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Understanding the non-Gaussianities in the Hubble-Lemaître diagram [1]

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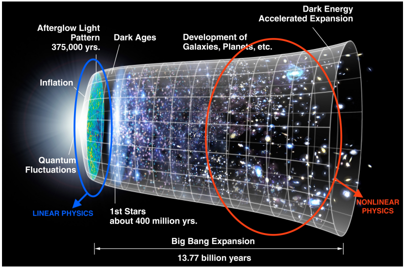
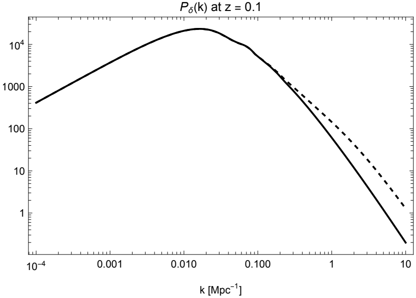
[1]based on
Di Dio, F, Schiavone *in preparation*

Two pictures of the Universe

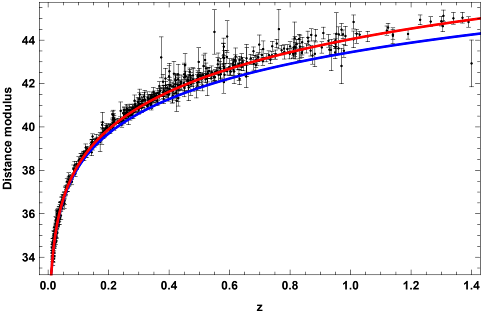




The Hubble-Lemaître diagram

- At the background level, we can write luminosity distance as (datapoints from Union2)

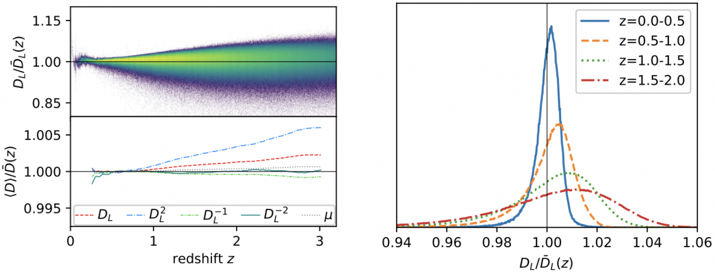
$$d_L = \frac{1+z}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_{m0}(1+z')^3 + \Omega_{\Lambda 0}}}$$



- For very low redshifts, these relations becomes independent on the chosen cosmology and leads to an estimator of the Hubble rate today as $H_0 = z/d_L$

Non-Gaussianities - Why?

- From *gevolution*[2] we have a significant shift in the mean value of the observed $d_L(z)$ in the real inhomogeneous Universe

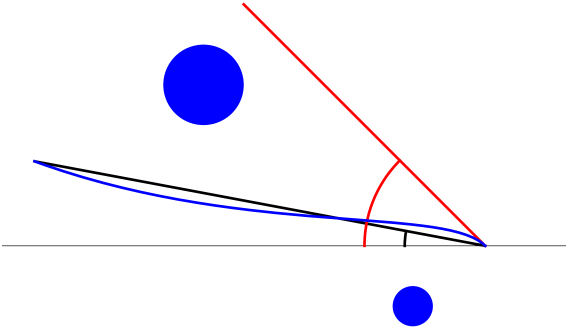


- Average and dispersion are in good agreement with theoretical estimations (Ben-Dayan, Gasperini, Marozzi, Nugier, Veneziano, 2013 Fleury, Clarkson, Maartens, 2017)
- Moreover we also have a significantly non-null skewness in the distribution of $d_L(z)$

[2] Adamek, Clarkson, Coates, Durrer, Kunz 2019

Non-Gaussianities - Physical reasons

- Initial conditions of the Universe are highly Gaussian. However non-Gaussianities are generated afterwards
- General Relativity is a non-linear theory
- Gravitational collapse breaks validity of linear regime at late time
- Light-cone is distorted by the non-linear structures
- Data binning can introduce a spurious non-Gaussianity in the data



Averaging observables - Motivations

- In order to understand the statistic, we need a well-posed framework to treat our theoretical observables
- Theoretical predictions can be done with different approaches: numerical simulations, perturbation theory, effective field theory
- The well-posedness of the theoretical computations are then crucial to correctly interpret observed data
- In this regard, a rigorous and well-posed prescription for evaluating statistics from theory in the distribution of the observable quantities is crucial
- In order to face these issues, we need to ask case by case what are the physical observations whose we want to study the statistics...

A well-posed prescription for averaging cosmological observables

- An observationally oriented prescription

$$\langle d_L^\alpha \rangle = \frac{J(d_L^\alpha)}{J(1)} = \frac{\int_{\Sigma_s} d\Omega\, \mathbf{d_A^2}\, \rho\, d_A^\alpha \frac{1}{\partial_\tau(1+z)}}{\int_{\Sigma_s} d\Omega\, \mathbf{d_A^2}\, \rho \frac{1}{\partial_\tau(1+z)}} \equiv \frac{\int d\mu\, d_A^\alpha}{\int d\mu}$$

  where Σ_s are constant redshift hyper-surfaces
- This number count weighted average has been proven to be gauge invariant and covariant even in the small redshift bin limit[3]
- Etherington relation $d_L(z) = (1+z)^2 d_A(z)$

[3] F. Gasperini, Marozzi, Veneziano, 2020

From exact to leading order results

- Assuming stochastic inhomogeneities of the linear relativistic gravitational potential ψ

$$ds^2 = a^2(\eta)\left[-\left(1+2\Phi\right)d\eta^2 + \left(1-2\Psi\right)\left(dr^2 + r^2 d\Omega^2\right)\right] \qquad (1)$$

where $\Phi \equiv \psi + \frac{1}{2}\phi^{(2)}$ and $\Psi \equiv \psi + \frac{1}{2}\psi^{(2)}$ which are delta correlated and with null mean value

$$\overline{\psi_{\vec{k}}} = 0 \qquad \overline{\psi_{\vec{k}}\,\psi_{\vec{k'}}} = \delta\left(\vec{k} + \vec{k'}\right)$$

the leading order for the average is the second one in the perturbations of the metric

- $$d_A \simeq d_A^{(0)}\left(1 + \sigma^{(1)} + \sigma^{(2)}\right) \qquad \text{and} \qquad d\mu \simeq d\mu^{(0)}\left(1 + \mu^{(1)} + \mu^{(2)}\right)$$

- If we define $I[f] \equiv \frac{\int d\mu^{(0)} f}{\int d\mu^{(0)}}$, then

$$\overline{\left\langle \left(\frac{d_A}{d_A^{(0)}}\right)^{\alpha} \right\rangle} = 1 + \alpha\, \overline{I[\mu^{(1)}\,\sigma^{(1)}]} + \alpha\, \overline{I[\sigma^{(2)}]} + \frac{\alpha\,(\alpha-1)}{2}\, \overline{I\left[{\sigma^{(1)}}^2\right]} - \alpha\, \overline{I[\mu^{(1)}]\, I[\sigma^{(1)}]}\,,$$

Skewness and kurtosis - theoretical expressions

- Within our framework, it is quite straightforward to evaluate standardised moments

$$\kappa_\alpha \equiv \frac{\mu_\alpha}{(\sigma^2)^{\alpha/2}} = \frac{1}{(\sigma^2)^{\alpha/2}} \overline{\left\langle \left(\frac{d_A}{d_A^{(0)}} - m \right)^\alpha \right\rangle}$$

- For the third moment, we get $\mu_3 = \mu_3^Q + \mu_3^{PB} + \mu_3^{LSS}$ where we defined

$$\begin{aligned}
\mu_3^Q \equiv& \frac{7}{2}\, \overline{I[\sigma^{(1)\,4}]} - \frac{15}{2} \left(\sigma^2\right)^2 \\
\mu_3^{PB} \equiv& 3 \left\{ \overline{I[\sigma^{(1)\,2}\Sigma^{(2)}]} - \sigma^2\, \overline{I[\Sigma^{(2)}]} \right\} \\
\mu_3^{LSS} \equiv& 3\, \overline{I\left[\sigma^{(1)\,2}\sigma^{(2)}_{LSS}\right]}
\end{aligned}$$

- The fourth moment instead is simply $\mu_4 = \overline{I[\sigma^{(1)\,4}]}$

Leading order terms

- Leading order expressions are independent of the measure in the small bin approximation and returns for the distance

$$\sigma^{(1)} = \int_0^{r_s} dr \frac{r - r_s}{r r_s} \Delta_2 \psi\,(r) \qquad , \qquad \sigma^{(2)} = \frac{1}{2}\sigma^{(1)\,2} + \Sigma^{(2)} + \sigma^{(2)}_{LSS}$$

- Non-linear leading terms are[4]

$$\begin{aligned}
\sigma^{(2)}_{LSS} \equiv & \frac{1}{4}\int_0^{r_s} dr \frac{r - r_s}{r r_s} \Delta_2 \left[\psi^{(2)} + \phi^{(2)}\right](r) \\
\Sigma^{(2)} \equiv & 2\int_0^{r_s} dr \frac{r - r_s}{r r_s} \partial_b \left[\Delta_2 \psi(r)\right] \int_0^{r_s} dr\, \frac{r - r_s}{r r_s} \bar{\gamma}_0^{ab} \partial_a \psi(r) \\
& + 2\int_0^{r_s} dr \left\{ \gamma_0^{ab} \partial_b \left[\int_0^{r} dr'\, \psi(r')\right] \int_0^{r} dr'\, \frac{r' - r}{r r'} \partial_a \Delta_2 \psi(r') \right\} \\
& + \int_0^{r_s} dr\, \frac{r - r_s}{r r_s} \Delta_2 \left[\gamma_0^{ab} \partial_a \left(\int_0^{r} dr'\, \psi(r')\right) \partial_b \left(\int_0^{r} dr'\, \psi(r')\right)\right]
\end{aligned}$$

[4] F., Gasperini, Marozzi, Veneziano 2015

Skewness - quadratic terms

- In the same manner, we can evaluate also the various terms in the third moment
- According to what we have already found, we have

$$\mu_3^Q = \frac{7}{2}\,\overline{I[\sigma^{(1)\,4}]} - \frac{15}{2}\left(\sigma^2\right)^2 = 3\left(\sigma^2\right)^2$$

- As a consequence, the skewness sourced by quadratic perturbations is

$$\kappa_3^Q = 3\,\sigma$$

- Recalling the results from the dispersion, we then have that

$$\kappa_3^Q \sim 10^{-2} z$$

hence positive

Skewness - post-Born terms

- Post-Born terms involve several nested line-of-sight integrals and this would provide in principle a lot of terms
- However, a careful evaluation simply returns

$$\mu_3^{PB} = 6\int_0^{r_s} dr_1 \frac{r_1 - r_s}{r_1 r_s} \int_0^{r_s} dr_2 \frac{r_2 - r_s}{r_2 r_s} \int_0^{r_s} \frac{dr}{r^2} \frac{r - r_s}{r r_s} \times \int_0^{r} dr_3 \int_0^{r} dr_4 \, \mathcal{L}(r_1, r_3)\, \mathcal{L}(r_2, r_4)$$

- Post-Born are sourced by the same kernel as the quadratic terms

$$\mathcal{L} = (r_1, r_2) \; \frac{9\, r_1 r_2 \mathcal{H}_0^4 \Omega_{m0}^2 D_1(r_1) D_1(r_2)}{a(r_1) a(r_2)} \left[2 r_1 r_2 I_2^2 \left(|r_1 - r_2|\right) + I_1^3 \left(|r_1 - r_2|\right) (r_1 - r_2)^2 \right]$$

where

$$I_\ell^n(s) = \int \frac{dq}{2\pi^2} q^2 P(q) \frac{j_\ell(qs)}{(qs)^n}$$

Skewness - Large Scale Structure

- μ_3^{LSS} is the most interesting term since it contains the actual information about the non-linearities in the LSS

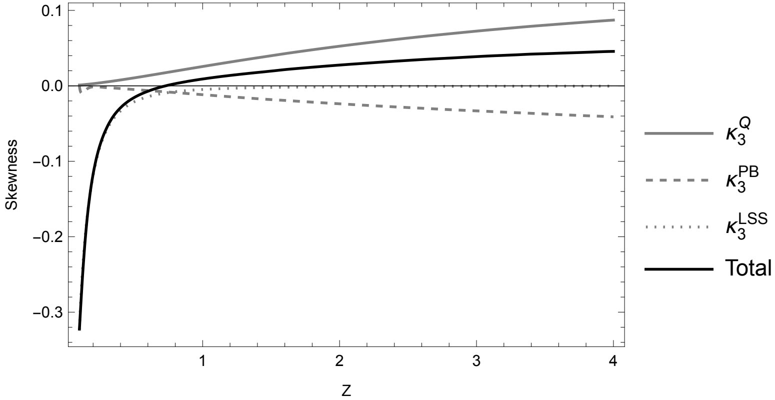
$$\psi^{(2)}_{\vec{k}}(\eta) = \int \frac{d^3k_1 d^3k_2}{(2\pi)^3 k^2 k_1^2 k_2^2} \delta\left(\vec{k} - \vec{k}_1 - \vec{k}_2\right) F_2\left(\vec{k}_1, \vec{k}_2\right) \delta\rho\left(\eta, \vec{k}_1\right) \psi_{\vec{k}_1}(\eta) \psi_{\vec{k}_2}(\eta)$$

- These non-linearities source also higher-order correlation functions, naturally sourcing non-Gaussian statistic since $\overline{\psi_{\vec{k}_1}\psi_{\vec{k}_2}\psi^{(2)}_{\vec{k}_3}} \neq 0$

$$\begin{aligned}
\mu_3^{LSS} =& \frac{3}{2} \sum_{\ell_1 \ell_2 \ell_3} \ell_1(\ell_1+1)\,\ell_2(\ell_2+1)\,\ell_3(\ell_3+1) \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix}^2 \\
& \times \frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{(4\pi)^2} \\
& \times \int_0^{r_s} dx\, \frac{C(x)}{x^4} \left(\frac{x - r_s}{x\, r_s}\right)^3 \frac{P\left(\widetilde{k}_1\right) P\left(\widetilde{k}_2\right)}{\widetilde{k}_1^{\,2}\,\widetilde{k}_2^{\,2}\,\widetilde{k}_3^{\,2}} F_2\left(\widetilde{k}_1, \widetilde{k}_2, \widetilde{k}_3\right)
\end{aligned}$$

where $\tilde{k}_i = \frac{\ell_i + 1/2}{x}$

Skewness - preliminary analytic results



- A competitive cancellation between quadratic and post-Born terms seems to emerge. Similar to CMB spectra (Marozzi, Fanizza, Di Dio, Durrer, 2016-2017-2018 Pratten, Lewis, 2016)
- The LSS term is strongly dependent on the small scales. Anyway, they point in the right direction to explain the simulations outcome

Conclusions

- A large competitive effect between linear perturbations seems to attenuate the skewness sourced by the linear gravitational potential and looks promising in order to get the leading effect mainly sourced by the correlation functions of the h.o. gravitational potential
- A better sampling of the non-linear perturbative scales is currently ongoing. An important enhancing of the effect seems to emerge at small redshifts
- Future developments are in order to a full comparison with data/observations
  - Finite-bin evaluations
  - Doppler, Redshift-Space-Distrortion
  - Higher-order moments (kurtosis)