A condensation transition

at equilibrium and out-of-equilibrium

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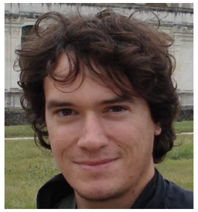


Gabriele Gotti Stefano Iubini

What is a condensation/localization?

It is a process/transition where a single site of the lattice accumulates a finite fraction of mass/energy/number of particles

Here: homogeneous localization in the real space

GRANULAR MATERIALS

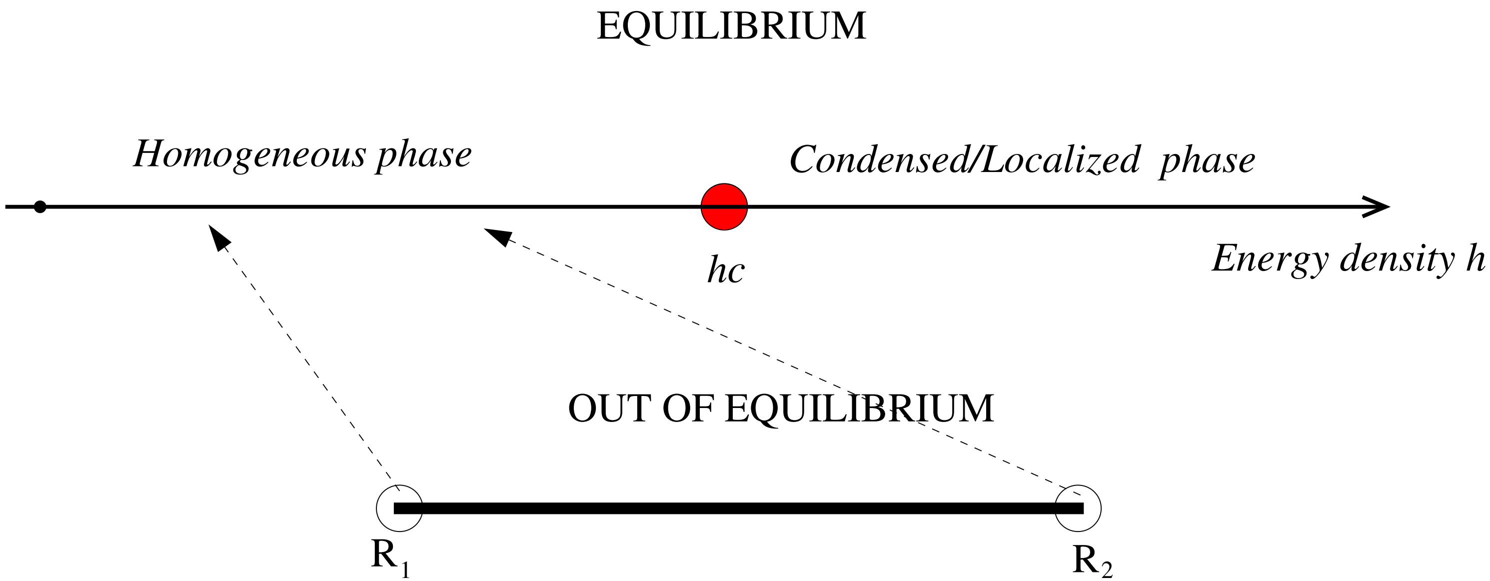
BUS–ROUTE MODELS

CONDENSATION PROCESSES

Discrete NonLinear Schroedinger (DNLS) equation

$$H = \sum_j (z_j^* z_{j+1} + z_j z_{j+1}^*) + \sum_j |z_j|^4 \qquad i\dot{z}_n = -\frac{\partial H}{\partial z_n^*} \qquad A = \sum_j |z_j|^2$$

What we are talking about:
a lattice model with two conserved quantities

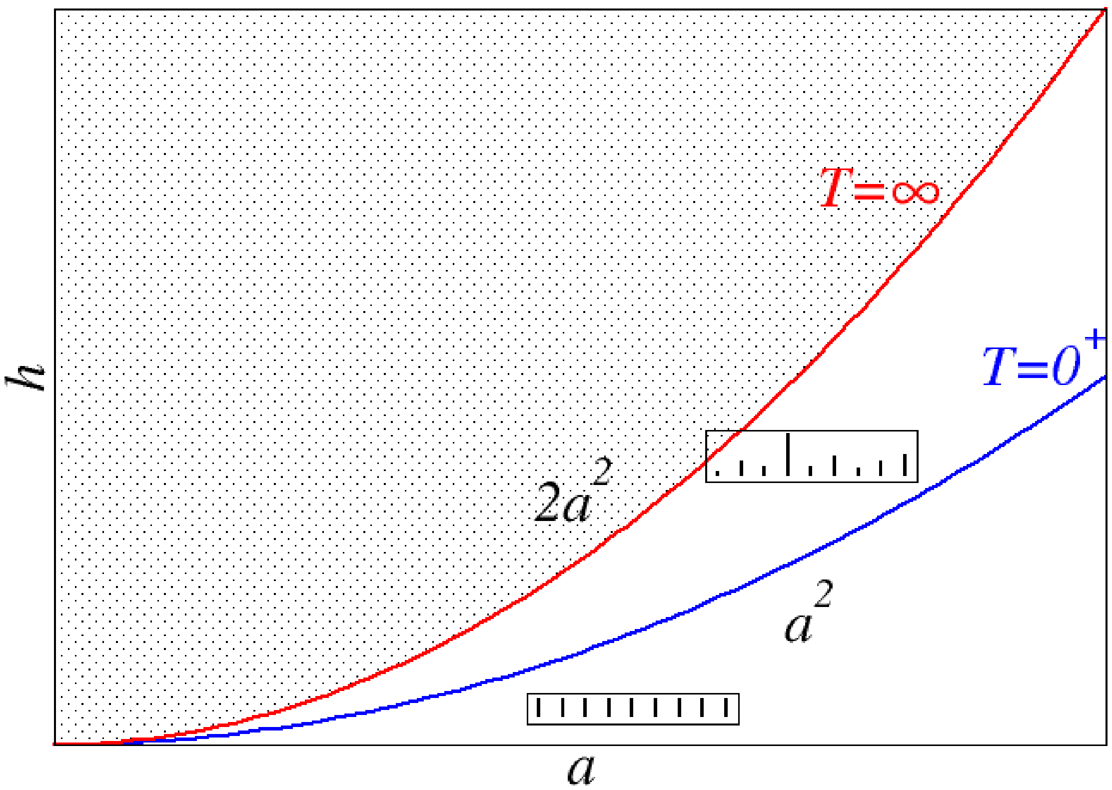


DNLS model (z_i) $\Longrightarrow$ C2C model ($c_i = |z_i|^2$)

$$A = \sum_i c_i \equiv Na$$

$$H = \sum c_i^2 = \sum \epsilon_i \equiv Nh$$

condensation $\leftrightarrow h > h_c = 2a^2$

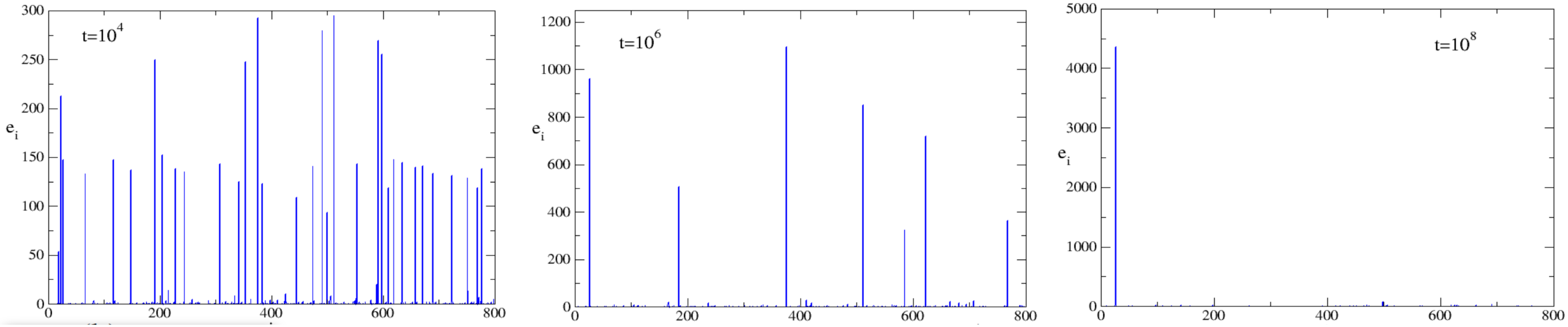


Region above $h = 2a^2$: Gradenigo, Iubini, Livi, Majumdar (2021)

Dynamics of a simple microcanonical MonteCarlo

$$a = 1, \quad h = 7.3, \quad N = 800$$

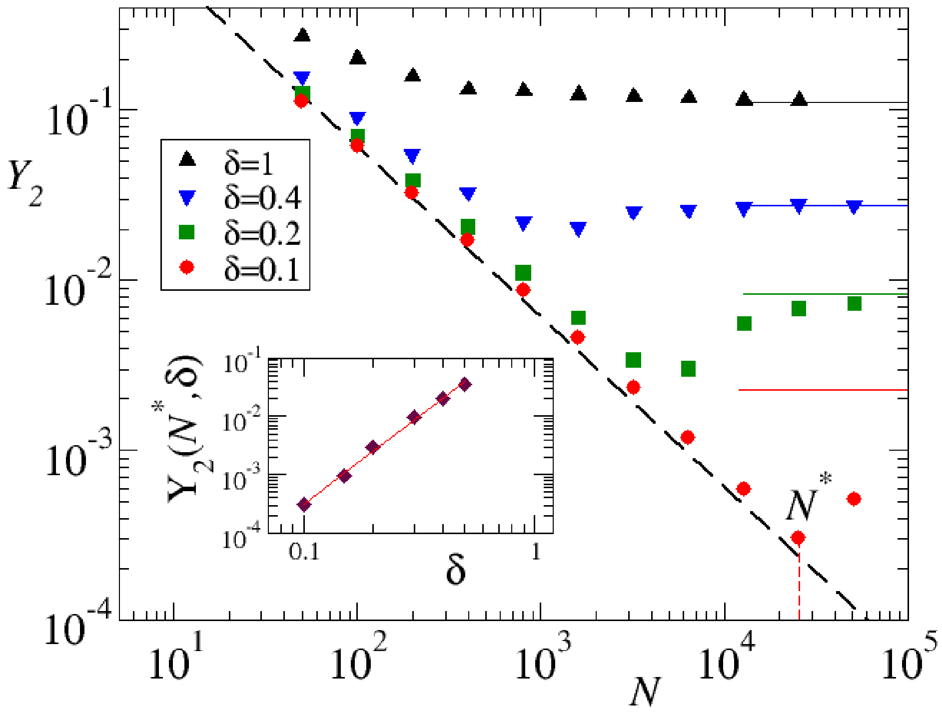
We choose a random triplet and we choose uniformly a new triplet having the same mass and energy



When we approach h_c things are more complicated

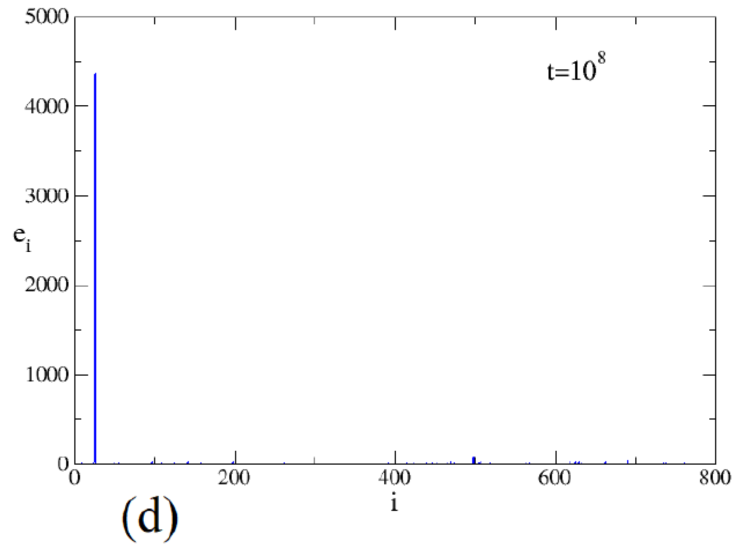
Order parameter: $Y_2 = \frac{\langle \epsilon^2 \rangle}{N \langle \epsilon \rangle^2} \equiv \frac{\langle \epsilon^2 \rangle}{N h^2}$

Control parameter: $\delta \equiv h - h_c$



$$Y_2^{min} \simeq \delta^{\gamma}, \qquad \gamma = 3$$

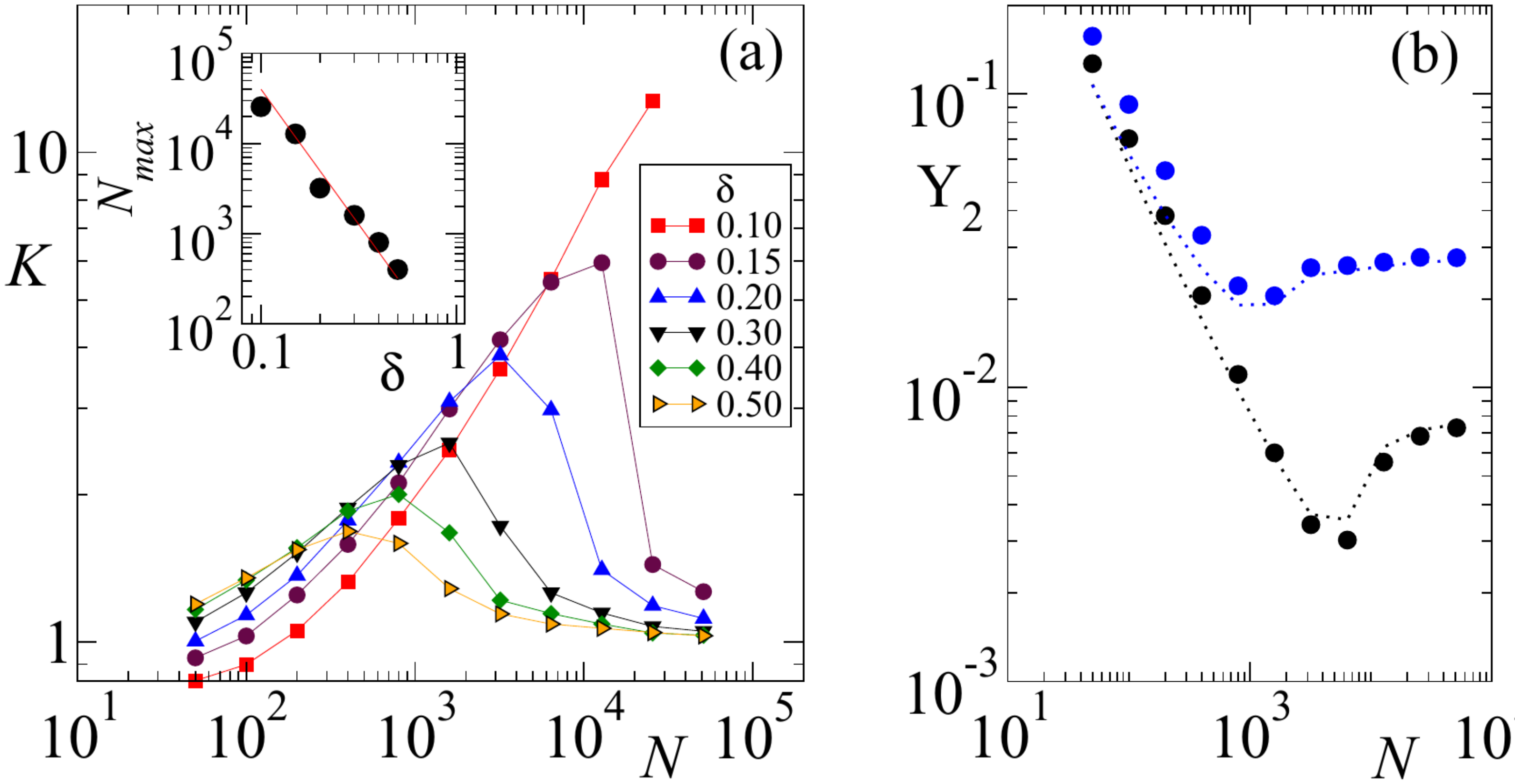
The minimum was surprising



$$Y_2(N, \delta) = \frac{c_0}{N} + \frac{\delta^2}{h^2}$$

$$K(N,\delta) \equiv \text{effective number of peaks}$$

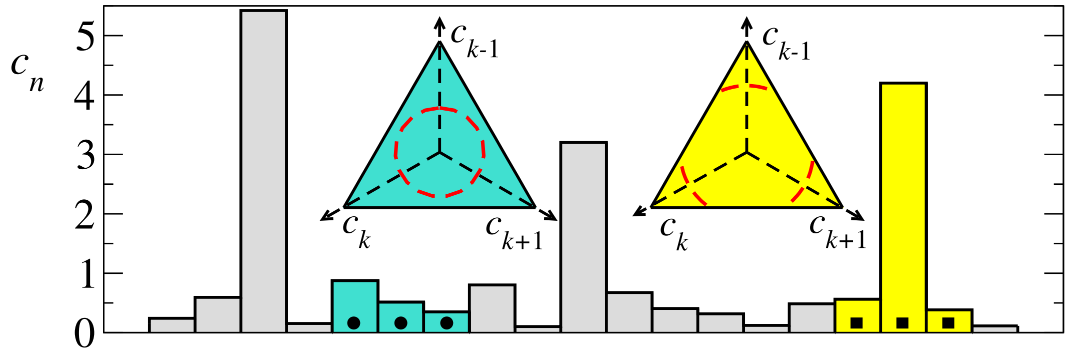
$$Y_2(N,\delta) = \frac{\mu_2}{Nh^2} + \frac{\delta^2}{K(N,\delta)h^2}$$



Equilibrium $\Longrightarrow$ Nonequilibrium

C2C

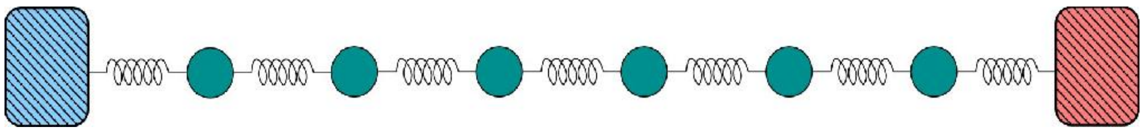
$$A = \sum_i c_i \quad H = \sum_i c_i^2$$



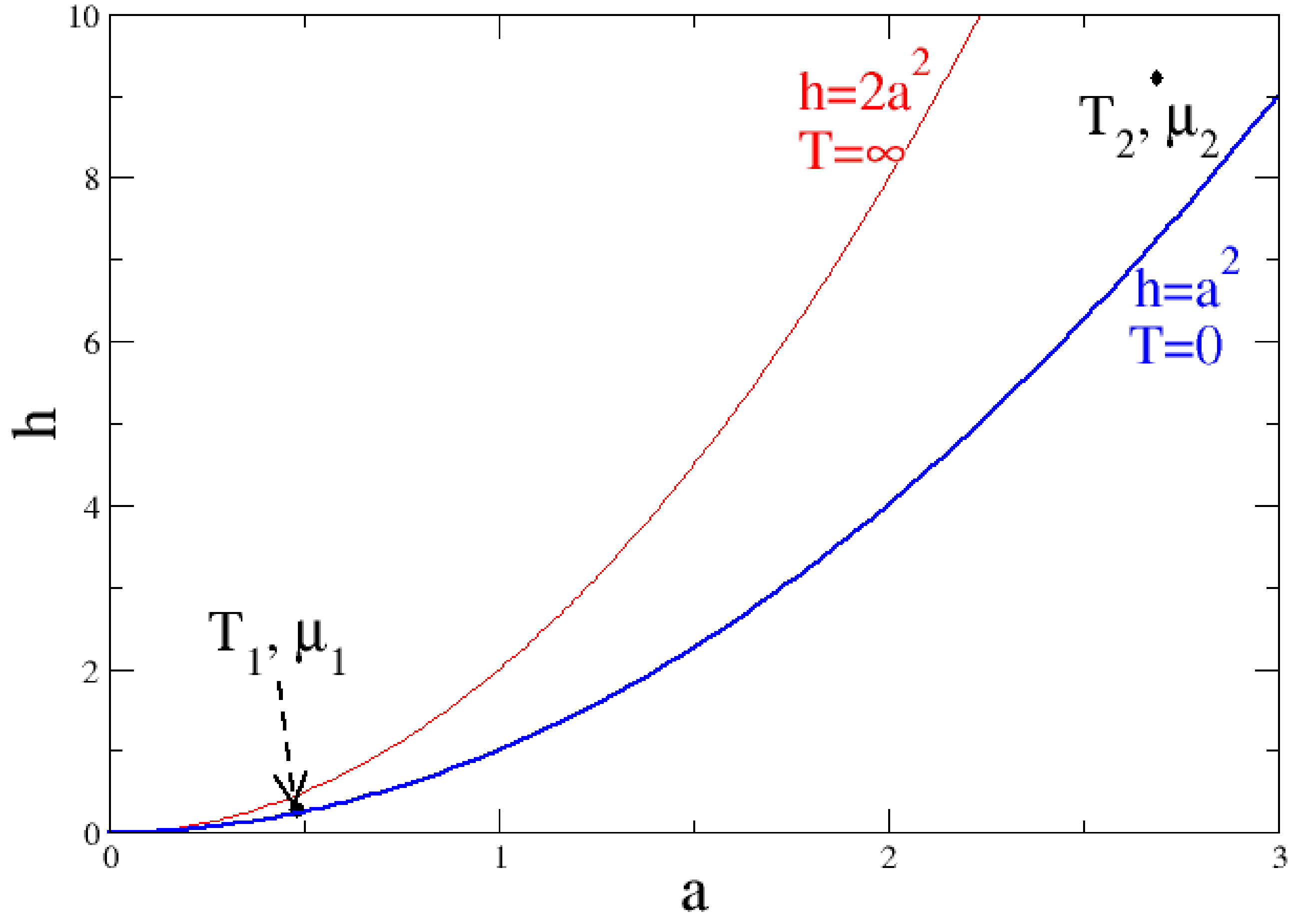
- No pinning (choice in any of the three arcs)

- Pinning (choice in the same arc)

Nonequilibrium set-up C2C model



(T_1, μ_1) (T_2, μ_2)



MonteCarlo
heat baths

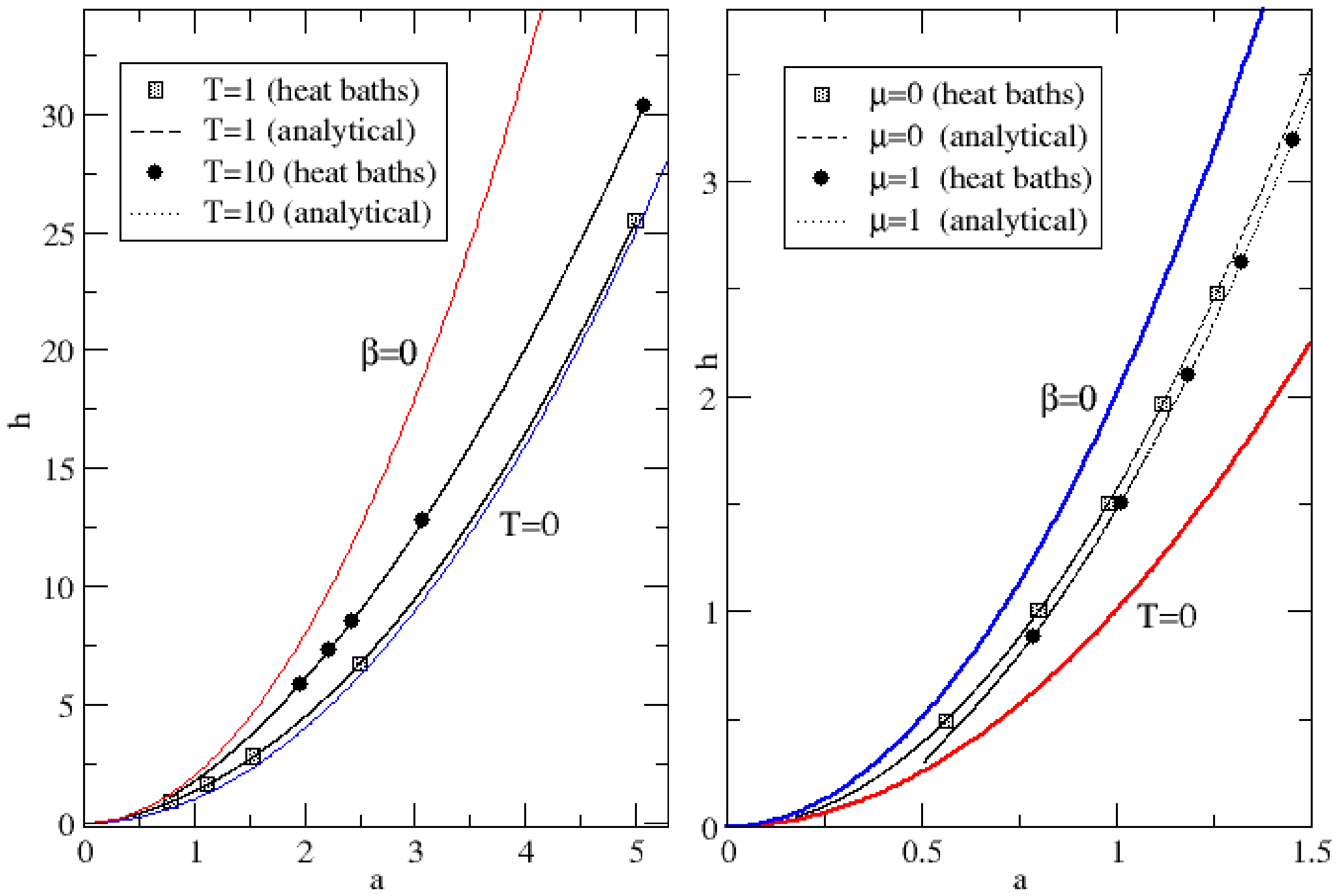
$$c_1 \rightarrow c_1 + \Delta A$$
$$c_1^2 \rightarrow (c_1 + \Delta A)^2 \equiv c_1^2 + \Delta E$$

$$W = e^{-\beta(\Delta E - \mu \Delta A)}$$

Heat baths work well

$$Z(\beta,\mu) = \prod_{n=1} N \int_0^\infty dc_n e^{-\beta(c_n^2 - \mu c_n)}$$

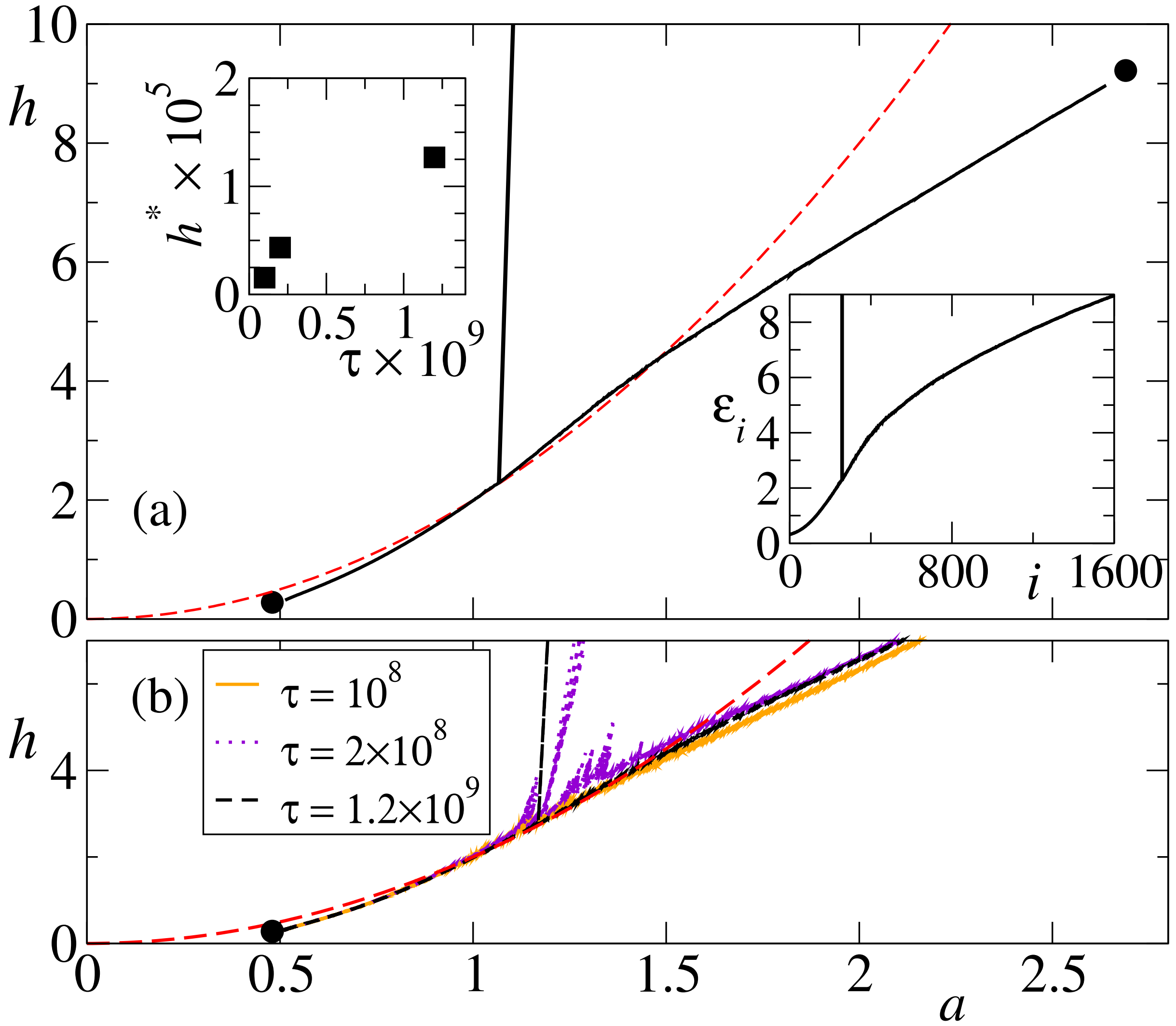
$$a = \frac{\mu}{2} + \frac{1}{\sqrt{\pi\beta}} \frac{e^{-\beta\mu^2/4}}{1 + \mathrm{erf}\left(\frac{\sqrt{\beta}\mu}{2}\right)} \qquad h = \frac{1}{2\beta} + \frac{1}{2} a\mu$$



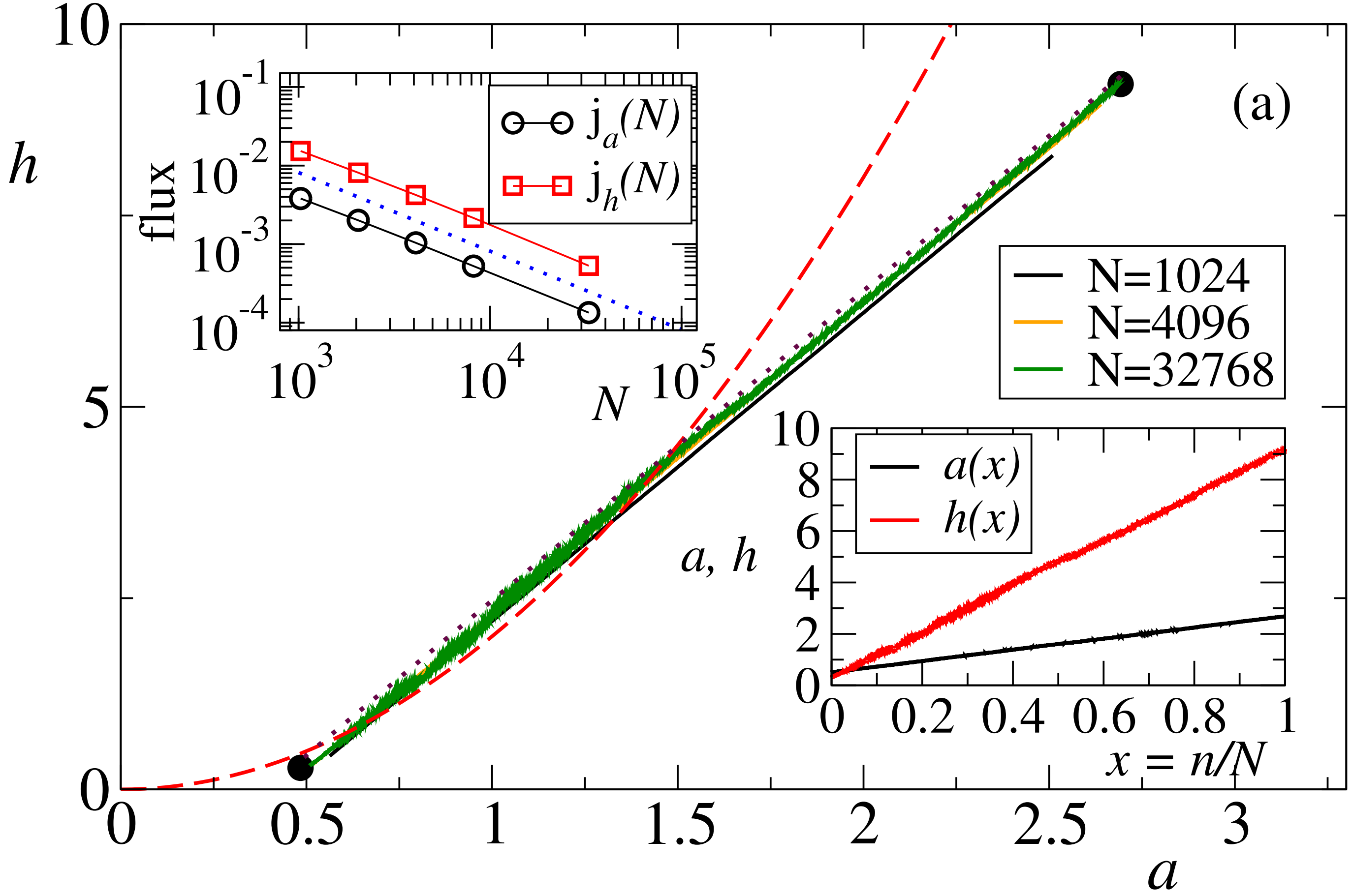
Out-of-equilibrium setup (C2C pinned)

For each site i we evaluate the time averages $a_i = \overline{c_i}, \quad h_i = \overline{c_i^2}$,

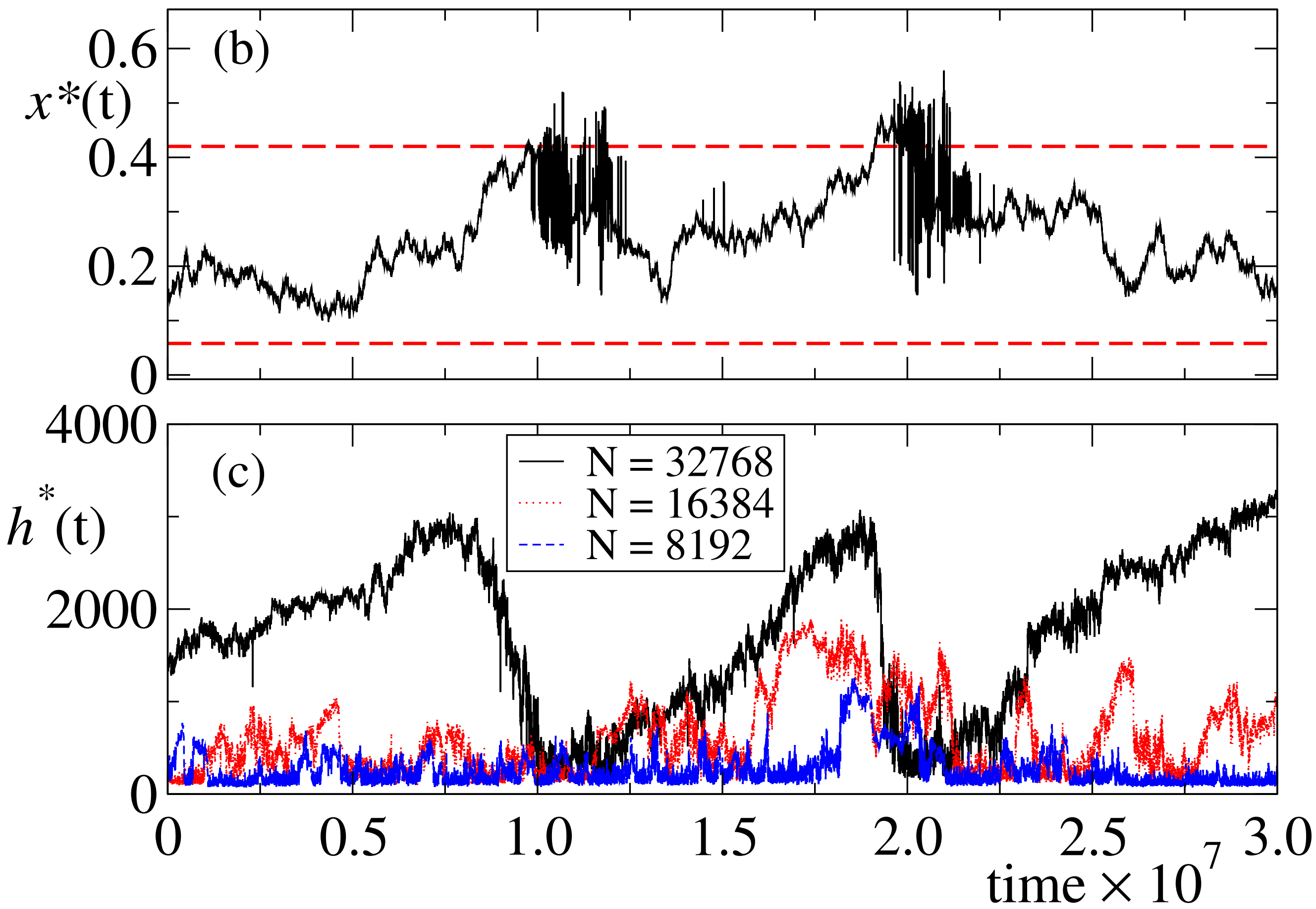
then we plot $h_i(a_i)$



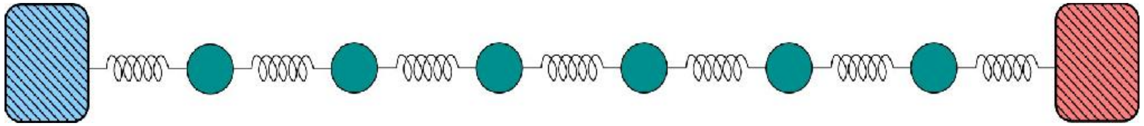
C2C unpinned: condensed steady state...



... because breathers diffuse



Why does nonequilibrium setup favour the condensation process?



$$(c_1, b, c_2) \longrightarrow (c_1 + x, b + \delta b, c_2 + y)$$

If $c_i = \bar{c}_i + \delta_i$ and $b \gg c_{1,2}$, if we perform an average over $\delta_{1,2}$,

$$\delta b \simeq -Q + \frac{1}{2b}\Delta(y - x), \quad Q > 0, \quad \Delta \equiv \bar{c}_1 - \bar{c}_2$$

Open questions

- We need to understand better the process of creation, diffusion and death of breathers in the unpinned case

- What does it happen to the Onsager coefficients when we cross the critical curve?

- What about the same setup for the DNLS equation?

Equilibrium

Finite-size localization scenarios in condensation transitions
Phys. Rev. E 103, 052133 (2021)

Nonequilibrium

Condensation induced by coupled transport processes
Phys. Rev. E 106, 054158 (2021)