SM&FT - The XIX Workshop on Statistical Mechanics
and nonperturbative Field Theory
19th - 21st December 2022, Bari (Italy)

SM&FT 2022 THE XIX WORKSHOP ON STATISTICAL MECHANICS AND NON PERTURBATIVE FIELD THEORY

Work fluctuations in the harmonic active Ornstein-Uhlenbeck particle model

Massimiliano Semeraro, Bari 20/12/2022
massimiliano.semeraro@uniba.it

Outline

-Introduction
-Active Matter
-Active Work
-Dynamical Phase Transitions

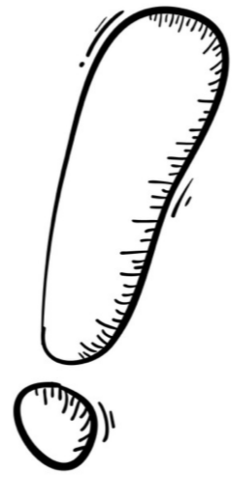
-Harmonically trapped AOUP
-Scaled Cumulant Generating Function
-Effective vs primary domain
-Singular rate function
-Trajectory characterisation

-Take home messages

Introduction

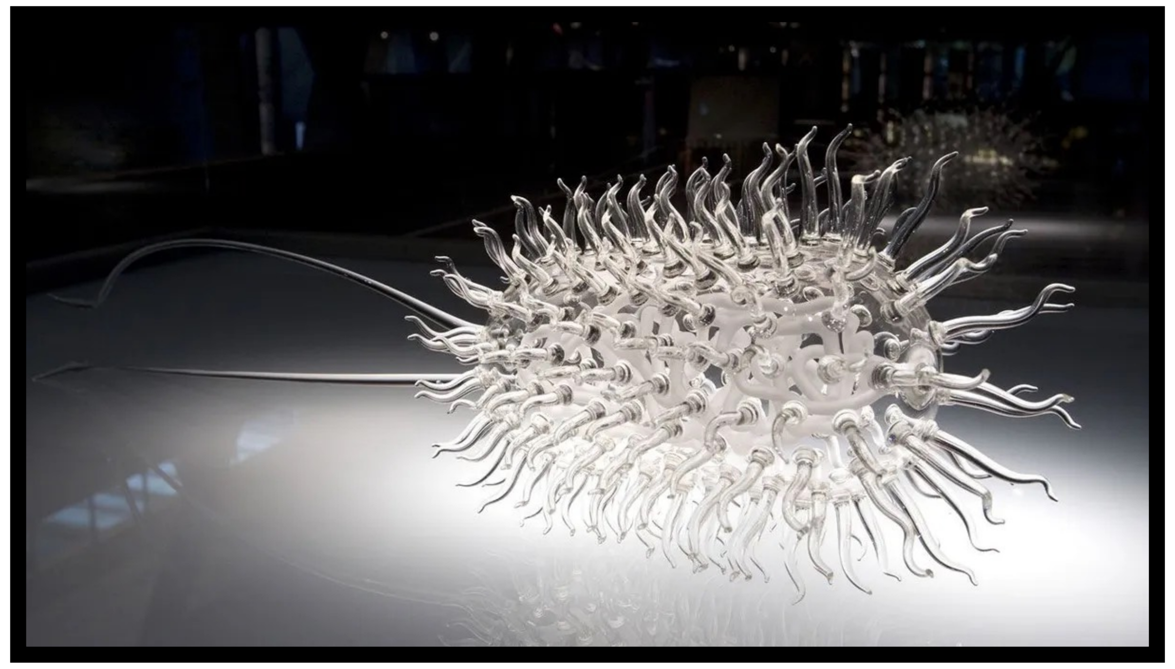
Non equilibrium systems

net macroscopic flows of matter or energy within a system or between systems



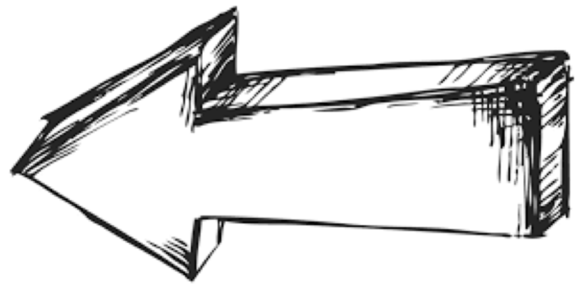
Relevant non equilibrium system

- Multi-bath systems
- Ratchet systems
- **Active matter**



Important observables

the most significant observable to monitor are the (trajectory-wise defined) entropy production and active work



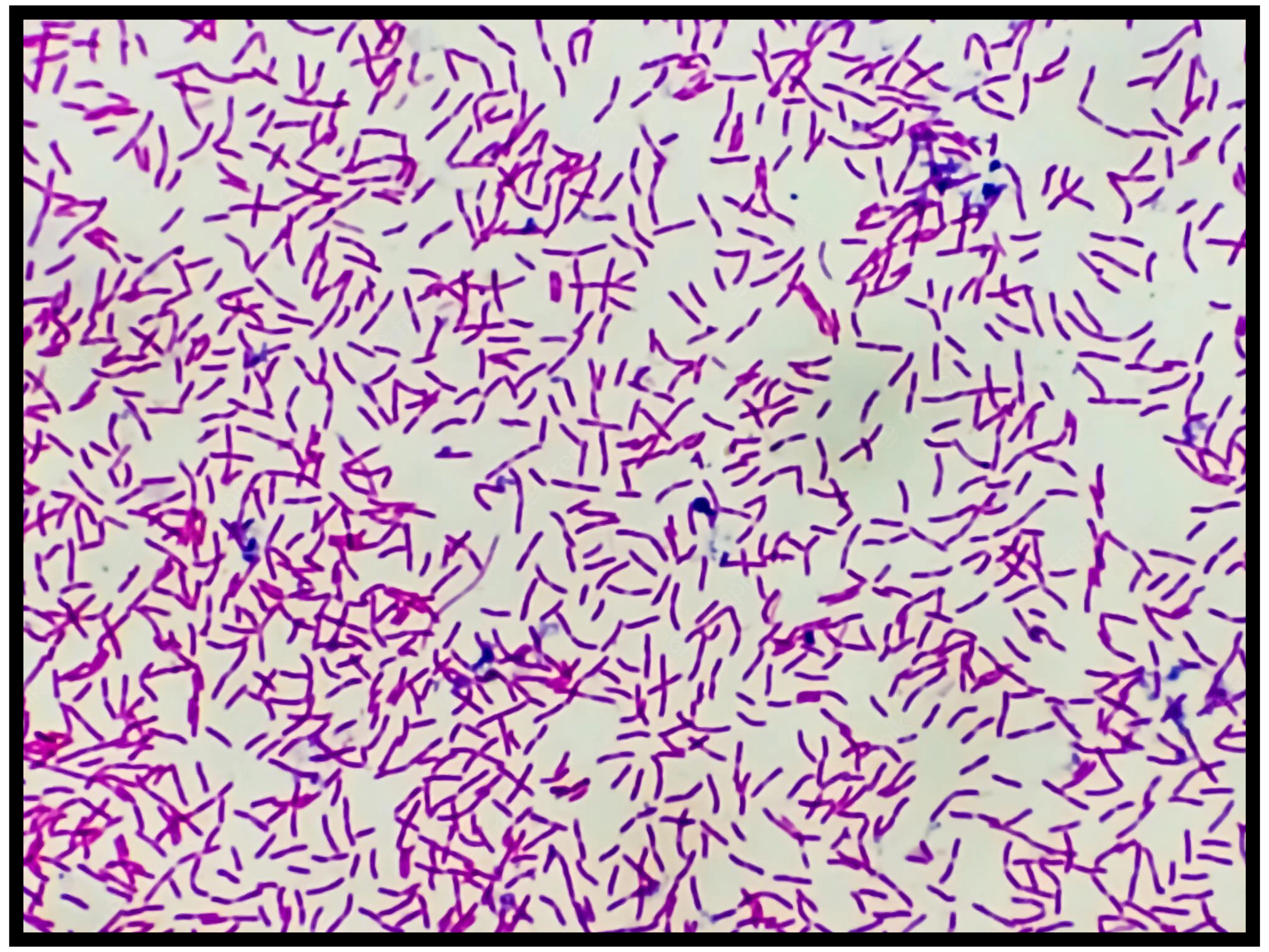
Active matter systems

- each single component continuously transforms energy from internal reservoirs or from the surroundings to self-propel

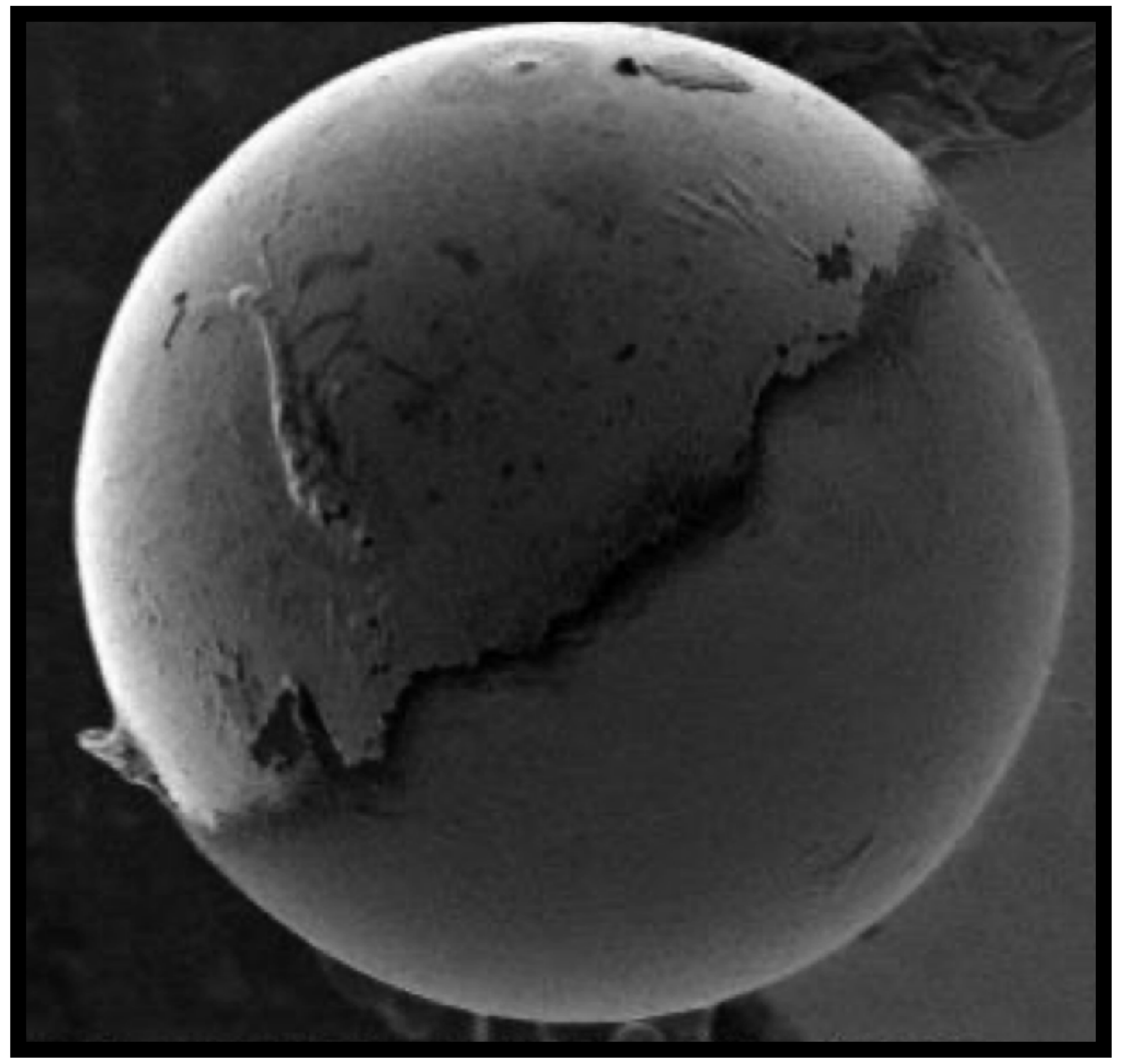
- Examples

flock of birds

bacteria



janus particles



- active matter systems are inherently out of equilibrium

Active Work

- Active particle dynamics

$$m\ddot{r}_i(t) = -\gamma\dot{r}_i + \boxed{F_i^a(t)} - \nabla \cdot U(x(t),t) + \eta_i(t)$$

- Active Work

$$W_\tau = \frac{1}{\tau}\int_0^\tau dt\ F_i^a(t) \cdot \dot{r}_i(t)$$

- Large deviation theory

$$P(W_\tau) \approx e^{-\tau\ I(W_\tau)}$$

$I(W_\tau)$ rate function

Dynamical phase transitions

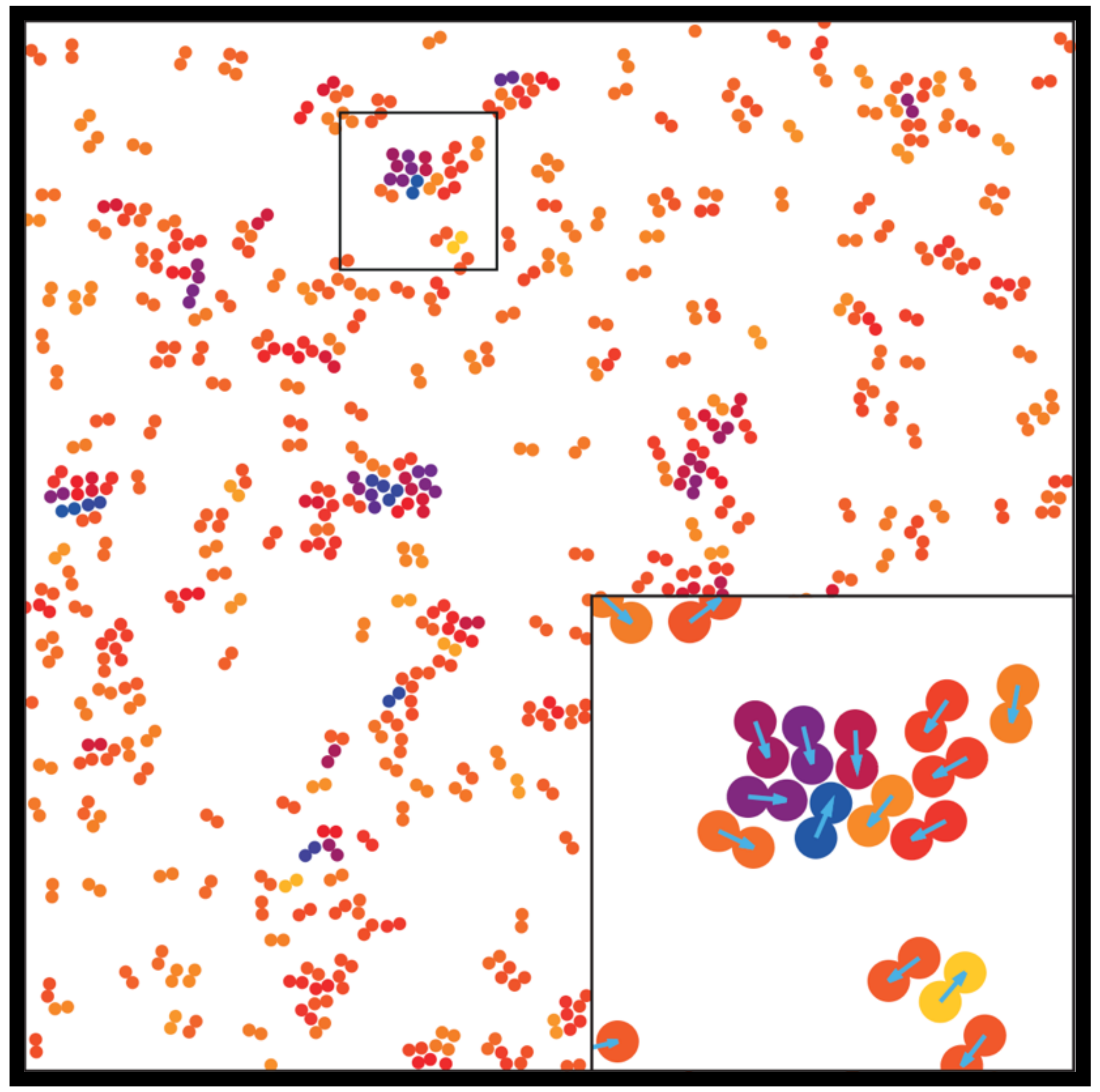
- Singularities in $I(W_\tau)$ ⟶ Dynamical Phase Transitions!

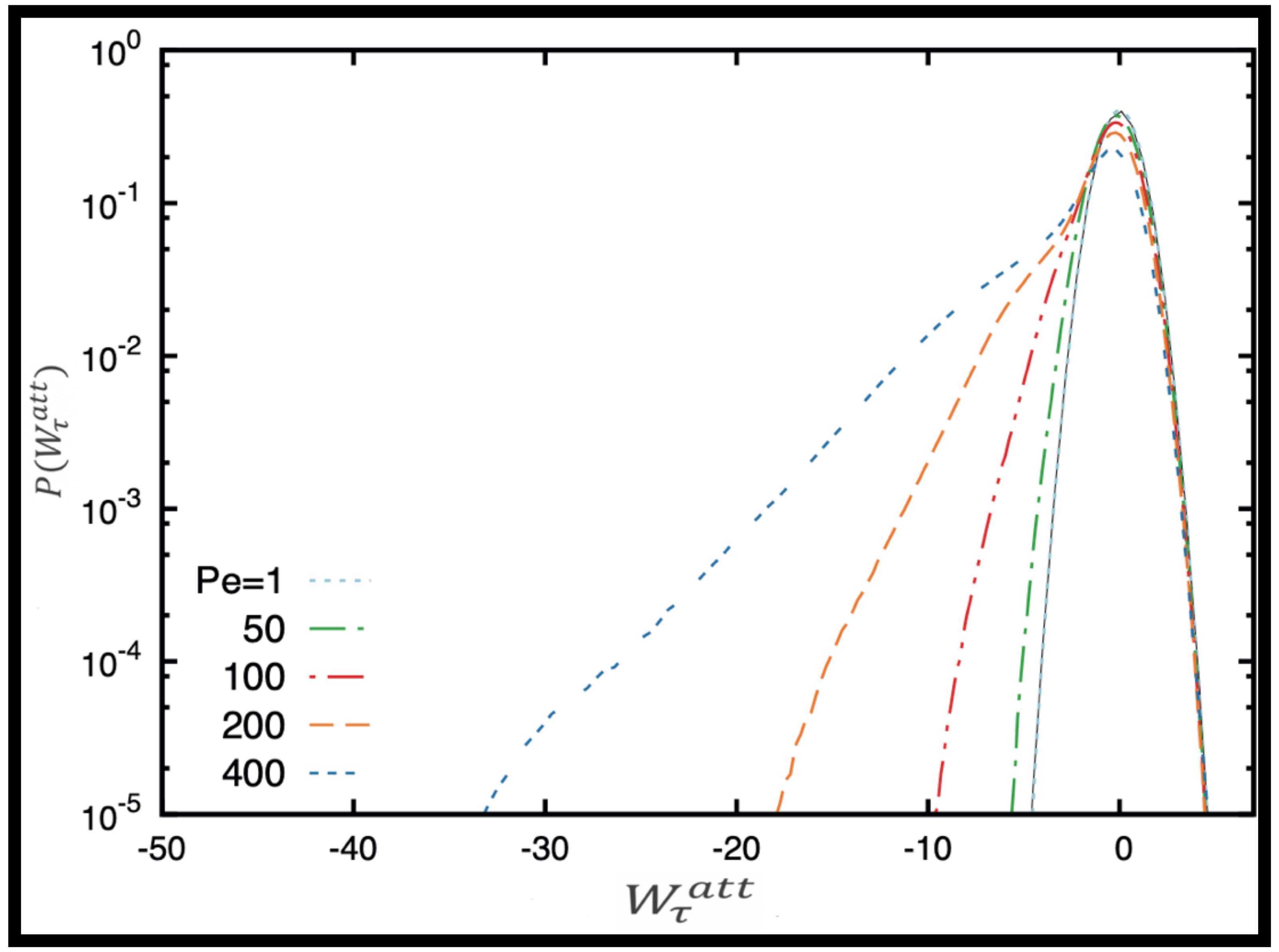
- Single free particle model (AOUP) ⟶ no singularities

J Stat Mech 2021, Semeraro et al

- System of interacting particles (ABPs) ⟶ singularities

PRL 2017, Cagnetta et al





Harmonically trapped AOUP

soon on arXiv,
Semeraro et al

- System equations $\begin{cases} \gamma \dot{r}(t) = a(t) - kr(t) + \sqrt{2\gamma k_B T}\ \xi(t) \\ \dot{a}(t) = -\nu a(t) + F\sqrt{2\nu}\ \eta(t) \end{cases}$

- W_τ distribution evaluated trough path integral techniques $(\gamma = k = T = \nu = 1)$

$$P(W_\tau = w\tau) = \int \delta(W_\tau - w\tau) P_\tau \mathscr{D}r\mathscr{D}$$

$$P_\tau \propto \left\{ -\frac{1}{2}(r(0)\ a(0))\Sigma_0^{-1} \begin{pmatrix} r(0) \\ a(0) \end{pmatrix} \right\}$$
$$exp \left\{ -\frac{1}{4}\int_0^\tau [\dot{r}(t) - a(t) + \kappa r(t)]^2\ dt \right\}$$
$$exp \left\{ -\frac{1}{4Pe^2}\int_0^\tau [\dot{a}(t) + a(t)]^2\ dt \right\}$$

$$Pe = \frac{Fd}{k_B T}$$

$$\kappa = \frac{kd^2}{k_B T}$$

- Direct approach fails, discretised version of the problem $W_\tau \longrightarrow W_N$

arXiv:2204.08059, Zamparo et al.

Scaled cumulant generating function

- Scaled cumulant generating function

$$\phi(\lambda) = \frac{1+\kappa}{2} - \frac{1}{2}\sqrt{(1+\kappa)^2 - 4Pe^2\lambda(1+\lambda)}$$

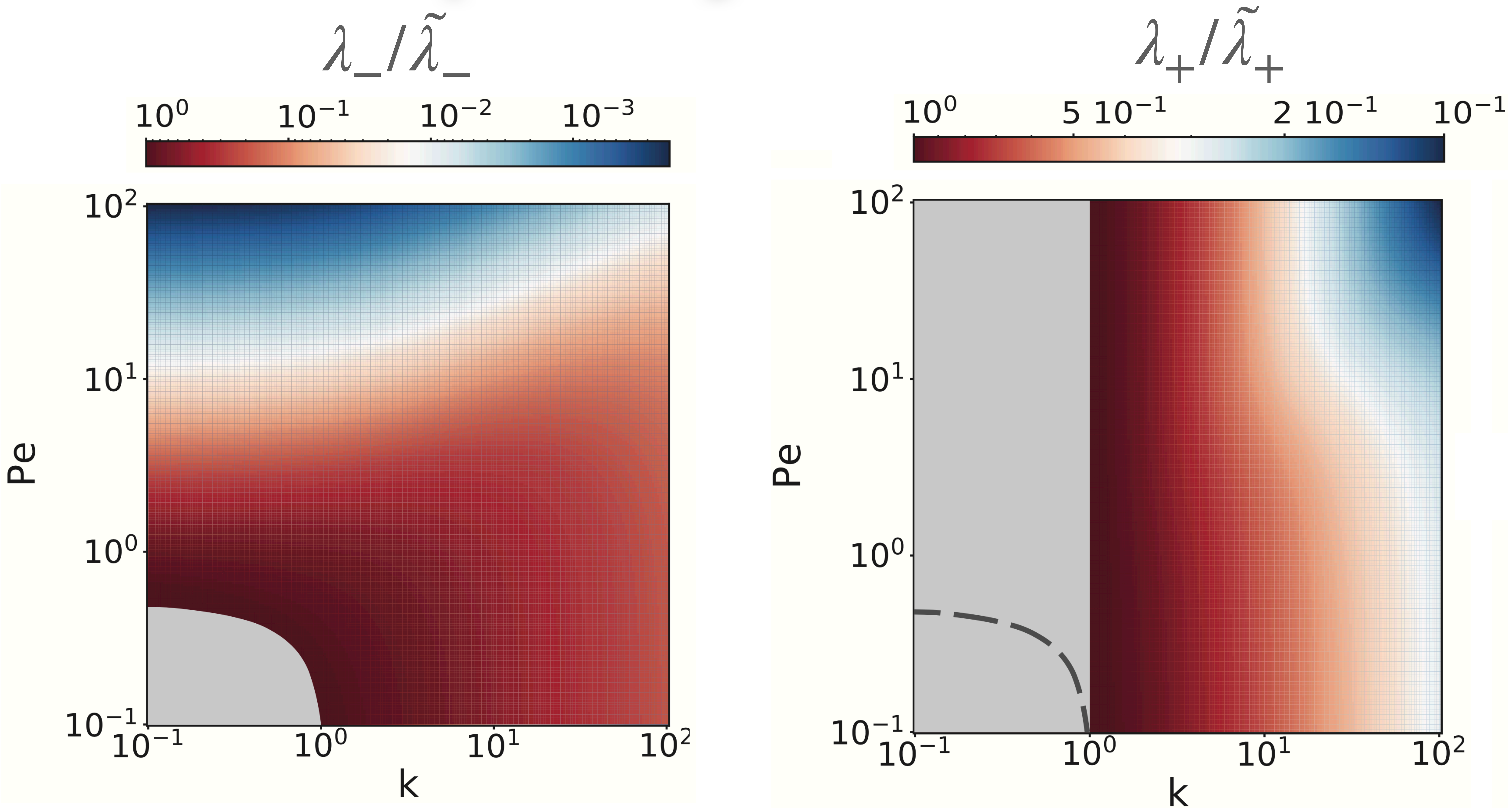
- Primary domain $(\tilde{\lambda}_-, \tilde{\lambda}_+)$

$$\tilde{\lambda}_\pm = -\frac{1}{2} \pm \sqrt{1 + \left(\frac{1+\kappa}{Pe}\right)^2}$$

- Effective domain (λ_-, λ_+) not always coinciding with the primary one $(\tilde{\lambda}_-, \tilde{\lambda}_+)$

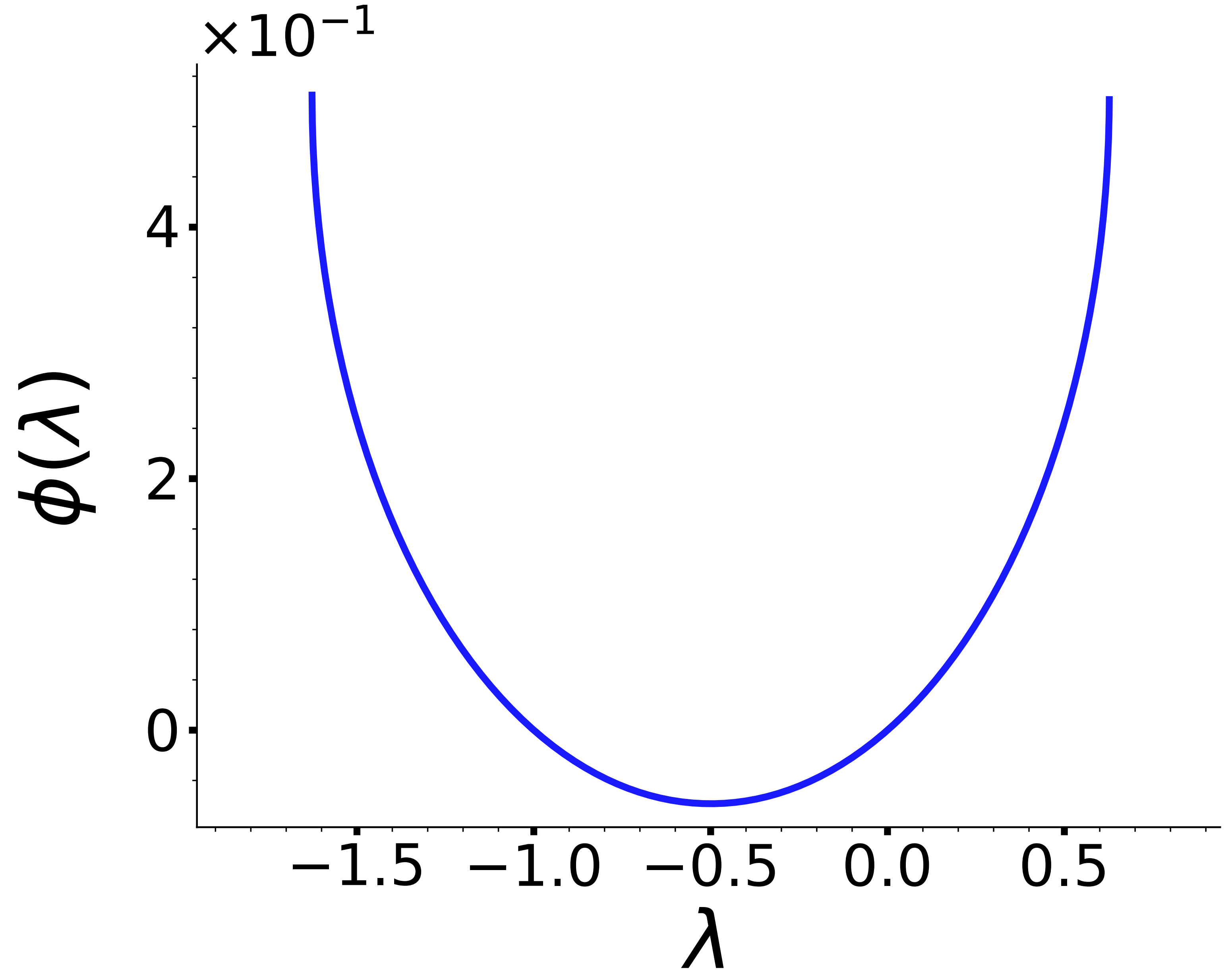
→ **Dynamical Phase Transition!**

Effective vs primary domain

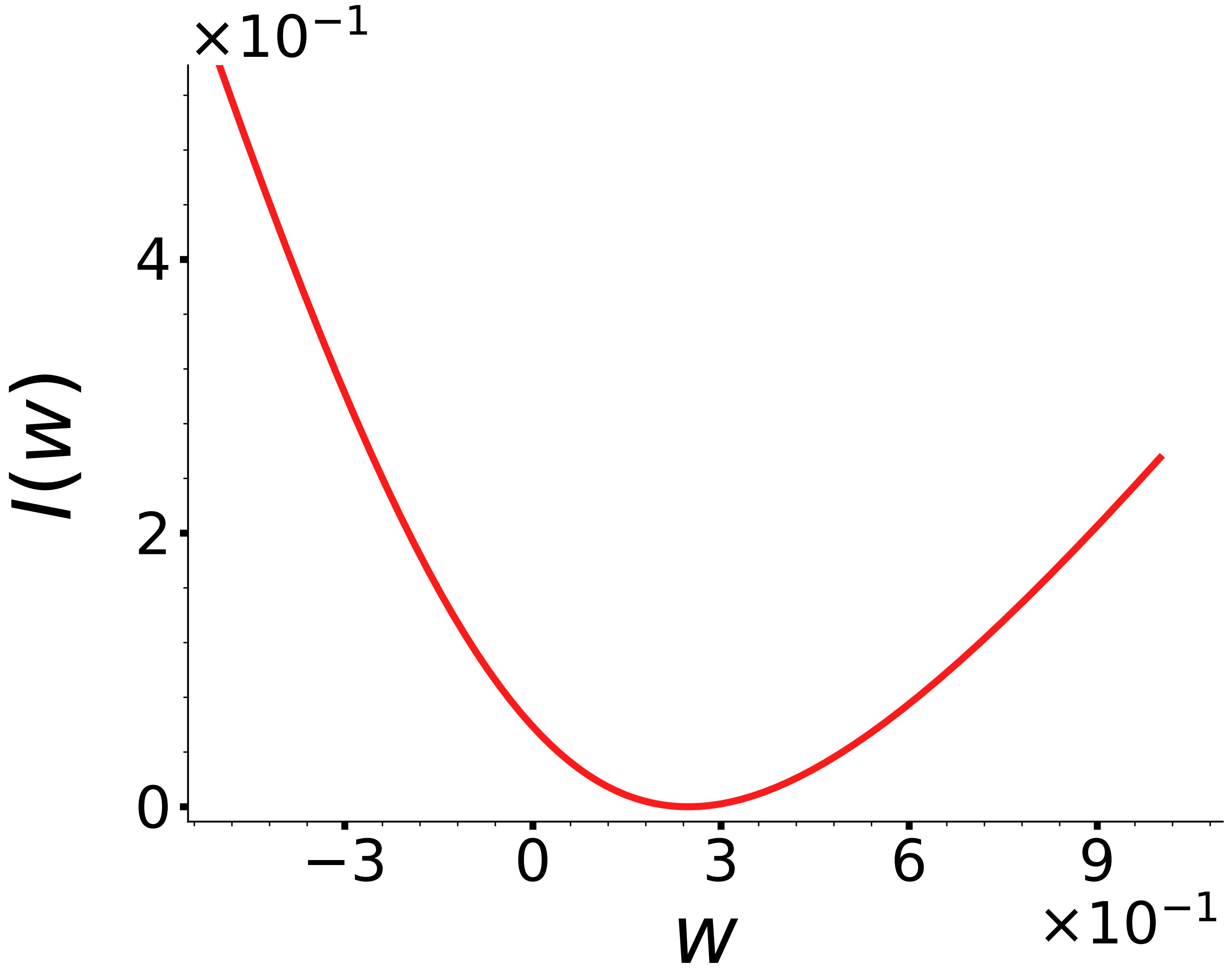


Singular rate function

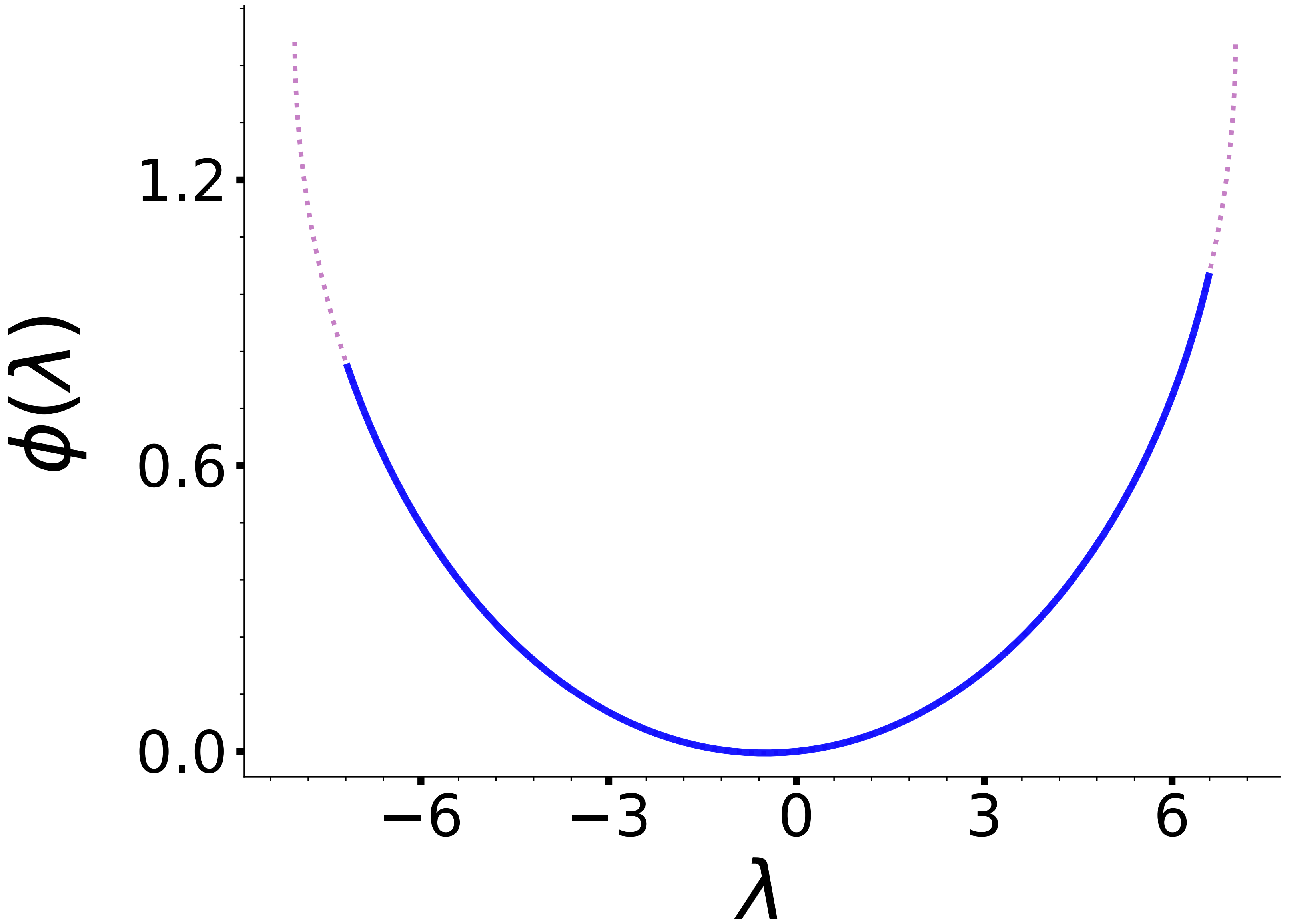
- $Pe = 0.5, \kappa = 0.01$
  stationary



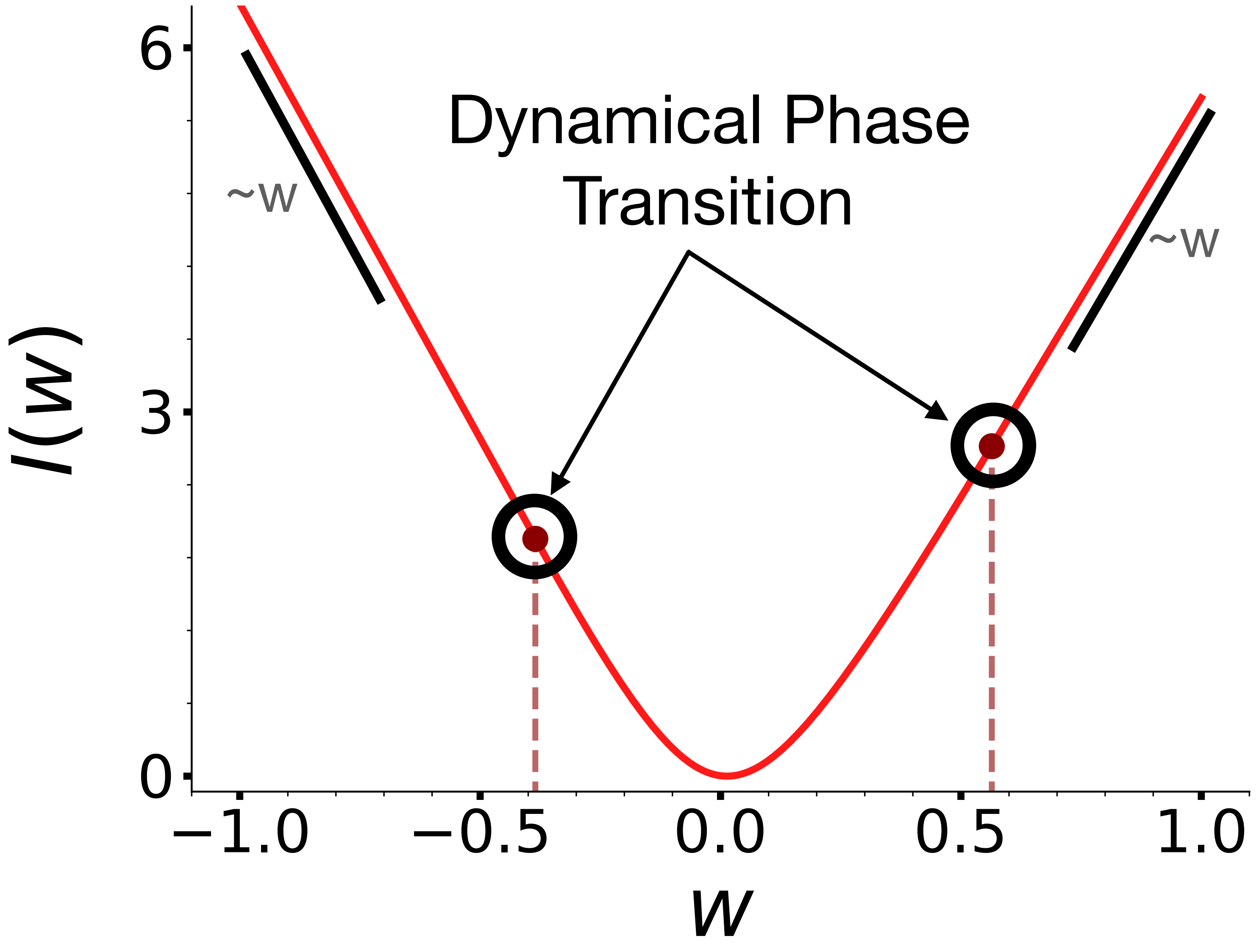
LFT



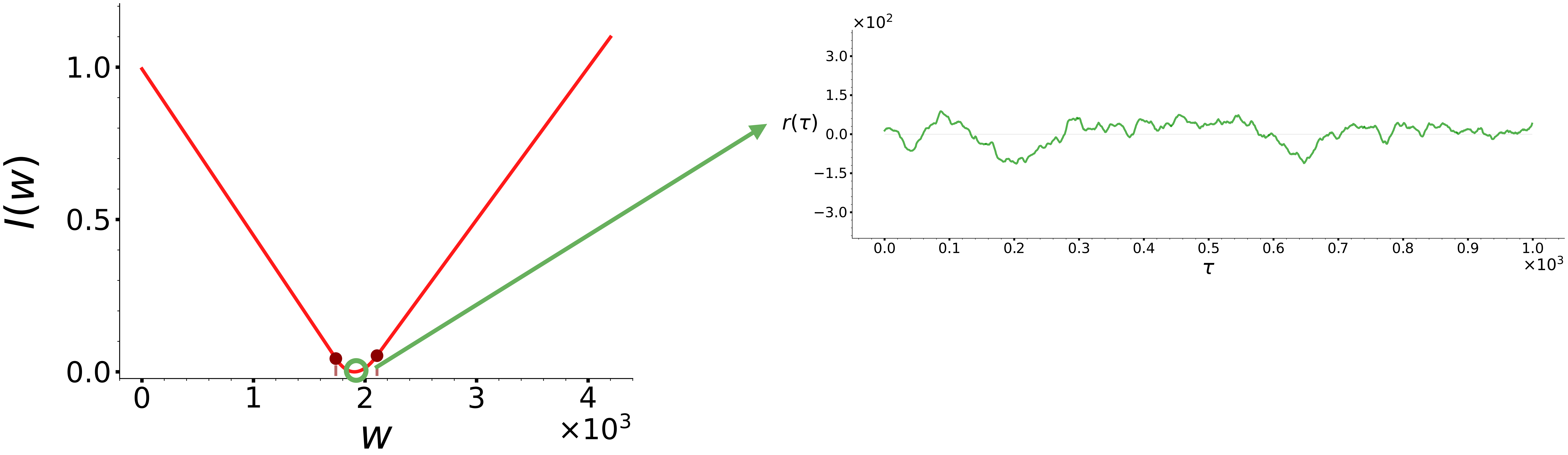
- $Pe = 0.2, \kappa = 2.0$
  stationary



LFT



Trajectory characterisation



Trajectory characterisation

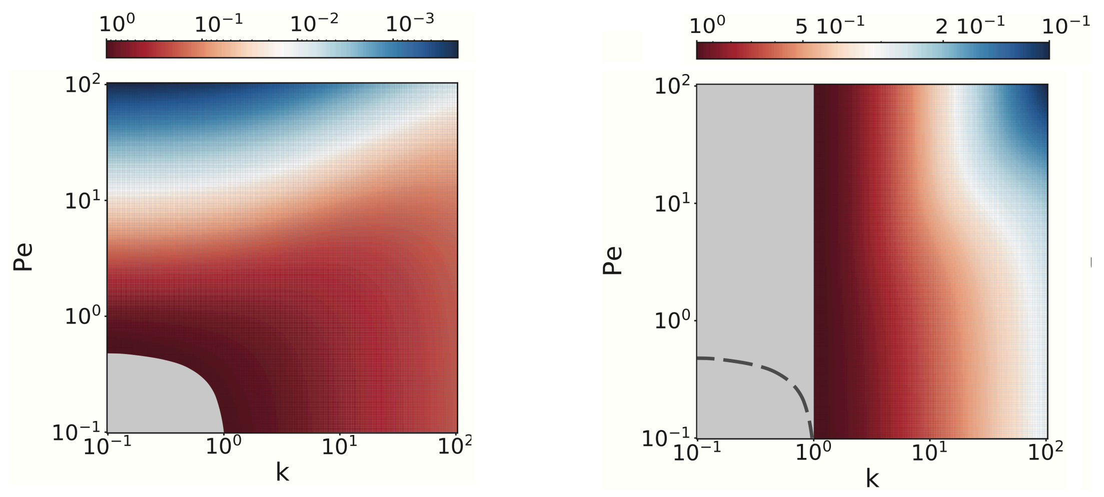
Trajectory characterisation

Take home messages

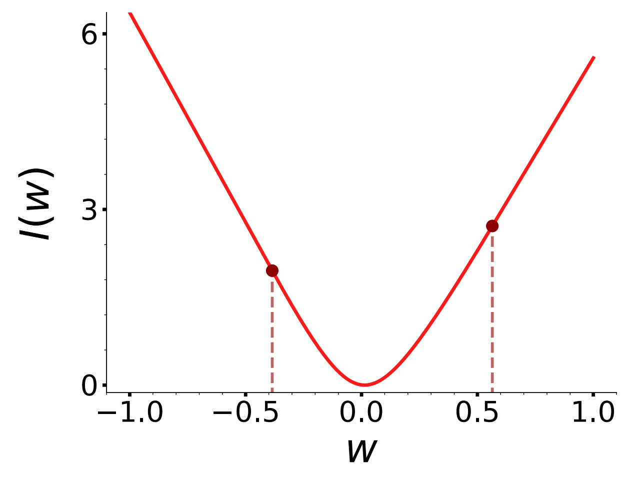
- Analytical evaluation of the scgf for a single harmonically trapped AOUP

$$\phi(\lambda) = \frac{1+\kappa}{2} - \frac{1}{2}\sqrt{(1+\kappa)^2 - 4Pe^2\lambda(1+\lambda)} \qquad \tilde{\lambda}_{\pm} = -\frac{1}{2} \pm \sqrt{1 + \left(\frac{1+\kappa}{Pe}\right)^2}$$

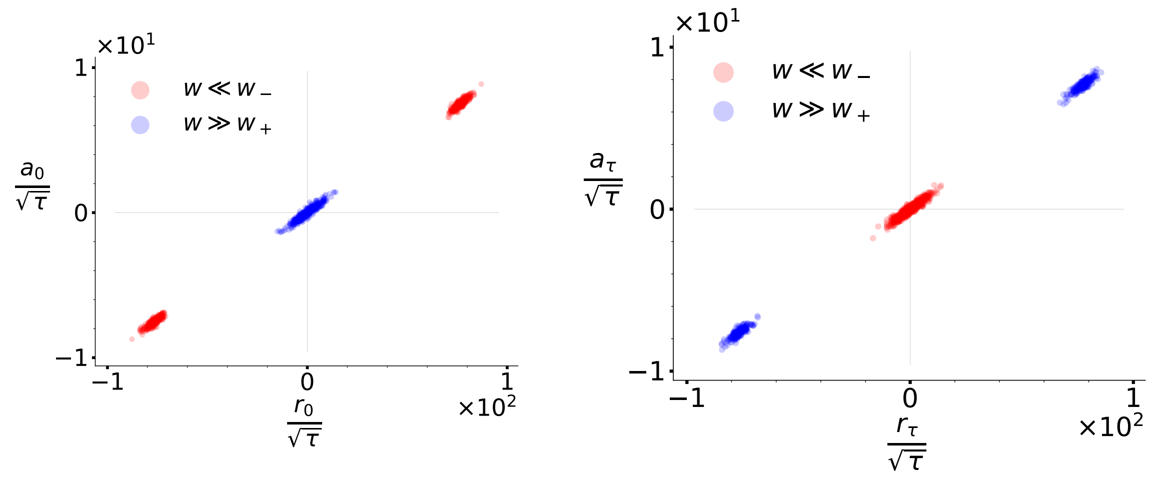
- System parameters and initial conditions influence the scgf effective domain



- Appearance of singularities in the rate function



- Trajectory characterisation



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SIGILLUM UNIVERSITATIS BARENSIS MCMXXV

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ReCaS

Thank you for your attention!