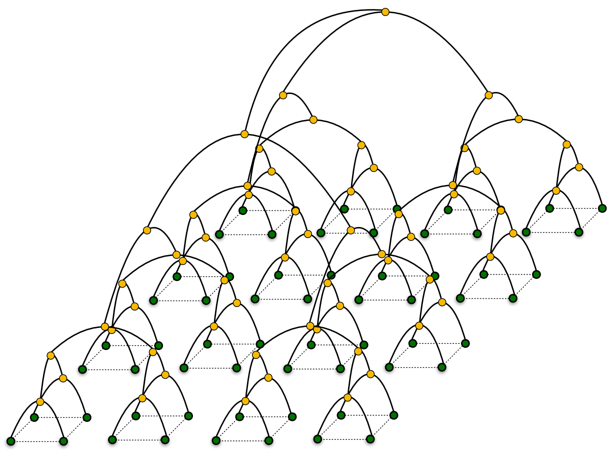
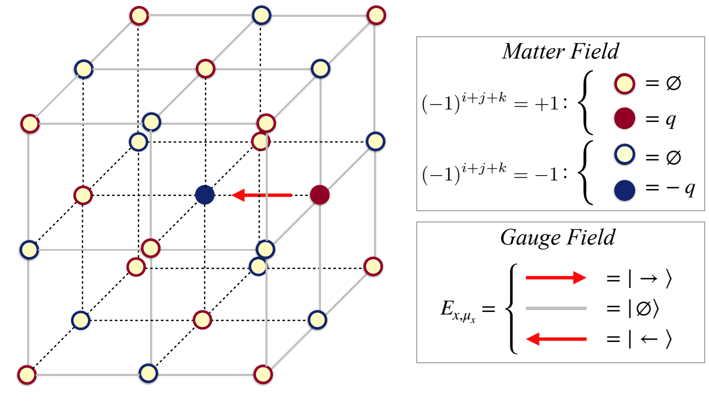
Lattice Gauge Theories at finite density with Tensor Networks





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Padua Quantum Technologies Research Center

quantum.dfa.unipd.it

qtech.unipd.it

SM&FT 2022 Frontiers in Computational Physics

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Dipartimento di Fisica e Astronomia Galileo Galilei

INFN

Nature Communications **12**, 3600 (2021)

TIMO FELSER
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PASQuanS

QuantHEP

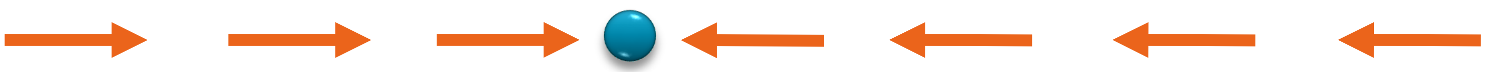
Lattice Gauge Theories (LGT)

1) Quantum Matter and Quantum Fields

2) Local symmetries, e.g. Gauss's law in QED

$$\nabla \cdot \mathbf{E} = \rho$$

$-e$



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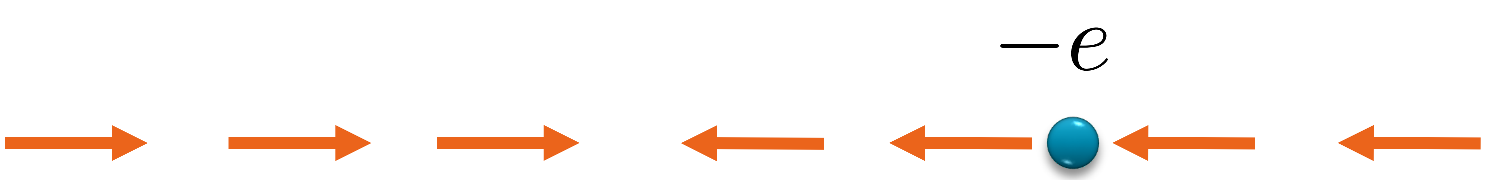

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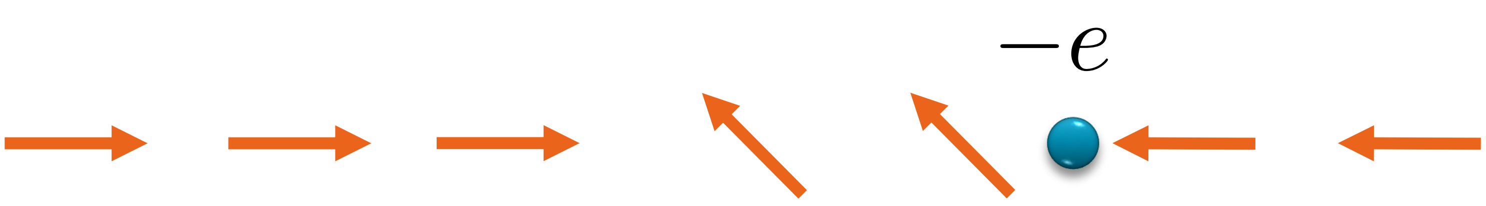

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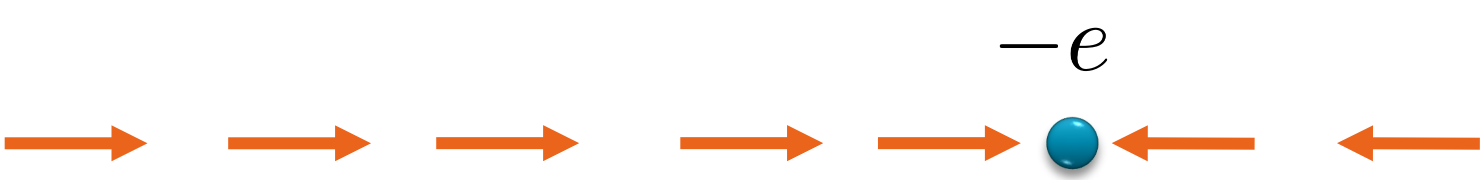
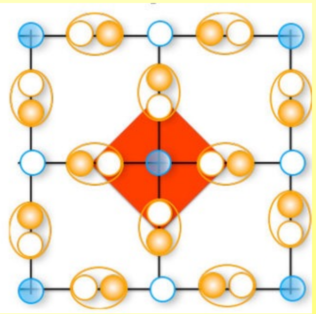
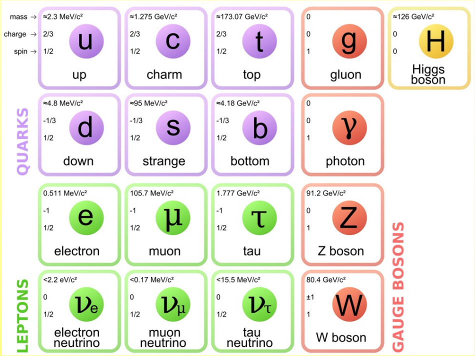


LGT are almost everywhere in physics!

As emergent theories in condensed matter: **high-Tc superconductors, frustrated systems, spin liquids.**



As fundamental description in particle physics: **Standard Model**



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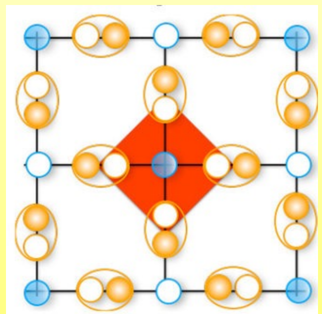

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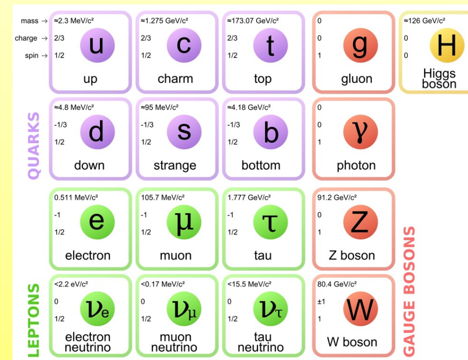
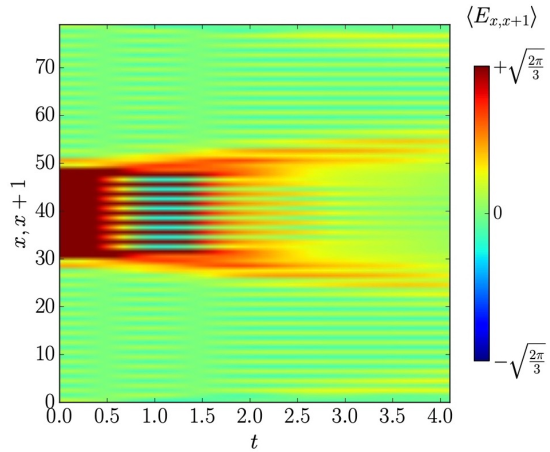


- They are **<u>extremely demanding</u>** from a numerical point of view.
- Powerful numerical methods, such as Monte Carlo, fail in several regimes of finite-density or for non-equilibrium phenomena (sign-problem).
- Ideal goal for quantum-inspired efficient algorithms and quantum simulation/computation!

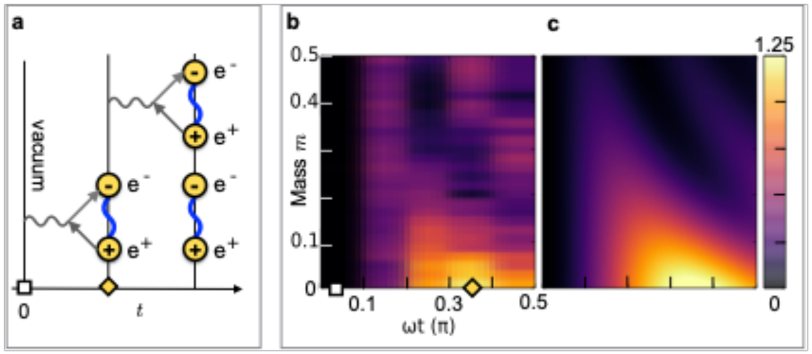
Quantum Technologies for LGT

Efficient quantum-inspired algorithms: Tensor Networks (no sign-problem)



Quantum **4**, 281 (2020)

First implementation of U(1) LGT on digital quantum computer



Nature **534,** 516–519 (2016).

Simulating Lattice Gauge Theories within Quantum Technologies

M.C. Bañuls[1,2], R. Blatt[3,4], J. Catani[5,6,7], A. Celi[3,8], J.I. Cirac[1,2], M. Dalmonte[9,10], L. Fallani[5,6,7], K. Jansen[11], M. Lewenstein[8,12,13], S. Montangero[7,14 a], C.A. Muschik[3], B. Reznik[15], E. Rico[16,17 b], L. Tagliacozzo[18], K. Van Acoleyen[19], F. Verstraete[19,20], U.-J. Wiese[21], M. Wingate[22], J. Zakrzewski[23,24], and P. Zoller[3]

Eur. Phys. J. D **74,** 165 (2020)

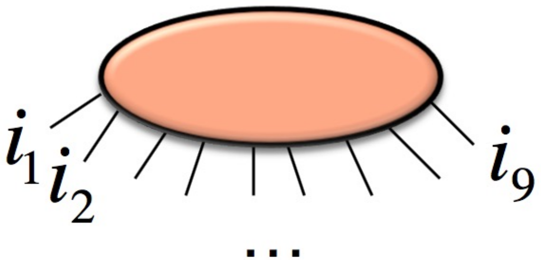
Tensor Networks

$$|\Psi\rangle = \sum_{i_1 i_2 \ldots i_N} C_{i_1 i_2 \ldots i_N} |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

d-level systems

Tensor (multidimensional array of complex numbers)

$C_{i_1 i_2 i_3 i_4 i_5 i_6 i_7 i_8 i_9}$



$O(d^N)$ representation, exponentially large in the system size. Inefficient.

Tensor Networks

$$|\Psi\rangle = \sum_{i_1 i_2 \ldots i_N} C_{i_1 i_2 \ldots i_N} |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

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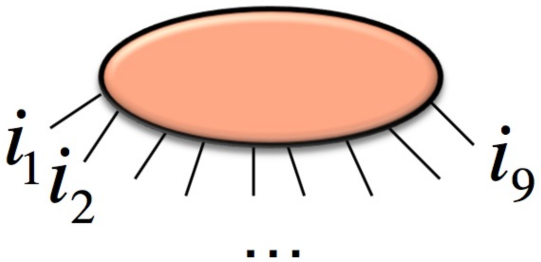
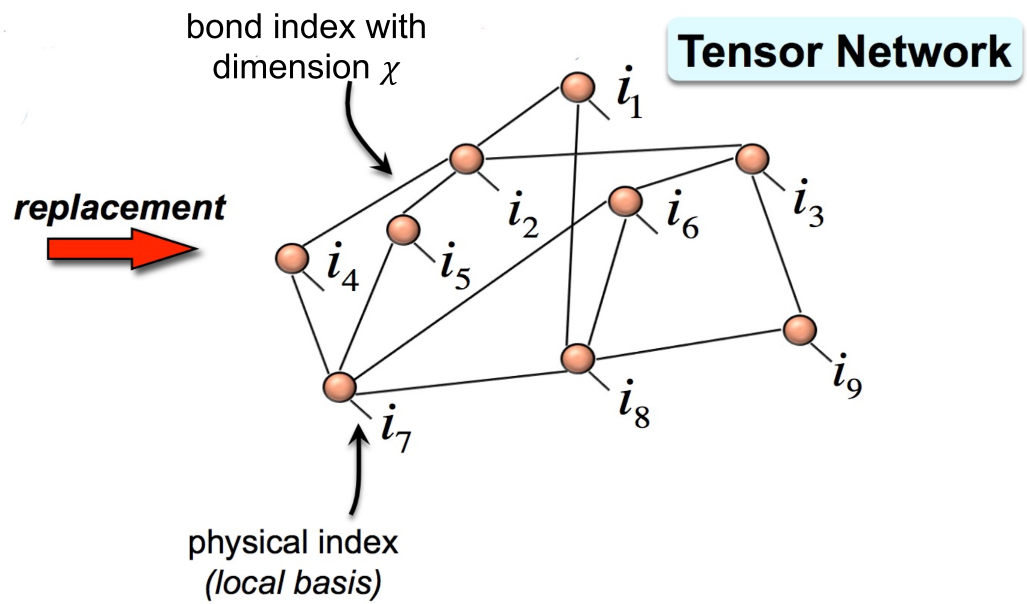


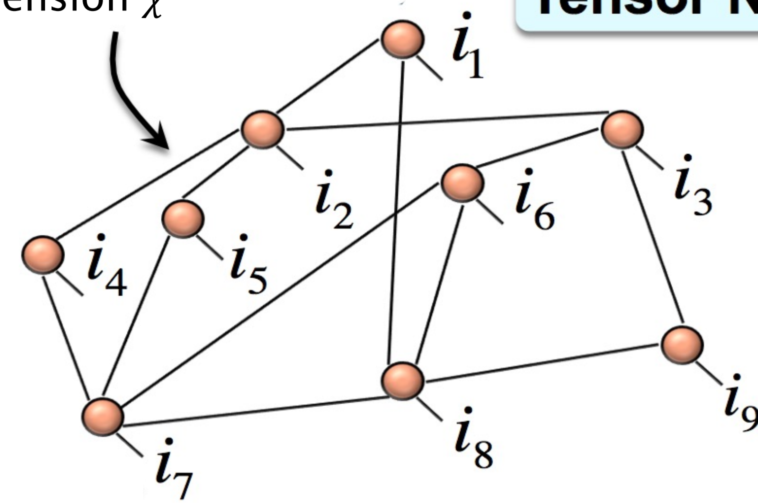


$O(d^N)$ representation, exponentially large in the system size. Inefficient.

$O(Nd\chi^c)$ representation, polynomial in the system size. Efficient.

Tensor Network

bond index with dimension χ



Tunable between mean field and exact

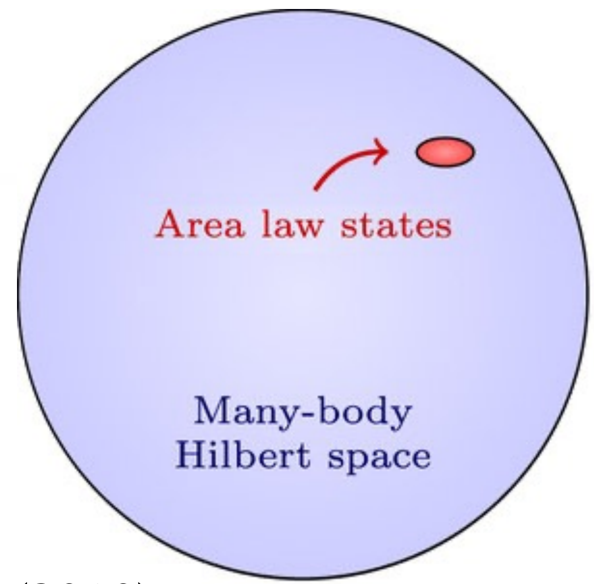
1  ∞

product state

χ

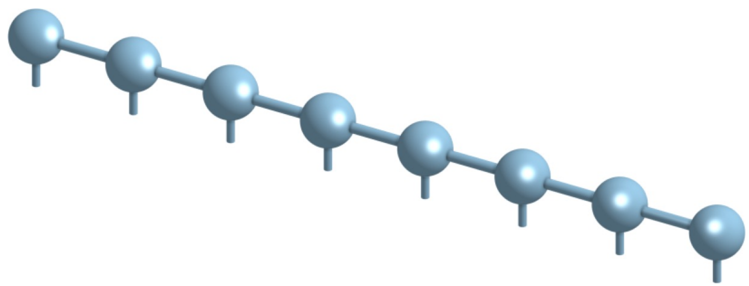
strongly entangled state

Low-Energy States



Rev. Mod. Phys. 82, 277 (2010)

Examples



Matrix Product States (MPS)

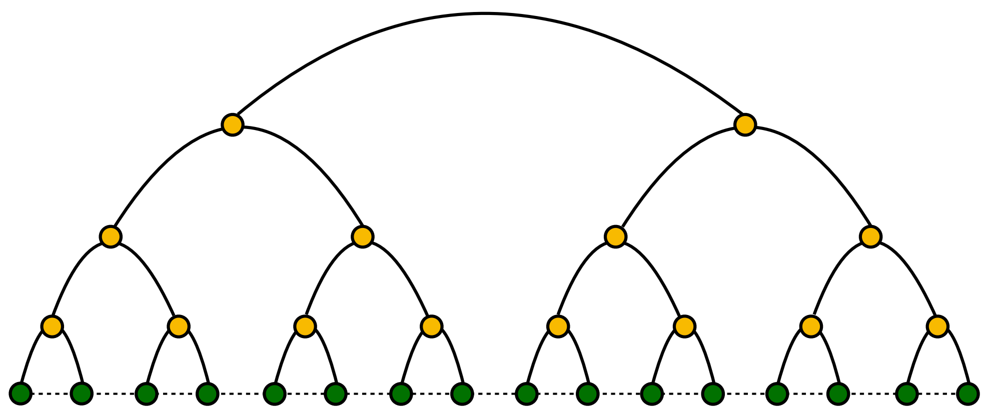
minimize $E = \langle\psi|H|\psi\rangle$

$$O(\chi^3)$$

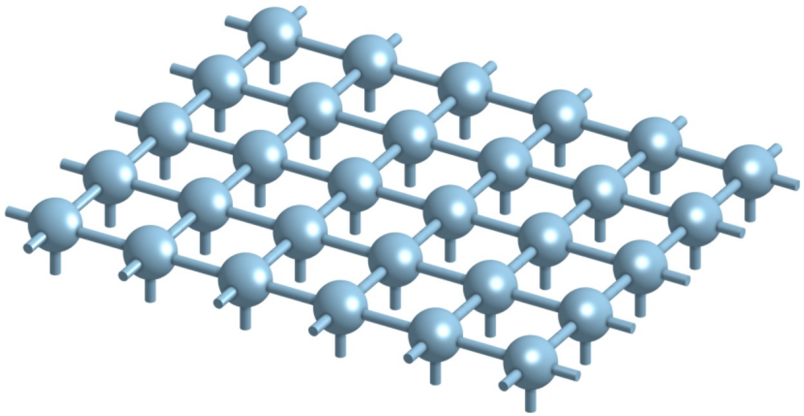
Tree Tensor Networks (TTN)

$$O(\chi^4)$$

- strong connectivity
- distance between two lattice sites scales logarithmically within the network



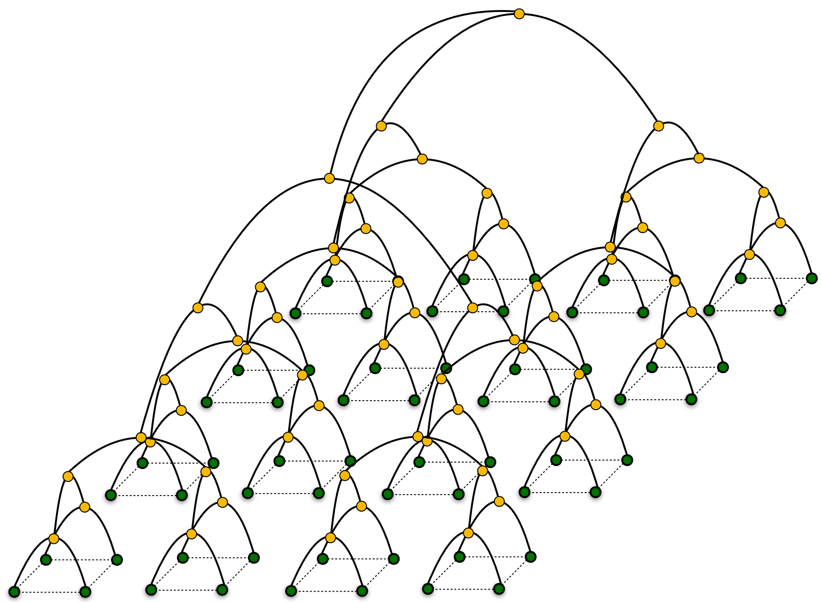
Projected Entangled Pair States (PEPS)



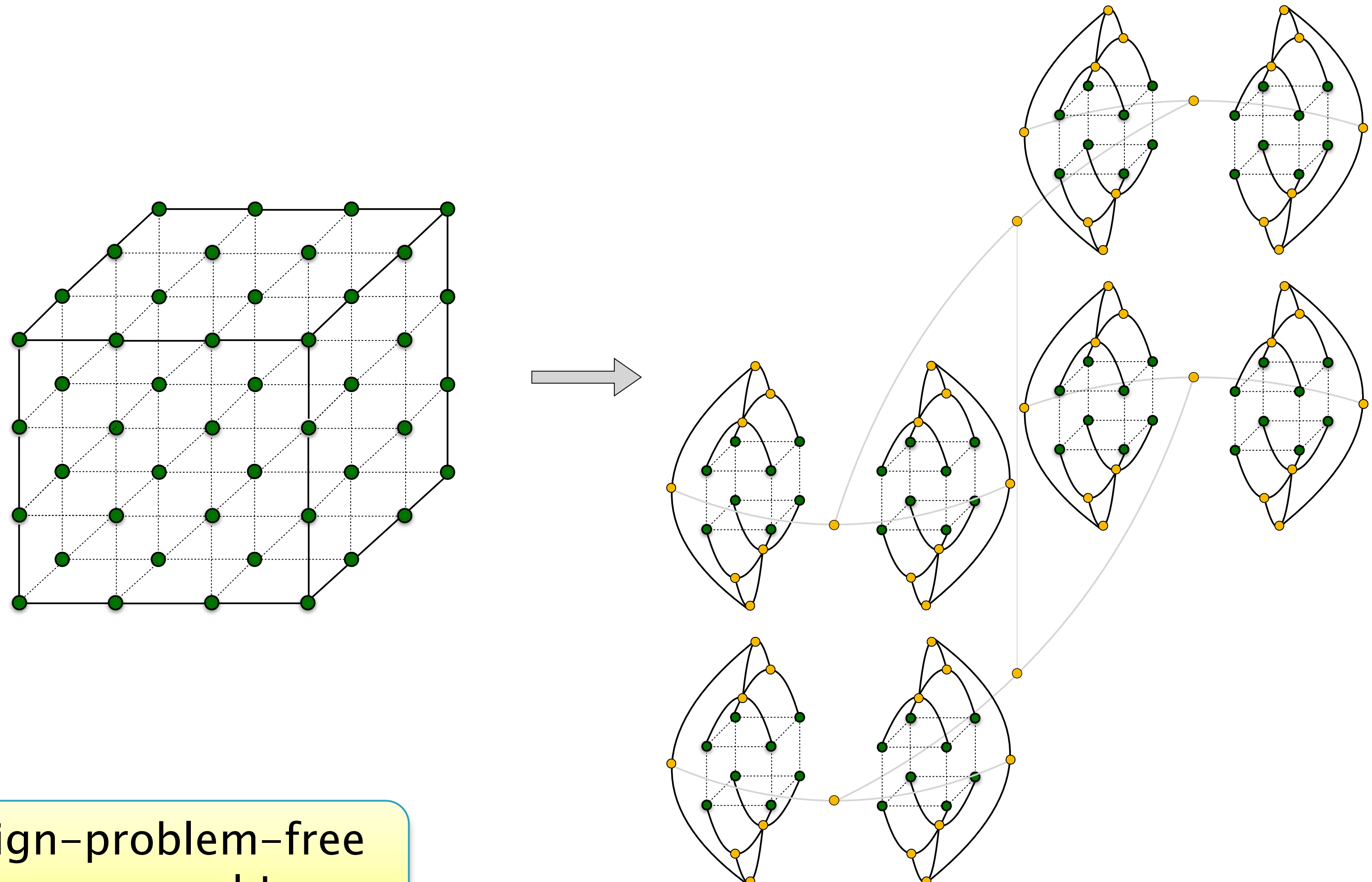
- they automatically reproduce the area-law of entanglement
- the optimization has a complexity $O(\chi^{10})$

Tree Tensor Networks (TTN)

- they do not automatically reproduce the area-law of entanglement
- the optimization has a complexity $O(\chi^4)$



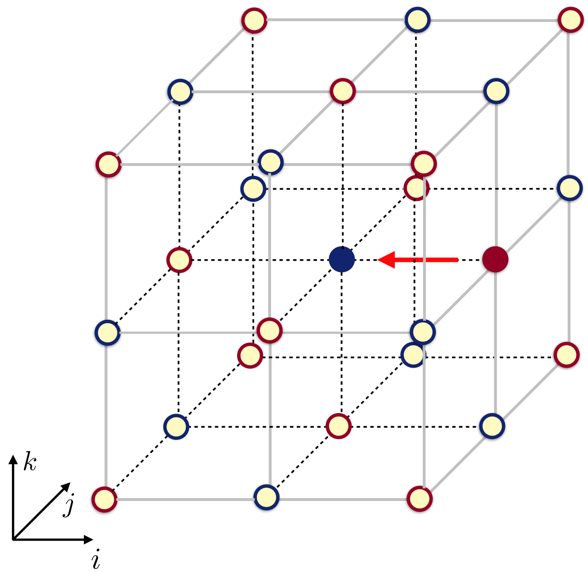
Tree Tensor Networks in 3D

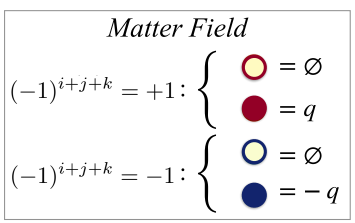


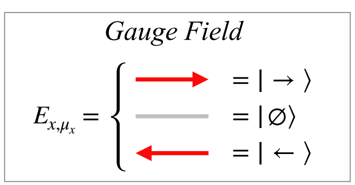
Sign-problem-free approach!

The optimization still has a complexity $O(\chi^4)$

Lattice QED in (3+1)D

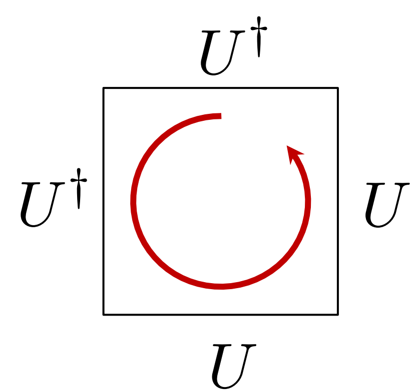




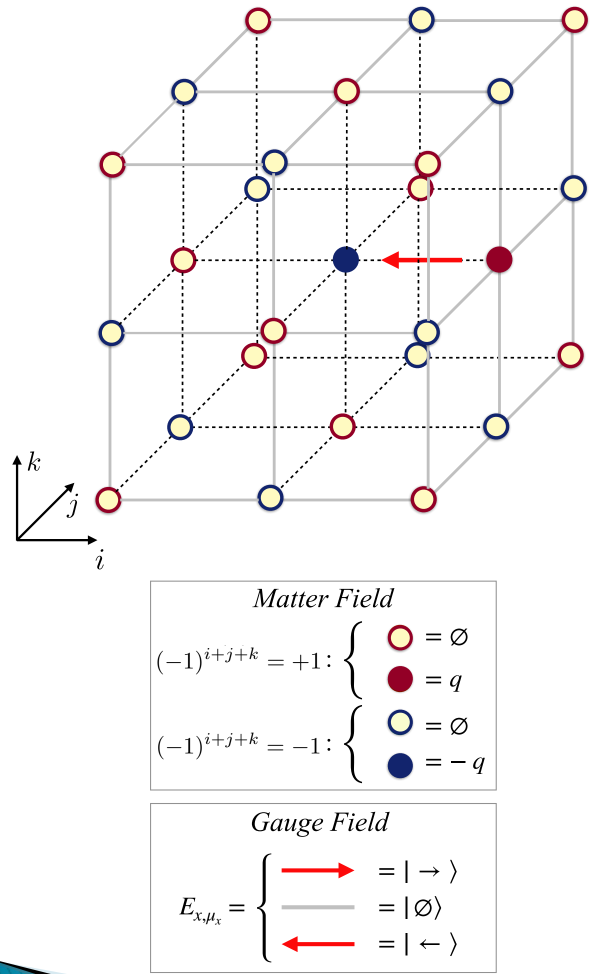


$$\hat{H} = -t \sum_{x,\mu} \left(\hat{\psi}_x^\dagger \, \hat{U}_{x,\mu} \, \hat{\psi}_{x+\mu} + \text{H.c.} \right) + m \sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g_e^2}{2} \sum_{x,\mu} \hat{E}_{x,\mu}^2 - \frac{g_m^2}{2} \sum_x \left(\Box_{\mu_x,\mu_y} + \Box_{\mu_x,\mu_z} + \Box_{\mu_y,\mu_z} + \text{H.c.} \right)$$

$$\Box_{\mu_x,\mu_y} = U_{x,\mu_x} U_{x+\mu_x,\mu_y} U^\dagger_{x+\mu_y,\mu_x} U^\dagger_{x,\mu_y}$$



Lattice QED in (3+1)D



$$\hat{H} = -t \sum_{x,\mu} \left(\hat{\psi}_x^\dagger \, \hat{U}_{x,\mu} \, \hat{\psi}_{x+\mu} + \text{H.c.} \right)$$

$$+ m \sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g_e^2}{2} \sum_{x,\mu} \hat{E}_{x,\mu}^2$$

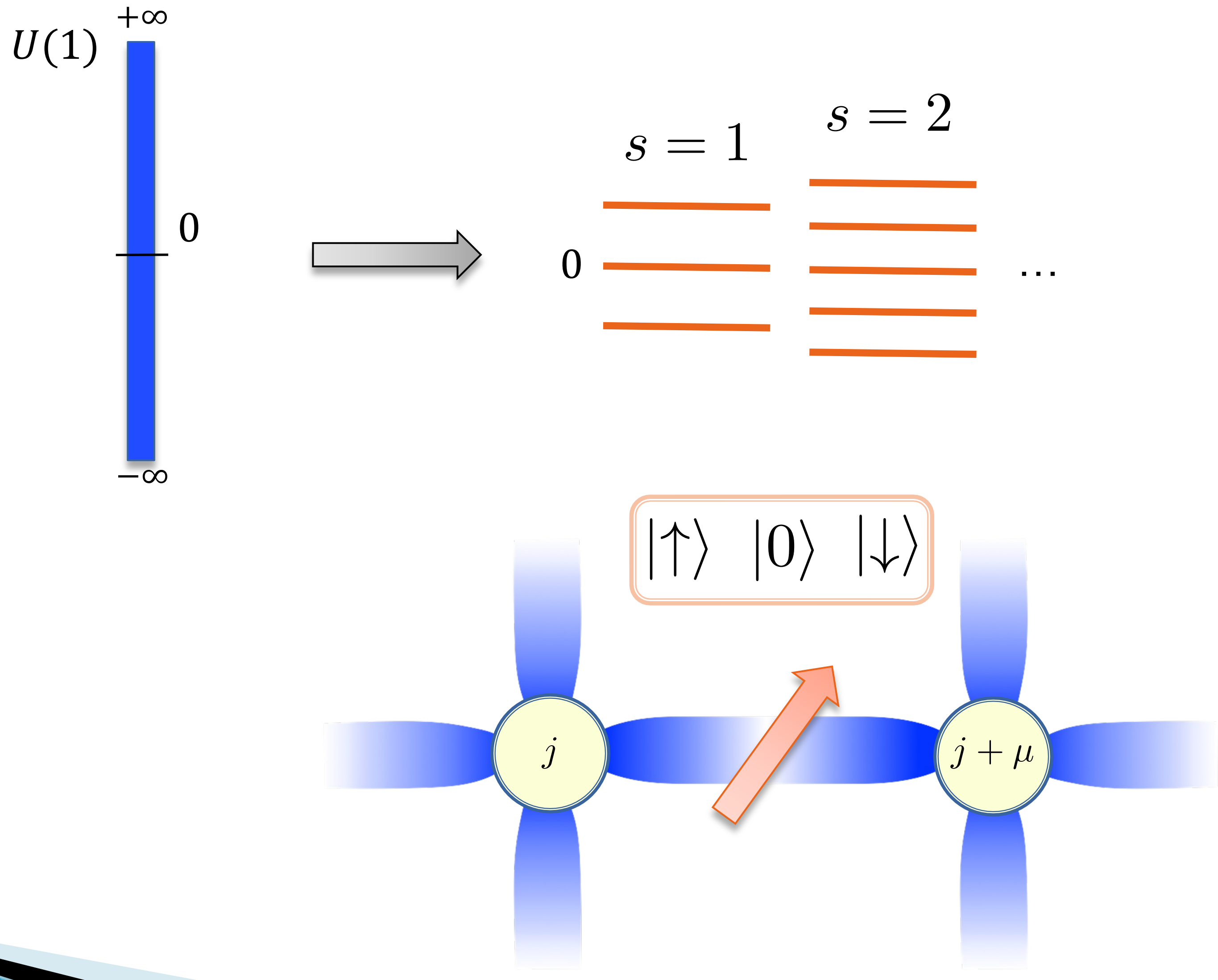
$$- \frac{g_m^2}{2} \sum_x \left(\Box_{\mu_x,\mu_y} + \Box_{\mu_x,\mu_z} + \Box_{\mu_y,\mu_z} + \text{H.c.} \right)$$

Quantum Link Model
discretization of Gauge Fields

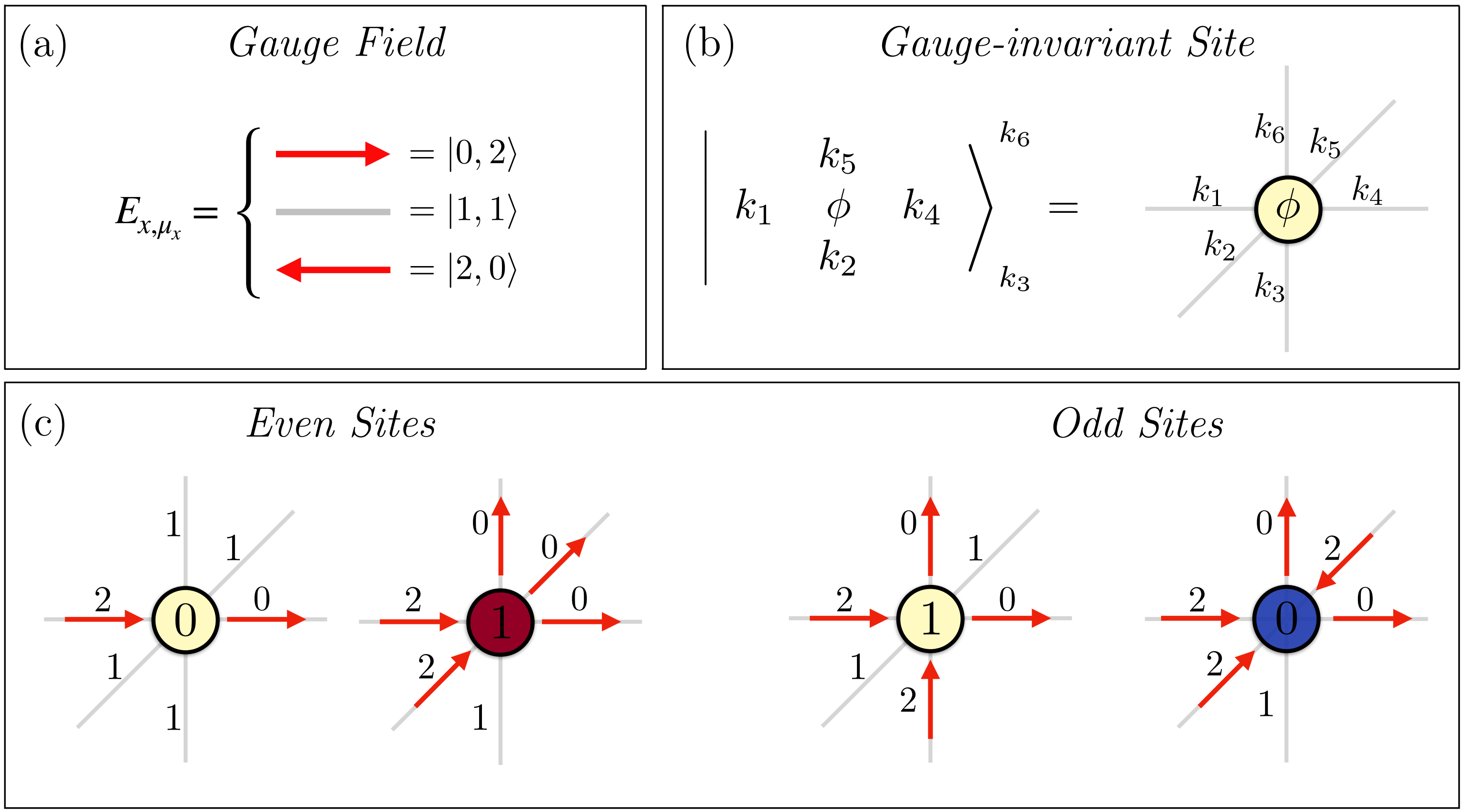
$$\hat{E}_{x,\mu} \rightarrow \hat{S}^z_{x,\mu}$$

$$\hat{U}_{x,\mu} \rightarrow \hat{S}^+_{x,\mu}/s,$$

$$\hat{E}_{x,\mu} \to \hat{S}^z_{x,\mu} \qquad \hat{U}_{x,\mu} \to \hat{S}^+_{x,\mu}/s,$$



$$\hat{G}_x = \hat{\psi}_x^\dagger \hat{\psi}_x - \frac{1 - p_x}{2} - \sum_\mu \hat{E}_{x,\mu} \qquad \hat{G}_x |\Phi\rangle = 0 \ \forall x$$



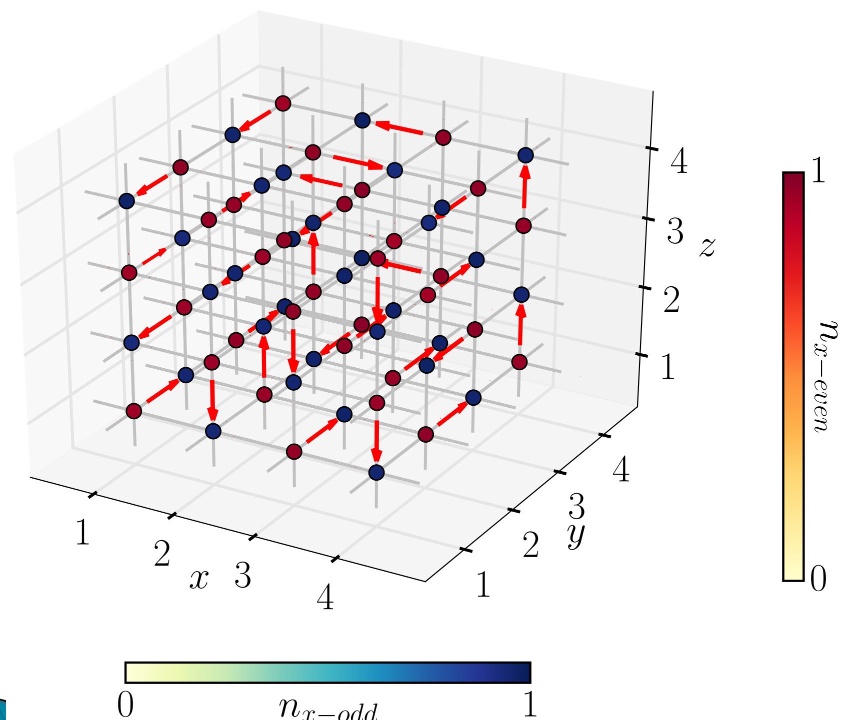
$$dimH_x = 267$$

just for comparison, the 8×8×8 system corresponds to 64×64×64 qubits

Local configurations of matter and gauge fields

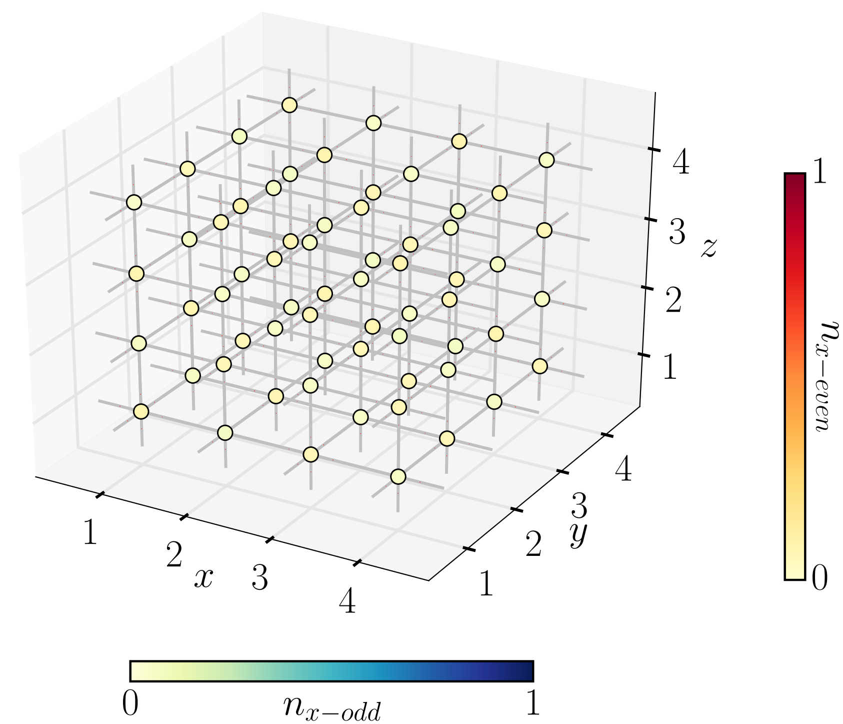
$$-2m \gg g_e^2/2 > 0$$

Charge-Crystal Phase: particle-antiparticle dimers



$$g_e^2/2 \gg 2|m|$$

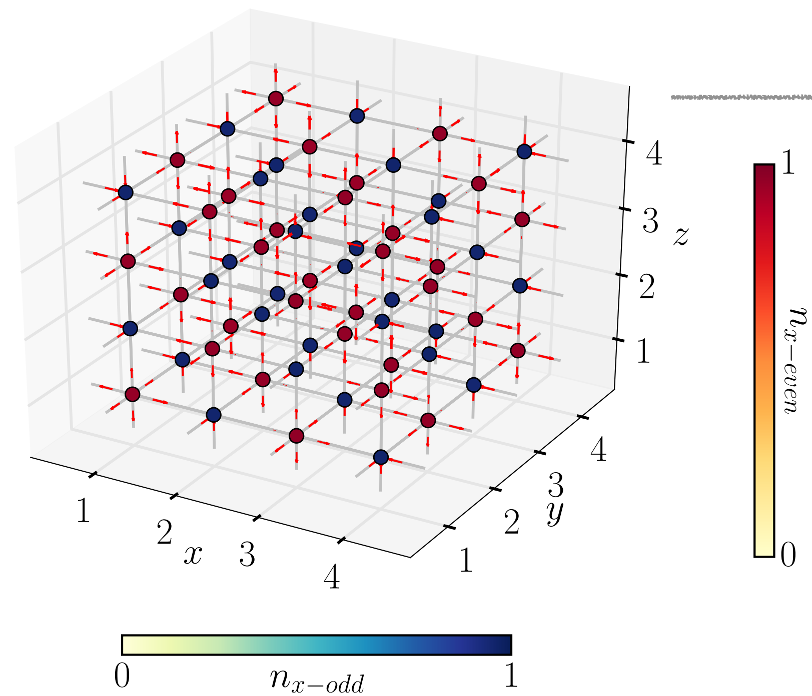
Vacuum phase: no particles, no field excitations

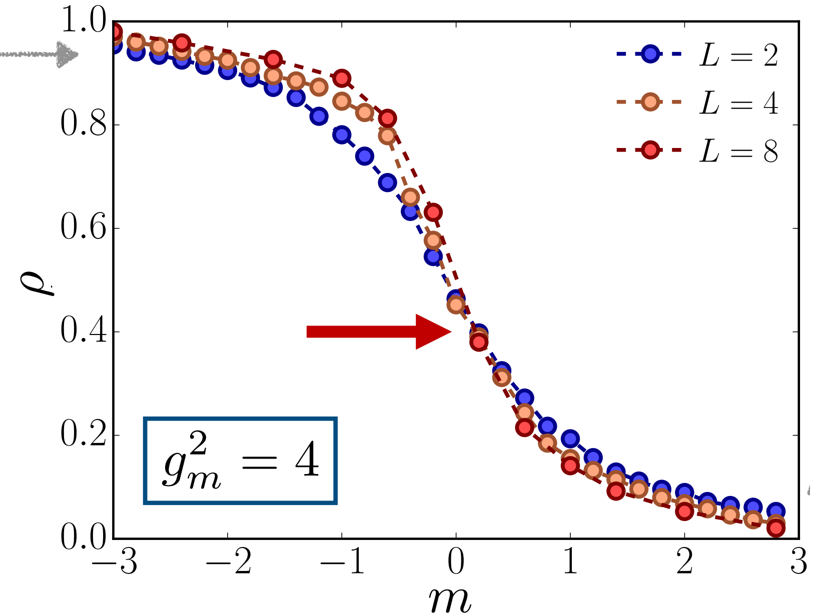


Ground state properties for $g_m^2 = \frac{8}{g_e^2} = 4$

$$\rho = \frac{1}{L^3} \sum_x \langle GS|\hat{n}_x|GS\rangle$$

$$\rho L^{\frac{\beta}{\nu}} = \lambda\left(L^{\frac{1}{\nu}}(m - m_c)\right)$$



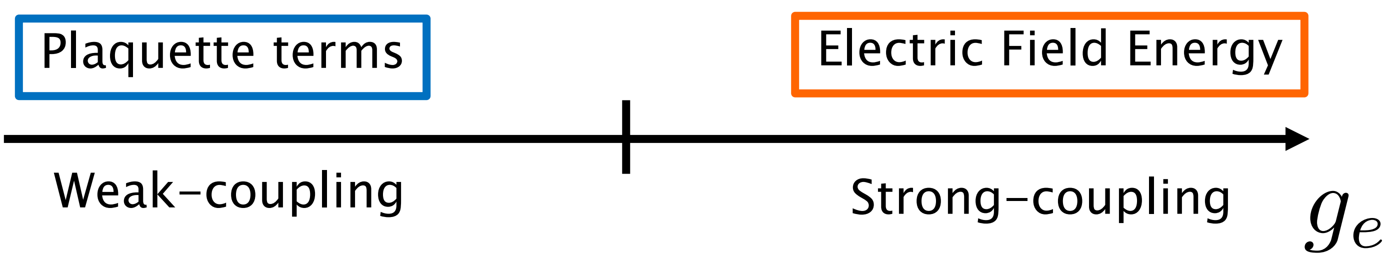


$$m_c = +0.22$$
$$\beta = 0.16 \quad \nu = 0.22$$

Confinement Properties

$$g_m^2 = 8/g_e^2$$

$$\hat{H} = -t \sum_{x,\mu} \left(\hat{\psi}_x^\dagger \, \hat{U}_{x,\mu} \, \hat{\psi}_{x+\mu} + \mathrm{H.c.} \right) + m \sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g_e^2}{2} \sum_{x,\mu} \hat{E}_{x,\mu}^2 - \frac{g_m^2}{2} \sum_x \left(\Box_{\mu_x,\mu_y} + \Box_{\mu_x,\mu_z} + \Box_{\mu_y,\mu_z} + \mathrm{H.c.} \right)$$



Confinement Properties

Plaquette terms | Electric Field Energy

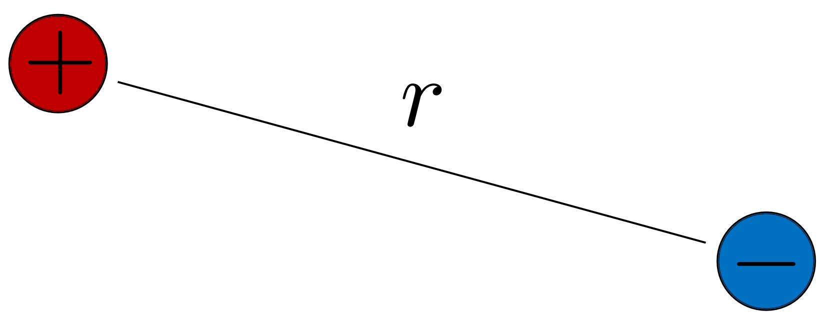
Weak-coupling | Strong-coupling | g_e

$$V(r) \propto 1/r$$

Deconfined phase

$$V(r) \propto r$$

Confined phase



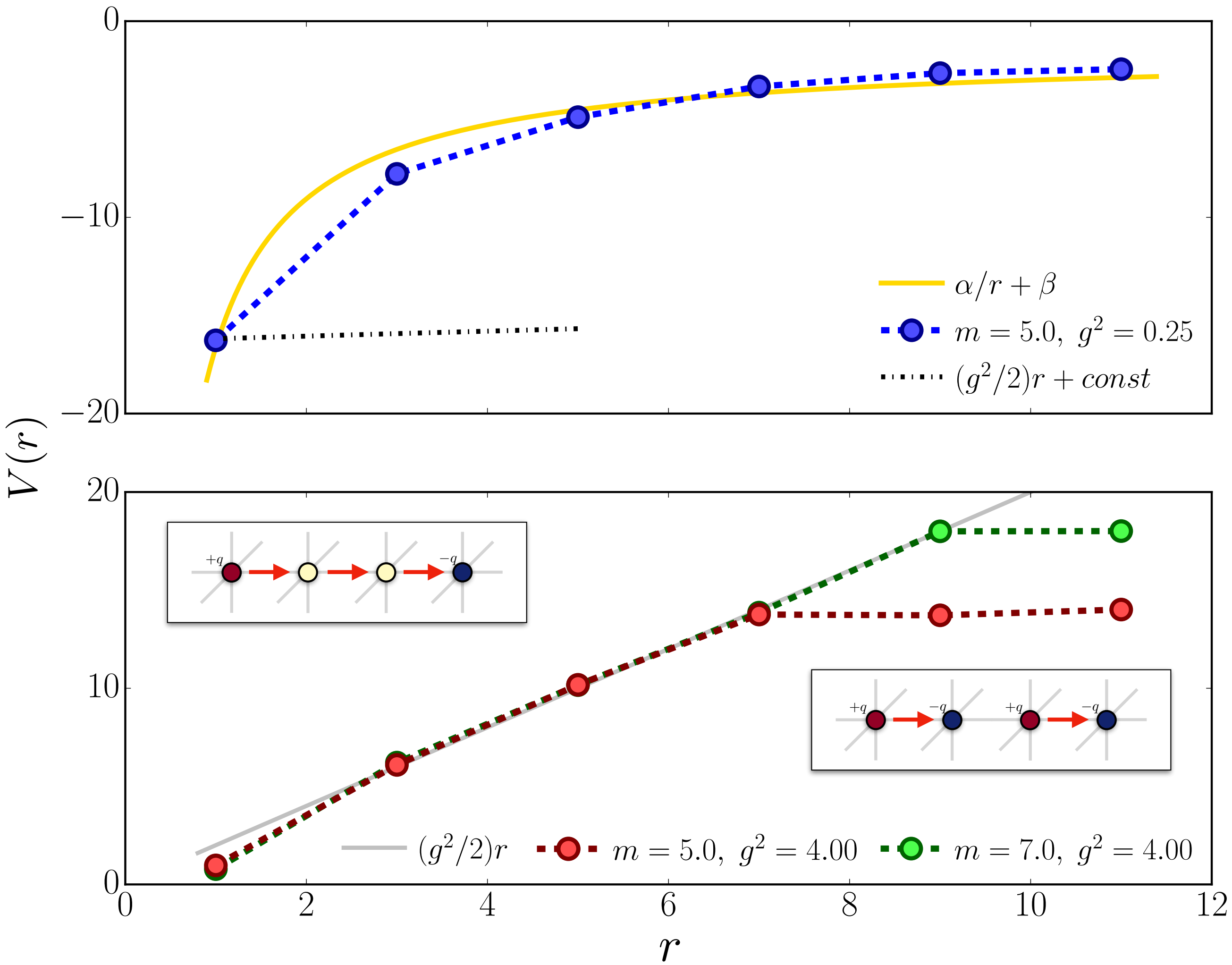
Kei-Ichi Kondo, Phys. Rev. D **58**, 085013 (1998)

Confinement Properties

$$V(r) = E(r) - E_0$$

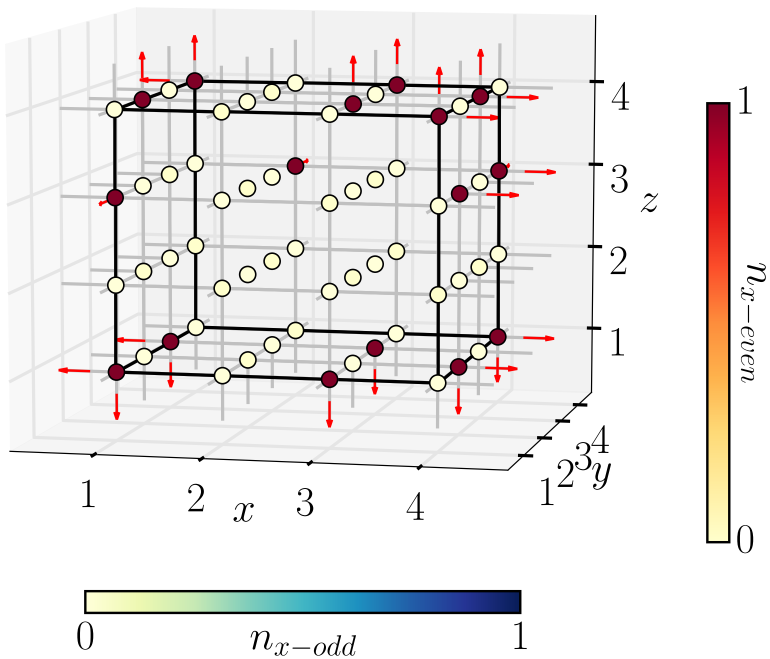
Weak-coupling

Strong-coupling

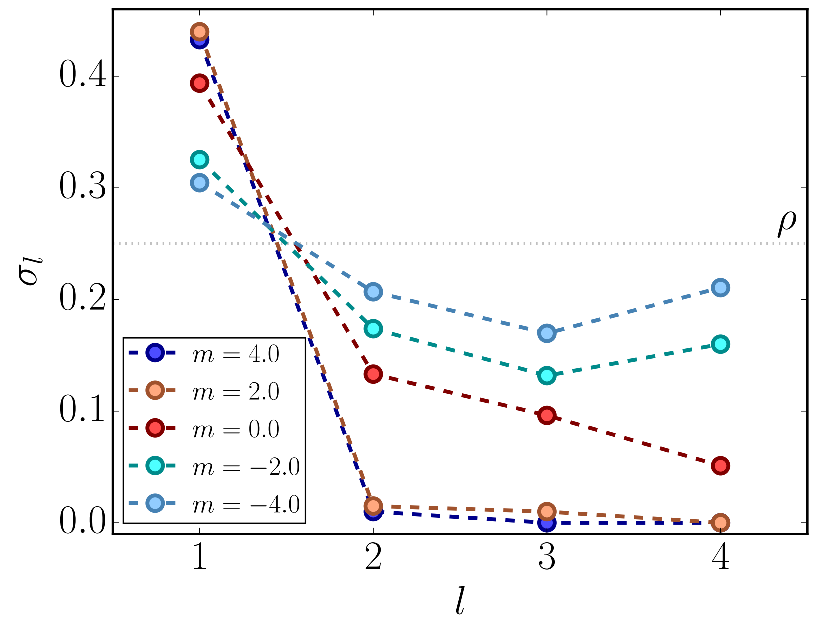


Finite Density

$$L = 4\,, Q = 16\,, \rho = 1/4$$



$$L = 8\,, Q = 128\,, \rho = 1/4$$



$$\sigma(l) = \frac{1}{A(l)} \sum_{x \in A(l)} \left\langle \hat{\psi}_x^\dagger \hat{\psi}_x \right\rangle$$

Tensor Networks – Scattering dynamics

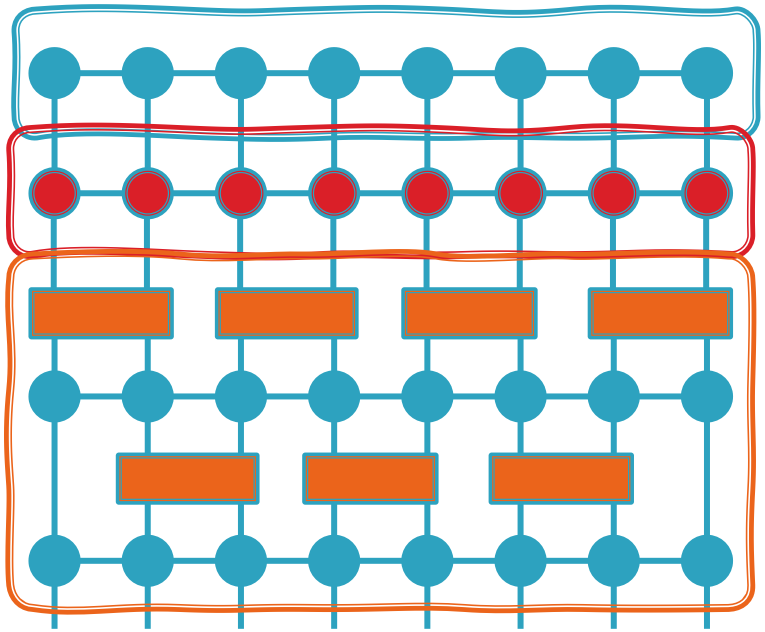
MARCO RIGOBELLO
University of Padua

$$\hat{H} = -t\sum_x \hat{\psi}_x^\dagger \hat{U}_{x,x+1}\hat{\psi}_{x+1} + \mathrm{h.c.} + m\sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g^2}{2}\sum_x \hat{E}_{x,x+1}^2$$

interacting vacuum MPS via DMRG

initial state via wave packet creation **MPOs**

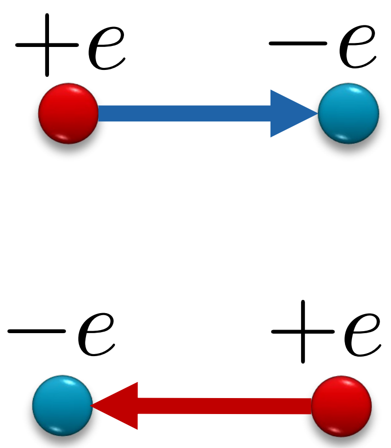
time evolution via TEBD & observables

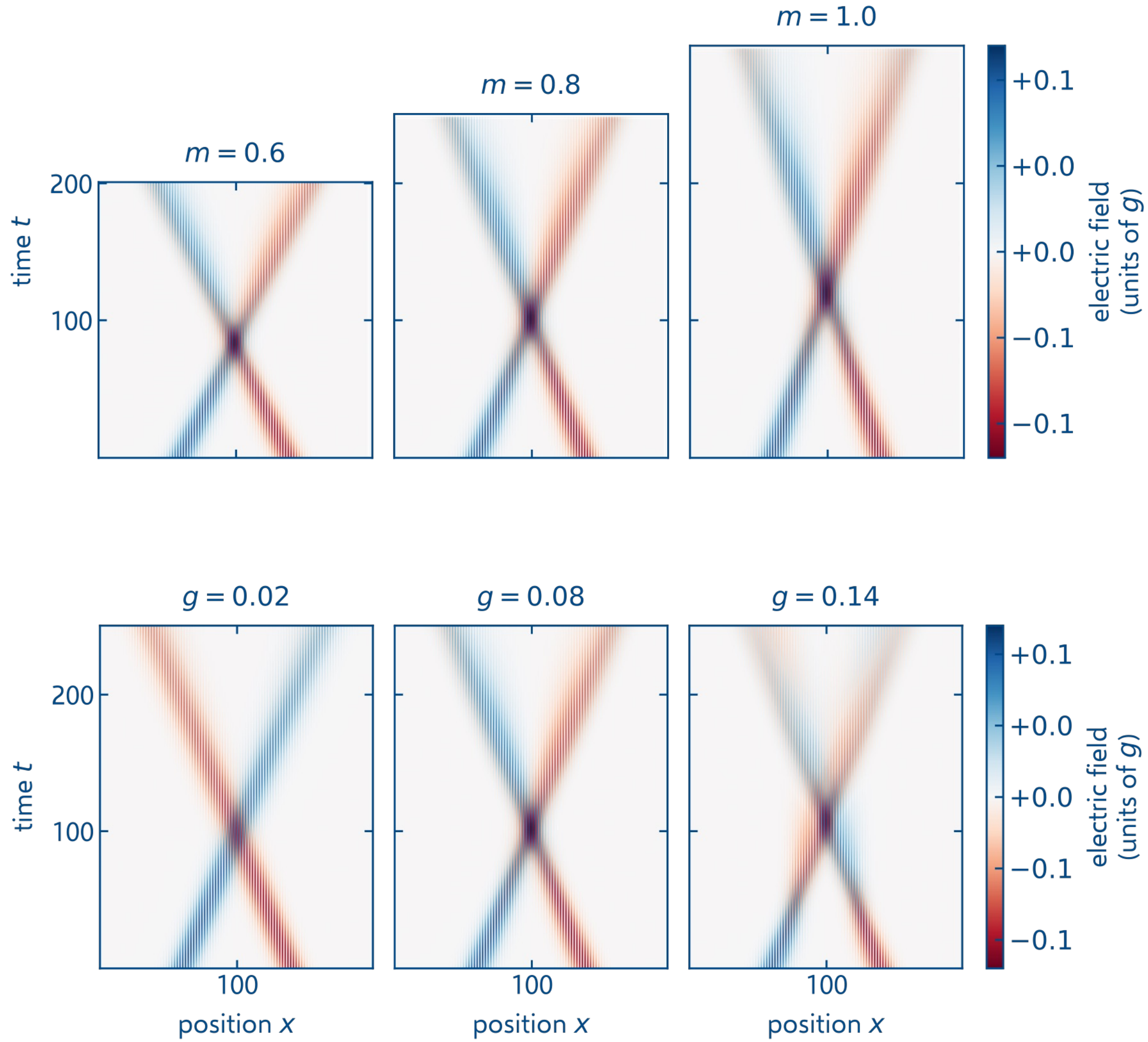


Phys. Rev. D **104**, 114501 (2021)

Tensor Networks – Scattering dynamics

Mesons with opposite momenta and internal electric fields





Phys. Rev. D **104**, 114501 (2021)

Tensor Networks – Scattering dynamics

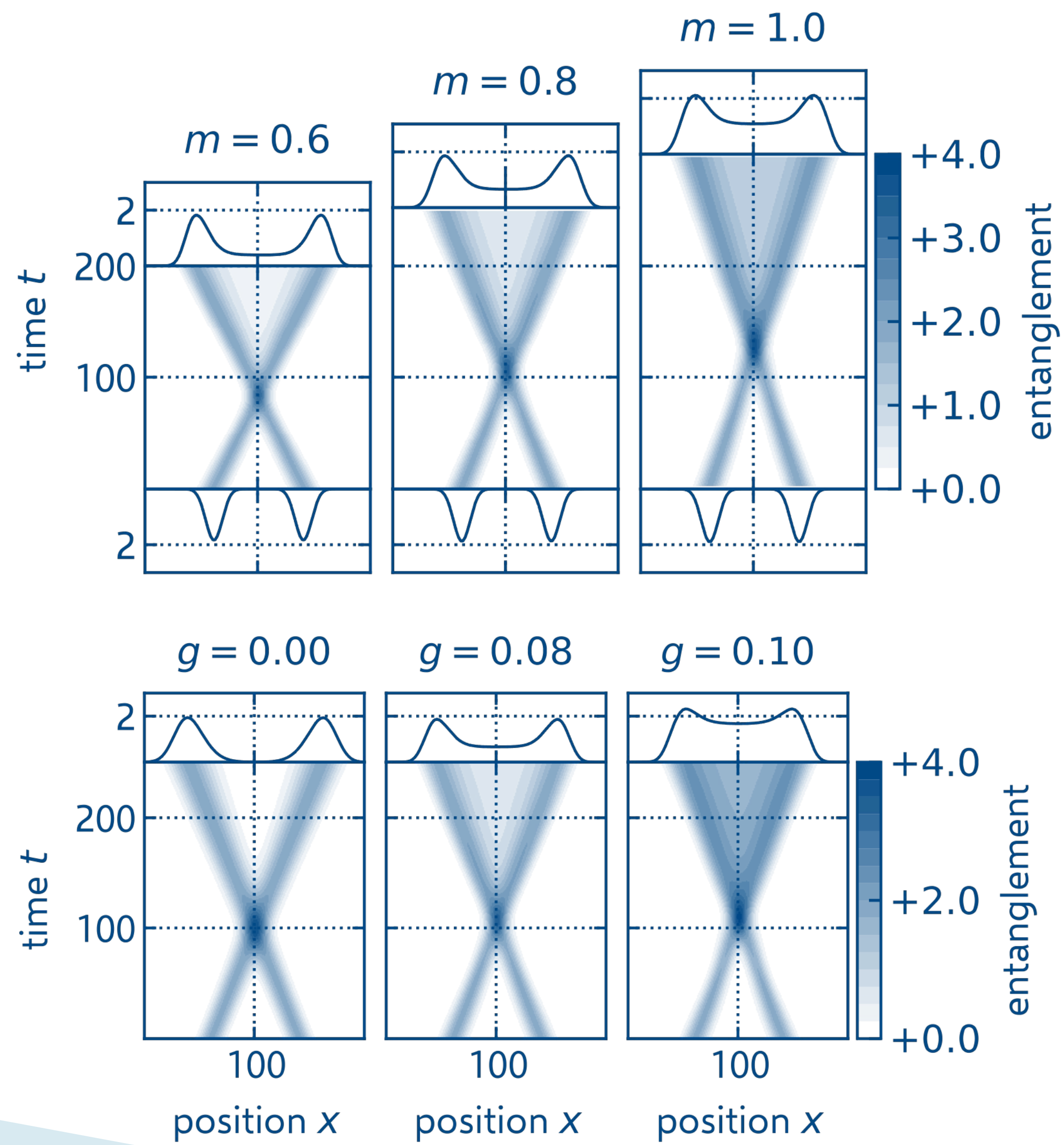
Vacuum

+

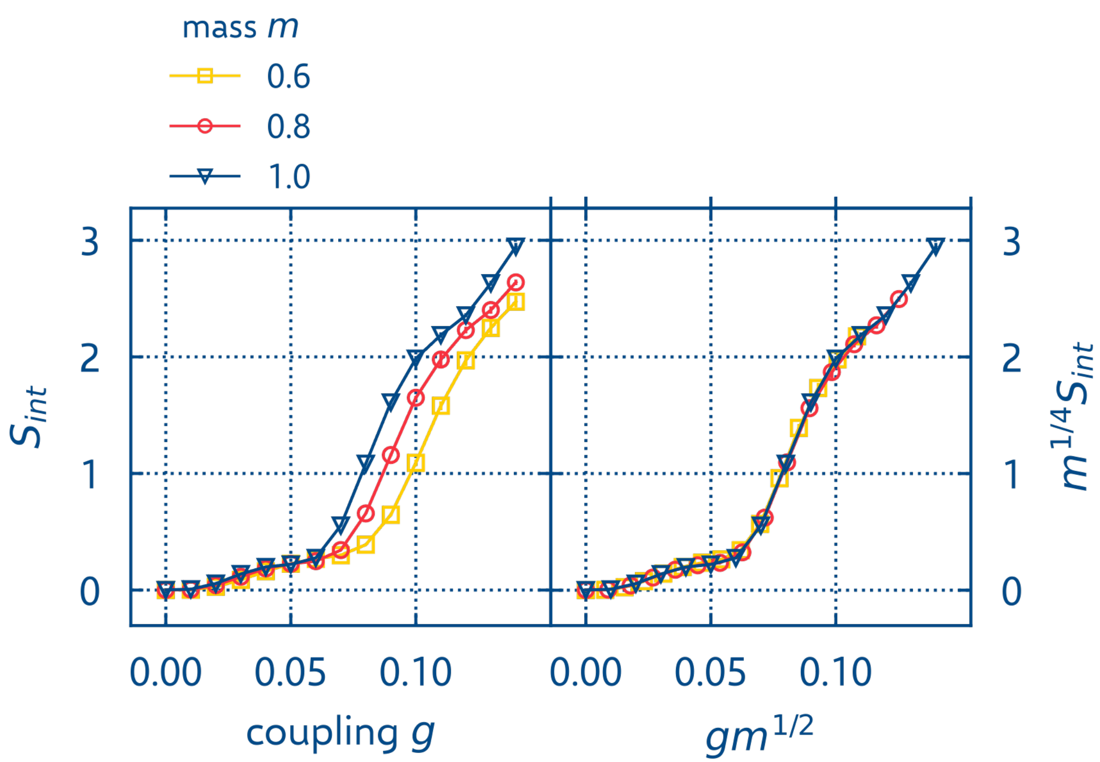
wave packets
intra–particle

+

interactions
intra–particle



Tensor Networks – Scattering dynamics



scaling relation

$$m^{1/4}S_{int}(m, g/\sqrt{m}) = S_{int}(1, g)$$

is m independent

two regimes

$g \lessgtr \boldsymbol{g}^{\star} \approx 0.06/\sqrt{m}$

appearance of new effective d.o.f.

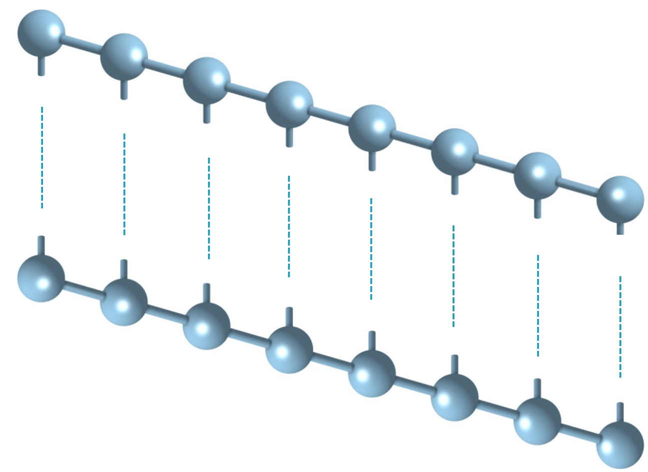
Phys. Rev. D **104**, 114501 (2021)

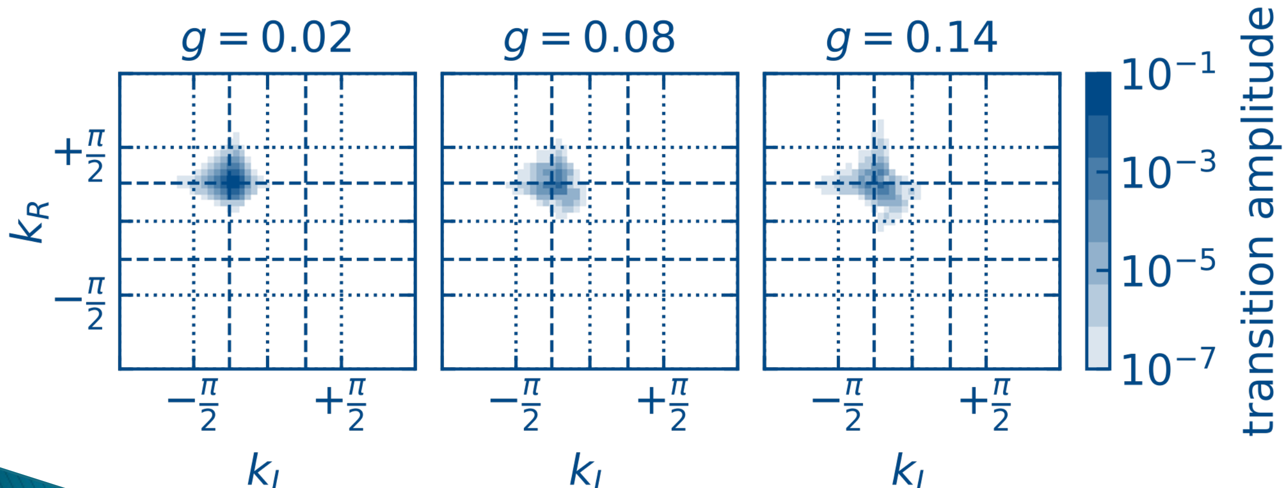
Tensor Networks – Scattering dynamics

overlap of final state with
pair of meson wave packet

$|\psi(t_f)\rangle$

$\langle\psi(t_i)|$





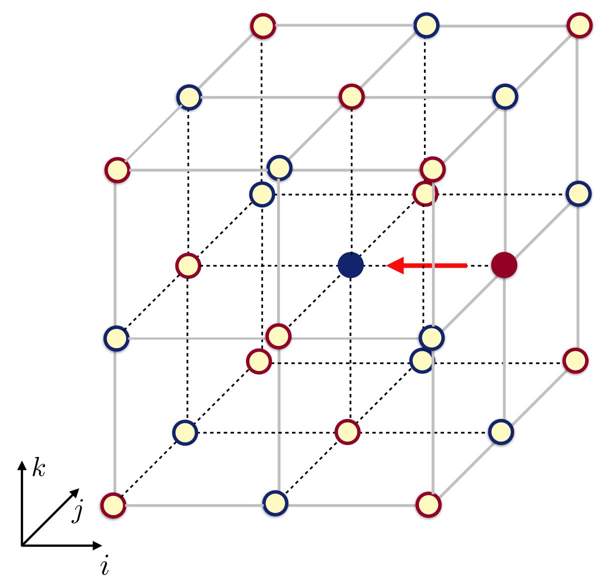
~ S-matrix elements

Phys. Rev. D **104**, 114501 (2021)

What's next?

Quantum Many-Body Physics ⟷ High-Energy Physics domain

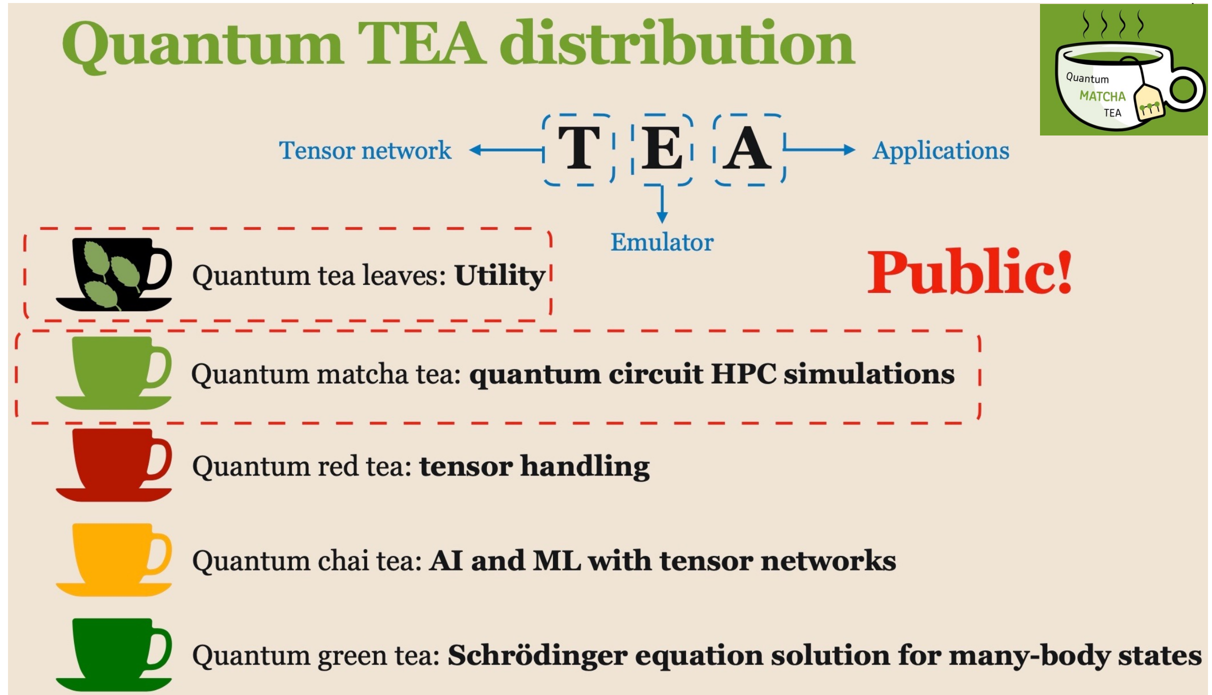
- Tensor networks can be used to develop, support, validate, quantum simulation and quantum computation protocols.
- Relevant interactions with High-Energy Physics.
- Long-term goal: tensor networks and quantum simulation of QCD (ongoing project: 1+1D QCD)

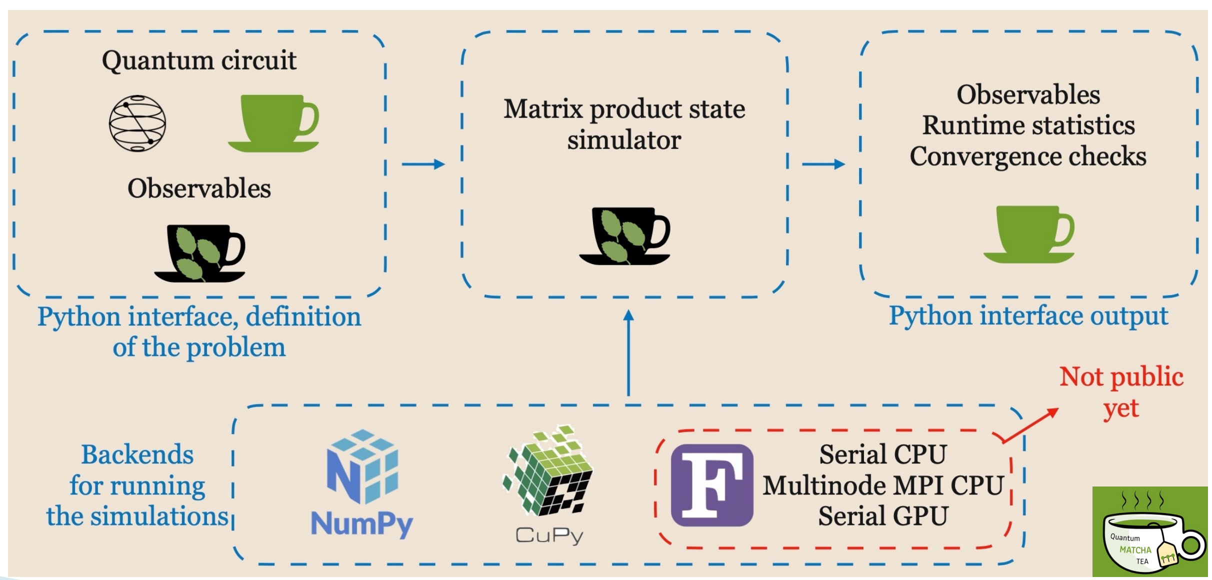


We just released some optmised TN codes and libraries!

Easy-to-use python frontend and fast HPC-ready backend (Both GPU and CPU)

Error analysis tools and efficient computations of observables optimised for the MPS representation





THANK YOU