

Parity-violating pion-nucleon coupling from Lattice QCD

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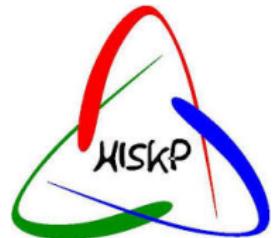
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Long-range nuclear parity violation ...

$\Delta I = 1$, LO, P -odd pion-nucleon interaction

$$\mathcal{L}_{\text{PV}}^{\text{w}} = -\frac{h_{\pi}^1}{\sqrt{2}} \bar{N} (\vec{\tau} \times \vec{\pi})^3 N + \dots = i h_{\pi}^1 (\bar{n} p \pi^- - \bar{p} n \pi^+) + \dots$$

with P -odd πN coupling

$$h_{\pi}^1 = -\frac{i}{2m_N} \lim_{p_{\pi} \rightarrow 0} \langle n \pi^+ | \mathcal{L}_{\text{PV}}^{\text{w}}(0) | p \rangle,$$

effective (at QCD scale) weak Lagrangian

$$\mathcal{L}_{\text{PV}}^{\text{w}} = -\frac{G_F}{\sqrt{2}} \frac{\sin^2(\theta_W)}{3} \sum_i \left(C_i^{(1)} \theta_i^{(\ell)} + S_i^{(1)} \theta_i^{(s)} \right)$$

$$\theta_1^{(\ell)} = \bar{q}_a \gamma_{\mu} \mathbb{1} q_a \bar{q}_b \gamma^{\mu} \gamma_5 \tau^3 q_b, \quad \theta_2^{(\ell)} = \bar{q}_a \gamma_{\mu} \mathbb{1} q_b \bar{q}_b \gamma^{\mu} \gamma_5 \tau^3 q_a$$

$$\theta_3^{(\ell)} = \bar{q}_a \gamma_{\mu} \gamma_5 \mathbb{1} q_a \bar{q}_b \gamma^{\mu} \tau^3 q_b$$

$$\theta_1^{(s)} = \bar{s}_a \gamma_{\mu} s_a \bar{q}_b \gamma^{\mu} \gamma_5 \tau^3 q_b, \quad \theta_2^{(s)} = \bar{s}_a \gamma_{\mu} s_b \bar{q}_b \gamma^{\mu} \gamma_5 \tau^3 q_a,$$

$$\theta_3^{(s)} = \bar{s}_a \gamma_{\mu} \gamma_5 s_a \bar{q}_b \gamma^{\mu} \tau^3 q_b, \quad \theta_4^{(s)} = \bar{s}_a \gamma_{\mu} \gamma_5 s_b \bar{q}_b \gamma^{\mu} \tau^3 q_a,$$

and known Wilson coefficients $C^{(1)}(\Lambda_{\chi})$, $S^{(1)}(\Lambda_{\chi})$

... with Parity-Conserving Lagrangian

Application of *Soft Pion Theorem* by Feng, Guo & Seng,

[Phys.Rev.Lett. 120 (2018) 18, 181801; Eur.Phys.J.C 79 (2019) 1, 22]

$$F_\pi h_\pi^1 = -\frac{(\delta m_N)_{4q}}{\sqrt{2}} + \text{ corrections} \quad (\text{PCAC \& } \chi\text{PT})$$

with induced proton-neutron mass shift

$$(\delta m_N)_{4q} = (m_n - m_p)_{4q} = \frac{1}{m_N} \langle p | \mathcal{L}_{\text{PC}}^w(0) | p \rangle$$
$$\mathcal{L}_{\text{PC}}^w = -\frac{G_F}{\sqrt{2}} \frac{\sin^2(\theta_W)}{3} \sum_i \left(C_i^{(1)} \theta_i^{(\ell)\prime} + S_i^{(1)} \theta_i^{(s)\prime} \right)$$

now with P -even operators

$$\theta_1^{(\ell)\prime} = \bar{q}_a \gamma_\mu \mathbb{1} q_a \bar{q}_b \gamma^\mu \tau^3 q_b, \quad \theta_2^{(\ell)\prime} = \bar{q}_a \gamma_\mu \mathbb{1} q_b \bar{q}_b \gamma^\mu \tau^3 q_a$$

$$\theta_3^{(\ell)\prime} = \bar{q}_a \gamma_\mu \gamma_5 \mathbb{1} q_a \bar{q}_b \gamma^\mu \gamma_5 \tau^3 q_b$$

$$\theta_1^{(s)\prime} = \bar{s}_a \gamma_\mu s_a \bar{q}_b \gamma^\mu \tau^3 q_b, \quad \theta_2^{(s)\prime} = \bar{s}_a \gamma_\mu s_b \bar{q}_b \gamma^\mu \tau^3 q_a,$$

$$\theta_3^{(s)\prime} = \bar{s}_a \gamma_\mu \gamma_5 s_a \bar{q}_b \gamma^\mu \gamma_5 \tau^3 q_b, \quad \theta_4^{(s)\prime} = \bar{s}_a \gamma_\mu \gamma_5 s_b \bar{q}_b \gamma^\mu \gamma_5 \tau^3 q_a,$$

Approach from Lattice QCD

Our milestones

- ① Method for Lattice QCD calculation of PC 4-quark operator matrix elements

$$\langle N | \theta_i^{(\ell)\prime} | N \rangle, \quad \langle N | \theta_i^{(s)\prime} | N \rangle$$

at non-zero lattice spacing ($a > 0$), in finite volume ($L < \infty$) and at (un-)physical pion mass ($m_\pi \geq m_\pi^{\text{phys}}$)

- ② Renormalization procedure for composite 4-quark operators on the lattice
- ③ Target simulation at physical pion mass with controlled continuum ($a \rightarrow 0$) & infinite volume ($L \rightarrow \infty$) extrapolation
- ④ Direct calculation (PV Lagrangian, 2-hadron $N\pi$, Lellouch-Lüscher)

We are in transition from 1 → 2.

Lattice How-To: Leading-order mass shift from LQCD

Lattice QCD simulations are pure $N_f = 2 + 1 + 1$ QCD
(mass-degenerate u , d and c , s)

Weak interaction treated as a perturbation with *Feynman-Hellmann-Theorem*
promoted by [Phys.Rev.D 96 (2017) 1, 014504] for nucleon couplings to quark-bilinears

$$S = S_{\text{LQCD}} + \lambda \sum_x \mathcal{L}_{\text{PC}}^{\text{w}}(x)$$

Effective mass of nucleon state in λ -vacuum $|0\rangle \rightarrow |\lambda\rangle$

$$C_\lambda(t) = \langle N(t) | \bar{N}(0) \rangle_\lambda \xrightarrow{t \text{ large}} \langle \lambda | N | p \rangle \frac{1}{2m_N(\lambda)} \langle p | \bar{N} | \lambda \rangle e^{-t m_N(\lambda)}$$

$$m_\lambda^{\text{eff}}(t | \tau) = \frac{1}{\tau} \log(C_\lambda(t)/C_\lambda(t+\tau)) \xrightarrow{t \text{ large}} m_N(\lambda)$$

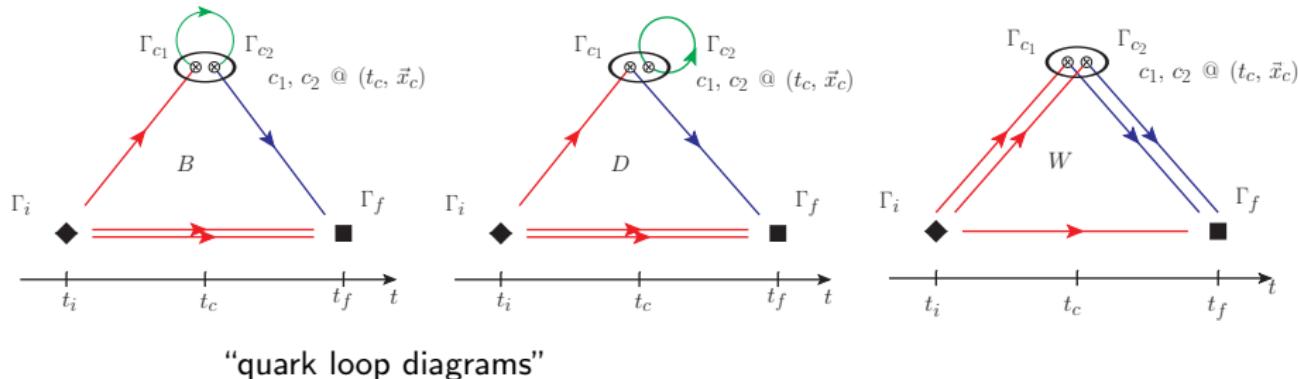
Leading-order perturbation

$$\left. \frac{dm_\lambda^{\text{eff}}(t | \tau)}{d\lambda} \right|_{\lambda=0} = \frac{1}{\tau} [\partial_\lambda C_\lambda(t) / C_{2pt}(t) - \partial_\lambda C_\lambda(t+\tau) / C_{2pt}(t+\tau)] \Big|_{\lambda=0}$$

$$\partial_\lambda C_\lambda(t)|_{\lambda=0} = -\langle N(t) \sum_{t_c, \vec{x}} \mathcal{L}_{\text{PC}}^{\text{w}}(t_c, \vec{x}) | \bar{N}(0) \rangle_{\lambda=0} \propto \sum_{t_c} C_{3pt}(t, t_c)$$

Quark flow diagrams

Wick contractions from 4-quark operator insertions



Each quark line = fully dressed quark propagator in a gauge field background
 $U = \{U_\mu(x)\}$

Solution of lattice Dirac equation $D_{\text{flavor}}[U] \psi = \eta$

Vertices for nucleons and 4-quark operators (product of quark-bilinesars)



$$\sim \epsilon_{abc} \bar{u}^a \Gamma_i \bar{d}^{bT} \bar{u}^c$$



$$\sim \epsilon_{abc} u^{aT} \Gamma_f d^b u^c$$



$$\sim \bar{q}(t_c, \vec{x}_c) \Gamma_{c1} q(t_c, \vec{x}_c) \bar{q}(t_c, \vec{x}_c) \Gamma_{c2} q(t_c, \vec{x}_c)$$

Our numerical evaluation in Lattice QCD

We study the FHT ratio for $f = \ell, s$ and operators $i = 1, 2, \dots$

$$R_i^{(f)\prime}(t, \tau) = \frac{\xi}{\sqrt{\xi^2 - 1}} \times \frac{1}{\tau} \left(\frac{C_{3pt}(t + \tau) + C_{3pt}(t - \tau)}{C_{2pt}(t + \tau) + C_{2pt}(t - \tau)} - \frac{C_{3pt}(t)}{C_{2pt}(t)} \right),$$

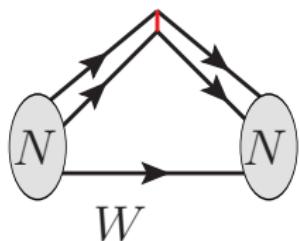
$$\stackrel{t \text{ large}}{\underbrace{\langle \mathbf{n} | \theta_i^{(f)\prime} | \mathbf{n} \rangle}}_{2m_N}$$

$$\xi = \frac{C_{2pt}(t + \tau) + C_{2pt}(t - \tau)}{2C_{2pt}(t)},$$

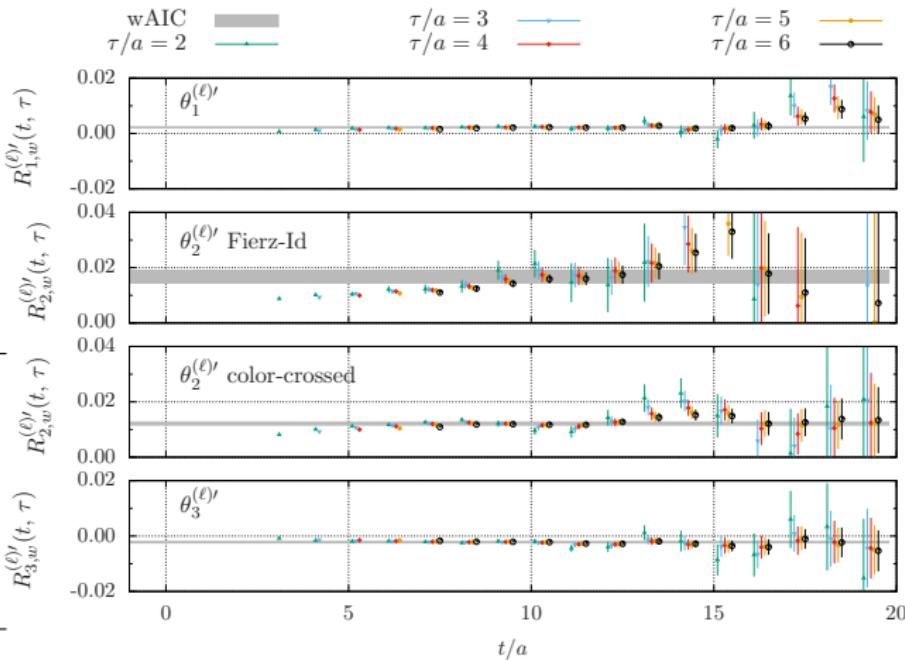
Gauge field ensemble by Extended Twisted Mass Collaboration

	$L^3 \times T$	a [fm]	L [fm]	m_π [MeV]	$m_\pi L$	m_N [MeV]
cA211.30.32	$32^3 \times 64$	0.097	3.1	261.1 (1.1)	4.01	1028 (4)
Wasem (2012)	$20^3 \times 256$	0.123	2.5	389	4.93	

W diagram for light quark operators (preliminary)



k	$\langle N \theta_{w,k}^{(\ell)\prime} N \rangle / (2m_N)$
1	$2.18^{+0.19}_{-0.17} \cdot 10^{-3}$
2	$1.66^{+0.24}_{-0.21} \cdot 10^{-2}$
3	$-2.18^{+0.22}_{-0.25} \cdot 10^{-3}$



$$h_\pi^1 = a^4 \frac{G_F \cdot (\hbar c/a)^2 \sin^2(\theta_W)}{3 a f_\pi^0} \times \sum_{i=1}^3 C_i^{(1)} \frac{\langle N | \theta_i^{(\ell)\prime}(0) | N \rangle}{2 am_N}$$

$$= 2.31^{+0.32}_{-0.24} \cdot 10^{-7} \quad (\text{W only, bare})$$

Comparison to other determinations (preliminary)

in units of 10^{-7}

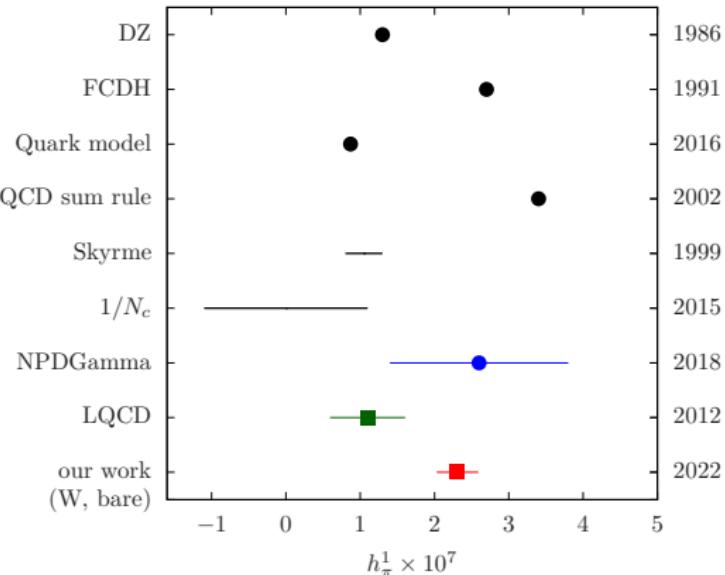
$$h_{\pi}^1 \text{ (NPDGamma)} = 2.6 \text{ (1.2)}$$

$$h_{\pi}^1 \text{ (Wasem 2012)} = 1.10 \text{ (0.51)}$$

$$h_{\pi}^1 \text{ (our work 2022)} = 2.31^{+0.32}_{-0.24}$$

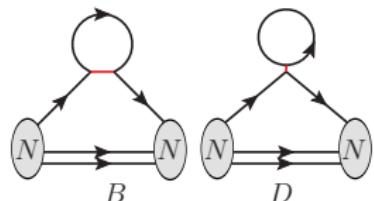
both LQCD feature

- W topology only
- no renormalization

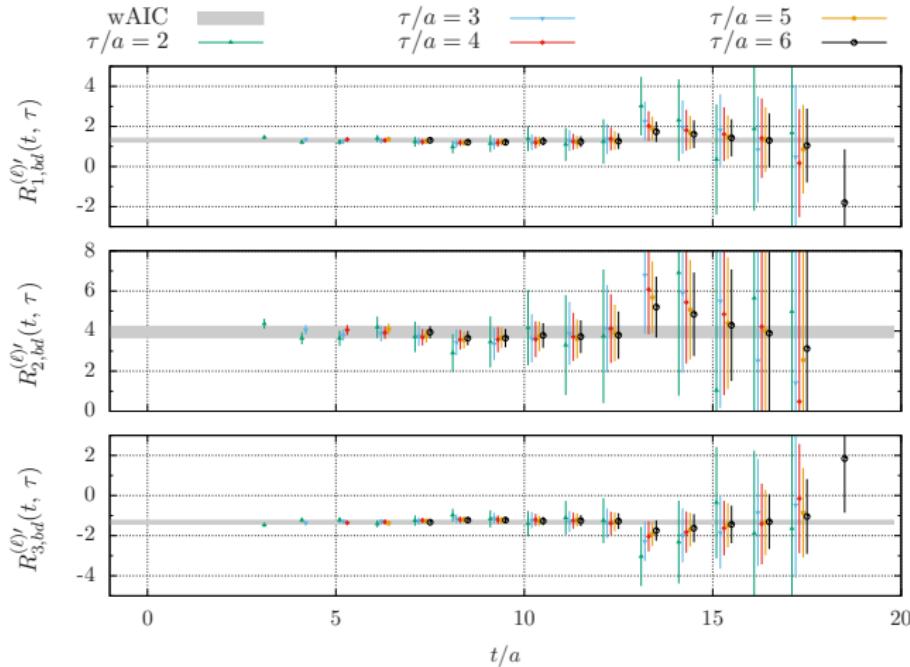


- Wasem: \mathcal{L}_{PV} , " πN " $\rightarrow N$, $m_{\pi} = 390$ MeV
- Us: \mathcal{L}_{PC} , $N \rightarrow N$, Soft Pion Theorem, FHT, $m_{\pi} = 260$ MeV

B+D diagram for light quark operators (preliminary)



k	$\frac{\langle N \theta_{bd,k}^{(\ell)\prime} N \rangle}{2m_N}$
1	$1.307^{+0.082}_{-0.094}$
2	$3.93^{+0.25}_{-0.28}$
3	$-1.320^{+0.097}_{-0.085}$



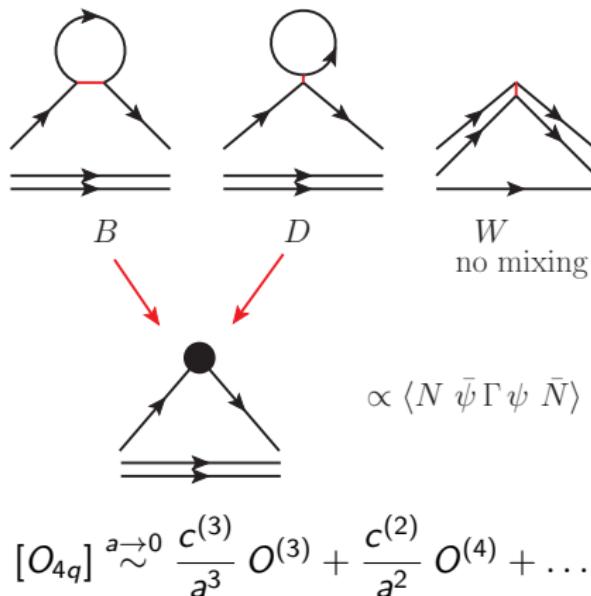
First LQCD estimates of for $\theta_k^{(\ell)\prime}$ B and D diagram.

2 orders of magnitude larger than W contribution.

Outlook towards non-perturbative renormalization

The trouble with 4-quark operators (work in progress)

- big challenge: power-divergent mixing under renormalization with operators of lower and equal dimension



- generated by explicit symmetry breaking in LQCD (parity, chiral symmetry)

4-quark operators and symmetries

$$\mathcal{O}_\Gamma = \bar{\psi} \Gamma \psi \bar{\psi} \Gamma \tau^3 \psi, \quad \Gamma = \{\gamma_\mu\}, \{\gamma_\mu \gamma_5\}, 1, \gamma_5$$

Lattice action (rotated to physical quark field basis):

$$\begin{aligned} S_{\text{tm}}^\ell &= a^4 \sum_{f=u,d} \sum_x \bar{\psi}_f(x) \left(\gamma_\mu \bar{\nabla}_\mu - i\gamma_5 W_{\text{cr}} + m_f + \frac{r_f c_{\text{SW}}}{4} \gamma_5 \sigma_{\mu\nu} G_{\mu\nu} \right) \psi_f(x) \\ \bar{\nabla}_\mu &= \frac{1}{2} (\nabla_\mu + \nabla_\mu^*) , \quad W_{\text{cr}} = -\frac{a r_f}{2} \nabla_\mu \nabla_\mu^* + M_{\text{cr}}(r_f) \end{aligned}$$

Transformations:

- Parity \mathcal{P} , Charge conjugation \mathcal{C} , time reversal \mathcal{T}
- \mathcal{R}_5 parity : $\psi \rightarrow \gamma_5 \psi$, $\bar{\psi} \rightarrow -\bar{\psi} \gamma_5$
- dimension counting \mathcal{D} : $U_\mu(x) \rightarrow U_\mu(-x - a\hat{\mu})^\dagger$, $\psi(x) \rightarrow e^{3\pi i/2} \psi(-x)$,
 $\bar{\psi}(x) \rightarrow e^{3\pi i/2} \bar{\psi}(-x)$
- exchange parity \mathcal{S} : $u \leftrightarrow d$, $\bar{u} \leftrightarrow \bar{d}$
- change of sign of quark masses $[m_f \rightarrow -m_f] = [-m_f]$

4-quark operators and symmetries

Reference symmetry quantum numbers (+ scalar & flavor content)

Operator	$\mathcal{PD}[-m_f]$	$\mathcal{TD}[-m_f]$	\mathcal{C}	\mathcal{DR}_5	\mathcal{PS}
$\mathcal{O}_\Gamma = \bar{\psi} \Gamma \psi \bar{\psi} \Gamma \tau^3 \psi$	+1	+1	+1	+1	-1

The mixing operator candidates (light quarks):

$$\text{dim 3 } \bar{\psi} \gamma_5 \otimes \mathbb{1} \psi$$

$$\text{dim 4 } m_f \bar{\psi} \mathbb{1} \otimes \tau^3 \psi, \quad \bar{\psi} \not{D} \otimes \tau^3 \psi$$

$$\begin{aligned} \text{dim 5 } & m_f^2 \bar{\psi} \gamma_5 \otimes \mathbb{1} \psi, \quad m_f \bar{\psi} \gamma_5 \not{D} \otimes \mathbb{1} \psi, \quad \bar{\psi} \gamma_5 D^2 \otimes \mathbb{1} \psi, \\ & \bar{\psi} \sigma \tilde{G} \otimes \mathbb{1} \psi, \quad m_f G \tilde{G} \end{aligned}$$

$$\begin{aligned} \text{dim 6 } & m_f^3 \bar{\psi} \mathbb{1} \otimes \tau^3 \psi, \quad m_f^2 \bar{\psi} \not{D} \otimes \tau^3 \psi, \quad m_f \bar{\psi} D^2 \otimes \tau^3 \psi \\ & m_f \bar{\psi} \sigma G \otimes \tau^3 \psi, \quad \bar{\psi} D^2 \not{D} \otimes \tau^3 \psi, \quad \bar{\psi} \sigma G \not{D} \otimes \tau^3 \psi \end{aligned}$$

Renormalization with Lüscher's Gradient Flow

Based on Gradient Flow [JHEP 08 (2010) 071] at non-zero flow time for Yang-Mills field $B_\mu(t, x)|_{t=0} = A_\mu(x)$ and quark-/anti-quark fields: $\chi|_{t=0} = \psi$, $\bar{\chi}|_{t=0} = \bar{\psi}$

$$\partial_t B_\mu = D_\nu G_{\nu\mu} + \alpha_0 D_\mu \partial_\nu B_\nu$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu], \quad D_\mu = \partial_\mu + [B_\mu, \cdot]$$

$$\partial_t \chi = \Delta \chi - \alpha_0 \partial_\nu B_\nu \chi, \quad \partial_t \bar{\chi} = \bar{\chi} \overset{\leftarrow}{\Delta} + \alpha_0 \bar{\chi} \partial_\nu B_\nu$$

$$\Delta = D_\mu D_\mu, \quad D_\mu = \partial_\mu + B_\mu$$

- fully *non-perturbative* subtraction of power divergent terms (as $a \rightarrow 0$)
[Phys.Rev.D 104 (2021) 7, 074516]

OPE at small but non-zero flow time $t = \text{Short Flow Time Expansion (SFTE)}$

$$[O_{4q}(x; t)]_R \stackrel{t \rightarrow 0}{\sim} c(t) m_f \bar{q} \tau^3 q(x), \quad c(t) \sim 1/t$$

- continuum limit at non-zero flow time \implies subtraction under continuum symmetries

Outlook towards a direct calculation

$$h_\pi^1 = -\frac{i}{2m_N} \lim_{p_\pi \rightarrow 0} \langle n\pi^+ | \mathcal{L}_{PV}^w(0) | p \rangle, \quad \mathcal{L}_{PV}^w = -\frac{G_F}{\sqrt{2}} \frac{\sin^2(\theta_W)}{3} \sum_i \left(C_i^{(1)} \theta_i^{(\ell)} + S_i^{(1)} \theta_i^{(s)} \right)$$

Matrix element for $2 \rightarrow 1$ transition amplitude $\langle n\pi^+ | \theta_i^{(f)\prime} | p \rangle$

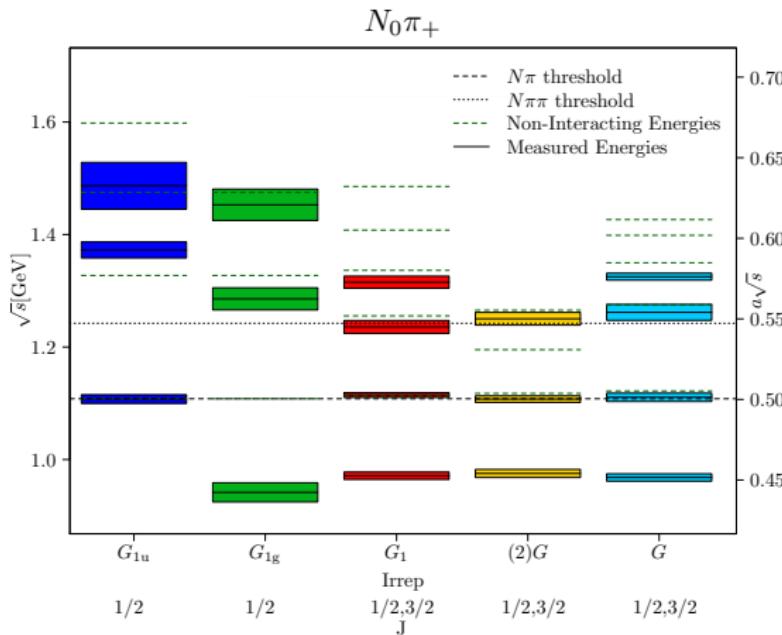
- mismatch of initial and final state energy level

$$\mathcal{L}_{PV}^w = h_\pi^1 (\bar{p}\pi^+ n - \bar{n}\pi^- p) + \hbar_E D_t (\bar{p}\pi^+ n - \bar{n}\pi^- p)$$

- $N\pi$ state: conversion from finite to infinite volume with Lellouch-Lüscher / BHWL formalism [PRD91 no. 3, (2015) 034501, PRD94 no. 1, (2016) 013008]
⇒ $N\pi$ scattering phase shift from lattice spectrum with Lüscher method
- increased complexity of quark-flow diagrams & signal-to-noise ratio challenge
- mixing with lower-dimensional operators excluded by \mathcal{P} , \mathcal{C} and \mathcal{S} symmetries

Outlook towards a direct calculation

First step: $I = 1/2$ nucleon-pion spectrum at physical pion mass (preliminary)



[F. Pittler @ Lattice 2022]

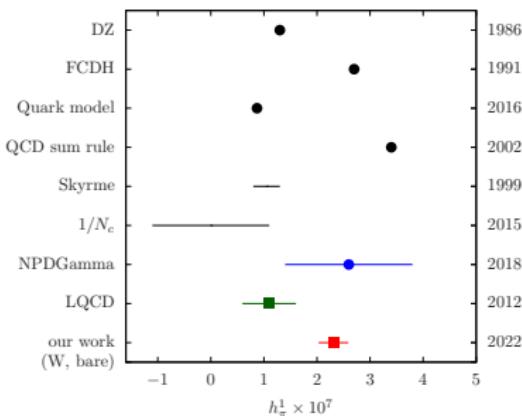
	$L^3 \times T$	a [fm]	L [fm]	m_π [MeV]	$m_\pi L$
cA2.09.48	$48^3 \times 96$	0.0913	4.3	134	3.0

We set our sails (both ways ...)



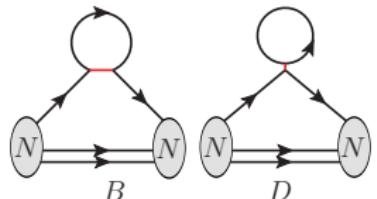
Summary

- efficient numerical method for $\langle N | \theta_{PC}^{(f)\prime} | N \rangle$ (incl. quark-loop diagrams)
- h_π^1 bare with W topology — pulled even with previous LQCD calculation agreement with NPDGamma experimental result
- scalable to *physical pion mass & larger volume*
- mixing and renormalization with Gradient Flow under way
- $N\pi$ $I = 1/2$ scattering analysis at physical pion mass as input for $N\pi \rightarrow N$ via \mathcal{L}_{PV}^W

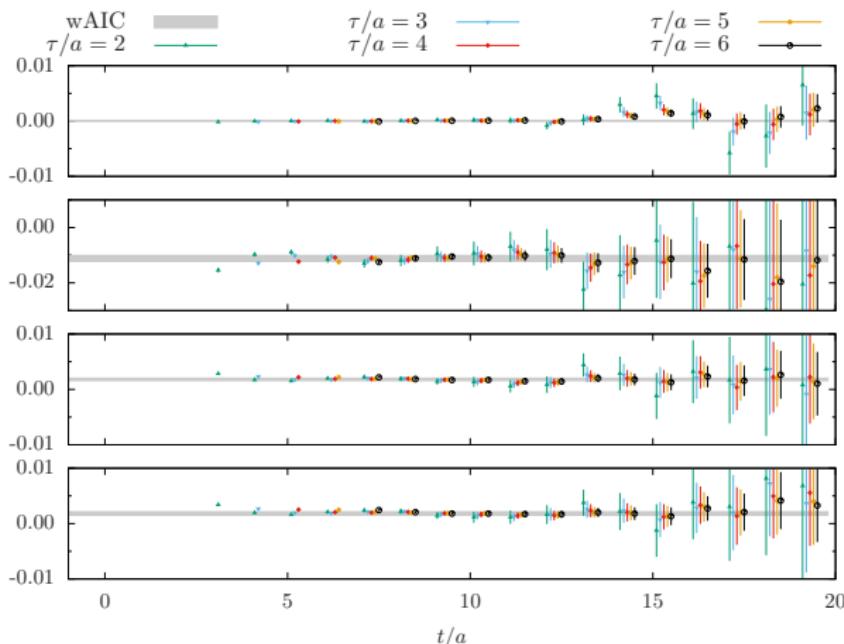


Thank you very much for your attention.

B and D for strange quark operators (preliminary)



k	$\frac{\langle N \theta_{b/d,k}^{(s)\prime} N \rangle}{2m_N}$	$R_{1,d}^{(s)\prime}(t, \tau)$	$R_{2,b}^{(s)\prime}(t, \tau)$	$R_{3,d}^{(s)\prime}(t, \tau)$	$R_{4,b}^{(s)\prime}(t, \tau)$
1	$0.002^{+0.010}_{-0.008} \cdot 10^{-2}$				
2	$-1.117^{+0.107}_{-0.100} \cdot 10^{-2}$				
3	$0.182^{+0.021}_{-0.024} \cdot 10^{-2}$				
4	$0.185^{+0.029}_{-0.032} \cdot 10^{-2}$				



First LQCD estimates of $\theta_k^{(s)\prime}$ B and D diagram.