

Role of a triangular singularity in the $\gamma p \rightarrow p \pi^0 \eta$ reaction

Volker Metag, Mariana Nanova,
II. Physikalisches Institut



for the CBELSA/TAPS Collaboration

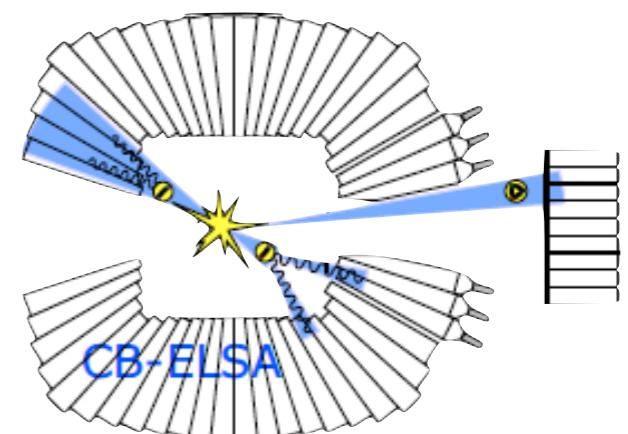
Outline:

- ❖ observation and characterisation of a structure at $M_{p\eta} \approx 1710$ MeV
- ❖ interpretation: triangular singularity at opening of the $\gamma p \rightarrow p a_0$ channel
- ❖ summary and conclusions

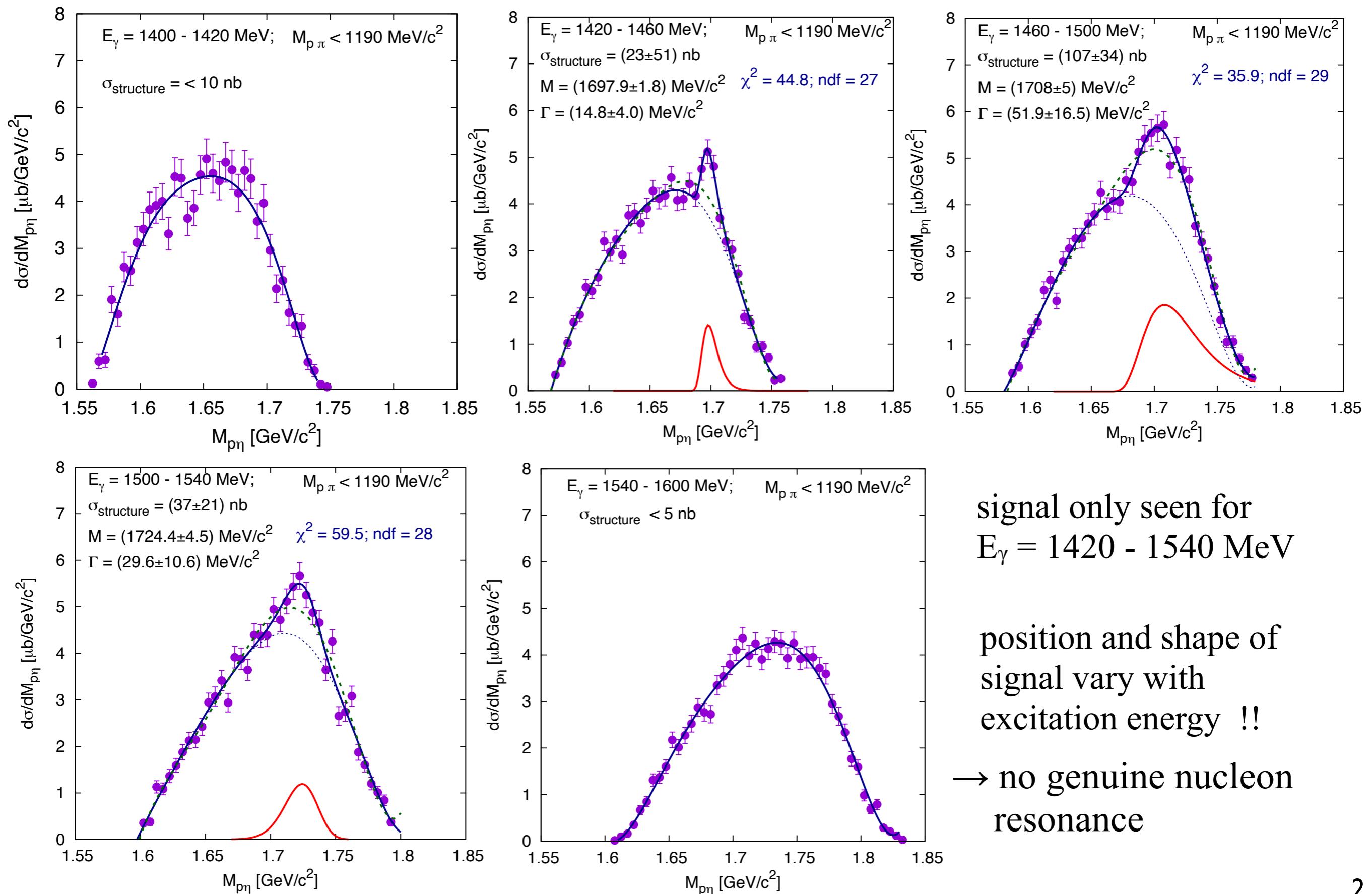


Experimental Physics II

NSTAR 2022
October 17-21, 2022
Santa Margherita Ligure, Genova, Italy



properties of structures as function of the incident photon energy



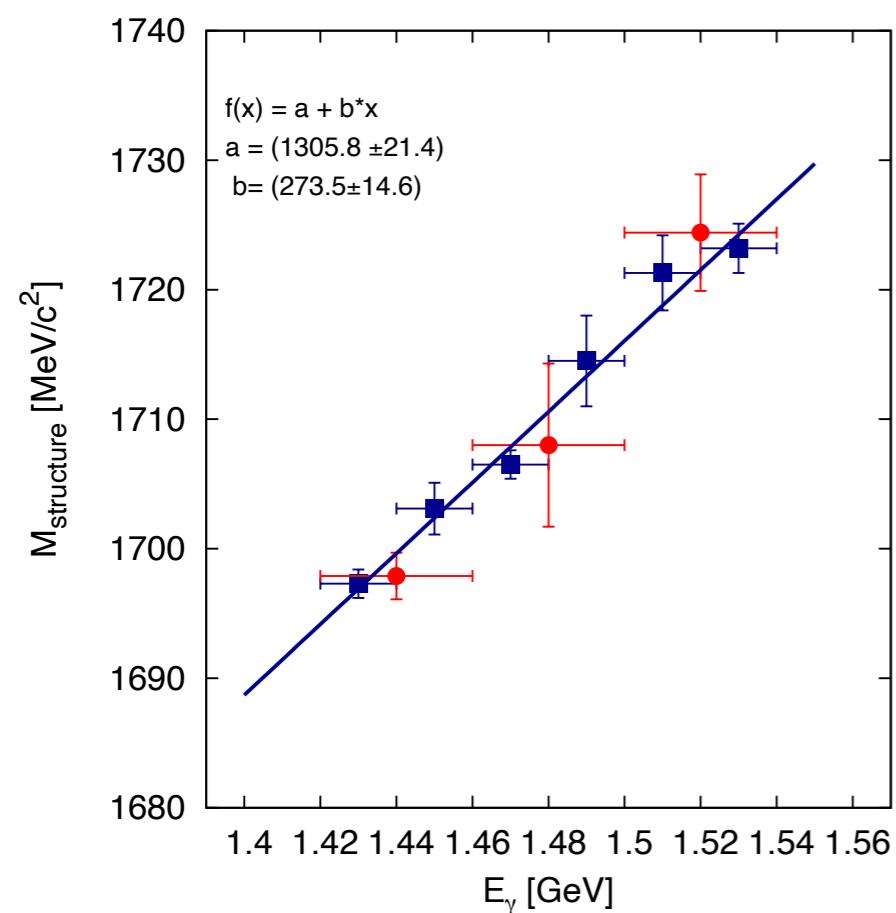
properties of the structure as function of the excitation energy

signal fitted with Novosibirsk function

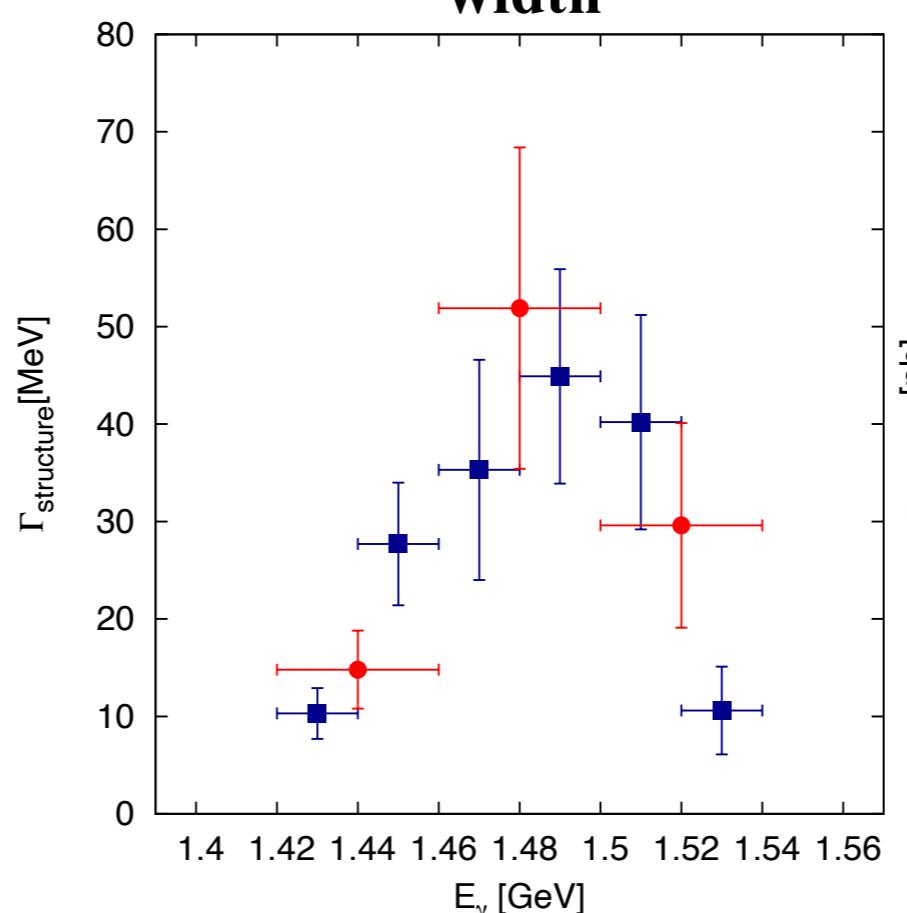
signal fitted with Gaussian function

systematic error of fits (different fit functions): $\leq 15\%$

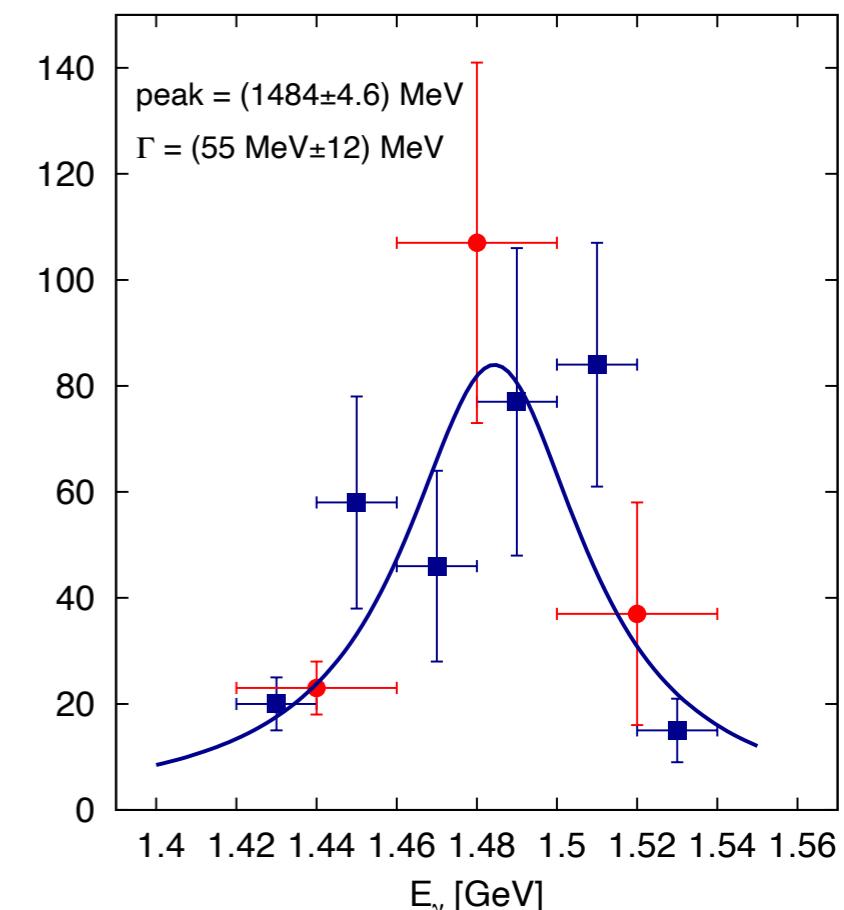
peak position



width



cross section



I. peak shifts with incident energy

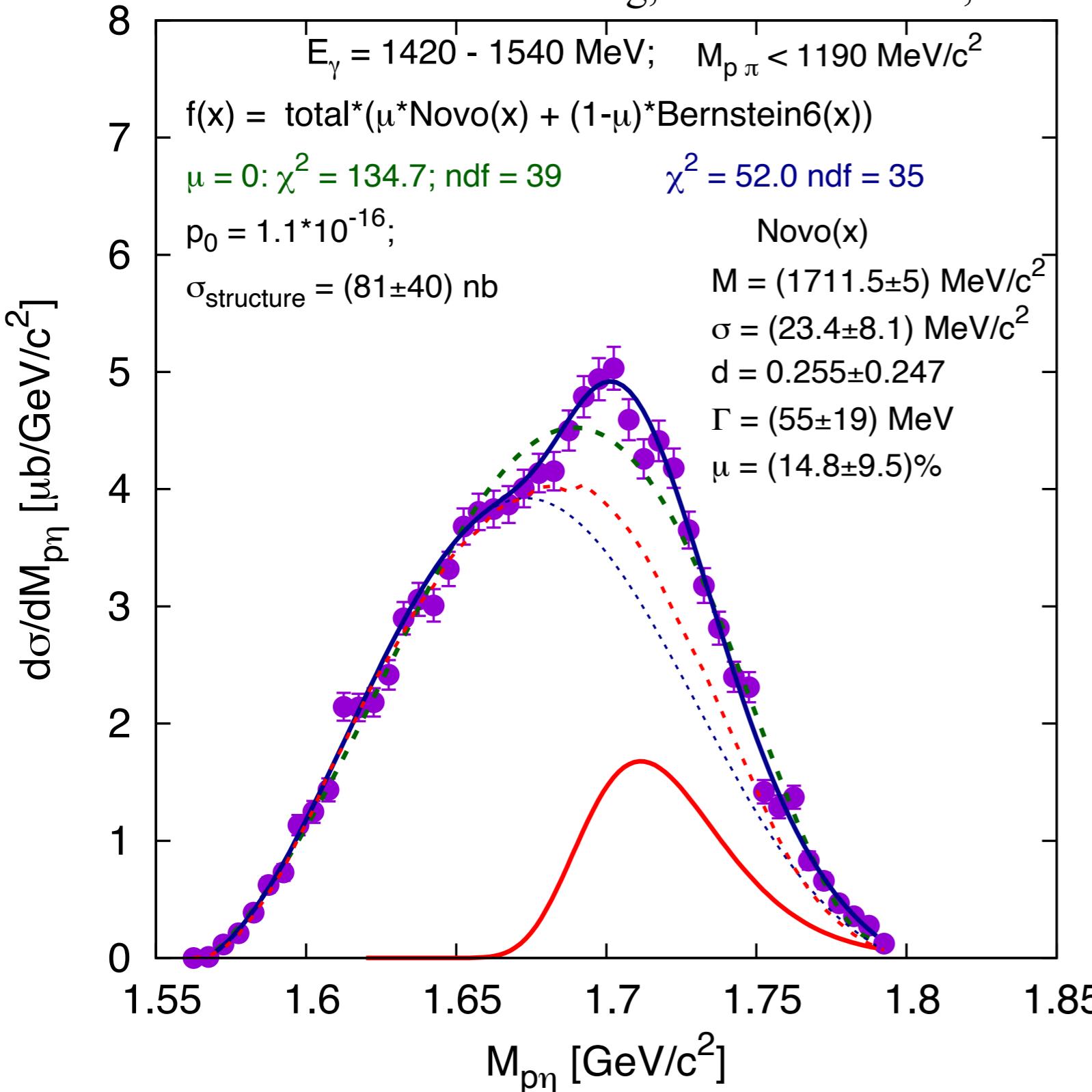
II. width: $\Gamma \leq 50 \text{ MeV}$

III. resonance-like cross section peaking at
 $E_\gamma = 1490 \text{ MeV} (\gamma p \rightarrow p a_0 \rightarrow p \pi^0 \eta \text{ threshold})$

observation of a structure at $M_{p\eta} \approx 1710$ MeV in the $\gamma p \rightarrow p \pi^0 \eta$ reaction

comparison to PWA: BnGa 2016-02 (normalized to data in $1550 \text{ MeV} < M_{p\eta} < 1680 \text{ MeV}$)

V. Metag, M. Nanova et al., EPJA 57 (2021) 325



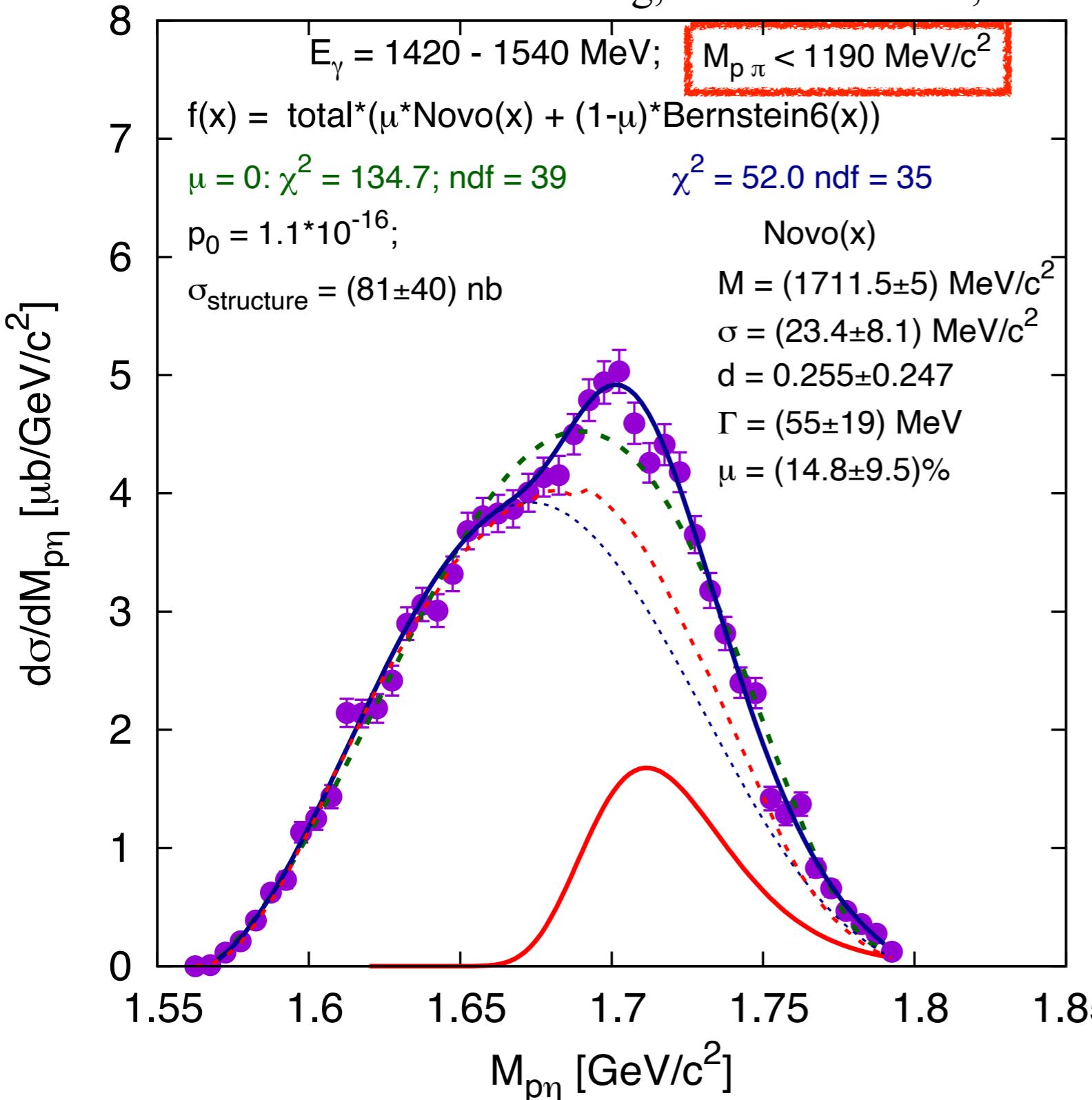
structure established with 6σ

characterisation of structure: { 1.) by fit of signal
2.) by deviation from PWA

observation of a structure at $M_{p\eta} \approx 1710$ MeV in the $\gamma p \rightarrow p \pi^0 \eta$ reaction

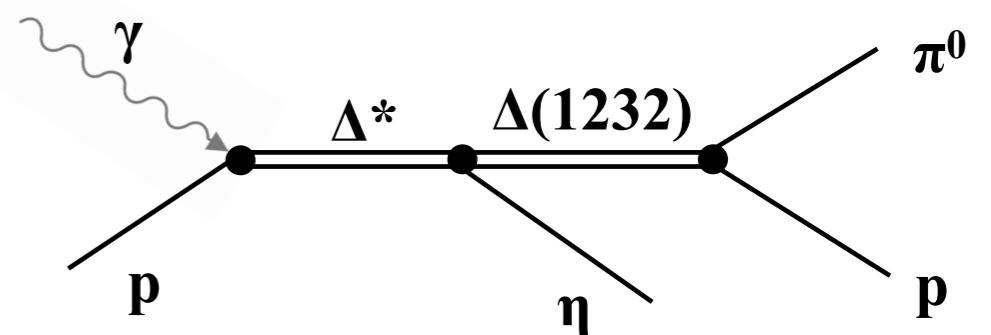
comparison to PWA: BnGa 2016-02 (normalized to data in $1550 \text{ MeV} < M_{p\eta} < 1680 \text{ MeV}$)

V. Metag, M. Nanova et al., EPJA 57 (2021) 325



$M_{p\pi}$ cut applied to suppress dominant decay chain via the $\Delta(1232)$ resonance

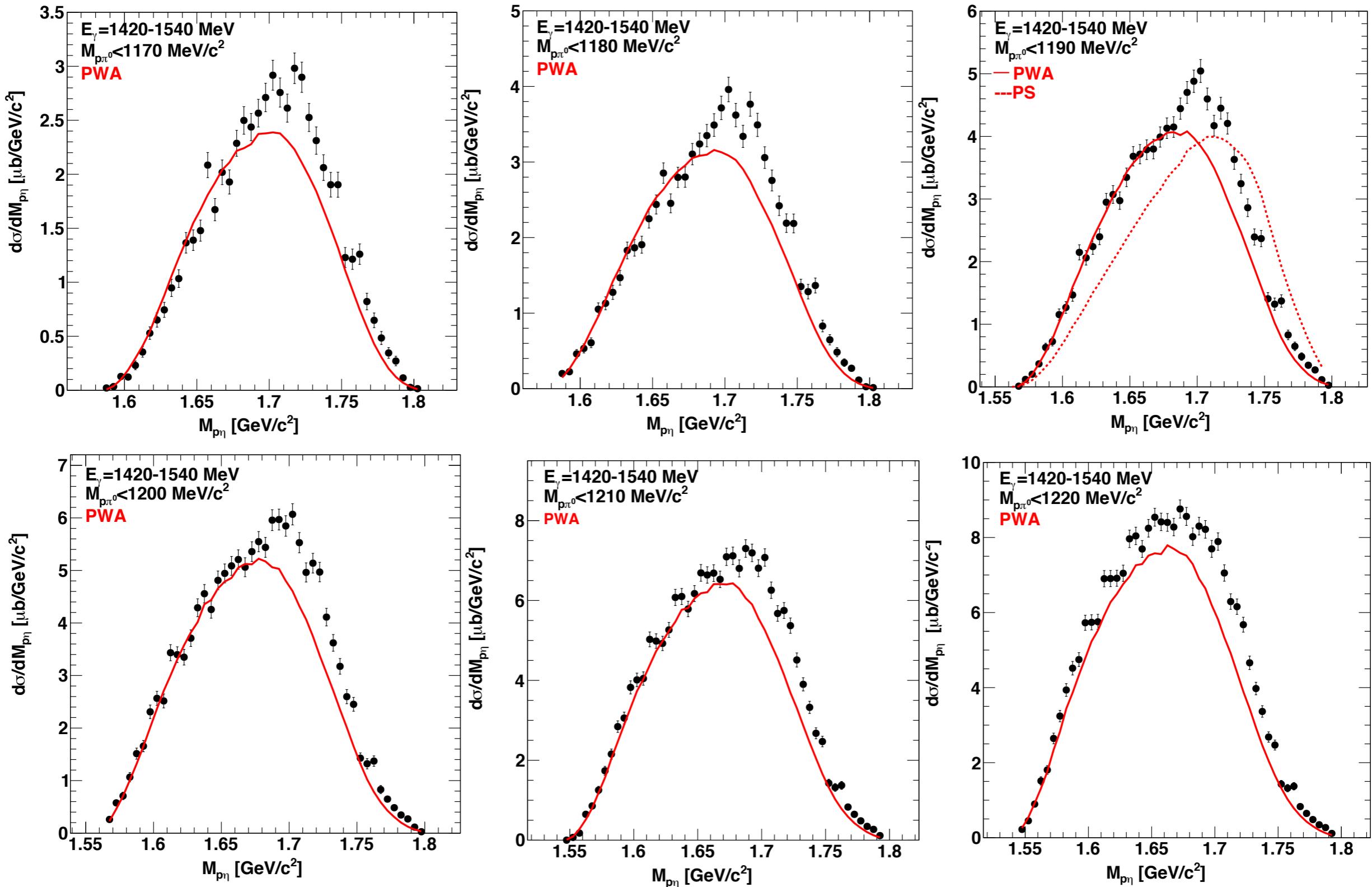
tree level diagram:



characterisation of structure: { 1.) by fit of signal
2.) by deviation from PWA

$\gamma p \rightarrow p \pi^0\eta; E_\gamma = 1420 - 1540 \text{ MeV}$

comparison to partial wave analysis (PWA) /and phase space distribution (PS)



structure not reproduced by PWA; excess also seen for different $M_{p\pi^0}$ cuts

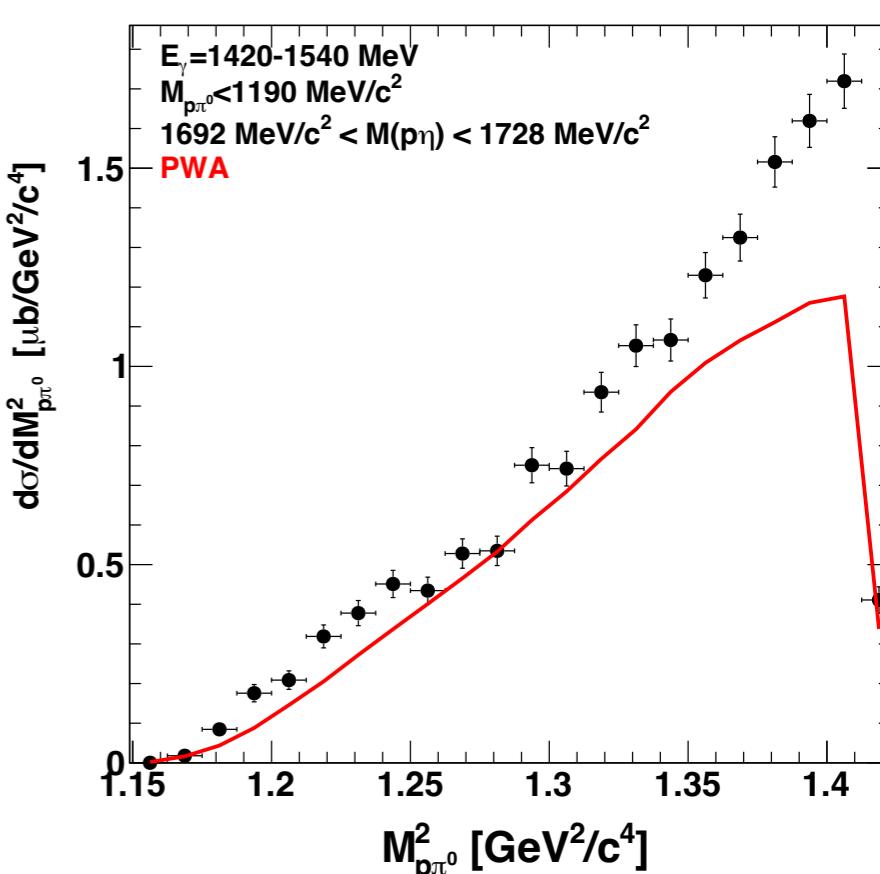
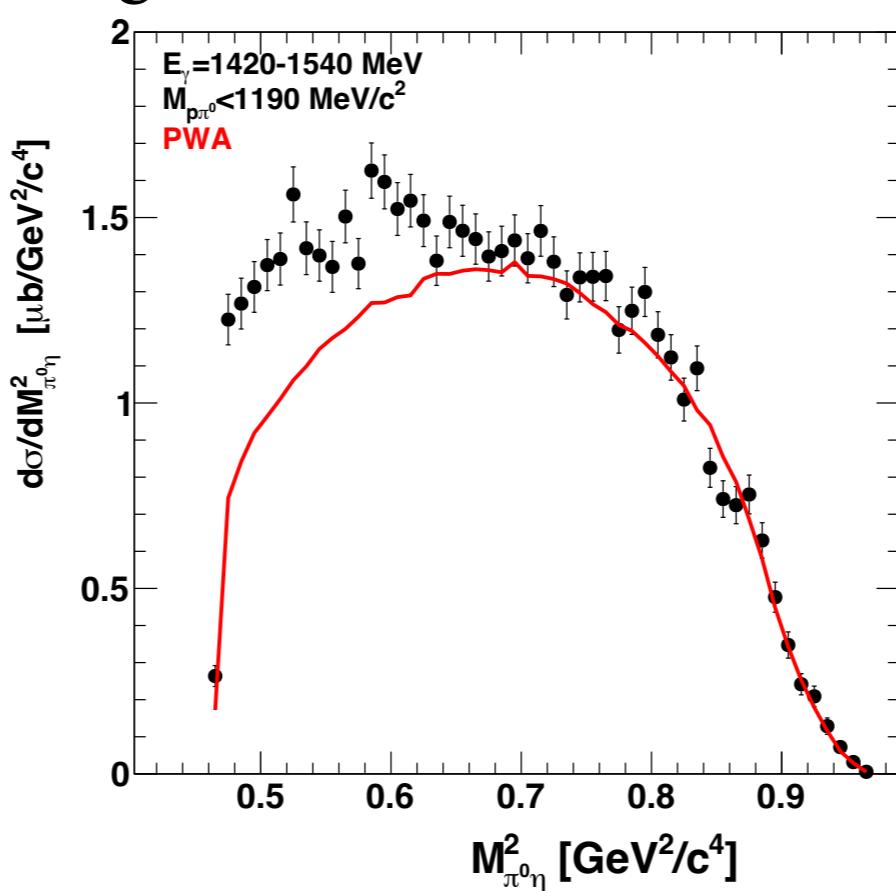
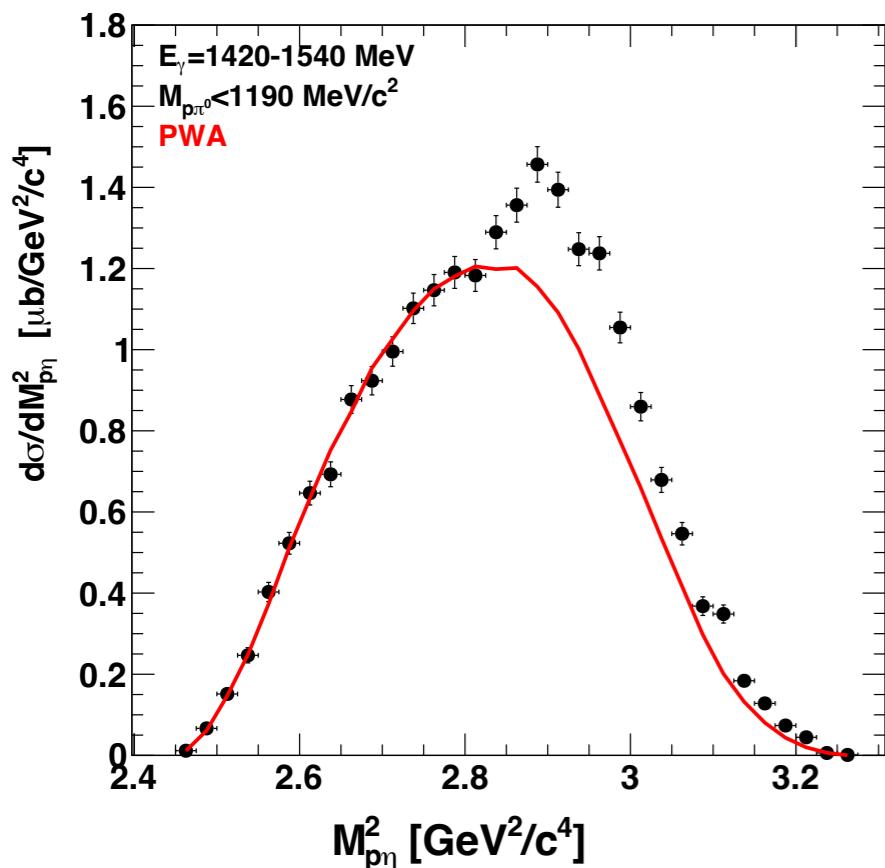
structure not caused by cut $M_{p\pi^0} < 1190 \text{ MeV}$

$M_{p\pi^0} < 1190 \text{ MeV}$

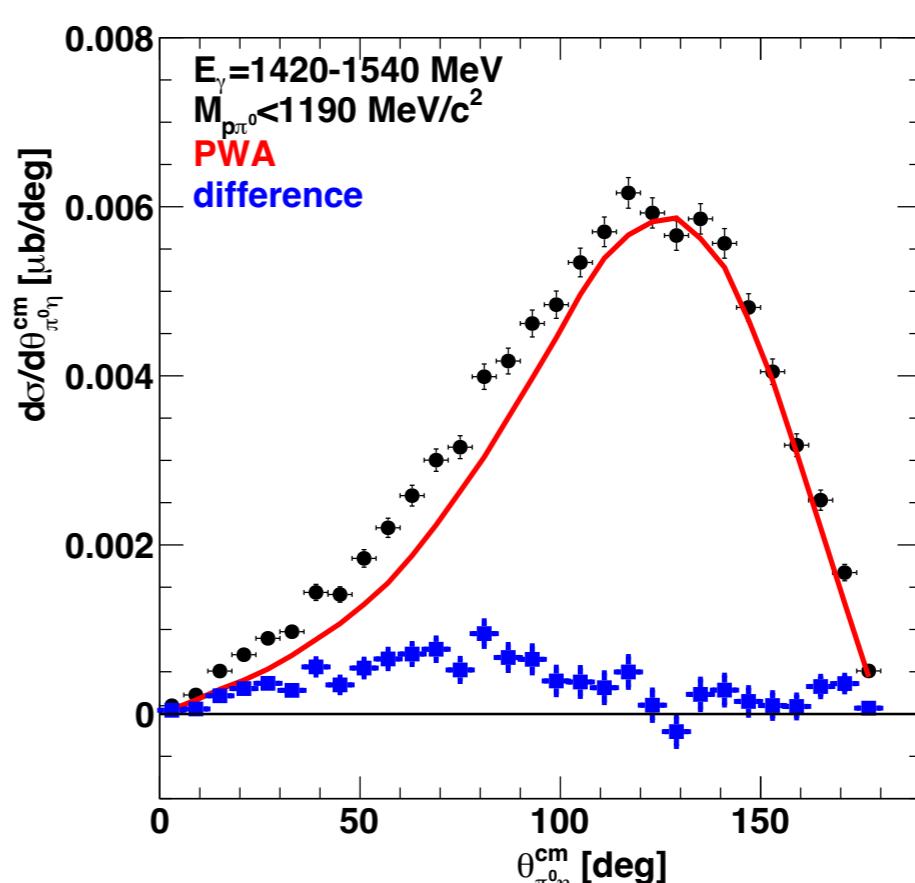
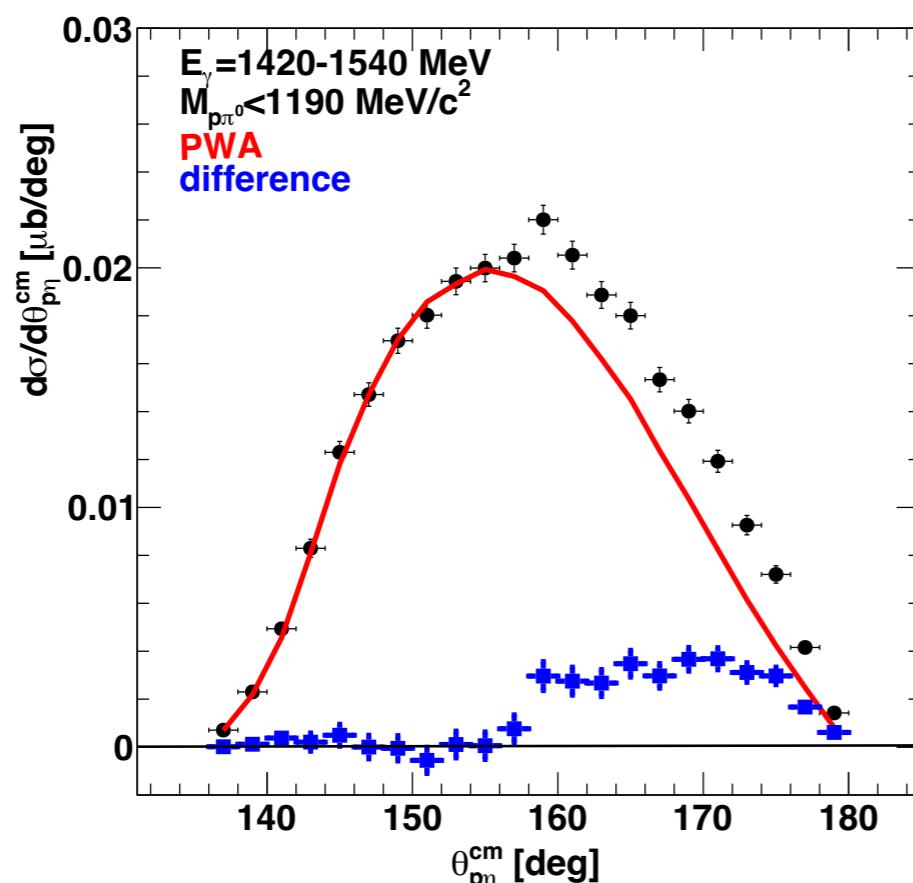
$\gamma p \rightarrow p \pi^0 \eta; E_\gamma = 1420 - 1540 \text{ MeV}$

$(1190 \text{ MeV})^2$

signal = deviation from PWA

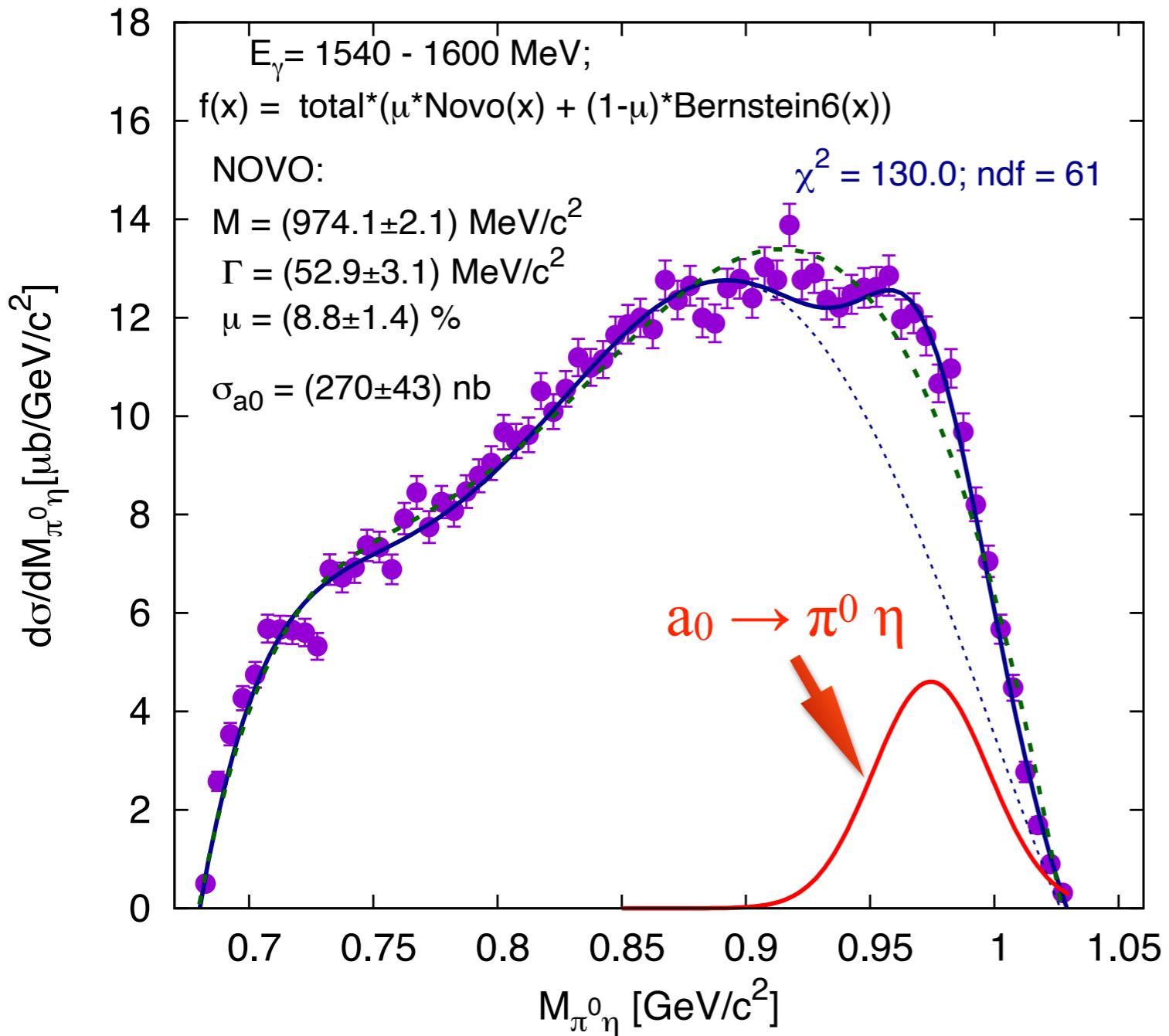


predominant
back-to-back
emission in
 γp -cm-system



deviation for
 $\theta_{\pi^0\eta} \approx 20^\circ - 90^\circ$
 γp -cm-system

γ p \rightarrow p $\pi^0\eta$; direct observation of an a₀ signal



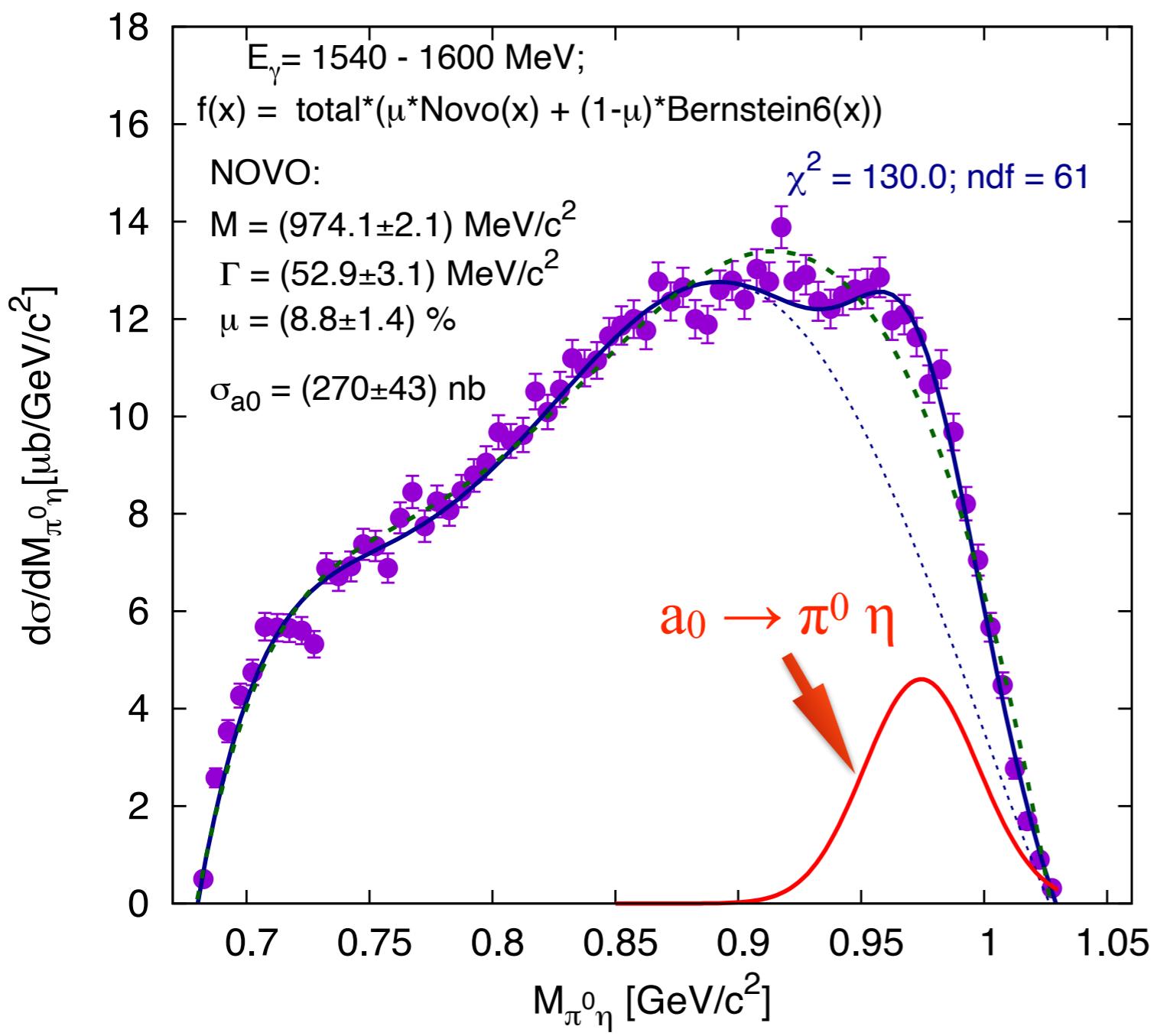
$$m_{a_0} = 974 \pm 2 \text{ MeV}; \quad \Gamma_{a_0} = 53 \pm 3 \text{ MeV}$$

$$\text{PDG: } m_{a_0} = 980 \pm 20 \text{ MeV}; \quad \Gamma_{a_0} = 50 - 100 \text{ MeV}$$

$$\sigma_{a_0} = 270 \pm 43 \text{ nb}; \quad \sigma_{a_0} (\text{PWA}) = 250 \text{ nb}$$

for $m_{a_0} > 990 \text{ MeV}$ $a_0 \rightarrow K^+K^-$ dominant

$\gamma p \rightarrow p \pi^0 \eta$; direct observation of an a_0 signal



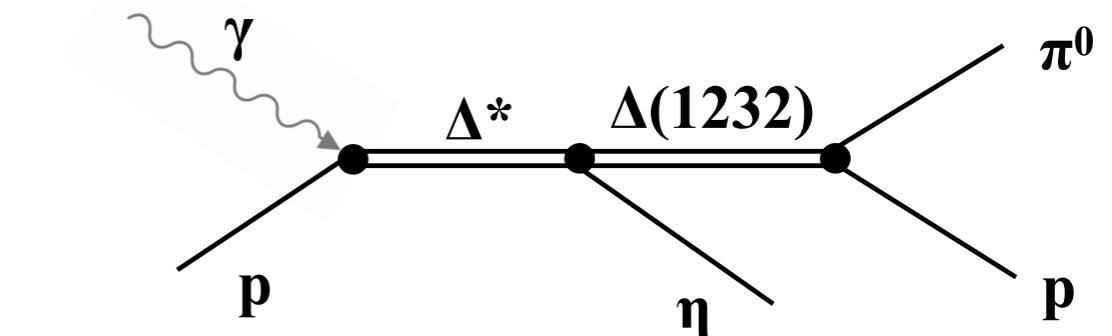
$$m_{a_0} = 974 \pm 2 \text{ MeV}; \quad \Gamma_{a_0} = 53 \pm 3 \text{ MeV}$$

$$\text{PDG: } m_{a_0} = 980 \pm 20 \text{ MeV}; \quad \Gamma_{a_0} = 50 - 100 \text{ MeV}$$

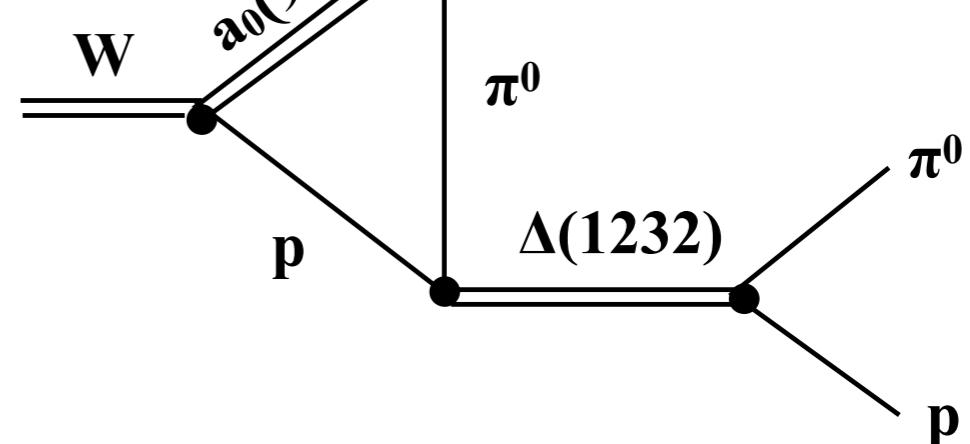
$$\sigma_{a_0} = 270 \pm 43 \text{ nb}; \quad \sigma_{a_0} (\text{PWA}) = 250 \text{ nb}$$

for $m_{a_0} > 990$ MeV $a_0 \rightarrow K^+K^-$ dominant

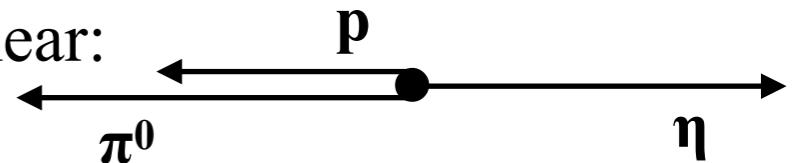
tree level diagram:
 (suppressed by $M_{p\pi} < 1190$ MeV cut)



triangular diagram:



the triangular diagram leads to an enhancement at the a_0 threshold;
 1.) energy/mom. balance: internal \leftrightarrow external
 2.) all internal particles almost on mass shell
 3.) p, π^0, η collinear:



Calculation of triangular singularity strength

comparison to data

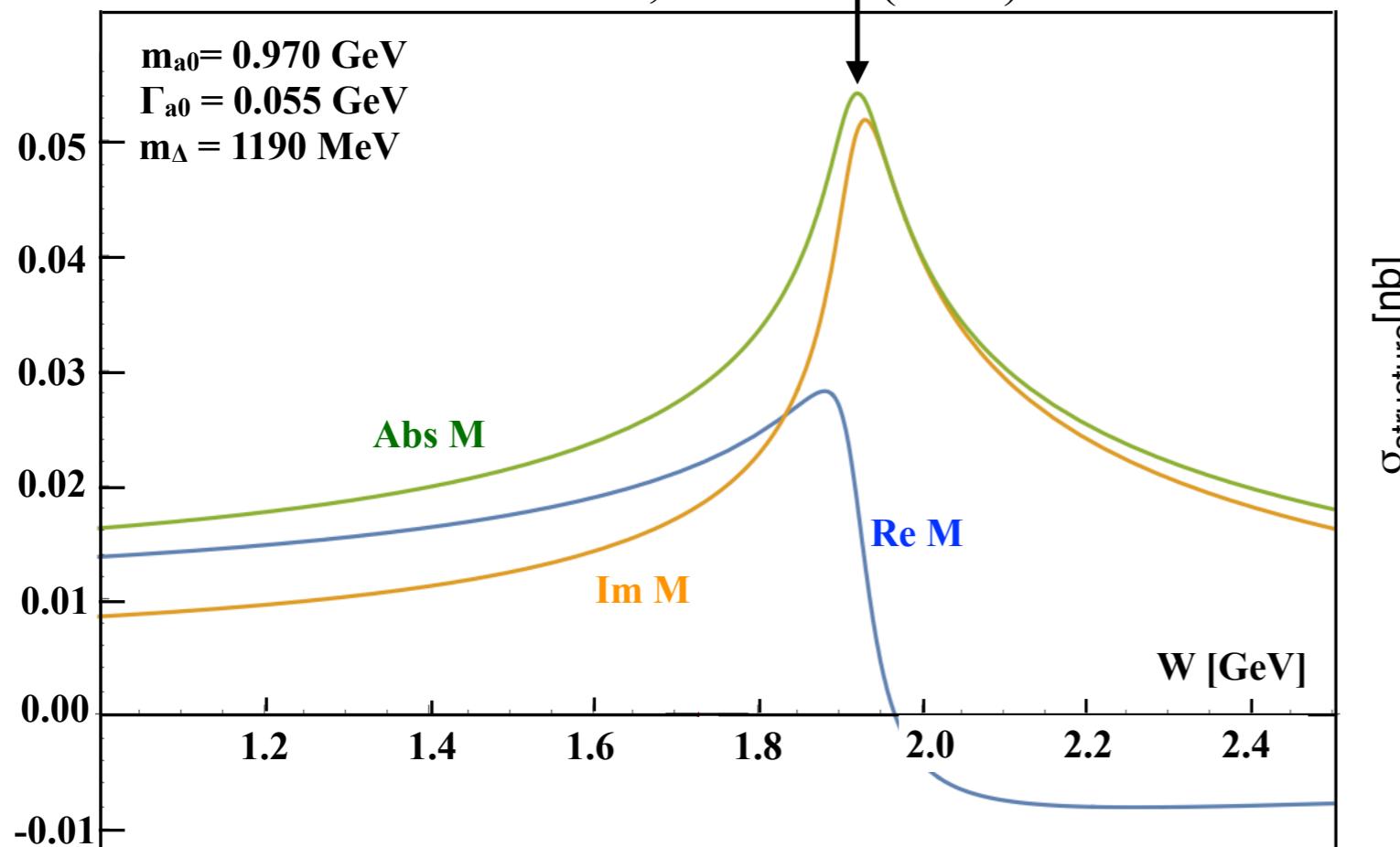
$W=1918 \text{ MeV}$

$E_\gamma = 1492 \text{ MeV}$

thanks to Mathias Wagner and Bernhard Ketzer (Univ. Bonn)

M. Mikhasenko et al., PRD 91 (2015) 094015

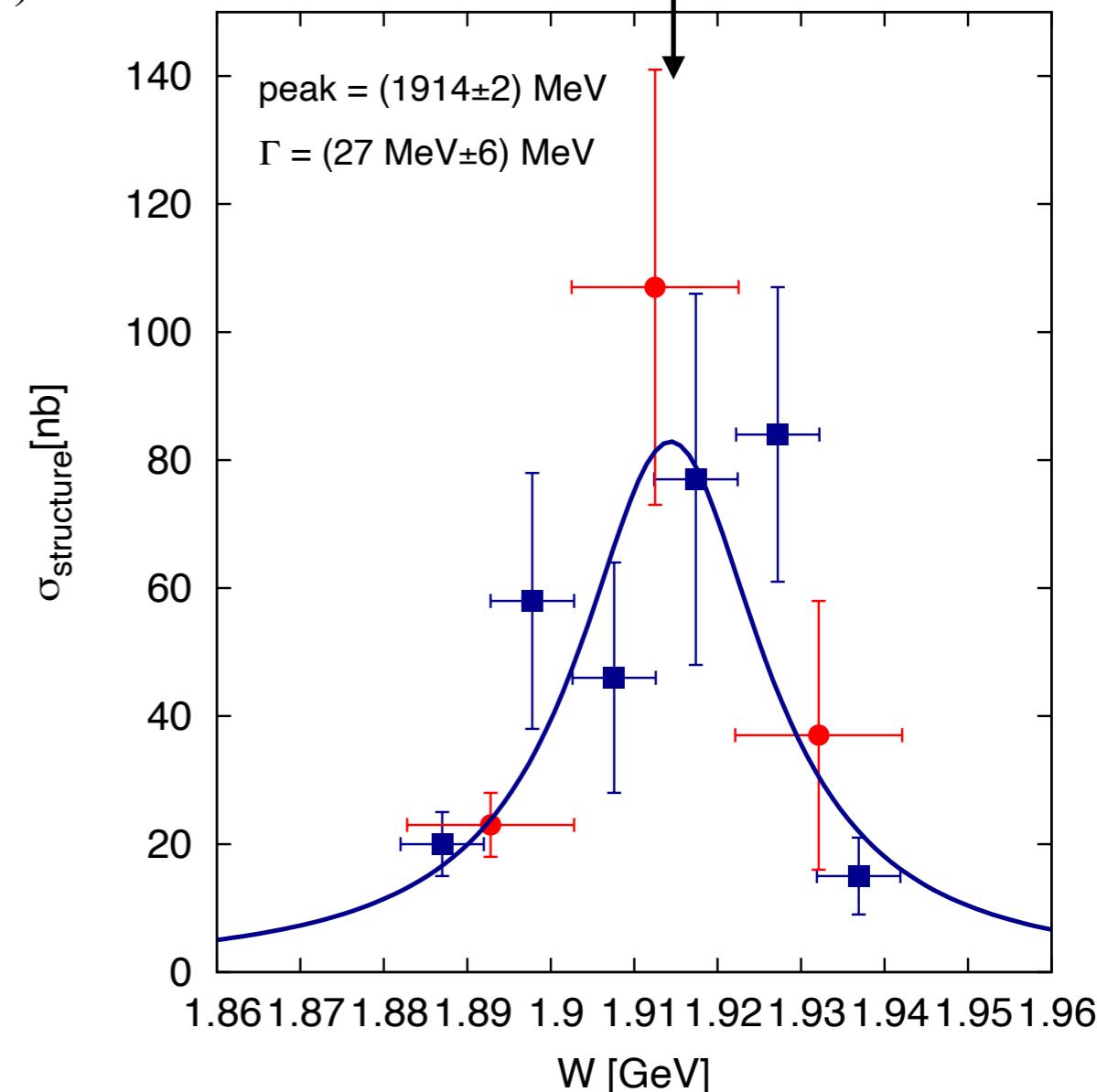
G.D. Alexeev et al., PRL 127 (2021) 082501



$m_p + m_{a0} = 1918 \text{ MeV}$

$W=1918 \text{ MeV}$

$E_\gamma=1492 \text{ MeV}$

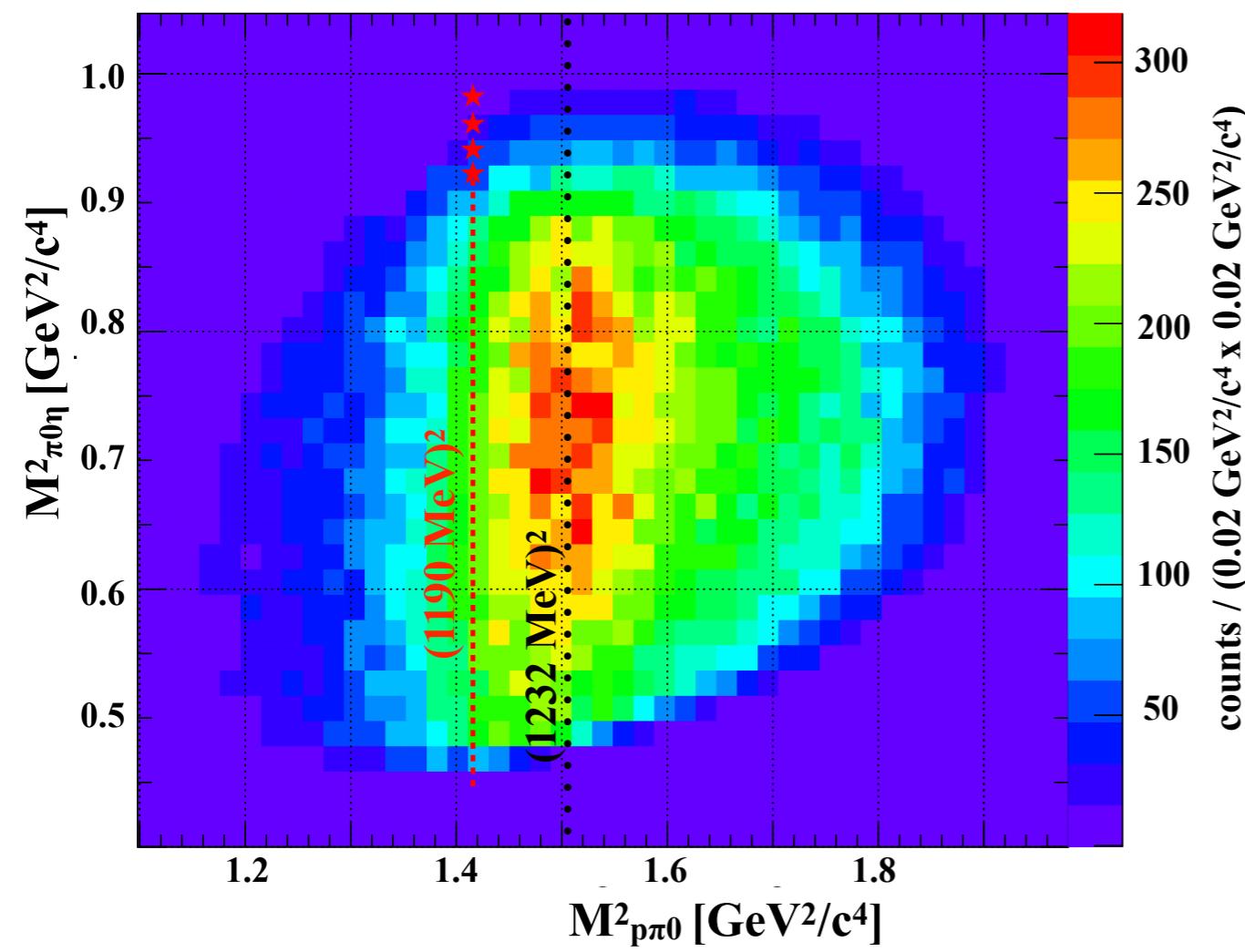


calculation shows enhancement at $W=1918 \text{ MeV}; E_\gamma = 1492 \text{ MeV}$
as observed experimentally

calculating triangular singularities

calculation of singularities for 4 incident energies near the a_0 threshold,
following Bayar et al. PRD 94 (2016) 074039

E_γ [MeV]	$M_{\pi^0\eta}$ [MeV]	$M_{p\eta}$ [MeV]	$M_{p\pi^0}$ [MeV]
1549	990	1609	1190
1524	980	1601	1190
1498	970	1591	1190
1473	960	1583	1190



calculating triangular singularities

calculation of singularities for 4 incident energies near the a_0 threshold,
following Bayar et al. PRD 94 (2016) 074039

E_γ [MeV]	$M_{\pi^0\eta}$ [MeV]	$M_{p\eta}$ [MeV]	$M_{p\pi^0}$ [MeV]
1549	990	1609	1190
1524	980	1601	1190
1498	970	1591	1190
1473	960	1583	1190



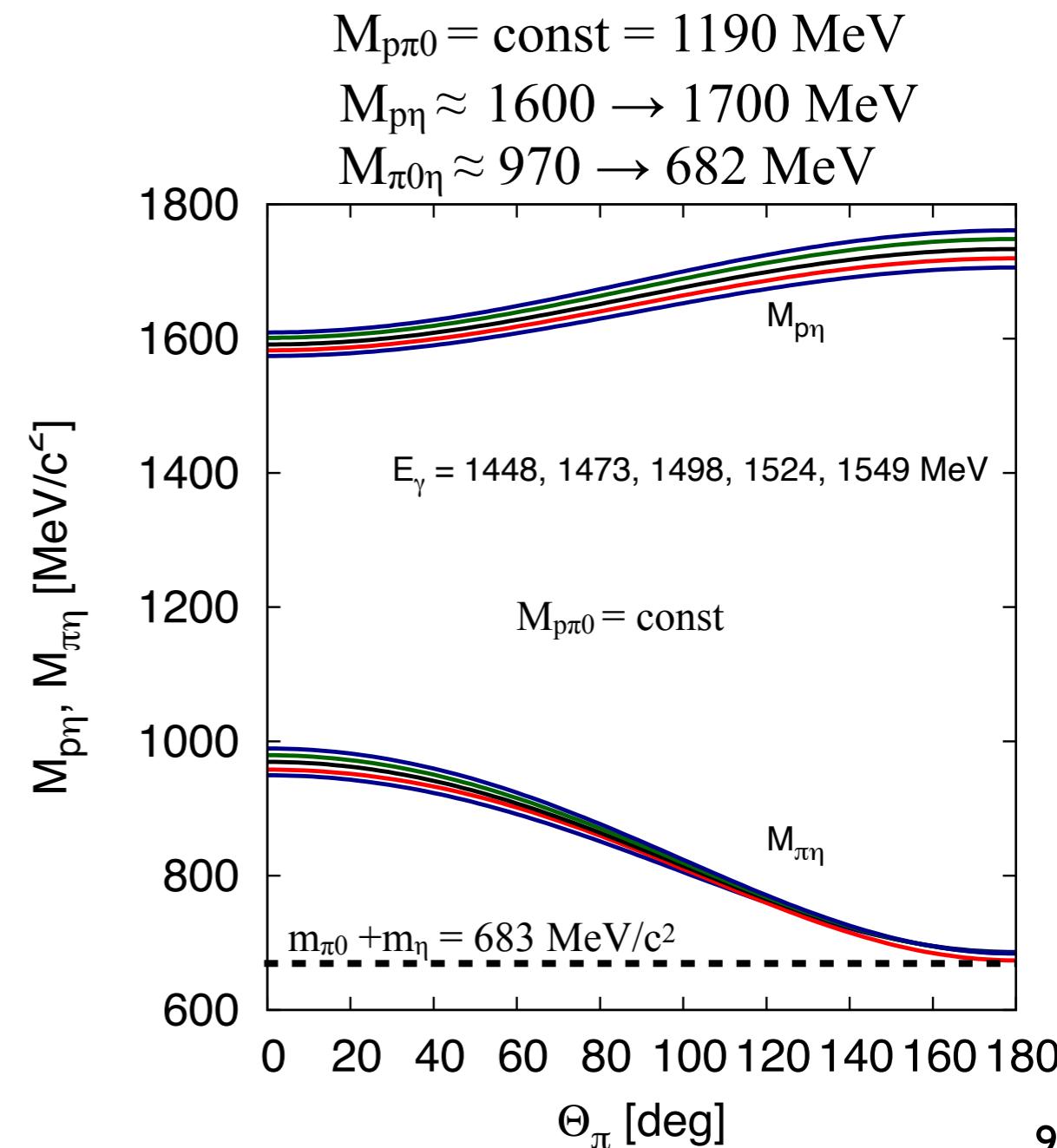
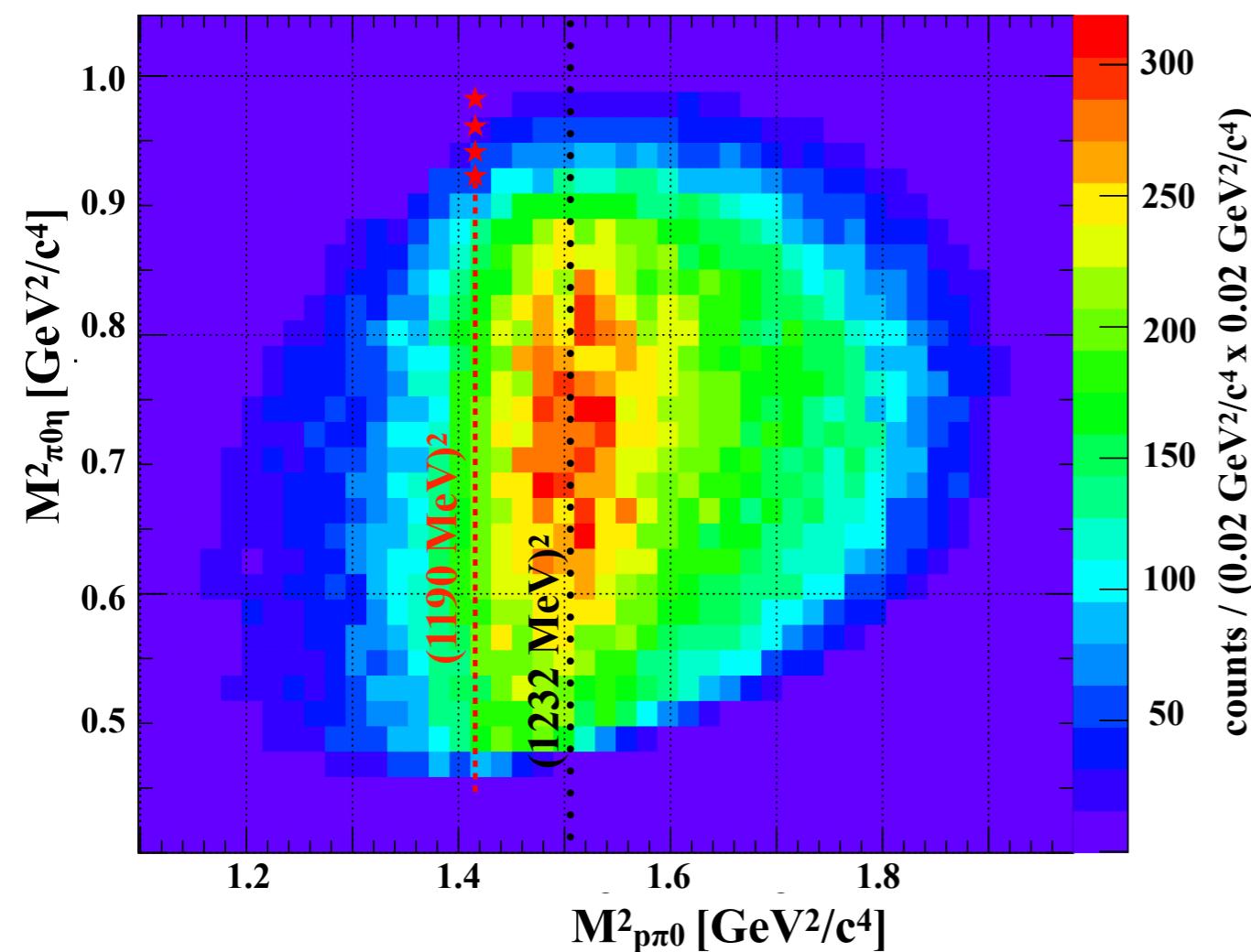
γ p-cm-system:

$p_p = 122.9 \text{ MeV}; \beta_p = 0.130$
 $p_\pi = 277.8 \text{ MeV}; \beta_\pi = 0.899$ $p_\eta = -400.7 \text{ MeV}$
 $\beta_\eta = -0.590$

π^0 faster than p

π^0 -p rescattering:

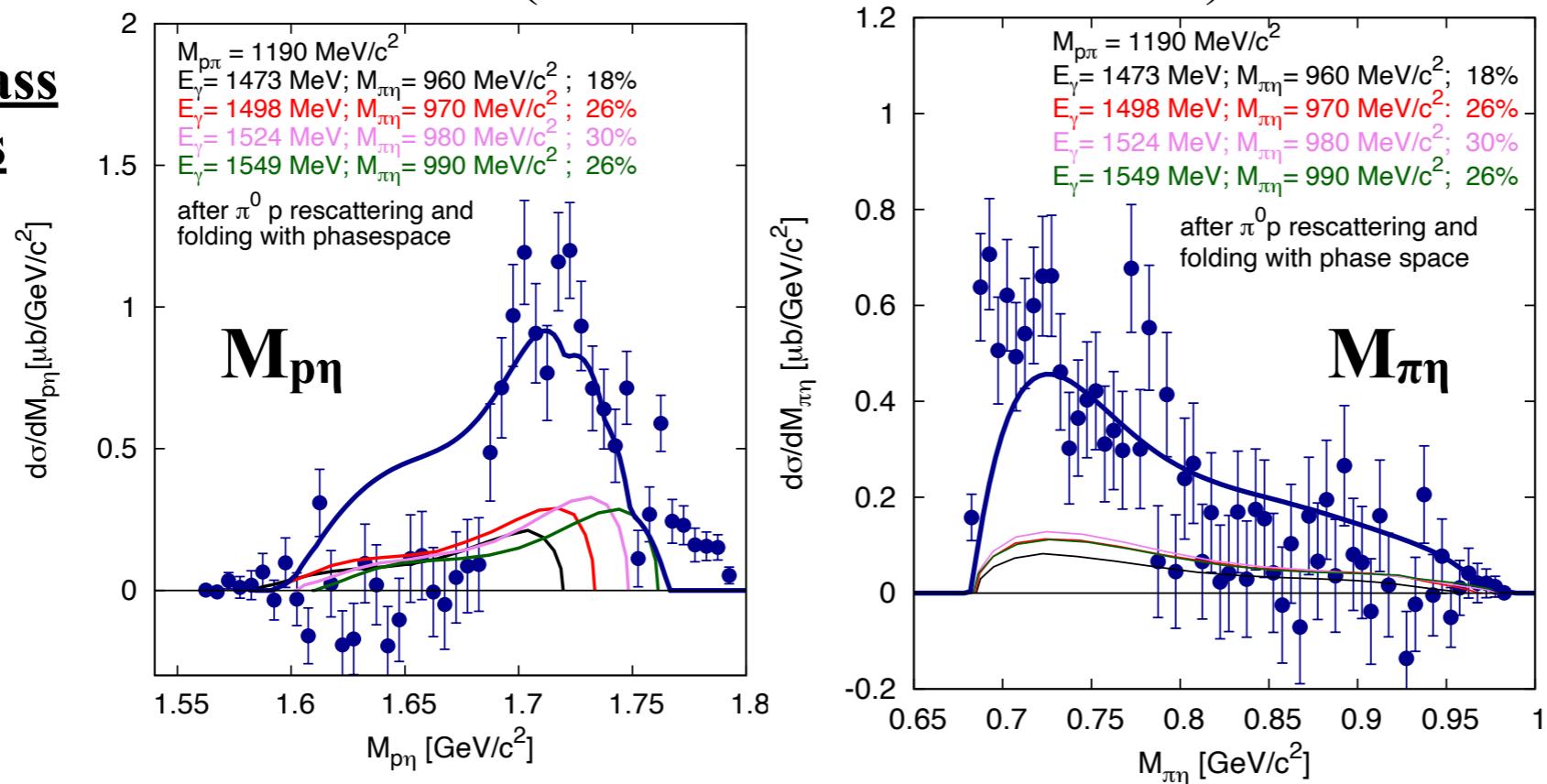
singularity events are re-distributed along
the dashed red line by **π^0 -p - rescattering**



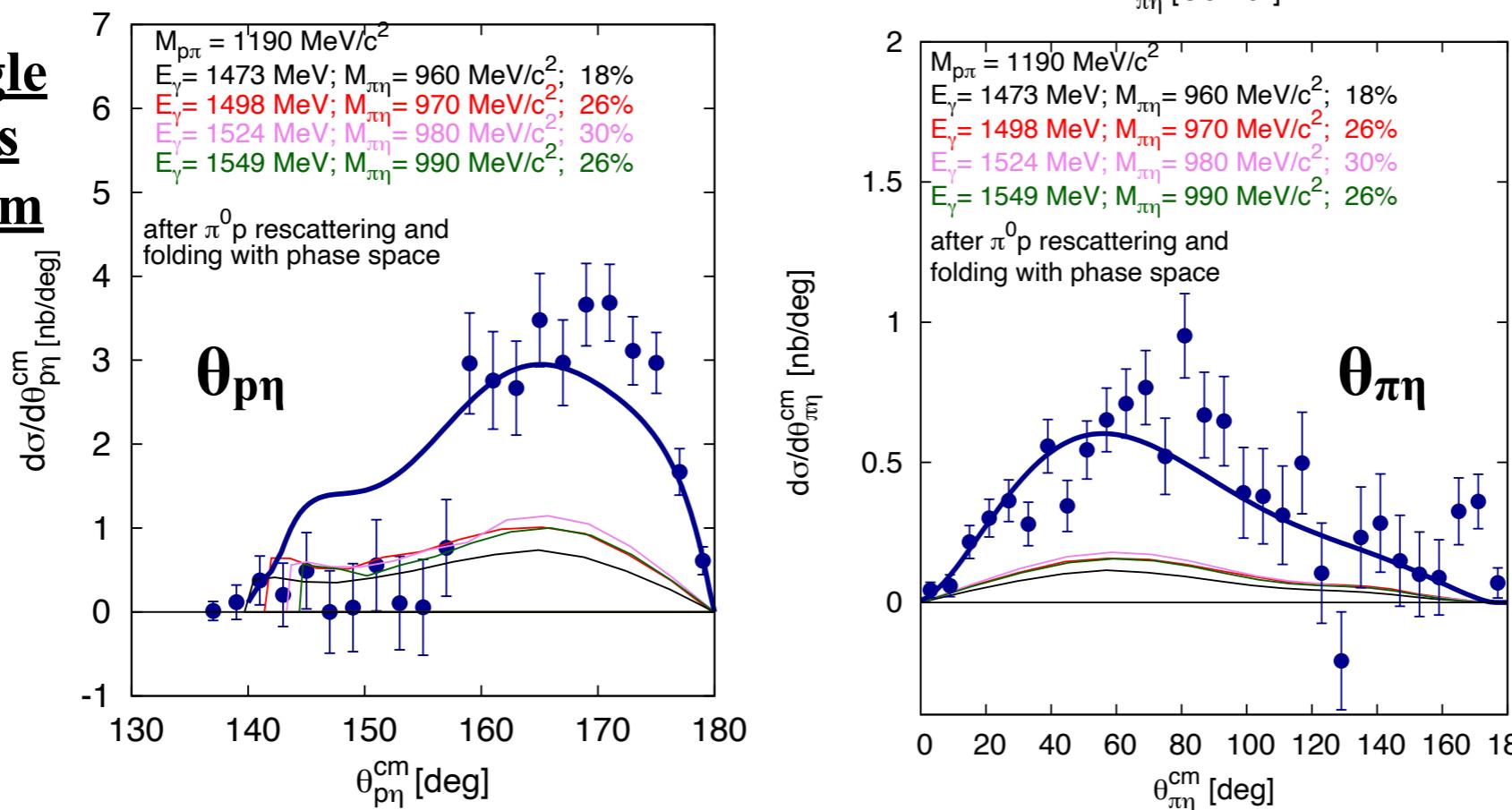
Comparison data (difference to PWA) \longleftrightarrow calculation

contributions of the 4 selected singularity points with weight given by a_0 line shape
 blue curve (sum of the 4 contributions) fitted to the data

invariant mass distributions



opening angle distributions γp -cm-system

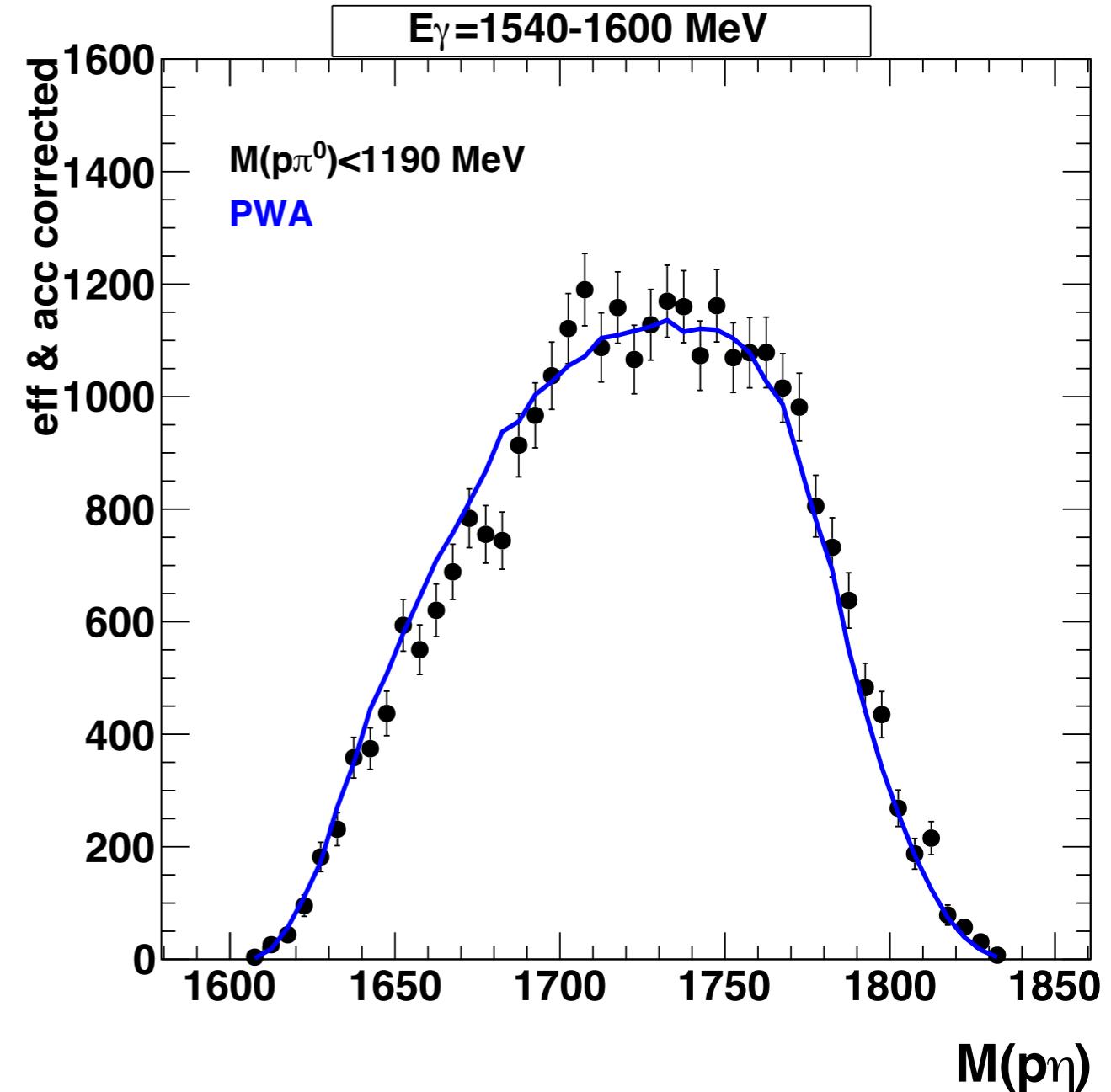
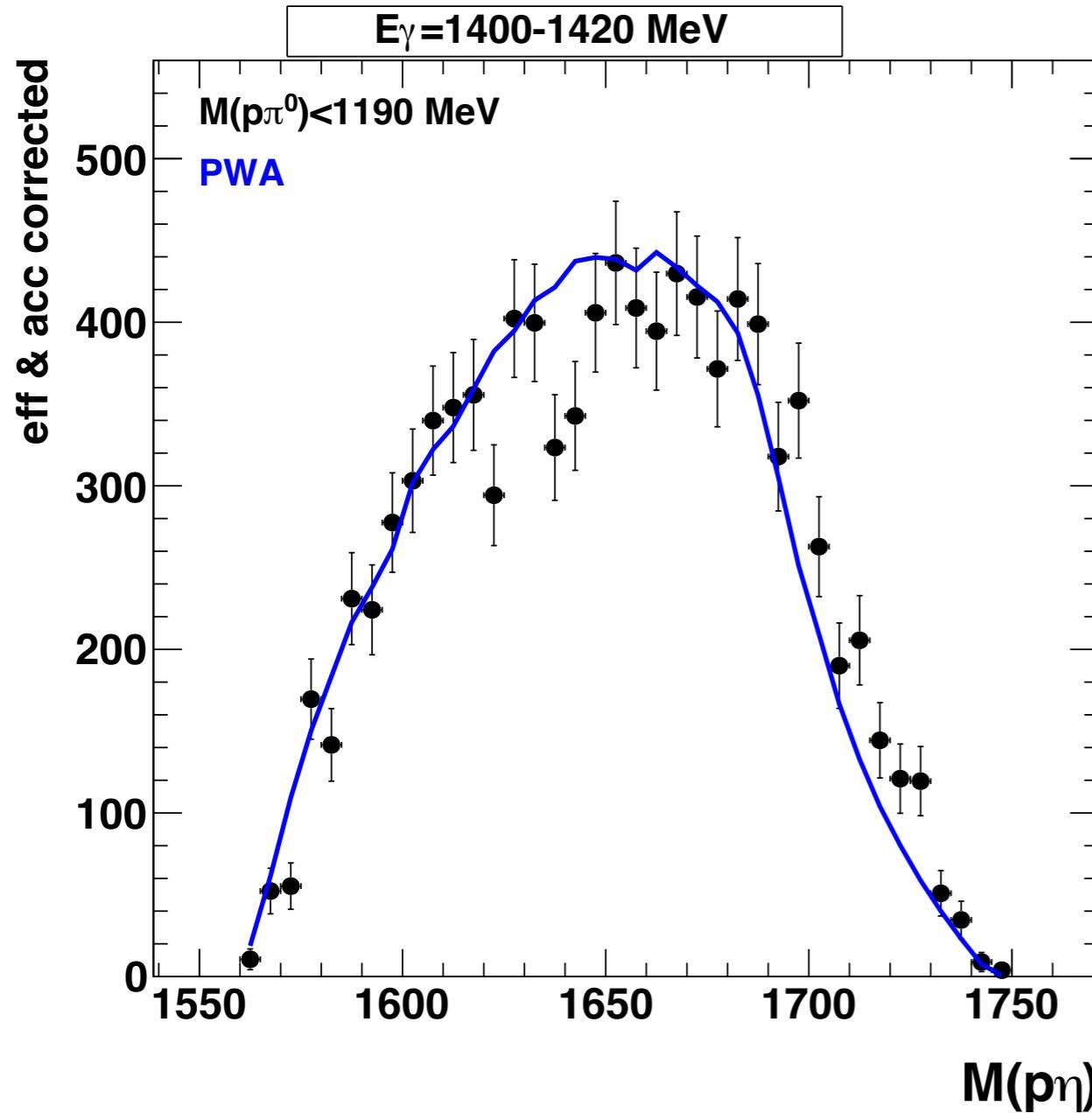


data qualitatively
 reproduced
 by calculations !!

summary and conclusions

- structure at $M_{p\eta} \approx 1710$ MeV established in $\gamma p \rightarrow p \pi_0 \eta$ reaction for $E_\gamma = 1400 - 1600$ MeV
- structure moves and changes shape with incident photon energy
→ no genuine nucleon resonance
- characteristics of structure qualitatively reproduced by calculation based on the triangular loop in the $\gamma p \rightarrow p a_0 \rightarrow p \pi_0 \eta$ reaction;
(EPJA 57 (2021) 325)
- loop diagrams and rescattering effects play an important role also in the **baryon sector** in the interpretation of structures in the excitation spectrum of the nucleon; important to distinguish kinematical singularities from genuine resonances
- not every bump in an invariant mass spectrum is a resonance !
- **improvements:**
calculation not only for 4 selected singularity points
→ **full partial wave analysis including the present data (in progress)**

backup



PWA describes data very well outside the signal region ($E_\gamma = 1420 - 1540$)

calculating triangular singularities

Bayar et al. PRD 94 (2016) 074039

energy-momentum balance within the loop has to match the energy-momentum balance of the initial and final state particles:

$$W = E_p(q) + E_\pi(q) + E_\eta$$

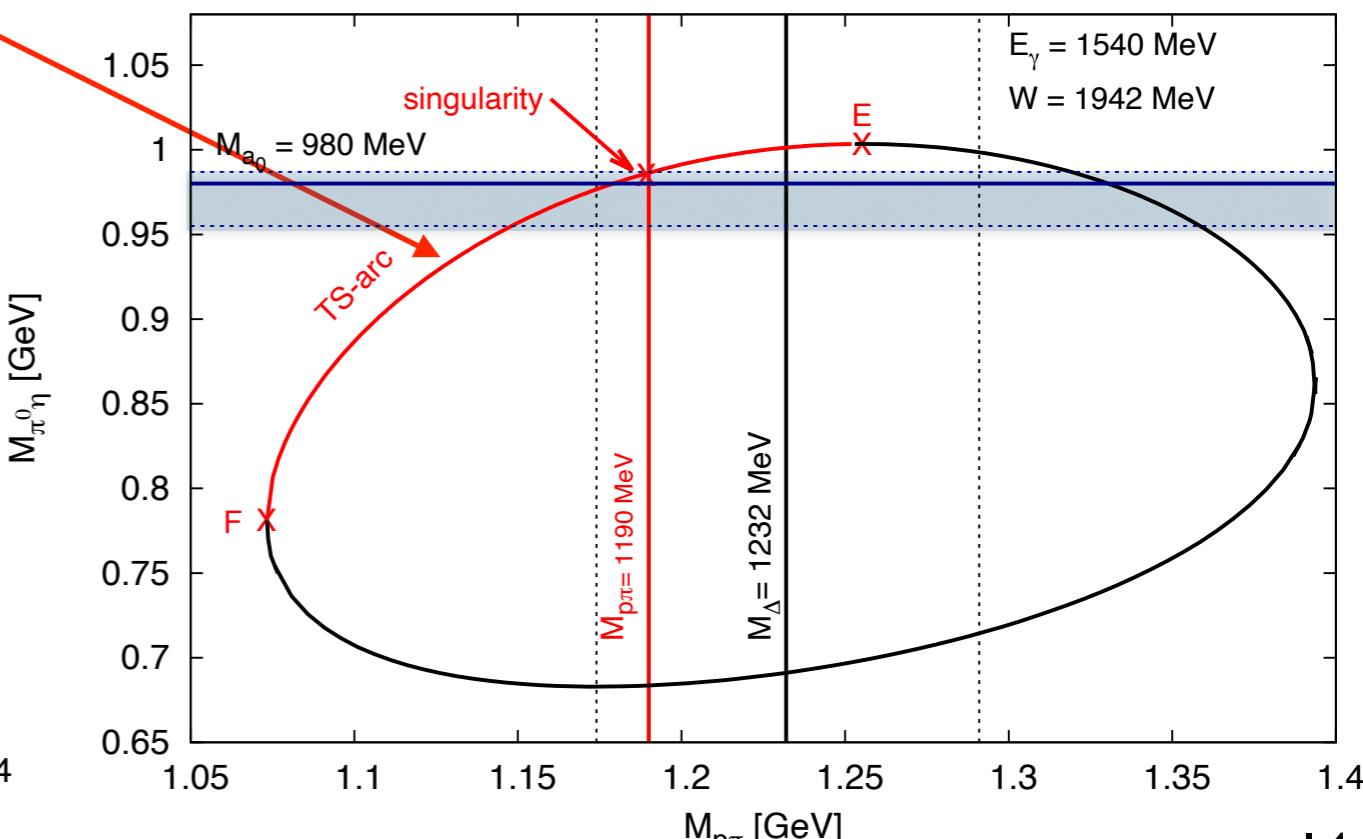
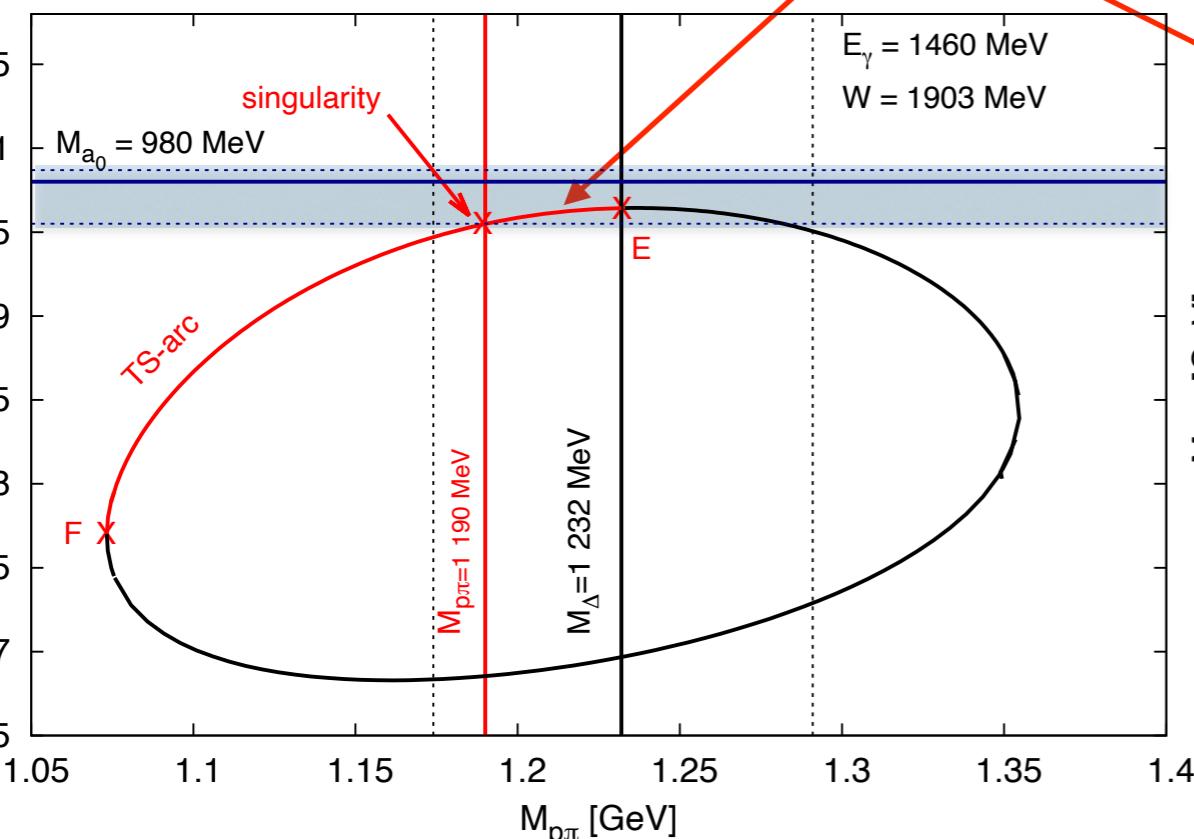
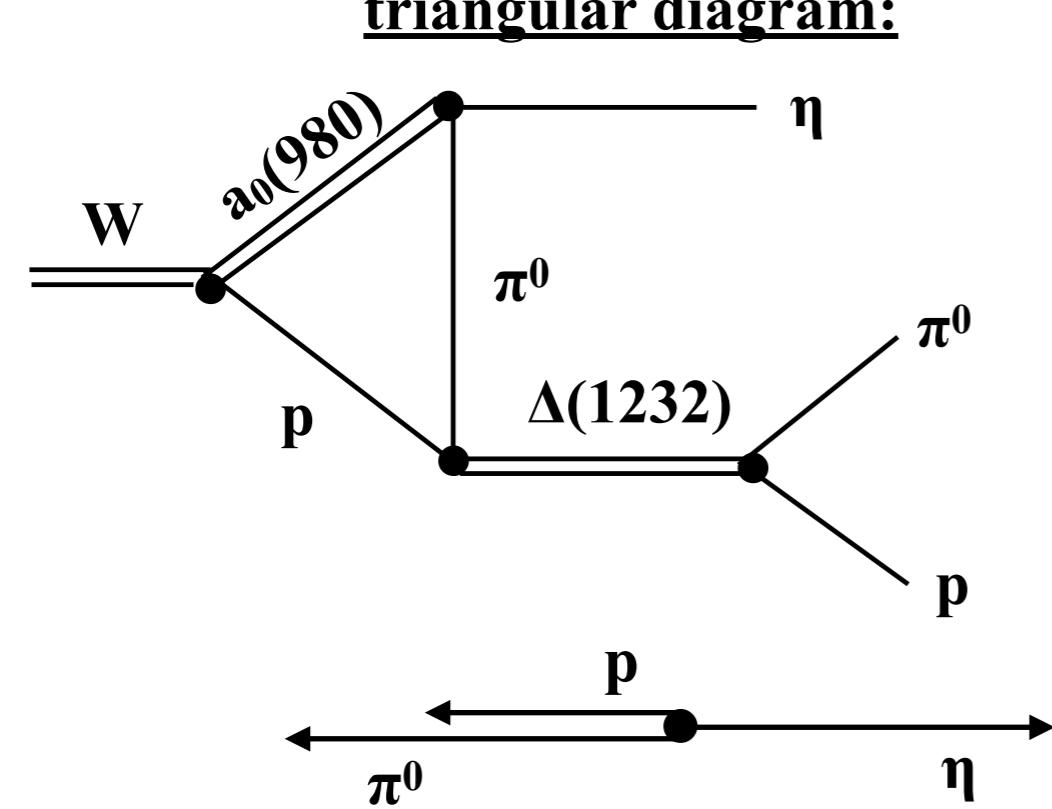
$$\underline{W = E_p(q) + E_{a0}(p_\eta - q)}$$

$$E_\eta + E_\pi(q) - E_{a0}(p_\eta - q) = 0$$

$$E_\eta + E_\pi(q) - \sqrt{m_{a0}^2 + (p_\eta - q)^2} = 0$$

q = proton momentum in loop

for given excitation energy W solutions only for certain $(M_{\pi^0\eta}, M_{p\pi^0})$ values and if all particles are almost on-mass shell and p, π^0, η are collinear



Interference of loop- (triangular) and tree-level amplitudes

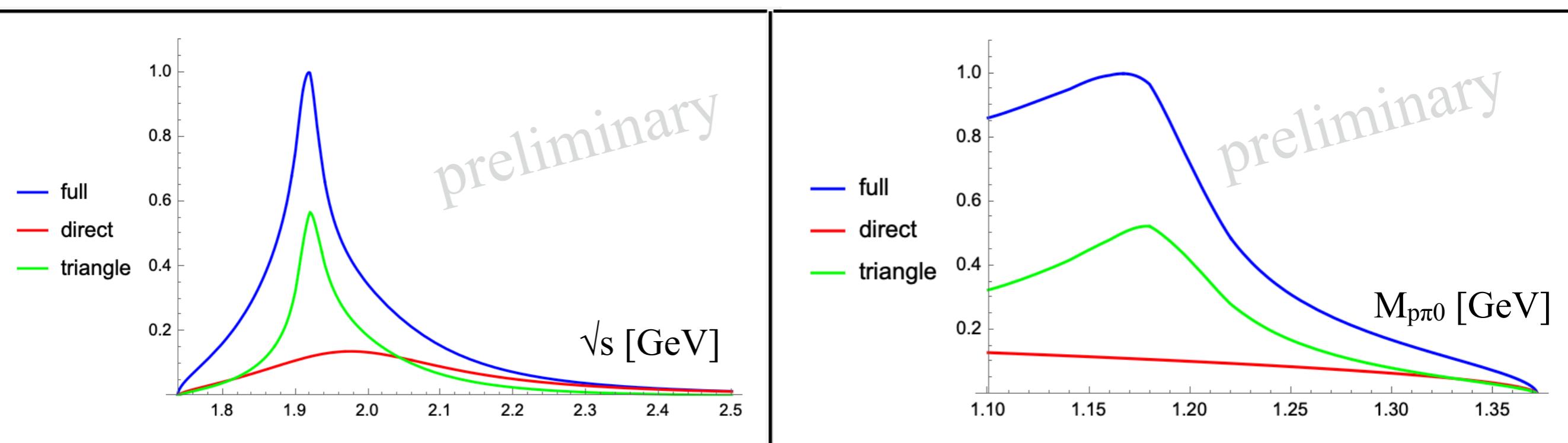
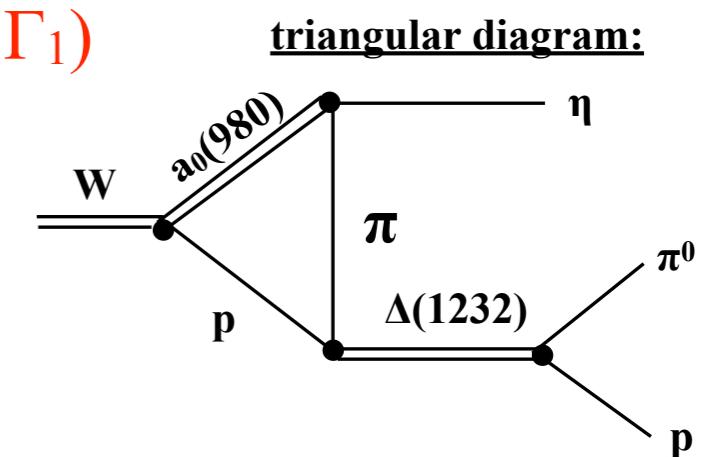
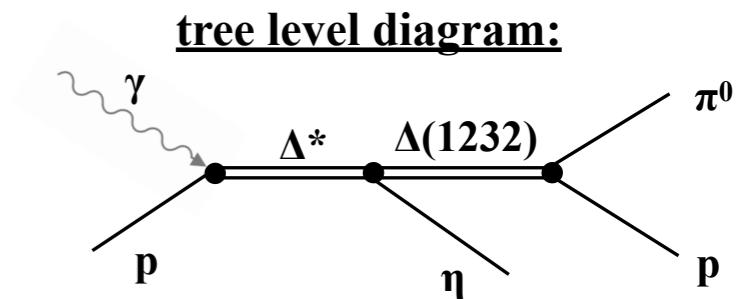
thanks to Mathias Wagner (Univ. Bonn)

initially populated nucleon resonance (source of p, a_0):
 $m_1 = 1.95 \text{ GeV}$; $\Gamma_1 = 0.350 \text{ GeV}$

$$f_{\text{tot}} = f_{\text{tree}} + f_{\text{loop}} = (\text{rel} + e^{i\varphi} * f_{\text{triangle}}(\sqrt{s}; \Gamma_{a_0}, m_{a_0}, m_\Delta)) * \text{BW}(\sqrt{s}; m_1, \Gamma_1)$$

intensity = $|M|^2 * \text{phasespace}$

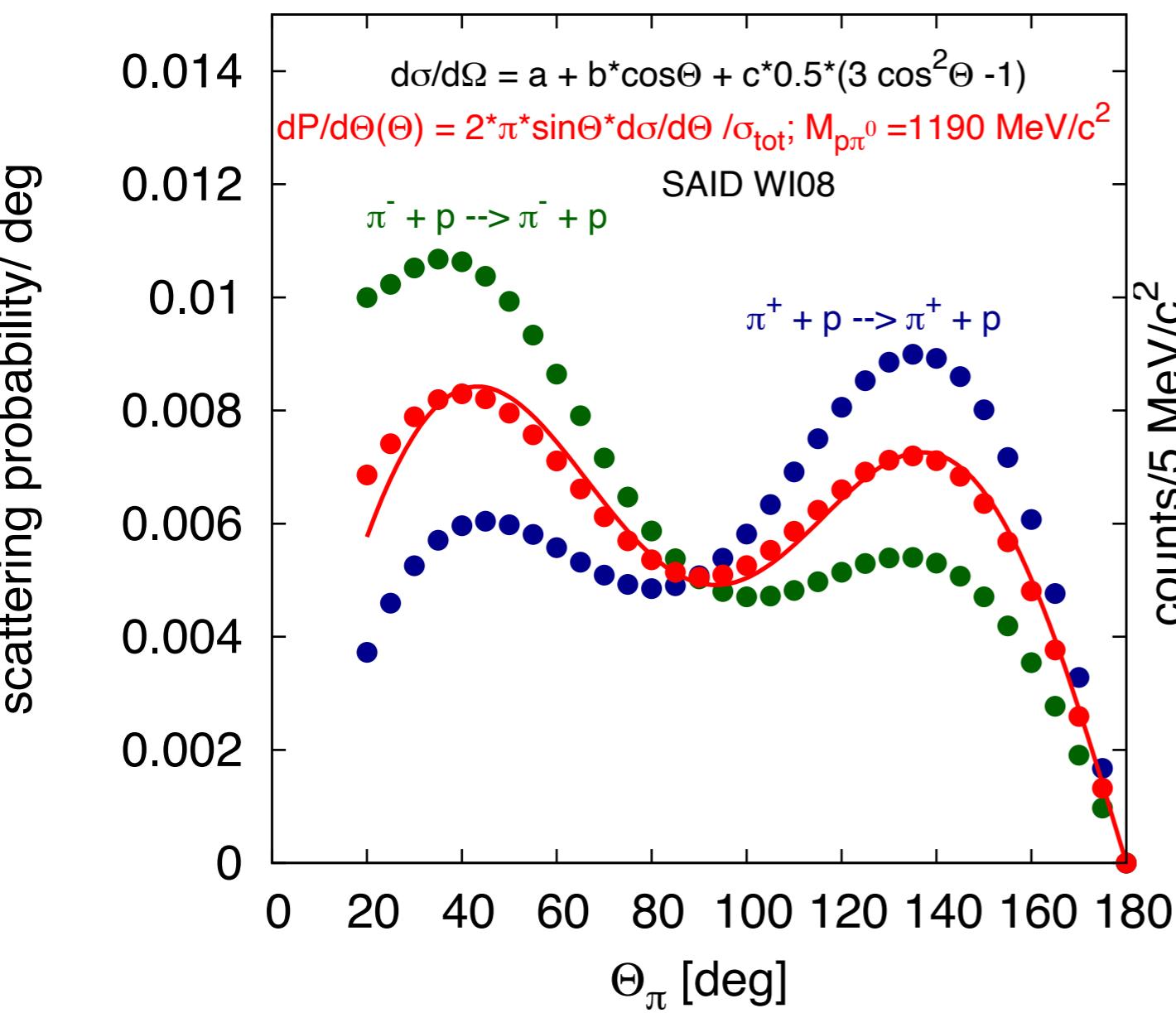
experimentally observed excitation function and
 $M_{p\pi^0}$ distribution reproduced for $\text{rel} = 0.036$; $\varphi = -0.5$



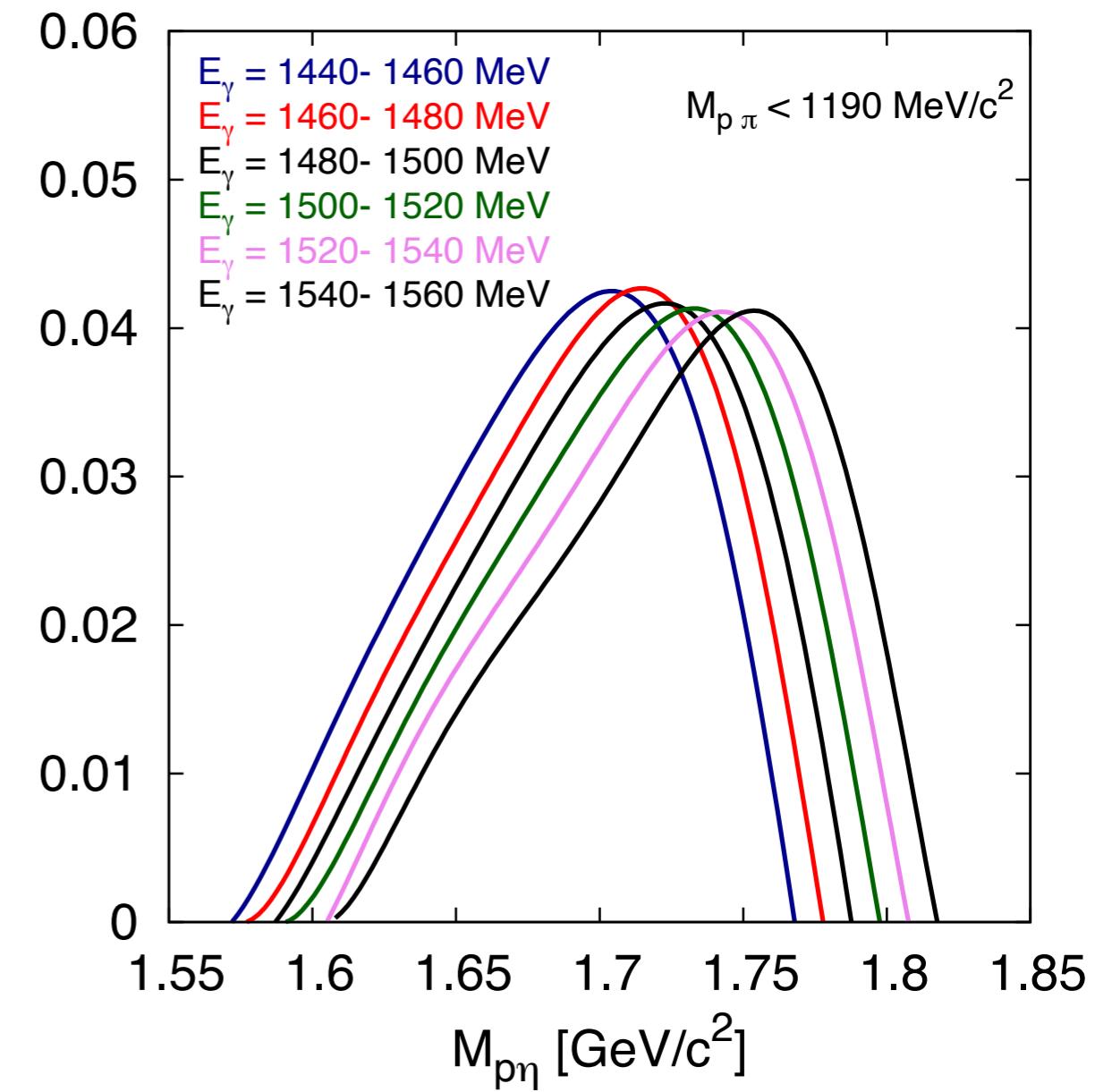
more detailed studies require partial wave analysis !!

probability of π^0 - p rescattering

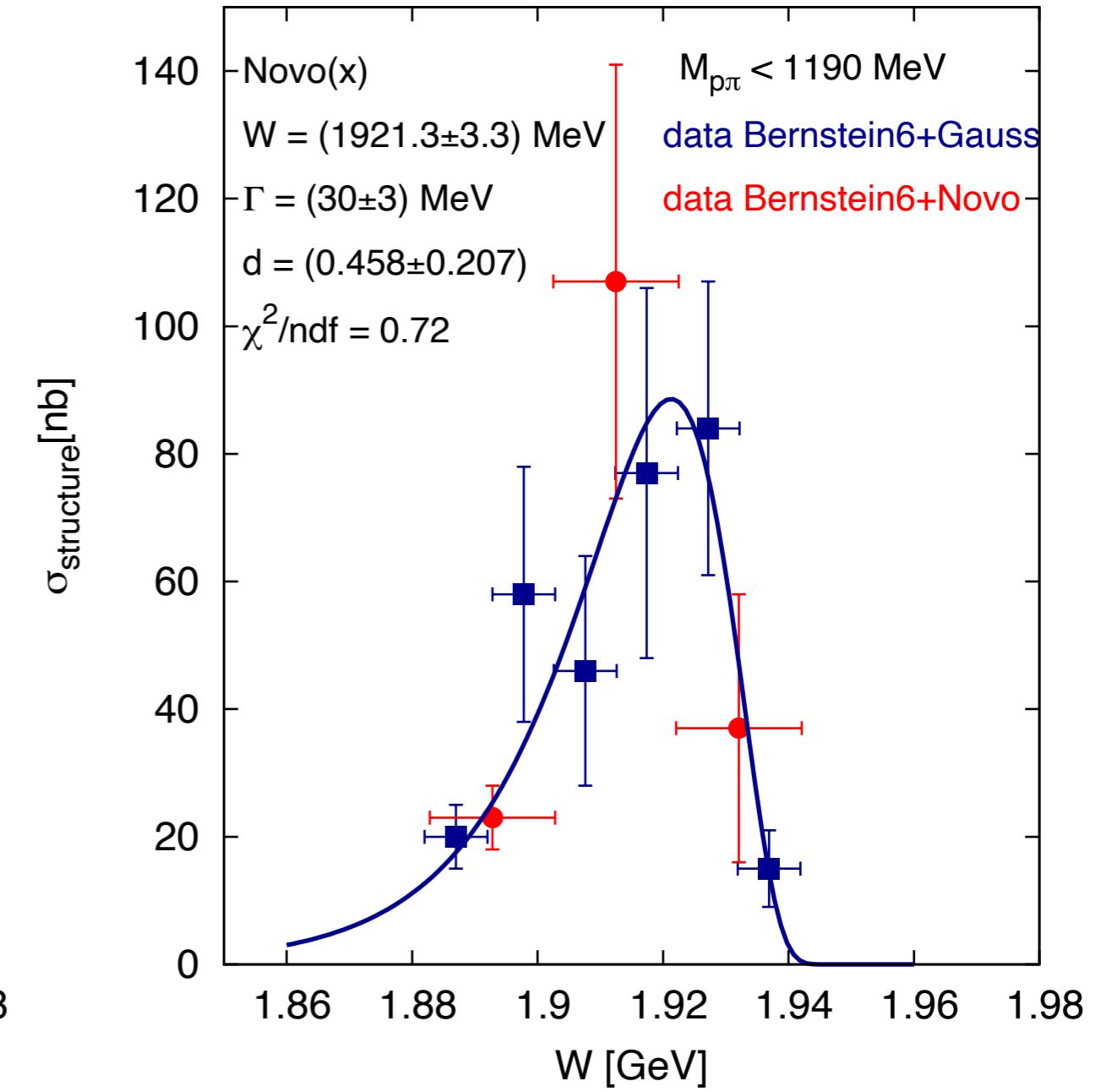
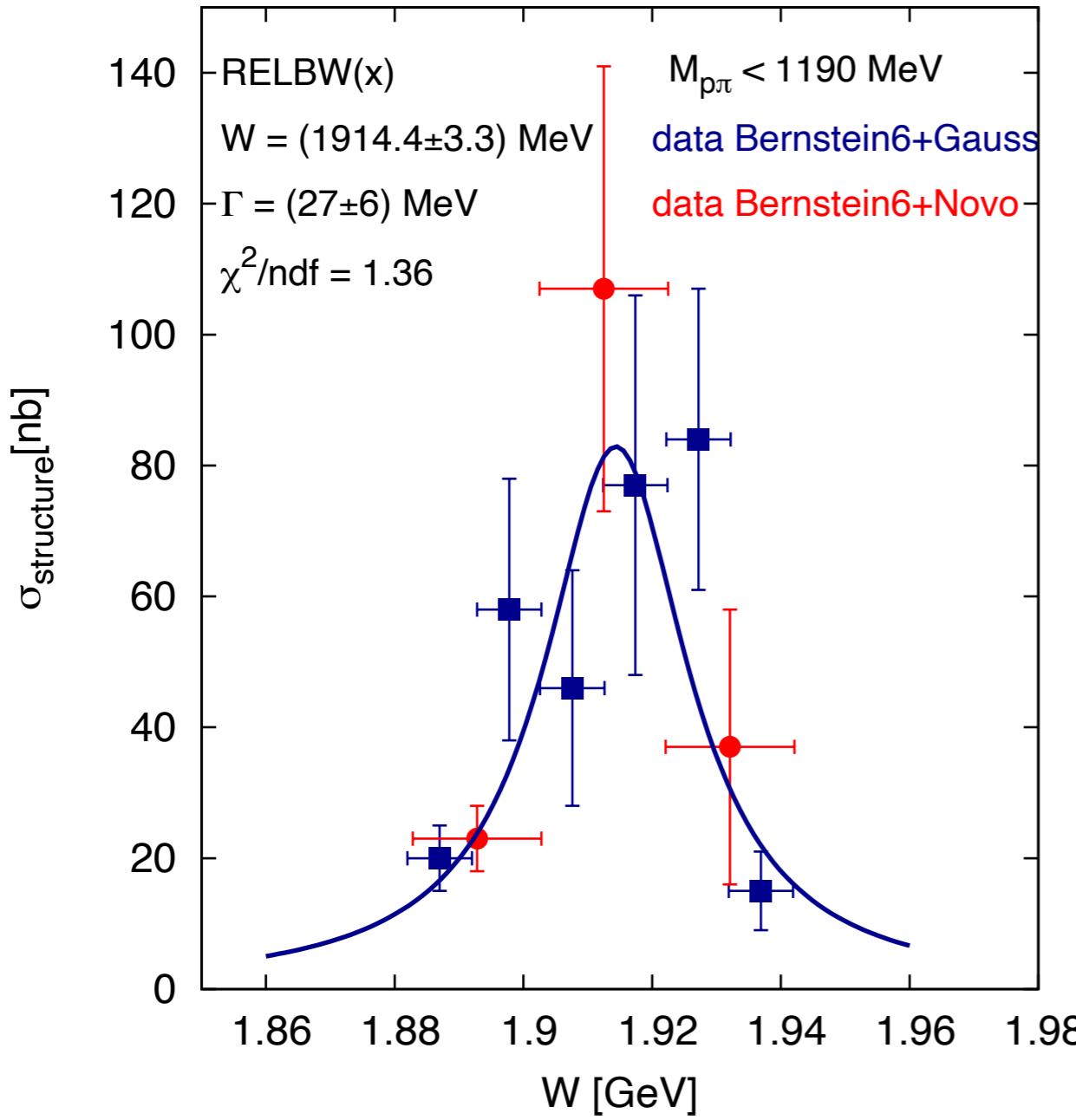
re-scattering probability



phase-space distribution

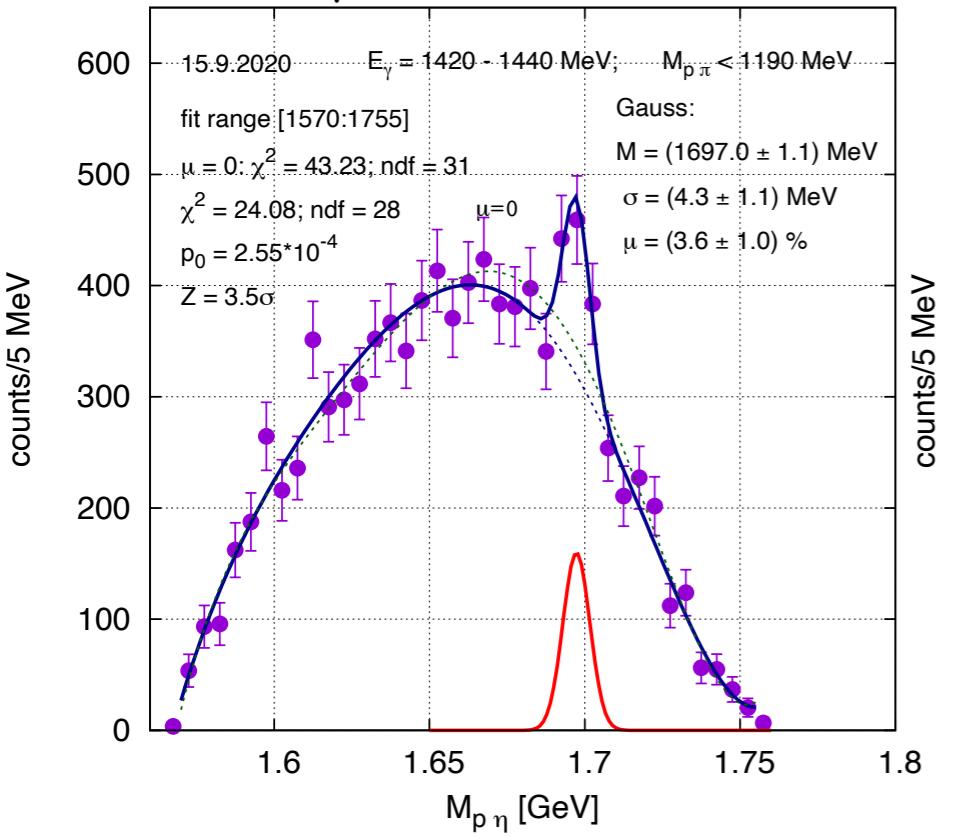


θ_π = scattering angle of π^0
in the π^0 -p cm system

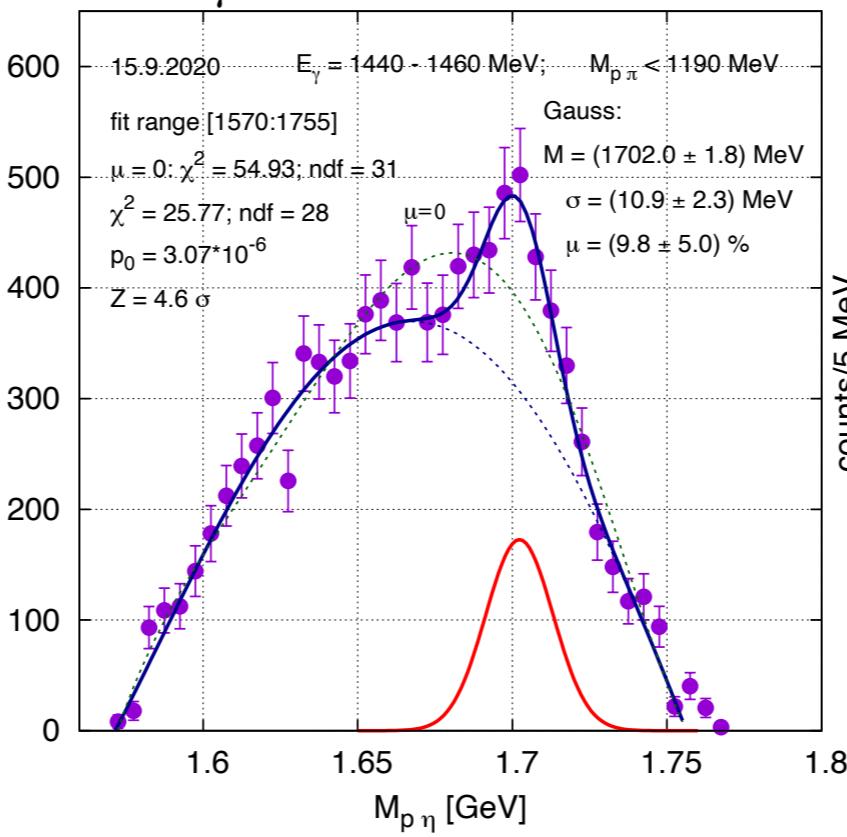


properties of structure as function of the incident photon energy; fit: Gauss

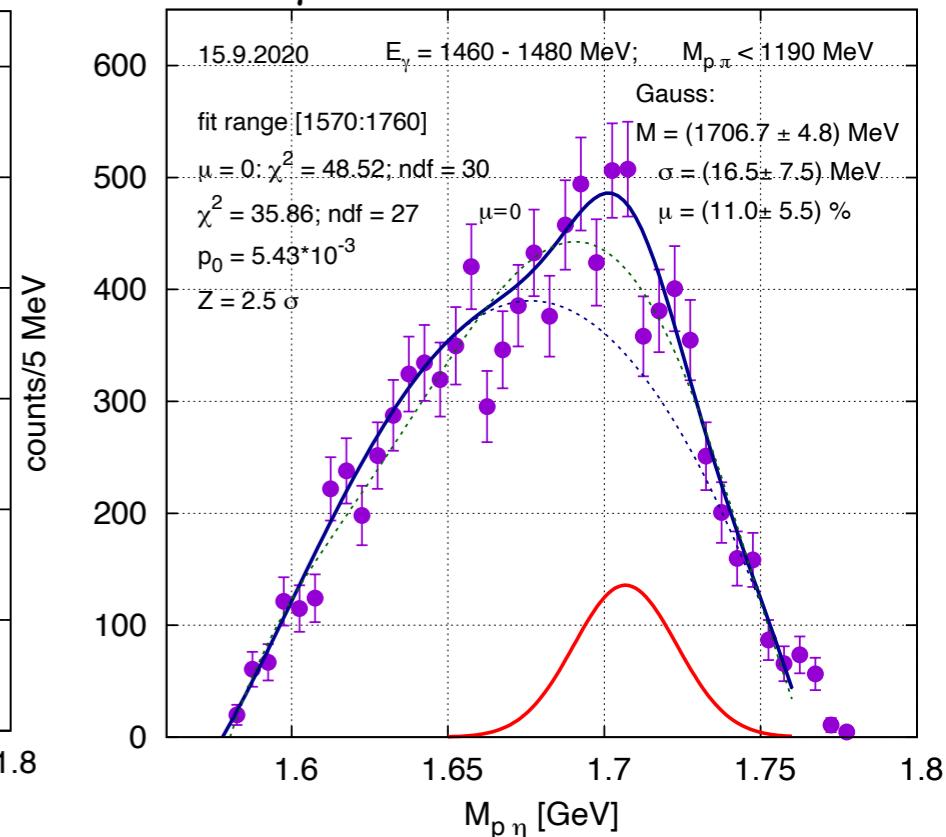
$E_\gamma = 1420 - 1440 \text{ MeV}$



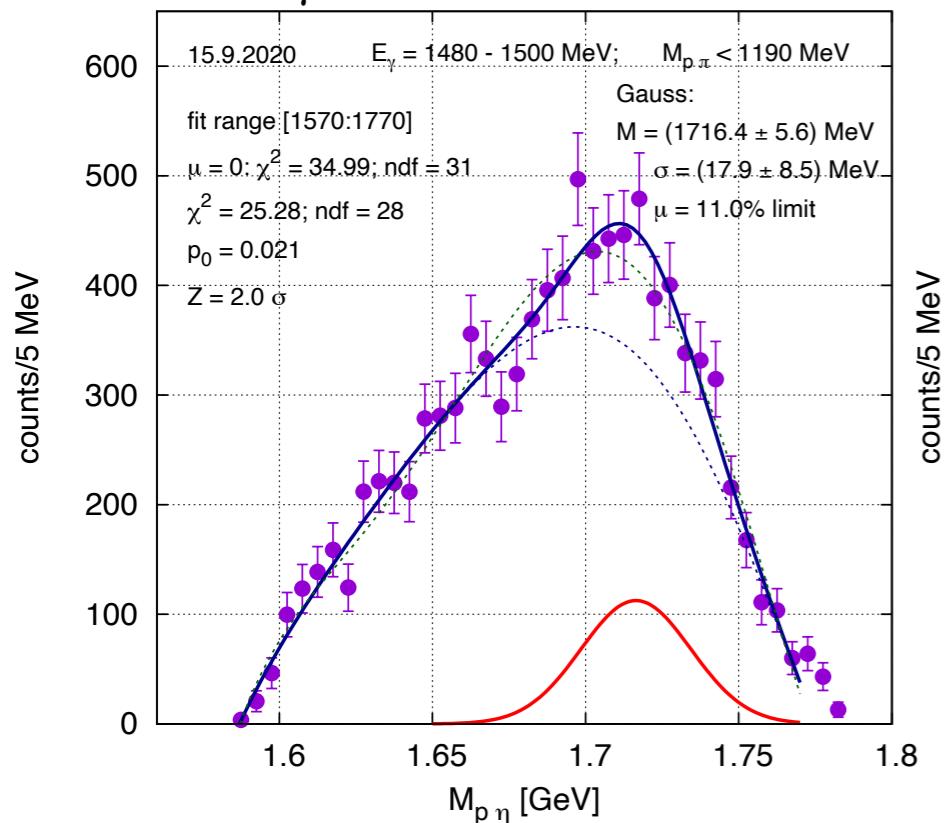
$E_\gamma = 1440 - 1460 \text{ MeV}$



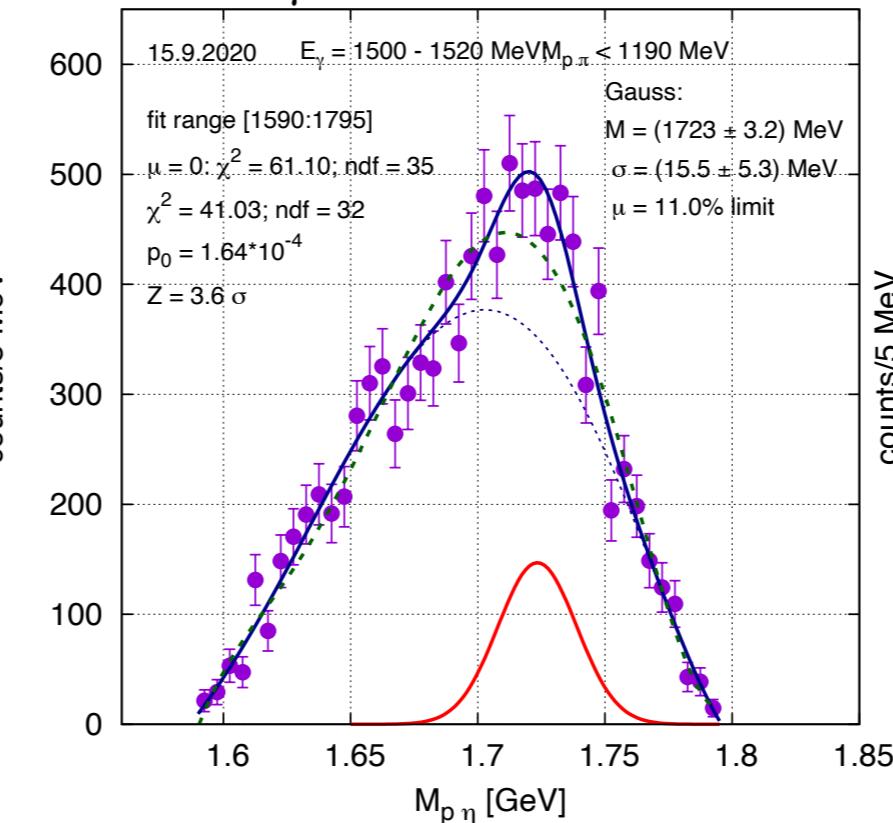
$E_\gamma = 1460 - 1480 \text{ MeV}$



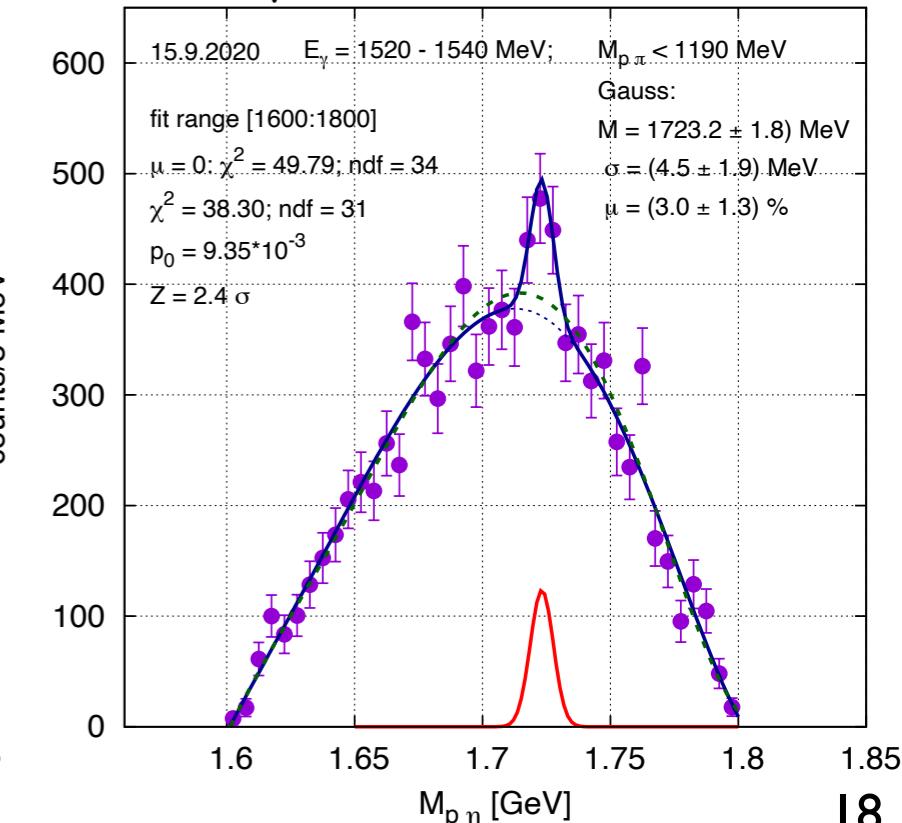
$E_\gamma = 1480 - 1500 \text{ MeV}$



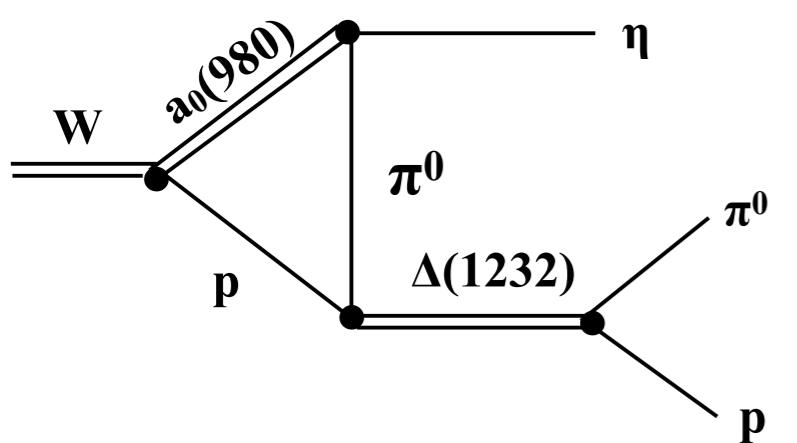
$E_\gamma = 1500 - 1520 \text{ MeV}$



$E_\gamma = 1520 - 1540 \text{ MeV}$

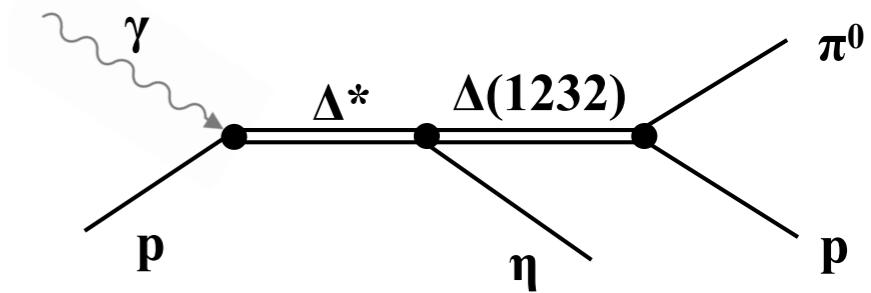


triangular diagram:



**interference of
tree-level and
triangular amplitudes**

tree level diagram:



resonance	I	J ^P	mass[MeV]	width [MeV]	br(p a0)	br(η Δ(1232))	reference
N(1880)	1/2	1/2 ⁺	1860	230	(3±2)%	—	Gutz/PDG
Δ(1910)	3/2	1/2 ⁺	1900	300	?	(5-13)%	Gutz/PDG
Δ(1920)	3/2	3/2 ⁺	1920	300	(4±2)%	(5-17)%	Horn/PDG
Δ(1940)	3/2	3/2 ⁻	2000	400	(2±1)%	(4-16)%	Horn/PDG

$N(1880)1/2^+ (1/2, 1/2^+) \rightarrow a_0(1, 0^+) + p(1/2, 1/2^+); L = 0$

$\Delta(1910)1/2^+ (3/2, 1/2^+) \rightarrow a_0(1, 0^+) + p(1/2, 1/2^+); L = 0$

$\Delta(1920)3/2^+ (3/2, 3/2^+) \rightarrow a_0(1, 0^+) + p(1/2, 1/2^+); L = 2$

$\Delta(1940)3/2^- (3/2, 3/2^-) \rightarrow a_0(1, 0^+) + p(1/2, 1/2^+); L = 1$

$W = N(1880)1/2^+ (I=1/2)$ would imply isospin violation since $\eta \Delta(1232)$ has $I=3/2$!!!

if $W = \Delta(1910)1/2^+ I=3/2$ interference with dominating tree level ($L_{\eta\Delta} = 1; I=3/2$); $L_{pa0} = 0$

if $W = \Delta(1920)3/2^+ I=3/2$ interference with dominating tree level ($L_{\eta\Delta} = 1; I=3/2$); $L_{pa0} = 2$

if $W = \Delta(1940)3/2^- I=3/2$ interference with dominating tree level ($L_{\eta\Delta} = 0, 2; I=3/2$); $L_{pa0} = 1$

motivation:

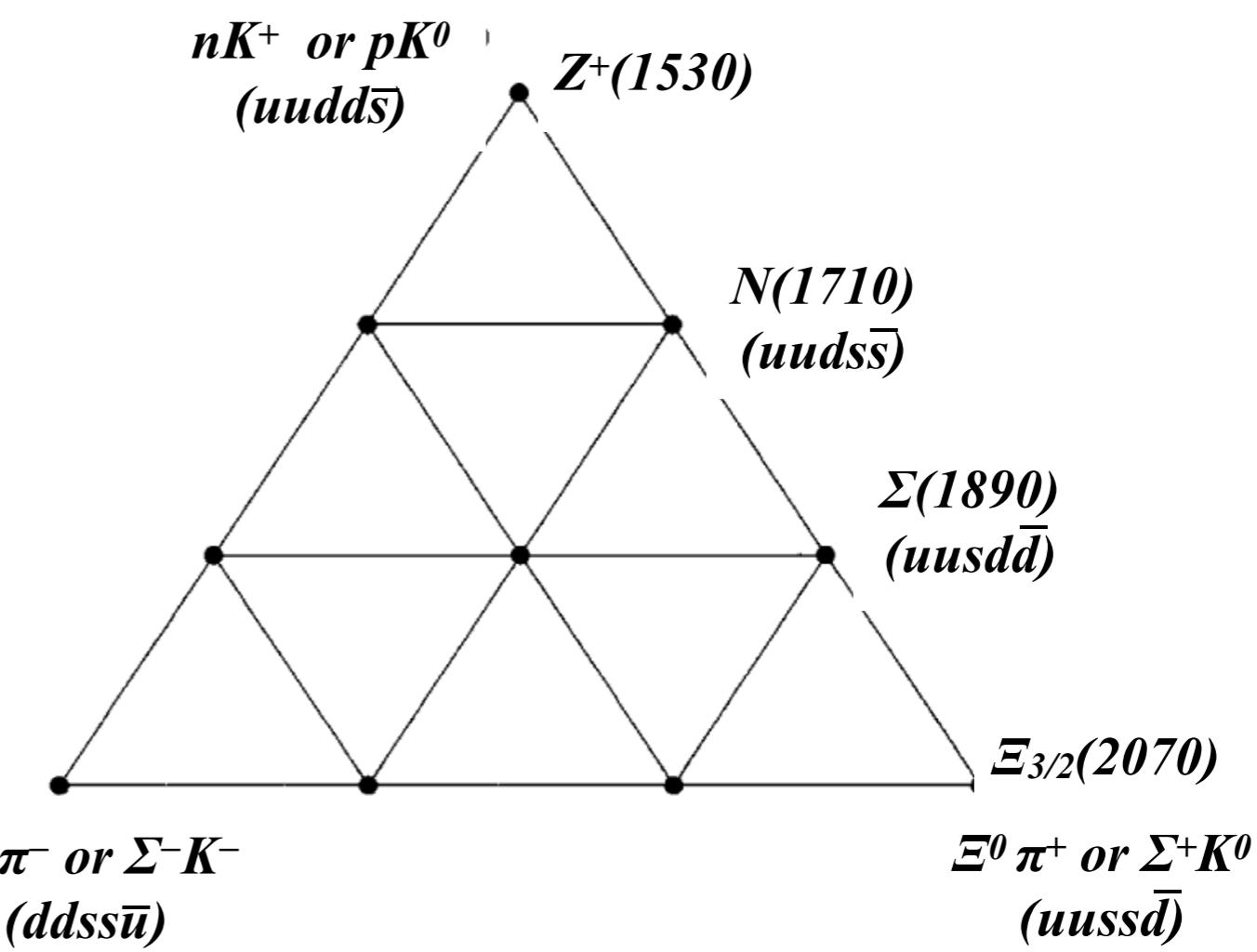
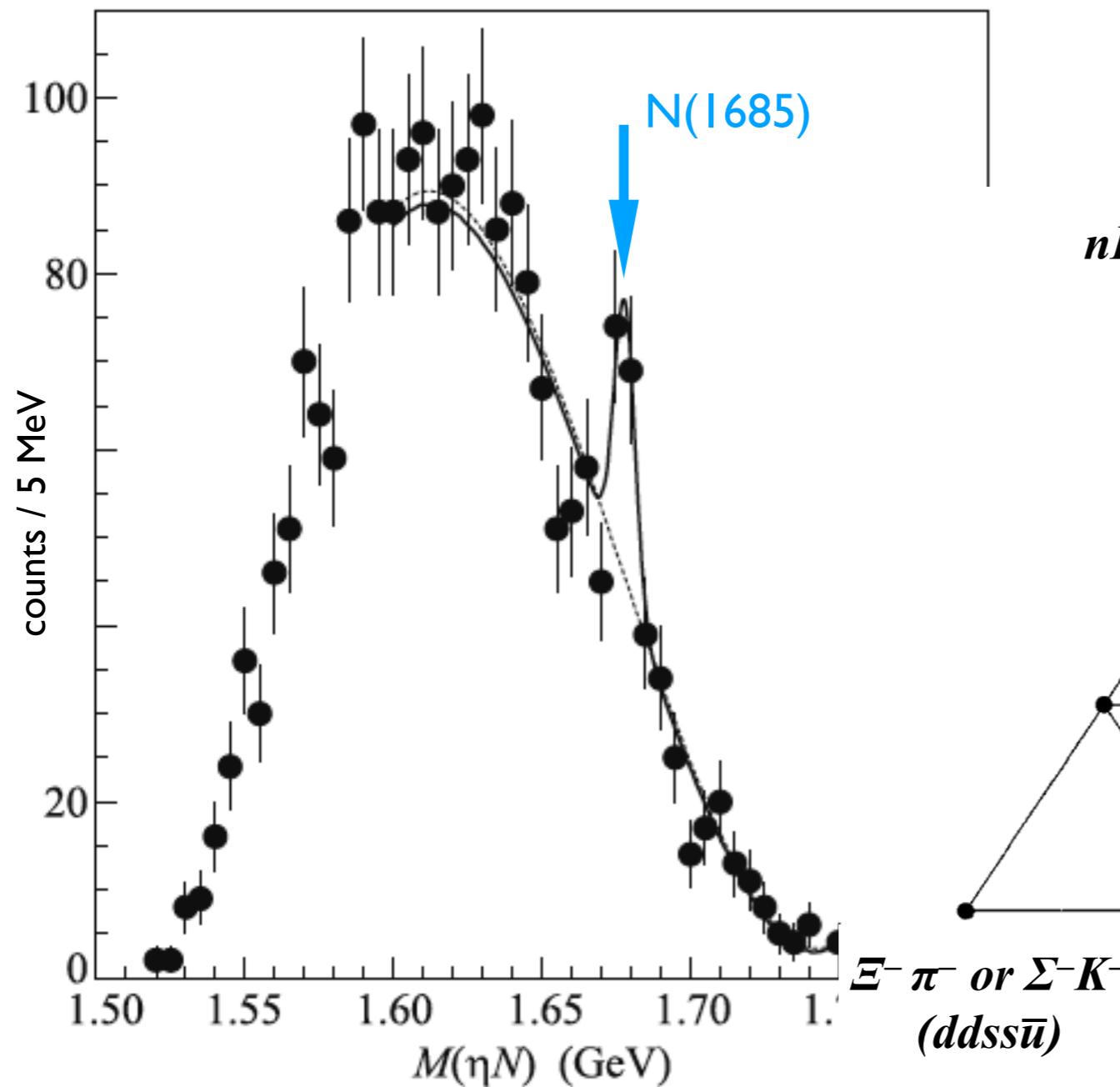
observation of narrow N(1685) resonances in $\gamma N \rightarrow \eta \pi N$ reactions

V. Kuznetsov et al., JETP Letters 106 (2017) 69

$E_\gamma = 1400 - 1500$ MeV

$\gamma N \rightarrow \pi \eta N$ - sum of all channels

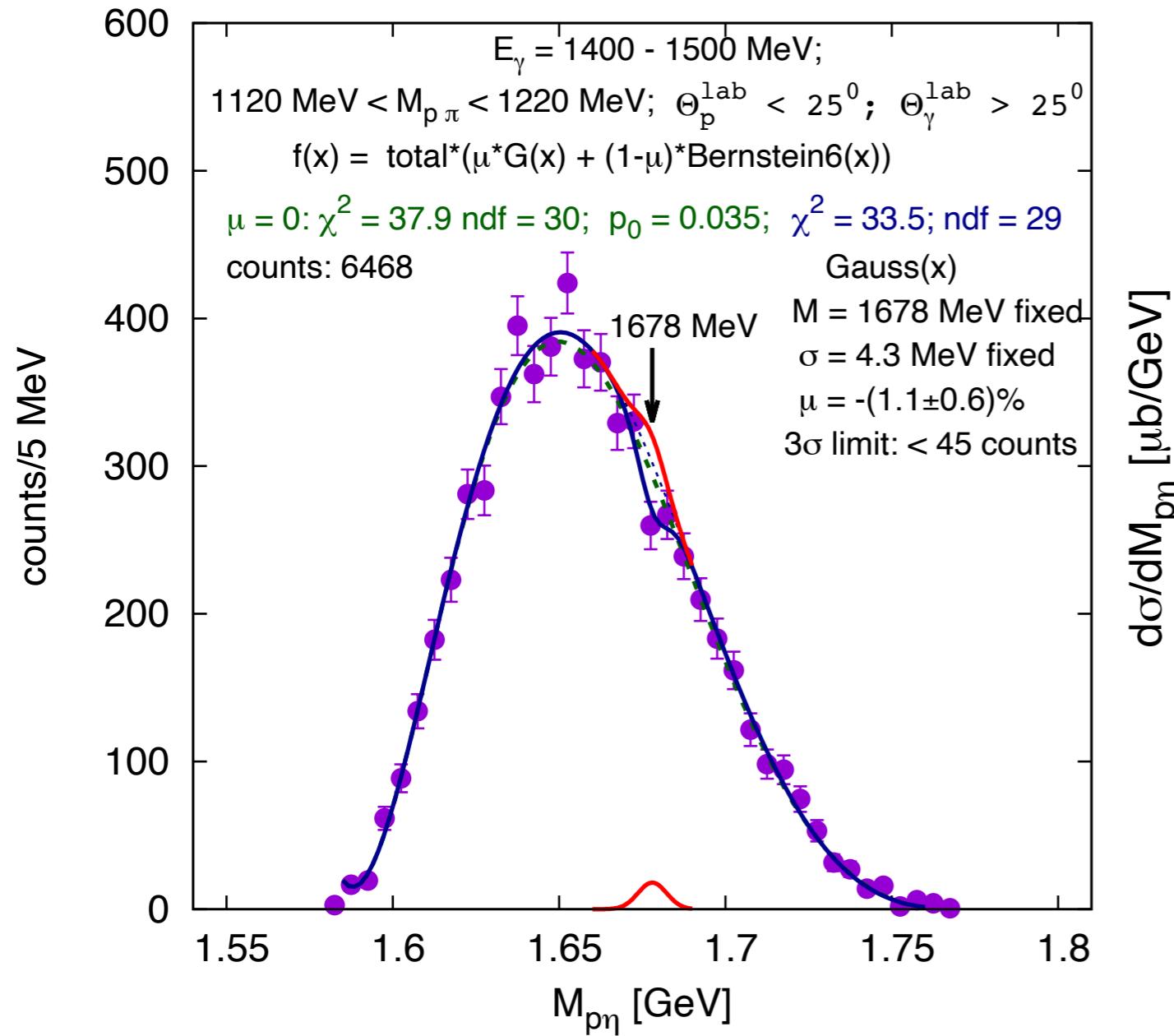
D. Diakonov, V. Petrov, and M.V. Polyakov,
Z. Phys. A 359 (1997) 305



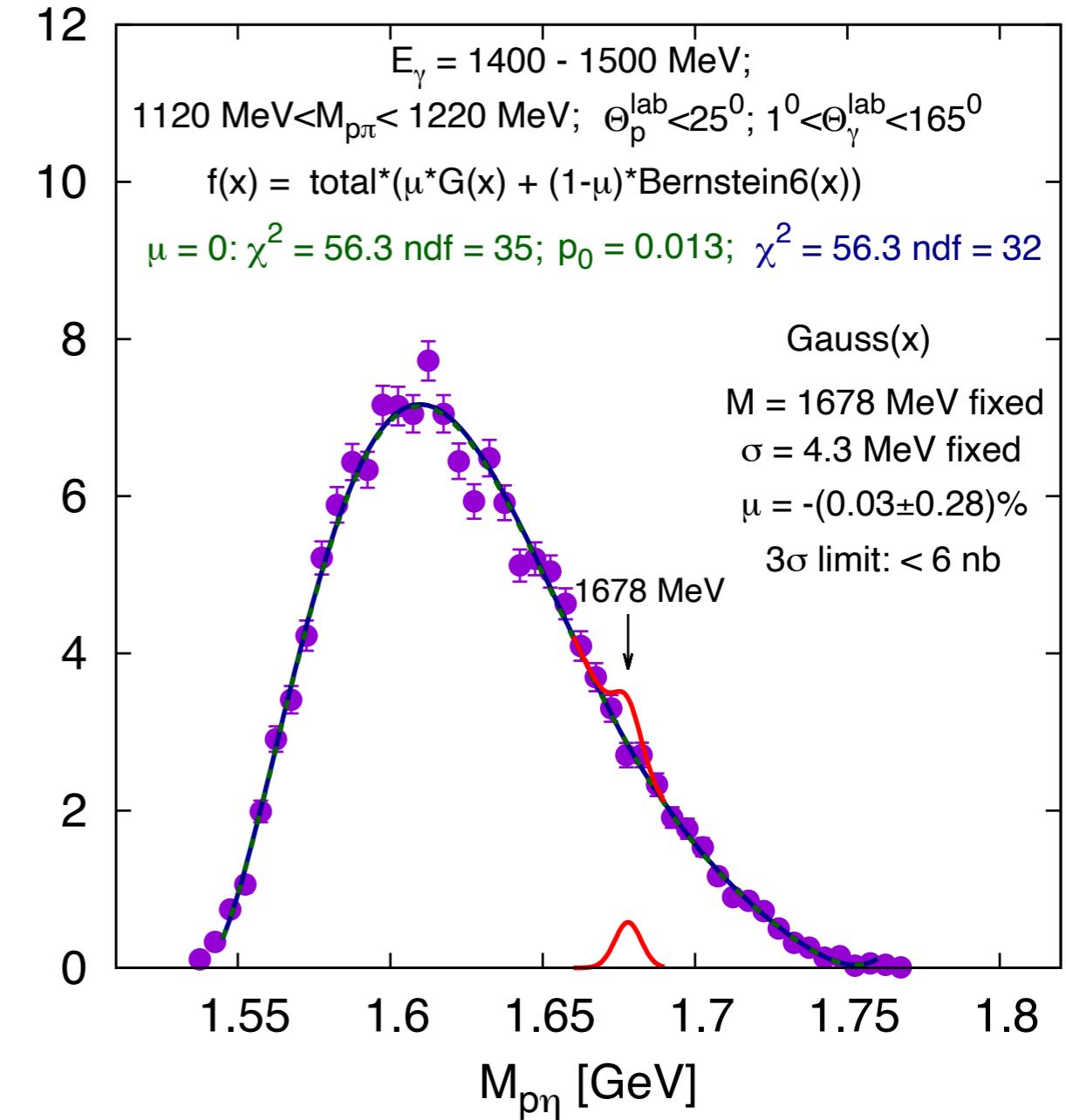
is the narrow structure in $M(\eta N)$ real?

search for a narrow structure in $M(p\eta)$ distribution around 1678 MeV

identical cuts as in JETP 106



acceptance corrected; $\theta_\gamma > 1^\circ$

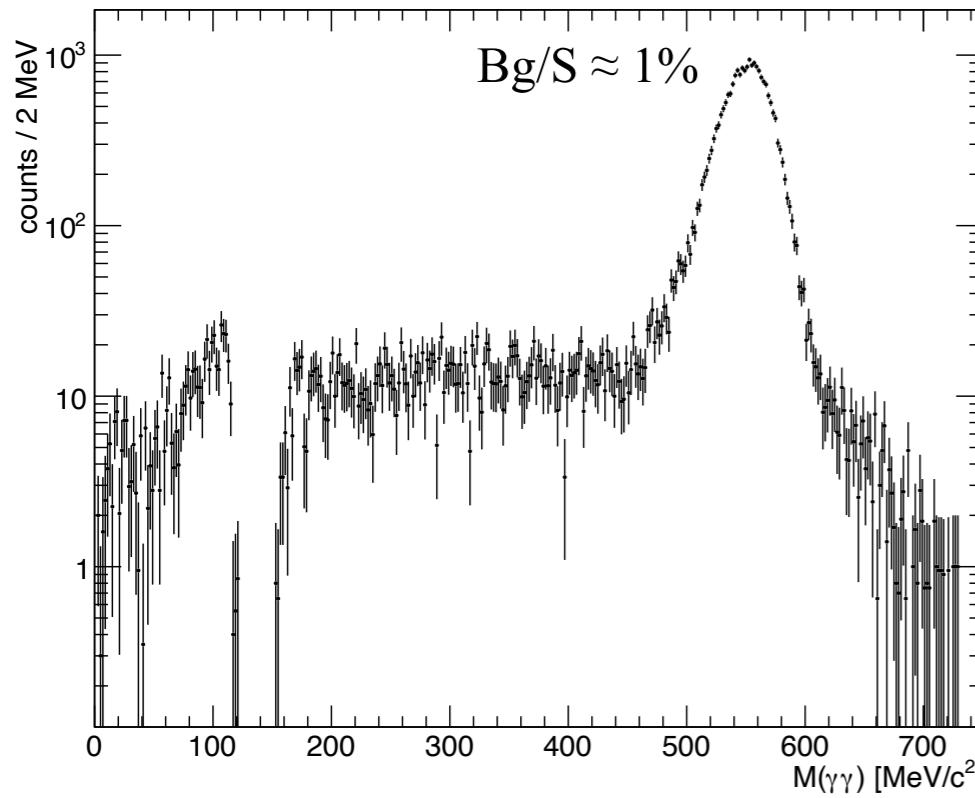


3σ limit: < 45 counts

≈ 4 times higher statistics

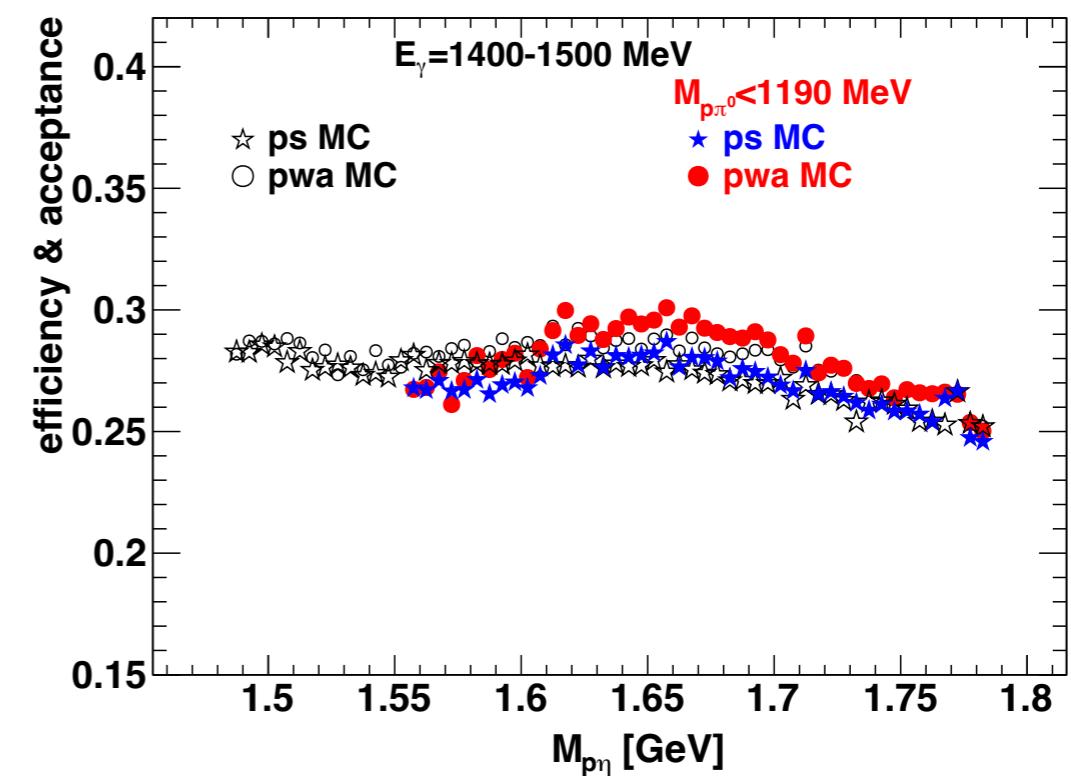
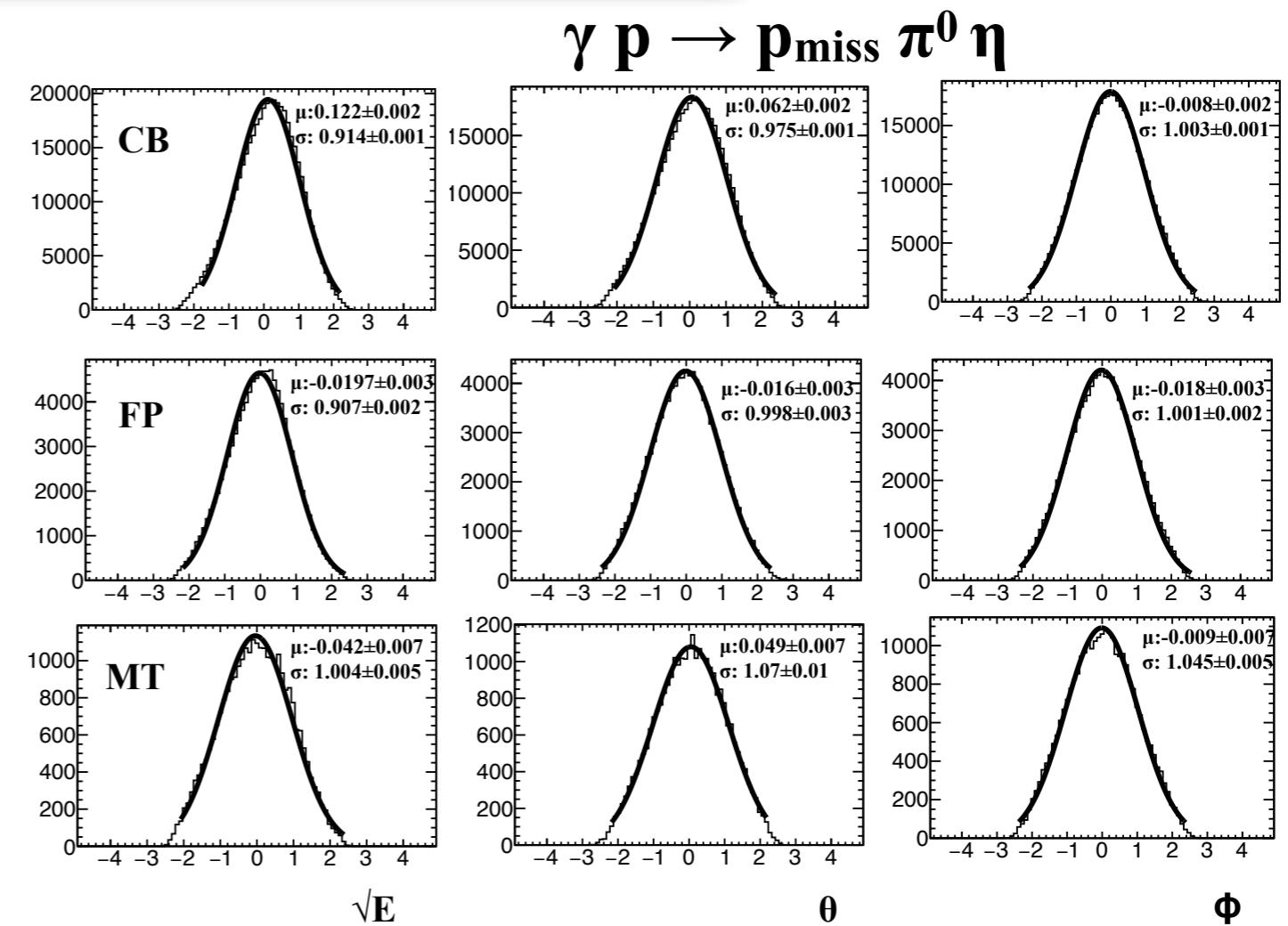
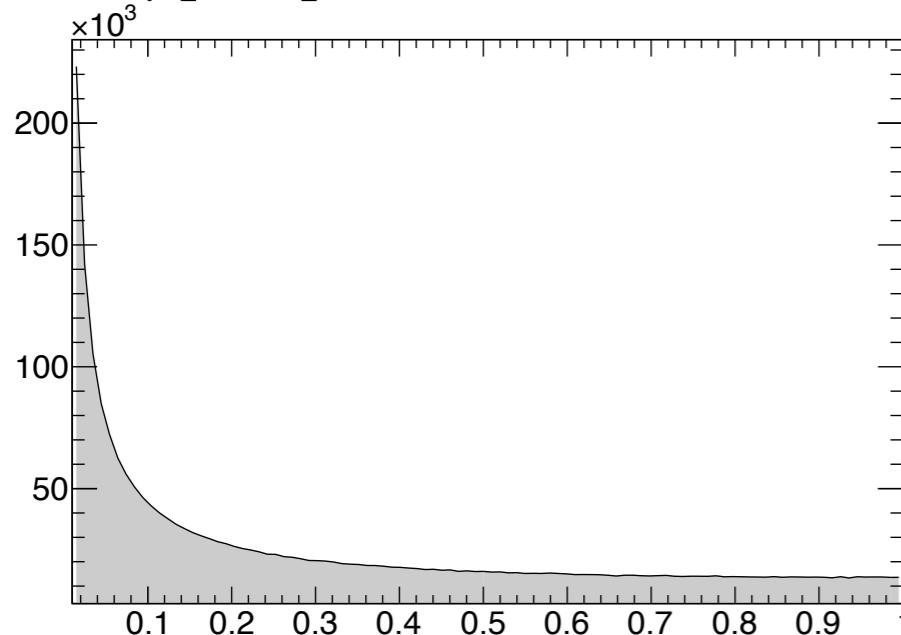
data analysis: kinematic fit

$\pi^0 \pi^0$ events removed from the data



CL: $\gamma p \rightarrow p_{\text{miss}} \pi^0 \gamma\gamma \geq 0.2$

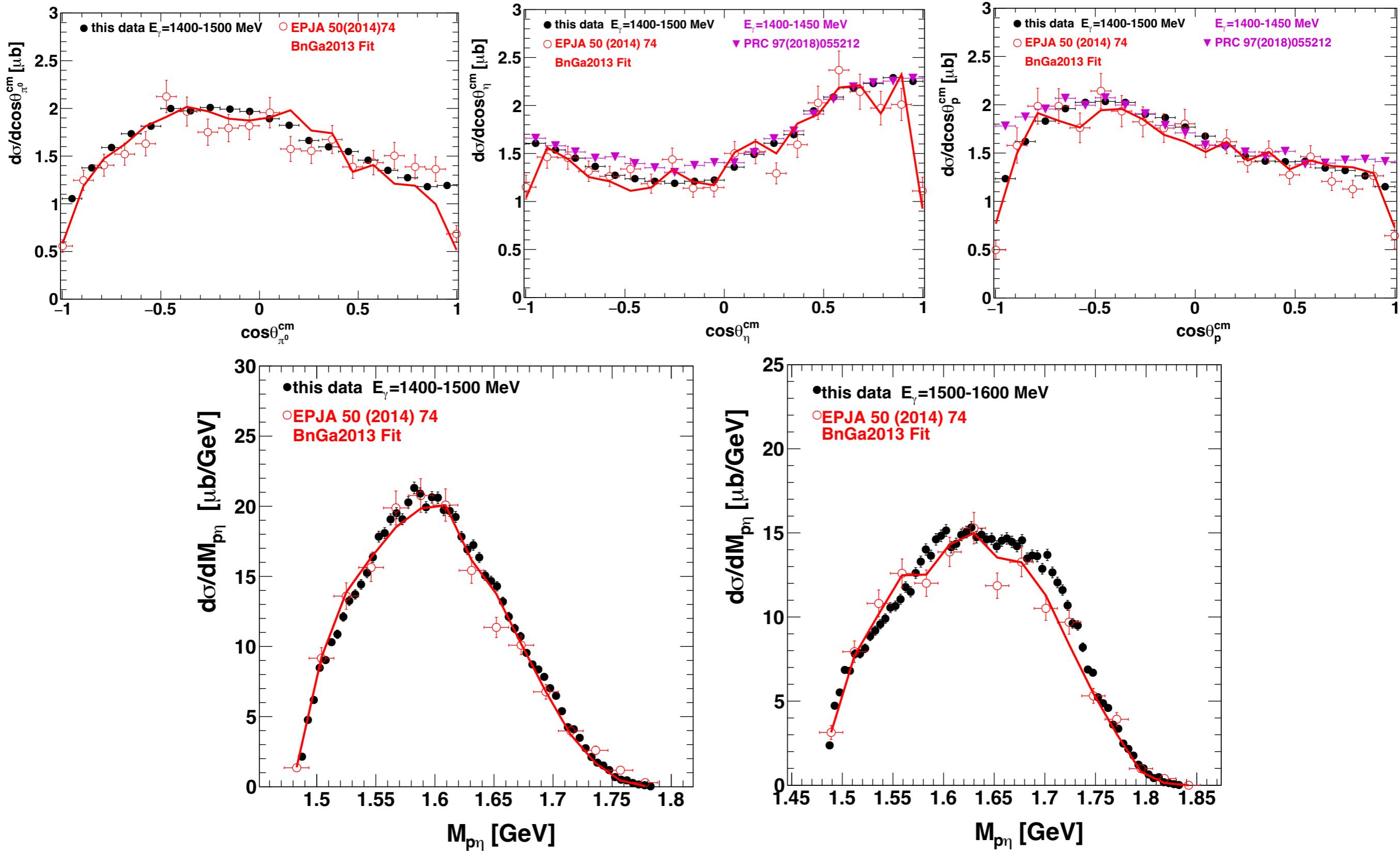
CL: $\gamma p \rightarrow p_{\text{miss}} \pi^0 \pi^0 \leq 0.01$



comparison to previous results

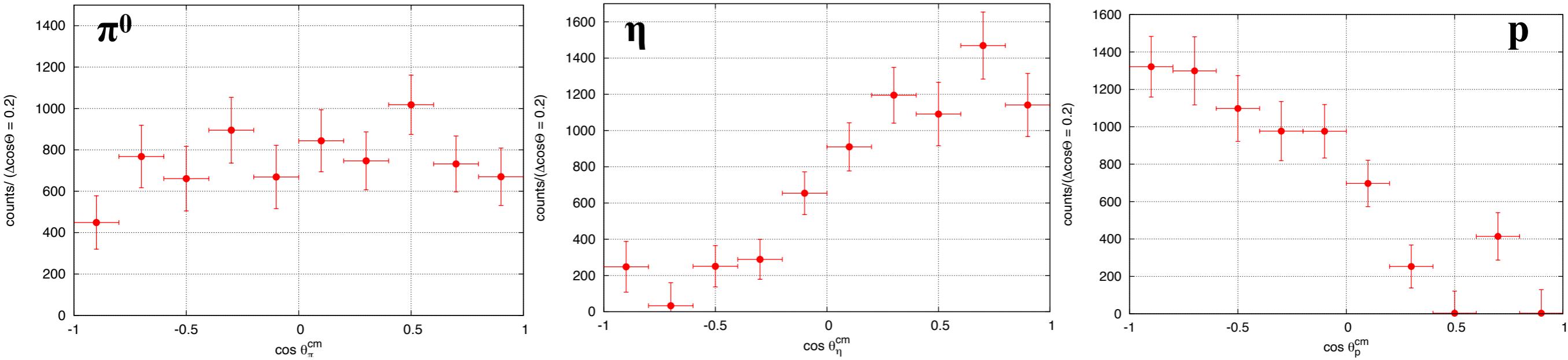
$\gamma p \rightarrow p \pi^0 \eta$

$E_\gamma = 1400 - 1500 \text{ MeV}$

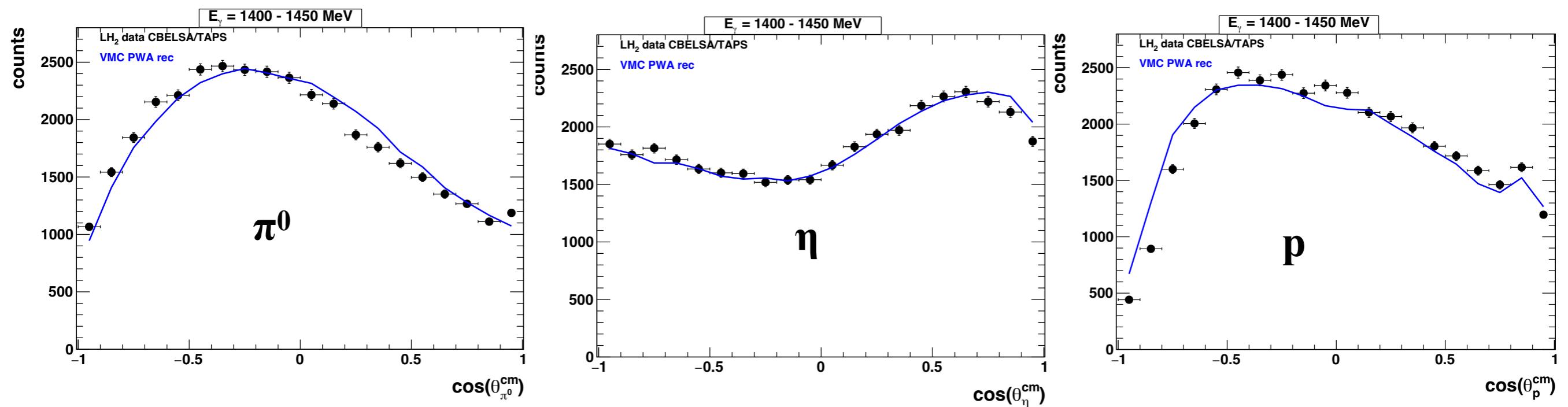


$$\gamma p \rightarrow p \pi^0 \eta$$

**angular distributions of events in structure for $E_\gamma = 1420 - 1540$ MeV and $M_{p\pi} < 1190$ MeV
in γp cm system**



angular distributions for $E_\gamma = 1400 - 1450$ MeV without $M_{p\pi}$ cut



interference of tree-level and triangular amplitude

V.R. Debastiani, S. Sakai, E. Oset
 Eur. Phys. J. C 79 (2019) 69

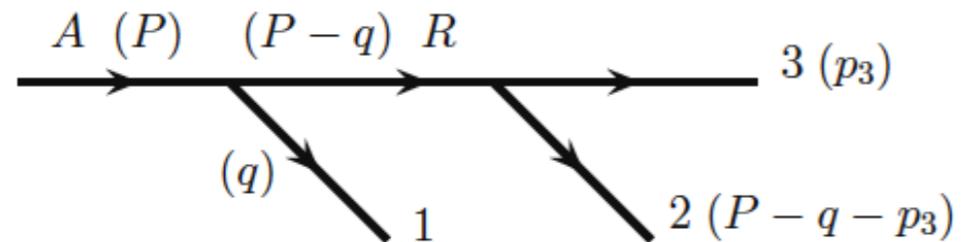


Fig. 1 Tree level diagram for the process $A \rightarrow 1 + 2 + 3$ mediated by a resonance R that decays into particles 2 and 3. In brackets the momenta of the particles

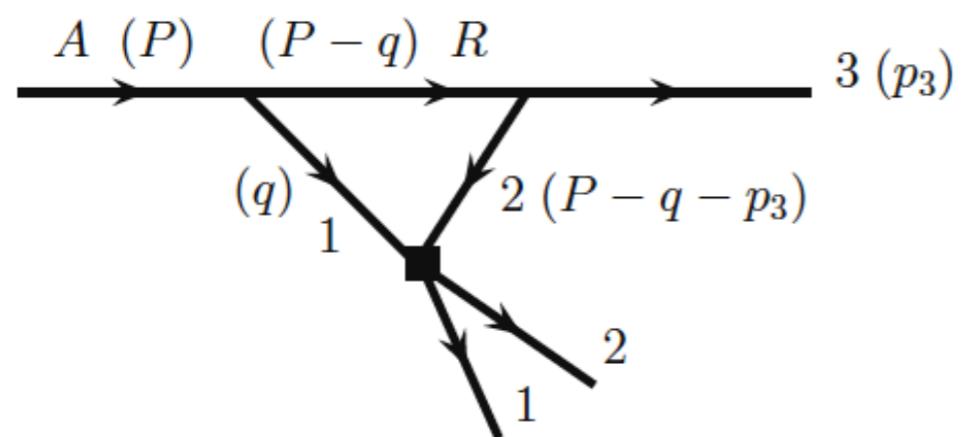


Fig. 2 Triangle mechanism emerging from the mechanism of Fig. 1, with final state interaction of particles 1 and 2. In brackets the momenta of the particles

**singularity for $M_A = 2154$ MeV,
 $M_R = 1600$ MeV, $\Gamma_R = 30$ MeV
 $M_{12} = 800$ MeV**

1-2 rescattering: $M_{BW} = 800$ MeV; $\Gamma_{BW} = 20$ MeV

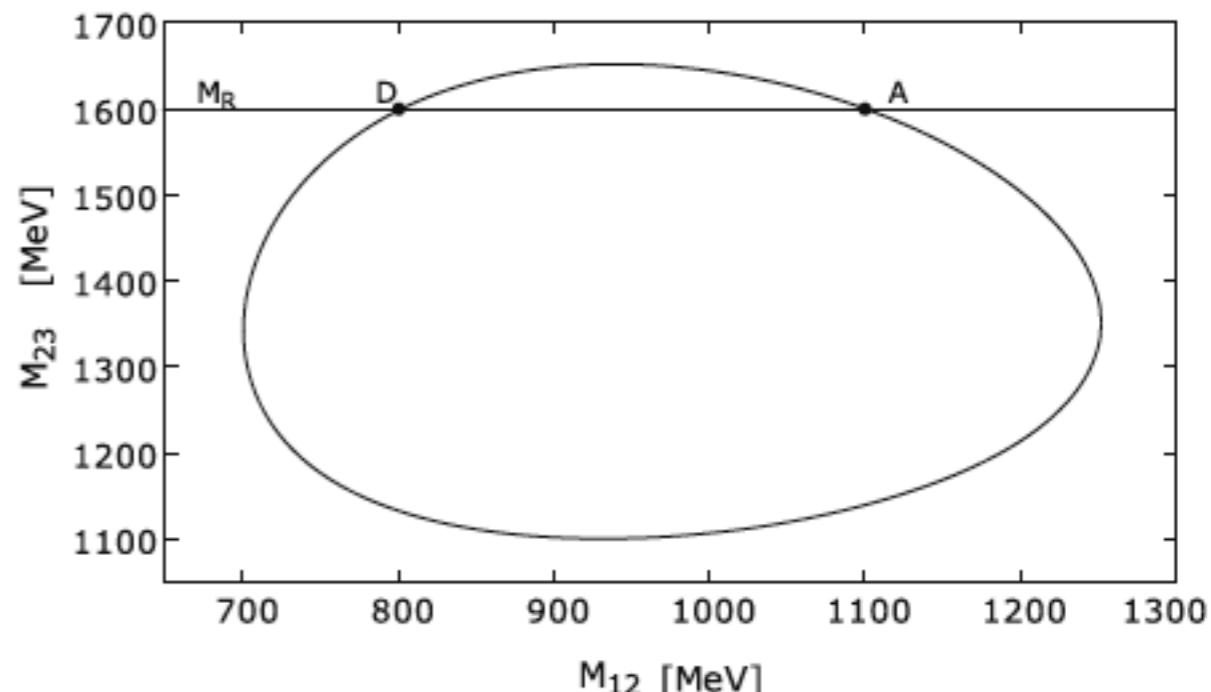


Fig. 7 Dalitz plot for $A \rightarrow 1 + 2 + 3$ with the parameters of Eqs. (63) and (64). Point D corresponds to the triangle singularity

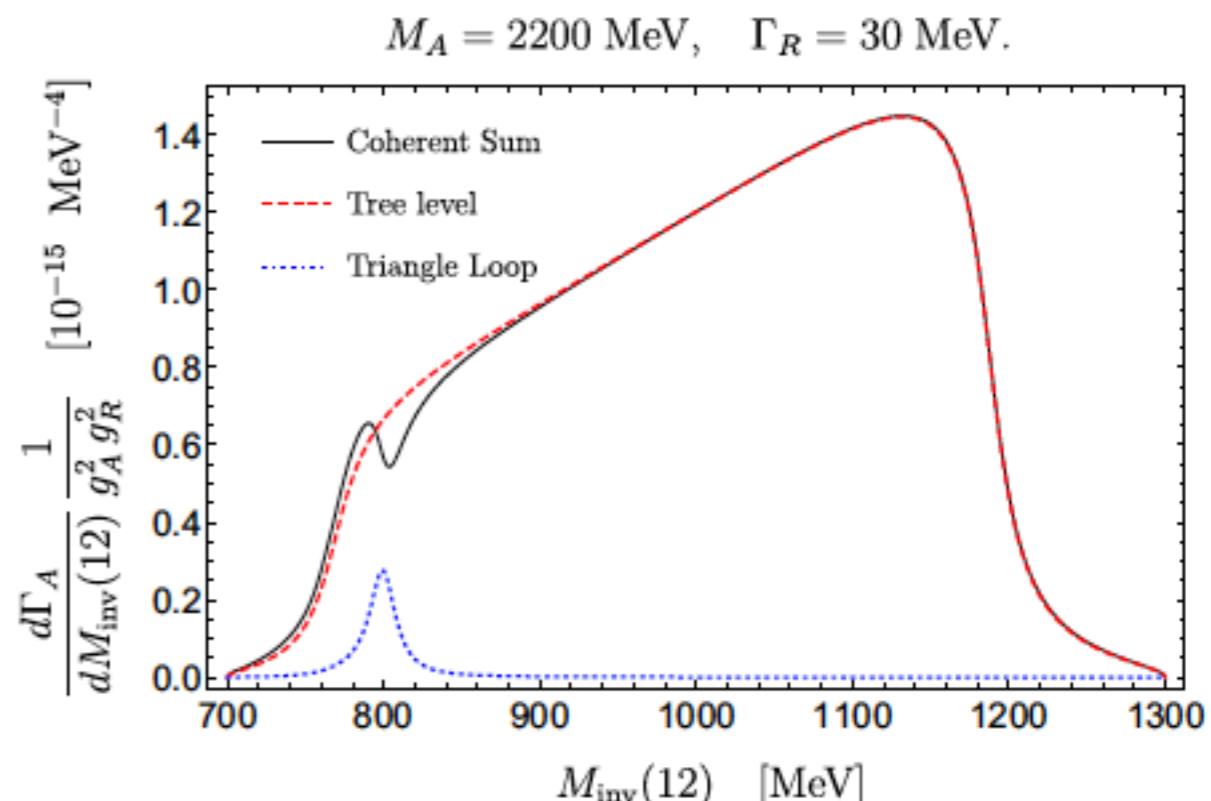
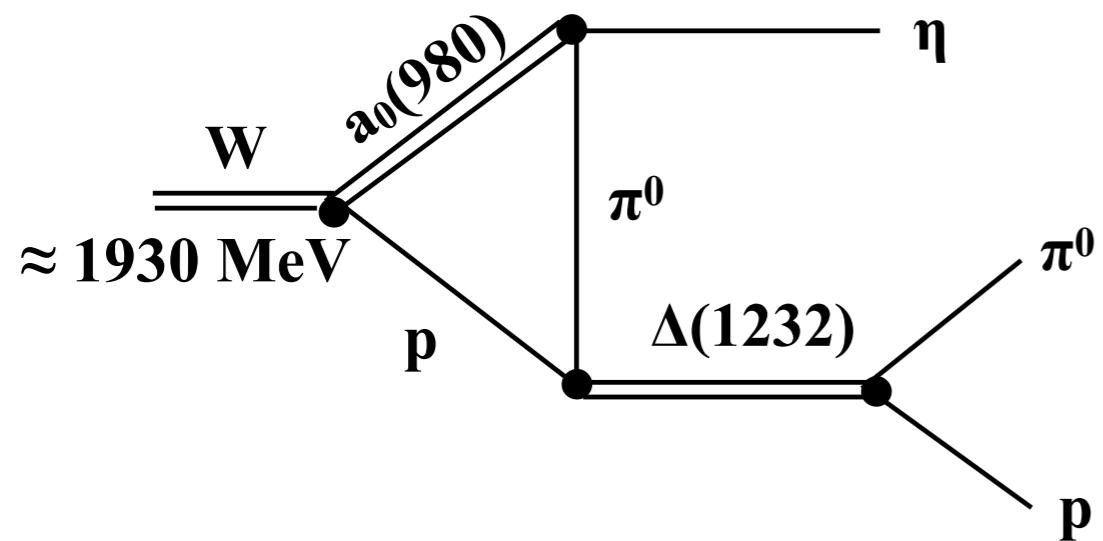


Fig. 8 $\frac{d\Gamma_A}{dM_{\text{inv}}(12)} \frac{1}{g_A^2 g_R^2}$ as a function of $M_{\text{inv}}(12)$ with $M_A = 2200$ MeV and $\Gamma_R = 30$ MeV

CBELSA/TAPS

$$\gamma p \rightarrow p \pi^0 \eta$$

triangular diagram:



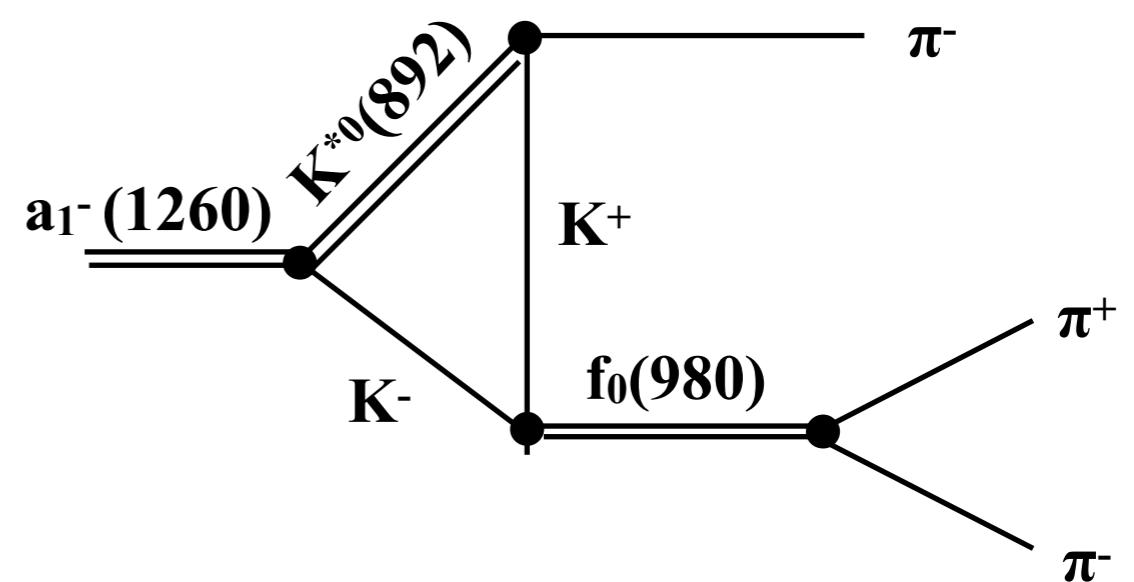
$a_0(980); \Gamma \approx 50 - 100 \text{ MeV}$

$\Delta(1232); \Gamma \approx 117 \text{ MeV}$

COMPASS

$$\begin{aligned} \pi^- p &\rightarrow a_1^-(1260) p \\ a_1^-(1260) &\rightarrow \pi^- \pi^+ \pi^- \end{aligned}$$

triangular diagram



$K^{*0}(892); \Gamma = 47.3 \pm 0.5 \text{ MeV}$

$f_0(980); \Gamma \approx 10 - 100 \text{ MeV}$

$$\pi^- p \rightarrow \pi^- \pi^- \pi^+ p$$

COMPASS
A triangular Singularity as Origin of the a1(1420)
arXiv:2006.05342

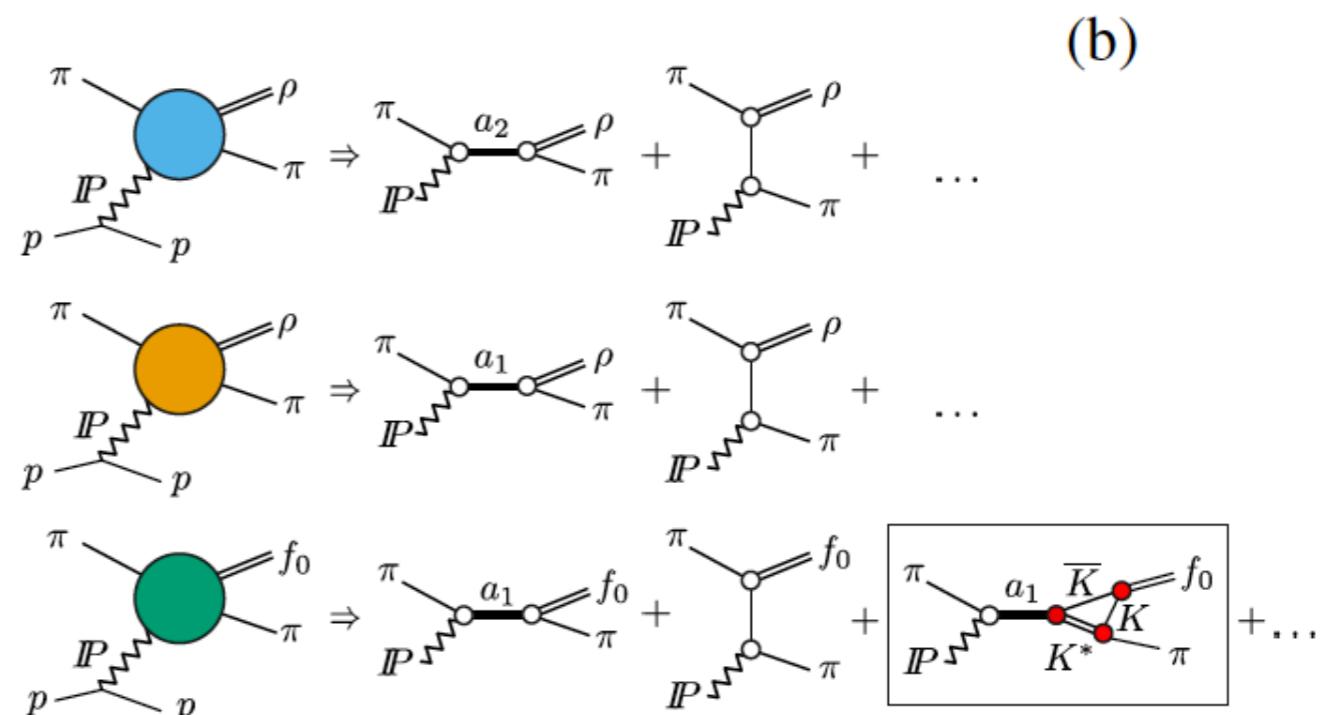
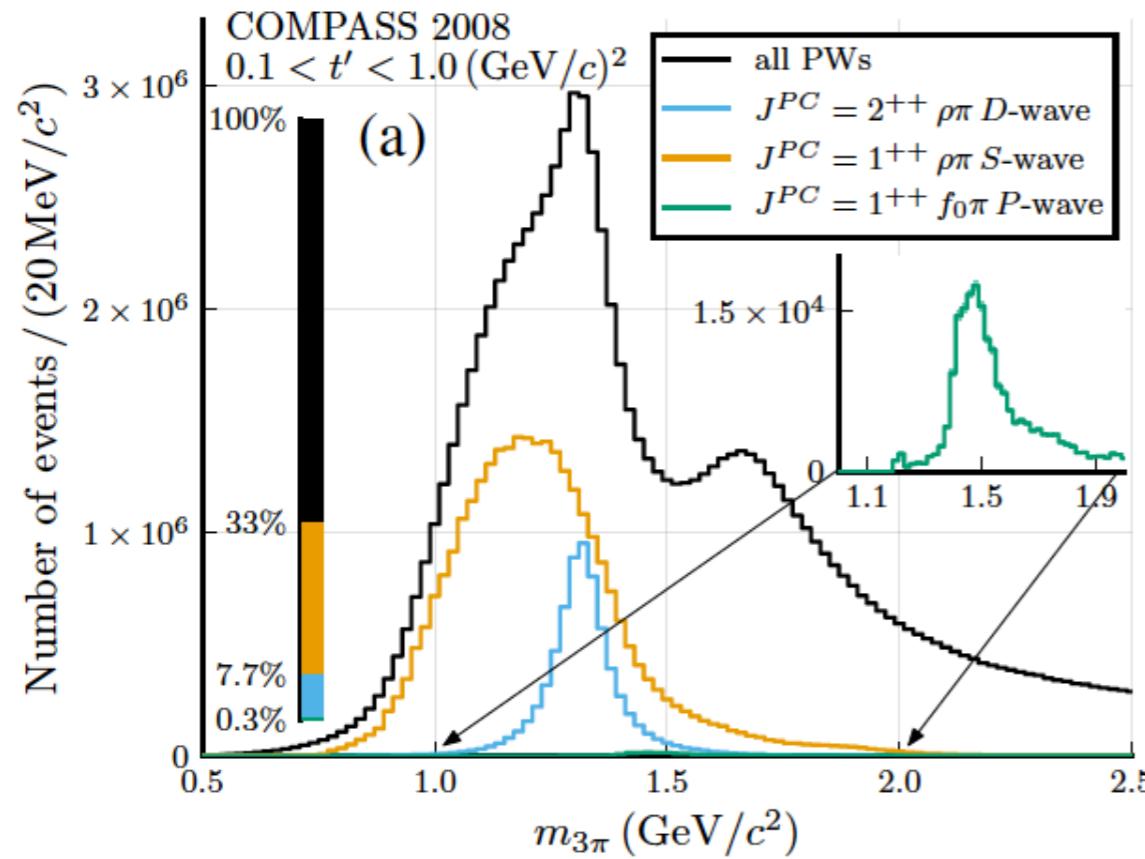
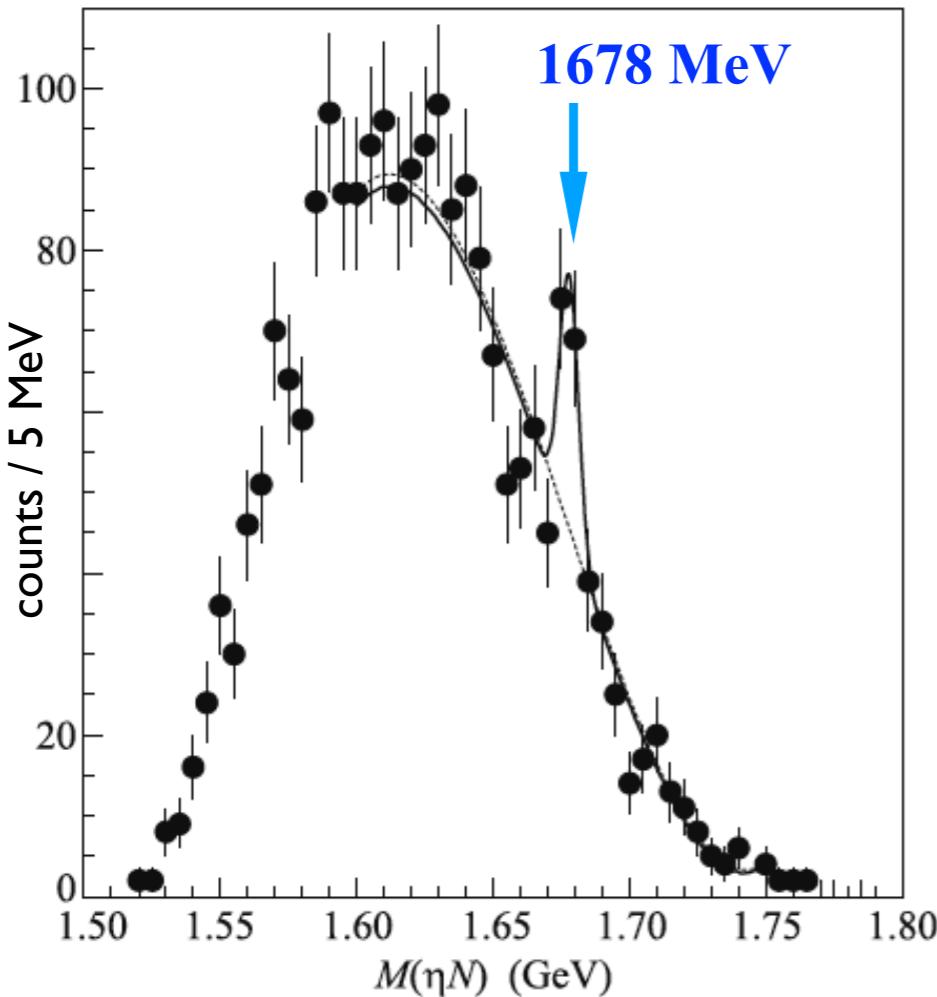


Fig. 1: (a) Intensities of selected waves from the PWA of the reaction $\pi^- + p \rightarrow \pi^- \pi^- \pi^+ + p$ [13]. The inset shows a zoomed view of the $1^{++} 0^+ f_0 \pi$ P-wave. The colored bar on the left indicates the contributions of the different waves to the total intensity. (b) Diagrams showing possible contributions to the $\rho(770)\pi$ and $f_0(980)\pi$ production amplitudes. The Pomeron is labelled IP , a_1 refers to the axial-vector ground state $a_1(1260)$, and a_2 to the tensor ground state $a_2(1320)$. The framed diagram shows the dominant contribution to the $a_1(1420)$ signal via the triangle diagram, as discussed in the present paper.

motivation: observation of narrow N(1685) resonances in $\gamma N \rightarrow \eta\pi N$ reactions

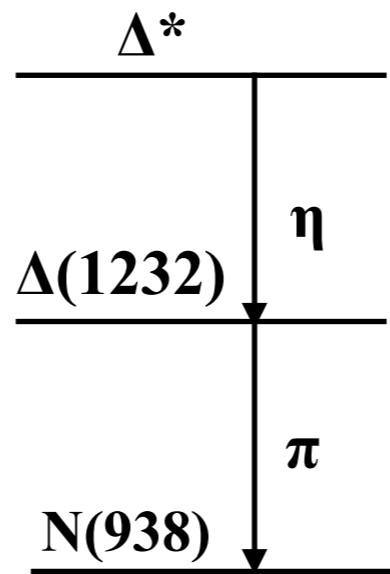
V. Kuznetsov et al.,
JETP Lett. 106 (2017) 693
exotic state predicted by
Chiral Soliton Model

D. Diakonov, V. Petrov, and M.V. Polyakov,
Z. Phys. A 359 (1997) 305



$E_\gamma = 1400 - 1500$ MeV
 $\theta_p < 25^\circ$; $25^\circ < \theta_\gamma < 155^\circ$

$1120 < M_{p\pi} < 1220$ MeV
to suppress
dominant decay

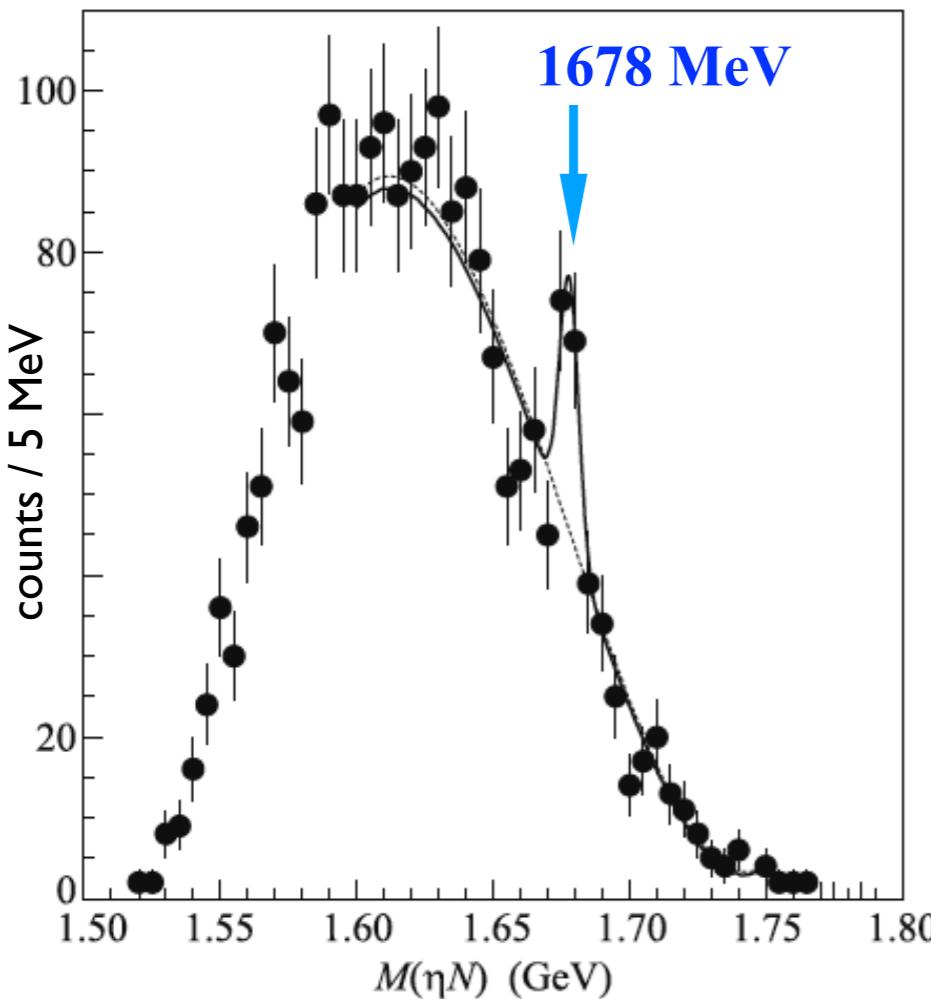


$M_{\eta N} = (1678 \pm 0.8(\text{stat}) \pm 10(\text{syst}))$ MeV;
 $\Gamma \approx 10$ MeV; significance 4.6σ

motivation: observation of narrow N(1685) resonances in $\gamma N \rightarrow \eta\pi N$ reactions

V. Kuznetsov et al.,
 JETP Lett. 106 (2017) 693
 exotic state predicted by
 Chiral Soliton Model

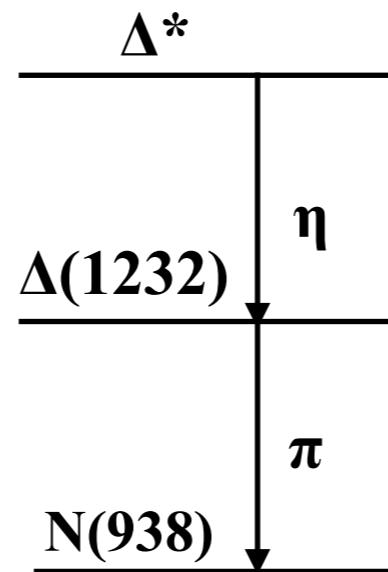
D. Diakonov, V. Petrov, and M.V. Polyakov,
 Z. Phys. A 359 (1997) 305



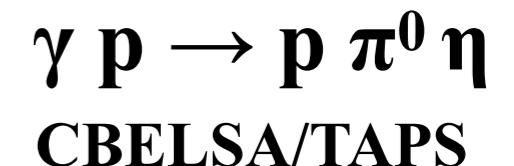
$M_{\eta N} = (1678 \pm 0.8(\text{stat}) \pm 10(\text{syst})) \text{ MeV};$
 $\Gamma \approx 10 \text{ MeV};$ significance 4.6σ

$E_\gamma = 1400 - 1500 \text{ MeV}$
 $\theta_p < 25^\circ; 25^\circ < \theta_\gamma < 155^\circ$

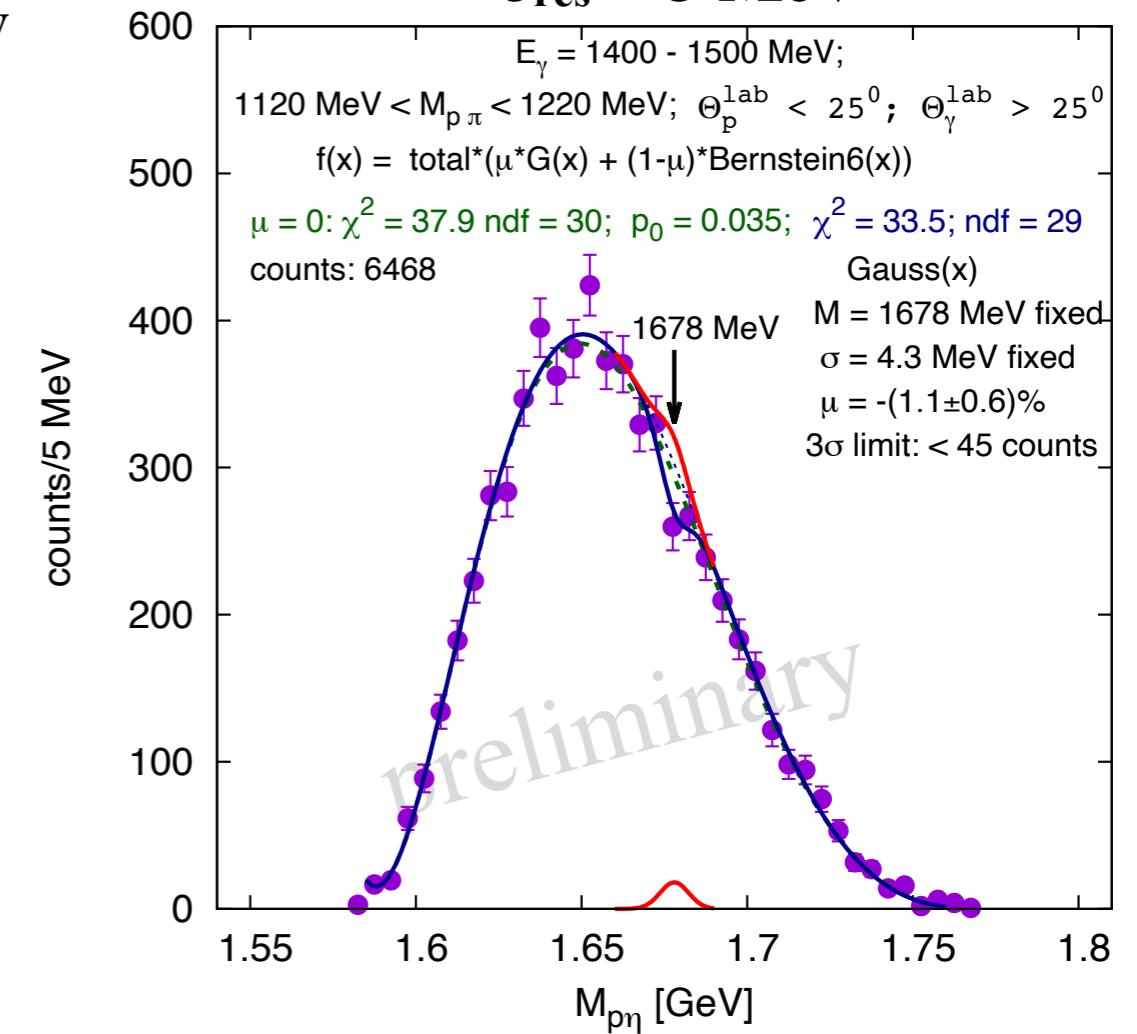
$1120 < M_{p\pi} < 1220 \text{ MeV}$
 to suppress
 dominant decay



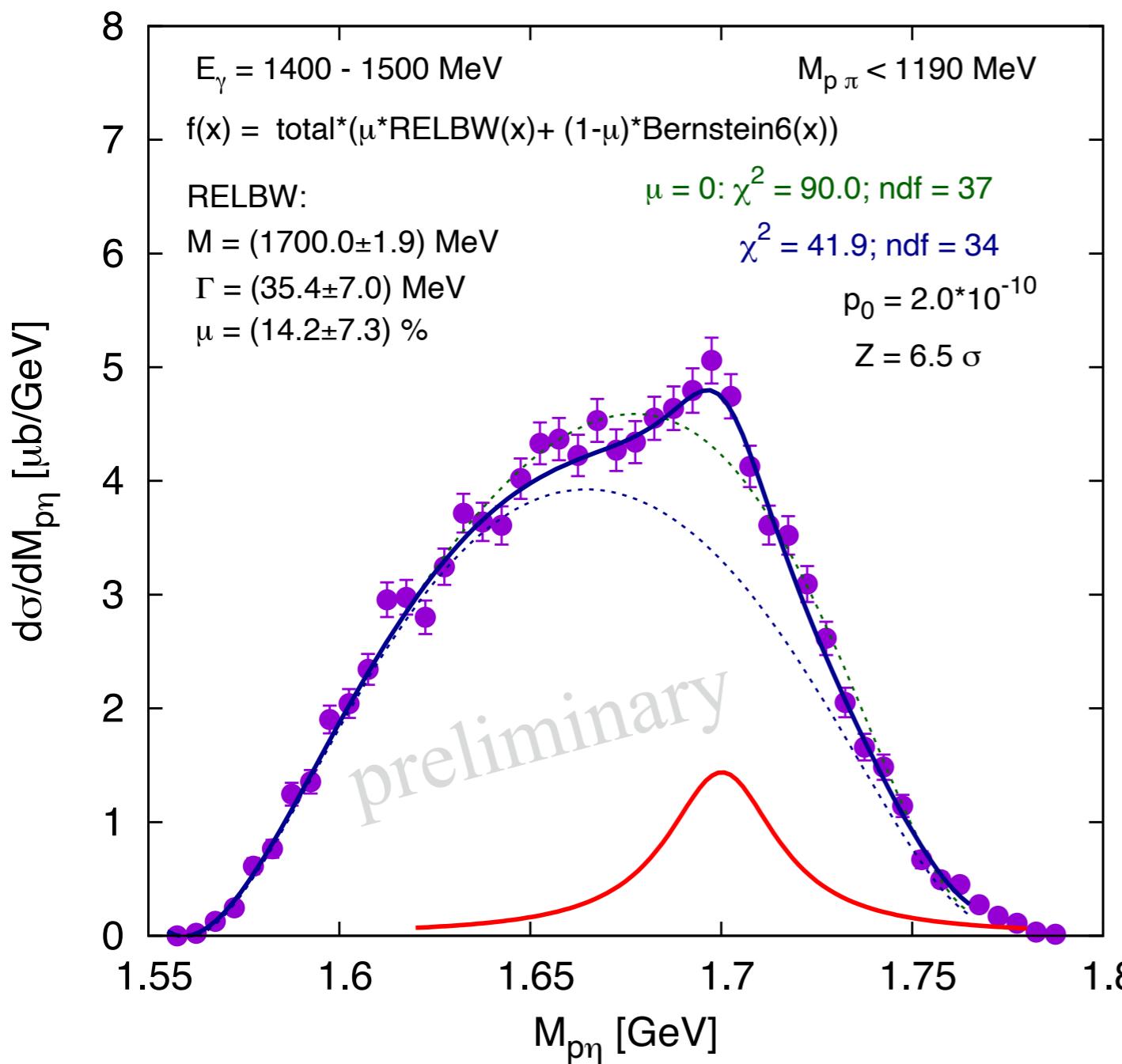
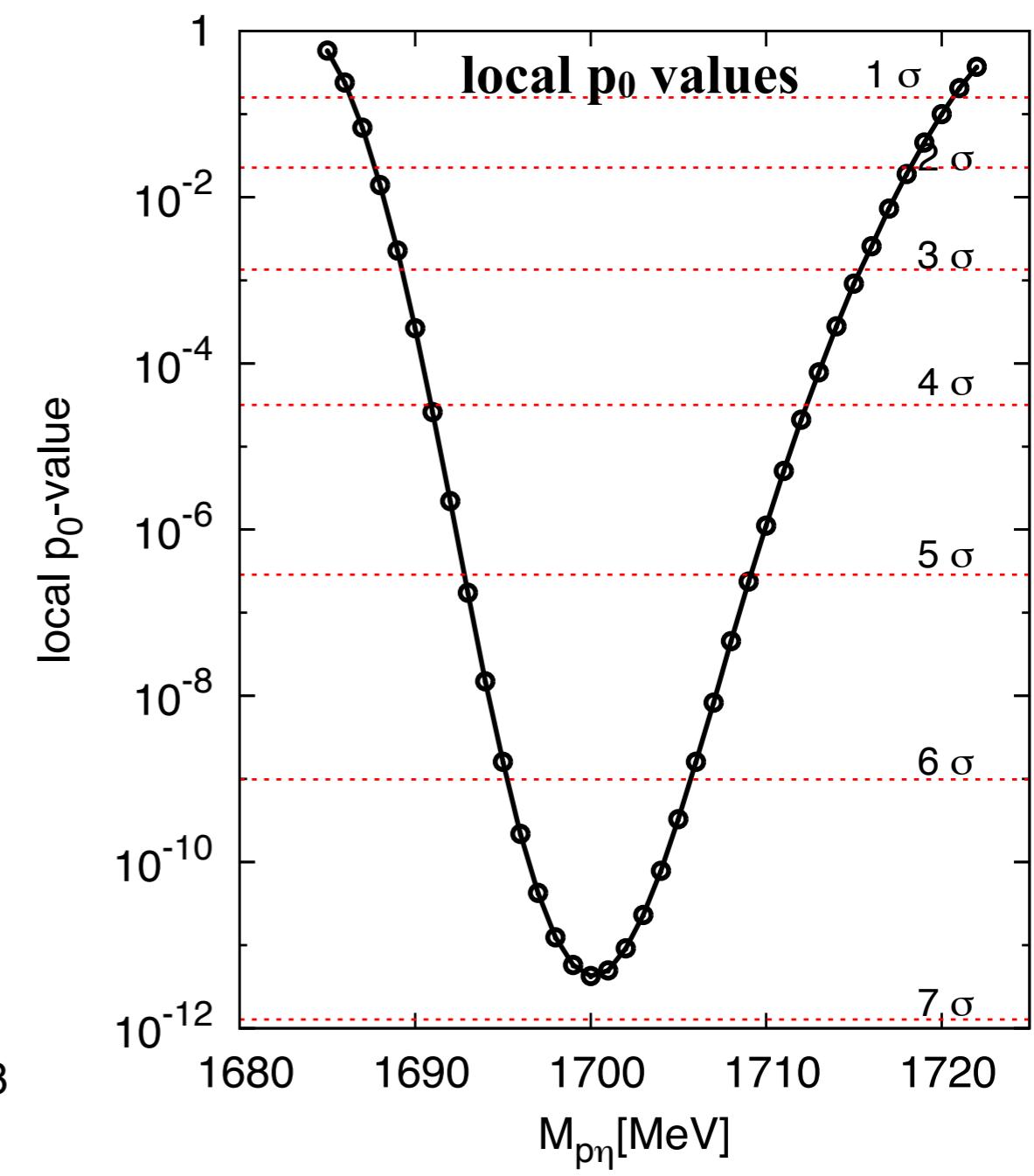
identical conditions:



4.5 times higher statistics;
 $\sigma_{\text{res}} = 5 \text{ MeV}$

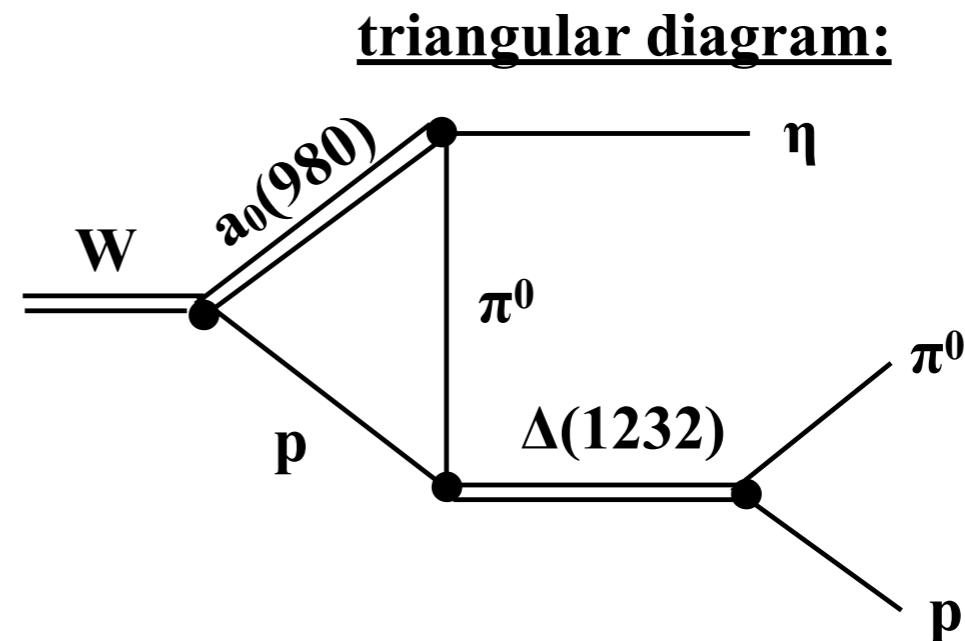
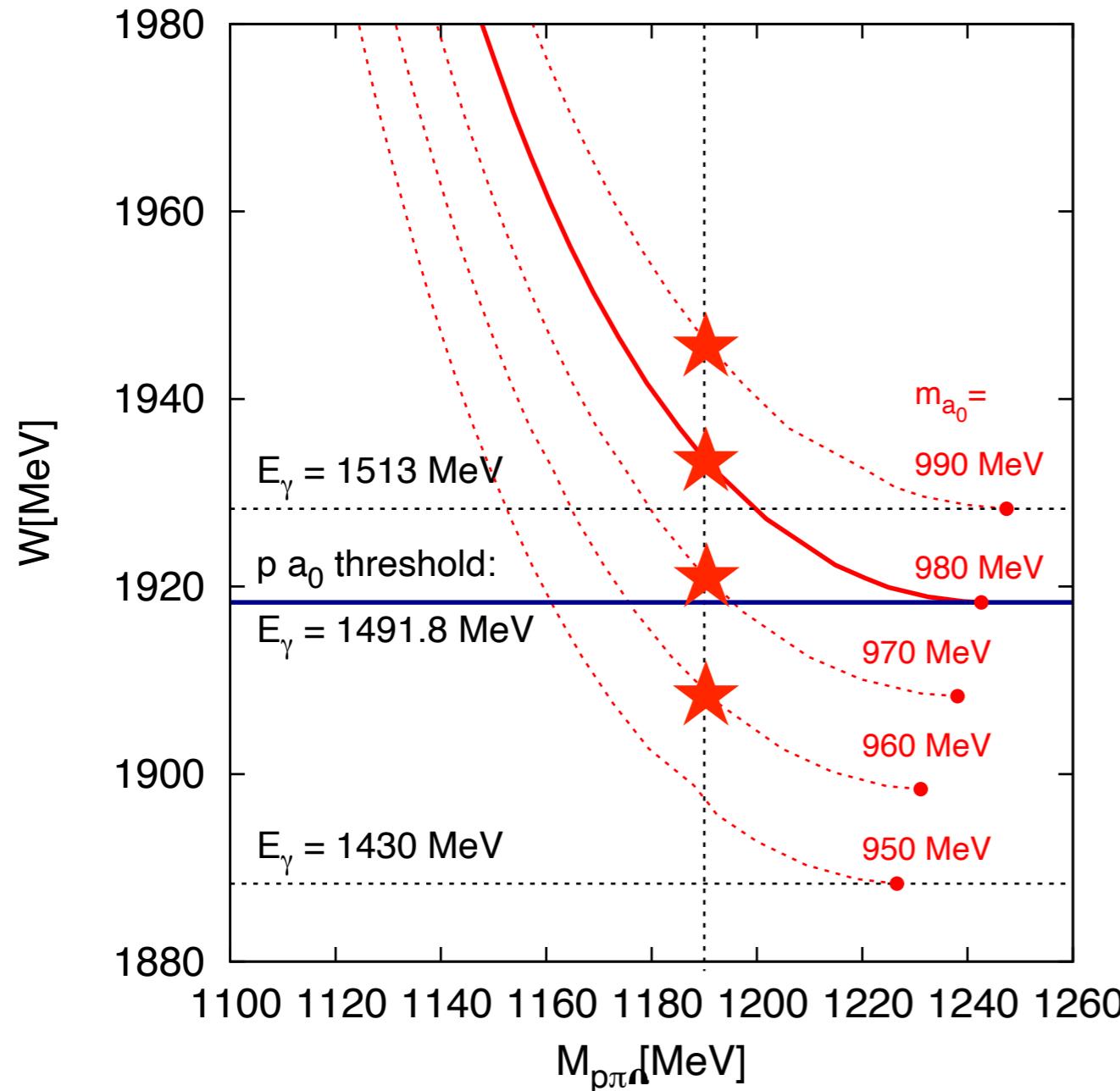


< 45 counts: $\sigma_{\text{structure}} < 6 \text{ nb (3}\sigma\text{)}$
structure cannot be confirmed !!!

$E_\gamma = 1400 - 1500 \text{ MeV}$ $M_{p\pi} < 1190 \text{ MeV}$ **statistical significant structure observed at $M_{p\eta} \approx 1700 \text{ MeV}$**  $M_{p\eta} = (1700 \pm 1.9) \text{ MeV}; \Gamma = (35.4 \pm 7.0) \text{ MeV}$ **structure established at 6.8σ**

calculation of triangular singularities

following M. Bayar et al.
PRD 94 (2016) 074039



the energy-momentum balance within the loop
has to match the energy-momentum
balance of the initial and final state particles;

$$W = E_p(q) + E_\pi(q) + E_\eta$$

$$\underline{W = E_p(q) + E_{a0}(p_\eta - q)}$$

$$E_\eta + E_\pi(q) - E_{a0}(p_\eta - q) = 0$$

$$E_\eta + E_\pi(q) - \sqrt{m_{a0}^2 + (p_\eta - q)^2} = 0$$

q = proton momentum in loop

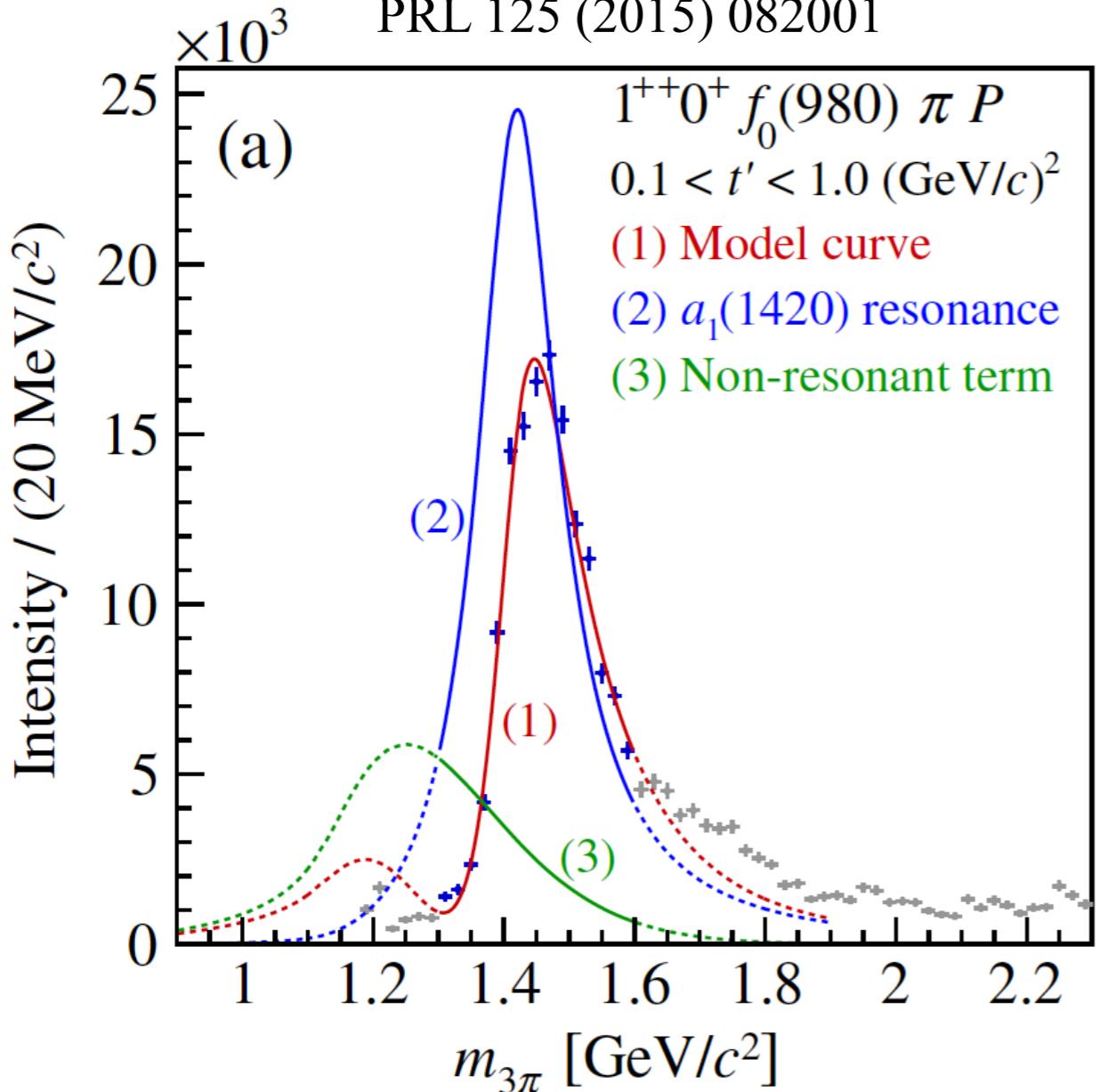
solution only for certain $(W, M_{p\pi^0})$ values and
if all particles are almost on-mass shell
and p, π^0, η are collinear

triangular singularities in hadronic reactions

diffractive $\pi^- p$ scattering at 190 GeV

COMPASS collaboration

PRL 125 (2015) 082001



resonance like signal in $\pi^- \pi^- \pi^+$ final state

genuine new state: $a_1(1420)$??
 difficult to explain within quark model
 tetra-quark resonance ??

M. Mikhasenko, B. Ketzer, A Sarantsev

PRD 91 (2015) 094105

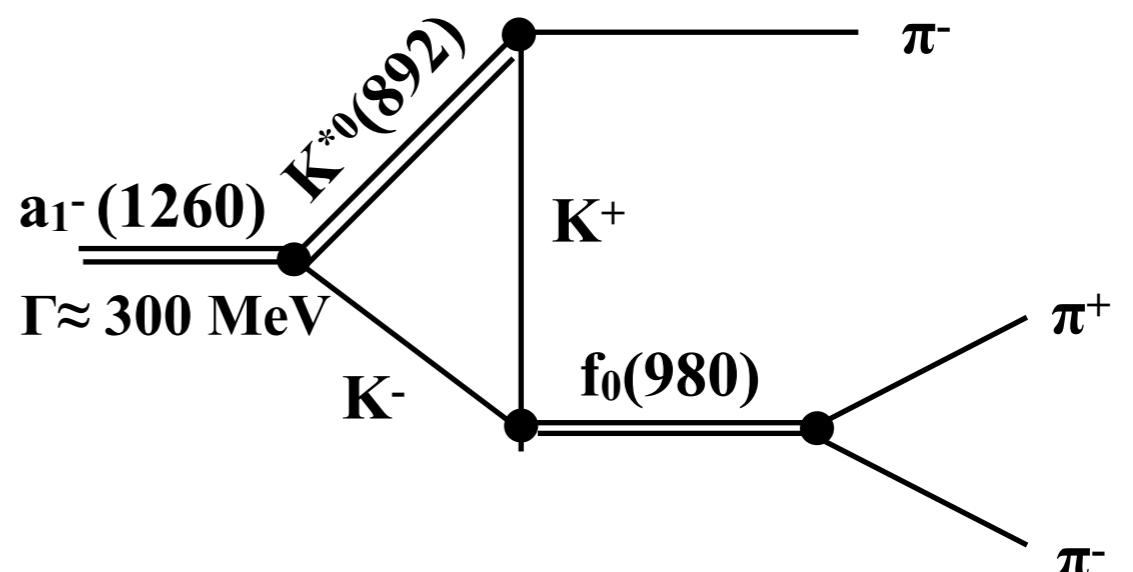
COMPASS collaboration

PRL 127 (2021) 082501

$\pi^- p \rightarrow a_1^- (1260) p$

$a_1^- (1260) \rightarrow \pi^- \pi^+ \pi^-$

triangular diagram



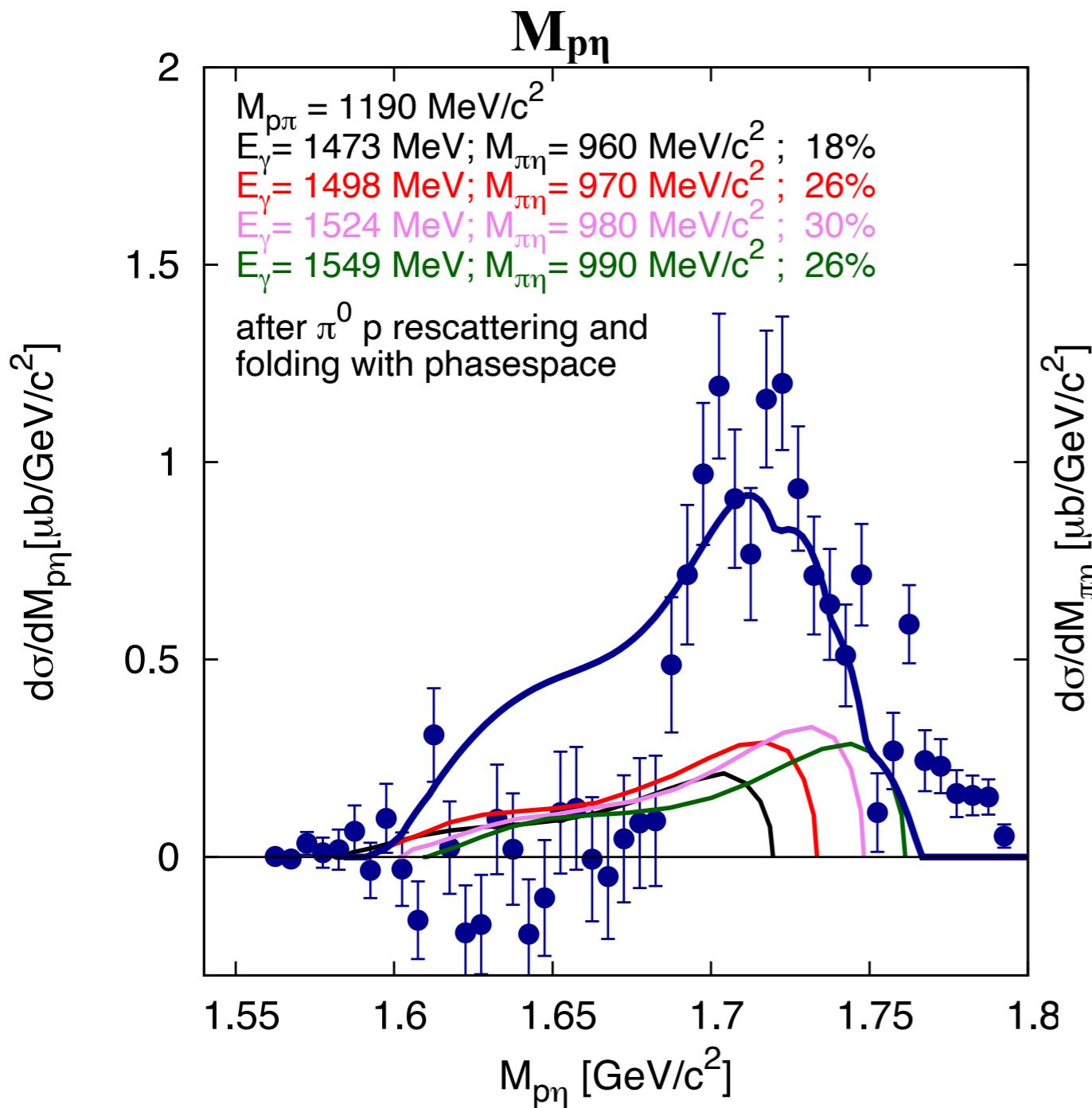
three point (triangular) loop generates structure (singularity) at $\sqrt{s} = 1420 \text{ MeV}$

many other cases in meson sector:

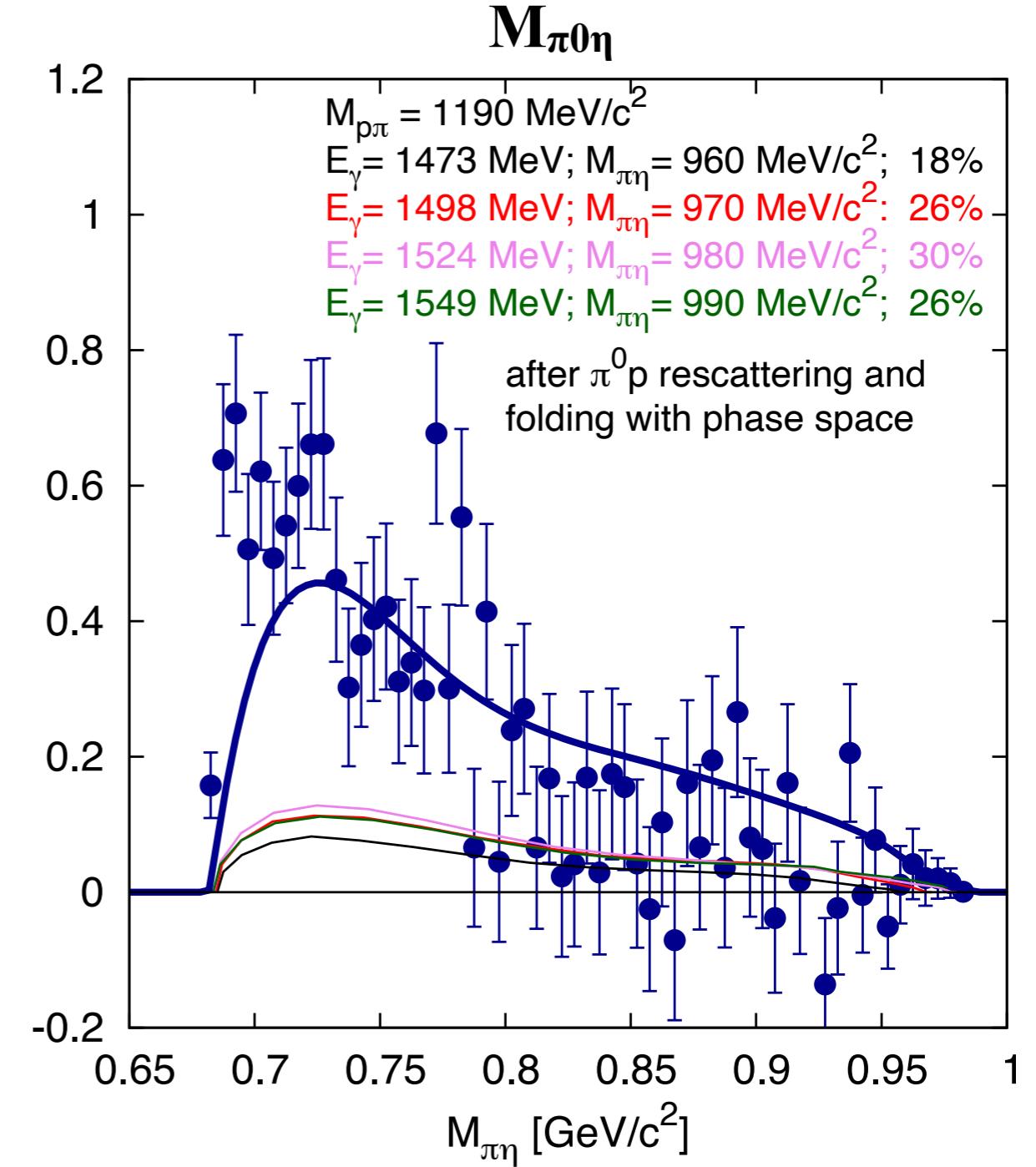
F-K.Guo, X-H.Liu, S.Sakai
 Prog. Part. Nucl. Phys. 112 (2020) 103757

Comparison data (difference to PWA) \longleftrightarrow calculation

contributions of the 4 selected singularity points with weight given by a_0 line shape
 blue curve (sum of the 4 contributions) fitted to the data



peak moves with excitation energy
 as observed experimentally



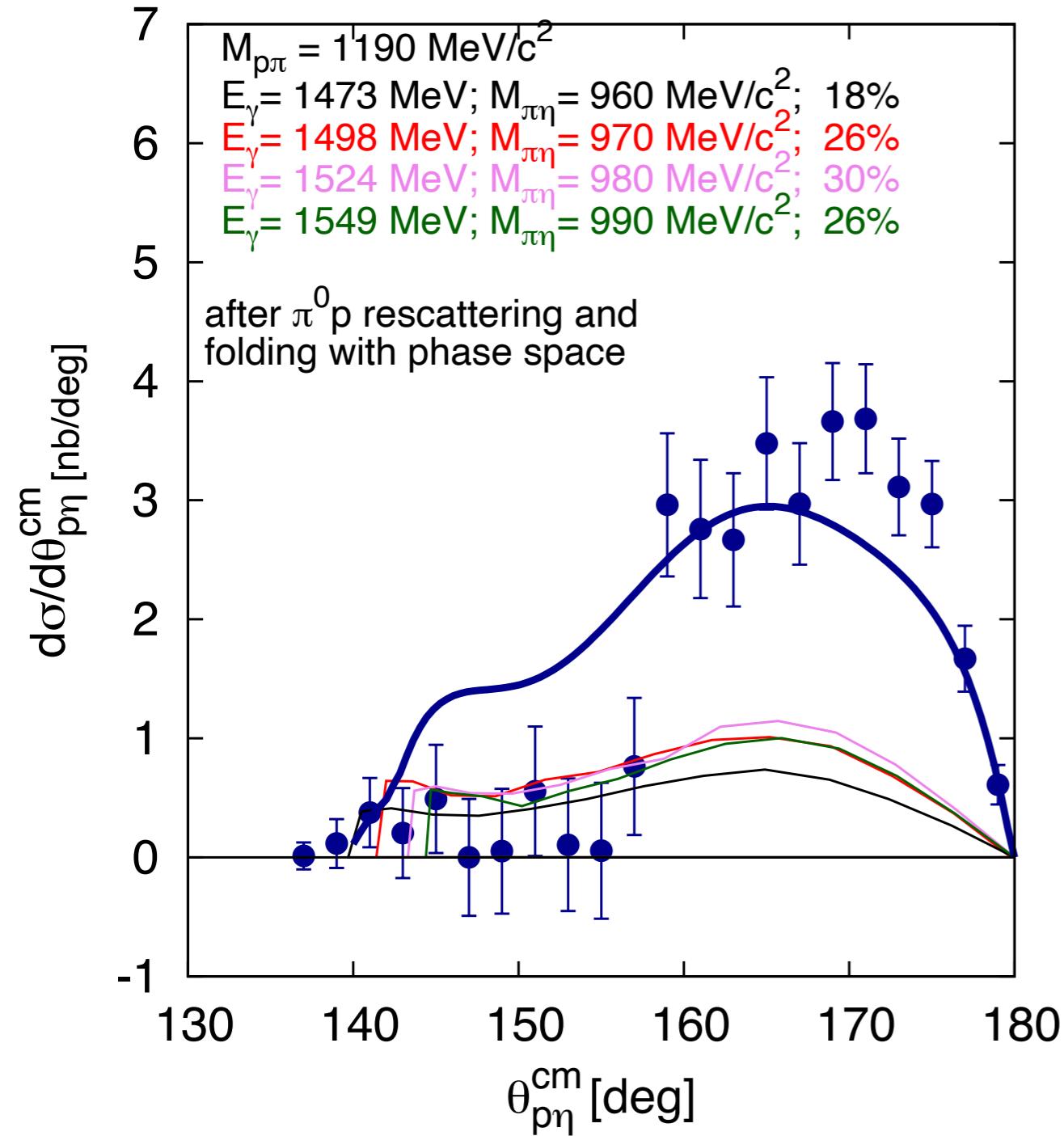
$M_{\pi 0\eta}$ distribution shifted towards kinematical limit $m_{\pi 0} + m_\eta = 682 \text{ MeV}$

calculation reproduces qualitatively the data !!

Comparison data (difference to PWA) \longleftrightarrow calculation

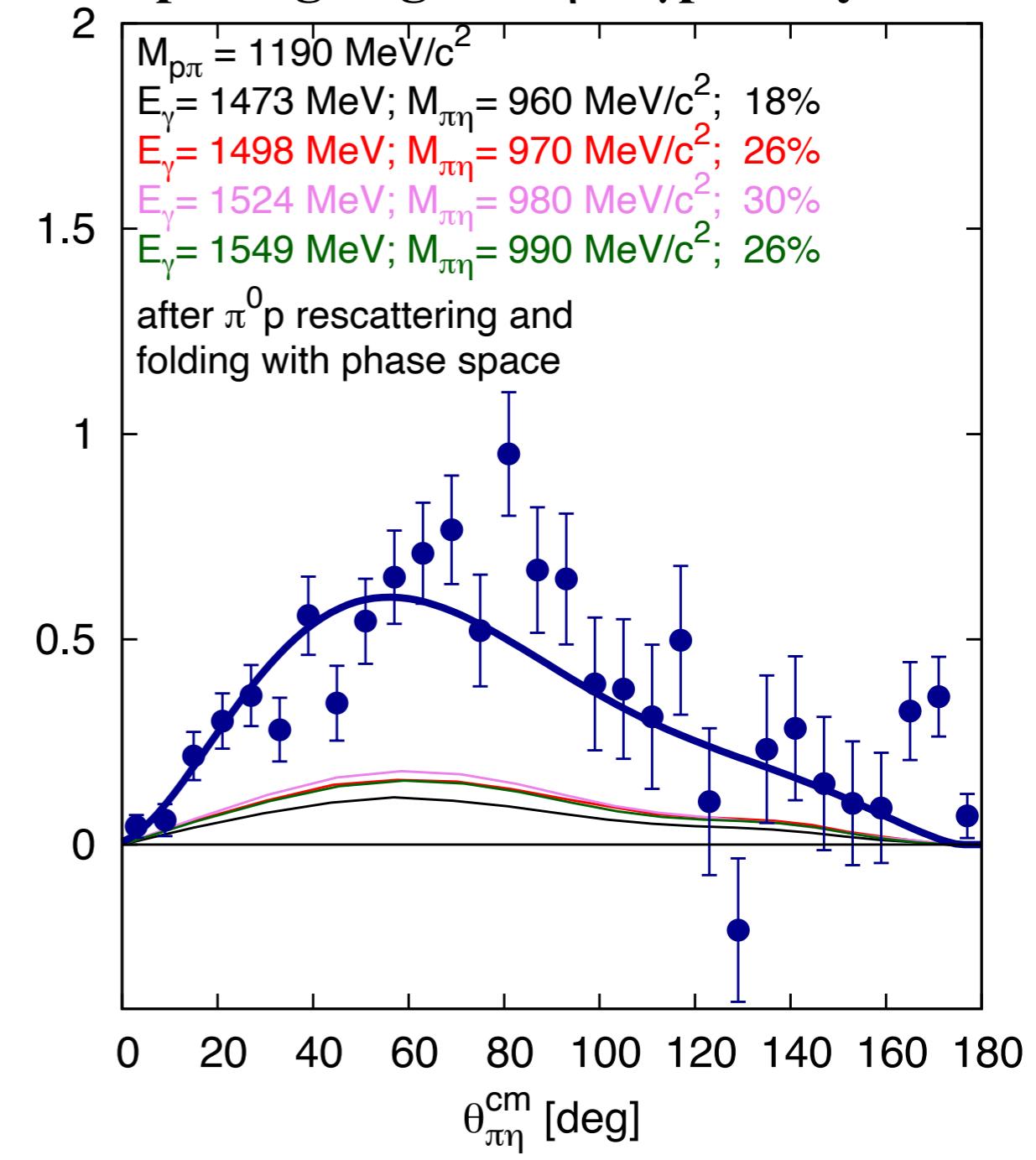
contributions of the 4 selected singularity points with weight given by a_0 line shape
 blue curve (sum of the 4 contributions) fitted to the data

opening angle $\theta_{p\eta}$ in γp cm system



opening angles $\theta_{p\eta}$ confined to $150^\circ - 180^\circ$

opening angle $\theta_{\pi 0\eta}$ in γp cm system



opening angles $\theta_{\pi 0\eta}$ show max at $\approx 60^\circ$

calculation reproduces qualitatively the data !!

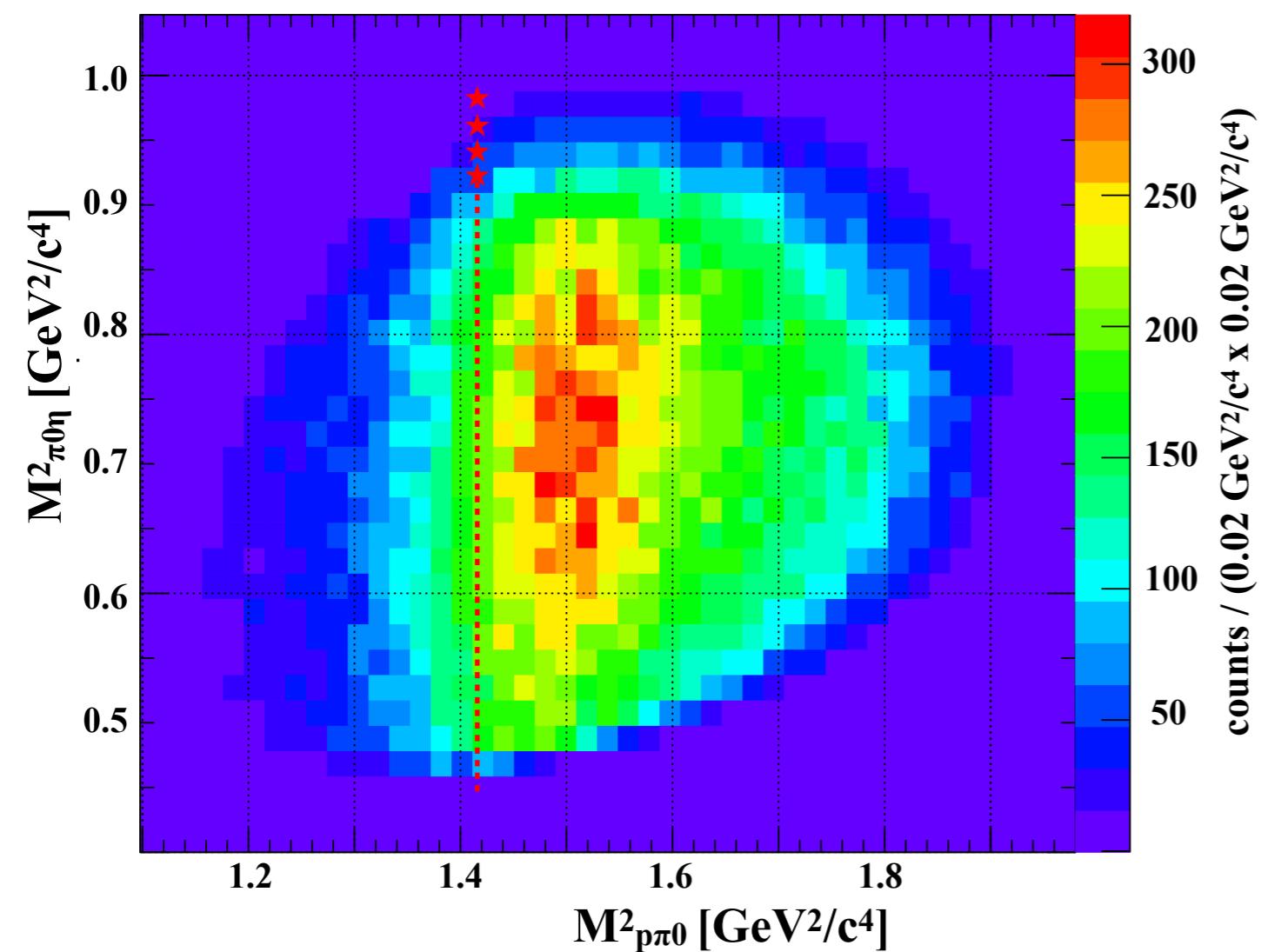
π^0 - p rescattering

kinematics in γp cm system
for $W=1934$ MeV, $m_{a0} = 980$ MeV

$$\begin{array}{l} p_p = 122.9 \text{ MeV}; \beta_p = 0.130 \\ p_\pi = 277.8 \text{ MeV}; \beta_\pi = 0.899 \\ \beta_\eta = -0.590 \\ p_\eta = -400.7 \text{ MeV} \end{array}$$

$M_{p\pi 0} = 1190$ MeV; $M_{p\eta} = 1601$ MeV; $M_{0\eta} = 970$ MeV

singularity events are re-distributed along the dashed red line by π^0 -p - rescattering



π^0 faster than p
 π^0 -p rescattering:

$$\begin{aligned} M_{p\pi 0} &= \text{const} = 1190 \text{ MeV} \\ M_{p\eta} &\approx 1600 \rightarrow 1700 \text{ MeV} \\ M_{\pi 0\eta} &\approx 970 \rightarrow 682 \text{ MeV} \end{aligned}$$

