From smeared spectral functions to inclusive semileptonic decays

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XIX Workshop on SM & FT
Università di Bari
20th of December 2022

Based on:
"*Lattice QCD study of inclusive semileptonic decays of heavy mesons*"
[hep-lat] 2203.11762, JHEP 2022, 83 (2022)
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- Shoji Hashimoto (KEK)
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- Silvano Simula (INFN Rome 3)
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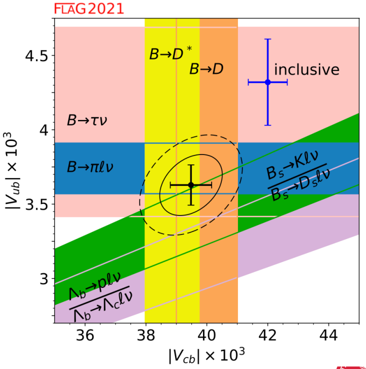
V_{cb} problem

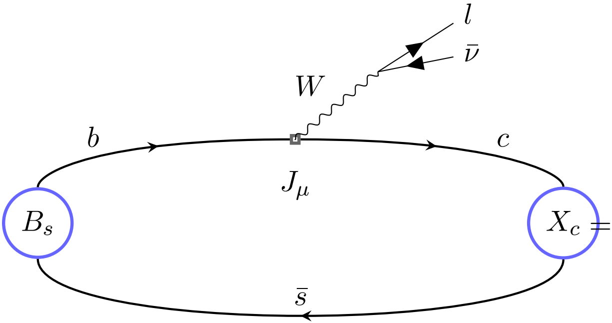
A persistent tension

The tension between the exclusive and inclusive determination of the CKM parameter V_{cb} is $\sim 3\sigma$

- exclusive determinations have reached an impressive level of precision thanks to lattice QCD calculations.
- Can we do the same for inclusive decays?



Inclusive $B_s \to X_c l\nu$



$= D_s, D_s^*, D_s\pi, \ldots$

$$\frac{d\Gamma}{dq^2 dq^0 dE_l} = \frac{G_F^2 |V_{cb}|^2}{8\pi^2} L_{\mu\nu} W^{\mu\nu}$$

with:

$$W_{\mu\nu}(\omega, \boldsymbol{q}) = \frac{(2\pi)^3}{2m_{B_s}} \langle \bar{B}_s(\mathbf{0}) | J_\mu^\dagger(0) \delta(\hat{H} - \omega) \delta^3(\hat{P} + \boldsymbol{q}) J_\nu | \bar{B}_s(\mathbf{0}) \rangle$$

Inclusive decays formalism

[Gambino,Hashimoto'20, Phys. Rev. Lett. 125, 032001, hep-lat/2005.13730]

$$\frac{24\pi^3}{|\boldsymbol{q}|G_F^2|V_{cb}|^2}\frac{d\Gamma}{d\boldsymbol{q}^2} = \sum_{l=0}^{2}(\sqrt{\boldsymbol{q}^2})^{2-l}Z^{(l)}(\boldsymbol{q}^2)$$

with

$$Z^{(l)}(\boldsymbol{q}^2) = \int_0^\infty d\omega\Theta^{(l)}(\omega_{max}-\omega)W^{(l)}(\omega,\boldsymbol{q}^2)$$

with:

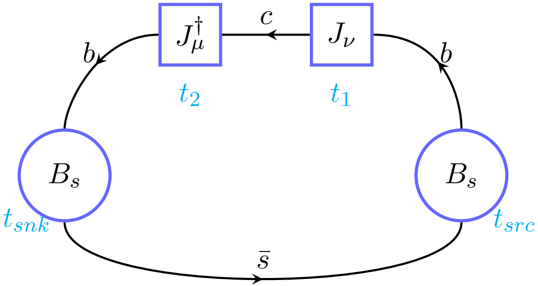
$$\Theta^{(l)}(\omega_{max}-\omega) = (\omega_{max}-\omega)^l\theta(\omega_{max}-\omega),$$

$$W^{(0)} = W^{00} + \sum_{i,j=1}^{3}\hat{n}_i\hat{n}_jW^{ij} - \sum_{i=1}^{3}\hat{n}_i(W^{0i}+W^{i0})$$

Inclusive decays on the lattice

[Gambino,Hashimoto'20, Phys. Rev. Lett. 125, 032001, hep-lat/2005.13730]

$$C_{\mu\nu}(t;\boldsymbol{q}) = \int_0^\infty d\omega \; W_{\mu\nu}(t;\boldsymbol{q}) \; e^{-\omega t}$$



Inverse problem

$$C_{\mu\nu}(t;\boldsymbol{q}) = \int_0^\infty d\omega \; W_{\mu\nu}(t;\boldsymbol{q}) \; e^{-\omega t}$$

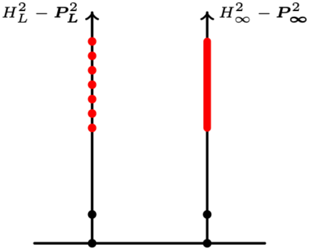
Hadamard conditions for well-posed problems

- **Existence**: For all suitable data, a solution exist
- **Uniqueness**: For all suitable data, the solution is unique
- **Stability**: The solution depends continuously on the data

Ill-posed inverse problem

first we ask: can we extract spectral densities from lattice correlators?

$$G(a\tau) = \int_0^\infty d\omega \; \rho(\omega)_L \; e^{-a\omega\tau}$$



Multidisciplinary problem

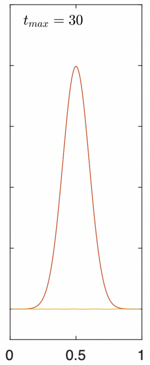
Ill-posed inverse problems affect many areas of science:

- Lattice QCD, image deblurring, medical tomography (MRI), aircraft stability, nuclear reactor physics, . . .
- the HLT method is based on the Backus-Gilbert method developed in the context of geophyiscal research about gross Earth data The resolving power of gross earth data, Geophysical Journal International 16 (1968), no. 2169–205
- It is also been developed independenlty in astrophysical research where it is known as SOLA method Pijpers & Thompson 1992, 1994
- many methods currently available: Backus-Gilbert, Bayesian reconstruction, Chebyshev polynomials, machine learining, . . .

HLT algorithm

The details can be found in:
[Hansen, Lupo, Tantalo '19, Phys. Rev. D 99, 094508] [hep-lat/1903.06476],
define smeared spectral function:

$$\hat{\rho}_\sigma(\omega) = \int_0^\infty d\omega \; \rho(w)_L \; \Delta_\sigma$$



with the resolution function given by

$$\Delta_\sigma(\omega) = \sum_{\tau=1}^{\tau_{max}} g_\tau e^{-a\omega\tau}$$

once the coefficients g_τ are known, then

$$\hat{\rho}_\sigma(\omega) = \sum_\tau g_\tau \; G(a\tau)$$

The coefficients are obtained minimizing

$$W_\lambda[g] = (1-\lambda)\frac{A[g]}{A[0]} + \lambda B[g]$$

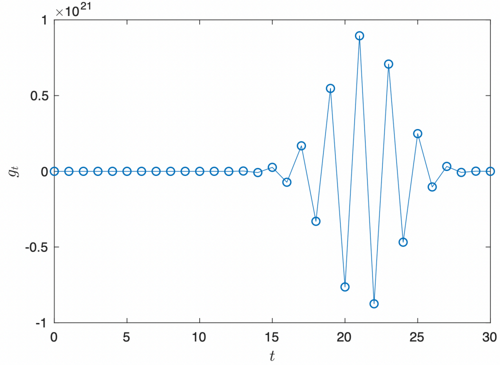
with

$$A[g] = a\int_{E_0}^{\infty} dw \left\{ \Delta_\sigma - \sum_{\tau=1}^{\tau_{max}} g_\tau e^{-aw\tau} \right\}^2,$$

$$B[g] = \sum_{\tau,\tau'=1}^{\tau_{max}} g_\tau g_{\tau'} \frac{Cov[G(a\tau), G(a\tau')]}{[G(0)]^2}$$

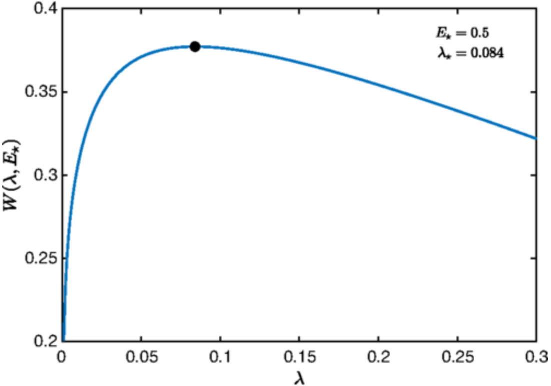
g_τ^λ defines the approximation of the kernel

$$\Delta_\sigma^\lambda = \sum_{\tau=1}^{\tau_{max}} g_\tau^\lambda e^{-aw\tau}$$



the best choice for λ can be found via:

$$\left.\frac{\partial W(\lambda)}{\partial \lambda}\right|_{\lambda=\lambda_\star} = 0$$



Ordered double limit

smeared spectral densities are smooth functions unlike the distribution $\rho(\omega)_L$, so their infinite-volume extrapolation at fixed values of the smearing is a well-posed problem

$$\rho(\omega) = \lim_{\sigma \to 0} \lim_{L \to \infty} \hat{\rho}_\sigma(\omega)$$

Now Remember...

$$\frac{24\pi^3}{|\boldsymbol{q}|G_F^2|V_{cb}|^2}\frac{d\Gamma}{d\boldsymbol{q}^2} = \sum_{l=0}^{2}(\sqrt{\boldsymbol{q}^2})^{2-l} Z^{(l)}(\boldsymbol{q}^2)$$

with

$$Z^{(l)}(\boldsymbol{q}^2) = \int_0^\infty d\omega \Theta^{(l)}(\omega_{max} - \omega) W^{(l)}(\omega, \boldsymbol{q}^2)$$

the kernel is defined as

$$\Theta^{(l)}(\omega_{max} - \omega) = (\omega_{max} - \omega)^l \theta(\omega_{max} - \omega)$$

and

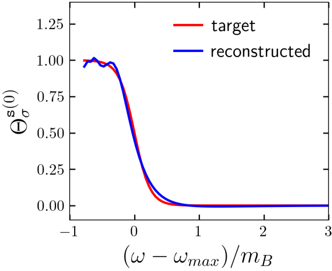
$$W^{(0)} = W^{00} + \sum_{i,j=1}^{3} \hat{n}_i \hat{n}_j W^{ij} - \sum_{i=1}^{3} \hat{n}_i (W^{0i} + W^{i0})$$

Kernel reconstruction method

$$\Theta_\sigma(\omega) = \sum_\tau g_\tau e^{-a\omega\tau}$$

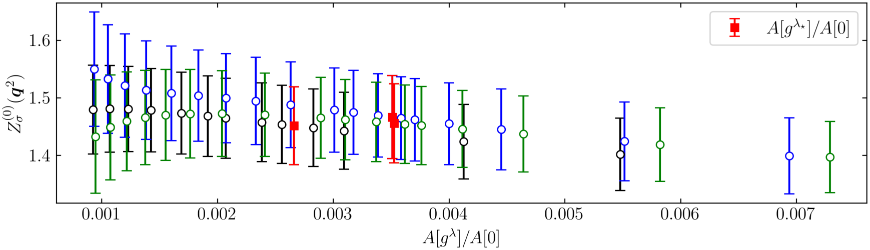
then:

$$\sum_\tau g_\tau G^{(l)}(a\tau; \boldsymbol{q}^2) = \sum_\tau g_\tau \int_0^\infty d\omega e^{-a\omega\tau} W^{(l)}(\omega; \boldsymbol{q})$$
$$= \int_0^\infty d\omega \left[g_\tau e^{a\omega\tau} \right] W^{(l)}(\omega; \boldsymbol{q})$$
$$= \int_0^\infty \Theta_\sigma(\omega) W^{(l)}(\omega; \boldsymbol{q})$$



$$\sum_\tau g_\tau G(a\tau; \boldsymbol{q}^2) = Z_\sigma^{(l)}(\boldsymbol{q}^2)$$

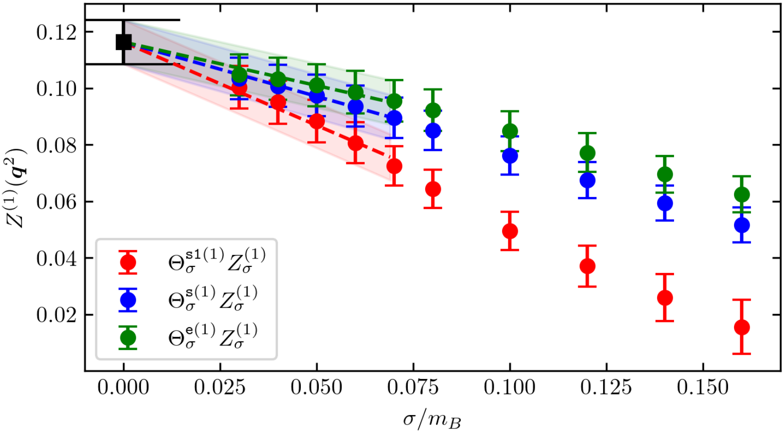
Smeared structure functions



$$Z^{(l)}(\boldsymbol{q}^2) = \lim_{\sigma\to 0}\left(\lim_{V\to\infty}\right)\int_0^\infty d\omega W_L^{(l)}(\omega,\boldsymbol{q}^2)\Theta_\sigma^{(l)}(\omega_{max}-\omega)$$

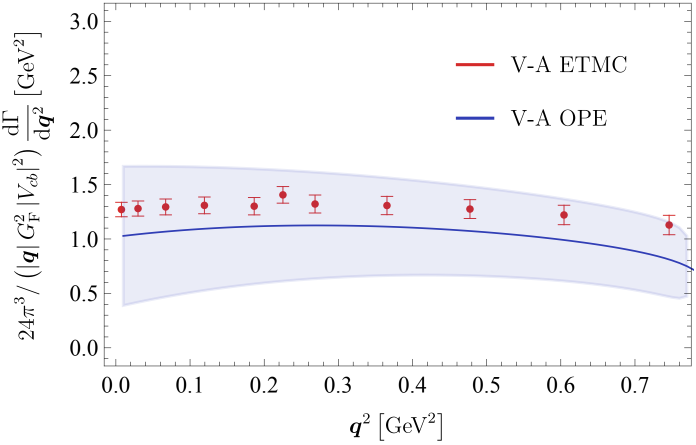
$$= \lim_{\sigma\to 0}\left(\lim_{V\to\infty}\right) m_{B_s}^l \sum_\tau^\infty g_\tau^{(l)}(\omega_{max},\sigma) G^{(l)}(a\tau,\boldsymbol{q})$$

Extrapolation to $\sigma = 0$



Differential decay rate comparison

After we take the $\sigma \to 0$ limit, we can confront our results with the analytic results from the OPE



Summary & outlook

Summary

- can study inclusive decays on the lattice by solving related inverse problem
- HLT method so far succesful in spectral reconstruction & inclusive decays

What's next?

Can we apply this method to other areas of physics?

- exists formalism for application to scattering amplitudes [Bulava, Hansen '19] & [Bruno, Hansen '21]
- promising applications to non-perturbative calculations in nuclear physics: deep inelastic scattering, parton distribution functions, nucleon hadronic tensor
- Something new?

BACKUP SLIDES

ETMC ensemble

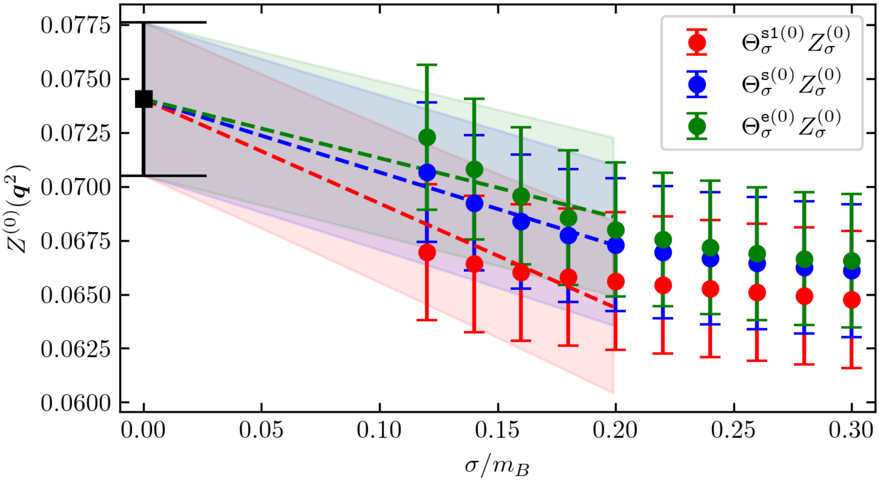
B55.32 ensemble

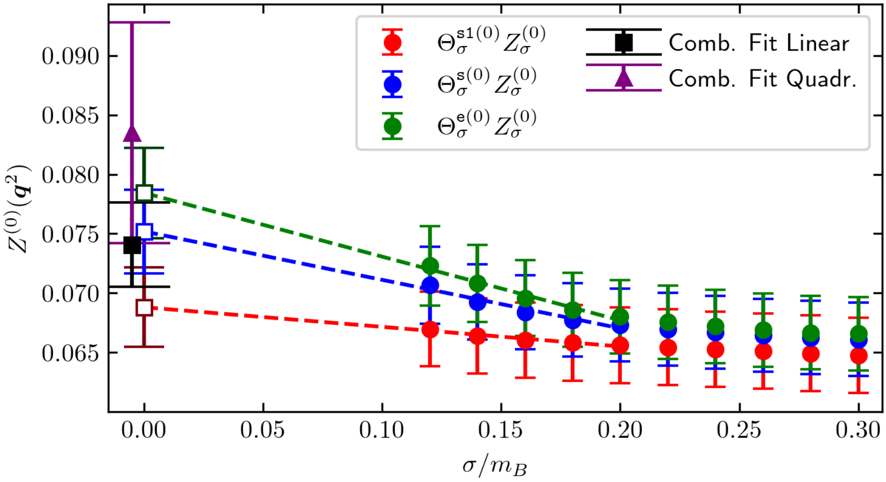
Uses the Twisted Mass QCD action with $N_f = 2+1+1$ sea quarks.

$L^3 \times T$	N_{cnfg}	$a\ (fm)$	
$32^3 \times 64$	150	0.0815(30)	
$a\mu_l$	$a\mu_\sigma$	$a\mu_\delta$	m_π (MeV)
0.0025	0.135	0.170	375(13)

Unphysically light B_s mass

Use of $m_c \simeq m_c^{phys}$ and $m_b = 2m_c$ which gives a meson mass $M_{B_s} = 3.08(11)$ GeV


0.0775
0.0750
0.0725
0.0700
0.0675
0.0650
0.0625
0.0600
$Z^{(0)}(\boldsymbol{q}^2)$
0.00
0.05
0.10
0.15
0.20
0.25
0.30
σ/m_B
$\Theta^{\mathrm{s1}(0)}_\sigma Z^{(0)}_\sigma$
$\Theta^{\mathrm{s}(0)}_\sigma Z^{(0)}_\sigma$
$\Theta^{\mathrm{e}(0)}_\sigma Z^{(0)}_\sigma$



$\Theta_\sigma^{s1(0)} Z_\sigma^{(0)}$
$\Theta_\sigma^{s(0)} Z_\sigma^{(0)}$
$\Theta_\sigma^{e(0)} Z_\sigma^{(0)}$
Comb. Fit Linear
Comb. Fit Quadr.
$Z^{(0)}(\boldsymbol{q}^2)$
0.090
0.085
0.080
0.075
0.070
0.065
0.00
0.05
0.10
0.15
0.20
0.25
0.30
σ/m_B