Thermalization with a multibath: an investigation in simple models

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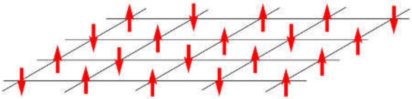
Spins on a lattice, out-of-equilibrium


2–D Ising Model

- Ising Hamiltonian

$$\mathcal{H} = -\sum_{ij} J_{ij} S_i S_j \ , \quad S_i = \pm 1;$$

S_i in contact with a thermal bath

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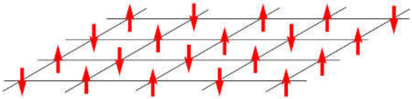

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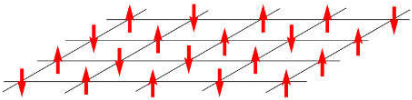

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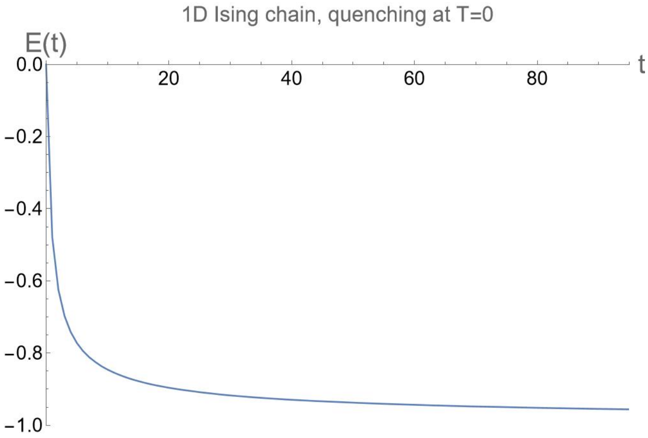
- for J_{ij} without fixed signs $\rightarrow$ long relaxation time

Aging



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- Idea introduced in the context of Langevin dynamics (Cugliandolo, Kurchan & Peliti, 1996, Cugliandolo & Kurchan, 1998, 1999)
- Naturally out of equilibrium, but rather simple models
- Lack of literature for lattice models (Piscitelli et al., 2008, 2009, Borchers et al., 2012, Contucci et al., 2020)

Nested partition function

Contucci et al. 2019, 2021

- Hamiltonian $\mathcal{H}(\mathbf{x}_1, \dots, \mathbf{x}_r)$, each group of variables in contact with a different thermal bath, temperatures $T_a = \beta_a^{-1}$, relaxation times $\tau_r \ll \tau_{r-1} \ll \dots \ll \tau_1$, $\zeta_a = \frac{\beta_a}{\beta_r}$

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- Nested partition function:

$$Z = \left\{ \int d\mathbf{x}_1 \left[\int d\mathbf{x}_2 \ldots \left[\int d\mathbf{x}_r e^{-\beta_r H(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \ldots, \mathbf{x}_r)} \right]^{\zeta_{r-1}/\zeta_r} \ldots \right]^{\zeta_1/\zeta_2} \ldots \right\}^{1/\zeta_1}$$

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- Bibath: $\vec{\sigma}$ in contact at T_1, $\vec{S}$ in contact at T_2, $\tau_2 \ll \tau_1$, $\zeta_1 = \frac{\beta_1}{\beta_2}$, $\zeta_2 = \frac{\beta_2}{\beta_2} = 1$

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- if $T_1 \to \infty$: $\zeta_1 \to 0$, $F = -\lim_{\zeta_1 \to 0} \frac{1}{\beta_2 \zeta_1} \log \left[\mathbb{E}_\sigma \left[\mathrm{tr}_S e^{-\beta_2 H} \right]^{\zeta_1} \right]$ (quenched)

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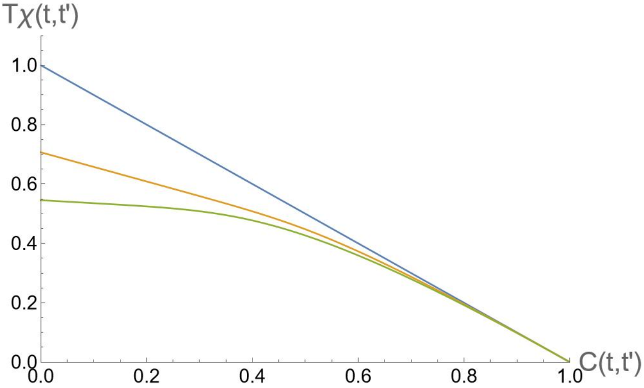
- Fluctuation-Dissipation Theorem: at equilibrium $T\chi(t,t') = 1 - C(t,t')$

Effective temperature

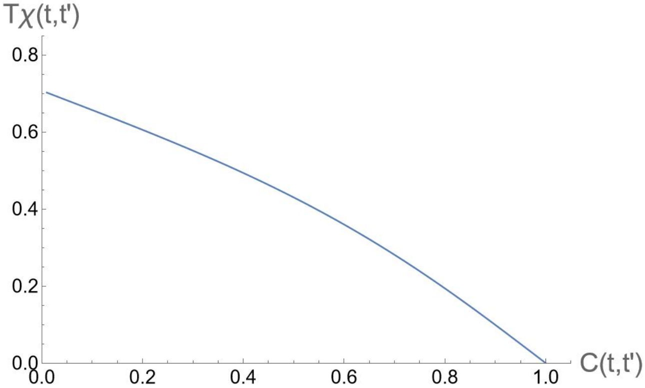


Effective temperature (Cugliandolo, Kurchan & Parisi, 1994):

$$T_{\text{eff}} = -\left(\frac{d(T\chi)}{dC}\right)^{-1}$$

Problematic?

- 1D Ising chain, quenching at $T = 0$ (Lippiello & Zannetti, 2000): $T\chi(t,t') = \frac{\sqrt{2}}{\pi}\text{arctan}\left(\sqrt{2}\cot\left(\frac{\pi}{2}C(t,t')\right)\right)$



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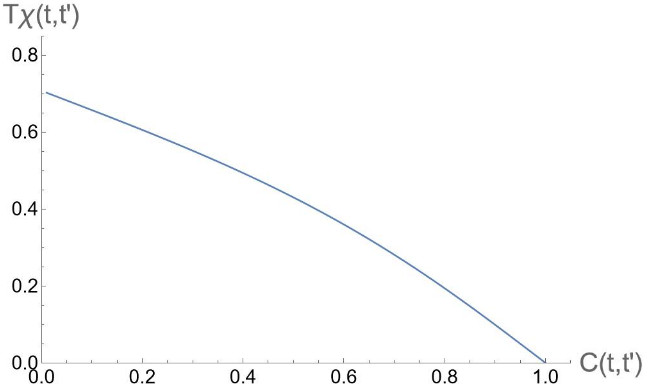

- This plot depends on the observables in the correlator (Sollich, Mayer & Fielding, 2002)

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  - σ_i evolve slower, with relative timescale $\tau > 1$.

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Basic models (with PBC):

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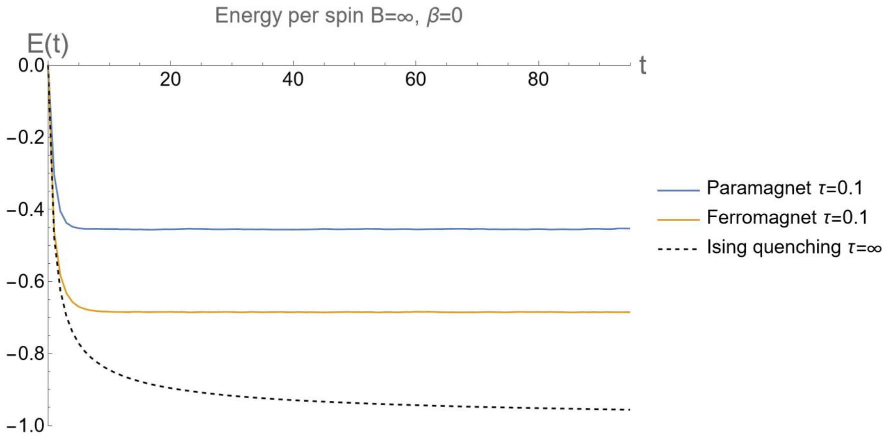
3. $\mathcal{H} = -\sum_i (S_i \sigma_i + \sigma_i S_{i+1})$ (alternated).

First result: stationarization

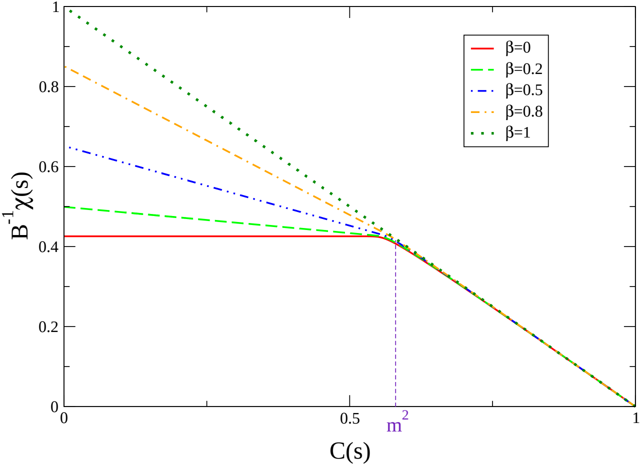


Paramagnet:

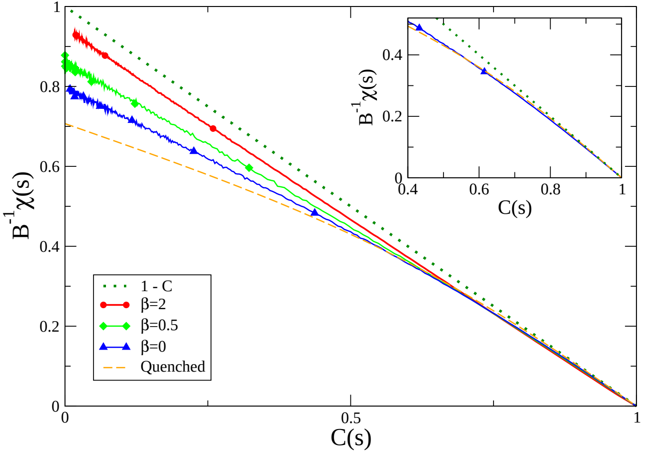
$$E(t) = -\frac{m + \frac{\mu}{\tau}}{1 + \frac{1}{\tau}} \left(1 - e^{-t\left(1+\frac{1}{\tau}\right)}\right) \;, \quad m = \tanh B \;, \quad \mu = \tanh \beta$$

Second result: effective temperatures

Paramagnet (analytical curves, $\tau = 100$, $B = 1$):



Ferromagnet (numerical simulation, $\tau = 20$, $B = \infty$):



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    2.2 effective temperatures:
        2.2.1 paramagnet: standard picture;
        2.2.2 ferromagnet: complicated picture.