Lindblad master equation approach to the topological phase transition in the disordered Su-Schrieffer-Heeger model

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The fate of the *mobility edge* and the *topological phase*
in presence of correlated - (non) chiral disorder

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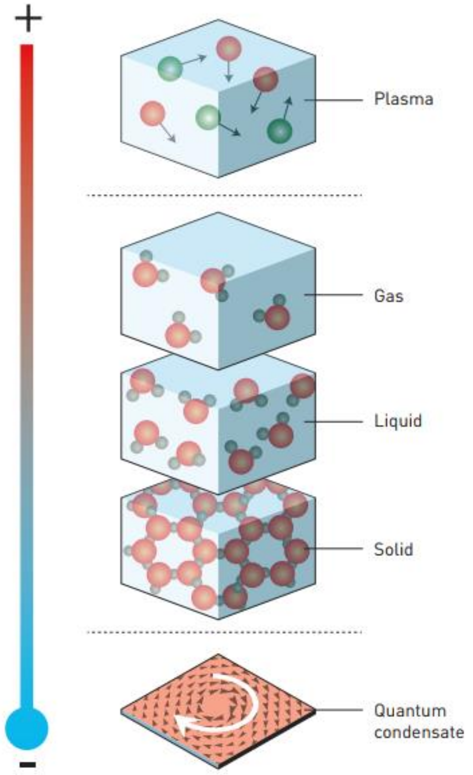
Topological phase transitions

Higher symmetry



Lower symmetry

PHASES OF MATTER

Different phases are characterized by different sorts of symmetry

Existence of a (local) order parameter “m”

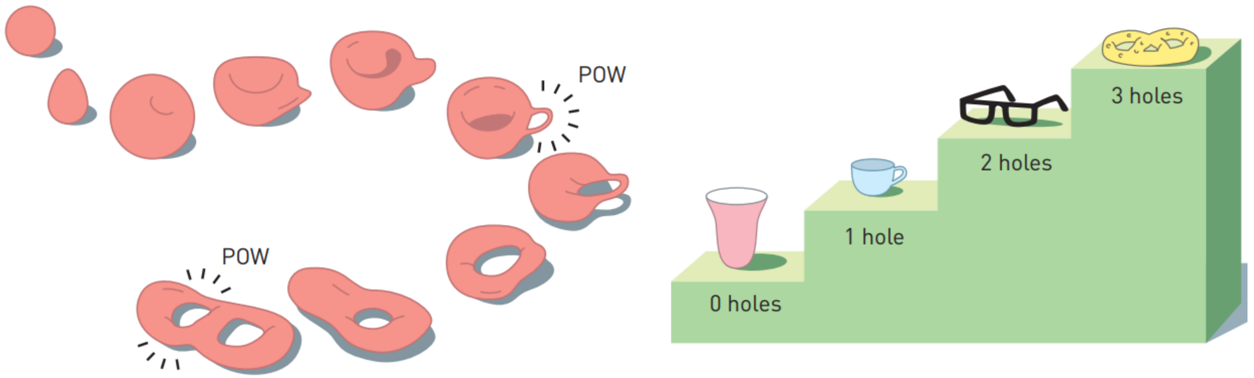
m=0 disordered phase

m≠0 ordered phase

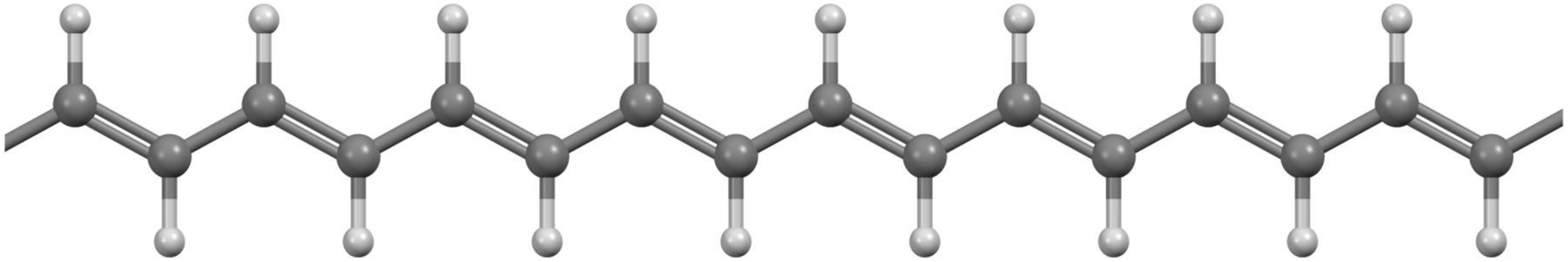
TOPOLOGICAL PHASES OF MATTER

No symmetry breaking mechanisms
Robust and insensitive to perturbations and impurities

Quantized and robust foundamental properties
(Chern number/Zak phase/number of edge states)



Su-Schrieffer-Heeger model (with periodic boundary conditions)



Introduced in 1979, to describe the increase of electrical conductivity of polyacetylene polymer.

↓

It is a quantum mechanical tight binding model, that describes the hopping of spinless electrons in a chain with two alternating types of bonds, J_{odd} and J_{even}.

↓

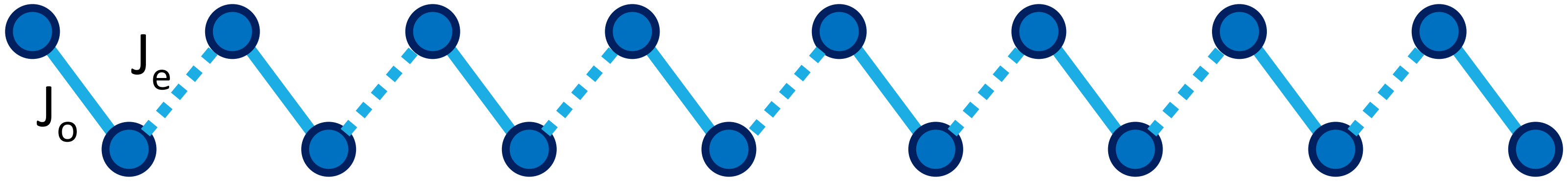
Undergoes a topological phase transition as a function of J_{odd}/J_{even}

↓

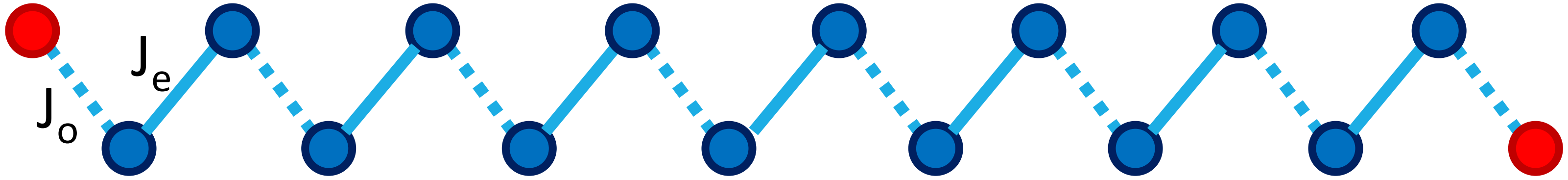
Quantized Berry phase integrated over the Brillouin zone (polarization, winding number)

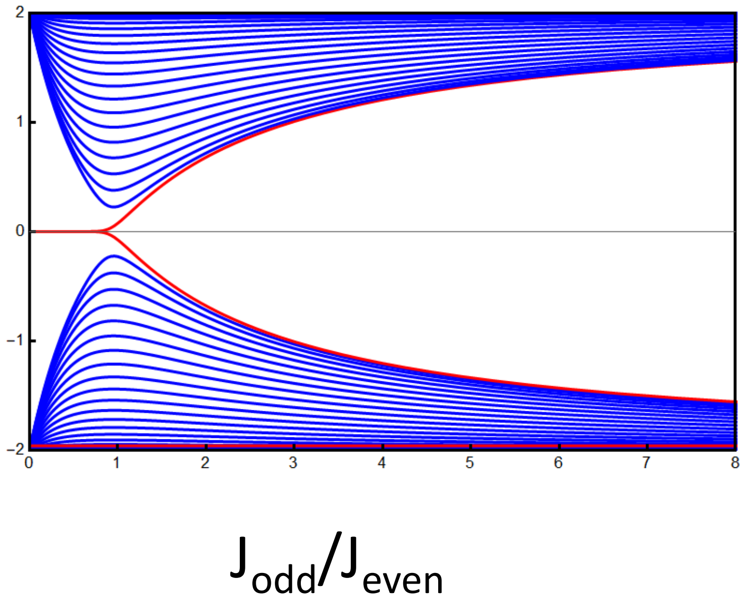
Su-Schrieffer-Heeger model (with open boundary conditions)

$J_{odd} > J_{even}$ Topological **trivial** phase



$J_{odd} < J_{even}$ **Topological** non-trivial phase





Disorder and Topology

UNCORRELATED DISORDER (in 1D)

→ Full localization of wavefunctions (insulating phase)

→ Reentrant topological phases

CORRELATED DISORDER (in 1D)

→ Existence of a «mobility edge» (localization/delocalization transition)

→ Induce topological phases transitions

Lindblad master equation

PHYSICAL DERIVATION

tracing out bath degree of freedom

$$\rho_{\mathrm{S}} = \mathrm{Tr}_{\mathrm{E}}\left[\rho_{S+E}\right]$$

Based on the following approximations:

- Born (small system-bath coupling)
- Markov (no memory effects)
- Rotating wave (no high oscillating terms)

MATHEMATICAL DERIVATION

Markovian CPT/Krauss map

$$\mathcal{V} : \rho_S \to \rho'_S$$

Based on the following requests:

- Markov (no memory effect)
- Trace preserving ($\mathrm{Tr}\left[\mathcal{V}\rho_S\right] = \mathrm{Tr}\left[\rho_S\right]$)
- Completely positive ($(\mathcal{V} \otimes \mathbb{I}_E)\, \rho_{S+E} \geq 0$)

*G. Lindblad - ON THE GENERATORS OF QUANTUM DYNAMICAL SEMIGROUPS

*H.-P. Breuer · F. Petruccione - THE THEORY OF OPEN QUANTUM SYSTEMS

Lindblad master equation

$$\dot{\rho}(t) = -i[H,\rho] + \sum_k \left(\mathcal{L}_k \rho(t) \mathcal{L}_k^\dagger - \frac{1}{2}\left\{ \mathcal{L}_k^\dagger \mathcal{L}_k, \rho(t) \right\} \right)$$

Liouvillian
(coherent evolution)

Lindbladian
(incoherent evolution)

Lindblad jump operators

$$\mathcal{L}_k$$

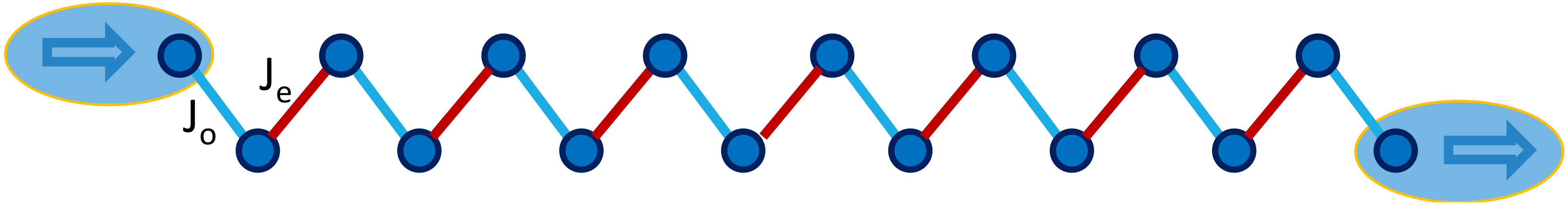
$|\psi_i\rangle$

Drift $P_D = 1 - P_J$ → $\left(\mathbb{I} - iH\delta t - \frac{1}{2}\mathcal{L}_k^\dagger \mathcal{L}_k \delta t\right)|\psi_i\rangle$

Jump $P_J = \langle\psi_i| \mathcal{L}_k^\dagger \mathcal{L}_k |\psi_i\rangle \delta t$ → $\dfrac{\mathcal{L}_k|\psi_i\rangle}{\sqrt{\langle\psi_i|\mathcal{L}_k^\dagger \mathcal{L}_k|\psi_i\rangle}}$

Lindblad master equation

We inject and drain electrons from the boundaries of the system



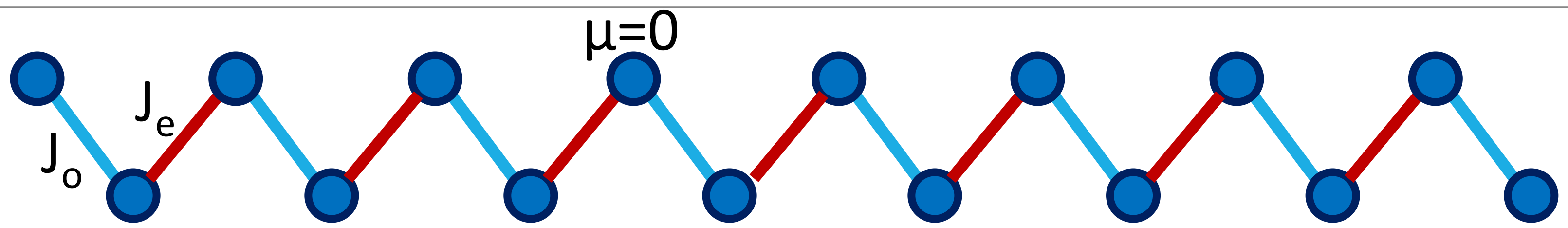
$$L_{in,1} = \sqrt{\Gamma_1}\, c_1^\dagger \qquad L_{out,L} = \sqrt{\gamma_L} c_L$$

and focus on transport properties and occupations numbers at the NESS

$$n_j(t) = \mathrm{Tr}[n_j \rho(t)] \qquad I_{out,L}(t) = \gamma_L n_L(t) \qquad I_{in,1}(t) = \Gamma_1 \left(1 - n_1(t)\right)$$

*G. Benenti, G. Casati, T. Prosen, D. Rossini, and M. Znidaric, Phys. Rev. B 80, 035110 (2009)
*A. Nava, M. Rossi, and D. Giuliano, Phys. Rev. B 103, 115139 (2021)

SSH – Clean Limit

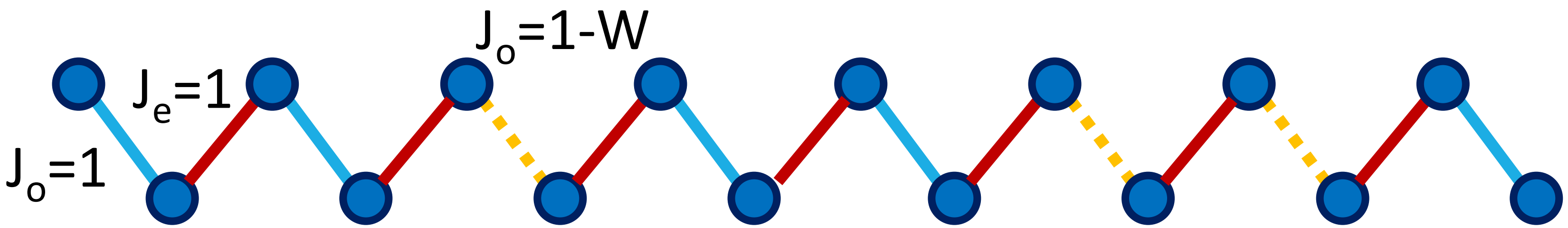


$$H_{\mathrm{SSH}} = -\sum_{j=1}^{L-1} J_{j,j+1}\{c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j\} - \sum_{j=1}^{L} \mu_j c_j^\dagger c_j \qquad J_{j,j+1} = \begin{cases} J_o \ , & \text{for } j \text{ odd} \\ J_e \ , & \text{for } j \text{ even} \end{cases}$$

If the chemical potential is set to 0 the spectrum of the Hamiltonian is invariant under the “chiral operator”

$$\mathbf{\Gamma} = \sum_{j=1}^{\frac{L}{2}} \{c_{2j-1}^\dagger c_{2j-1} - c_{2j}^\dagger c_{2j}\} \qquad \{\mathbf{\Gamma}, H_{\mathrm{SSH}}\} = 0$$

SSH – Correlated bond disorder

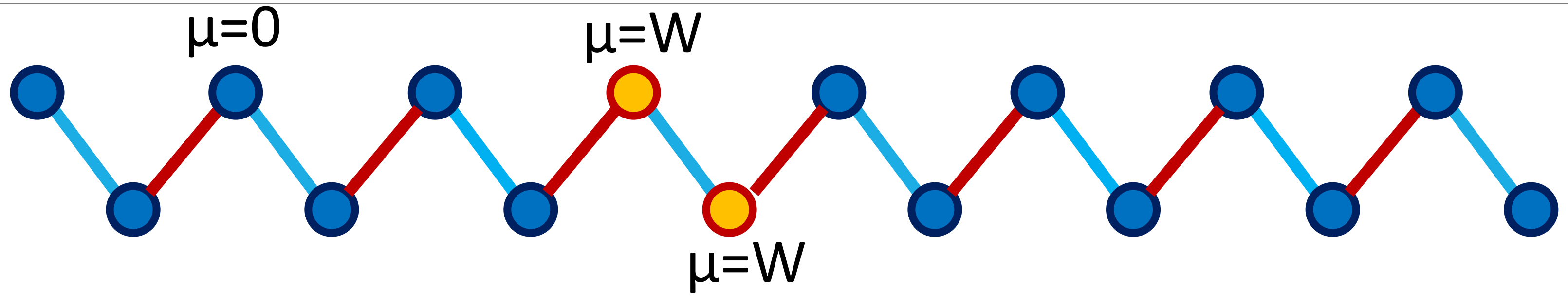


At each odd bond of the chain we may have single electron hopping Jo either equal to 1, or to (1−W), with probability respectively given by σ and 1 − σ

$$\mathcal{P}_{\rm b}[J_o] = \sigma\delta\left(J_o - 1\right) + \left(1 - \sigma\right)\delta\left(J_o - 1 + W\right)$$

This kind of disorder preserves the chiral invariance

SSH – Dimer disorder



We randomly assign to the chemical potential at both sites of each elementary cell (that is, two consecutive odd and even sites) either one of two selected values: 0 or W

$$\mathcal{P}_{\mathrm{d}}[\mu_j] = \begin{cases} \sigma\delta\left(\mu - 0\right) + \left(1-\sigma\right)\delta\left(\mu - W\right) & j \text{ odd} \\ \mu_{j-1} & j \text{ even} \end{cases}$$

This kind of disorder breaks the chiral invariance

SSH – partecipation ratio

- The most effective way of probing the disorder-induced localization in one-dimensional systems is through dc current transport measurements
- In the linear response regime, the current is proportional to the zero-energy transmission coefficient *T* across the chain and is therefore exponentially suppressed with *L*
- However, due to the presence of the dimerization gap, the SSH chain is insulating even in the absence of disorder
- It has been proposed to look at the normal- and at the inverse-participation ratio

$$\mathrm{NPR} = \frac{1}{N}\sum_{s=1}^{N}\frac{1}{L}\sum_{n}\left(L\sum_{j=1}^{L}|\langle j|\psi_{n,s}\rangle|^4\right)^{-1}$$

$$\mathrm{IPR} = \frac{1}{N}\sum_{s=1}^{N}\frac{1}{L}\sum_{n}\sum_{j=1}^{L}|\langle j|\psi_{n,s}\rangle|^4$$

- In the thermodynamic limit, NPR = 0 corresponds to localization of all the states, while IPR = 0 corresponds to all the states being delocalized.

SSH – Mobility Edge

- Driving the chain to the large-bias limit allows for using charge transport to probe the localization transition even for the insulating system

- Instead, once the system is driven toward the NESS, the stationary current keeps finite as *L increases* even if the system is gapped

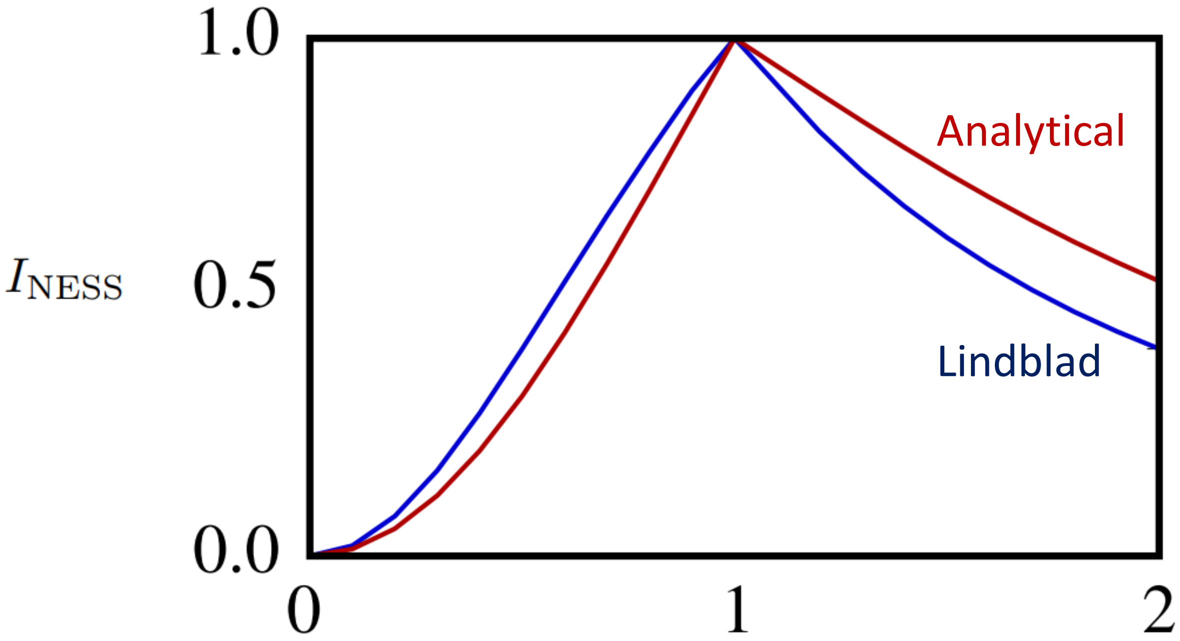
- On turning on the disorder, *the current* is suppressed, due to the strong localization effect of random disorder in onedimensional systems, thus signaling the onset of the delocalization/localization transition in the electronic states in the chain

- The existence of a “Mobility edge” is indeed "naturally" revealed by the stationary current maintaining a finite value, even on increasing L

*P. H. Guimaraes, G. T. Landi, and M. J. de Oliveira, Phys. Rev. E 94, 032139 (2016)

SSH – Mobility Edge in clean limit

- In absence of disorder



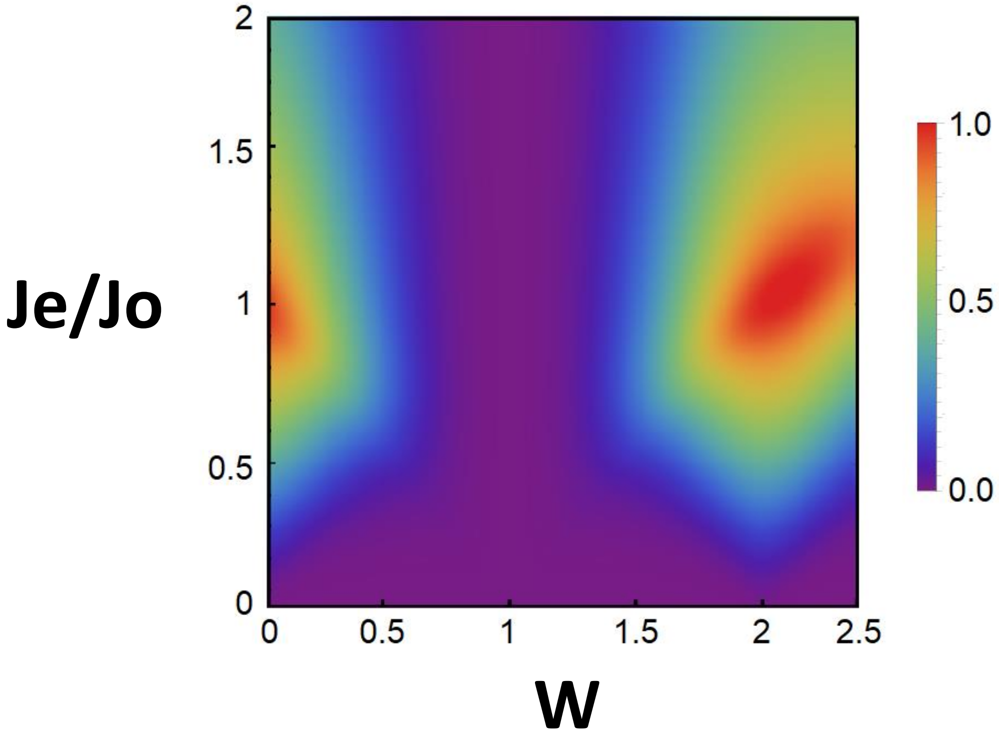
$$\frac{3}{(3\pi+1)}\mathcal{I}_L \leq I_{\text{NESS}} \leq \mathcal{I}_L$$

$$\mathcal{I}_{L\to\infty} = \left\{ \frac{\gamma g^2 \mathcal{J}^2}{2\sqrt{(J_e^2 + J_o^2 + \gamma^2)^2 - 4J_e^2 J_o^2}} \right\}$$

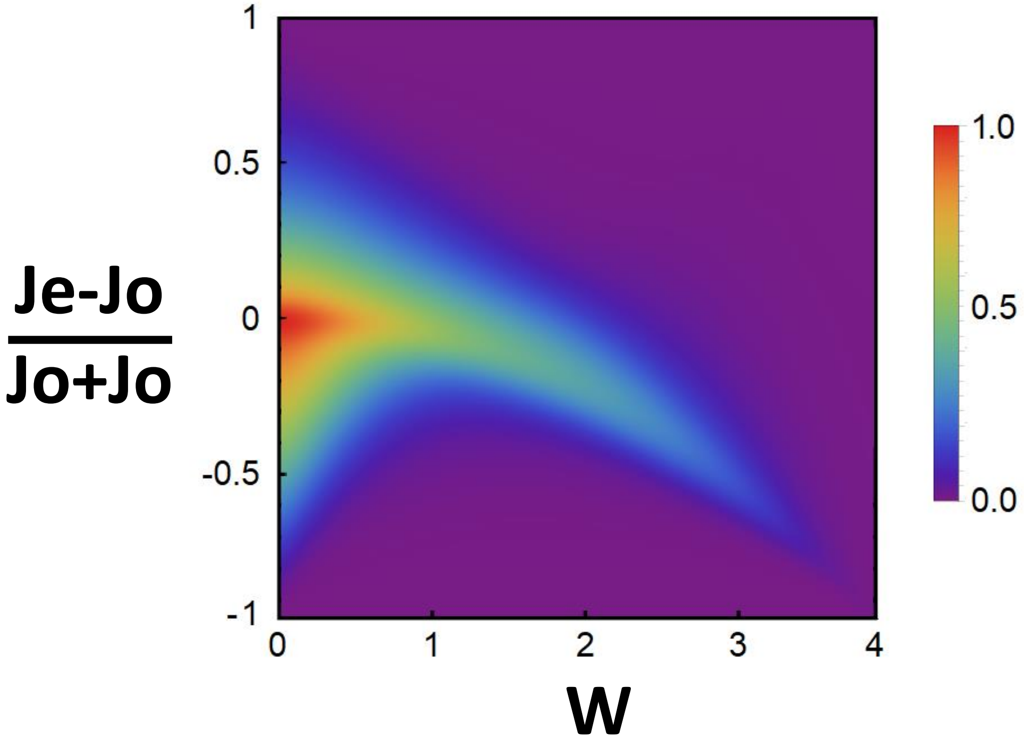
$$\mathcal{J} = \min\{J_o, J_e\}$$

*P. H. Guimaraes, G. T. Landi, and M. J. de Oliveira, Phys. Rev. E 94, 032139 (2016)

SSH – Mobility Edge disorder L=20

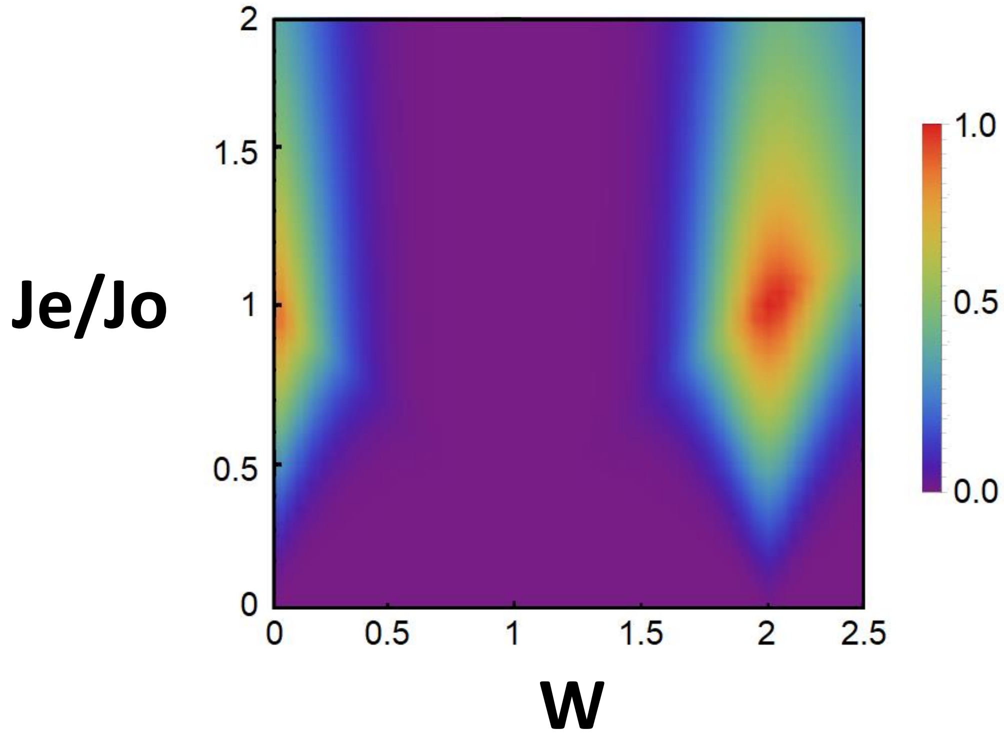


Bond dirsorder

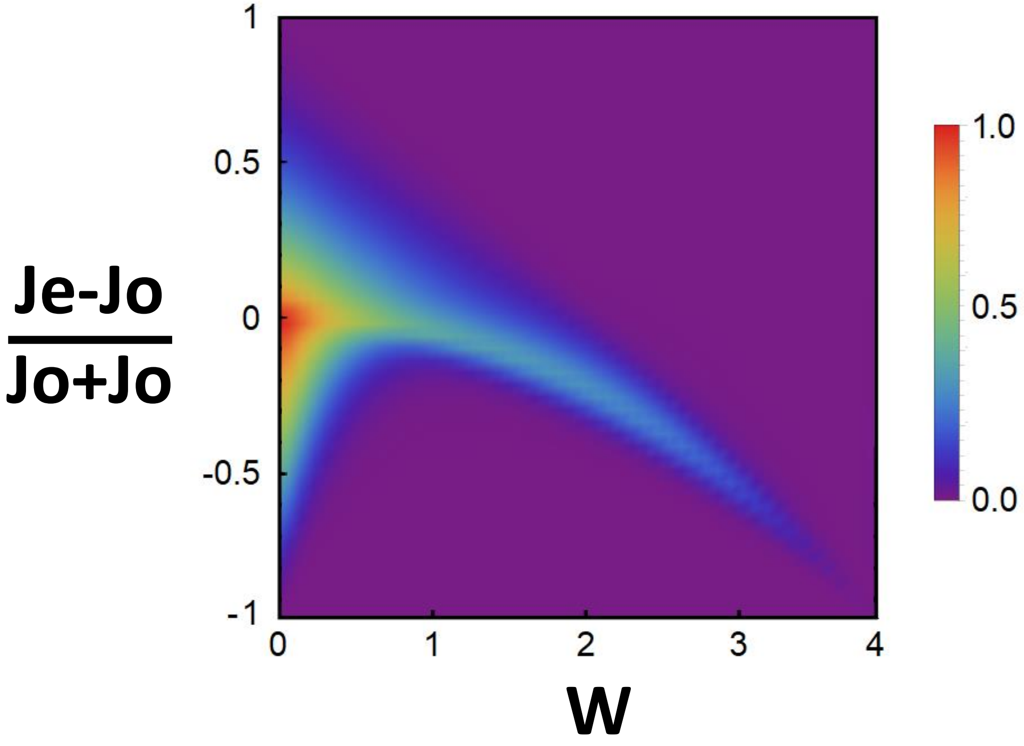


Dimer dirsorder

SSH – Mobility Edge disorder L=80



Bond dirsorder



Dimer dirsorder

SSH – Topology

- When the system is at equilibrium, the onset of the topological phase corresponds to a nonzero value of the winding

- Alternative physical quantities sensible to the onset of nontrivial topology have been proposed. The DAWN has been proposed in the presence of disorder described by a potential that preserve chiral invariance

$$\delta_\nu = \frac{1}{N}\sum_{s=1}^{N}\frac{1}{L}\,\mathrm{Tr}\{\Gamma Q_s[Q_s, X]\}$$

$$Q_s = \sum_n \{|\psi_{n.s}\rangle\langle\psi_{n.s}| - \mathbf{\Gamma}|\psi_{n,s}\rangle\langle\psi_{n.s}|\mathbf{\Gamma} \qquad \mathbf{\Gamma} = \sum_{j=1}^{\frac{L}{2}}\{c^\dagger_{2j-1}c_{2j-1} - c^\dagger_{2j}c_{2j}\}$$

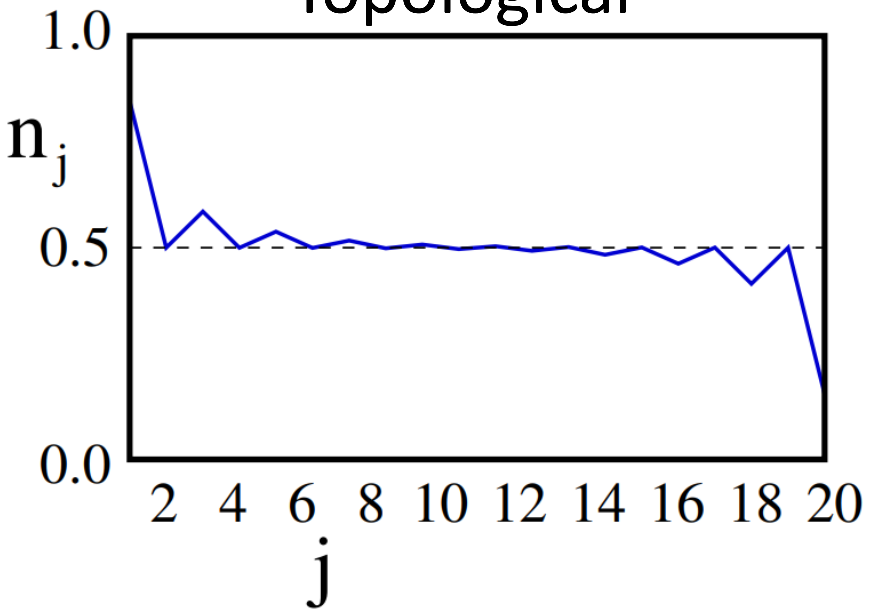
- The DAWN does not work if the disorder does not anticommute with the chiral operator

*S.-N. Liu, G.-Q. Zhang, L.-Z. Tang, and D.-W. Zhang, Physics Letters A 431, 128004 (2022)

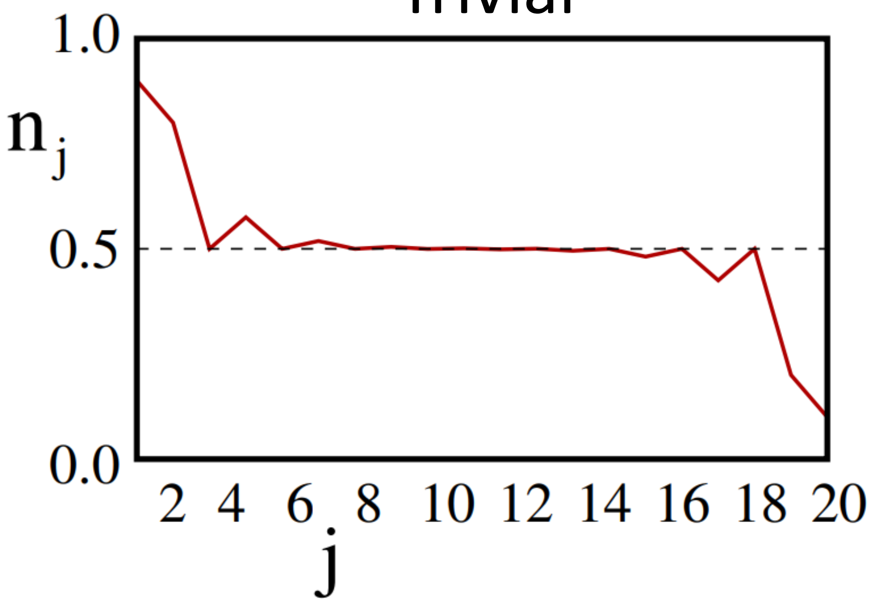
SSH – Even-Odd occupance

- We introduce $\bar{\nu} = \mathrm{Tr}[\boldsymbol{\Gamma}\rho]$
- The EOD measures the net average occupancy of the odd sites minus the one of the even sites of the chain

Topological



Trivial



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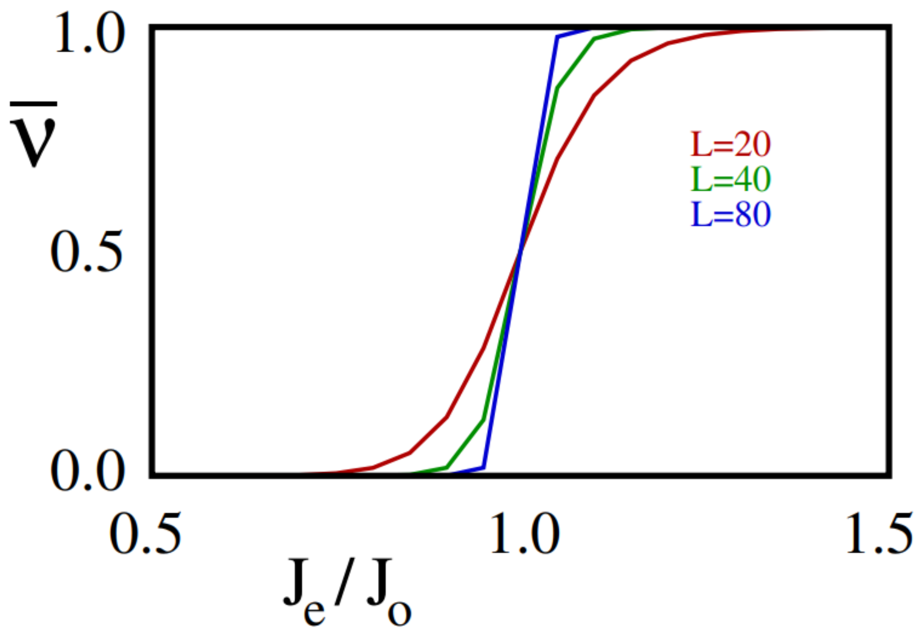
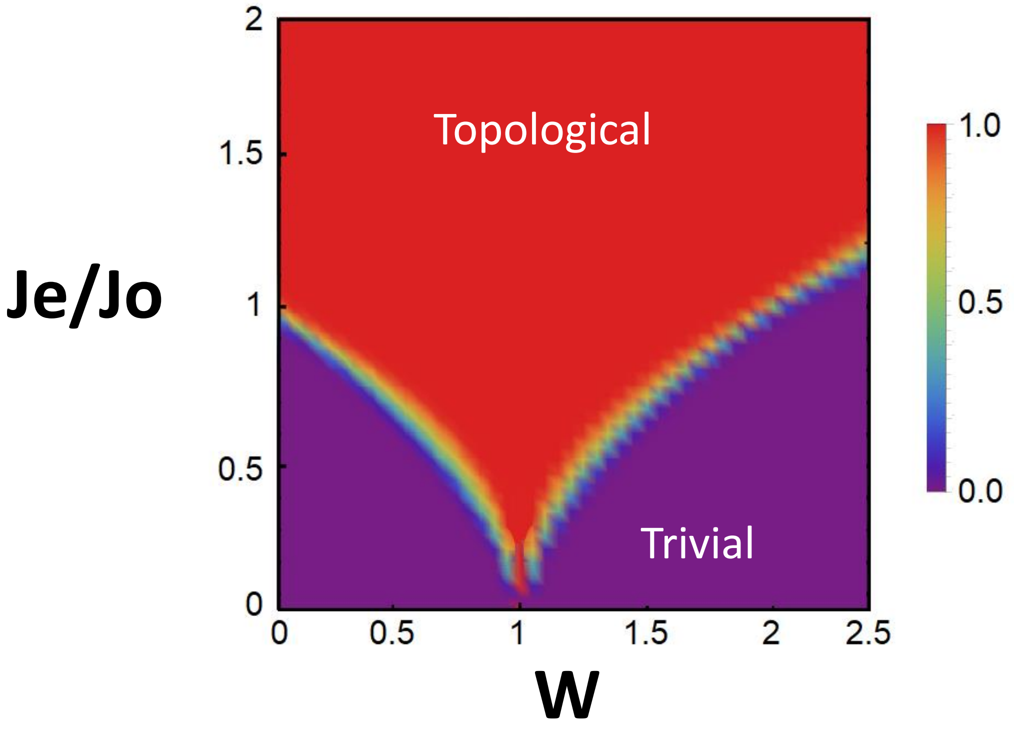
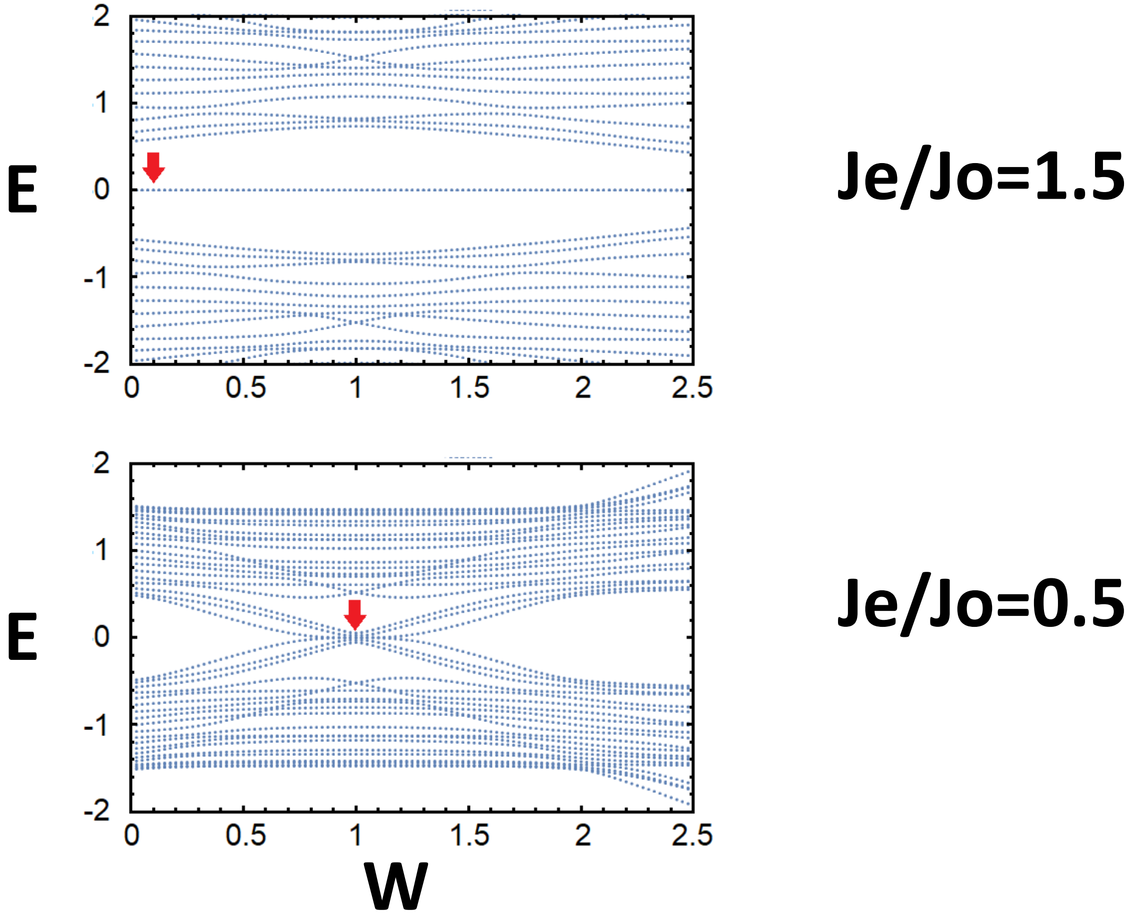


SSH – Topological/trivial transition L=80

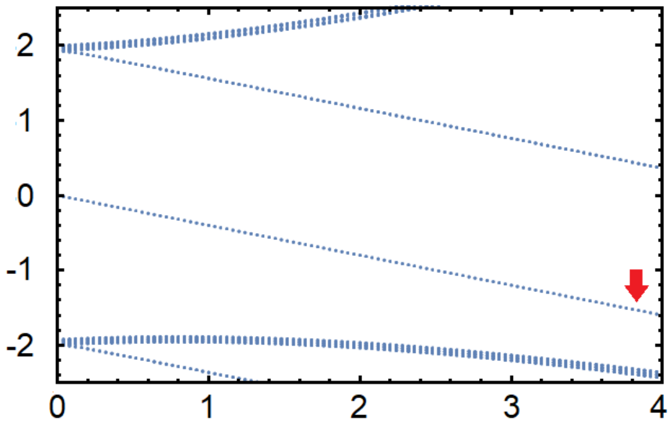


Bond dirsorder

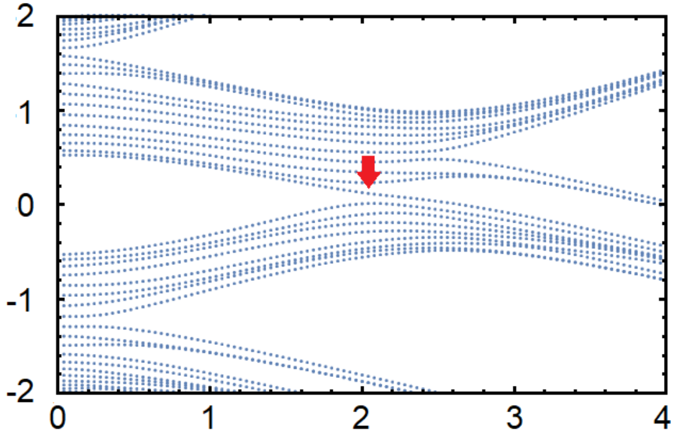


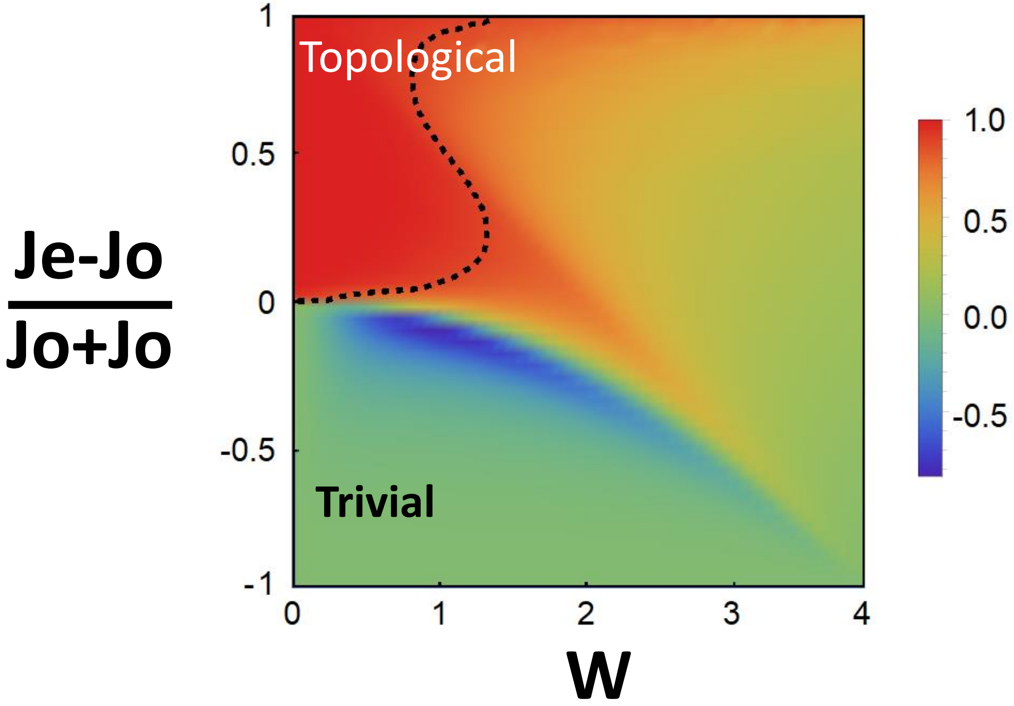
SSH – Topological/trivial transition L=80

$\frac{Je-Jo}{Jo+Jo}=0.95$



$\frac{Je-Jo}{Jo+Jo}=-0.25$





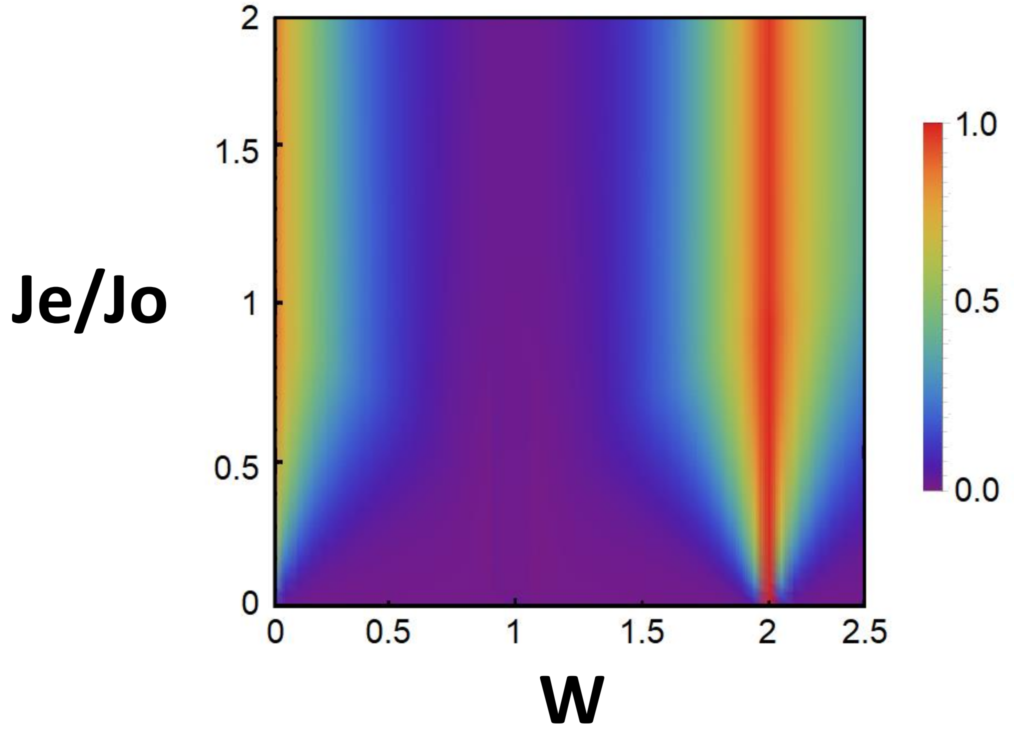
Dimer dirsorder

Conclusions

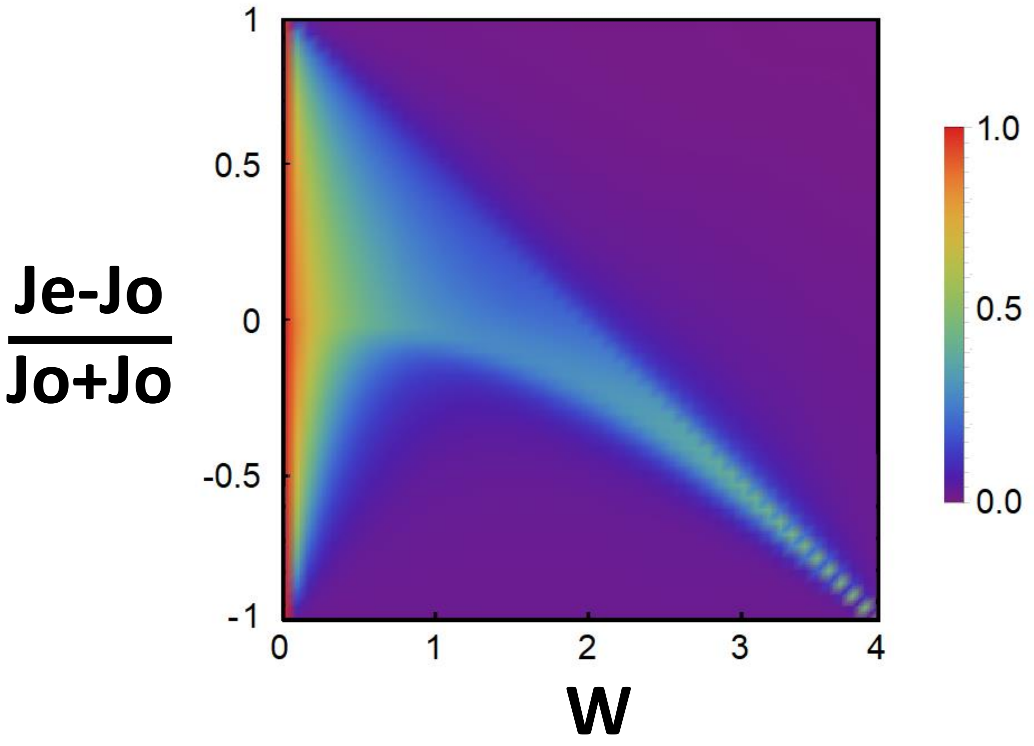
- We have applied the Lindblad equation method to derive the phase diagram of an open SSH chain connected to two external baths in the large bias limit, in the presence of bond and of dimer disorder
- Biasing the external baths has allowed us to stabilize a non-equilibrium steady state, characterized by a steady current
- A simple transport measurement, combined with an appropriate scaling analysis, maps out the localization/delocalization transition in the disordered chain
- The even-odd differential occupancy allows to distinguish between topologically trivial and nontrivial phases even in presence of chiral invariance breaking disorder

THANK YOU

SSH – NPR vs Current



Bond dirsorder



Dimer dirsorder