UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

INFN Istituto Nazionale di Fisica Nucleare

Dynamical quantum phase transitions of the Schwinger model: real-time dynamics on IBM Quantum

REGIONE PUGLIA

QUANTERA

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QuantHEP

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SM&FT2022

Frontiers in Computational Physics

Outline

- Theoretical and experimental framework
  - Discretized gauge group
  - Register encoding
  - Dynamical quantum phase transitions
  - Quantum state tomography
- IBM Quantum
  - Input state preparation
  - Real-time evolution
  - Noise models
  - Estimation of noise reduction

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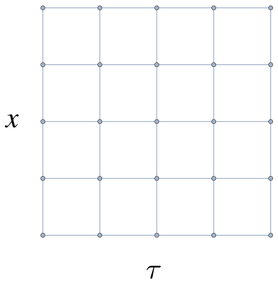
Lattice gauge theories

Lagrangian LGT:

- imaginary time $\tau = \mathrm{i}t$
- discrete spacetime for Monte Carlo methods

$$Z = \int \mathcal{D}\phi \; \mathrm{e}^{-S(\phi)}$$



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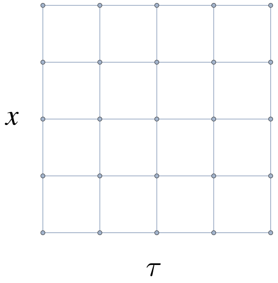
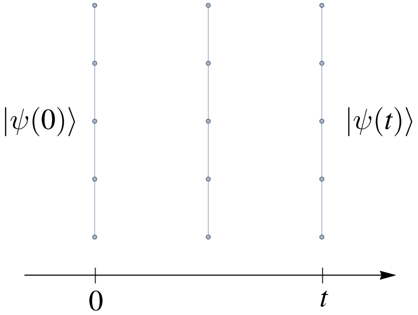


Hamiltonian simulation:

- discrete space
- input preparation, evolution, measurements

$$|\psi(t)\rangle = \mathrm{e}^{-\mathrm{i}Ht} |\psi(0)\rangle$$



Hamiltonian for the lattice with spacing a in one dimension:

$$\mathcal{H} = \frac{\mathrm{i}}{2a}\sum_x \left[\psi^\dagger_{x+1} U^\dagger_{x,x+1}\psi_x - h.c.\right] + m\sum_x (-1)^x \psi^\dagger_x \psi_x + \frac{ag^2}{2}\sum_x E^2_{x,x+1}$$

Kogut-Susskind staggered fermions: solution for the doubling.

Notarnicola et al. J. Phys. A: Math. Theor. **48**, 30FT01 **2015**

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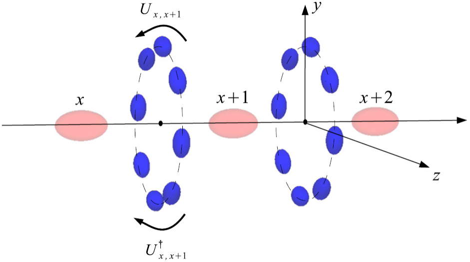

Cyclic group $\mathbb{Z}_n$
discretization $\{|e_\ell\rangle\}$:

$$U\,|e_\ell\rangle = |e_{\ell+1}\rangle$$
$$U\,|e_n\rangle = |e_1\rangle$$
$$U^n = \mathbb{1}$$

Gauss law constraint for the physical states subspace:

$$G_x\,|\phi\rangle = \left(\psi^\dagger_x\psi_x + \frac{(-1)^x - 1}{2} - \sqrt{\frac{n}{2\pi}}(E_{x,x+1} - E_{x-1,x})\right)|\phi\rangle = 0$$

Notarnicola et al. J. Phys. A: Math. Theor. **48**, 30FT01 **2015**

Register encoding

- Jordan-Wigner transformation maps the spinor field into a spin system
- Minimal complexity: $\mathbb{Z}_2$ with $E_{x,x+1}\left|\uparrow\right\rangle = \frac{\sqrt{\pi}}{2}\left|\uparrow\right\rangle$ and $E_{x,x+1}\left|\downarrow\right\rangle = -\frac{\sqrt{\pi}}{2}\left|\downarrow\right\rangle$

$$\mathcal{H} = \mathcal{H}_J + \mathcal{H}_m = \frac{J}{2}\sum_{x=0}^{N-1}\left(\sigma_x^- U_{x,x+1}\sigma_{x+1}^+ + \text{H.c.}\right) - \frac{m}{2}\sum_{x=0}^{N-1}(-1)^x Z_x,$$

with $\sigma^\pm = \frac{X \pm \mathrm{i}Y}{2}$, Pauli matrices X, Y, Z and occupied sites referred to $\left|\downarrow\right\rangle$.

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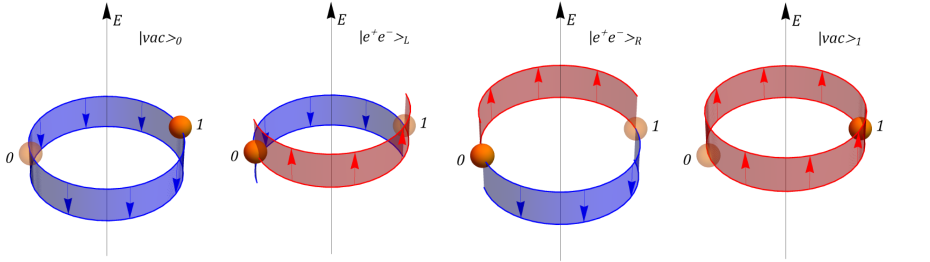
Physical states for a periodic lattice with $N = 2$ matter sites



Dynamical quantum phase transitions

Quench protocols: parameters dependent Hamiltonian $\mathcal{H}(\gamma)$

$$\mathcal{H}_0 = \mathcal{H}(\gamma_0) \qquad\qquad \mathcal{H} = \mathcal{H}(\gamma_f)$$

with ground state $|\psi(0)\rangle = |\psi_g\rangle \longrightarrow$ real-time evolution $|\psi(t)\rangle = \mathrm{e}^{-\mathrm{i}\mathcal{H}t}\,|\psi_g\rangle$

Loschmidt amplitude: $\mathcal{G}(t) = \langle\psi_g|\psi(t)\rangle \implies \mathcal{L}(t) = |\mathcal{G}(t)|^2 = \mathrm{e}^{-N\lambda(t)}$

↑ rate function

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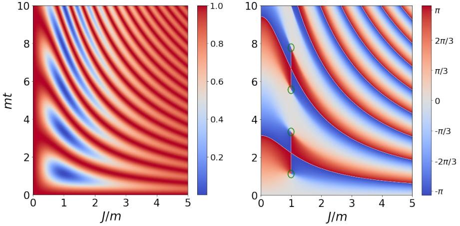
↑ rate function

Schwinger model: $\mathcal{H}_0 = \mathcal{H}(m, J) \longrightarrow \mathcal{H} = \mathcal{H}(-m, J)$

for $J = m$

$\mathcal{L}(t)$ zeros at

$$t_j = \frac{(2j+1)\pi}{2\sqrt{2}\,m}$$



$\varphi(J, t) = \arg \mathcal{G}(t)$

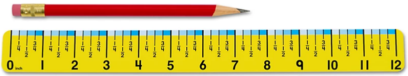
winding number

$$\nu = \frac{1}{2\pi}\oint_{\mathcal{C}} \mathrm{d}\mathbf{s}\cdot\nabla\varphi,$$

Quantum state tomography

Pure state projector:
$P = |\psi\rangle\langle\psi|$, $P^2 = P$

- it is possible to determine the state using a measurement just in one direction

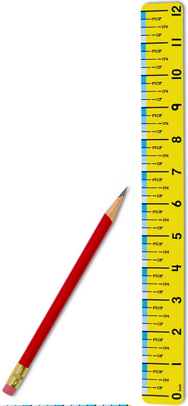


Quantum state tomography

Pure state projector:

$P = |\psi\rangle\langle\psi|$, $P^2 = P$

- it is possible to determine the state using a measurement just in one direction
- in general such direction is unkown a priori (e.g. random noise processes)



Example:

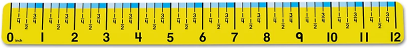
single qubit case

$$\rho = \frac{\mathrm{Tr}\{\rho\}\mathbb{1} + \mathrm{Tr}\{X\rho\}X + \mathrm{Tr}\{Y\rho\}Y + \mathrm{Tr}\{Z\rho\}Z}{2}$$

N qubit case

$$\rho = \sum_{\mathbf{v}} \frac{\mathrm{Tr}\{\sigma_{v_1} \otimes \cdots \otimes \sigma_{v_N}\rho\}\sigma_{v_1} \otimes \cdots \otimes \sigma_{v_N}}{2^N}$$

with $\sigma_0 = \mathbb{1}$, $\sigma_1 = X$, $\sigma_2 = Y$, $\sigma_3 = Z$.



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Ground state preparation

The encoding assigns each degree of freedom to a qubit q_i, $i = 0, 1, 2, 3$

- matter site $0 \rightarrow q_1$
- matter site $1 \rightarrow q_2$
- gauge link $0, 1 \rightarrow q_0$
- gauge link $1, 0 \rightarrow q_3$

$$|q_0 q_1 q_2 q_3\rangle \Longrightarrow$$

- $|vac\rangle_0 = |1011\rangle$
- $|e^+e^-\rangle_L = |0101\rangle$
- $|e^+e^-\rangle_R = |1100\rangle$
- $|vac\rangle_1 = |0010\rangle$

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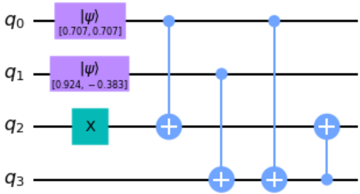
Ground state $|\psi_g\rangle = a_g(|vac\rangle_0 + |vac\rangle_1) + b_g(|e^+e^-\rangle_L + |e^+e^-\rangle_R)$

Each physical state is identified by the first two qubits $|q_0 q_1\rangle$

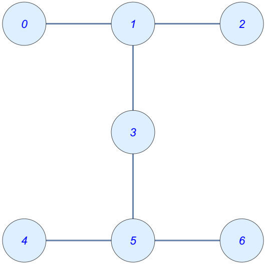
$$|\psi'_g\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \sqrt{2}(a_g\,|0\rangle + b_g\,|1\rangle) \left\{ \right.$$



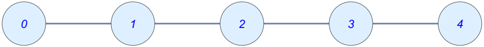
IBM Quantum NISQ devices

Comparison of the built-in command: `QuantumCircuit.initialize`

ibm_nairobi



ibmq_manila



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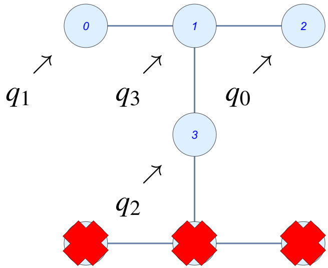

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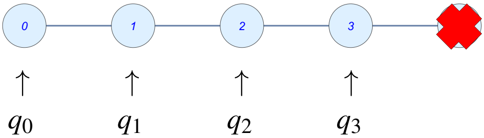
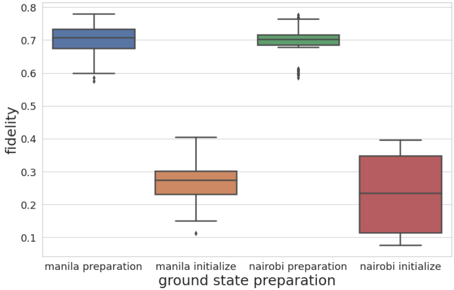




80 runs statistics

- median values of fidelities approximately equal to 0.7
- higher interquartile range for ibmq_manila
- ibm_nairobi with limited fluctuations for lower values

Noise models for ground state prearation (I)

Simulated circuits include an error probability related with each gate:

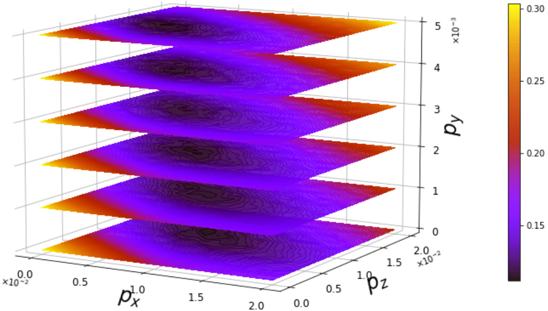
single qubit $K_0 = \sqrt{1 - \sum_{i=x,y,z} p_i}\, \mathbb{1}, K_1 = \sqrt{p_x}\, X, K_2 = \sqrt{p_y}\, Y, K_3 = \sqrt{p_z}\, Z$

double qubit $\widetilde{K}_{ij} = K_i \otimes K_j$

- shared parameters (p_x, p_y, p_z)

trace distance

$$T = \frac{1}{2} || \rho_{\text{ibmq}} - \rho_{\text{sim}} ||_1$$



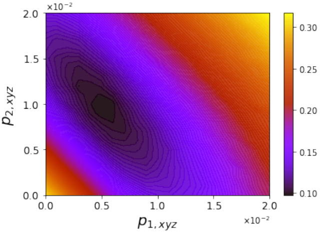
Noise models for ground state prearation (II)

- single qubit gates with $p_1 = p_x = p_y = p_z$
  two qubits gate with $p_2 = p_x = p_y = p_z$

- same two parameters (p_1, p_2),
  without errors along Y $(p_y = 0)$

trace distance

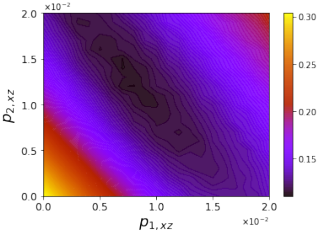
$$T = \frac{1}{2}||\rho_{\text{ibmq}} - \rho_{\text{sim}}||_1$$



scale factor $\frac{3}{2}$

$\longrightarrow$

conserved overall error probability



Equal minima: negligible loss of information without noise along Y

Cartan decomposition in Trotter evolution

Trotter product formula:

$$\mathrm{e}^{-\mathrm{i}\mathcal{H}t} = \mathrm{e}^{-\mathrm{i}\sum_x \mathcal{H}_x t} = \left(\overleftarrow{\prod_x} \mathrm{e}^{-\mathrm{i}\mathcal{H}_x \Delta t}\right)^{\frac{t}{\Delta t}} + \mathcal{O}(\Delta t^2)$$

where $\overleftarrow{\prod}_x \mathrm{e}^{-\mathrm{i}\mathcal{H}_x \Delta t} = \ldots\ \mathrm{e}^{-\mathrm{i}\mathcal{H}_1 \Delta t} \mathrm{e}^{-\mathrm{i}\mathcal{H}_0 \Delta t}$, with fermionic hopping

$$\mathcal{H}_J = \sum_{x=0}^{N-1} \mathcal{H}_{J,x} = \frac{J}{4} \sum_{x=0}^{N-1} \left[X_{x,x+1} \left(X_x X_{x+1} + Y_x Y_{x+1}\right)\right]$$

because $U_{x,x+1} = U^\dagger_{x,x+1} = X_{x,x+1}$.

Wiebe et al. arXiv:2002.11146 **2020**

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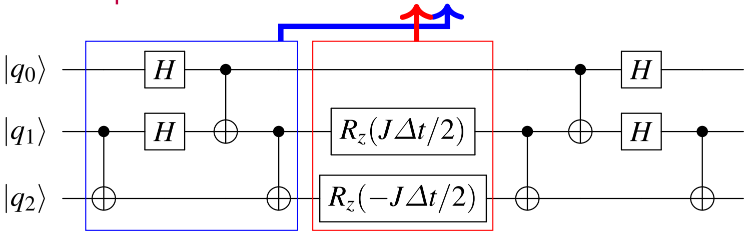
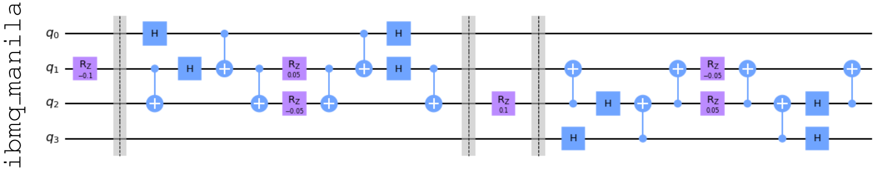
Cartan decomposition: $e^{-i\mathcal{H}_{J,0}\Delta t} = K^\dagger A\ K$



Wiebe et al. arXiv:2002.11146 **2020**

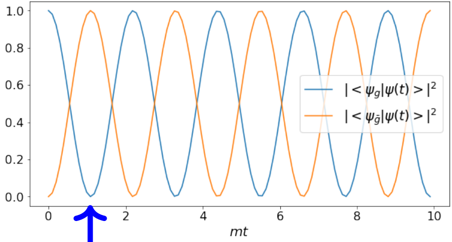
Simulated evolution

Trotter step evolution: $e^{-i\mathcal{H}\Delta t} \approx e^{-i\mathcal{H}_{J,1}\Delta t} e^{-i\mathcal{H}_{m,1}\Delta t} e^{-i\mathcal{H}_{J,0}\Delta t} e^{-i\mathcal{H}_{m,0}\Delta t}$



DQPTs conditiontion: $J = m$ yielding a Rabi model in the positive parity sector spanned by $\{|\psi_g\rangle, |\psi_{\bar{g}}\rangle\}$

evolution using time step $\Delta t = 0.1$

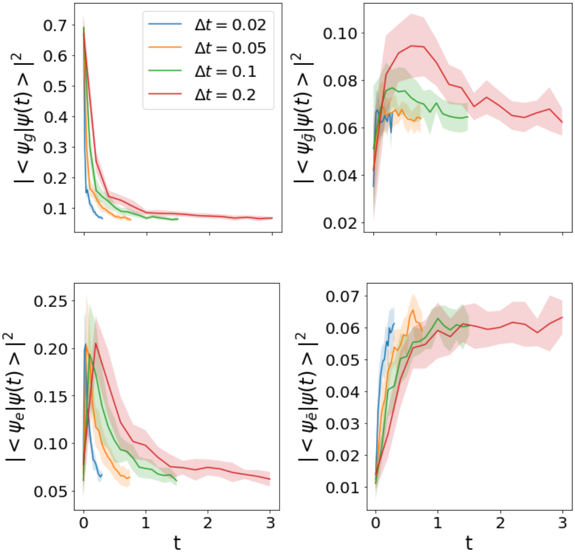


negligible error for noiseless Trotterization

first DQPT at $t_0 \approx \frac{1.11}{m}$

Real-time evolution on IBM Quantum

- Time evolution leads to the maximally mixed state
- Longer time steps with reduced decoherence

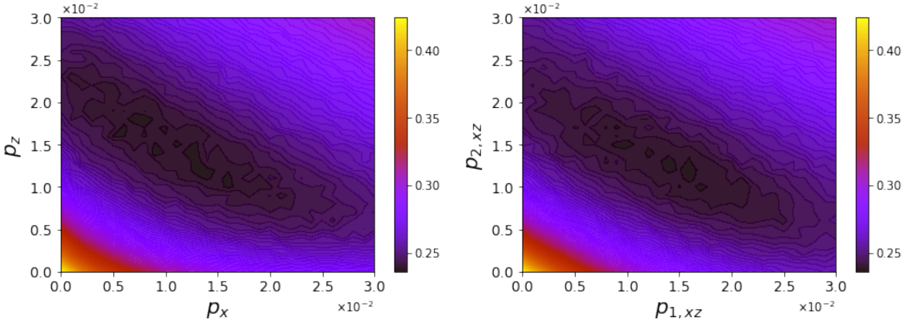


IBM Quantum Hub at CERN

Noise models for evolution

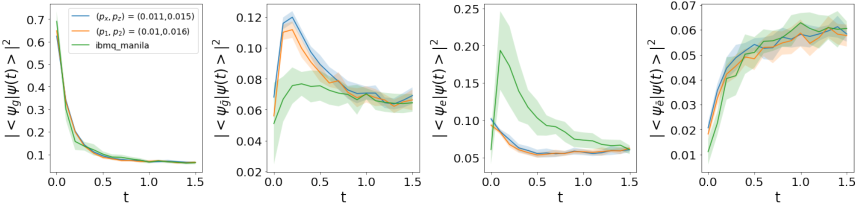
Noise models: compared with $\overline{T} = \frac{1}{3\Delta t}\sum_{\ell=0}^{2} T(\rho_{\text{ibmq}}(\ell\ \Delta t), \rho_{\text{sim}}(\ell\ \Delta t))\Delta t$

- single and double qubit gate share (p_x, p_z)
- two parameters (p_1, p_2), without errors along Y $(p_y = 0)$



Estimation for noise reduction

Two minima combined with optimal ground state preparation model:



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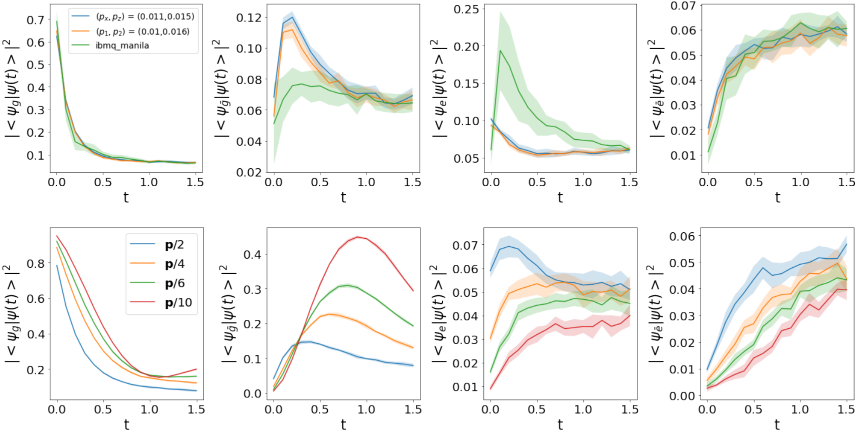

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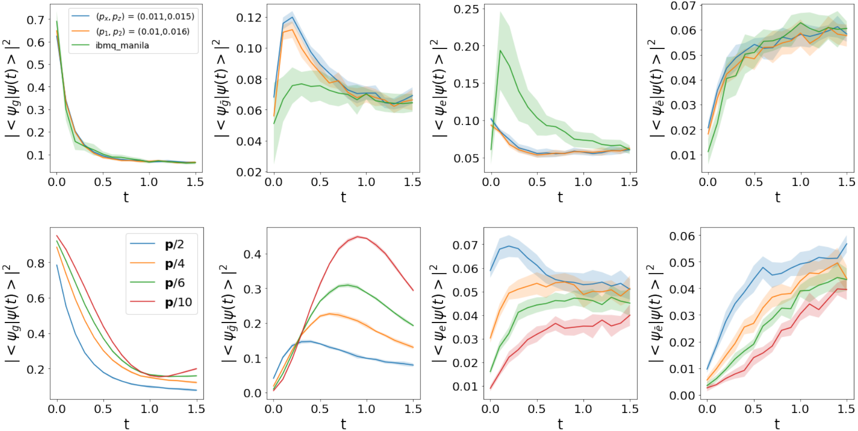

`ibmq_manila`: dephasing time $T_2 \approx 60\ \mu s$, gate time $\approx 370 \pm 80\ ns$

$$\text{max circuit moments} = \frac{T_2}{\text{gate time}} \approx 160$$

- Trotter step with 20 moments
- ground state preparation

Conclusions

Schwinger model real-time dynamics on IBM Quantum:

- outperforming ground state preparation;
- negligible noise along the Y direction;
- DQPTs phenomena for reduced error probabilities.

Our results could be improved:

- in devices with a higher number of qubits (not free access);
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Thank you!

Questions/Comments?