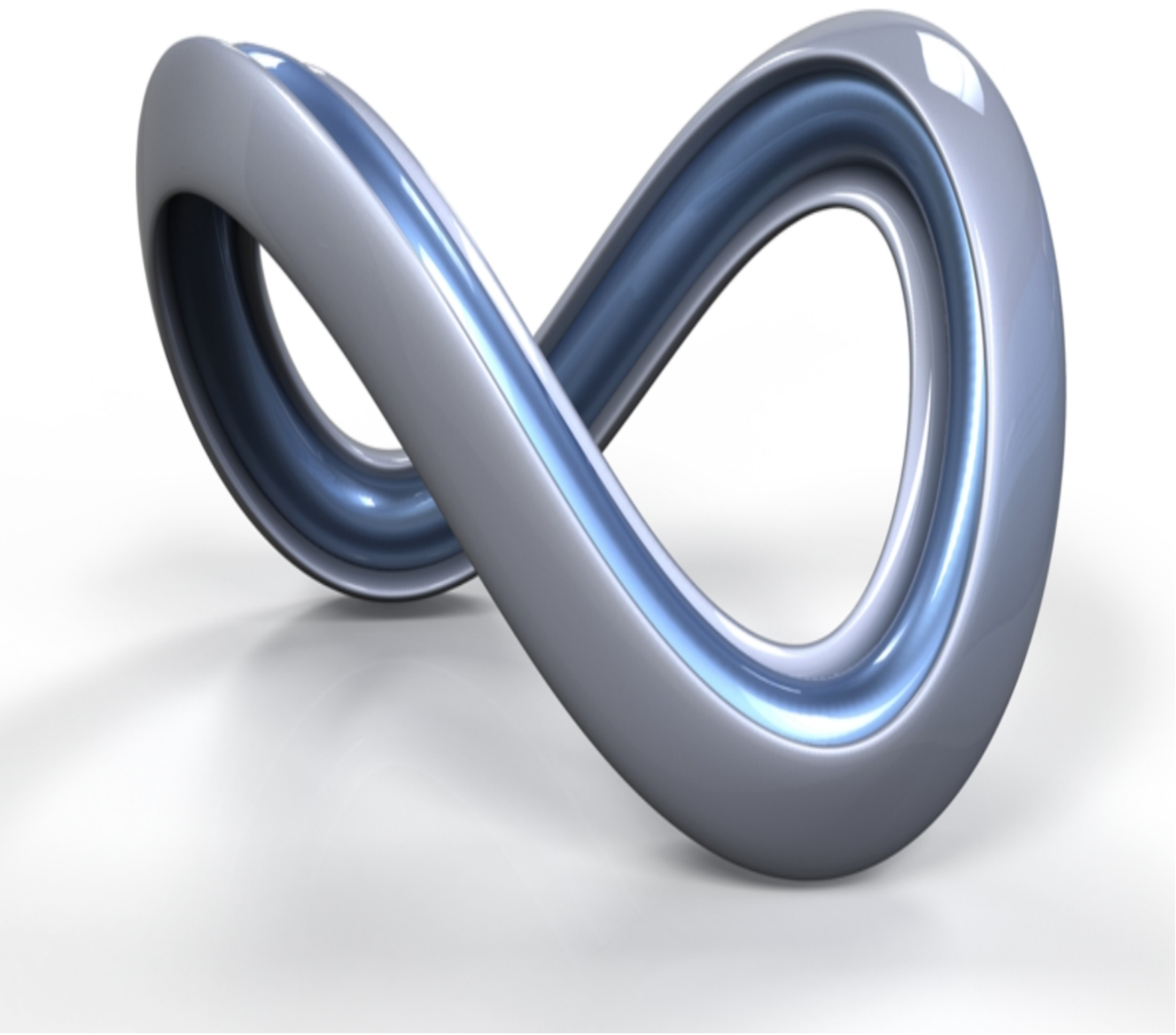
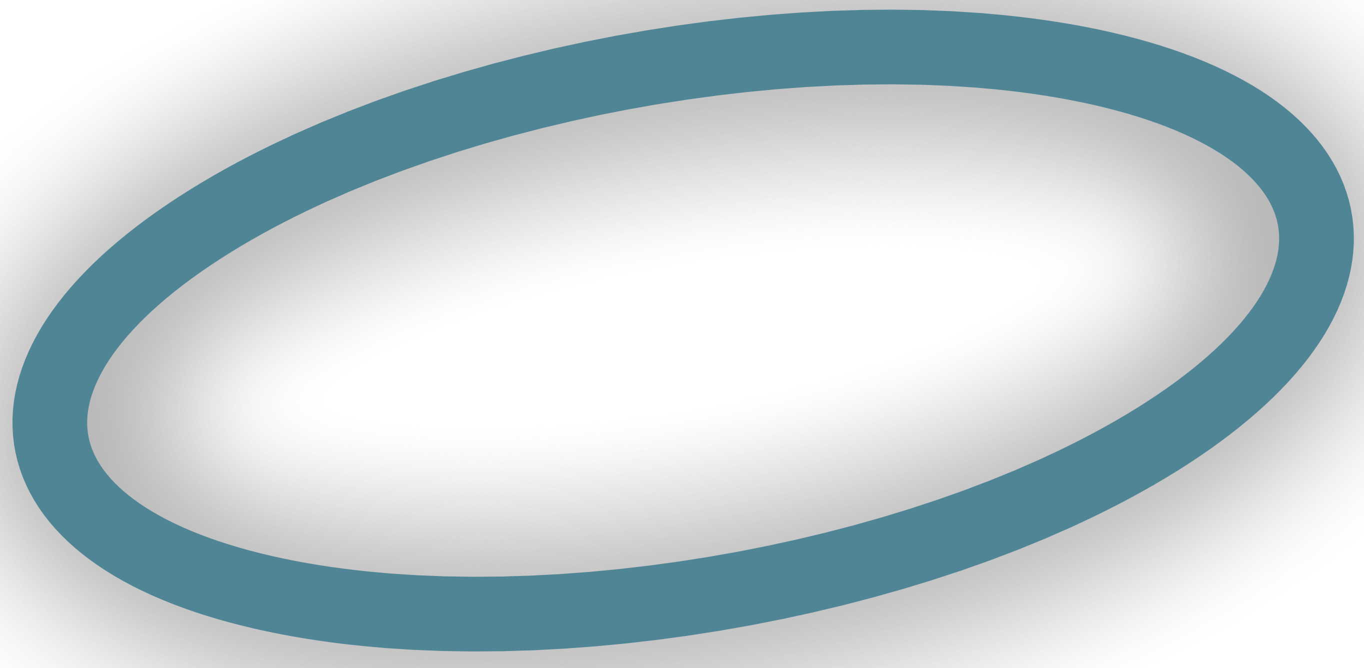
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455

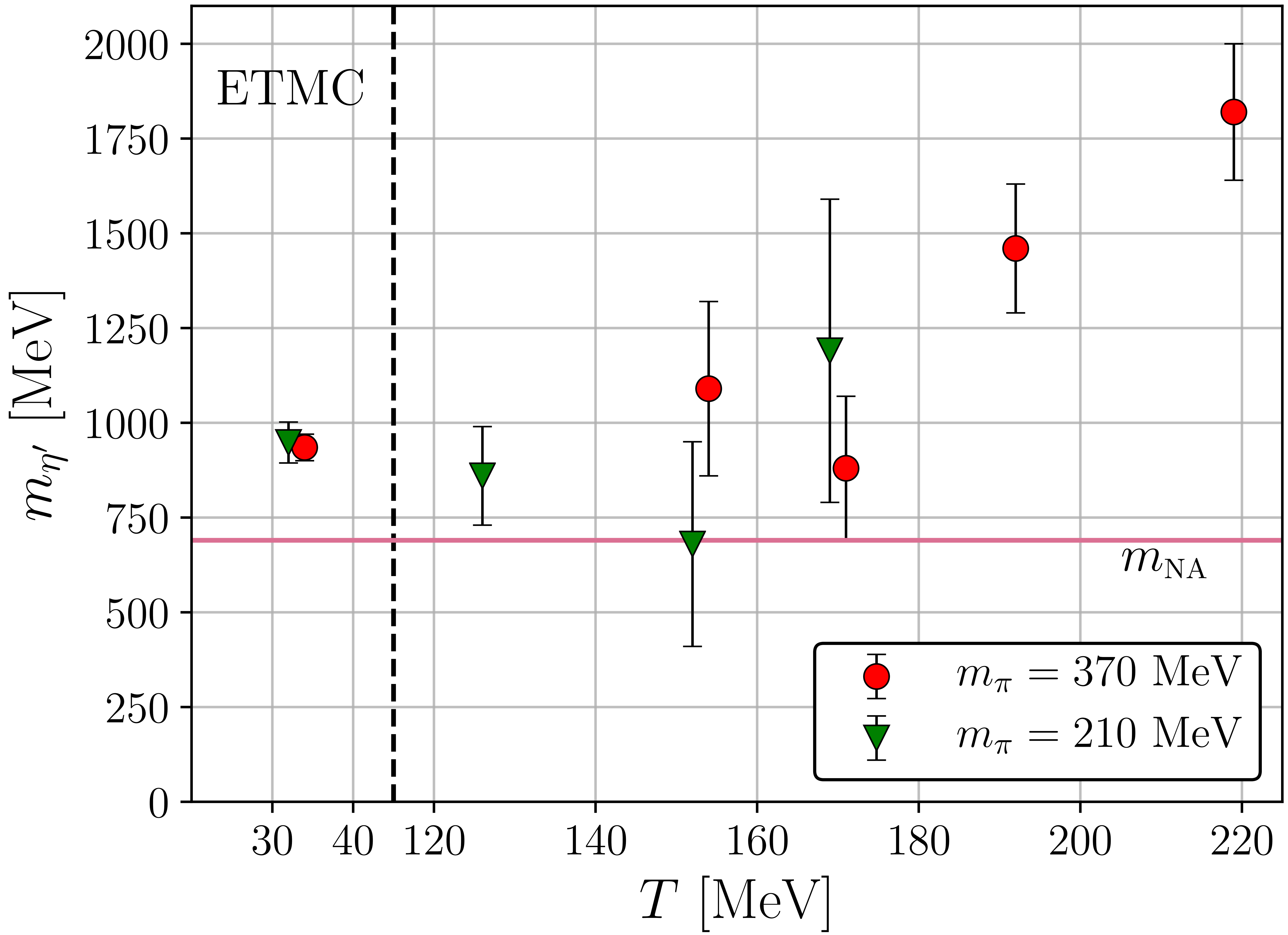
(Received 14 July 1995)

We propose that the mass of the η' meson is a particularly sensitive probe of the properties of finite energy density hadronic matter and quark-gluon plasma. We argue that the mass of the η' excitation in hot and dense matter should be small, and, therefore, that the η' production cross section should be much increased relative to that for pp collisions. This may have observable consequences in dilepton and diphoton experiments.





Evidence for topology suppression in the spectrum



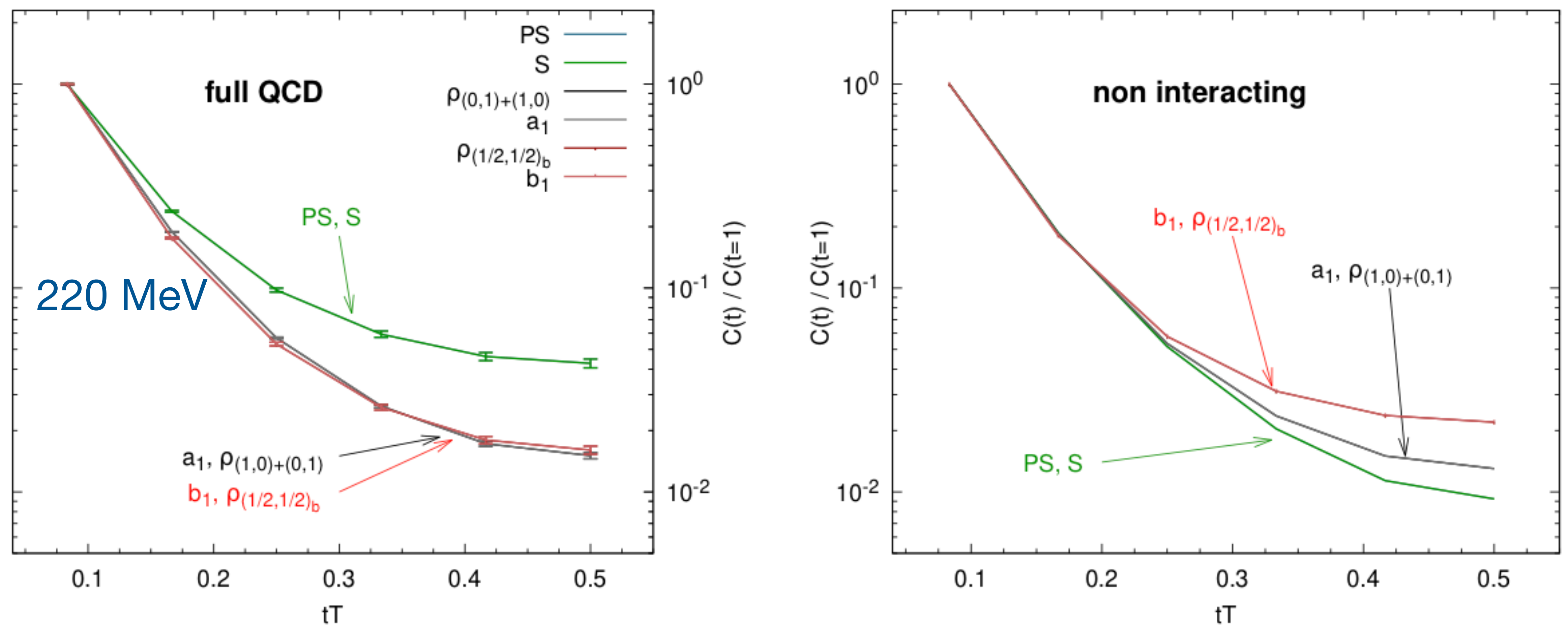
Non anomalous:

$\eta' \simeq 700$ MeV

(strange only)

Kotov, Trunin, MpL (2019)

Spectrum and the anomalous threshold



C. Rohrhofer, Y. Aoki, L.Y. Glozman and S. Hashimoto, 2021

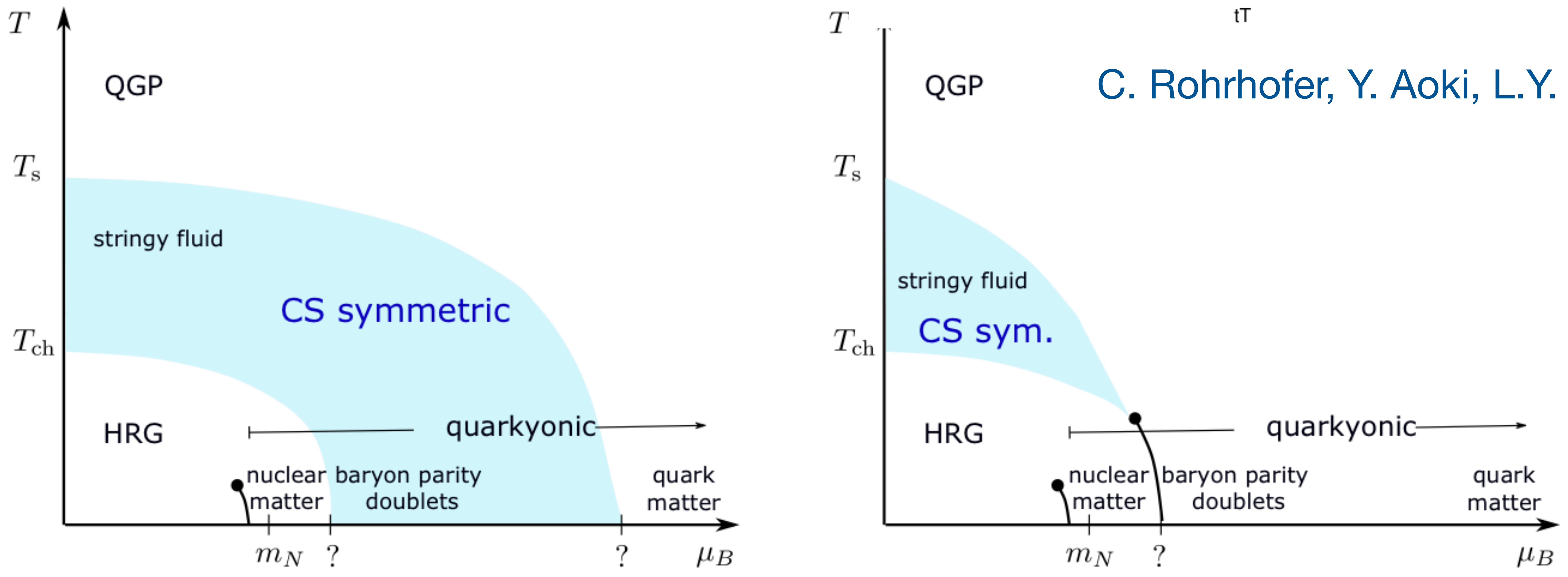


Figure 8: Possibilities for the QCD phase diagram with a chiral spin and $SU(4)$-symmetric band.

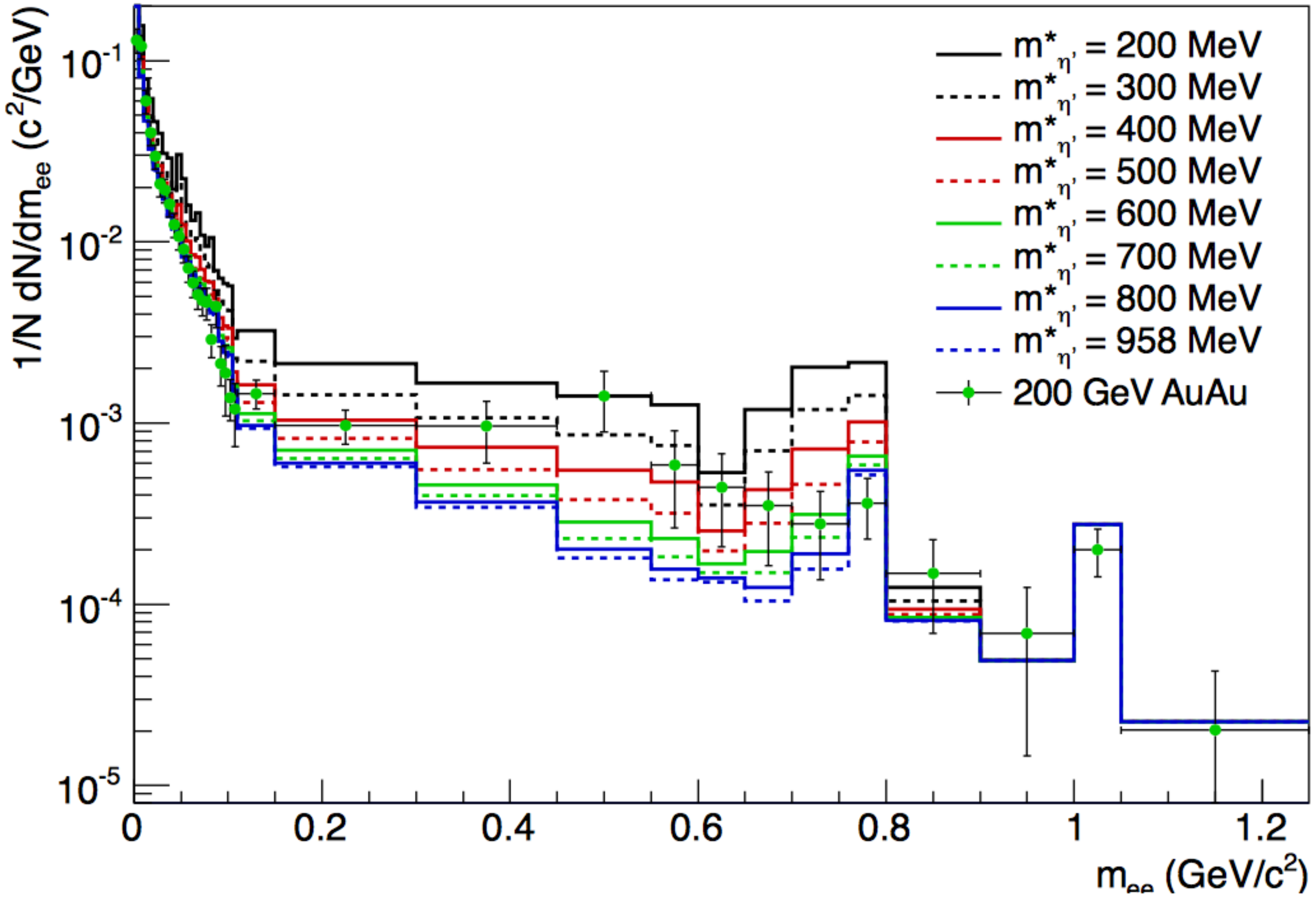
Philipsen et al. 2022

From low to high densities - Experimental results and first theoretical analysis

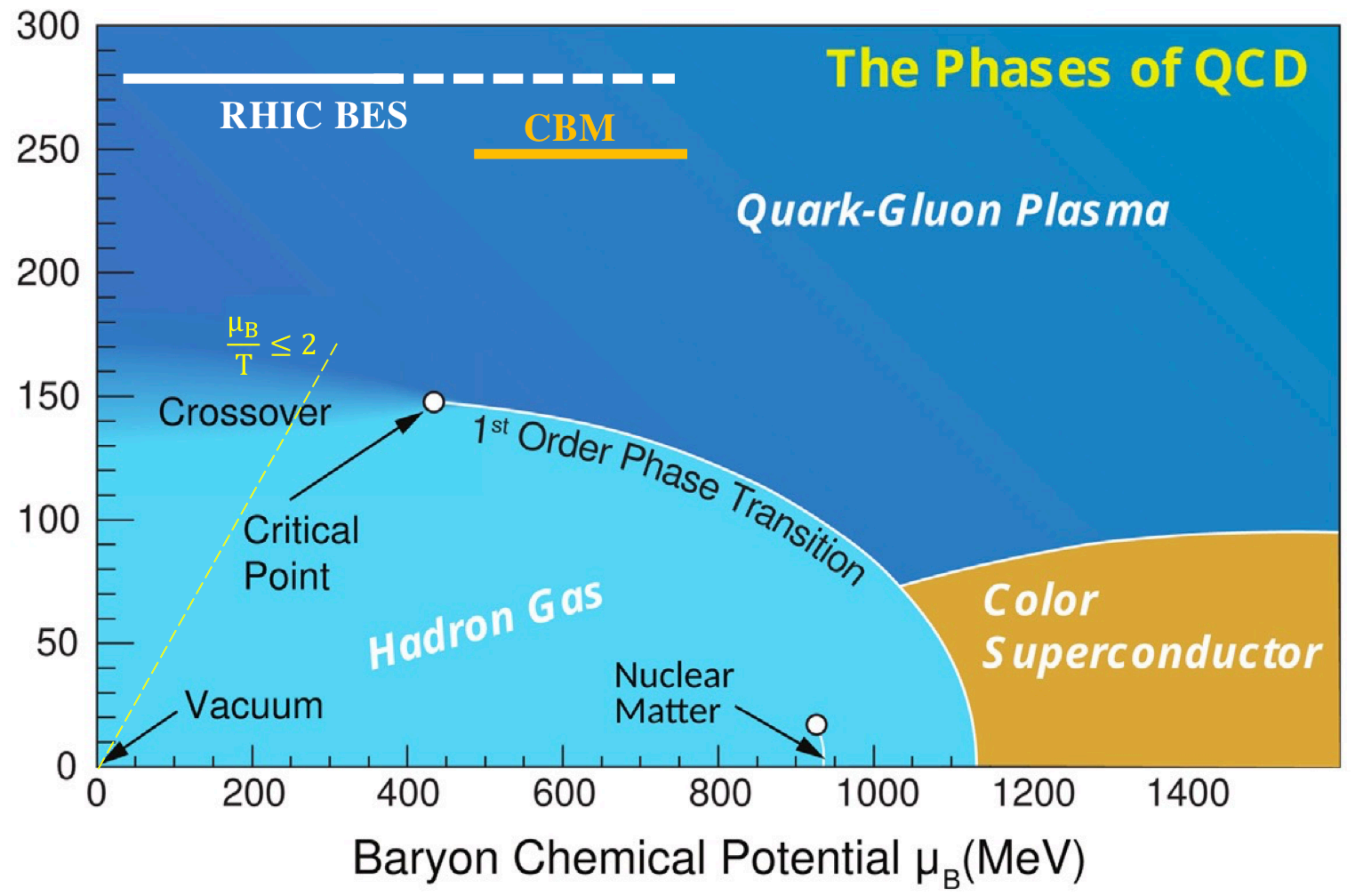
Indication of topology suppression in PHENIX@RHIC

Effects of chain decays, radial flow and $U_A(1)$ restoration on the low-mass dilepton enhancement in $\sqrt{s_{NN}}$=200 GeV Au+Au reactions

Márton Vargyas[a,b,1], Tamás Csörgő[b,2], Róbert Vértesi[b,c,3]

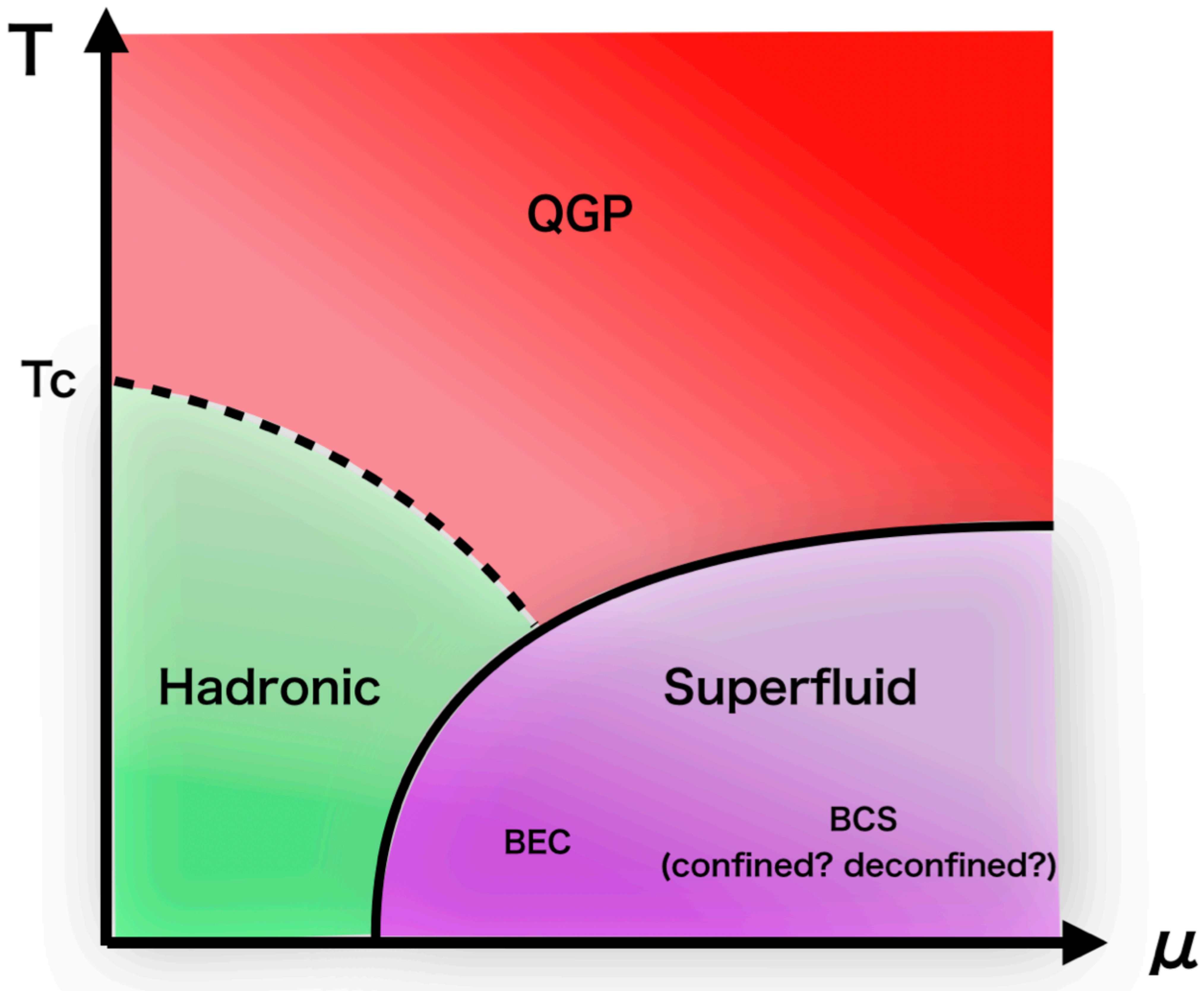


This is at finite density!



The sign problem hampers simulations in dense matter at low temperature

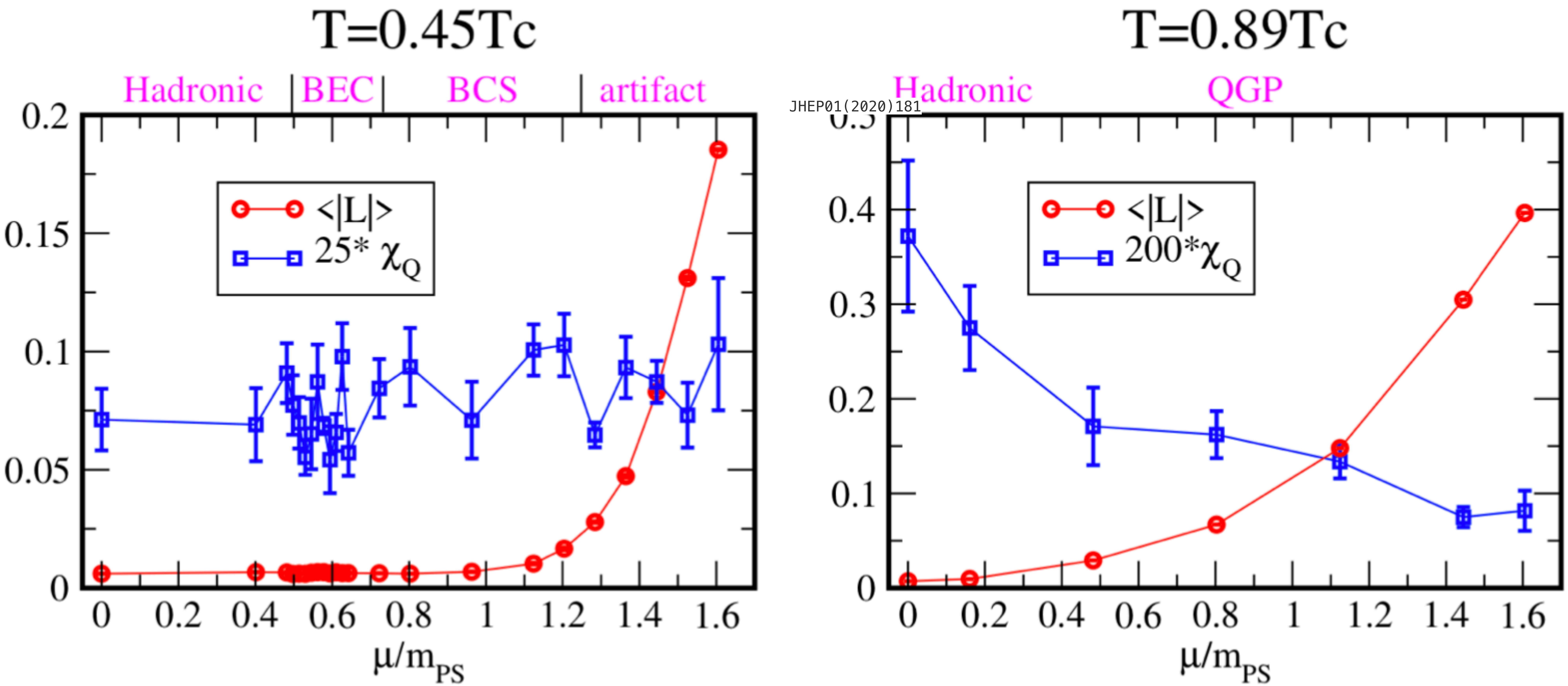
Some lessons may be learned from two colors QCD:



Iida, Itou, Lee 2021

Topology in two-color QCD: susceptibility ‘survives’ beyond the transition!

Role of instanton molecules?
Trivial in QCD2?
Need to go beyond susceptibility?



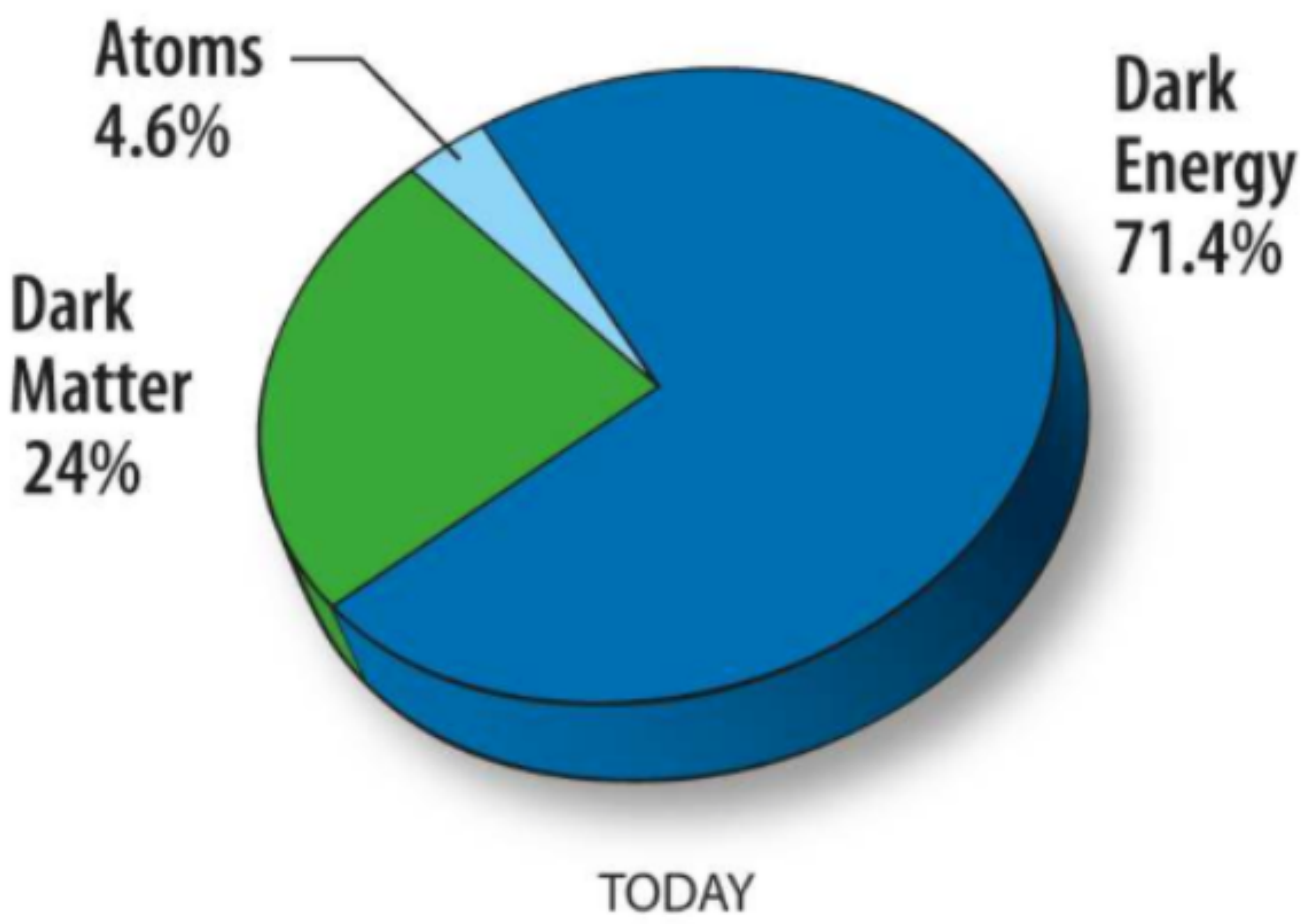
Iida, Itou, Lee 2021

3. Topology and axions

the strong CP problem in QCD

..can be solved by introducing the AXION

a new particle which is a viable **dark matter candidate**

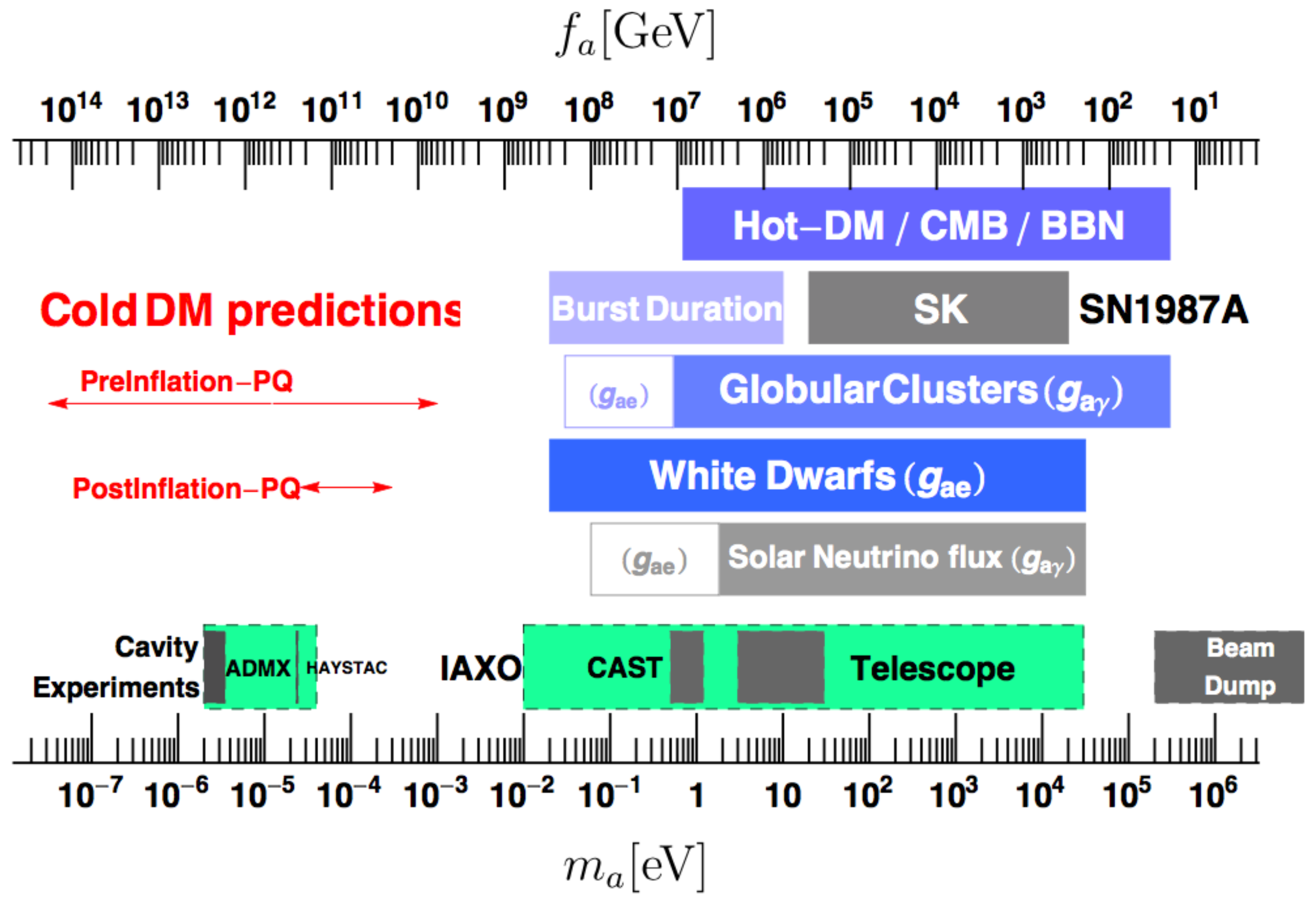


Crucial ingredient: $< Q^2(T) >$

The axion mass

At leading order in $1/f_A$ – well justified as $f_A \gtrsim 4 \times 10^8$ GeV

$$m_A^2(T) f_A^2 = \left. \frac{\partial^2 F(\theta, T)}{\partial \theta^2} \right|_{\theta=0} \equiv \chi_{top}(T)$$



At low temperature ChPT gives:

$$m_A^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_A^2},$$

In general

$$m_A^2 f_A^2 = \chi_{top},$$

$$m_A = 5.70(6)(4)\,\mu\text{eV} \left(\frac{10^{12}\,\text{GeV}}{f_A} \right);$$

valid for any temperature.

Temperature

Time from Big Bang

Hubble parameter

$$H(T) \simeq T^2/M_P$$

Axions's freezout

3H(T) = m_a(T)

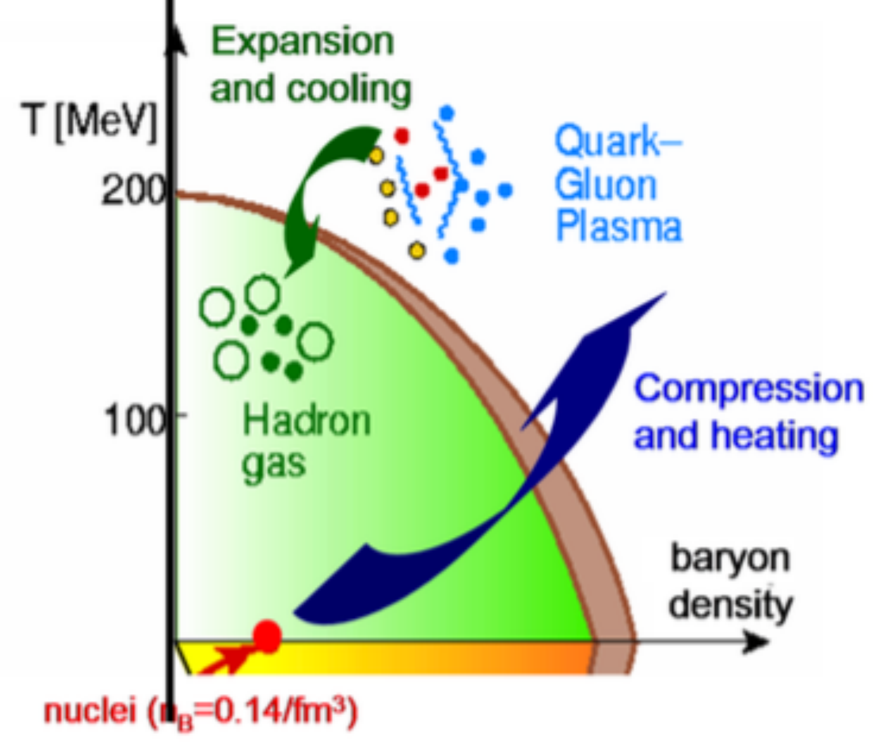
m_a(T) $= \sqrt{\chi(T)}/f_a$

Quark Gluon Plasma: Topology

Axions' mass and density today

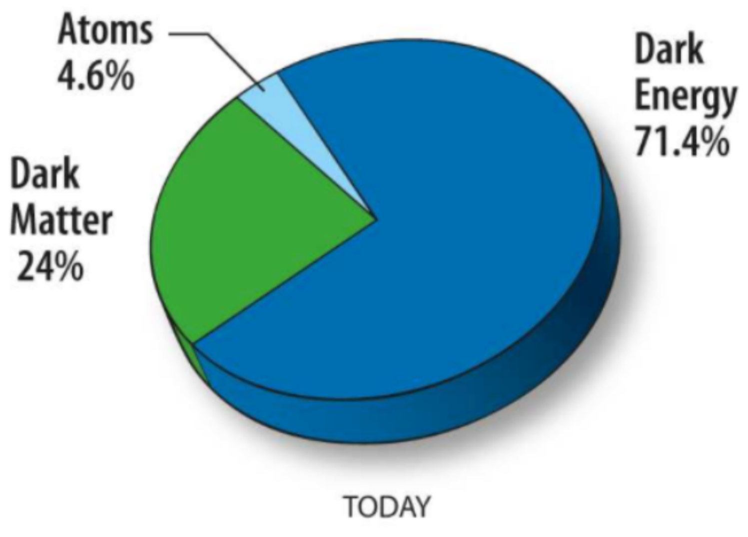
After freezout $\frac{n_a}{s}$ constant

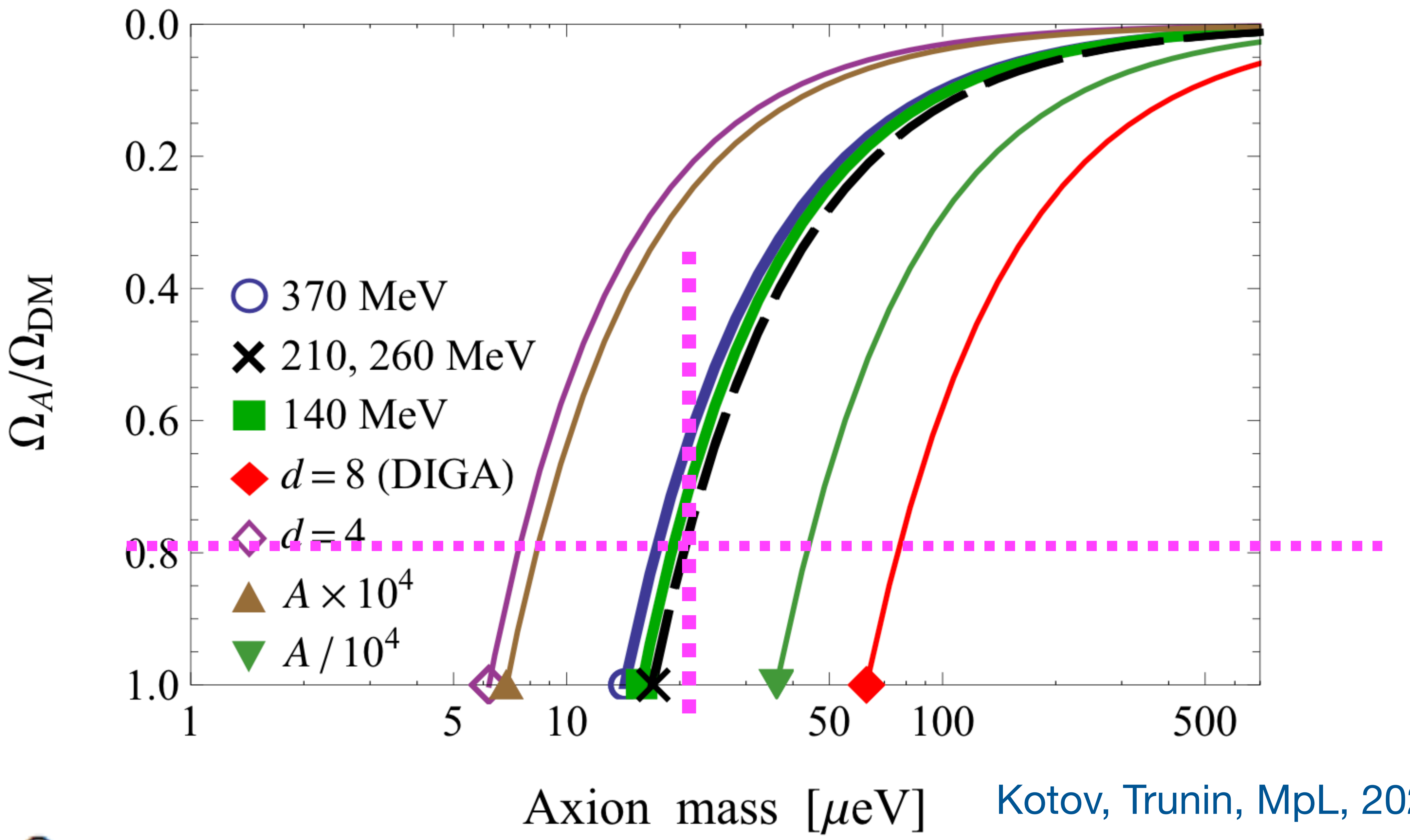
$$\rho_{a,0} = \frac{n_a}{s} m_a s_0$$



Wantz, Shellard 2010

$$\Omega_A = F(A, d, \ldots)\, m_A^{-\frac{3.053+d/2}{2.027+d/2}}$$





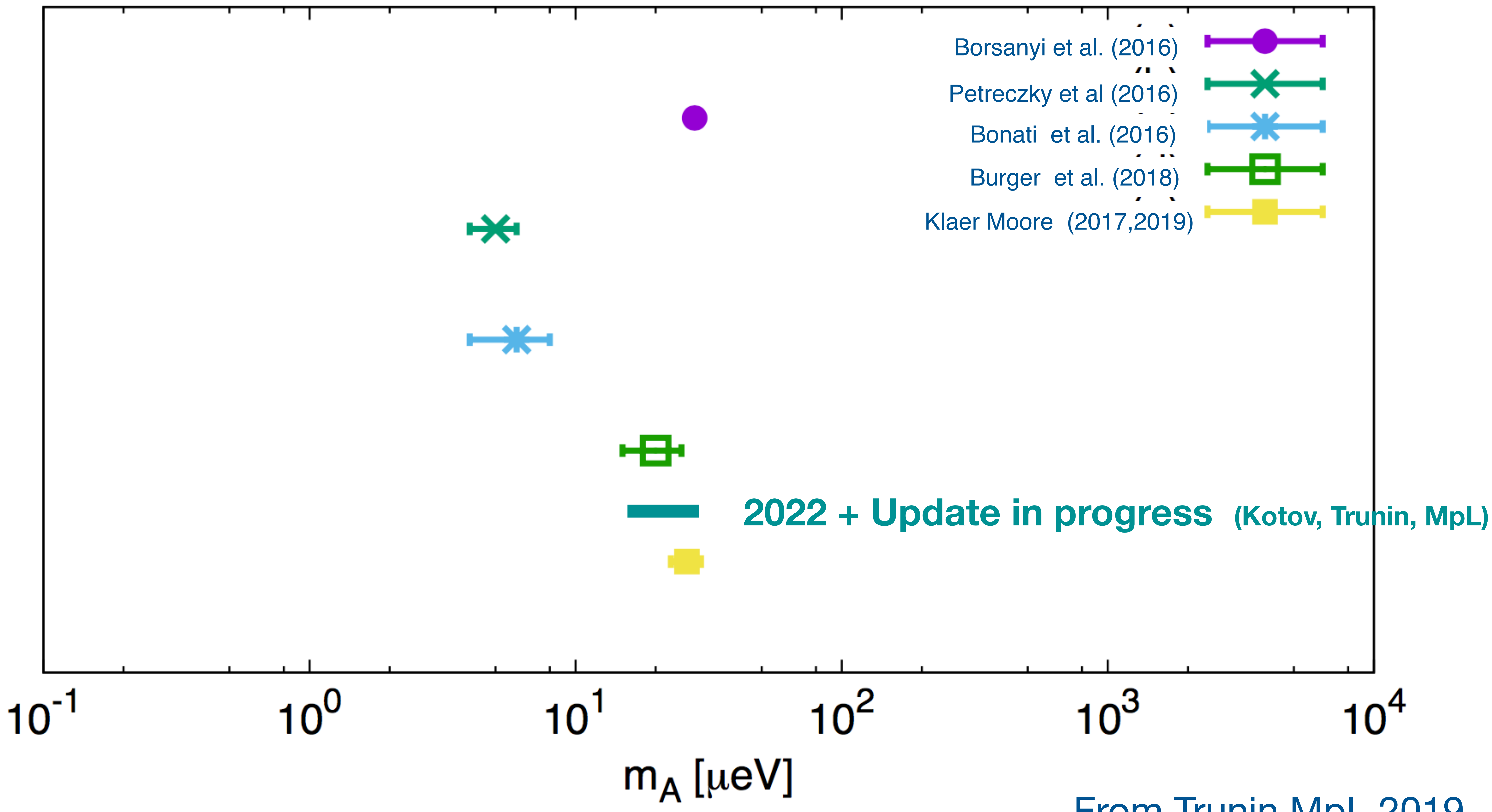
Kotov, Trunin, MpL, 2022

Burger,Trunin, Ilgenfritz,Mueller-Preussker,MpL 2019

$$\Omega_a = \frac{\rho_{a,0}}{\rho_c};$$

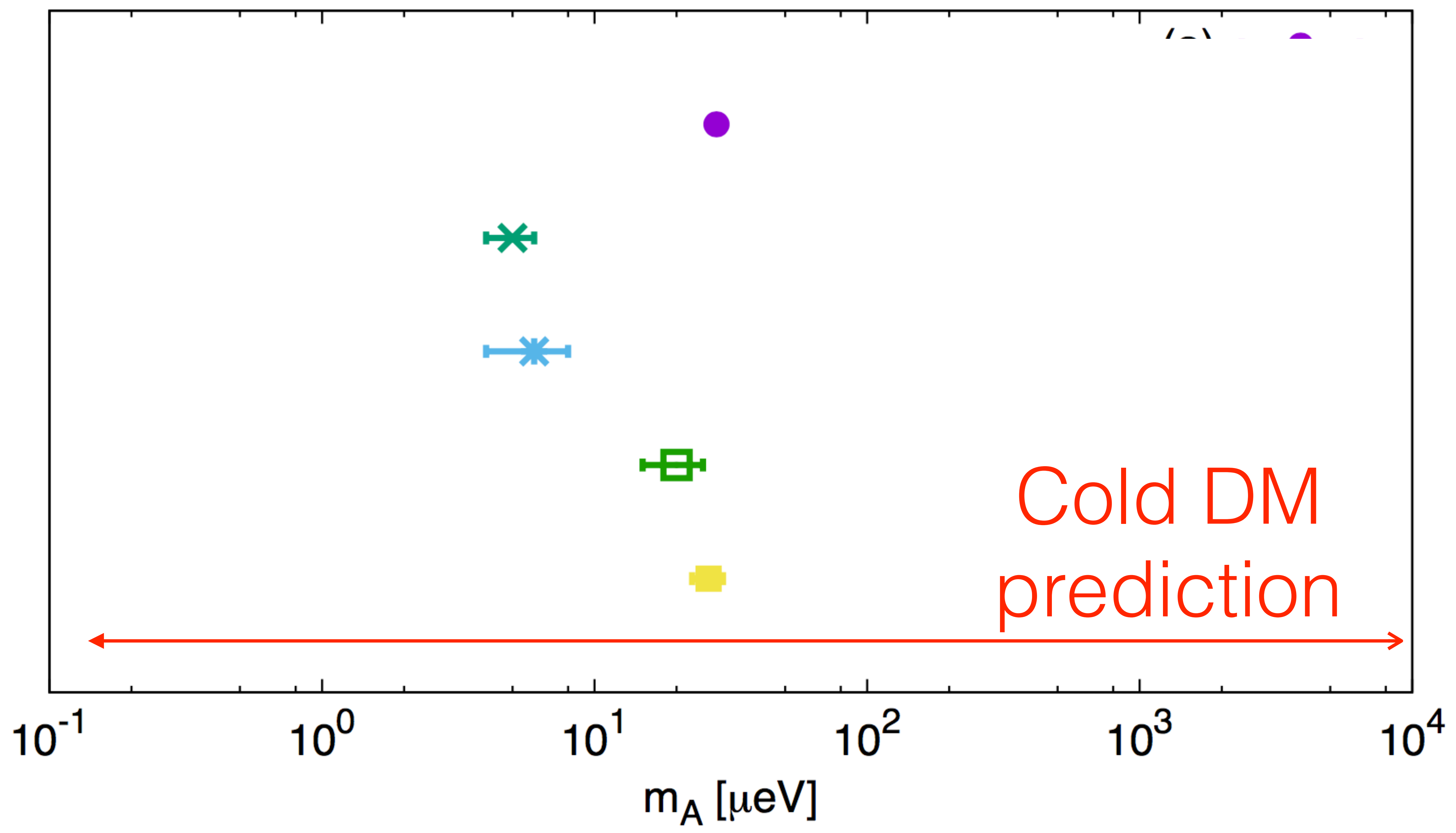
Example: if axions constitute 80% DM, our results give a lower bound for the axion mass of $\simeq 30\mu\mathrm{eV}$

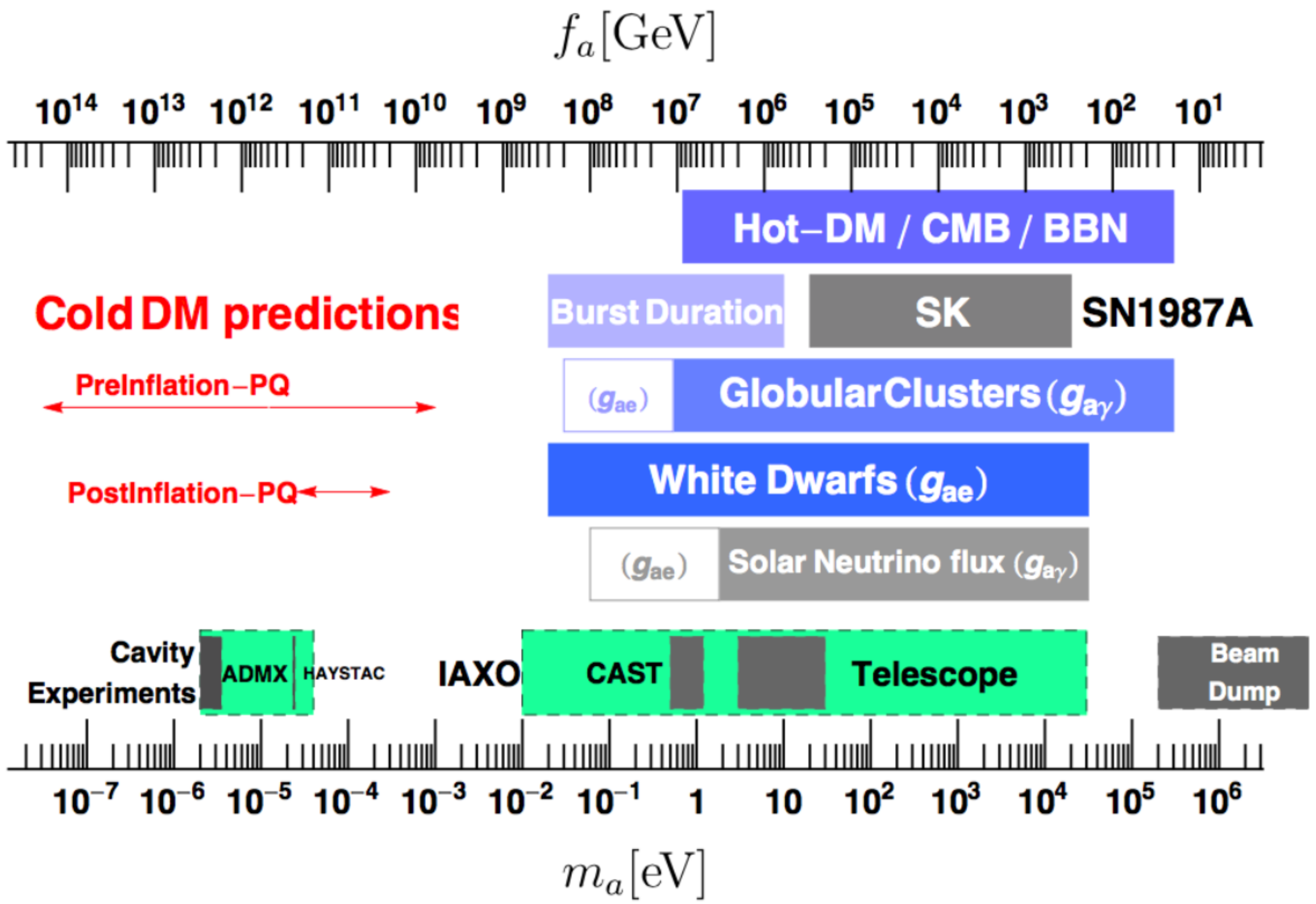
Lower limits on post-infl. axion mass from lattice QCD



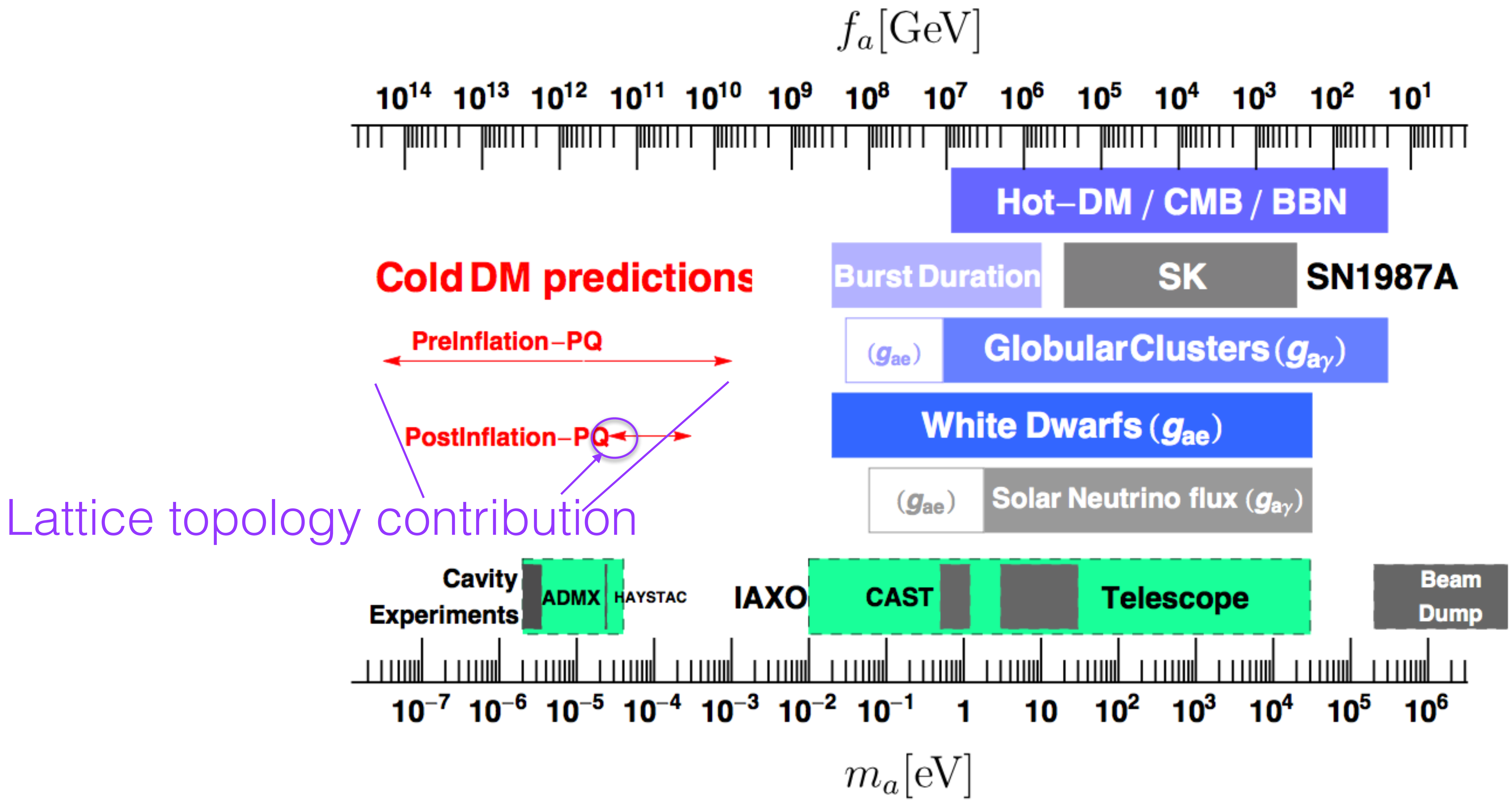
From Trunin MpL 2019

Lower limits on post-inflationary axion mass





Limits on the axion mass



Summary

Topology in QCD

1. From low to high temperatures — a threshold in the plasma?

   Common temperature for the crossover for topology and the limit of the scaling window

2. From low to high densities - experimental results and first theoretical analysis

   Experimental observation of topology suppression
   Lattice results confirm this to some extent, but with puzzling features at low temperatures in two colors QCD

3. Topology and the QCD axion

   Limits on the post-inflationary axion mass; results not entirely settled ; extrapolation to high temperature regime may be subtle and needs further studies; Axion potential?