

Topological susceptibility in high- T $N_f = 2 + 1$ QCD via staggered spectral projectors

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Peccei-Quinn mechanism: one of the most promising solution of strong CP-problem

New symmetry in the SM: $U(1)_{\text{PQ}} \xrightarrow{\text{SSB}} \text{axion}$

- Axion couplings $\propto 1/f_a$ with f_a very large \implies **dark matter candidate**
- knowledge of axion effective potential $V_{\text{eff}}(a; T) \sim F\left(\theta + \frac{a(x)}{f_a}; T\right)$ fixes all effective parameters which enter the axion field evolution (e.g. mass, couplings) and axion relic abundance
- mass term for the axion given by QCD topology

$$m_a^2(T) = \left. \frac{\partial^2 V_{\text{eff}}(a; T)}{\partial a^2} \right|_{\langle a \rangle = -\theta f_a} = \frac{\chi(T)}{f_a^2} = \frac{\langle Q^2 \rangle_{T, \theta=0}}{V f_a^2}$$

Chiral symmetry and lattice artifacts

Index theorem:

$$Q = \text{Tr}\{\gamma_5\} = n_+ - n_-$$

Path-integral weight of a $Q \neq 0$ configuration:

$$\det[\not{D} + m_q] = \prod_i [m_q + i\lambda_i]; \quad \lambda_{min} = m_q$$

Staggered discretization breaks chiral symmetry at finite lattice spacing

$$\lambda_{min} = m_q \longrightarrow m_q + i\lambda$$

On the lattice, $Q \neq 0$ configurations are less suppressed \implies large lattice artifacts for χ

- Problem observed with standard gluonic definition (see, e.g., [C. Bonati *et al.*, 2018](#)), faced by using the eigenvalue reweighting technique in ([S. Borsányi *et al.*, 2016](#))
- We adopt a fermionic definition based on **spectral projectors** ([L. Giusti and M. Lüscher, 2009](#); [C. Alexandrou *et al.*, 2018](#); [C. Bonanno *et al.*, 2019](#))

Lattice version of index theorem for staggered fermions (\mathbb{P}_M is the projector on the space spanned by the eigenstates of \mathbb{D}_{stag} with eigenvalues $|\lambda| \leq M$):

$$Q_{\text{SP, bare}}^{(\text{stag})} = \frac{1}{2^{d/2}} \text{Tr}\{\Gamma_5 \mathbb{P}_M\} = \frac{1}{2^{d/2}} \sum_{|\lambda_k| \leq M} u_k^\dagger \Gamma_5 u_k; \quad \mathbb{D}_{stag} u_k = i\lambda_k u_k$$

Multiplicative renormalization of $Q_{\text{SP, bare}}^{(\text{stag})}$ (C. Bonanno *et al.*, 2019):

$$Q_{\text{SP}}^{(\text{stag})} = Z_{\text{SP}} Q_{\text{SP, bare}}^{(\text{stag})} = \sqrt{\frac{\langle \text{Tr}\{\mathbb{P}_M\} \rangle}{\langle \text{Tr}\{\Gamma_5 \mathbb{P}_M \Gamma_5 \mathbb{P}_M\} \rangle}} Q_{\text{SP, bare}}^{(\text{stag})}$$

Topological susceptibility: $\chi_{\text{SP}} = \frac{1}{V} \left\langle \left(Q_{\text{SP}}^{(\text{stag})} \right)^2 \right\rangle$

Choice of cut-off mass M

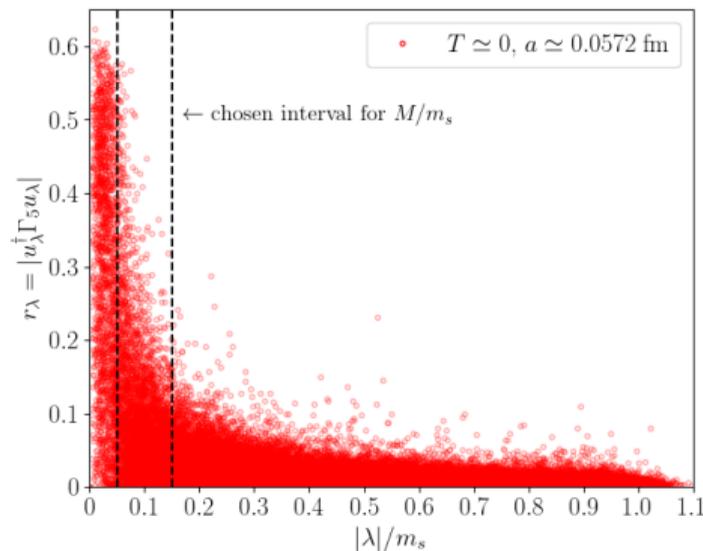
- Continuum limit does not depend on the choice of M , lattice artifacts do
- $M_R = M/Z_S^{(s)}$ has to be fixed when performing the continuum limit
- If a LCP is known $\implies M_R/m_{q,R} = M/m_q$ can be fixed
- Idea! Choose M/m_q to include modes with the highest value of chirality

Example:

run $T \simeq 0$, $a = 0.0572$ fm

$r_\lambda \equiv |u_\lambda^\dagger \Gamma_5 u_\lambda|$ vs $|\lambda|/m_s$ for the first 200 eigenmodes

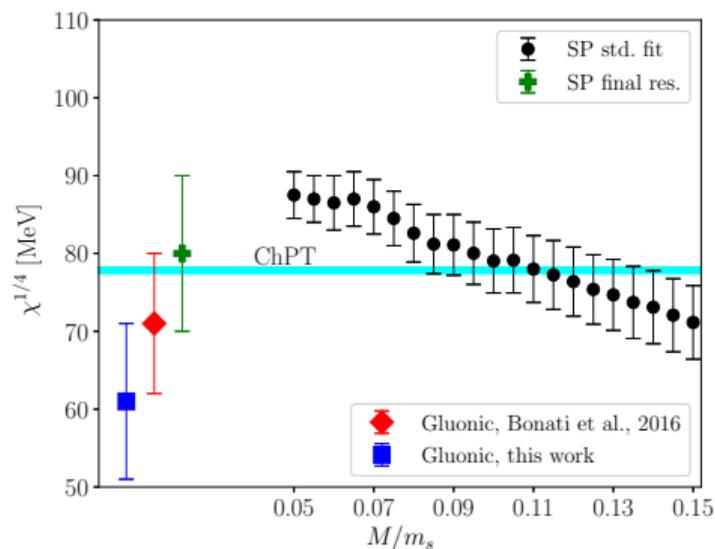
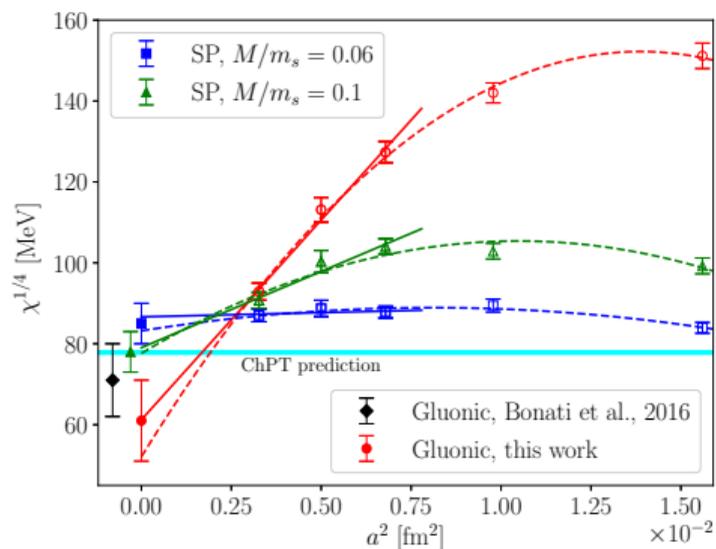
M -range: $M/m_s \in [0.05, 0.15]$



$T \simeq 0$ result

Lattice setup: $N_f = 2 + 1$, rooted stout staggered discretization, physical point

Cont. scaling: $\chi_{\text{SP}}^{1/4}(a, M_R) = \chi_{\text{SP}}^{1/4} + c_{\text{SP}}(M_R)a^2 + o(a^2)$



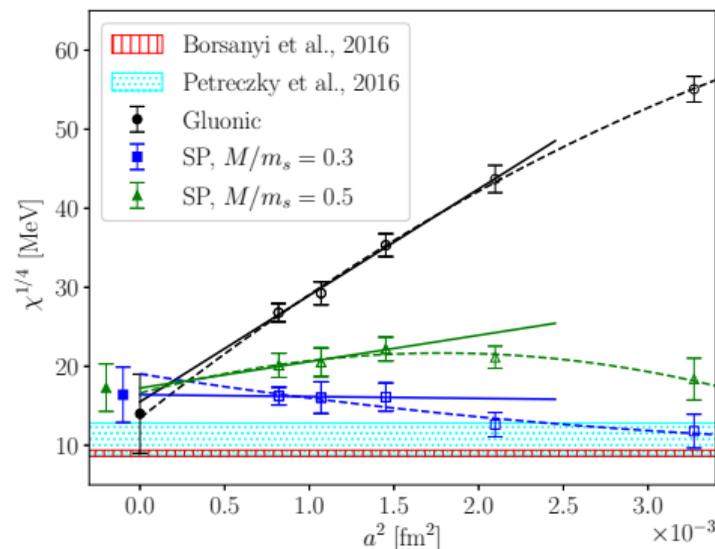
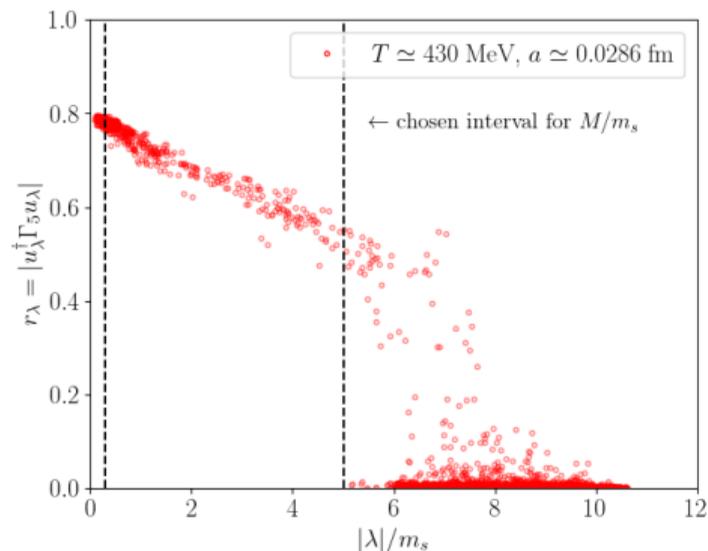
Final estimation at $T \simeq 0$: $\chi_{\text{SP}}^{1/4} = 80(10)$ MeV in agreement with $\chi_{\text{ChPT}}^{1/4} = 77.8(4)$ MeV

Topological susceptibility at finite temperature

At high T , $\langle Q^2 \rangle = \chi V \ll 1 \implies$ multicanonical approach (Bonati *et al.*, 2018; P. T. Jahn *et al.*, 2018)

Results for $T \simeq 430$ MeV, M -range: $M/m_s \in [0.3, 5]$

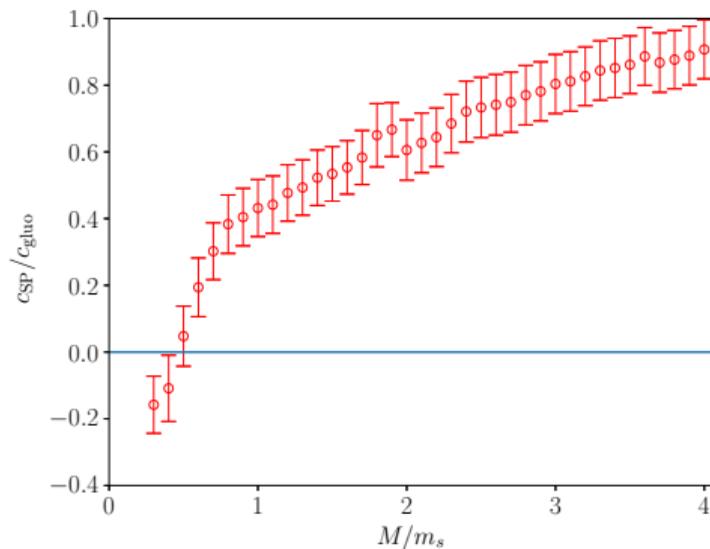
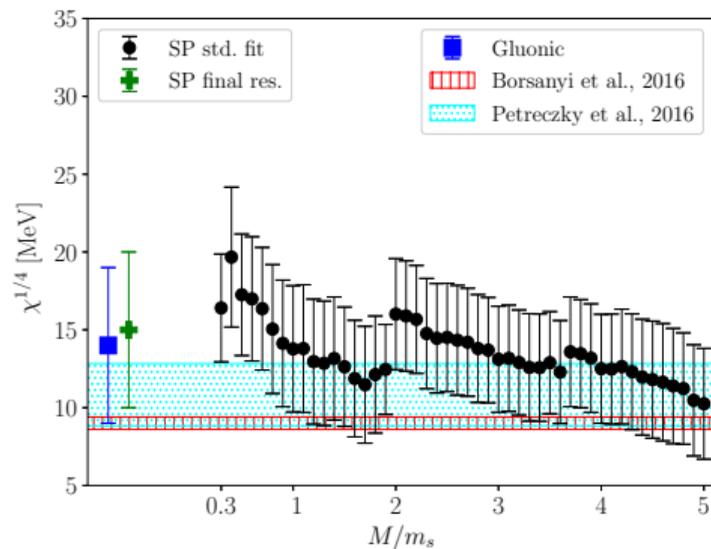
Clearer separation between high and low chirality modes



Topological susceptibility at finite temperature

Agreement between SP determinations in the M -range

Lattice artifacts can be reduced if M/m_s is chosen properly



Final estimation at $T \simeq 430$ MeV: $\chi_{\text{SP}}^{1/4} = 15(5)$ MeV, $\chi_{\text{gluo}}^{1/4} = 14(5)$ MeV

Temperature dependence of the topological susceptibility

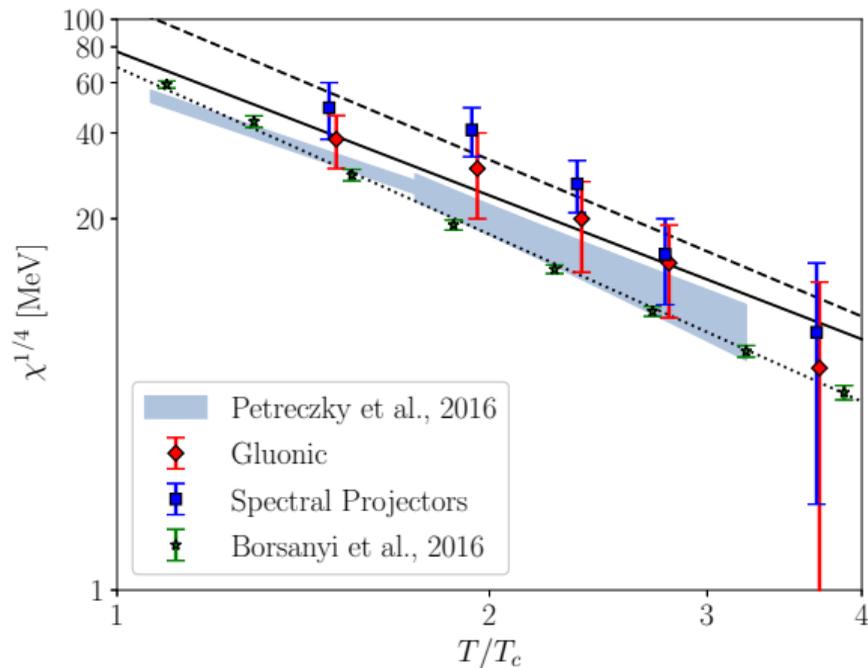
According to DIGA: $\chi^{1/4} \propto T^{-b}$ ($b_{\text{DIGA}} \sim 2$) when $T \gg T_c$

Agreement with DIGA power-law behaviour:

$$b_{\text{SP}} = 1.82(43) \quad b_{\text{gluo}} = 1.67(51)$$

With values in (S. Borsányi *et al.*, 2016), for $T \gtrsim 170$ MeV:

$$b_{\text{BW}} = 1.945(23)$$



Summary

- SP method allows to reduce lattice artifacts with respect to the gluonic definition
- The introduction of the free parameter M provides a better control of the systematics in the continuum extrapolation
- 2-3 std. dev. tension with values of (S. Borsányi *et al.*, 2016) and (P. Petreczky *et al.*, 2016) in the range $300 \text{ MeV} \lesssim T \lesssim 400 \text{ MeV}$
- Agreement with DIGA power-law behaviour

Future perspectives

- Refine our analysis in the range $300 \text{ MeV} \lesssim T \lesssim 400 \text{ MeV}$ by adding a finer lattice spacing
- Exploration of region around $T \sim 1 \text{ GeV} \implies$ very small lattice spacing ($\sim 0.01 \text{ fm}$)
 \implies **topological freezing**. A promising solution: *parallel tempering on boundary conditions* (M. Hasenbusch, 2017; C. Bonanno *et al.*, 2021, 2022, see also talk by C. Bonanno)

Backup slides

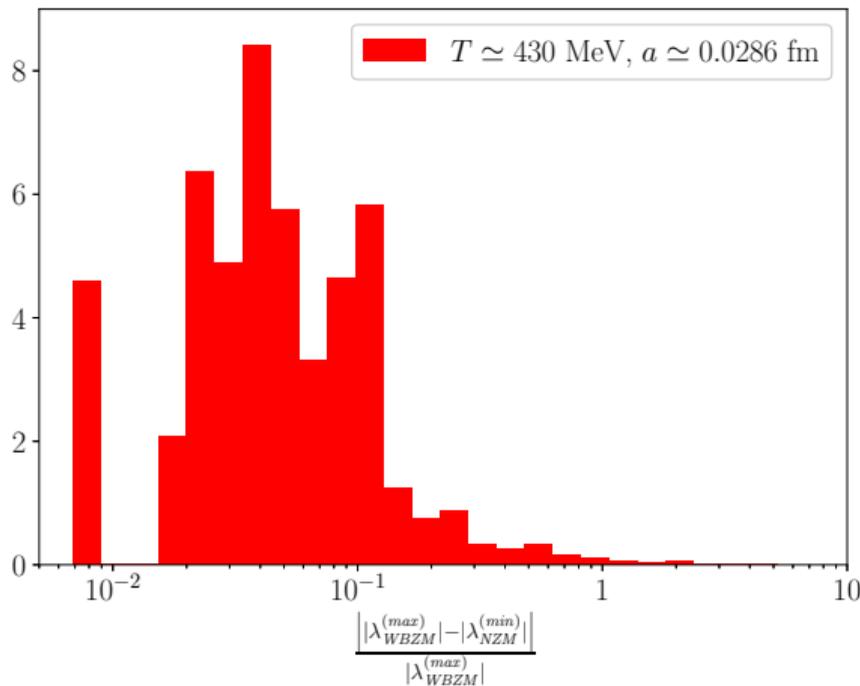
Identification of Would-Be Zero Modes (WBZM)

run $T \simeq 430$ MeV, $a \simeq 0.0286$ fm

$|\lambda_{\text{WBZM}}^{(\max)}|$: largest eigenvalue in the set of WBZM candidates, identified with the same criterium in (S. Borsányi *et al.*, 2016)

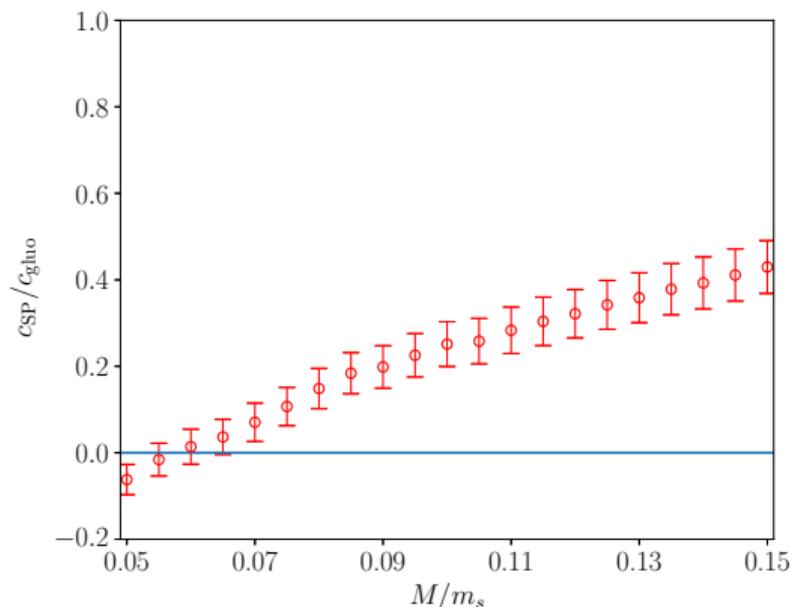
$|\lambda_{\text{NZM}}^{(\min)}|$: smallest eigenvalue in the set of Non-Zero Modes (NZM)

A sharp separation can not unambiguously established already at the level of the single configuration

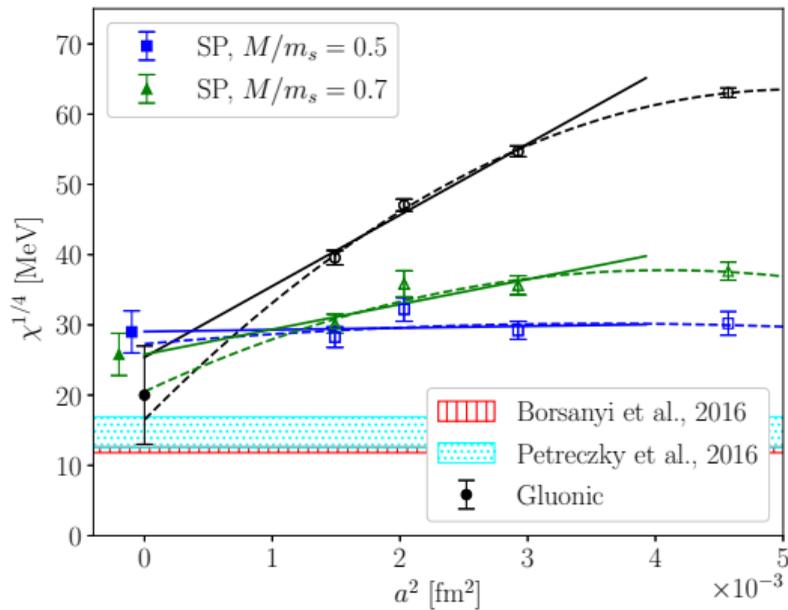
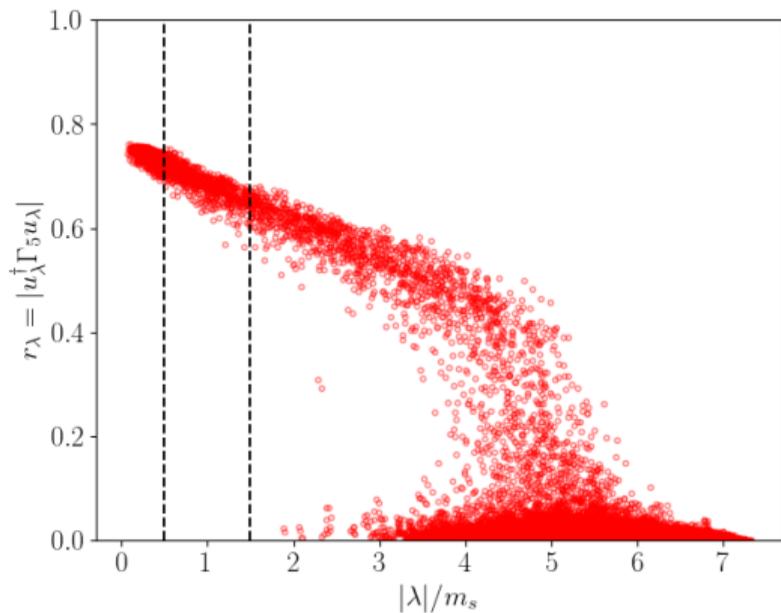


Behavior of $c_{\text{SP}}/c_{\text{gluo}}$ as a function of the cut-off M/m_s in the optimal range

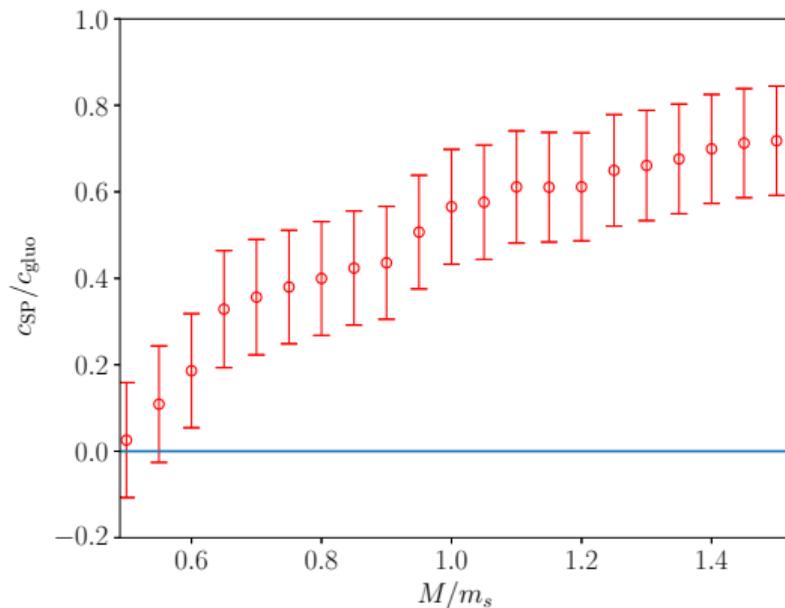
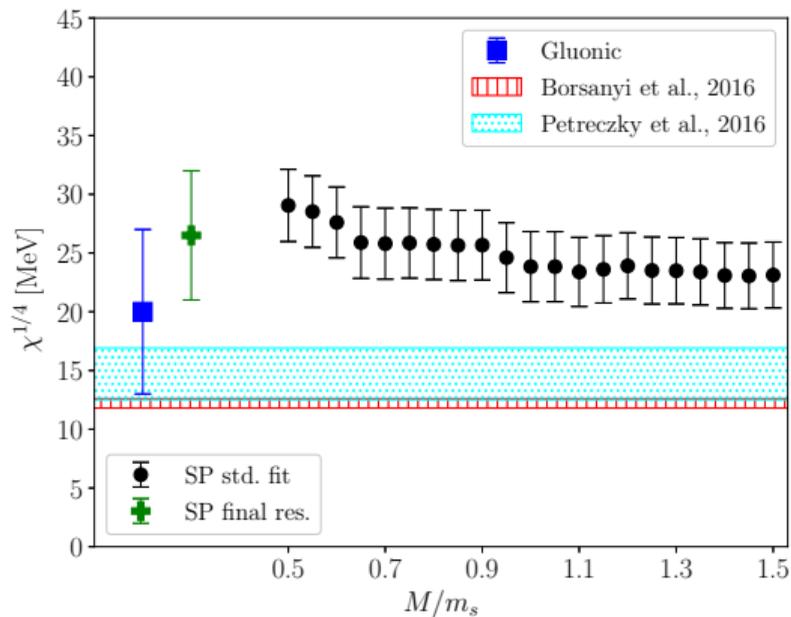
SP corrections grow when M/m_s is increased, getting closer to the gluonic ones



M -range: $M/m_s \in [0.5, 1.5]$



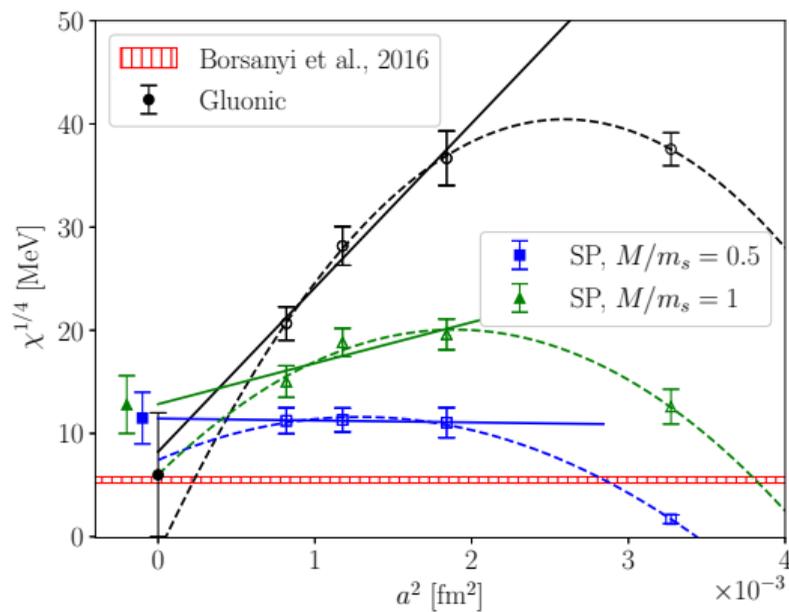
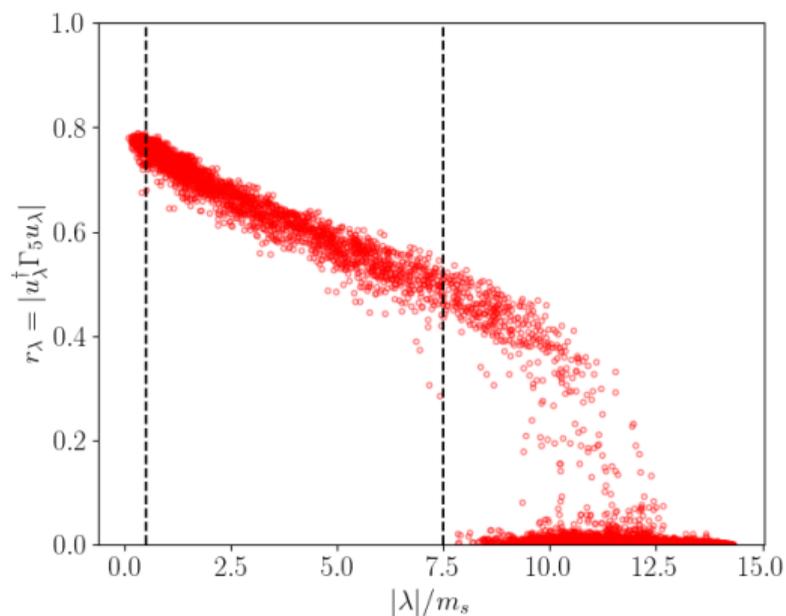
$T \simeq 365$ MeV results



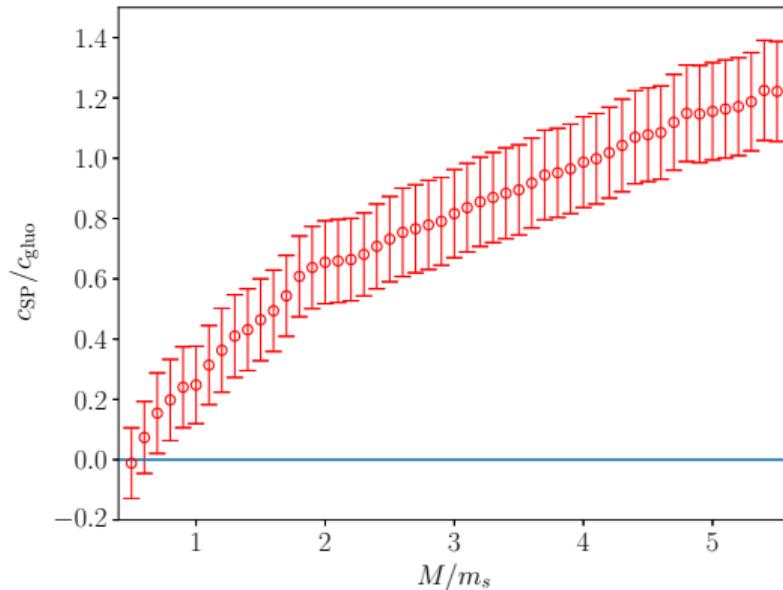
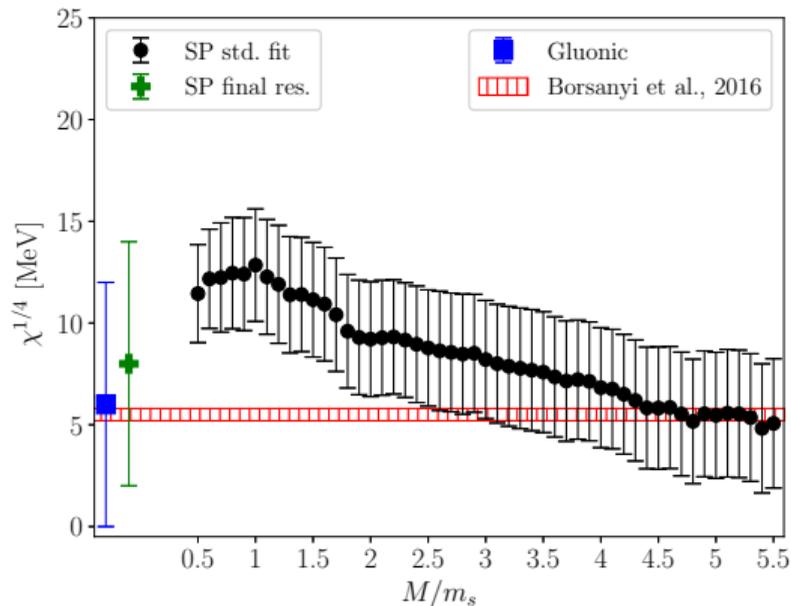
Final estimation at $T \simeq 365$ MeV: $\chi_{SP}^{1/4} = 26.5(5.5)$ MeV, $\chi_{gluo}^{1/4} = 20(7)$ MeV

$T \simeq 570$ MeV results

M -range: $M/m_s \in [0.5, 7.5]$



$T \simeq 570$ MeV results



Final estimation at $T \simeq 570$ MeV: $\chi_{\text{SP}}^{1/4} = 8(6)$ MeV, $\chi_{\text{gluo}}^{1/4} = 6(6)$ MeV

Summary of $\chi(T)$ values

T [MeV]	T/T_c	$\chi_{\text{SP}}^{1/4}$ [MeV]	$\chi_{\text{gluo}}^{1/4}$ [MeV]
230	1.48	49(11)	38(8)
300	1.94	41(8)	32(10)
365	2.35	26.5(5.5)	20(7)
430	2.77	15(5)	14(5)
570	3.68	8(6)	6(6)

Results for the fourth root of the topological susceptibility as a function of T . For the crossover temperature T_c we adopted the reference value $T_c = 155$ MeV.