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# A new formulation of GR resembling EM


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# Conclusions

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We derived dGREM, a new formulation of the EFE with striking properties:

☛ *Olivares et al., PRD, June 2022*

For NR applications:

- fully first-order
- flux-balanced form
- allowing for exact suppression of constraint violations

From a theoretical viewpoint:

- derived in terms of differential forms
- related to gauge theories
- striking resemblance with EM equations

# Introduction

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EM equations as an IVBP $\rightarrow$ evolution equations + constraint equations $(\nabla \cdot \mathbf{B} = 0)$

Well-established methods to handle constraint violations numerically:

- divergence cleaning (Dedner's method *[Dedner et al., 2002]*)
- Yee algorithm *[Yee, 1966]*, constrained-transport schemes (staggered grids) *[Evans and Hawley, 1988]*

What about GR?

- we have the Hamiltonian and momentum constraints (+ formulation-specific ones)
- "divergence cleaning" (Z4 family of formulations)
- **But no such thing as CT for GR!** (An attempt in [Meier, 2003])

# Introduction

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Not enough to guess were to store fields

👉 Write Eqs. in terms of **differential forms** and integrate on **appropriate submanifolds**: the desired **cancellations** emerge!

Example: the wave equation. Take a scalar field (0-form) with exterior derivative $\boldsymbol{J} = \boldsymbol{d}\phi$,

The wave equation:

$$-\star^{-1}\boldsymbol{d}\star\boldsymbol{J} = 0\,.$$

$\boldsymbol{J}$ has to satisfy constraints of the form

$$\boldsymbol{dJ} = \boldsymbol{dd}\phi = 0\,,$$

i.e. $\partial_i\partial_j\phi = \partial_j\partial_i\phi$.

Recovering evolution equations:

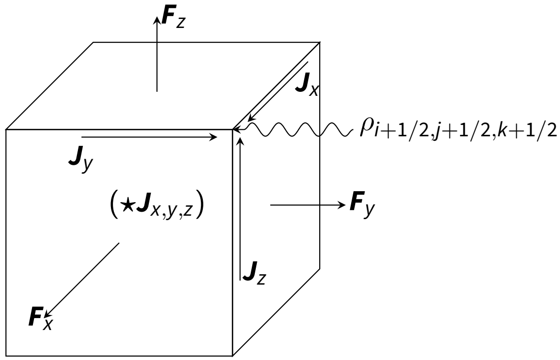
$$\partial_t\boldsymbol{J} = -\boldsymbol{d}\rho\,,$$

$$\partial_t\star\boldsymbol{J} = \boldsymbol{d}\left(\varepsilon_{ijk}\,j^i\,\boldsymbol{d}x^j\wedge\boldsymbol{d}x^k\right)\,.$$

# Introduction

$$\partial_t \boldsymbol{J} = -\boldsymbol{d}\rho\,, \qquad \partial_t \star \boldsymbol{J} = \boldsymbol{d}\left(\varepsilon_{ijk}\, j^i\, \boldsymbol{d}x^j \wedge \boldsymbol{d}x^k\right) := \boldsymbol{dF}\,.$$

We are evolving a 1-form (3-form): it is a **natural integrand** on curves (volumes).
Applying **Stokes' theorem** to the RHS, we evaluate a 0-form (2-form) on the **boundary** of the curve (volume).
Adjacent elements contribute with the **opposite sign**: conservation is built into the scheme!



# The dGREM formulation

# Derivation

Start from the Sparling equation (equivalent to the EFE):

$$\boldsymbol{du}_a = \boldsymbol{t}_a + k\boldsymbol{T}_a$$

$\rightarrow$ project along/perpendicularly to $n_\mu$ get evolution eqs. $\partial_t\sqrt{\gamma}D^k_a = \dots$

Conservation of matter+gravity energy-momentum is

$$\boldsymbol{ddu}_a = \boldsymbol{d}(\boldsymbol{t}_a + k\boldsymbol{T}_a) = 0$$

$\rightarrow$ get eqs. $\partial_t\sqrt{\gamma}\rho_a = \cdots$ and $\partial_t\sqrt{\gamma}P_a = \cdots$.

Need eqs. for other quantities $\rightarrow$ need to choose a connection. We choose the spin-connection.

$\rightarrow$ get eqs. $\partial_t A^{\hat{i}}_i = \cdots$

Because $\boldsymbol{ddA} = \boldsymbol{dF} = 0$ get further evolution eqs. $\partial_t\sqrt{\gamma}B^{\hat{a}i} = \cdots$

Finally: add an evolution eq. for $\sqrt{\gamma}$ (for convenience).

# Final system

First Cartan structure equations

$$\partial_t A^{\hat{i}}{}_i - \partial_i \beta^{\hat{i}} = -\alpha E^{\hat{i}}{}_i - \epsilon_{ilk} \beta^l B^{\hat{i}k}$$

First Bianchi identities

$$\partial_t \sqrt{\gamma} B^{\hat{\alpha}k} + \partial_i \sqrt{\gamma} (\alpha \epsilon^{ijk} E^{\hat{\alpha}}_j - \beta^i B^{\hat{\alpha}k} + \beta^k B^{\hat{\alpha}i}) = 0$$

Einstein evolution equations

$$\partial_t \sqrt{\gamma} D^{\;k}_{\hat{\alpha}} - \partial_i \sqrt{\gamma} (\alpha \epsilon^{kij} H_{\hat{\alpha}j} + \beta^i D^{\;k}_{\hat{\alpha}} - \beta^k D^{\;i}_{\hat{\alpha}}) = -\sqrt{\gamma} (j^k{}_a + \kappa J^k{}_a)$$

Conservation of gravitational energy-momentum

$$\partial_t \sqrt{\gamma} \rho_{\hat{\alpha}} + \partial_i \sqrt{\gamma} j^i{}_{\hat{\alpha}} = -\kappa \sqrt{\gamma} Q_{\hat{\alpha}}$$

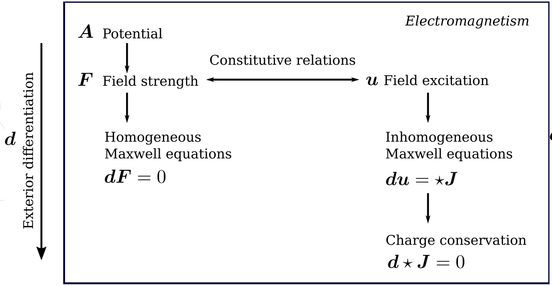
Conservation of ‘matter’ energy-momentum

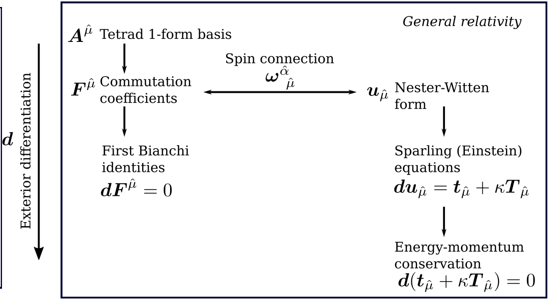
$$\partial_t \sqrt{\gamma} P_{\hat{\alpha}} + \partial_i \sqrt{\gamma} J^i{}_{\hat{\alpha}} = \sqrt{\gamma} Q_{\hat{\alpha}}$$

Auxiliary evolution equation for $\sqrt{\gamma}$

$$\partial_t \sqrt{\gamma} - \partial_i \sqrt{\gamma} \beta^i = \frac{5}{2} \alpha \sqrt{\gamma} D^{\;\hat{k}}_{\hat{k}}$$

# Correspondence with EM



# A possible CT scheme

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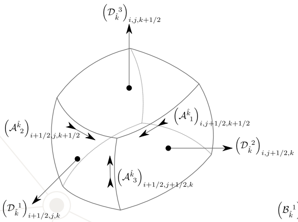
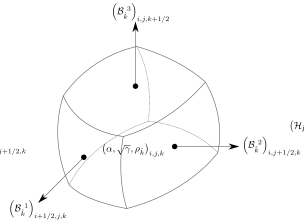
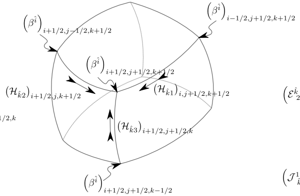
### Evolved



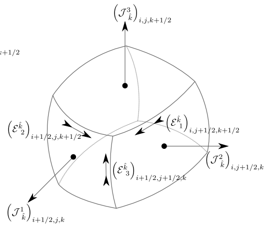
### Auxiliary



### Boundary



### Sources



$$\partial_t \mathcal{A}^{\hat{i}} - \boldsymbol{d}\beta^{\hat{i}} = -\mathcal{E}^{\hat{i}} \,, \qquad \partial_t \mathcal{D}_{\hat{\alpha}} - \boldsymbol{d}\mathcal{H}_{\hat{\alpha}} = -\mathcal{J}_{\hat{\alpha}} \,.$$

# Conclusions

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# Further developments

What's missing?

- Proof of hyperbolicity
  (**We got it!** Upcoming paper by Ilya Peshkov)
- Proof of linear degeneracy (i.e. no shock waves)
- Conversion to and from `ADM` variables ($\alpha$, $\beta^i$, $\gamma_{ij}$ and $K_{ij}$)
- ☛ Implementation in a code and numerical tests

# The TEONGRAV specific initiative at INFN

# The TEONGRAV initiative

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TEONGRAV

Theory of Gravitational Wave Sources

- Firenze (Luca Del Zanna)
- Milano-Bicocca (Bruno Giacomazzo)
- Napoli (Mariafelicia De Laurentis)
- Padova (Michela Mapelli)
- Roma1 (Leonardo Gualtieri)
- TIFPA (Albino Perego)
- Trieste (Enrico Barausse)

# The TEONGRAV initiative

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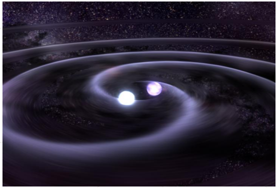
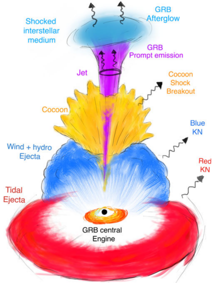
- GW sources modelling via semi-analytical and numerical methods;
- Extract information (with theory and observations) on the EOS of NS;
- Study of the dynamics of BH formation;



*Credit: NASA*




*[Ascenzi et al., 2021]*

- Study of electromagnetic counterparts of GW signals;
- Study of modified gravity theories.

# The TEONGRAV initiative

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Numerical/computational techniques in HPC are a big part of our work, in particular large scale DNS of BBH/BNS/BHNS
(agreement between INFN and CINECA, have access to CINECA HPC resources)

We work on improved realism...

- general relativistic gravitation
- magnetohydrodynamics *e.g. [Cipolletta et al., 2021]*
- Microphysics and EOS *e.g. [Loffredo et al., 2022]*
- neutrino transport *e.g. [Radice et al., 2022]*
- etc.

... and increasing performance/efficiency:

- "better" algorithms *e.g. This work*
- more efficient parallelism
- leveraging cutting edge technology (e.g. GPUs!)

Thank you for your attention!

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