

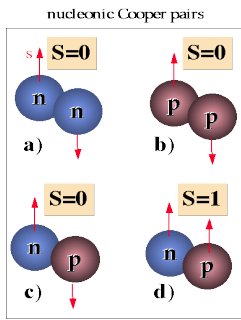
# N~Z and isospin symmetry

**Silvia M. Lenzi**

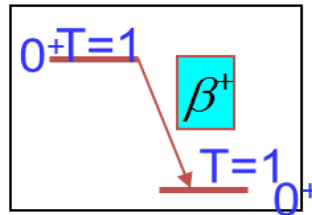
University of Padova and INFN, Padova, Italy

# The N=Z richness

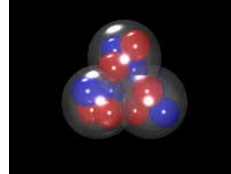
## p-n pairing



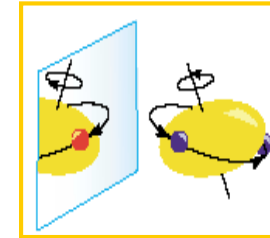
## fundamental interactions



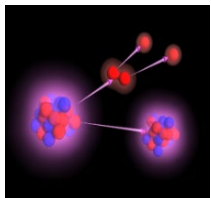
## alpha clusterization



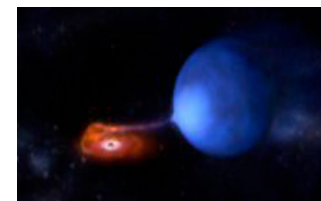
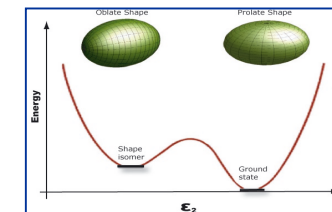
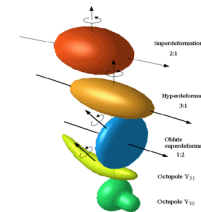
## Isospin symmetry breaking



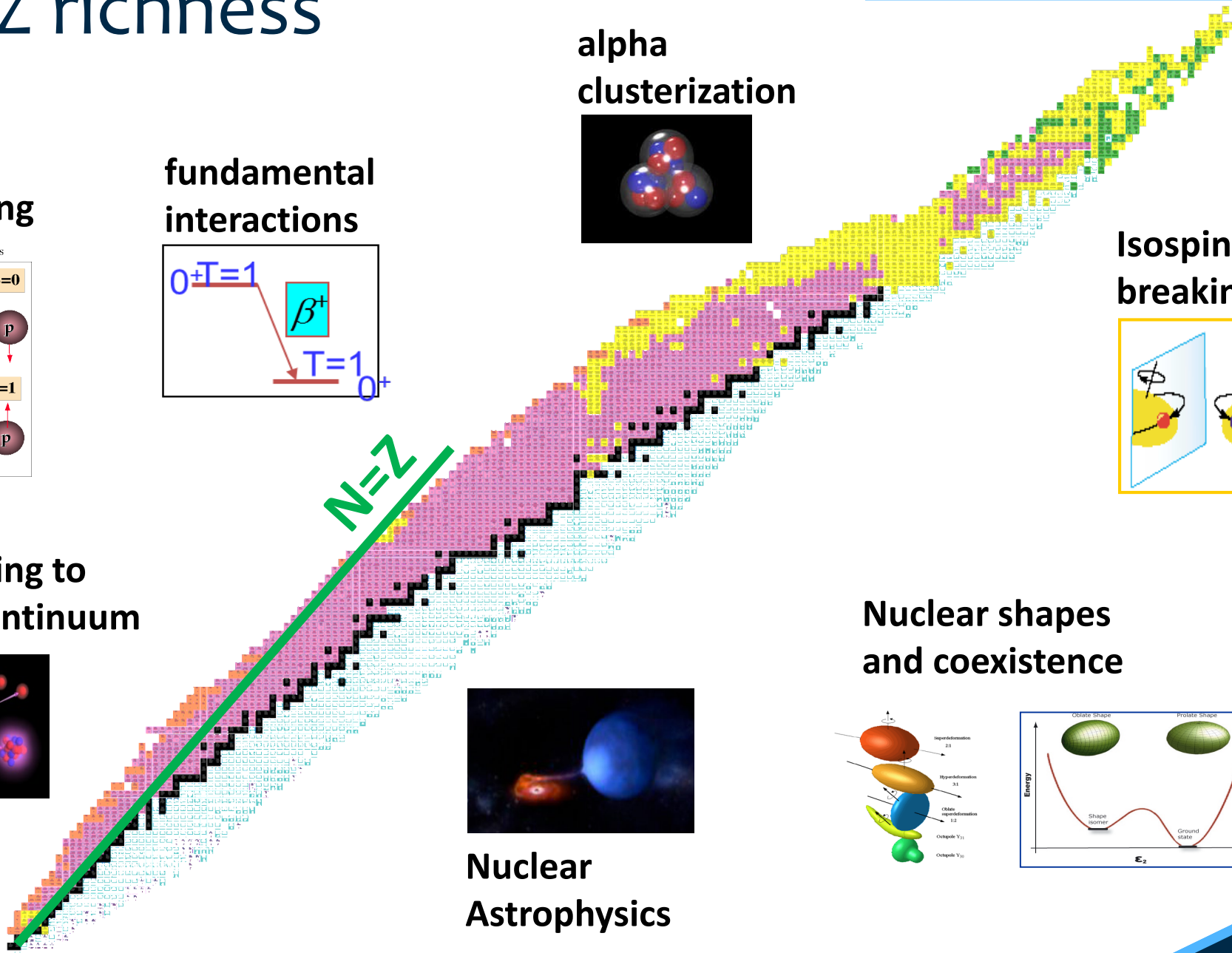
## Coupling to the continuum



## Nuclear shapes and coexistence



## Nuclear Astrophysics



# Open problems in nuclear structure of $N \sim Z$ nuclei

Understanding the nuclear interaction

Some properties of the nuclear interaction are unique or enhanced in  $N \sim Z$  nuclei

- Quadrupole+ Pairing interplay
  - Quadrupole  $\rightarrow$  deformation
  - Pairing  $\rightarrow$  Role of p-n  $T=0$  pairing
- Isospin symmetry (**breaking**)
- Fundamental interactions

**Working group:**

**Marlène Assié (Orsay)**

**Michael Bentley (York)**

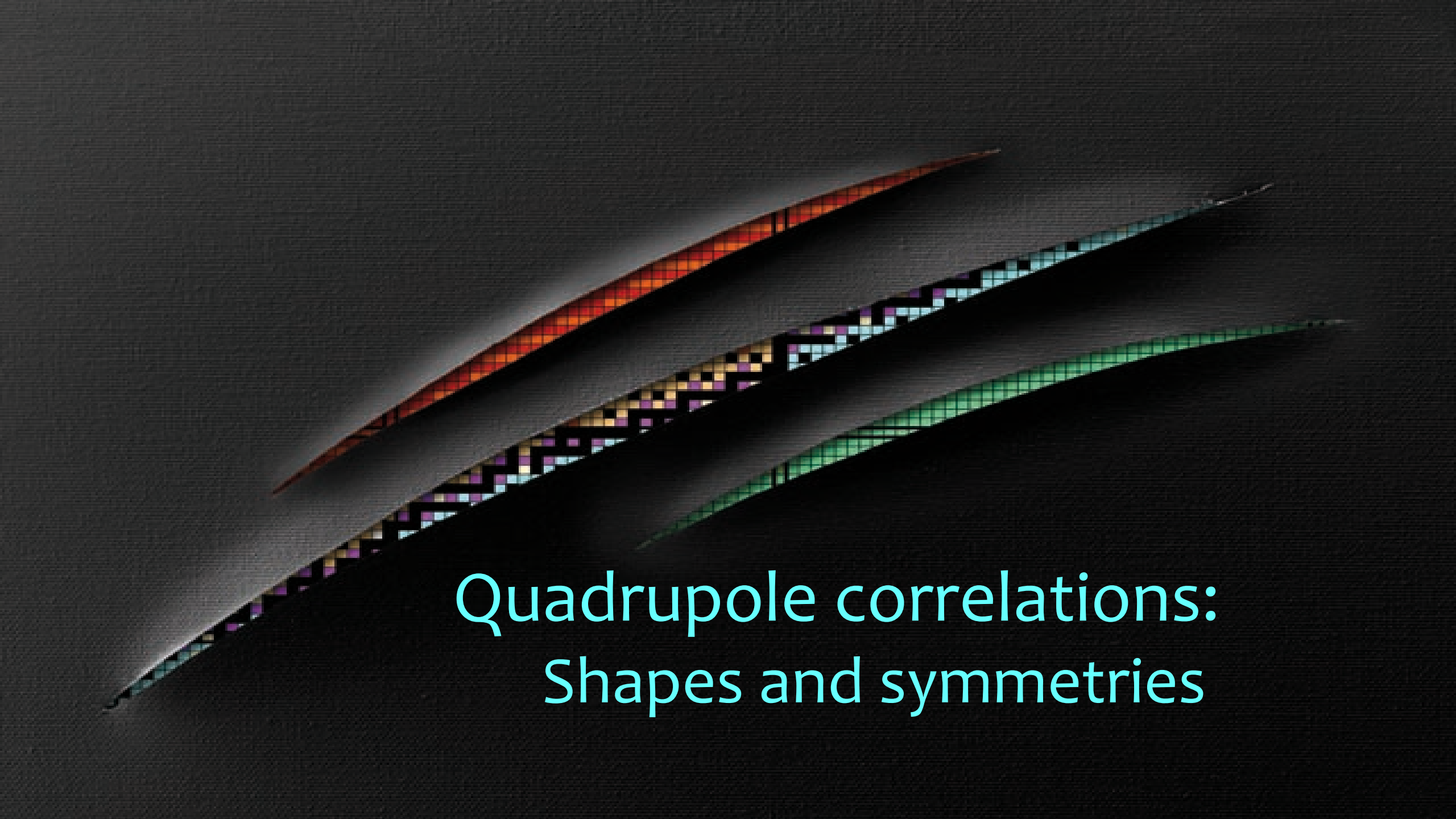
**Augusto Macchiavelli (Oak Ridge)**

**Francesco Recchia (Padova)**

**Dirk Rudolph (Lund)**

**Silvia Lenzi (Padova)**





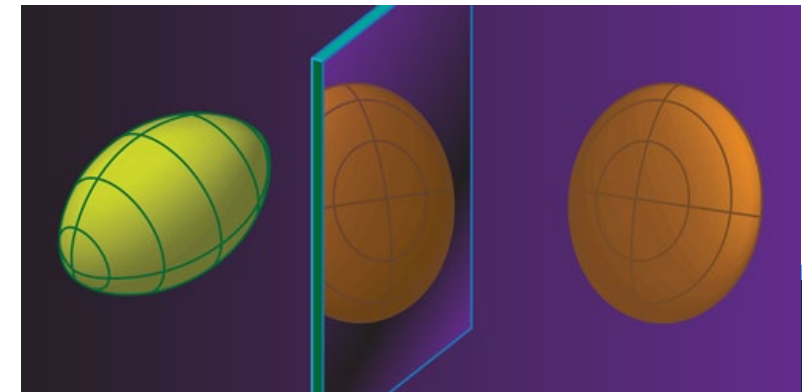
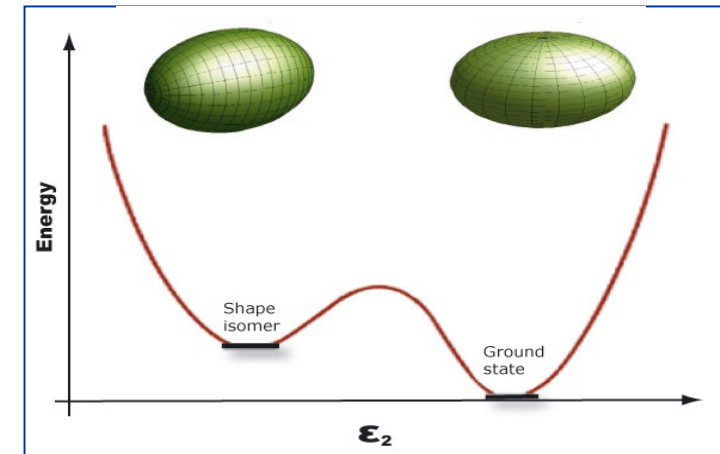
Quadrupole correlations:  
Shapes and symmetries

# Quadrupole collectivity in N=Z nuclei

Along the N=Z line quadrupole correlations are quite strong and in most of the cases govern the nuclear structure properties.

This gives rise to **different nuclear shapes** and their coexistence.

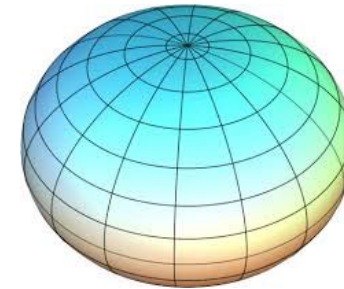
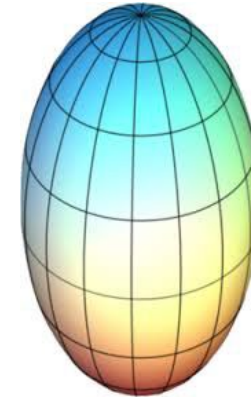
The development of nuclear deformation is a **key property** central to our **understanding** of the nuclear force.



# Quadrupole deformation: a simple model

The spherical nuclear field is close to the harmonic oscillator potential.

In the limit of **degeneracy of the single-particle energies** of a major harmonic oscillator shell, and in the presence of an **attractive Q.Q proton-neutron interaction**, the ground state of the many-body nuclear system is **maximally deformed**.



## Elliott SU(3) in the sd shell

So, at low energy, nuclear states tend to maximize the intrinsic quadrupole moment

# Quadrupole moment in N=Z nuclei

The nuclear quadrupole moment is the sum of the single-particle quadrupole moments

$$q_{sp} = (2n_z - n_x - n_y)$$

where the principal quantum number  $N = (n_x + n_y + n_z)$

In the  $sd$  shell  $N = 2$

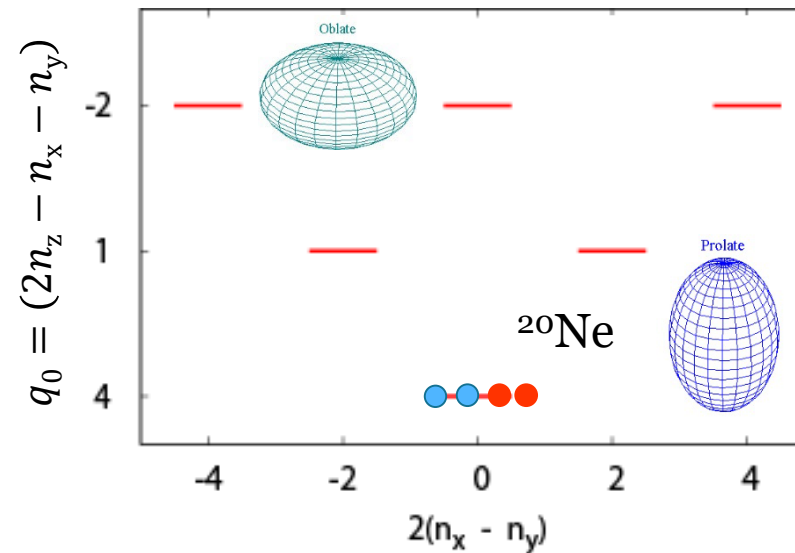
$$N = n_x + n_y + n_z = 2$$

there are 3 possible values:

$$q_{sp} = 4, 1, -2$$

We obtain the nuclear quadrupole moment by filling these fourfold ( $2p + 2n$ ) degenerate “orbits” along the  $N=Z$  line

The “intrinsic orbits” in SU3



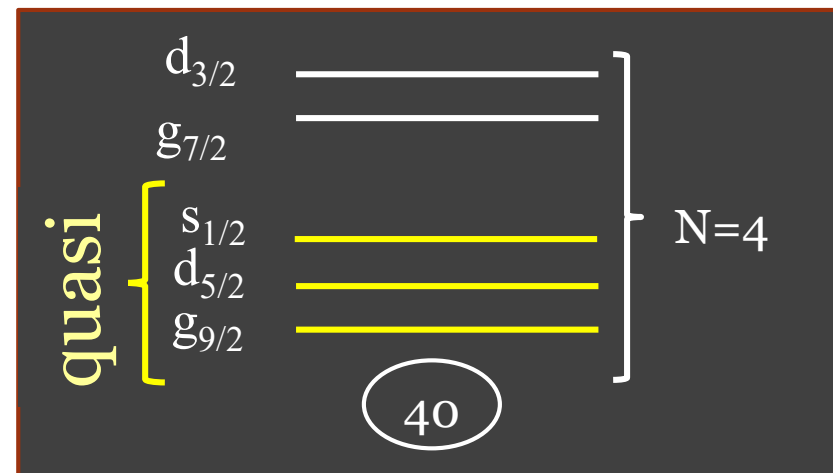
- start filling from below → prolate deformation
- start filling from above → oblate deformation

# SU<sub>3</sub> approximate symmetries

Elliott's SU<sub>3</sub> works well in the *sd* shell but fails for upper shells where the spin-orbit interaction introduces large energy shifts

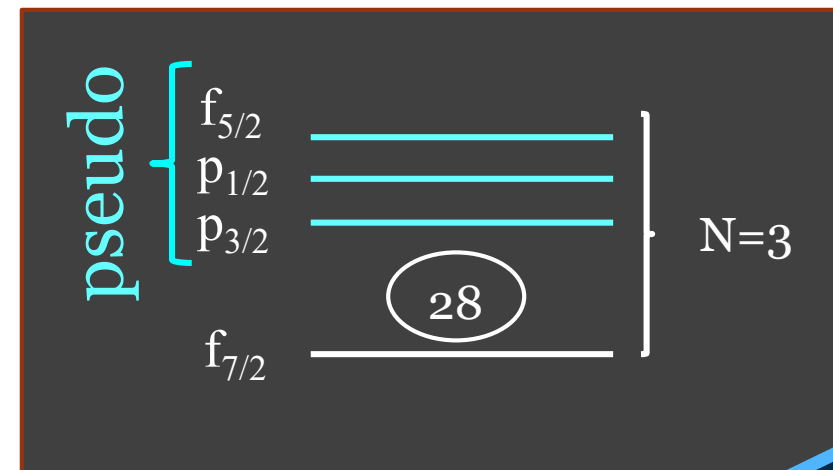
## Quasi SU<sub>3</sub>

applies to the lowest  $\Delta j = 2, \Delta l = 2$  orbits in a major HO shell



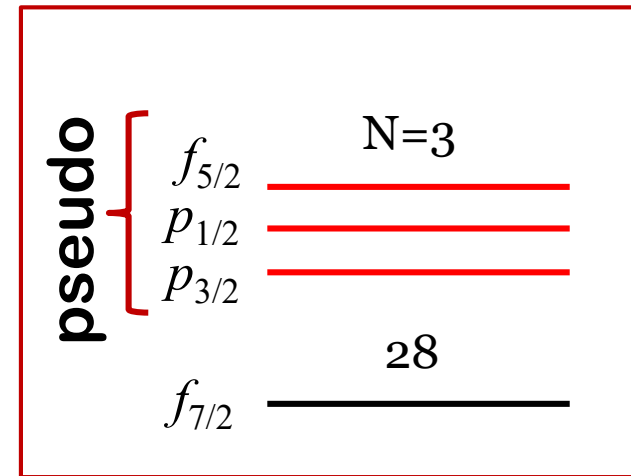
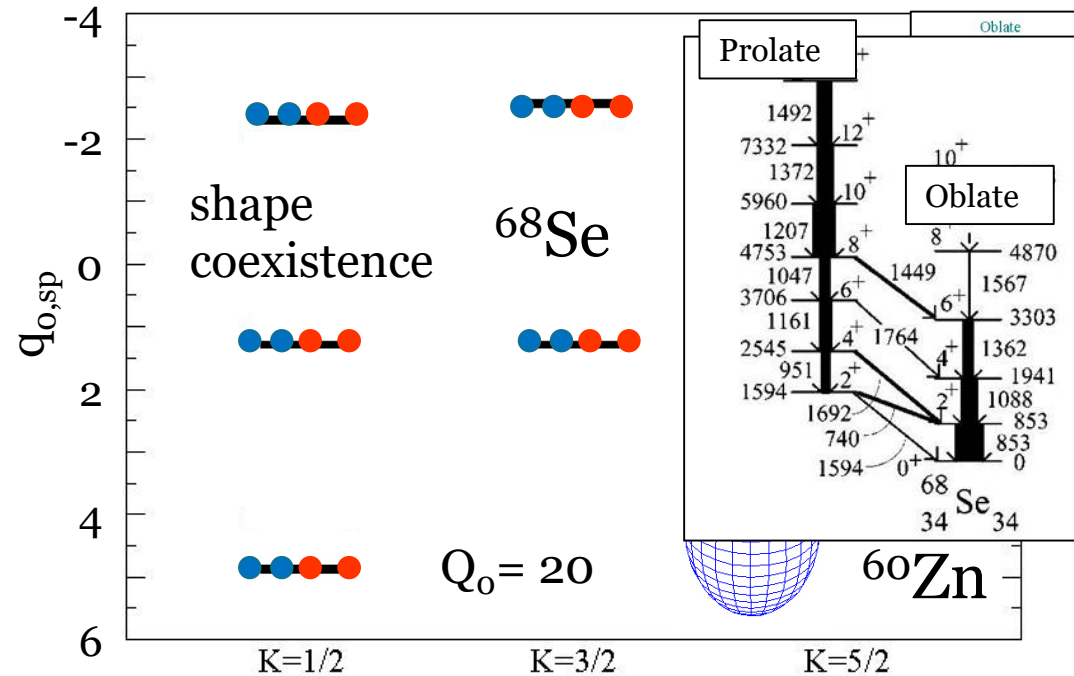
## Pseudo SU<sub>3</sub>

applies to a HO space where the largest *j* orbit has been removed.





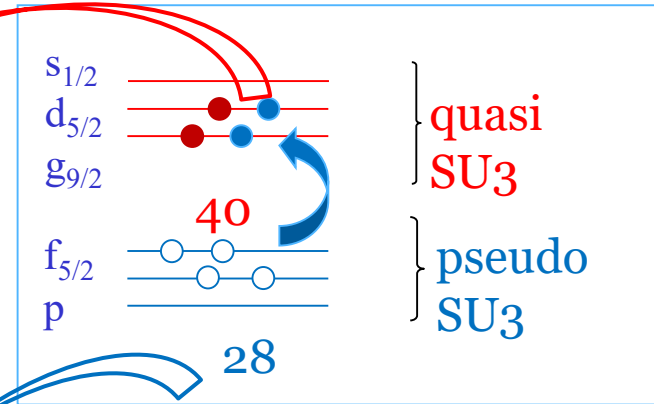
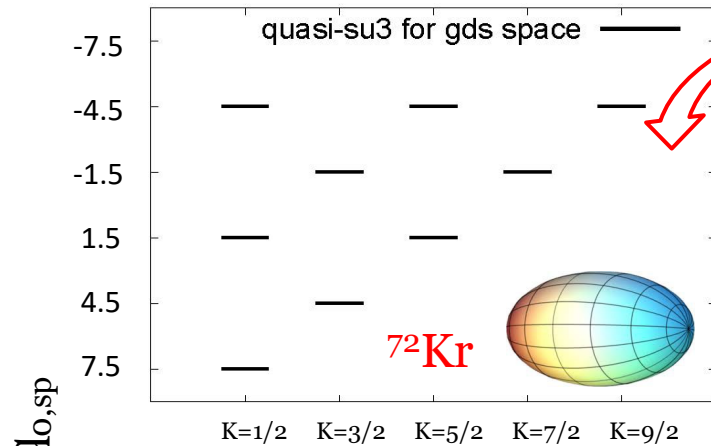
# Quadrupole moment in Pseudo SU<sub>3</sub> space



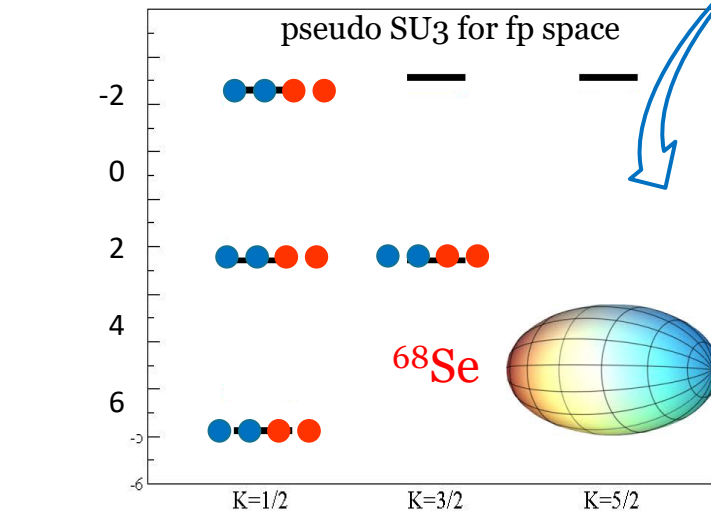
We obtain  $Q_0$  by summing those of the single particles/holes in each “orbit”

# Quadrupole moment in Pseudo+Quasi SU<sub>3</sub> tandem

quasi



pseudo



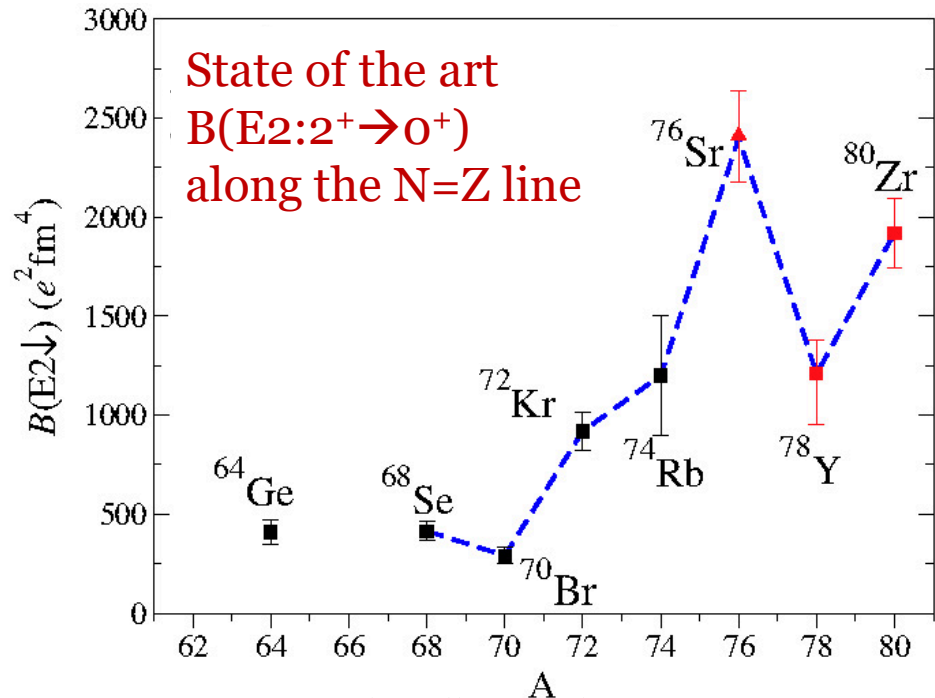
Particle-hole excitations in the pseudo + quasi space maximize the quadrupole moment.

The quadrupole correlation energy results much larger than the energy cost to promote the particles

Based on these dynamical symmetries we can predict different shapes in N=Z nuclei

# Shapes and shape coexistence in $N=Z$ at LNL

A way to test the degree of deformation is to measure the  $B(E2)$  of excited states



R. D. O. Llewellyn et al.,  
*Phys. Rev. Lett.* 124, 152501 (2020)

With the exception of  $^{72}\text{Kr}$ , in  $N=Z$  nuclei with  $A > 64$  only the  $B(E2: 2^+ \rightarrow 0^+)$  has been measured so far.

Shape coexistence is predicted in most of  $N=Z$  nuclei.

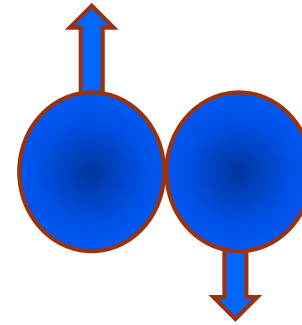
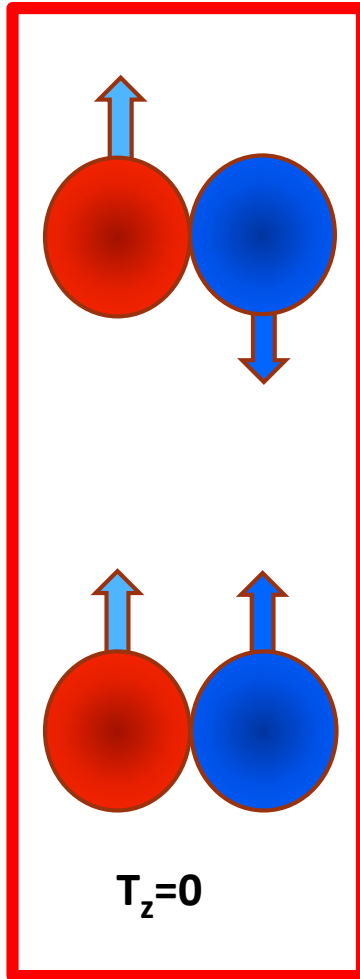
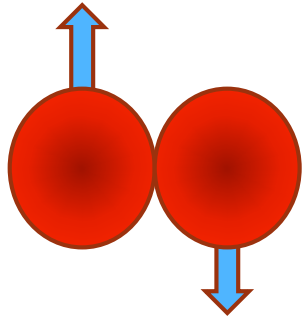
Lifetimes of excited states in  $N \sim Z$  nuclei can be measured using stable beams in fusion-evaporation and multinucleon transfer reactions

→ combining AGATA + NEDA + charged-particle det. + (Plunger)  
or AGATA+ PRISMA + differential plunger

The image features a dark, textured background with three prominent diagonal bands of colored squares. The top band is orange, the middle band is a mix of purple, yellow, and blue, and the bottom band is green. The squares are arranged in a grid-like pattern, creating a sense of depth and movement. The overall aesthetic is modern and abstract.

Pairing: the role of  $T=0$  pn

# Pairing interaction



$T=1$   
 $S=0$

**Isovector  
Pairing**

$T=0$   
 $S=1$

**Isoscalar  
Pairing**

# Role of T=0 pn pairing

Does isoscalar pairing give rise to collective modes?

Possible signatures:

- Binding energy differences
- Low-lying states of odd-odd self-conjugate nuclei
- Rotational properties: moments of inertia, alignments
- Beta decay
- **DIRECT REACTIONS**

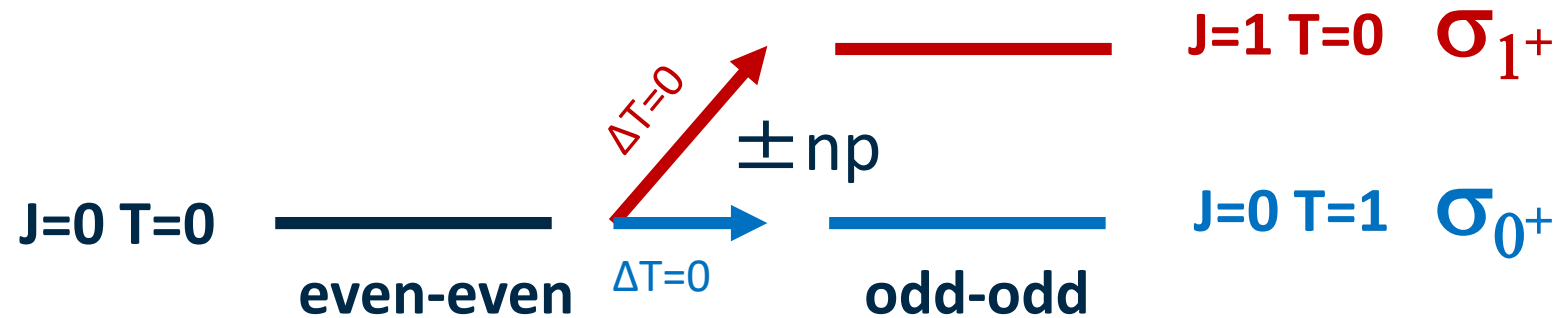
# Pure np transfer

$(p, {}^3\text{He}), ({}^3\text{He}, p) \quad \Delta T=0,1$

$(d, \alpha), (\alpha, d) \quad \Delta T=0$

$(\alpha, {}^6\text{Li}), ({}^6\text{Li}, \alpha) \quad \Delta T=0$

$+ ({}^3\text{He}, p)$   
 $- (p, {}^3\text{He})$



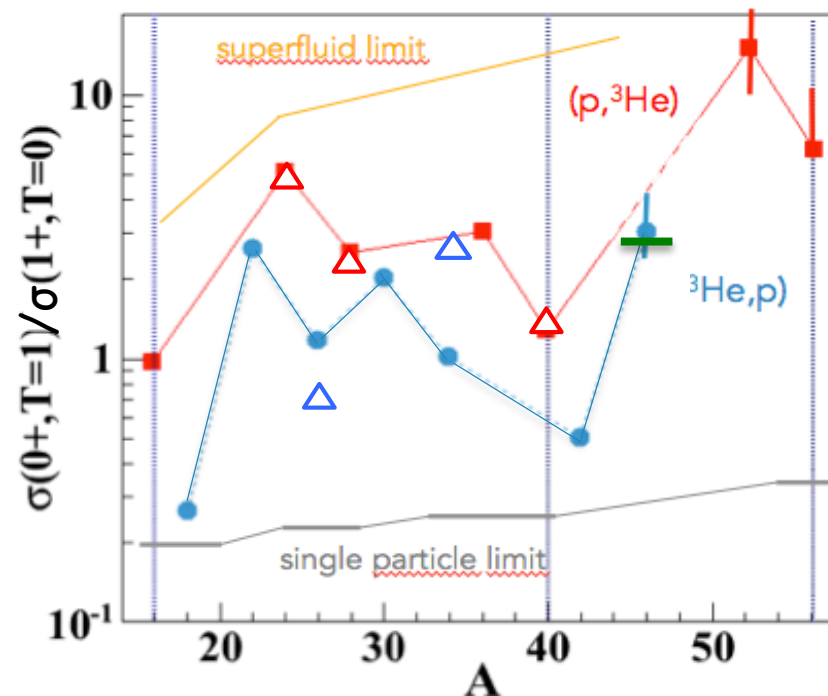
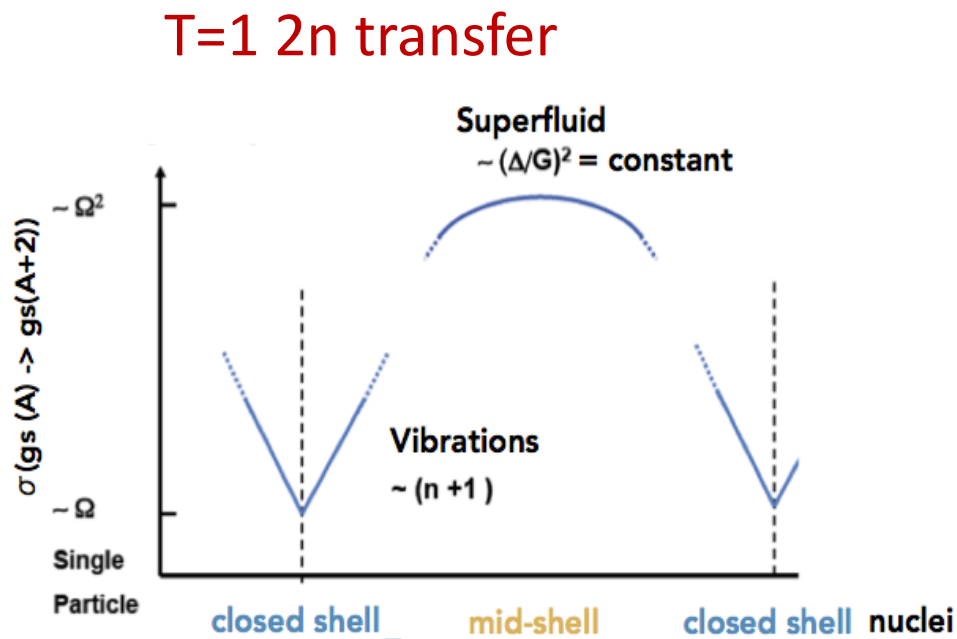
$({}^3\text{He}, p)$   $L=0$  transfer – forward peaked

Measure the  $np$  transfer cross sections to  $T=1$  and  $T=0$  states

Both **absolute**  $\sigma(J=0)$  and  $\sigma(J=1)$  and **relative**  $\sigma(J=0) / \sigma(J=1)$  tell us about the character and strength of the pn correlations

# T=0 vs T=1 pn transfer

compilation of T=0/T=1 pn transfer



Y. Ayyad et al., PRC96 (2017)

From 2n transfer the T=1 cross section, as a function of N, shows a parabolic behaviour along a shell of degeneracy  $\Omega$  with the superfluid regime at a plateau in the middle of the shell

## Warning!

- Ratios obtained in different experiments and at different energies --> **effect of the reaction mechanism**
- L=0 and L=2 contributions overlapping --> **angular distributions needed**



# Short and Mid-term opportunities at LNL

$^{36}\text{Ar}(p, ^3\text{He})$  and  $(^3\text{He}, p)$

complete systematics on the *sd* shell

$^{46}\text{V}$ ,  $^{50}\text{Mn}(p, ^3\text{He})$  and  $(^3\text{He}, p)$

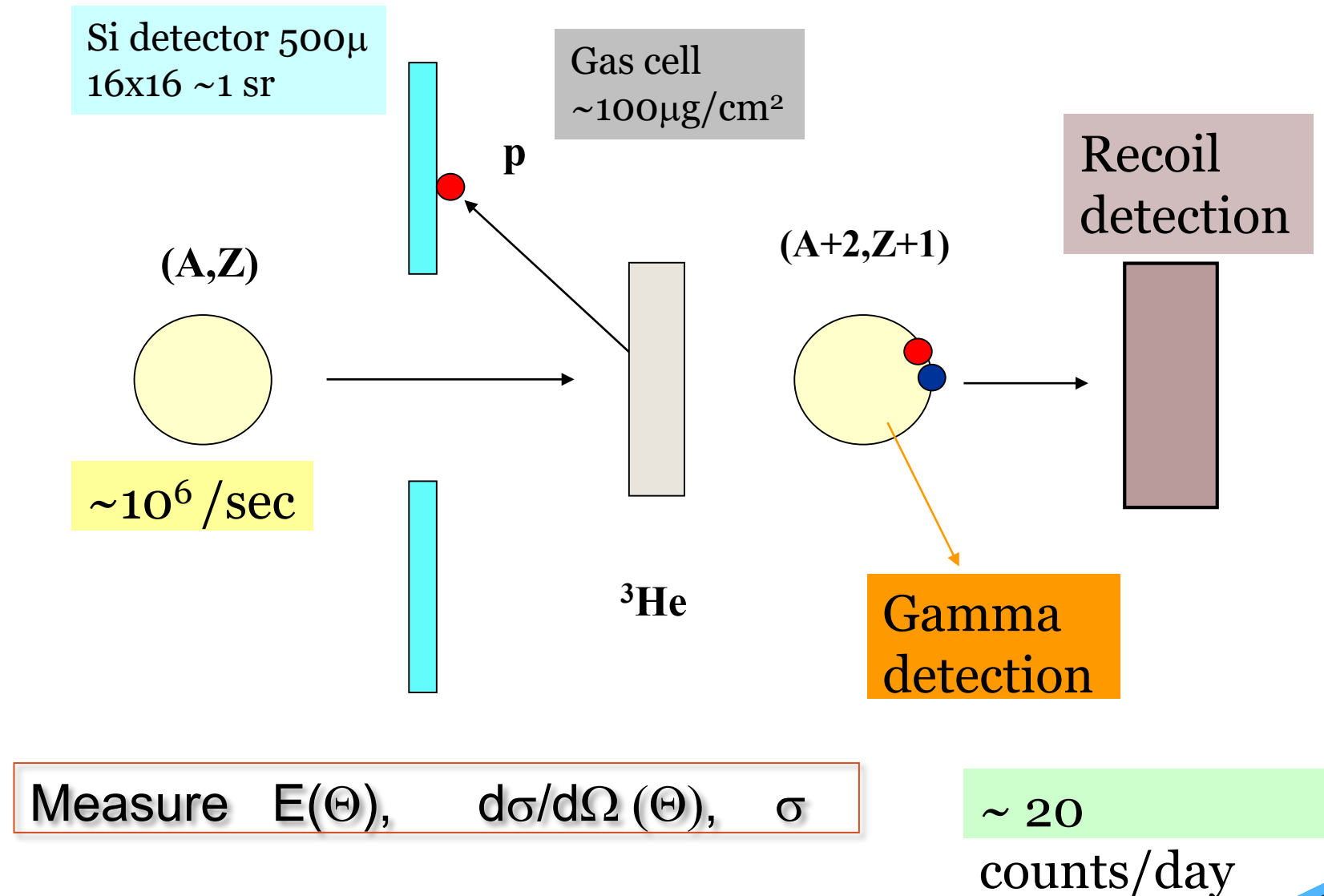
complementary studies from odd-odd  $T=1$  ground states to  $T=0, 1$  final states in even-even

$^{46}\text{V}$ ,  $^{50}\text{Mn}(d, \alpha)$  and  $(\alpha, d)$ ;  $(\alpha, ^6\text{Li})$  and  $(^6\text{Li}, \alpha)$

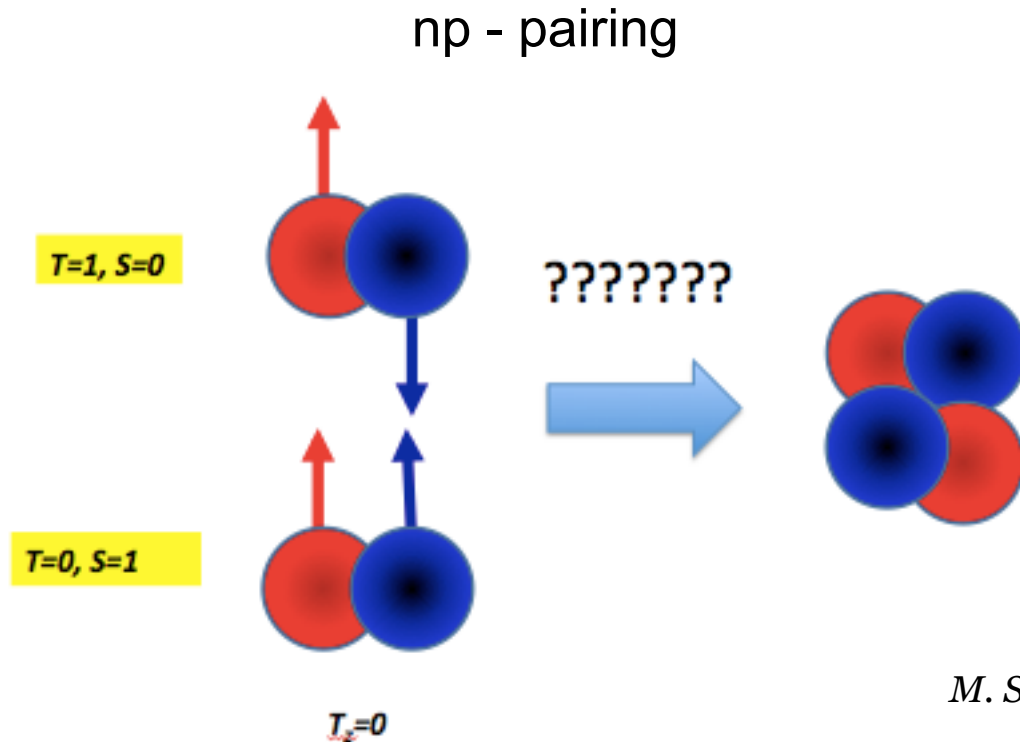
complementary studies to the reactions above



# Possible experimental setup



# Quartet condensate



**Alpha-like quartets** are predicted to appear in the ground state of alpha-conjugate nuclei as a collective state

*M. Sambataro and N. Sandulescu, Eur. Phys. J. A 53, 47 (2017)*

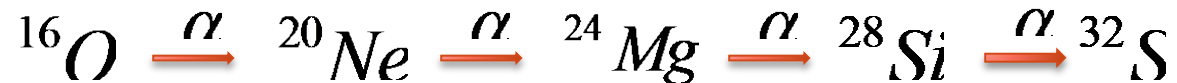
**ground state of even-even  $N=Z$  nuclei are close to a phase transition between an alpha boson-condensed gas and a quantum liquid**

*Lattice effective field theory calculations by S. Elhatisari et al, arXiv: 1602.04539 (2016); Nature 528, 111 (2015)*

# Short-term opportunities: Quarteting

- test of quartet condensation *alpha particle transfer along N=Z line*

$$\langle QCM(A+4) | Q^+ | QCM(A) \rangle \quad | QCM \rangle \equiv (Q^+)^{n_q} | - \rangle$$

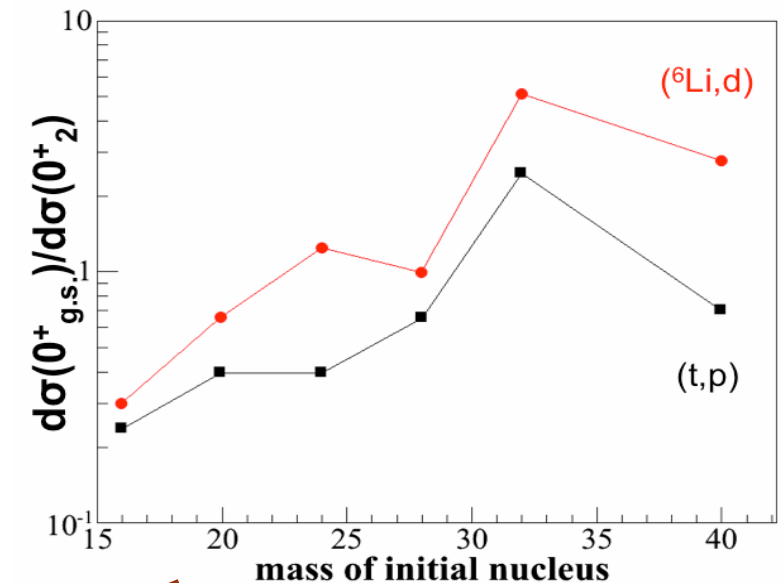


Revisit the *sd* shell:



(<sup>6</sup>Li,d) reactions: a powerful tool due to its clusterization  
(<sup>6</sup>Li, <sup>4</sup>He) reaction will test also T=0 pn pairing

M. Sambataro and N. Sandulescu, *Eur. Phys. J. A* 53, 47 (2017)



Data obtained with different energy regimes from 26 to 75 MeV and with different experimental setups: **questionable consistency of the systematics**

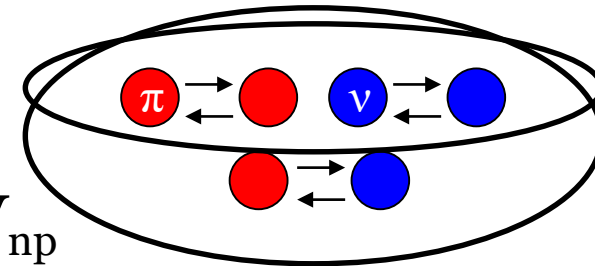


Isospin symmetry (breaking)

# Neutron-proton exchange symmetry

Charge Symmetry:  $V_{pp} = V_{nn}$

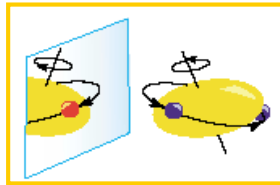
Charge Independence:  $(V_{pp} + V_{nn})/2 = V_{np}$



Deviations are small

The electromagnetic interaction lifts the degeneracy of the analogue states, but does not generally affect the underlying symmetry.

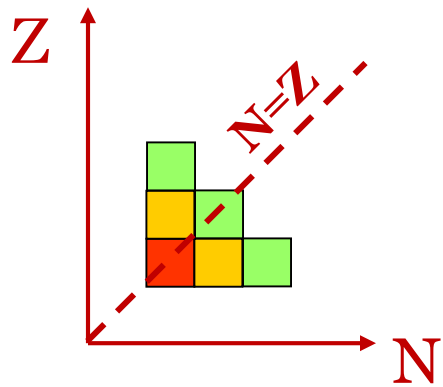
# Energy differences between analogue states



Mirror Energy Differences (MED)

$$\text{MED}_J = E_{x_{J, T_z = -T}} - E_{x_{J, T_z = +T}}$$

Test the charge symmetry of the interaction



Triplet Energy Differences (TED)

$$\text{TED}_J = E_{x_{J, T_z = -T}} + E_{x_{J, T_z = +T}} - 2E_{x_{J, T_z = 0}}$$

Test the charge independency of the interaction



Extensive studies of these **energy differences** have given important information on:

- ✿ Evolution of radii (deformation) along a rotational band
- ✿ The configuration of the states
- ✿ Isospin non-conserving terms of the interaction
- ✿ Estimate the neutron skin

A mature research field that relies on the isospin purity of the states

# Testing the EM transition (isospin) rules

A crucial next phase → to move to precision tests of the isospin symmetry through spectroscopic methods that probe the wave functions across isobaric multiplets.

The total transition matrix element  $M = \langle J_f M_f; T_f T_z | M_{jm}^0 + M_{jm}^1 | J_i M_i; T_i T_z \rangle$

with both isoscalar ( $M^0$ ) and isovector ( $M^1$ ) multipole operators.

Applying the Wigner-Eckart theorem ( $T_f = T_i = T$ )

$$M_{jm} = \frac{1}{\sqrt{2T+1}} \left[ \underbrace{M_{jm}^0}_{\text{isoscalar}} + \frac{T_z}{\sqrt{(T+1)T}} \underbrace{M_{jm}^1}_{\text{isovector}} \right]$$

for  $\Delta T = 0$  transitions

$$M(T_z) = a + bT_z$$

matrix elements **linear** in  $T_z$

## Isospin “selection rules”

- E1 transitions in mirror nuclei must be identical (no isoscalar component)
- E1 transitions forbidden in N=Z nuclei
- E2 transitions in T=1 isobaric triplets should lie on a line



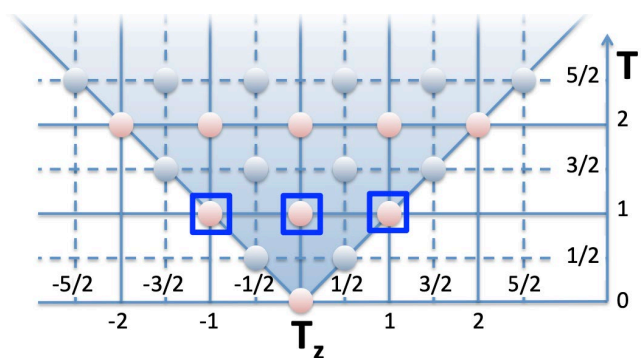
# Linearity of E2 matrix elements

$$M(T_z) = a + bT_z$$

Can only be tested in multiplets of  $T \geq 1$

Ideal case:  $B(E2)$  for the  $2^+_{T=1} \rightarrow 0^+_{T=1}$   
 Yields the proton matrix element,  $M_p$

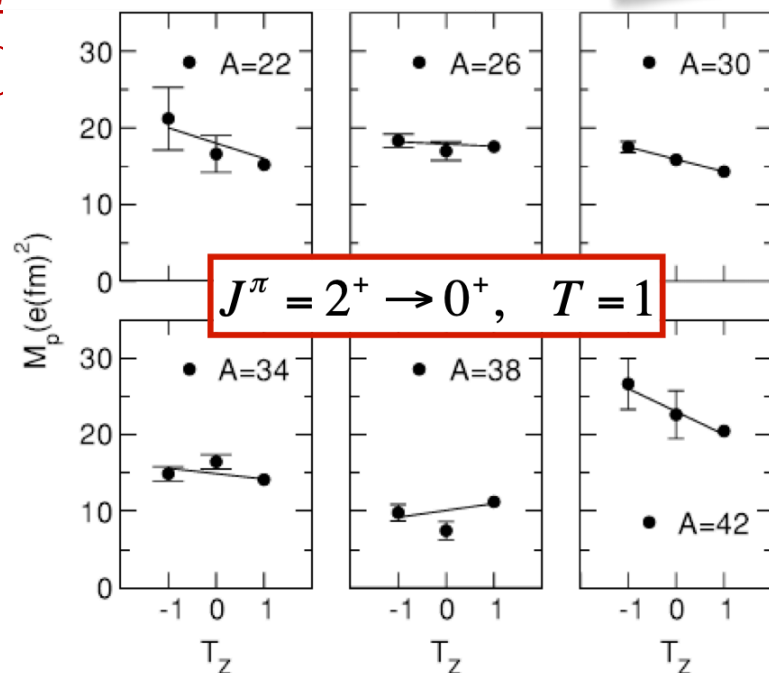
$$M_p = \sqrt{(2J_i + 1)B(E2, J_i \rightarrow J_f)}$$



Bentley's triangle

Proton matrix  
 element for isospin  
 triplets:

Experimentally  
 challenging



- High precision (e.g. lifetime) measurements required in exotic nuclei
- Systematic errors usually large (~10%)
- Isovector matrix element is small (not much variation with  $T_z$ )
- Odd-odd  $N=Z$  nuclei can be challenging

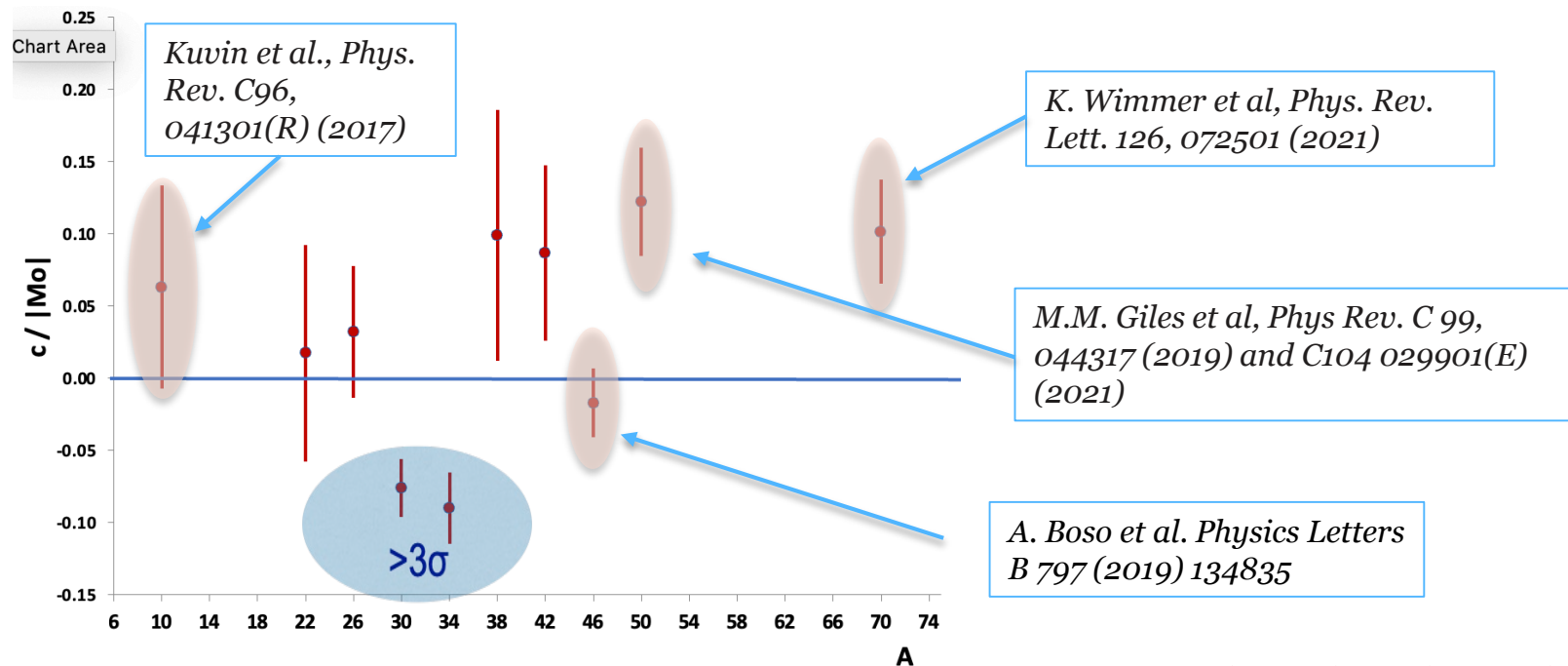
# Testing the linearity in T=1 Triplets

Deviations can be measured through fitting quadratic function to the transition ME in a multiplet

- Requires  $B(M\lambda)$  or  $B(E\lambda)$  measurements in  $\geq 3$  members of a multiplet
- All 3 members of a T=1 Triplet (10 examples to date)
- 4 members of a T=3/2 Quartet (no examples!)

$$M(T_Z) = a + bT_Z + [cT_Z^2]$$

From  $B(E2, 2_{T=1}^+ \rightarrow 0_{T=1}^+)$   
In all known T=1 triplets



Compilation by Mike Bentley (2021)

# Opportunities at LNL

## Measure the $B(E2)$ in isobaric triplets

### Reactions with stable beams:

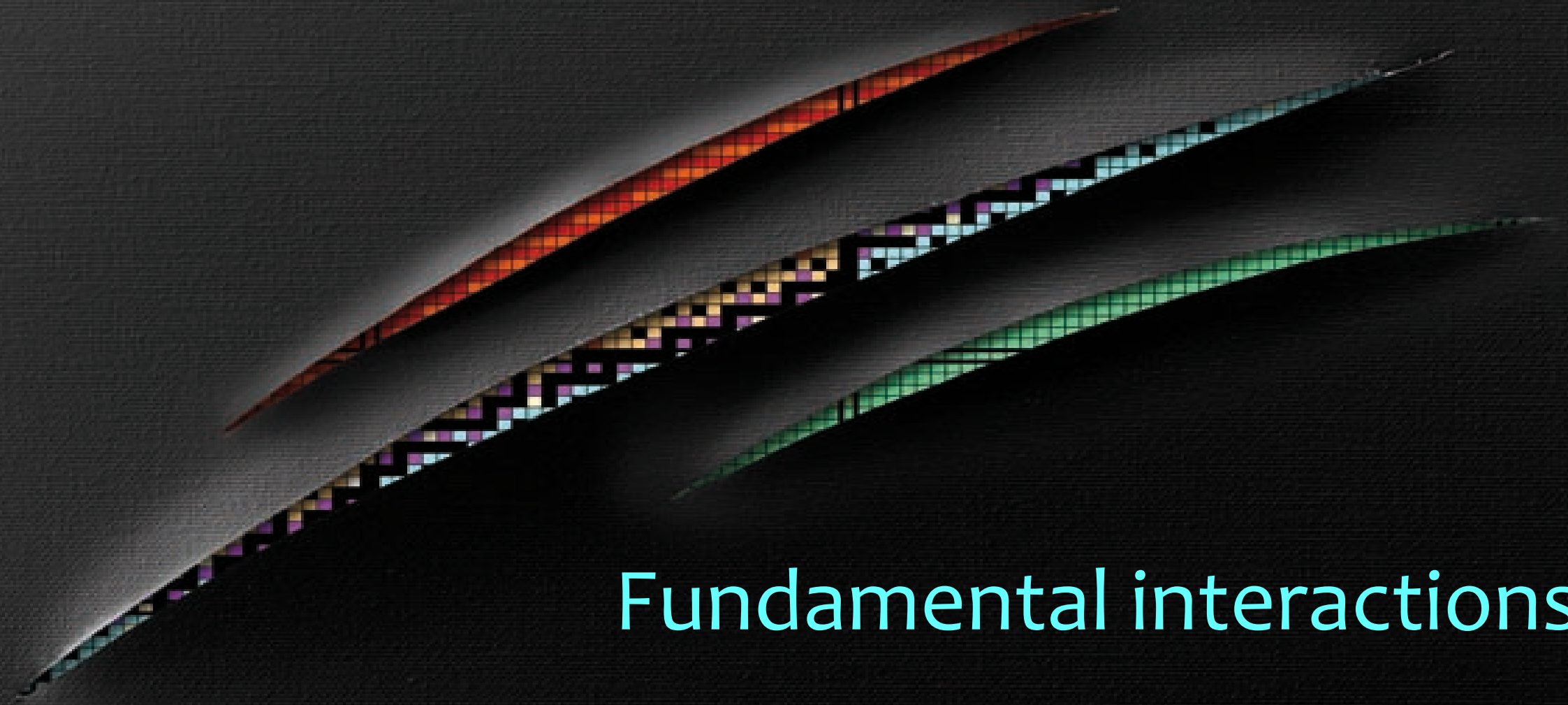
- Fusion-evaporation reactions (-2n evaporation channels)
- Reactions with solid  $^3\text{He}$  targets ( $^3\text{He},n$ )
- Selected cases may be done in multinucleon transfer with  $N=Z$  beam/target combinations utilizing PRISMA

**AGATA:** High-efficiency, gamma-gamma capability, position sensitivity (essential for high-velocity reactions and line-shapes)

**NEDA:** High-efficiency neutron detection

**Doppler-shift methods:** Lineshape analysis and Plunger-methods

**Cases chosen to aim for high-statistics high-precision measurements across multiplets (reduce systematic errors).**



Fundamental interactions

# Test the Standard Model:

## Looking for «new physics»

### Unitarity of the CKM three-generation quark mixing matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates      Cabibbo Kobayashi Maskawa (CKM) matrix      mass eigenstates

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \quad ?$$

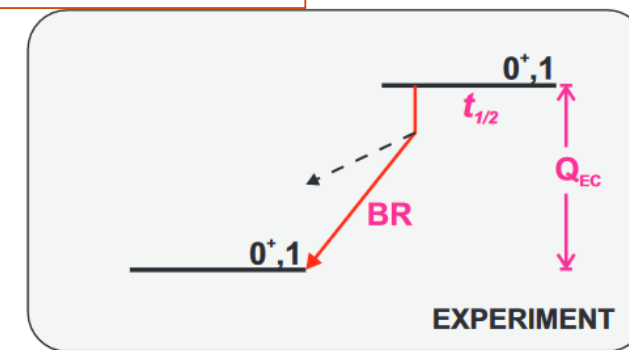
Information on  $V_{ud}$  can be obtained from:

- super-allowed Fermi beta decay
- free neutron lifetime
- mirror beta decay
- pion decay

### super-allowed Fermi beta decay

From transition strength of super-allowed Fermi beta decay between nuclear analogue states of  $I^\pi = 0^+ T = 1$ : vector transition

➤ vector coupling constant



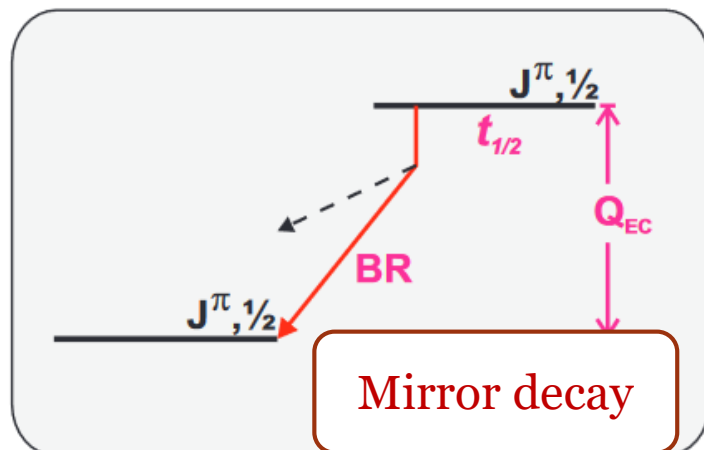
$$Ft \equiv ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C) = \frac{K}{2G_V^2(1 + \Delta_V^R)}$$

$Q$        $T_{1/2}, BR$       ← Experimental values

$$V_{ud} = \frac{G_V}{G_\mu}$$

These measurements have already arrived to a very good precision

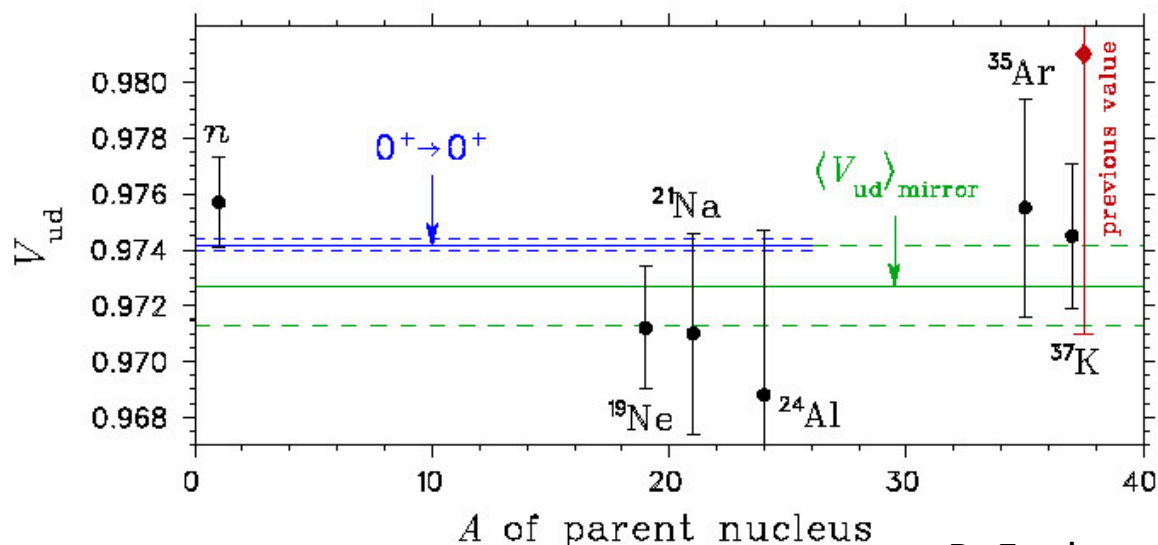
# Mirror beta transitions



Involve the vector (Fermi) and the axial component of the weak current (Gamow Teller)

$$ft = ft (1 + \delta'_R) [1 - (\delta_C - \delta_{NS})] = \frac{K}{G_V^2 (1 + \Delta_R) (1 + \lambda^2 \langle \sigma \tau \rangle^2)}$$

$$\lambda = G_A / G_V$$



From mirror decay:

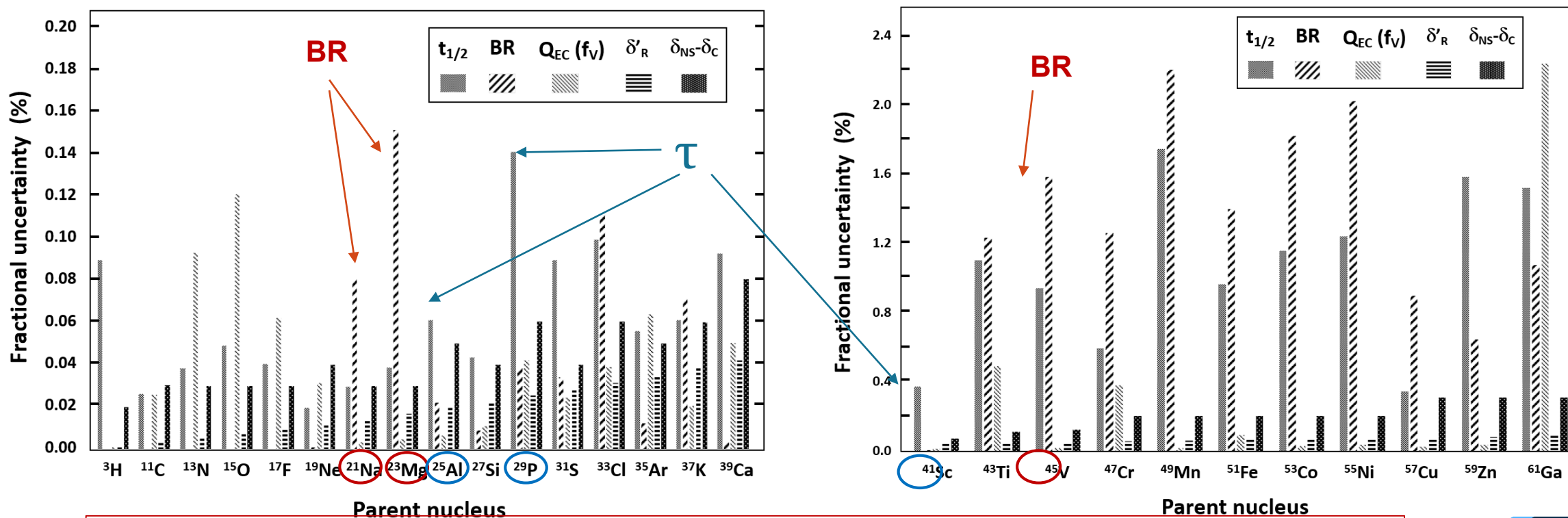
$$V_{ud} (T=1/2) = 0.9727 + 0.0014$$

From super-allowed:

$$V_{ud} (0^+ \rightarrow 0^+) = 0.9742 + 0.0002$$

# Precision measurements to look for new physics

Fractional contribution of the experimental and theoretical input factors to the error of mirror  $Ft$  values

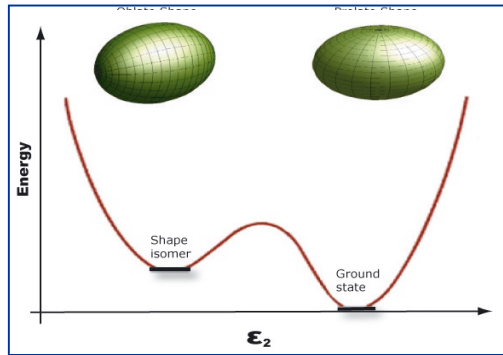


Precision measurements of the **branching ratio** and/or **lifetimes** at the beta-decay station with SiC and TiC targets in SPES

# Perspectives with $N \sim Z$ nuclei

## Understanding the nuclear interaction

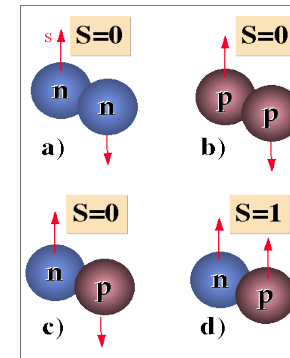
### Shapes and shape coexistence



Lifetime measurements  
of excited states in  $N \sim Z$   
nuclei

Stable beams  
AGATA + NEDA +  
plunger

### $pn$ $T=0$ pairing and quarteting



Transfer reactions

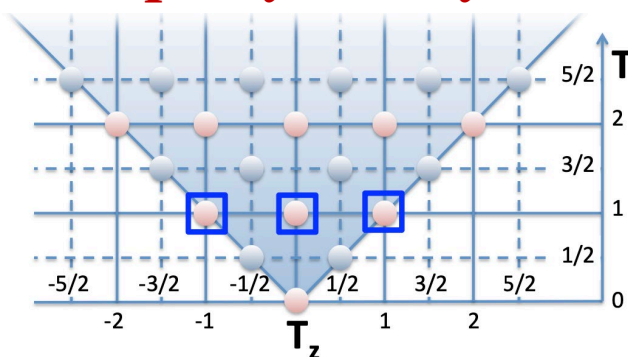
$({}^3\text{He}, p)$ ,  $({}^4\text{He}, d)$ ,  $({}^6\text{Li}, d)$

Stable beams ( $sd$  shell)

SPES beams ( $fp$  shell)

GRIT+ Ge det + recoil det.

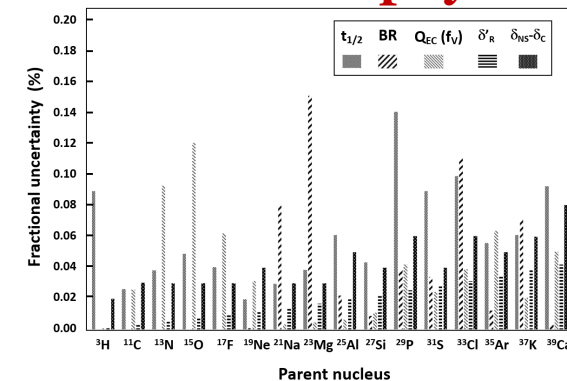
### Isospin symmetry breaking



Lifetime measurements  
of the  $2+$  state in  $T=1$   
isobaric triplets

Stable beams  
AGATA + NEDA +  
plunger

### Search of new physics with Mirror beta decay



Lifetime and BR  
measurements in mirror  
beta decay

SPES Beams

Beta-decay station +  
Beta + Ge det.



# Thank you!

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