Electromagnetic conductivity of quark-gluon plasma at finite baryon chemical potential

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- Why to study conductivity?

- Lattice setup and strategy;

- Inverse problem;

- Results and interpretation;

- Conductivity in presence of strong magnetic fields;

- Conclusions.

The e.m. conductivity is a transport coefficient which parametrizes the charge transport phenomena.

Challenging → heavy-ion collision experiments

During these collisions ***nonzero baryon density.***

How nonzero baryon density influences the e.m. conductivity of QGP?

↓

We made its first lattice QCD study.

$N_f = 2 + 1$ dynamical staggered quarks at physical quark masses with chemical potentials

$$\mu_u = \mu_d = \mu_B/3, \quad \mu_s = 0.$$

a, fm	L_s	N_t	T, MeV	$m_l a$	$m_s a$	$\mu_I/3\pi T$	$c(T)$
0.0988	48	10	200	0.0014	0.0394	0.0, 0.14, 0.20, 0.245, 0.285	0.0060(14)
0.0788	48	10	250	0.001119	0.031508	0.0, 0.14, 0.20, 0.245, 0.285	0.0086(14)
0.0820	48	12	200	0.001168	0.032872	0.0, 0.14, 0.20, 0.245, 0.285	0.0076(14)
0.0657	48	12	250	0.000917	0.025810	0.0, 0.14, 0.20, 0.245, 0.285	0.0084(10)

To measure the conductivity we went trough two principal steps

Step 1: the measure of the correlation functions

Step 2: spectral function extraction and conductivity computation via Kubo formulas

$$C_{ij}(\tau) = \frac{1}{L_s^3}\langle J_i(\tau) J_j(0)\rangle$$

$$J_i(\tau) = \frac{1}{4} e \sum_f q_f \sum_{\vec{x}} \eta_{x;i} \left(\bar{\chi}^f_x U^{(2)}_{x,i} \chi^f_{x+i} + \bar{\chi}^f_x U^{(2)\dagger}_{x,i} \chi^f_{x+i} \right)$$

$\eta_i(x) = (-1)^{x_1 + \ldots x_{i-1}}$ where $x = (\tau, \vec{x})$ and i=1,2,3

$\bar{\chi}^f_x, \chi^f_x$ are staggered fermion fields of f=u,d,s flavours.

This correlators correspond to two different operators for the even $\tau = 2n \times a$ and odd $\tau = (2n+1) \times a$ slices. In the continuum limit it reads

$$C^{e,o}_{ij}(\tau) = \sum_{\vec{x}} \left(\langle A_i(x) A_j(0)\rangle - s^{e,o} \langle B_i(x) B_j(0)\rangle \right)$$

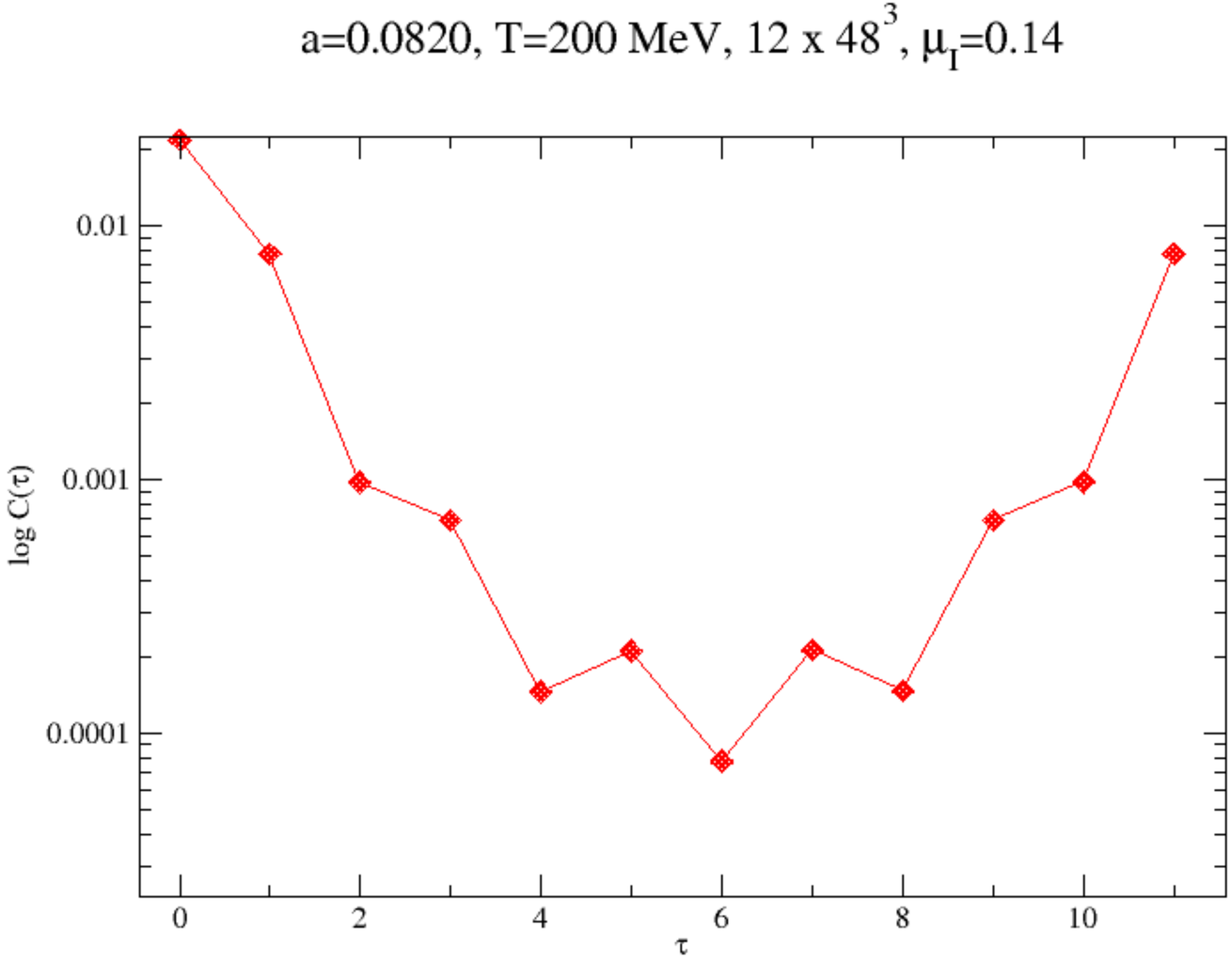
$A_i = e \sum_f q_f \bar{\psi}^f \gamma_i \psi^f$ corresponding to the electromagnetic current in the continuum limit

$s^{e,o} = (-1)^\tau$

$B_i = e \sum_f q_f \bar{\psi}^f \gamma_5 \gamma_4 \gamma_i \psi^f$

The figure shows an example of the result we obtain for a correlation function at nonzero imaginary chemical potential.

The correlator is symmetric. Furthermore, the analysis is made independently for the even and odd values of the Euclidean time.



Having computed the correlators for all the lattice spacing for the even and odd slices, we can study the conductivity.

How?

Let's recall

$$C_{ij}^{e,o}(\tau) = \int_0^\infty \frac{d\omega}{\pi} K(\tau,\omega)\rho_{ij}^{e,o}(\omega)$$

where $K(\tau,\omega) = \dfrac{\cosh \omega(\tau - \beta/2)}{\sinh \omega\beta/2}$

If we are able to invert this relation and then extract $\rho_{ij}^{e,o}(\omega)$, then

$$\frac{\sigma_{ij}}{T} = \frac{1}{2T} \lim_{\omega\to 0} \frac{1}{\omega}\left(\rho_{ij}^{e}(\omega) + \rho_{ij}^{o}(\omega)\right)$$

The problem of the inversion of the correlation function to extract the spectral function is well known in literature.

Smearing function techniques

$$\delta(\bar{\omega}, \omega) = \sum_i q_i(\bar{\omega}) K(\tau_i, \omega)$$

in such a way that once the coefficients $q_i(\bar{\omega})$ are known we can extract the smeared spectral density as

$$\hat{\rho}(\bar{\omega}) = \int_0^\infty d\omega \rho(\omega) \delta(\bar{\omega}, \omega) = \sum_i q_i(\bar{\omega}) C(\tau_i) \,.$$

But how do we fix the coefficients $q_i(\bar{\omega})$?

Backus-Gilbert method ***[G. Backus and F. Gilbert, Geophysical Journal International 16, 169 (1968)]***
A modified version of it recently proposed in [***M. Hansen, A. Lupo, and N. Tantalo, Physical Review D 99 (2019)].***

We considered the difference $\Delta C^{e,o} = C^{e,o}_{\mu_I} - C^{e,o}_{\mu_I=0}$

The UV regime starts around $\omega \sim 2GeV$ and $\mu_I \ll \omega$ for frequencies in the UV regime and baryon chemical potential.

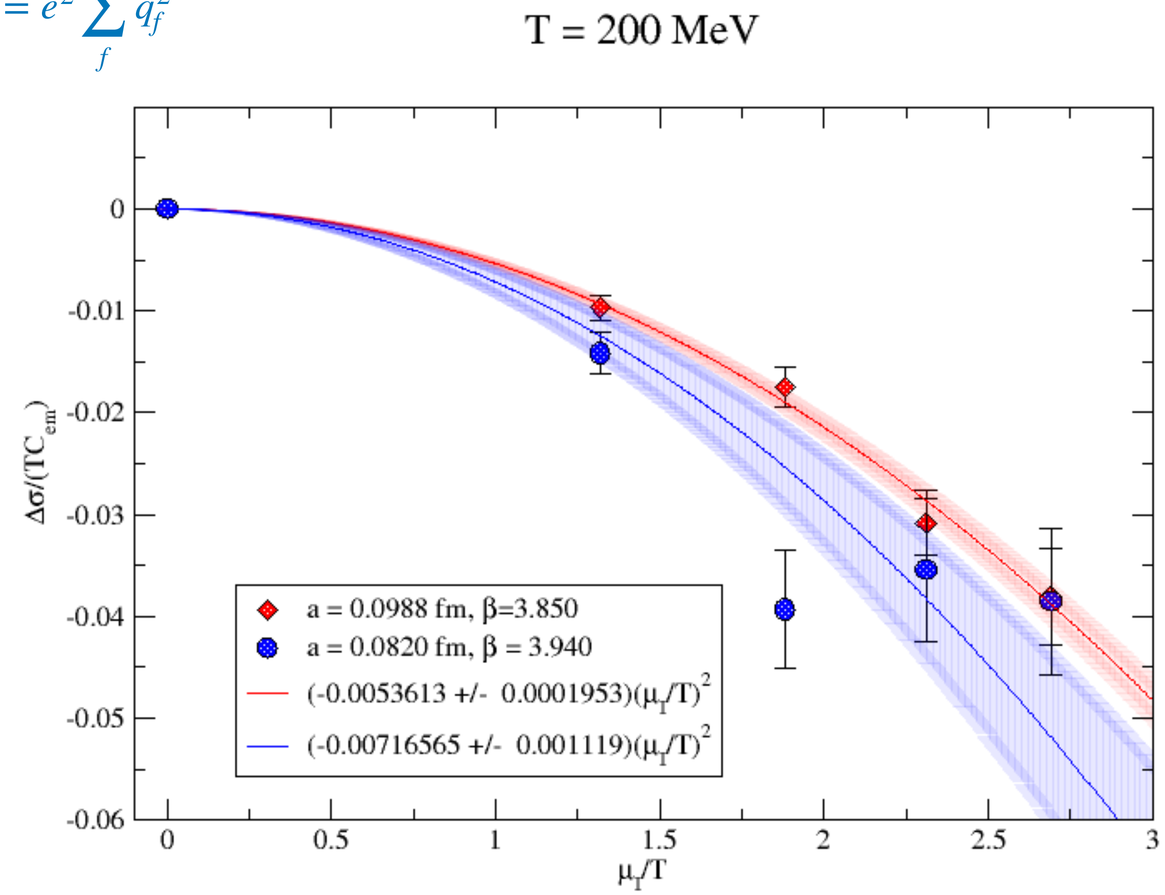
UV spectral function independent on the imaginary chemical potential and assume that $\Delta C^{e,o}$ does not contain UV contributions.

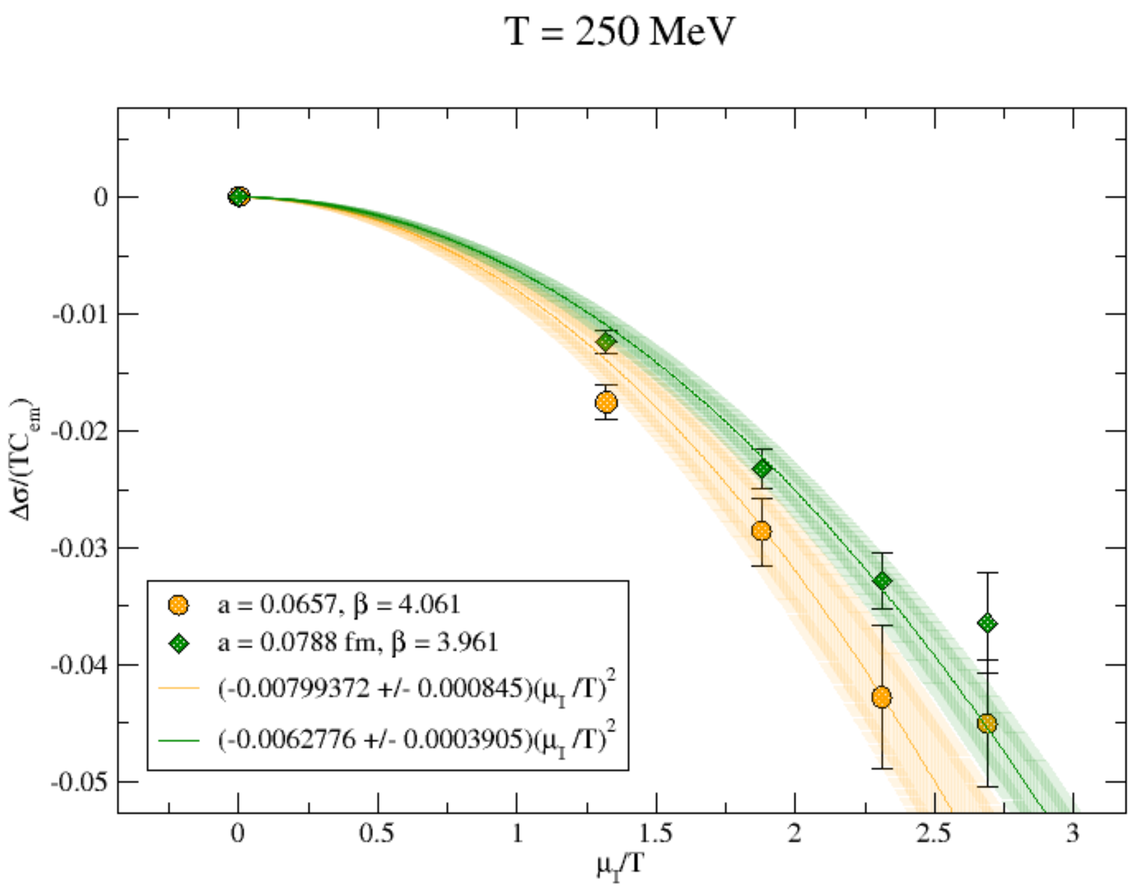
We extract the conductivity $\Delta\sigma$ using the Kubo formula

$$\frac{\Delta\sigma_{ij}}{T} = \frac{1}{2T}\lim_{\omega\to 0}\frac{1}{\omega}\left(\Delta\rho^{e}_{ij}(\omega) + \Delta\rho^{o}_{ij}(\omega)\right).$$

In Figure we show the change of the e.m. conductivity $\Delta\sigma = \sigma_{\mu_I} - \sigma_{\mu_I=0}$ as a function of imaginary chemical potential.

$$C_{em} = e^2 \sum_f q_f^2$$





$$\frac{\Delta\sigma}{TC_{em}} = -\,c(T)\left(\frac{\mu_I}{T}\right)^2 \longrightarrow \frac{\Delta\sigma}{TC_{em}} = c(T)\left(\frac{\mu_B}{T}\right)^2$$

a, fm	T, MeV	c_{standard}	c_{improved}
0.0988	200	0.0066(12)	0.0061(9)
0.0788	250	0.0077(12)	0.0089(6)
0.0820	200	0.0077(16)	0.0080(18)
0.0657	250	0.0082(12)	0.011(1)

By looking at the results of the fit showed in the table we can make the following observations:

- No dependence on the lattice spacing;

- Different methods are compatible;

- The temperature dependence of the $c(T)$ is within the uncertainties;

- The coefficients for all the lattice parameters are positive and so the conclusion is that real ***baryon density enhances the e.m. conductivity.***

In fact it could generate Chiral Magnetic Effect (CME) which is particularly important to understand heavy ion collision phenomenology.

Conductivity in presence of large magnetic fields ($eB \sim 4,\ 9\ GeV^2$) at different temperatures.

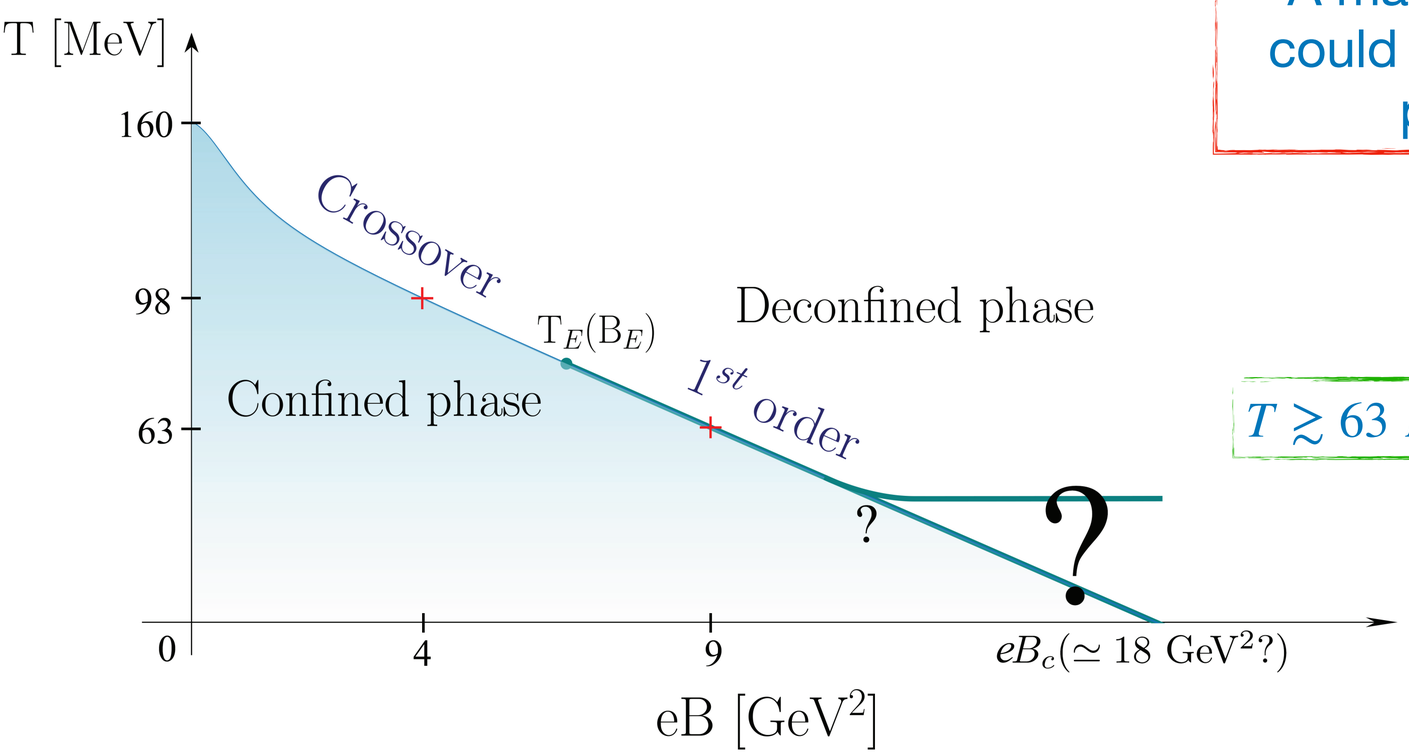
We used the same configurations adopted in ***Phys. Rev. D 105 (2022) no.3, 034511***..

The direction of the magnetic field is **along the z axis**.

$eB[\mathrm{GeV}^2]$	b	$a[\mathrm{fm}]$	β	am_s	N_s	N_t
9	93	0.0572	4.140(6)	0.0224	48	12,14,16,18,20,22,26,28, 30,32,34,36,38,40,44
		0.0858(2)	3.918	0.0343	32	12,14,16,18,20,22, 24,26,30
		0.1144(3)	3.787	0.0457	24	12,14,16,18,20,22
4	41	0.0572	4.140(6)	0.0224	48	12,14,16,18,20, 30,32,34,36,38,40
		0.0858(2)	3.918	0.0343	32	12,14,16,18,20,22, 24,26,28
		0.1144(3)	3.787	0.0457	24	12,14,16,18,20,22

See Lorenzo Maio's talk for further details

Note that in ***Phys. Rev. D 105 (2022) no.3, 034511*** a new hint for the phase diagram of the QCD in the (eB,T) plane was proposed.



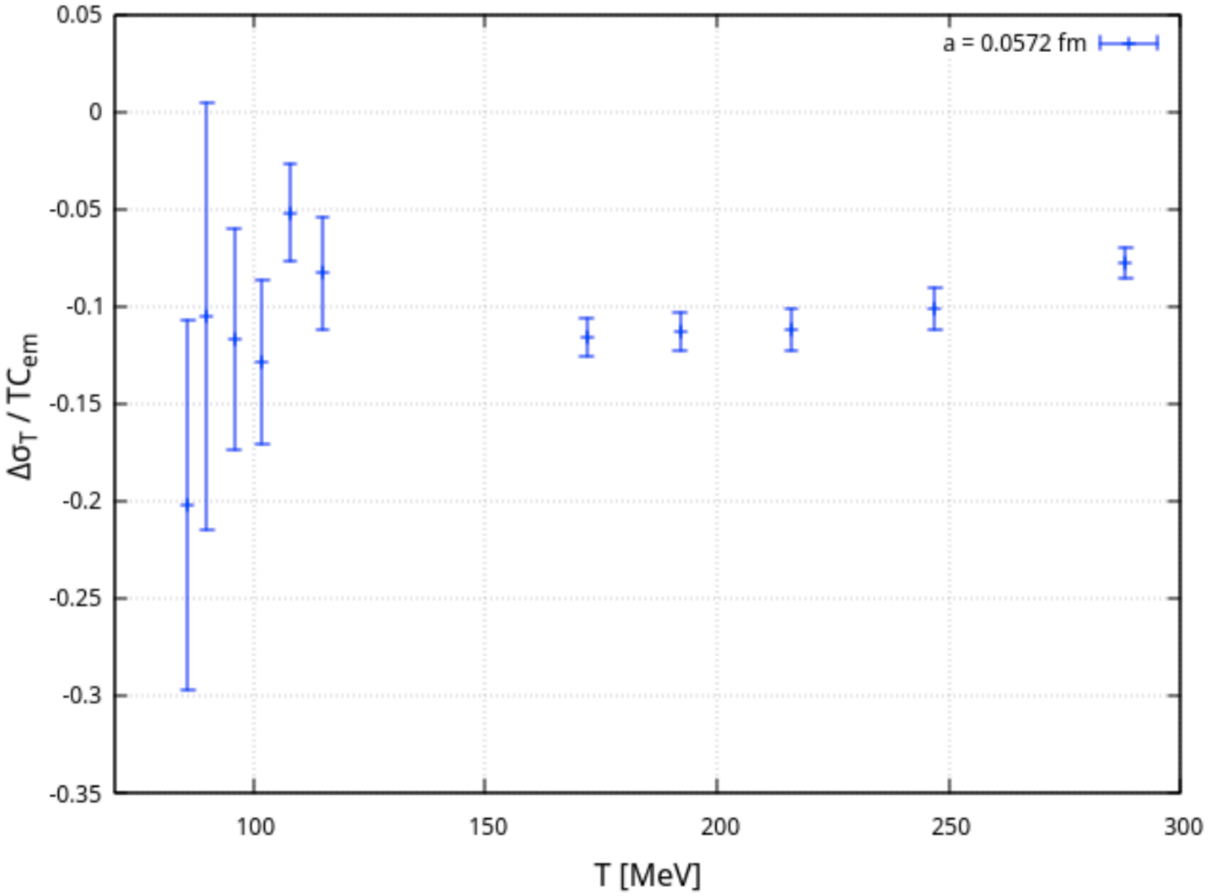
A manifestation of the Chiral Magnetic Effect (CME) could appear as a rising of the conductivity along the parallel direction in the deconfined phase.

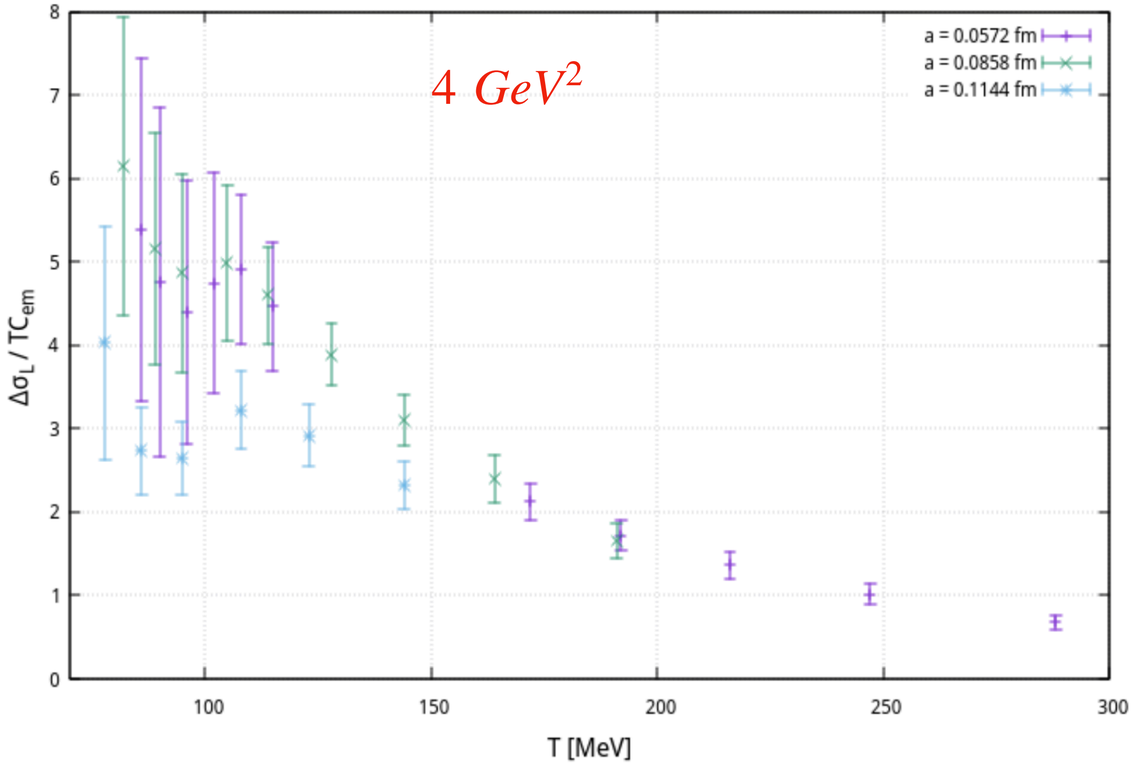
$T \gtrsim 63\ MeV$ for $eB = 9\ GeV^2$ and $T \gtrsim 98\ MeV$ for $eB = 4\ GeV^2$

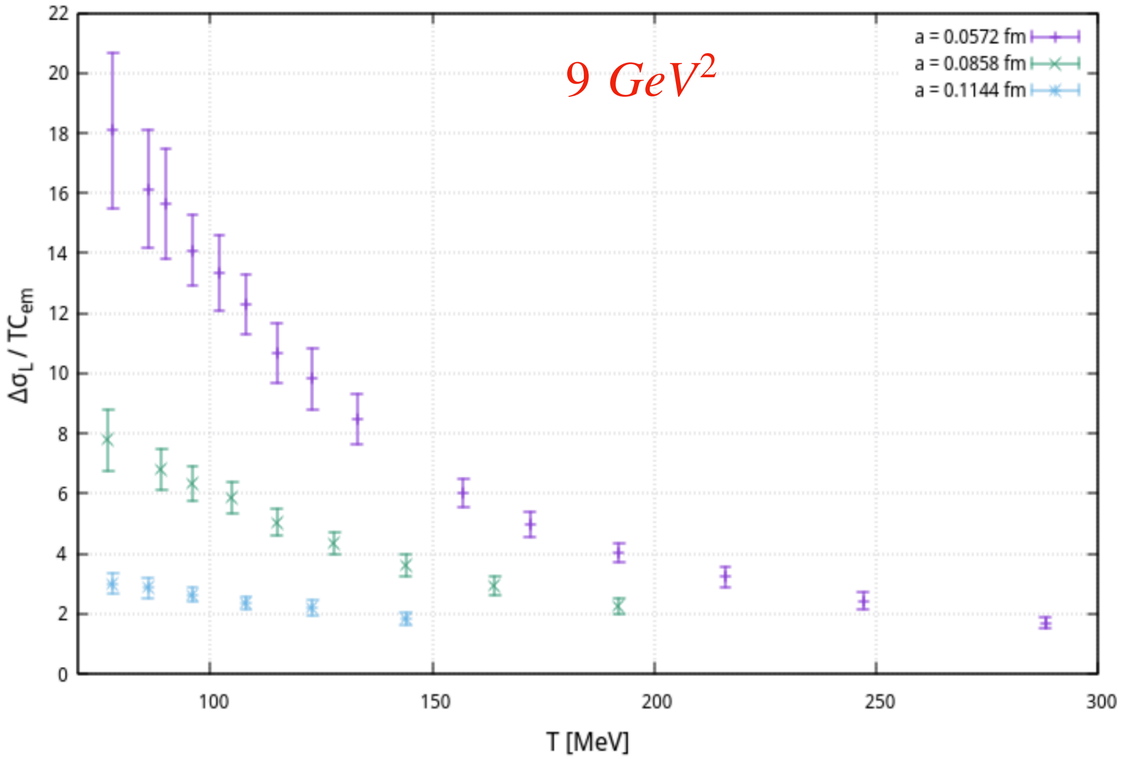
The figure shows the results for the perpendicular component of the conductivity as a function of $T = 1/(N_t a)$ for $eB = 4GeV^2$ and $a = 0.0572fm$.

The magnetic field has the clear effect of reducing the conductivity which is the effect of the magnetoresistance of the QGP on the lattice.

Around the transition ($\simeq 98\ MeV$) the errors are too large and we are not able to make any conclusion.



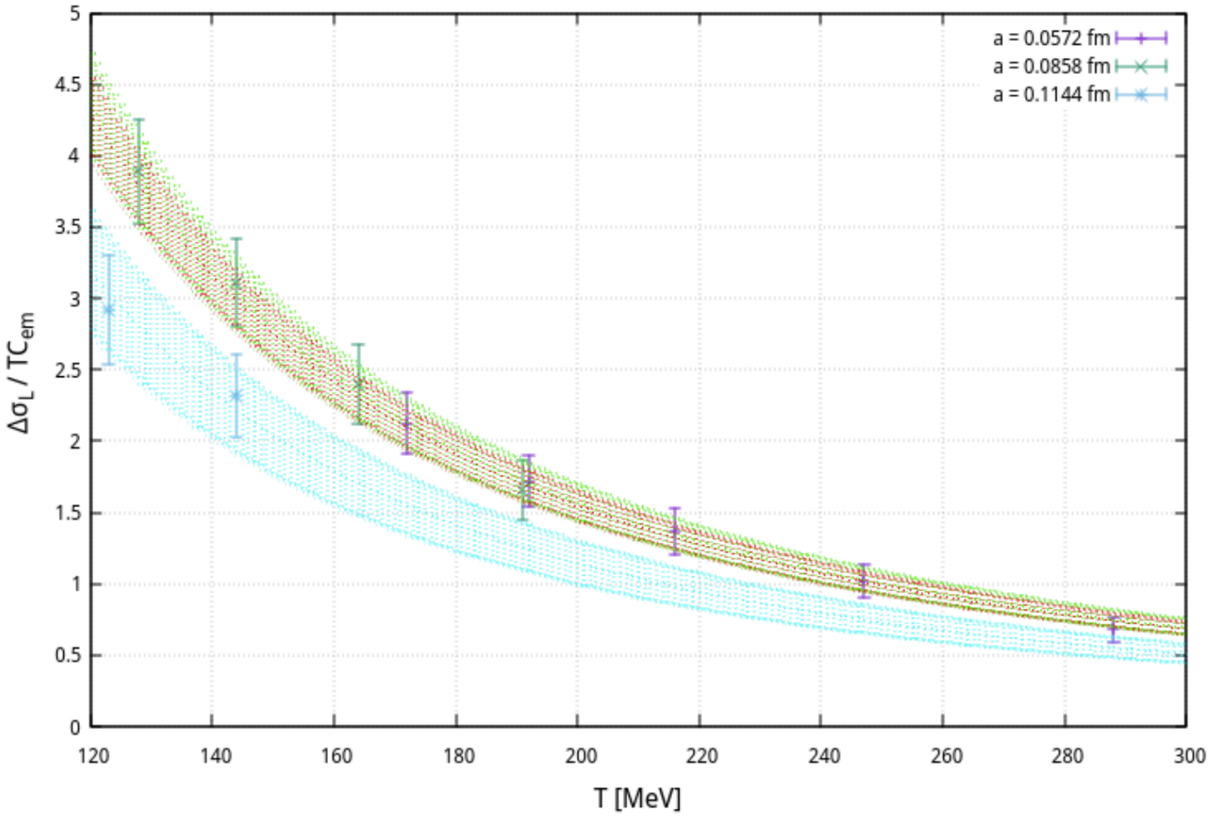


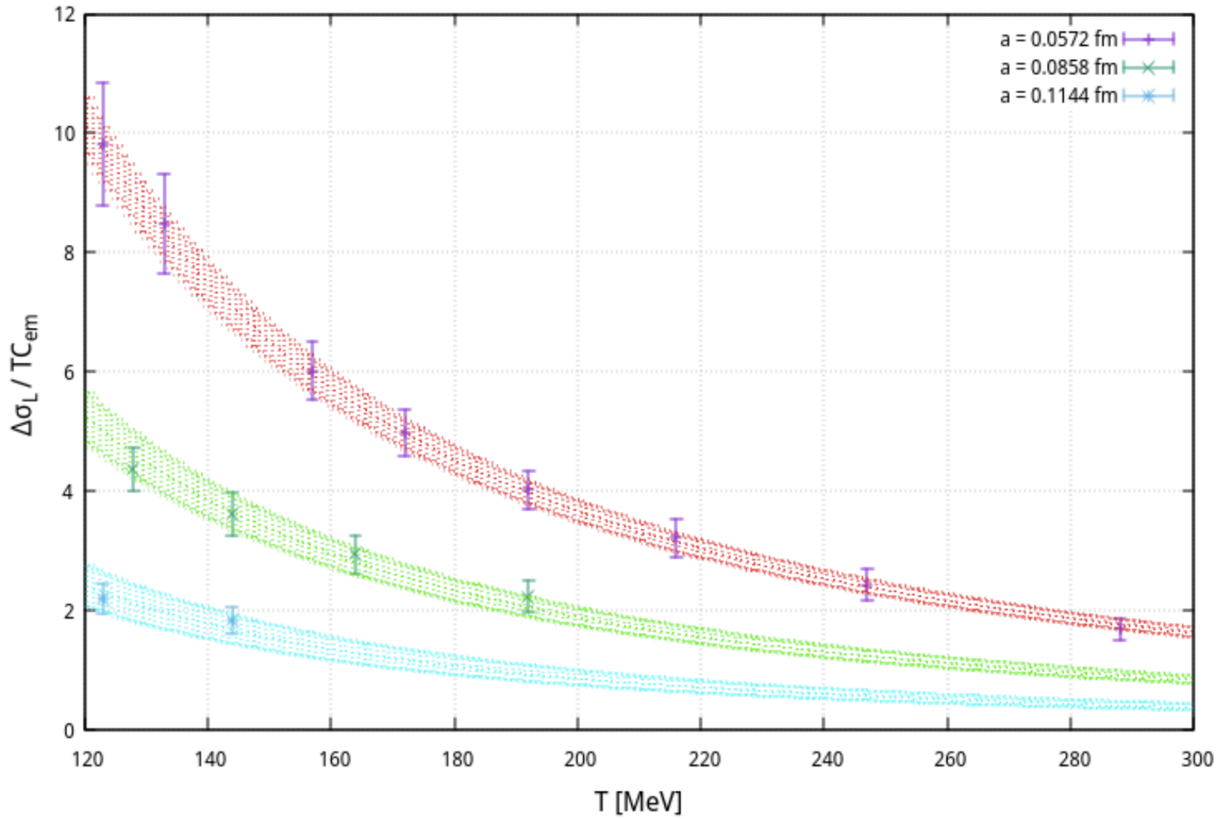


Manifestation of the CME in the QGP.

In the case of $9\ GeV^2$ the points are all in the deconfined regime ($T_C \simeq 63 MeV$), while for $4\ GeV^2$ the transition is around $98\ MeV$.

Parallel case: $1/T$ behaviour





For both the magnetic fields at high temperature ($T > 120 MeV$) we observe a $1/T$ behaviour.

To check this we performed a fit using the function

$$f_B(T) = eB \times C_B/T^2$$

$eB[\mathrm{GeV}^2]$	$a[\mathrm{fm}]$	C_B
9	0.0572	0.0164(7)
	0.0858(2)	0.0096(6)
	0.1144(3)	0.0038(5)
4	0.0572	0.0154(9)
	0.0858(2)	0.0159(11)
	0.1144(3)	0.0115(12)

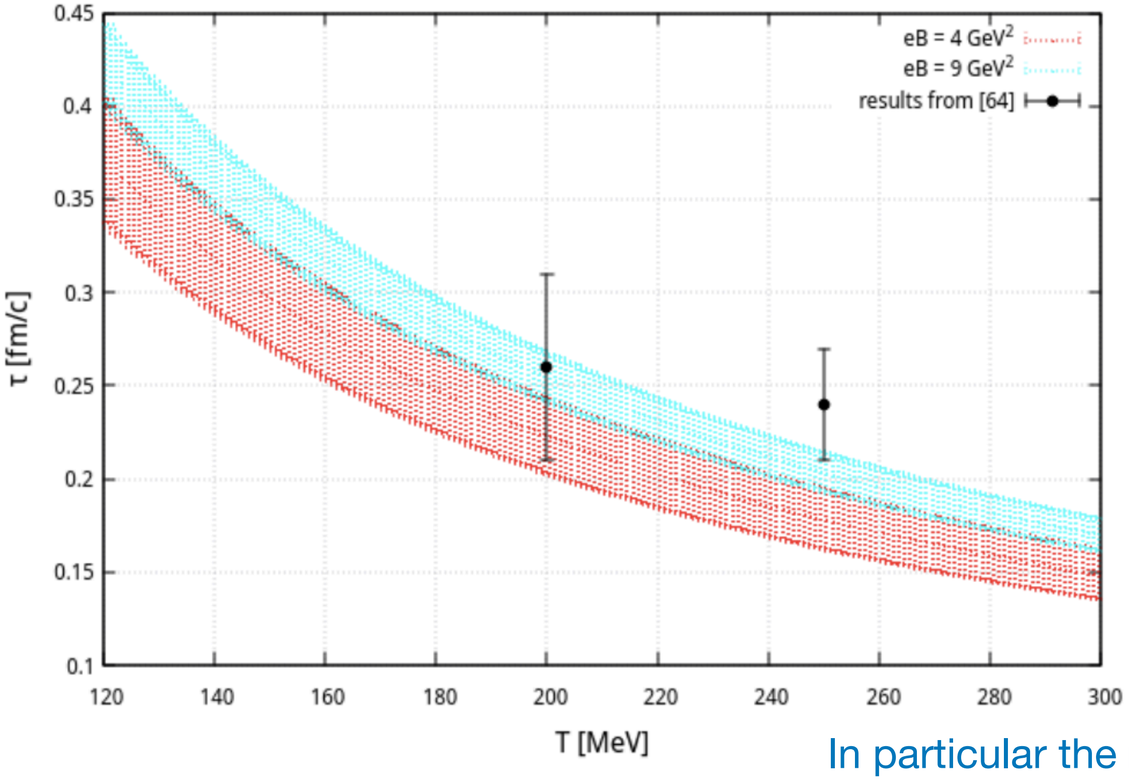
If the main contribution to the conductivity comes from the CME

$$C = N_c \sum_f q_f^2 \qquad \frac{\Delta\sigma_L}{TC_{em}} = \frac{C^2}{C_{em}} \frac{e^2}{4\pi^4} \frac{eB}{N_c \sum_f |q_f|/2\pi^2} \frac{\tau}{T} = eB \times C_B/T^2$$

relaxation time

That means that we found a behaviour for the relaxation time

$$\tau \sim 1/T$$



By performing the continuum limit on the coefficients C_B we can extract the behaviour of the relaxation time as a function of the temperature.

$$\tau = C_t/T \rightarrow \begin{cases} C_\tau(4\ GeV^2) = 0.226 \pm 0.020 \\ C_\tau(9\ GeV^2) = 0.258 \pm 0.013 \end{cases}$$

In particular the results for $T = 200,\ 250\ MeV$ are

4 GeV² $\tau(200\ MeV) = 0.223 \pm 0.020\ fm/c\ ;\ \tau(250\ MeV) = 0.223 \pm 0.020\ fm/c$

9 GeV² $\tau(200\ MeV) = 0.255 \pm 0.013\ fm/c\ ;\ \tau(250\ MeV) = 0.203 \pm 0.010\ fm/c$

which are compatible with the results from ***arXiv:1910.08516.***

1) *Conductivity in presence of non zero baryon density*

- We studied the electromagnetic conductivity in dense quark-gluon plasma obtained within lattice simulations with $N_f = 2 + 1$ dynamical quarks;
- The simulations were performed at imaginary chemical potential and to reconstruct the e.m. conductivity we employed the Backus-Gilbert method, both in the normal and modified versions.
- Our results were analytically continued to real values of baryon chemical potential.

The study indicates that e.m. conductivity of QGP raises with real baryon density and this dependence is quite strong.

2) *Conductivity in presence of strong magnetic field*

- ***Possible manifestation of the Chiral Magnetic effect*** in the deconfined phase;
- We observed a 1/T behaviour at large values of T that indicates a ***1/T behaviour for the relaxation time*** τ;

Sphalerons?

Thanks for your attention!

Back-up slides

In the case of the Backus Gilbert method, to fix the coefficients the idea is to introduce the functional

$$W[\lambda, q] = (1-\lambda)A[q] + \lambda B[q]$$

$A[q] = \int_0^\infty d\omega(\omega - \bar{\omega})^2\{\delta(\bar{\omega}, \omega)\}^2$ that measures the width of the smearing function.

$B[q] = q^T Cov q$, being Cov the covariance matrix of the correlators. It takes into account the fact that the correlators have uncertainties.

$\lambda \in [0,1]$ is a tradeoff between the systematic error (related to the width of the smearing) and the statistic one.

By minimizing it and imposing the unit area constraint, $\int_0^\infty d\omega\delta(\bar{\omega}, \omega) = 1$, we can extract an expression for the coefficients

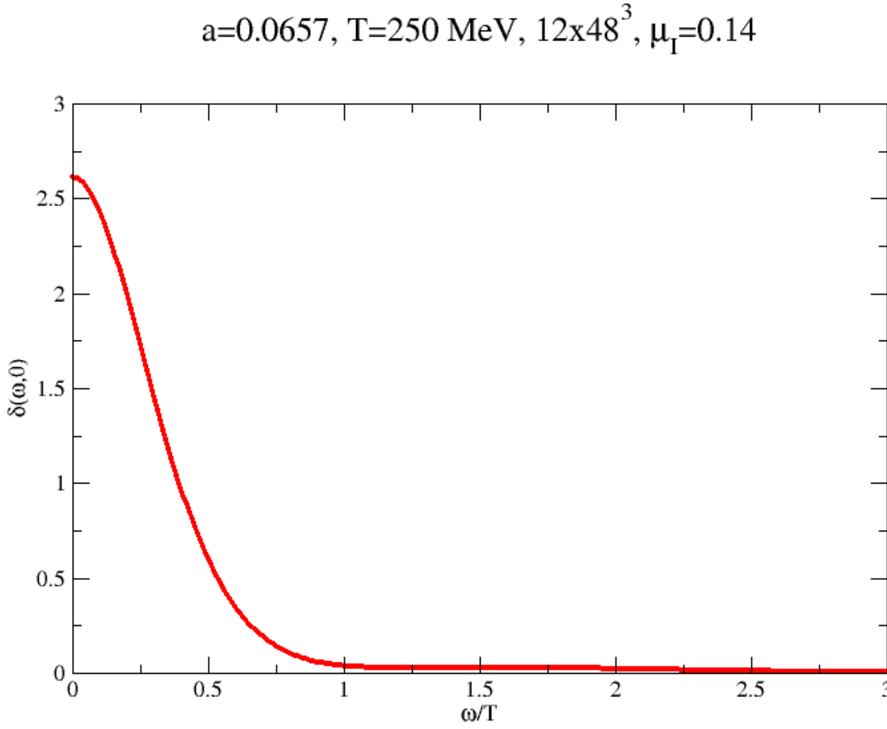
$$q(\bar{\omega}, \lambda) = \frac{W^{-1}(\lambda, \bar{\omega})R}{R^T W^{-1}(\lambda, \bar{\omega})R}$$

$W_{ij}(\lambda, \bar{\omega}) = (1-\lambda)A_{ij} + \lambda Cov_{ij}$, being

$A_{ij}(\bar{\omega}) = \int_0^\infty d\omega(\omega - \bar{\omega})^2 K(\tau_i, \omega)K(\tau_j, \omega)$

$R_i = \int_0^\infty d\omega K(\tau_i, \omega)$

through which we can extract the smearing function.



Thus, fixed λ, this allows us to extract the smearing function and then the value of the spectral function in the generic point $\bar{\omega}$:

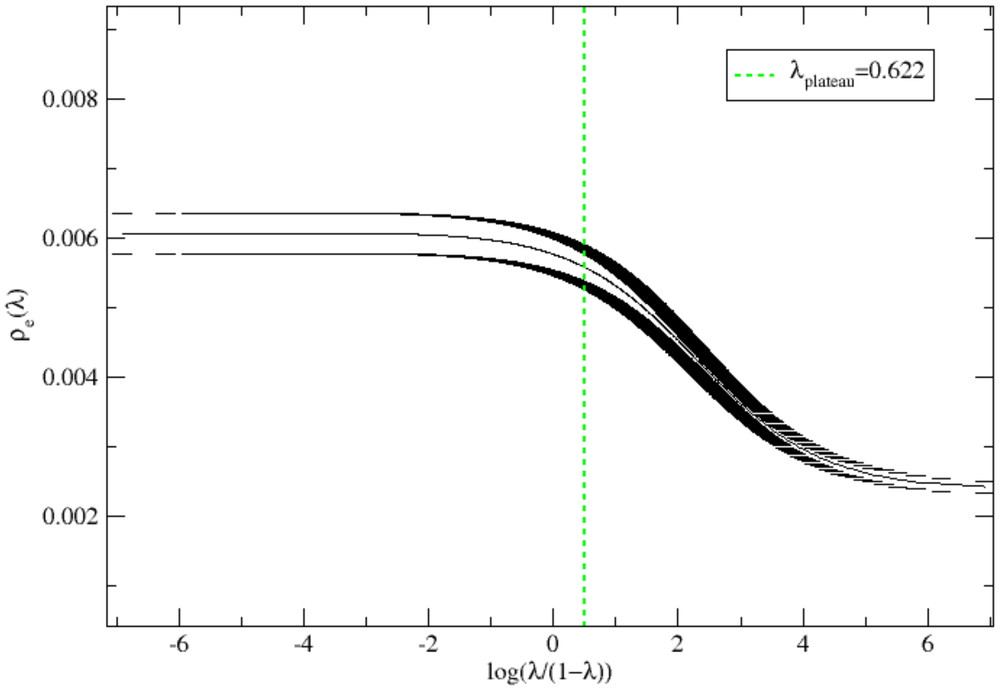
$$\hat{\rho}_\lambda(\bar{\omega}) = \sum_i q_i(\bar{\omega}, \lambda) C(\tau_i)$$

But how do we fix λ?

We look at spectral function for different values of the λ parameter and then choose the value in correspondence of the first point (starting from larger values of λ) in which a plateau, namely the region where the value stabilizes inside the errors, begins.

Once we fixed λ we can extract the value of the spectral function in the unknown point $\bar{\omega}$.

a=0.0656, T=250MeV, $12x48^3$, μ_I=0.245



In the case of the modified BG the main difference is the change of the choice of the first term of the functional

$$A[q] = \int_0^\infty d\omega(\omega - \bar{\omega})^2\{\delta(\bar{\omega},\omega)\}^2 \longrightarrow A[q] = \int d\omega\rho(\omega)(\delta(\omega,\bar{\omega}) - \delta_0(\omega,\bar{\omega}))^2$$

The idea is that instead of minimizing the width of the smearing, we minimize the deviation from a reference chosen function $\delta_0(\omega,\bar{\omega})$.

This leads to an analogous expression for the coefficients

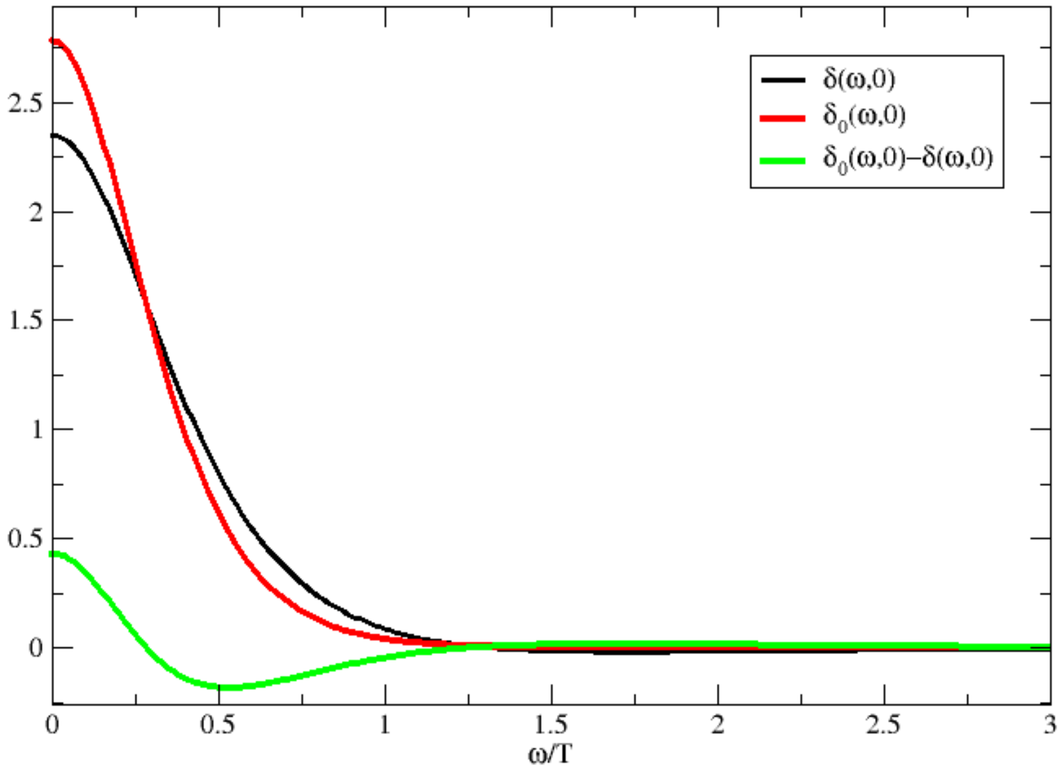
$$\hat{\rho}_\lambda(\bar{\omega}) = \sum_i q_i(\bar{\omega},\lambda)C(\tau_i).$$

where λ will be fixed with the same strategy as before.

Here we'll have a new uncertainty which results from the deviation of the resolution function $\delta(\omega,\bar{\omega})$ from the target one $\delta_0(\omega,\bar{\omega})$ that can be computed as

$$\Delta_{syst} = |r|\bar{\rho}(\bar{\omega})$$

where $r = 1 - \dfrac{\delta(\bar{\omega},\bar{\omega})}{\delta_0(\bar{\omega},\bar{\omega})}$ is the relative deviation at the peak.



Chemical potentials μ_f $(f = u, d, s)$ are coupled to quark number operators, $\mathcal{Z}(T, \mu_u, \mu_d, \mu_s)$, in a setup for which

$$\mu_u = \mu_d = \mu_B/3,\ \mu_s = 0$$

The path integral formulation $\mathcal{Z}(T, \mu_B)$, discretized via improved rooted staggered fermions and using exponentiated implementation of the chemical potentials, reads

$$\mathcal{Z}(T, \mu_B) = \int \mathcal{D}U e^{-S_{YM}} \prod_{f=u,d,s} \det \left[M^f_{st}(U\,.\,u_f) \right]^{1/4}$$

where

$$S_{YM} = -\frac{\beta}{3} \sum_{i,\mu\neq\nu} \left(\frac{5}{6} W^{1\times1}_{i;\mu,\nu} - \frac{1}{12} W^{1\times2}_{i;\mu\nu} \right)$$

is the Symanzik improved action and the staggered fermion matrix is defined as

$$M^f_{st}(U, \mu_f) = a m_f \delta_{i,j} + \sum_{\nu=1}^{4} \frac{\eta_{i;\nu}}{2} [e^{a\mu_f \delta_{\nu,4}} U^{(2)}_{i;\nu} \delta_{i,j-\hat{\nu}} - e^{-a\mu_f \delta_{\nu,4}} U^{\dagger}_{i-\hat{\nu};\nu} \delta_{i,j+\hat{\nu}}]$$

The naivest way we can think is to fix them such that the smearing function is as more peaked as possible. Then, we could fix the coefficients by minimizing the deterministic functional

$$A[q] = \int_0^\infty d\omega(\omega - \bar{\omega})^2 \{\delta(\bar{\omega}, \omega)\}^2$$

that can be interpreted as a measure of the width of the smearing function.

Minimizing the functional by imposing the unit area constraint, namely $\int_0^\infty d\omega \delta(\bar{\omega}, \omega) = 1$, we can extract an expression for the coefficients

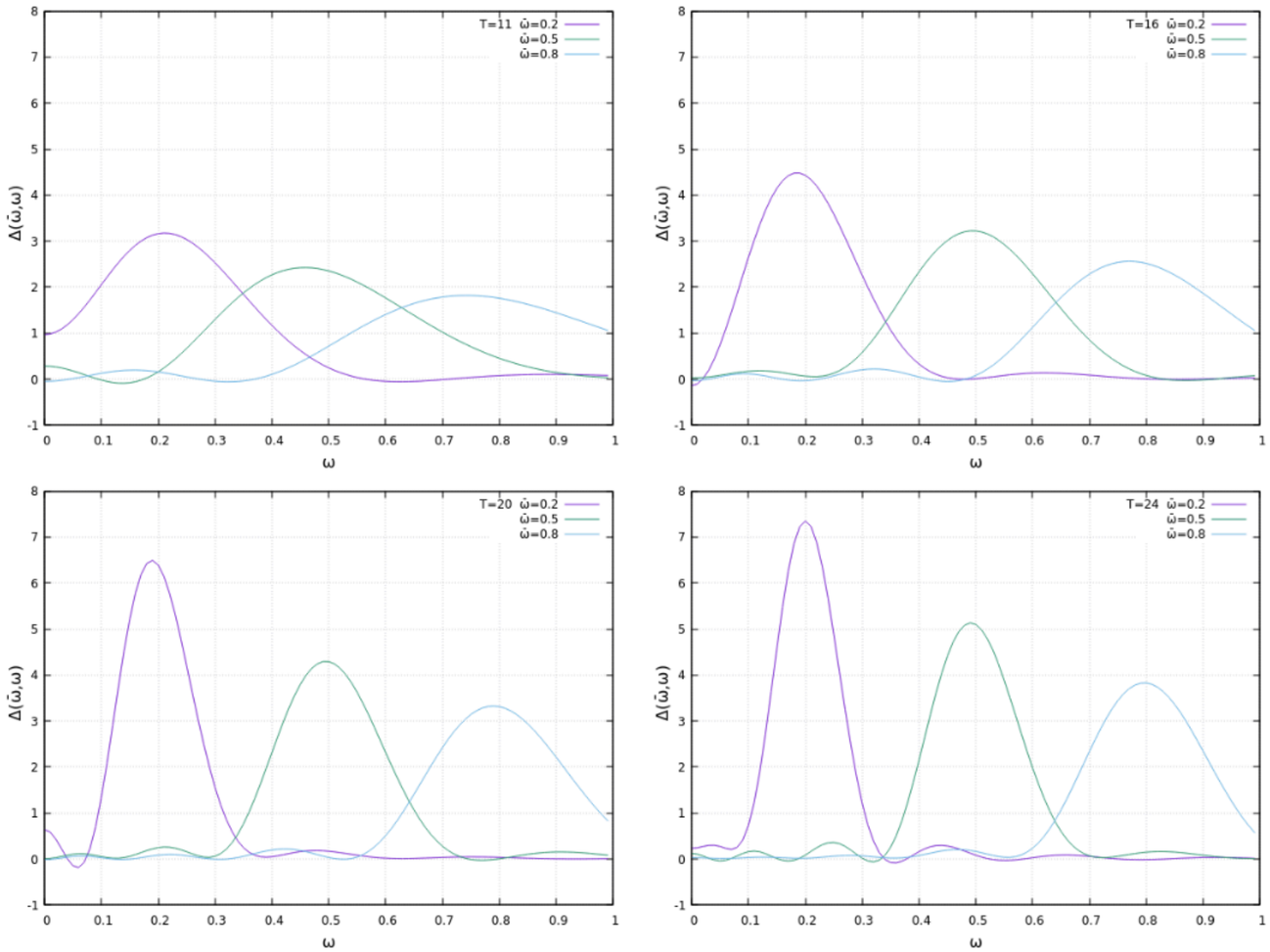
$$A_{ij}(\bar{\omega}) = \int_0^\infty d\omega(\omega - \bar{\omega})^2 K(\tau_i, \omega) K(\tau_j, \omega)$$

$$q(\bar{\omega}) = \frac{A^{-1}(\bar{\omega})R}{R^T A^{-1}(\bar{\omega})R}$$

$$R_i = \int_0^\infty d\omega K(\tau_i, \omega)$$

This would allow us to compute the smeared spectral function in the point $\bar{\omega}$ by using the relation $\hat{\rho}(\bar{\omega}) = \sum_i q_i(\bar{\omega}) C(\tau_i)$.

Note that the coefficients only depend on the point $\bar{\omega}$ where we want extract information and on the Euclidean time. Obviously the more point we have the tighter will be the smearing function $\delta(\bar{\omega}, \omega)$.



We show the smearing functions for different values of the point $\bar{\omega}$ in correspondence of which we want to extrapolate and for different number of points N_t.

At fixed $\bar{\omega}$, the smearing becomes more similar to a Dirac-δ function increasing N_t. This is due to the fact that, in absence of the second term, the functional tends to reduce the width of the smearing function as much as possible:

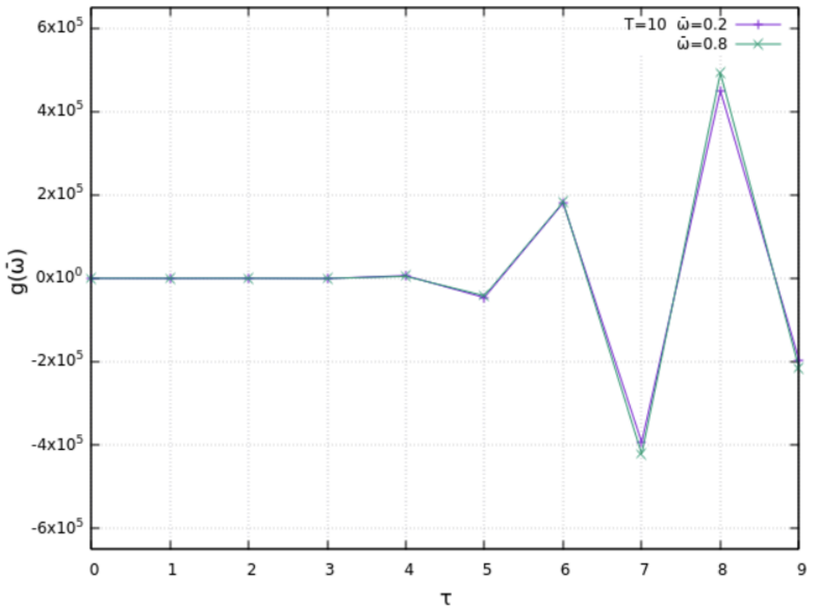
$$\lim_{N_t \to \infty} \Delta(\omega, \bar{\omega}) = \delta(\omega - \bar{\omega}) \, .$$

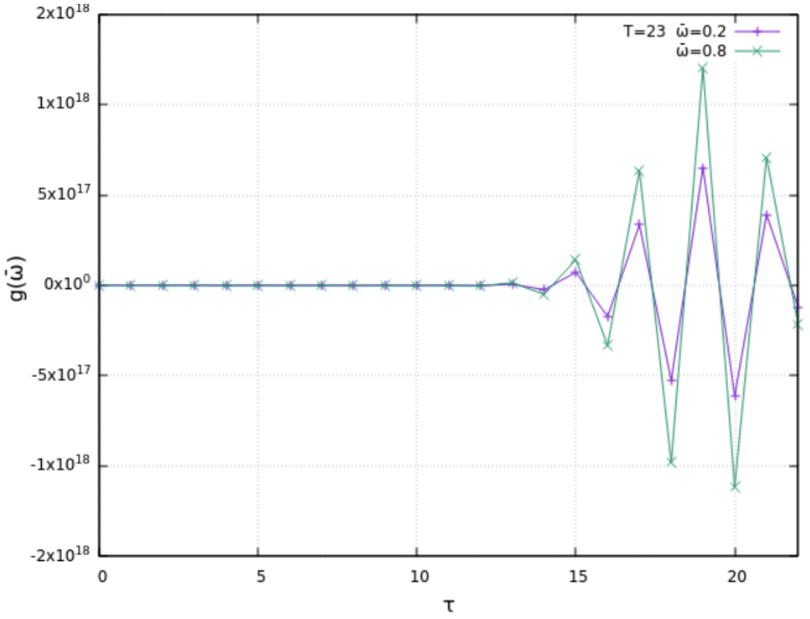
However there is a problem. In fact in the real case, where the correlators are affected by uncertainties, if the width of the smearing function is too small then the method becomes unstable and susceptible to noise in the data.

In fact when N_t becomes large, then also the coefficients become large and oscillating.

Thus, if we consider the errors associated to the correlators $C(t_i)$, we obtain that

$$\sum_i g_i(\bar{\omega})(C(\tau_i) + \delta C(\tau_i)) = \hat{\rho}(\bar{\omega}) + \sum_i g_i(\bar{\omega})\delta C(\tau_i)$$





Being the coefficients g_i large, also this term will be large and then the final error will be unacceptably large.

For this reason, we need to add the second term in the expression of the functional to take into account that we want a balance also with the statistical uncertainties.

While in the in the BG method we add the second term that takes into account uncertainties in the TR scheme it's regularized the Singular Value Decomposition (SVD) of

$$A^{-1} = VDU^T$$

The diagonal matrice $D = diag(\sigma_1^{-1}, \sigma_2^{-1}, \ldots, \sigma_n^{-1})$ might have large entries that represent the susceptibility of the data to noise.

Thus, the regularization is done by adding the regularizer γ to all the entries as

$$\tilde{D} = diag((\sigma_1\gamma)^{-1}, (\sigma_2 + \gamma)^{-1}, \ldots, (\sigma_n + \gamma)^{-1})$$

In this way, small σ_i will be smoothly cut-off.

Extrapolation of $\rho(\omega)/\omega$ for $\omega \to 0$

For the Kubo formula

$$\frac{\sigma_{ij}}{T} = \frac{1}{2T} \lim_{\omega \to 0} \frac{1}{\omega} \left(\rho^e_{ij}(\omega) + \rho^o_{ij}(\omega) \right)$$

we want to compute the ratio of the $\rho(\omega)/\omega$ for $\omega \to 0$.

We directly extract it by using the BG method. In fact, we have that

$$C^{e,o}_{ij}(\tau) = \int_0^\infty \frac{d\omega}{\pi} K(\tau,\omega) \rho^{e,o}_{ij}(\omega) = \int_0^\infty \frac{d\omega}{\pi} \tilde{K}(\tau,\omega) \frac{\rho^{e,o}_{ij}(\omega)}{\omega}$$

$$\tilde{K}(\tau,\omega) = \omega \frac{\cosh \omega(\tau - \beta/2)}{\sinh \omega\beta/2}$$

In this way, we can find the smeared spectral function as

$$\hat{\rho}_\lambda(\bar{\omega}) = \bar{\omega} \sum_i q_i(\bar{\omega},\lambda) C(\tau_i)$$

Fixing $\bar{\omega} = 0$, this allows us to directly extract $\rho(\omega)/\omega$ for $\omega \to 0$.

To treat the statistical uncertainties, we made a binned bootstrap analysis.

We produced a number of new correlators samples by resampling the original sample. For each sample, we carried the analysis arriving at the final determination of the even and odd spectral functions and of the value of the conductivity. Then, we estimated the statistical error on the conductivity by monitoring it fluctuation over the different bootstrap samples.

In the error bar we reported, in the case of the modified BG, also the systematic error for both even and odd spectral function by using the definition

$$\Delta_{syst} = |r|\bar{\rho}(\bar{\omega})$$

where $r = 1 - \dfrac{\delta(\bar{\omega},\bar{\omega})}{\delta_0(\bar{\omega},\bar{\omega})}$ is the relative deviation at the peak.

When we summed the even and odd spectral functions to obtain the conductivity, we also summed in modulus the systematic errors. Then, we summed in modulus the final value of the systematic error to the statistical uncertainty coming from the bootstrap analysis.

The Backus-Gilbert, such as the Tikhonov regularization procedure, has three main problems:

- The resolution function depends on the lattice data and then it's non strictly correct to compare the spectral functions obtained at different lattice spacings;

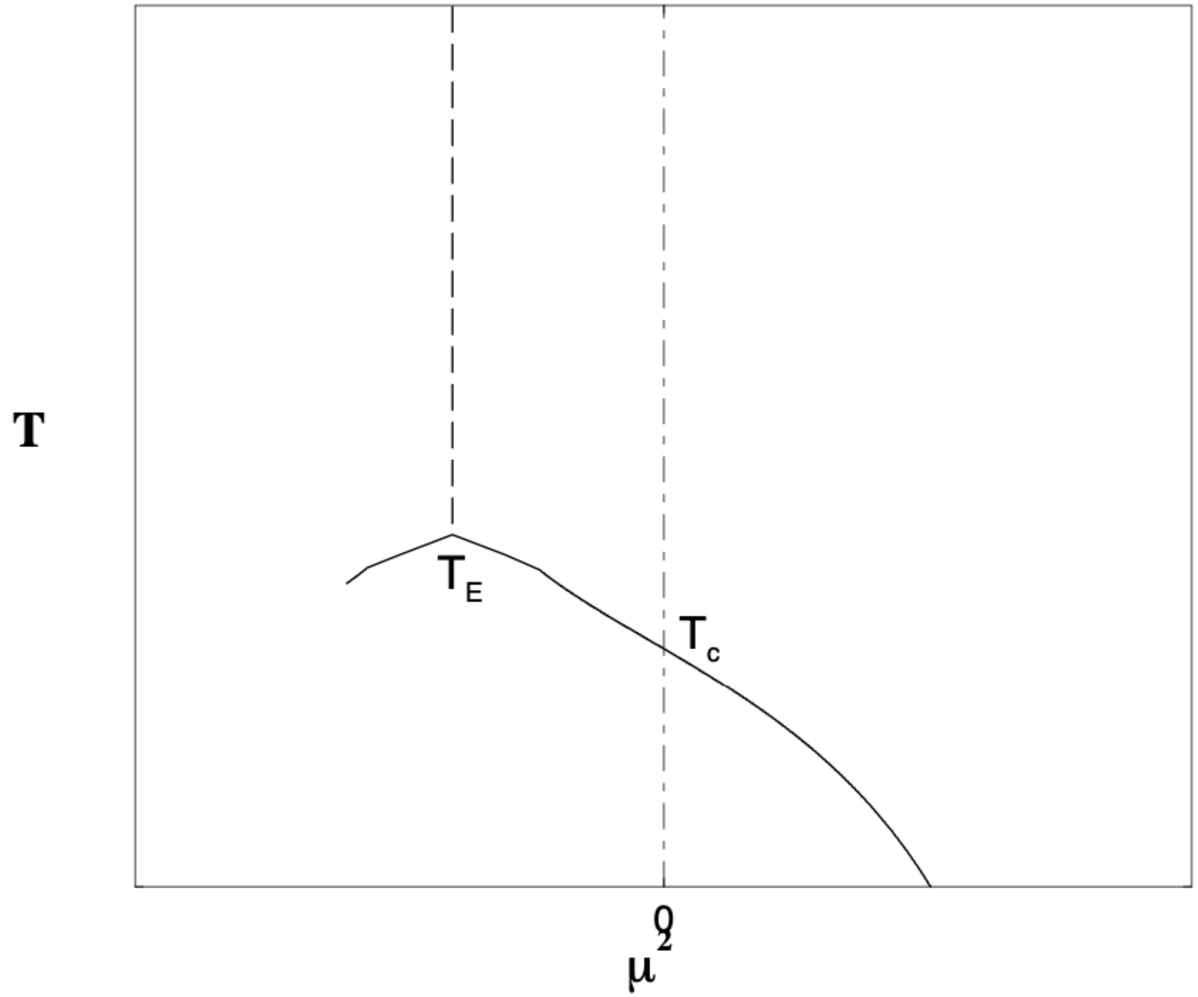
- There is a systematic uncertainty related to the choice of the parameter λ which is difficult to estimate;

- The resolution function in the BG approach is not a simple Ansatz, but it's an output of the method. This complicates the comparison of the obtained results with different models.

These problems can be solved within the modified approach that also allow us to fix a common target function for different lattice spacings. This allows us also to perform the continuum limit.

The solid line is the chiral transition, the dashed line is the Roberge Weiss transition. Simulations can be carried out a $\mu^2 \leq 0$ and results continued to the physical domain $\mu^2 \geq 0$

However, as we approach the critical point and we decrease the temperature, we have less space to extract information from the region $\mu^2 \leq 0$.



If we consider QGP in presence of parallel electric and magnetic fields then they will generate a chiral density, due to the anomaly, given by

$$C = N_c \sum_f q_f^2$$

$$\frac{d\rho_5}{dt} = C\frac{e^2}{2\pi^2}\mathrm{E}\cdot\mathrm{B} - \frac{\rho_5}{\tau}$$

Chirality changing processes with relaxation time τ

Chiral anomaly term

which has stationary solution for

$$\rho_5 = C\frac{e^2}{2\pi^2}\mathrm{E}\cdot\mathrm{B}\tau$$

We can parametrize the chemical potential μ_5 using the equation of state $\rho_5 = \rho(\mu_5)$. Using the linear response theory and considering the electric field as a perturbation, the chemical potential is small and the EoS is

$$\rho_5 = \mu_5\chi(T,B) + O(\mu_5^3)$$

Since we'll consider large magnetic fields ($q_f eB \gg T^2$), the chiral density is governed by the lowest Landau level degeneracy and then

$$\chi \propto eB$$

The CME generates the electric current

$$j_{CME} = C\frac{e^2}{2\pi^2}\mu_5 \mathrm{B}$$

Thus one finds that the conductivity due to CME is given by

$$j^i_{CME} = \sigma^{ij}E^j,\ \sigma^{ij}_{CME} = C^2\frac{e^4}{4\pi^4}\frac{\tau}{\chi(T,B)}B^iB^k$$

It is assumed that the magnetic field is along the z axis.

In addition there is also the Ohmic current. Then the total conductivity will be $\sigma = \sigma_{CME} + \sigma_O$.

Now, if the electric field is applied along the x axis, then the Lorentz force reduces the transverse conductivity σ^O_{xx} while the σ^{CME}_{xx} is zero in this case. The decrease of σ_{xx} in external magnetic field is called ***magnetoresistance***.

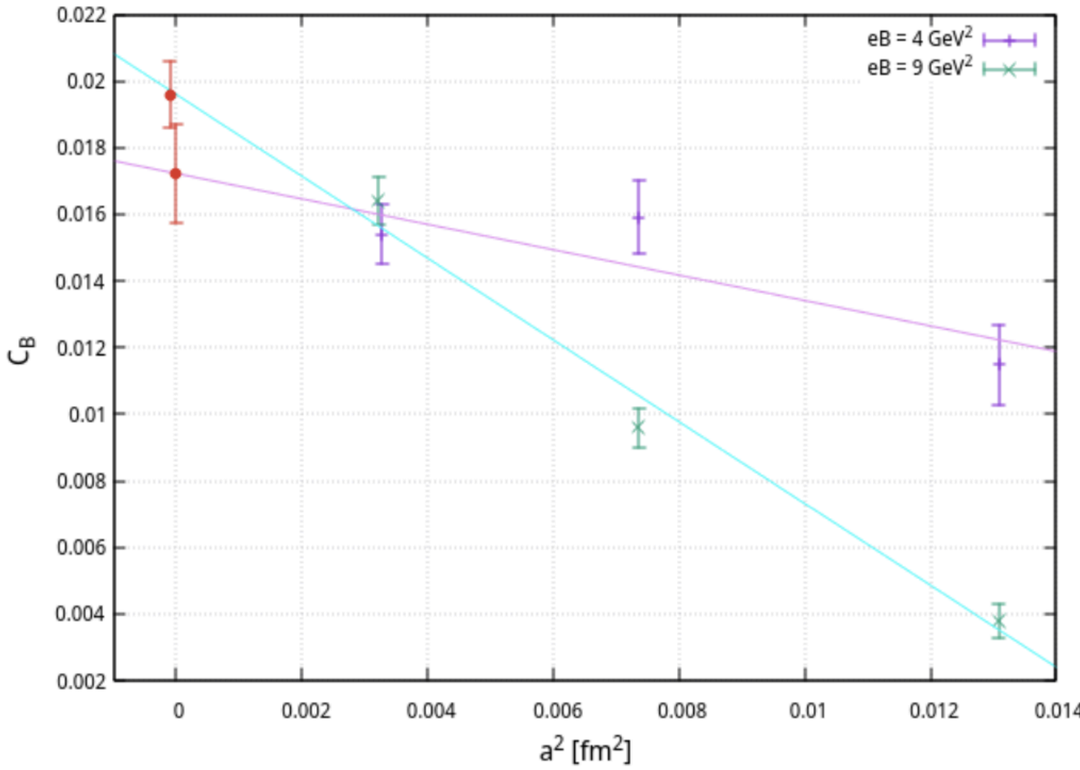
At the same time σ^{CME}_z is a rising function of the magnetic field which can be a manifestation of the CME!

To obtain the result of the relaxation time we performed a continuum extrapolation of the fit parameter C_B and we found

$$C_B = (4\ GeV^2) = 0.0172 \pm 0.0015$$

$$C_B = (9\ GeV^2) = 0.0196 \pm 0.0010$$

The results for the two values of eB are compatible inside the errors and this is a confirmation of the linear dependence as a function of the magnetic field.



By defining $\tau = C_t/T$ we can match our parameterization with the expression of the CME conductivity

$$\frac{\Delta\sigma_L}{TC_{em}} = \frac{C^2}{C_{em}}\frac{e^2}{4\pi^4}\frac{eB}{N_c\sum_f |q_f|/2\pi^2}\frac{C_\tau}{T^2} = eB\frac{C_B}{T^2}$$

From this we find

$$C_\tau = \frac{4}{3}\pi^2 C_B$$

$$C_\tau(4\ GeV^2) = 0.226 \pm 0.020$$

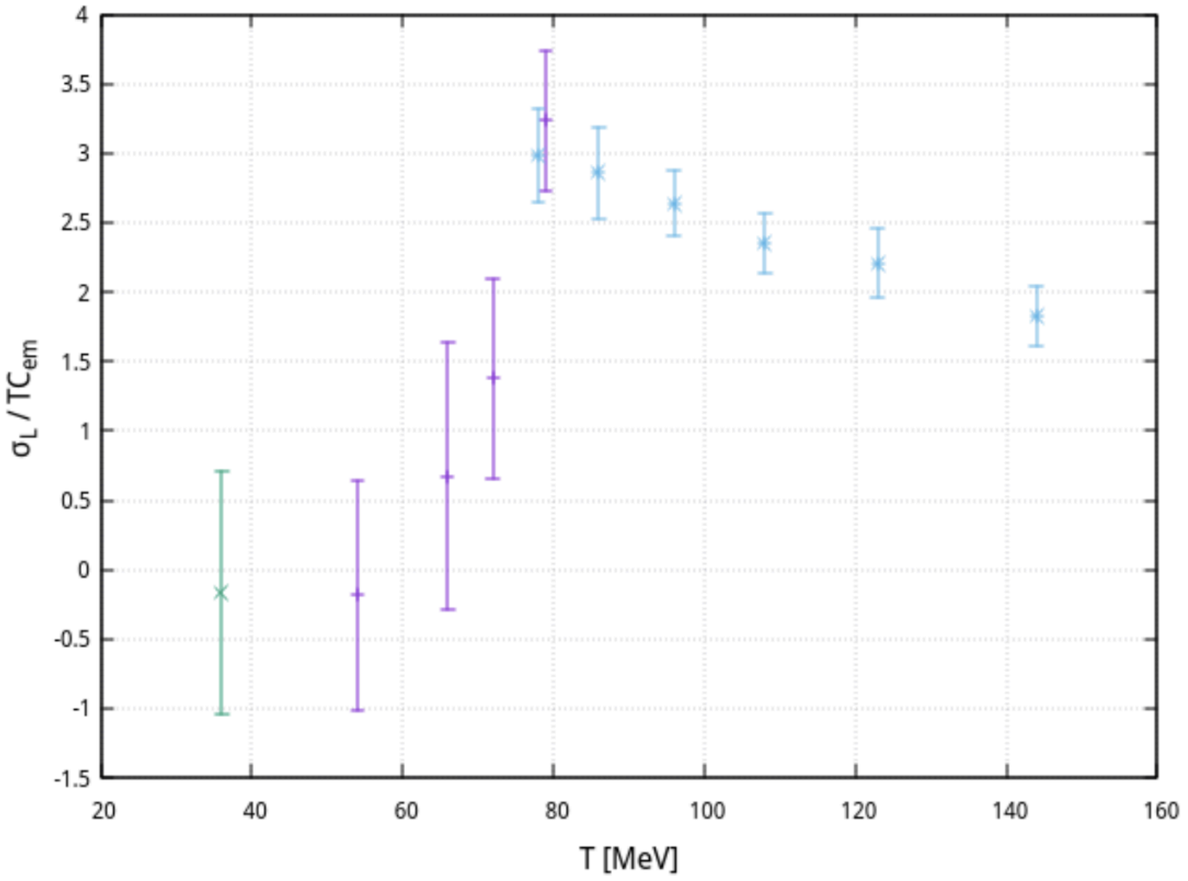
$$C_\tau(9\ GeV^2) = 0.258 \pm 0.013$$

This allows us to compute the values of the relaxation time as a function of the temperature.

Transition at $eB = 9\ GeV^2$

We also checked that the conductivity drops in the confined phase since no chiral symmetry means no CME.

We are not able to draw any reliable conclusion in the case of $eB = 4\ GeV^2$ because of the large uncertainties.
Differently, in the case of $eB = 9\ GeV^2$ we expect a better situation since a first order phase transition takes place.



Thus, we explored smaller temperatures since the transition for $eB = 9\ GeV^2$ is expected around $T \simeq 60\ MeV$.

So we study the parallel conductivity on the lattices 22, 24, 26, 32×36^3 and 48×24^3 with lattice spacing a = 0.1144 fm.

As it can be seen the conductivity ***strongly drops*** when the temperature approaches the transition one, moreover after the expected transition temperature it is compatible with zero.

The conductivity rapidly changes as we would expect in the case of a first order phase transition.

A possible explanation of this behaviour may be found in the chirality-flipping generated by sphaleron transitions, namely topological transitions which change the winding number of the gauge fields configuration.

What sphaleron transitions are?

It is well-known that Yang–Mills theory has a non-trivial topological structure.
This theory has an infinite number of energetically degenerate vacua differing in the topological characteristic-integer Chern-Simons number

$T = 0$ → Transitions between different vacua are possible (instantons) but they are suppressed.

$T \neq 0$ → Thermal fluctuations can excite saddle solutions and change the Chern-Simons numbers (sphalerons).

The relaxation time of the sphaleron rate is inversely related to the sphaleron rate

$$\frac{1}{\tau_S} \sim \frac{\Gamma_S}{\chi T}$$

If the sphaleron transitions are responsible of this temperature behaviour, we would find a guess for the sphaleron rate dependence both on magnetic field and temperature

$$\Gamma_S \sim eBT^2$$

This result, obtained as a consequence of the temperature dependence of the conductivity, has to be deeper investigated.

The interesting point is that the sphaleron rate can be computed solving an inverse problem.

$$G_{ij}(\tau) = \int_0^\infty \frac{d\omega}{\pi} K(\tau,\omega)\rho_{ij}(\omega)$$

where now

$$G(\tau) = \int_0^\infty d^3x \ \langle q(t,\bar{x})q(0,\bar{x})\rangle, \qquad q(x) = \frac{1}{32\pi^2} Tr\left\{ F_{\mu\nu}(x)\tilde{F}_{\mu\nu}(x) \right\}$$

The sphaleron rate can be computed again as

$$\Gamma_{sphal} = 2T \lim_{\omega\to 0} \frac{\rho(\omega)}{\omega}$$

Our charge carriers are right-/left- handed fermions.

Then, one could think that the relaxation time is due to the increased spin misalignment, with respect to the magnetic field, increasing temperature.

In order to check this one can simply study the thermal average of a spin coupled to the magnetic field.

One finds that the spin thermal average

$$\langle s \rangle = \tanh\left(\frac{eB}{mT}\right).$$

And in our limit $eB \gg T^2 \gg m^2$. Then

$$\langle s \rangle \sim 1$$

Other possibilities are that the chirality-flipping is generated by a finite quark mass or sphaleron transitions.

The first one is easy to understand since a finite quark mass breaks explicitly the chiral symmetry.

The second one is a bit more complicated: a sphaleron transition is a topological transition, which changes the winding number of the gauge fields configuration, in this way it affects the value of the chiral anomaly through the variation of the Chern-Simons number.

We have to see which of these two effects can influence our system in our physical conditions.

Let's compare the relaxation time of the two processes.

$$\frac{1}{\tau_S} \sim \frac{\Gamma_S}{\chi T} \qquad \frac{1}{\tau_m} \sim \frac{\alpha_s m^2}{T}$$

The first one depends on the magnetic field from Γ_S and χ while the second one is the same with or without it.

At zero magnetic field one has $\chi \sim T^2$ and for the sphaleron rate $\Gamma_S \sim \alpha_s T^4$. Since our coupling is of the order $\alpha_s \sim O(1)$ we have

$$\alpha_s m^2 \ll \alpha_s^5 T^2$$

thus the sphaleron inverse relaxation time dominates on the finite quark mass one.

If this condition is maintained also for non zero magnetic fields, we are done.