

# DATA-DRIVEN

# POLE DETERMINATION

# OF (OVERLAPPING)

# RESONANCES

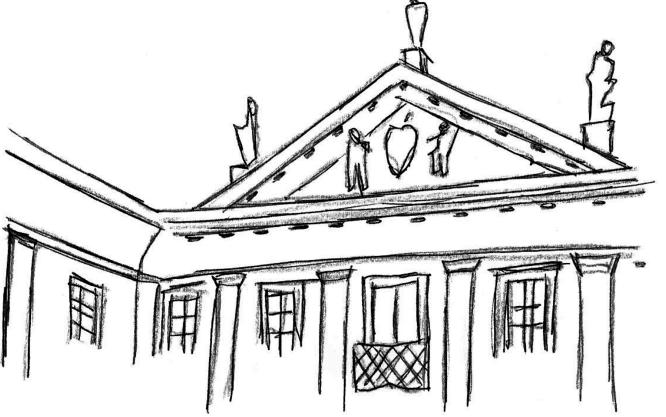
NSTAR 2022

S. Margherita Ligure

OCTOBER 17 - OCTOBER 21 2022

DANIELE BINOSI

ECT\* - FONDAZIONE BRUNO KESSLER



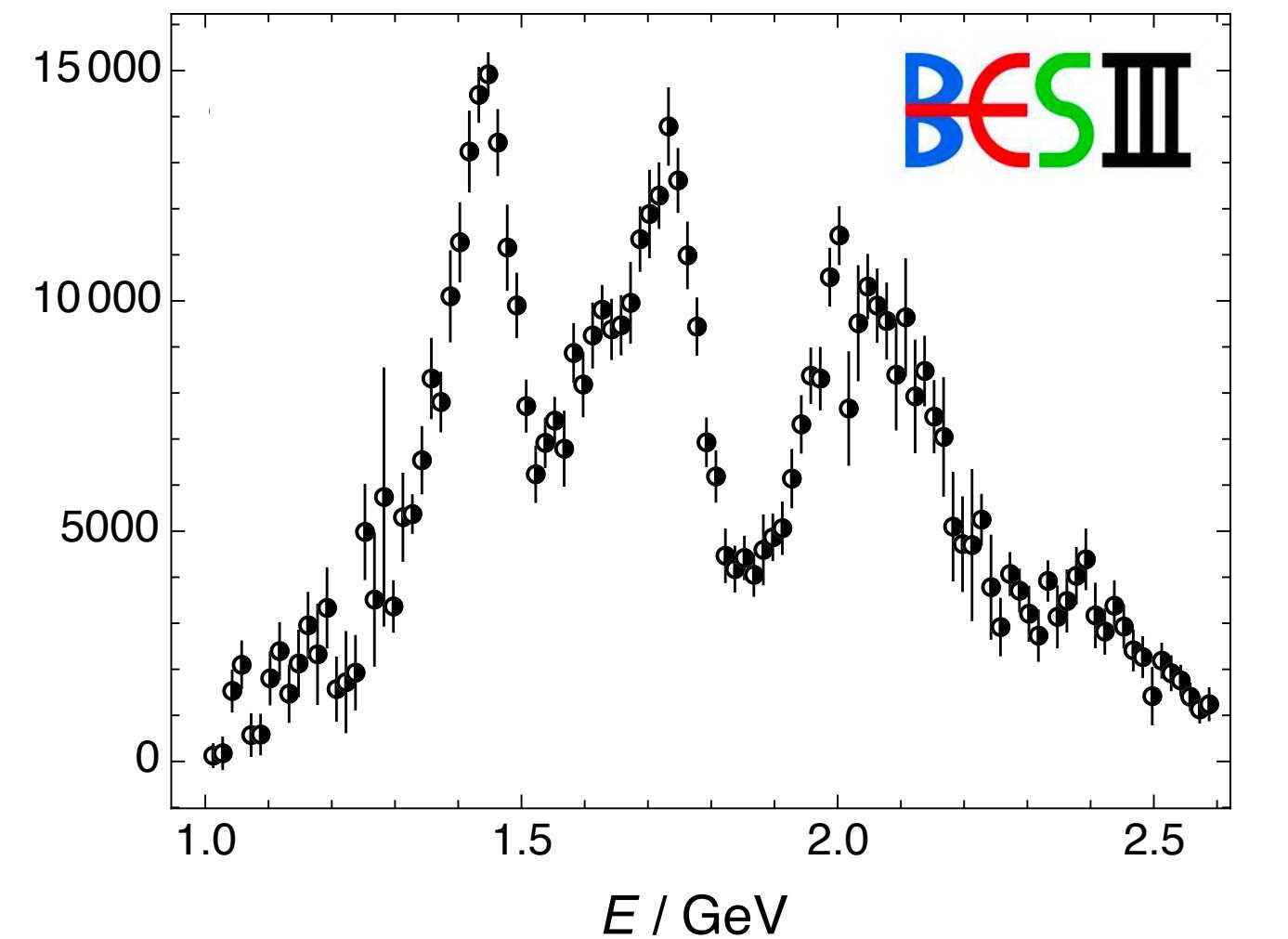
**ECT\***  
EUROPEAN CENTRE  
FOR THEORETICAL STUDIES  
IN NUCLEAR PHYSICS AND RELATED AREAS  
FONDAZIONE BRUNO KESSLER



$J/\psi \rightarrow \gamma\pi^0\pi^0$

**S-WAVE  
INTENSITY**

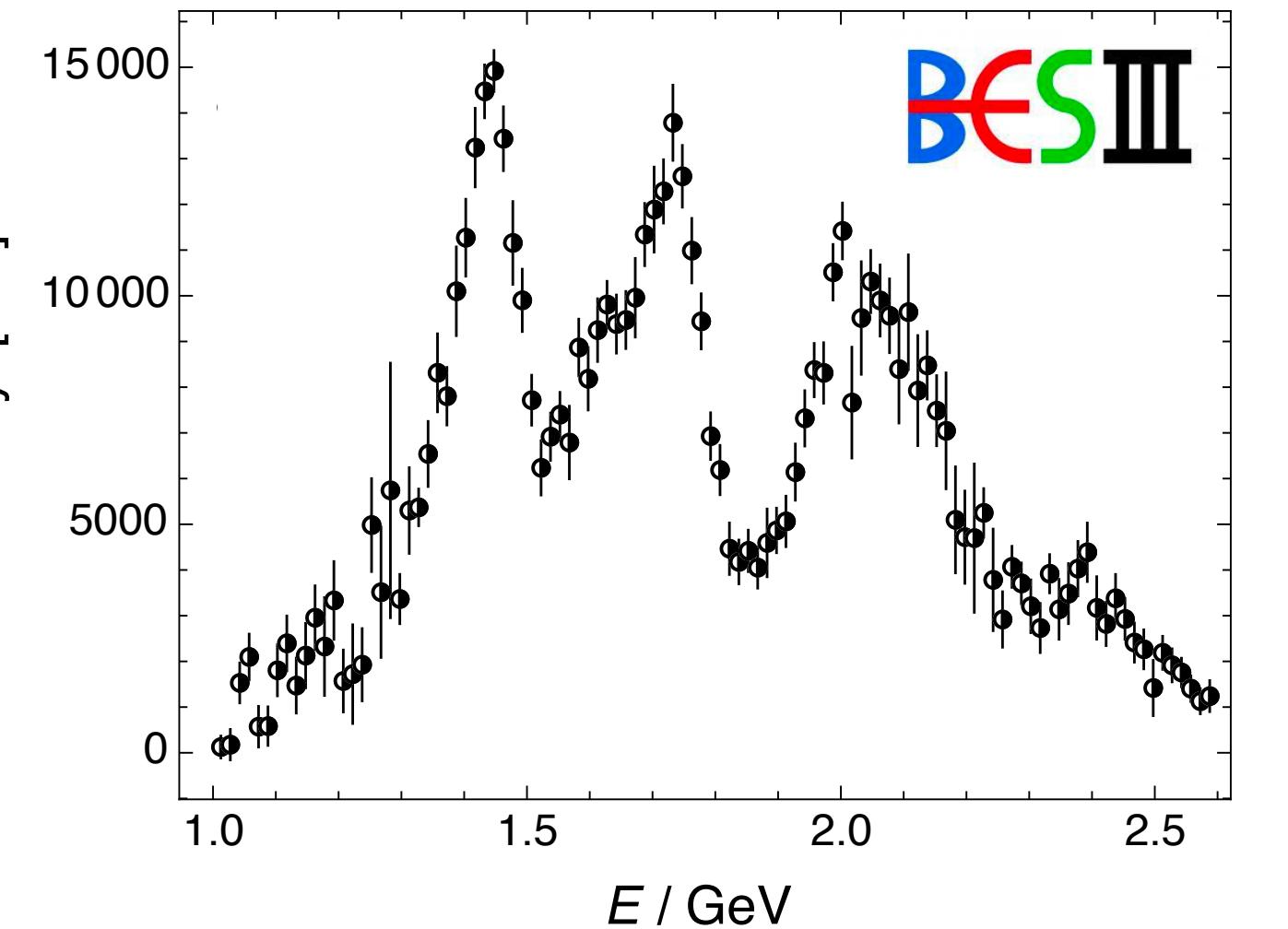
**BESIII data**



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**S-WAVE  
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## AMPLITUDE RELATION

The intensity is related to the amplitude via

$$I(E) = \rho(E) |f(E)|^2$$

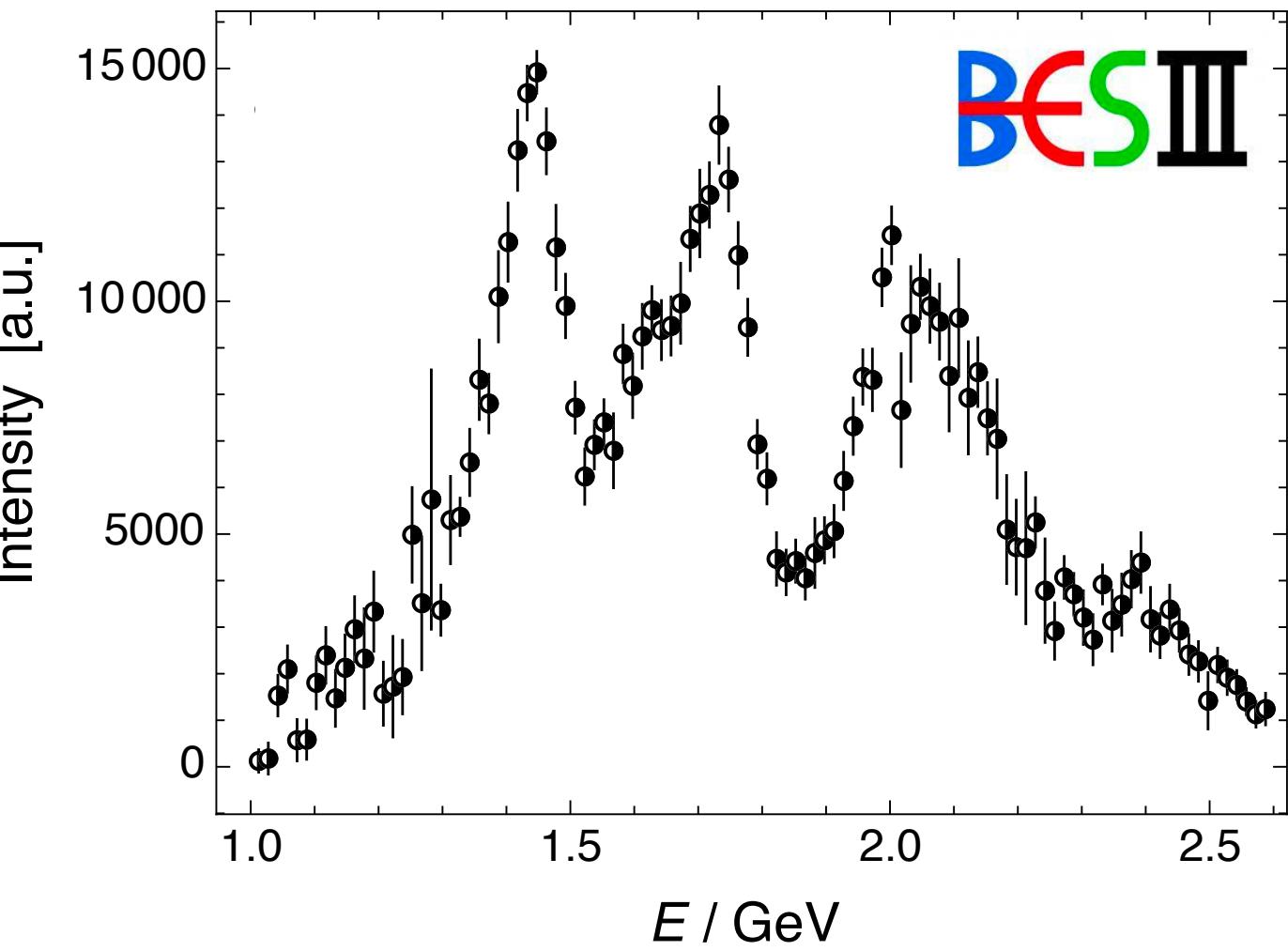
$f$ : right hand cut at threshold and possibly other branch cuts (opening of channels)

**Analytic continuation** can access the unphysical Riemann sheet where resonant poles are found

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## SPM INTERPOLATOR

$$\mathcal{D}_N = \{(E_i, I_i = I(E_i)), i = 1, \dots, N\}$$

Select a subset  $\mathcal{D}_M \subseteq \mathcal{D}_N$  to construct the **interpolator**

$$C_M(E) = \frac{I_1}{1 + \frac{a_1(E - E_1)}{1 + \frac{a_2(E - E_2)}{1 + \frac{\dots}{1 + \frac{\dots}{a_{M-1}(E - E_{M-1})}}}}}$$

with  $C_M(E_i) = I_i \forall E_i \in \mathcal{D}_M$

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## SPM ANALYTIC CONTINUATION

$$C_M(E) = \frac{P(E)}{Q(E)}$$

and simply let  $E$  take on complex values!

Pole structure will provide an approximation to the one of the original intensity

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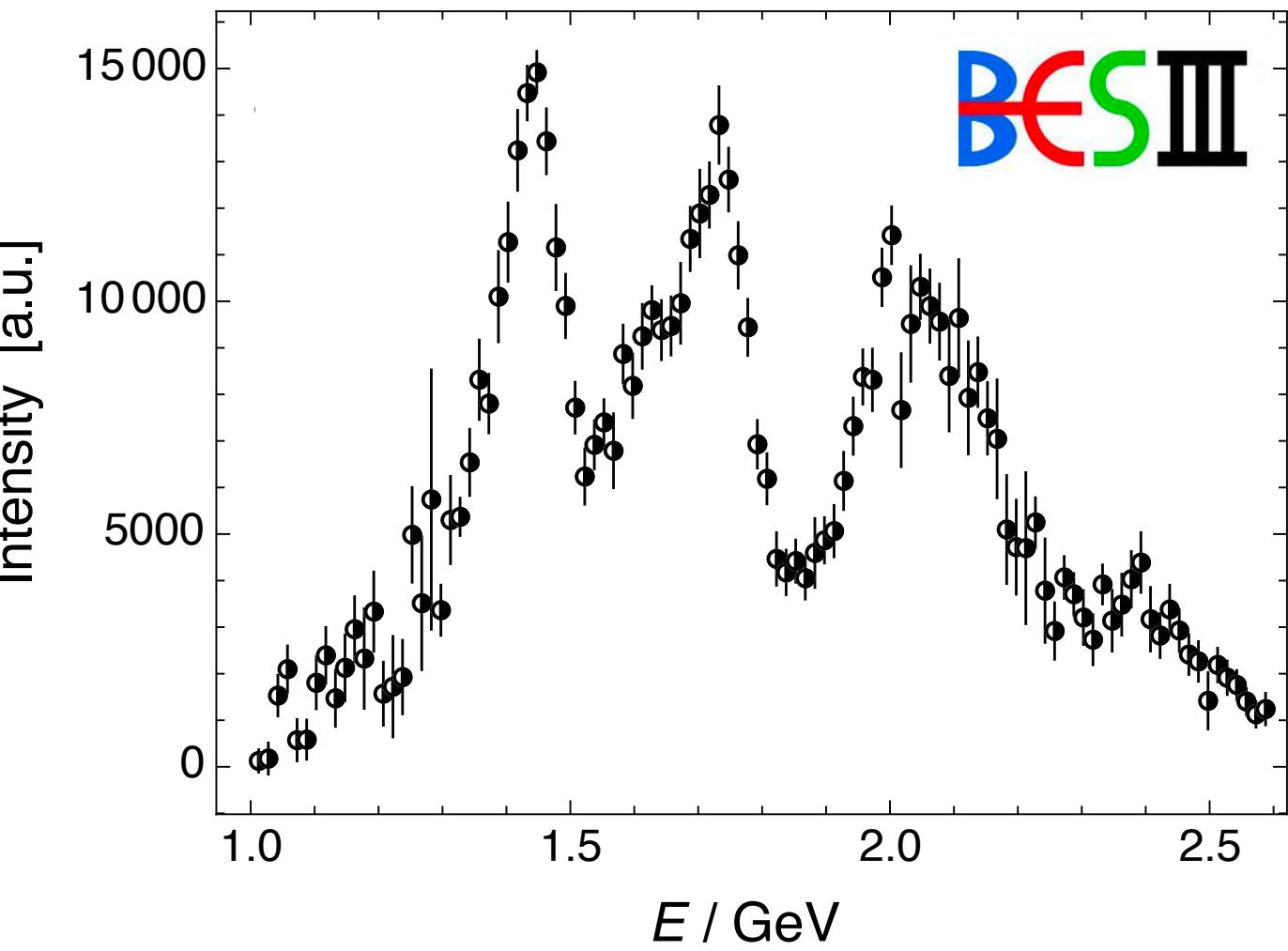
Propagate **uncertainties** through **resampling**

Need to reconstruct the analytic structure of the intensity from values at energy bins

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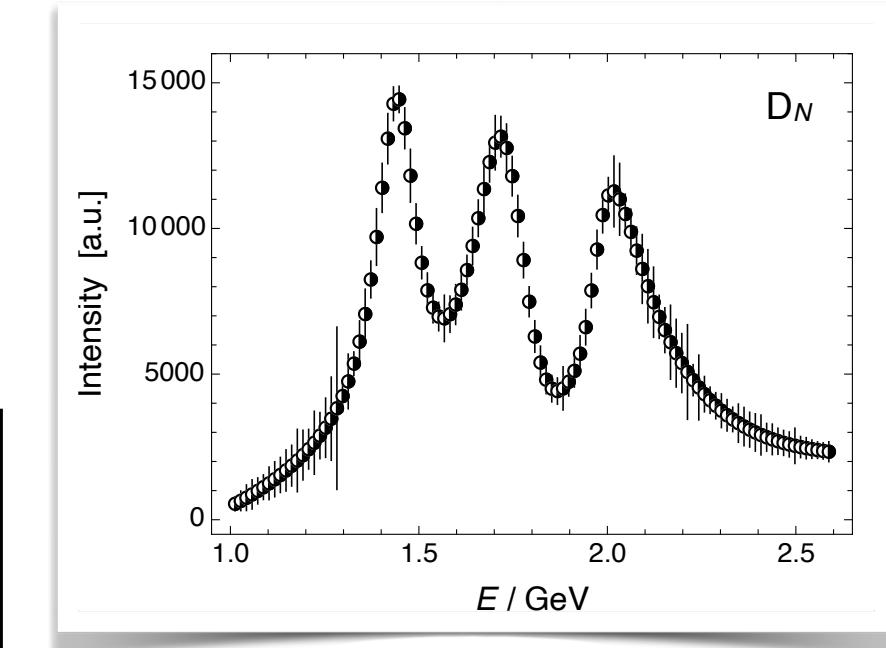
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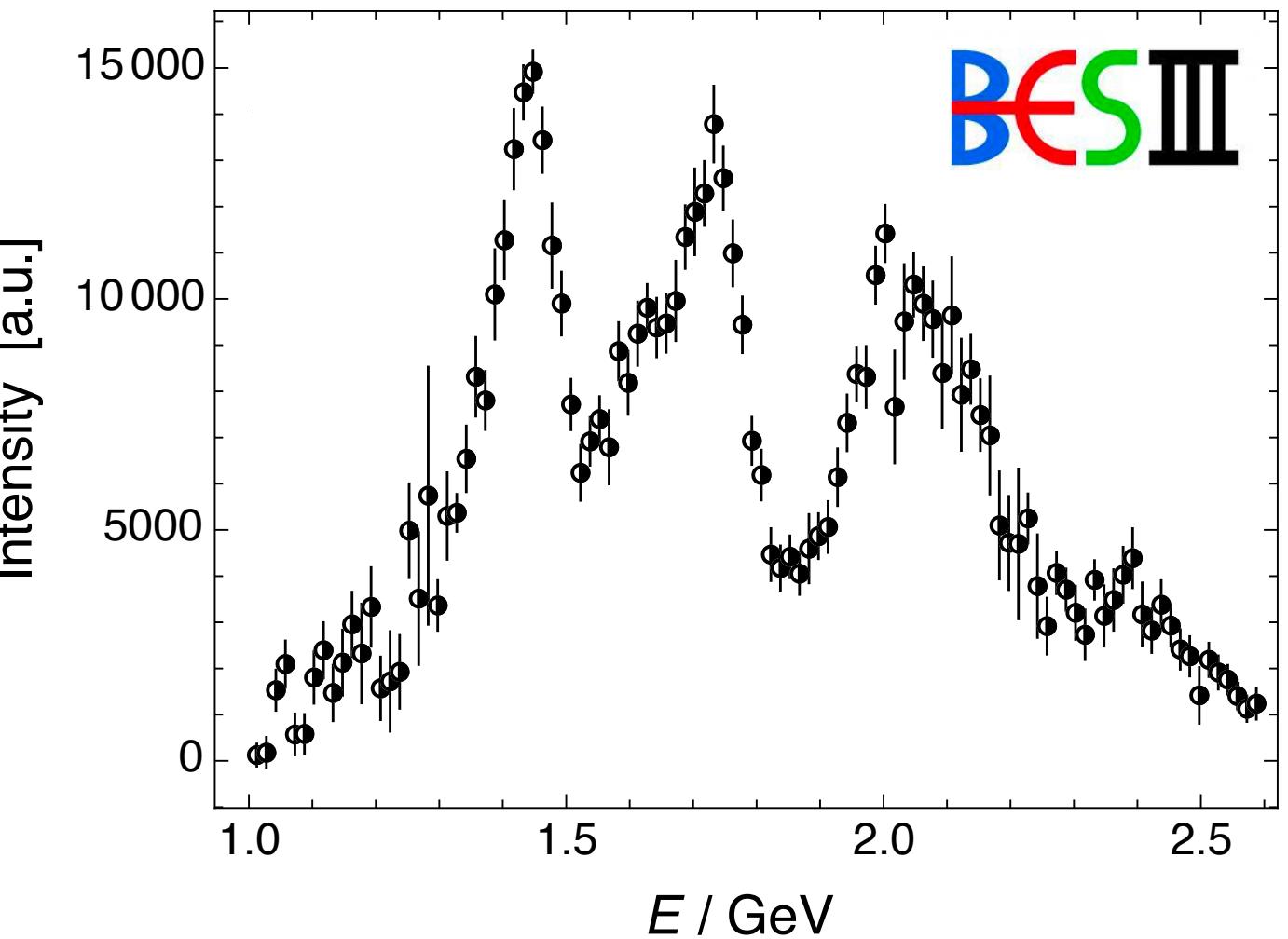
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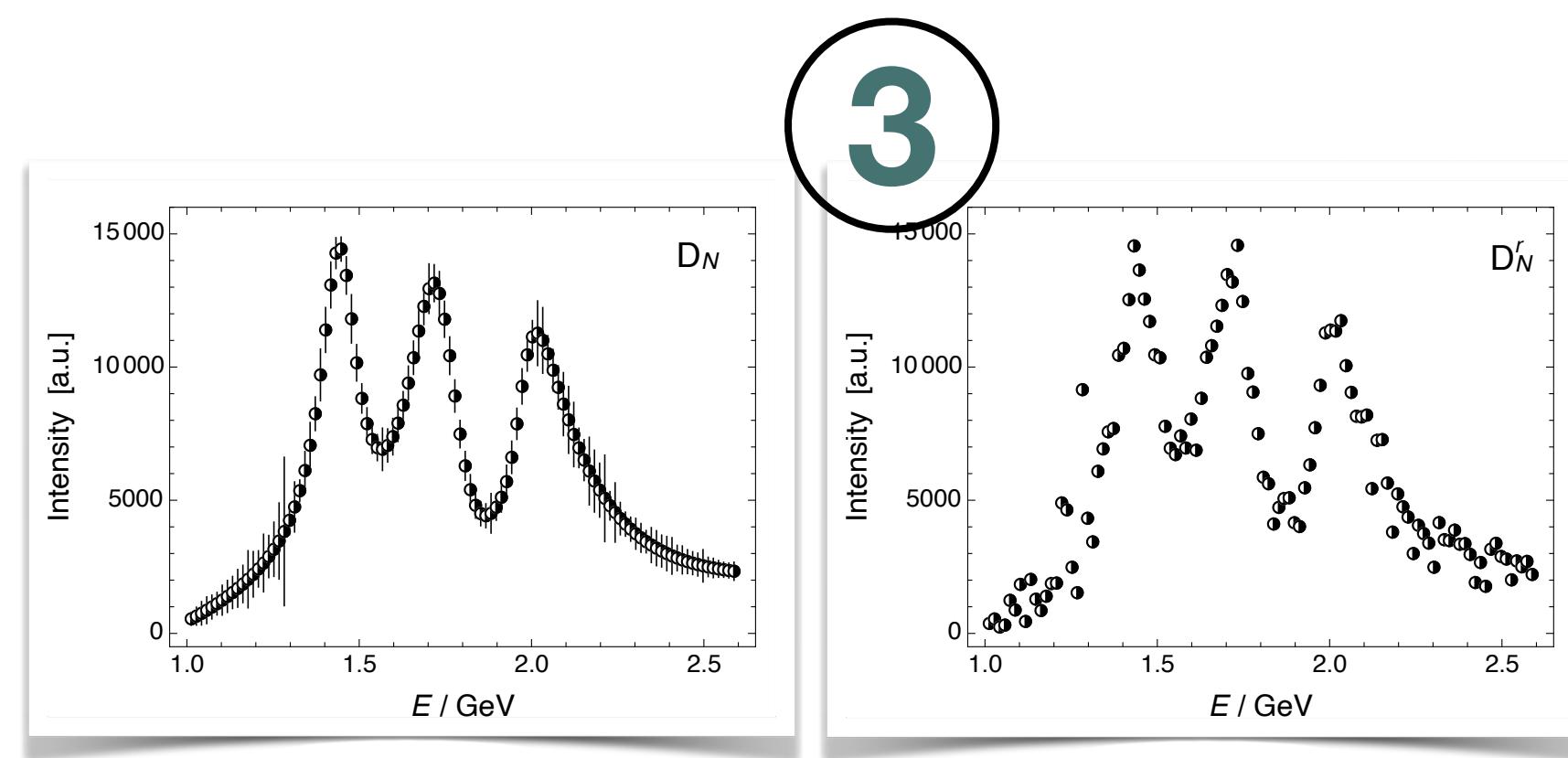
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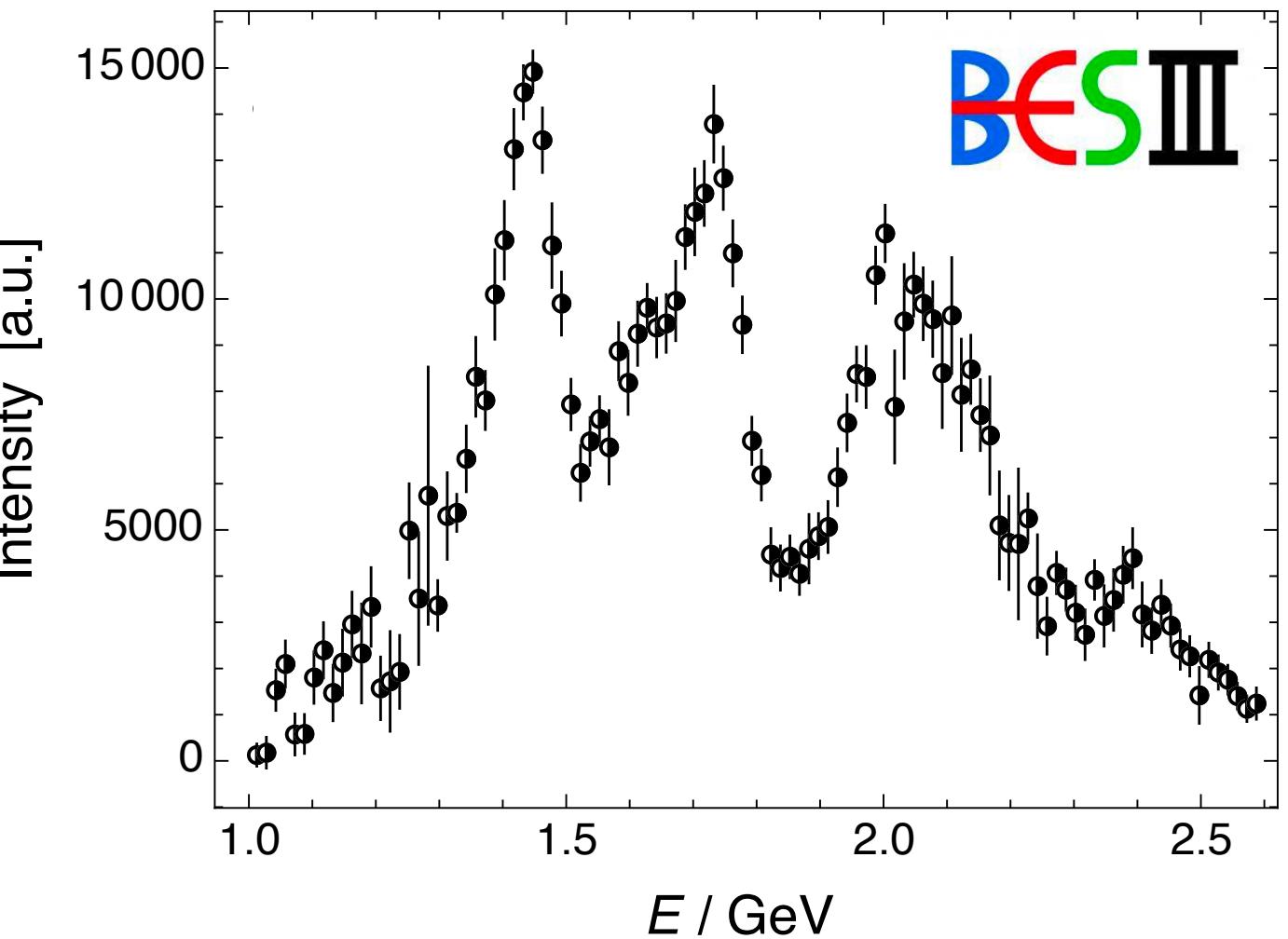


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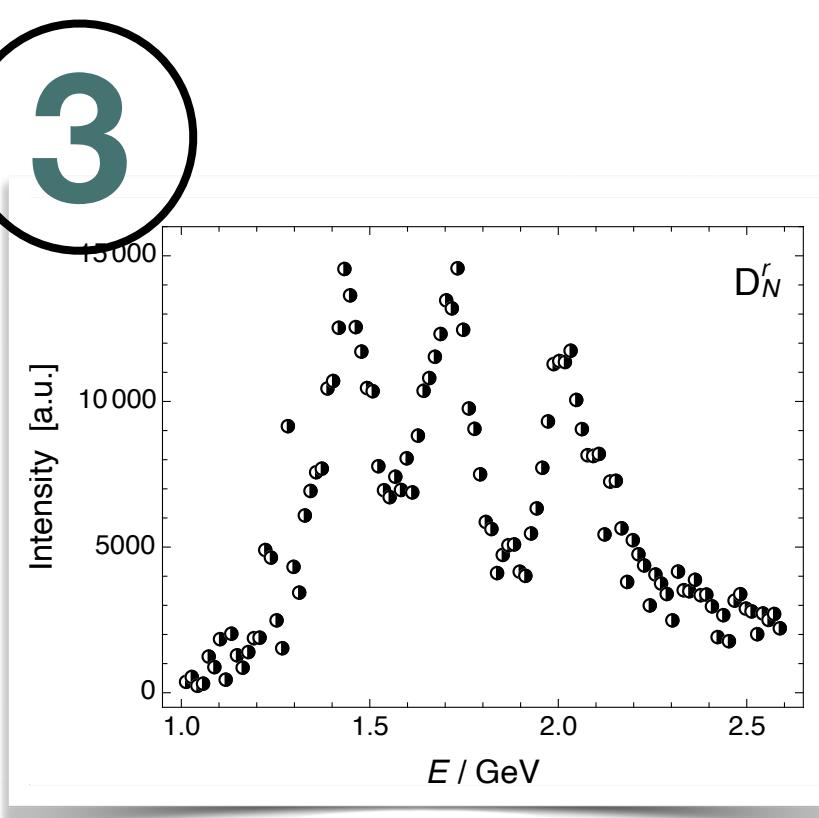
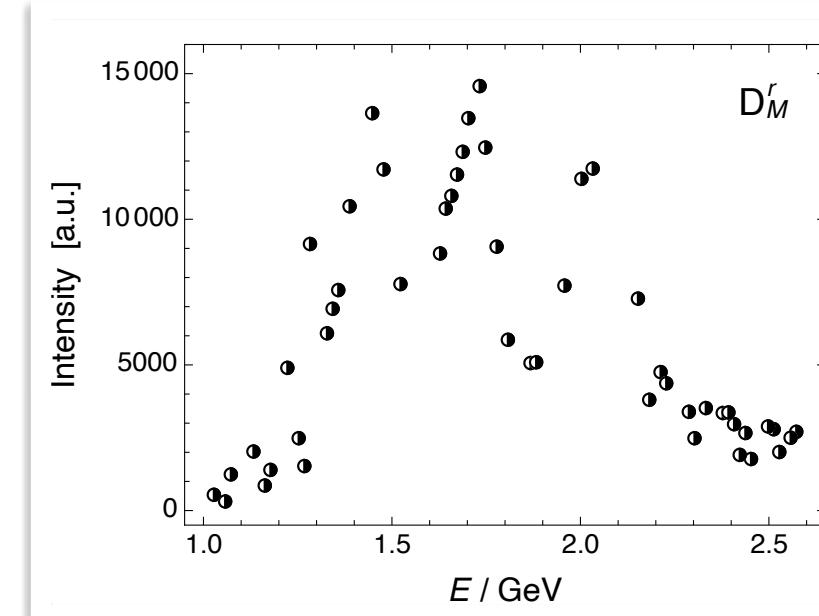
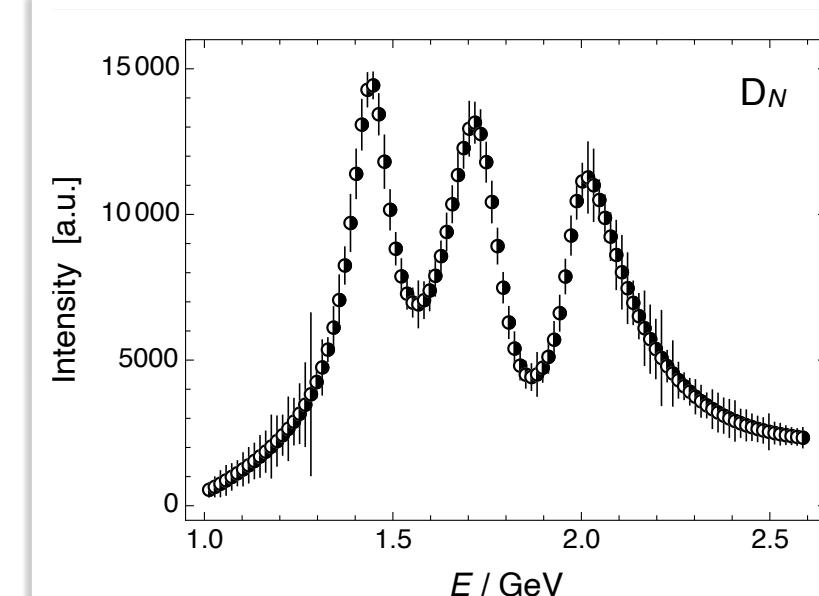
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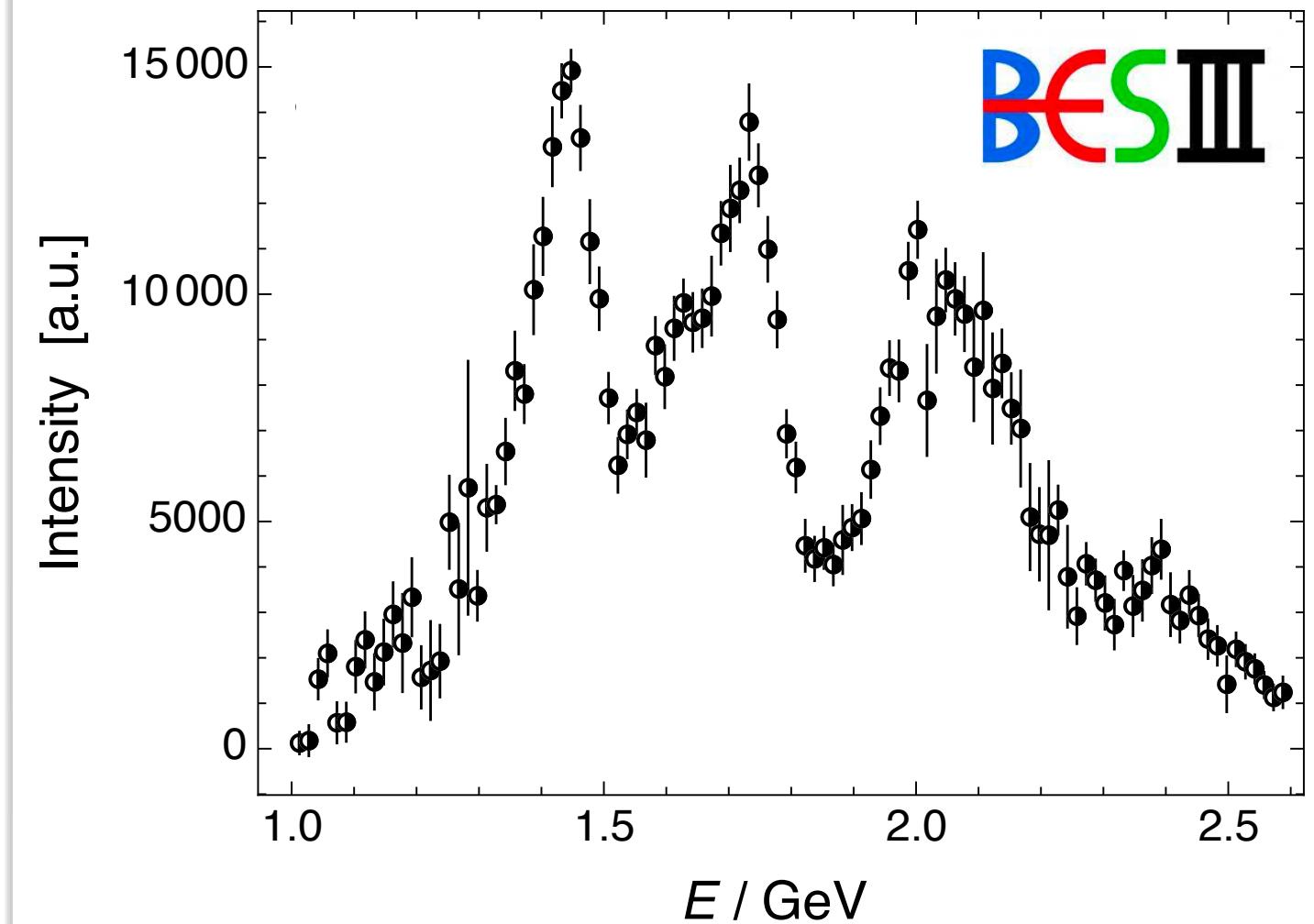
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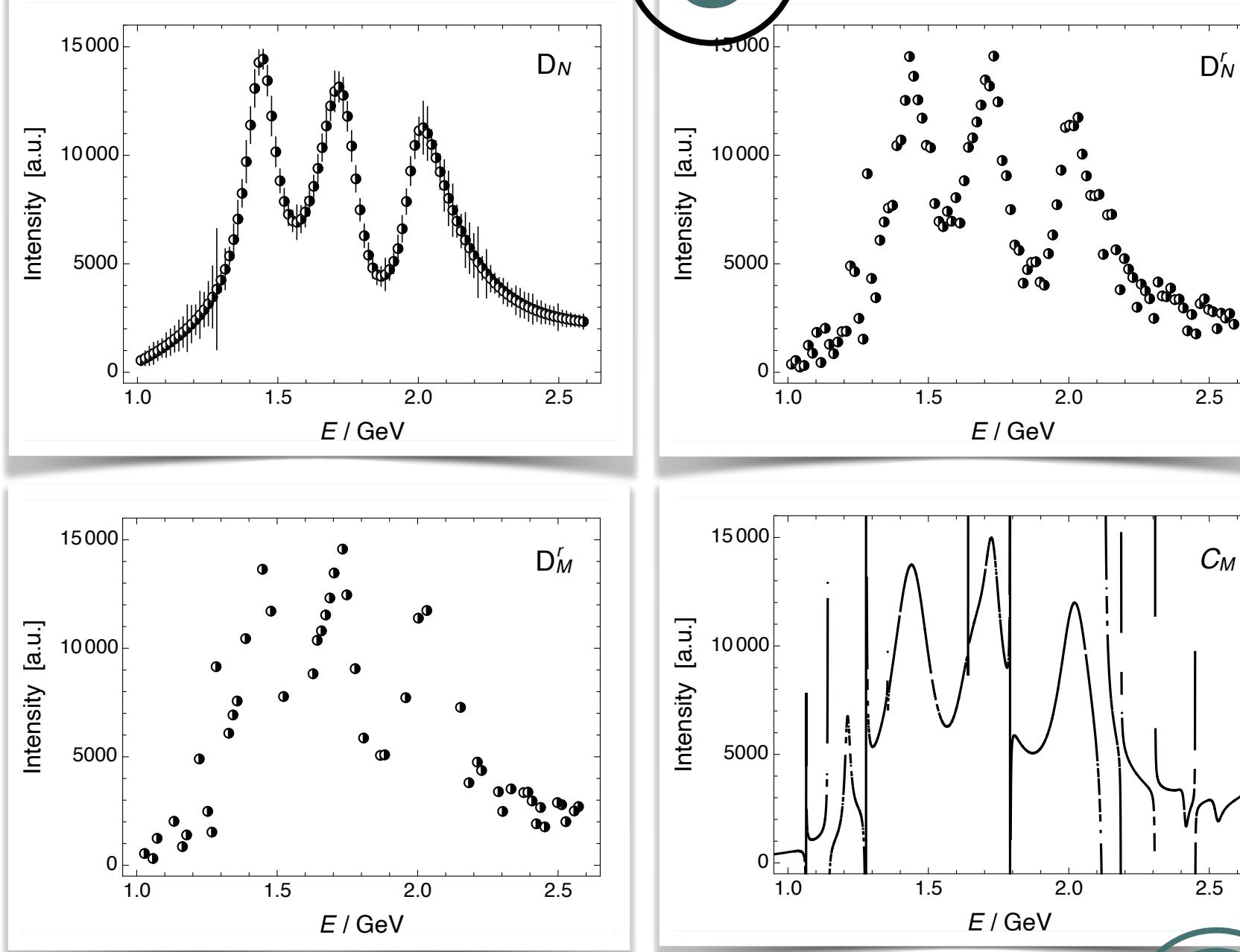
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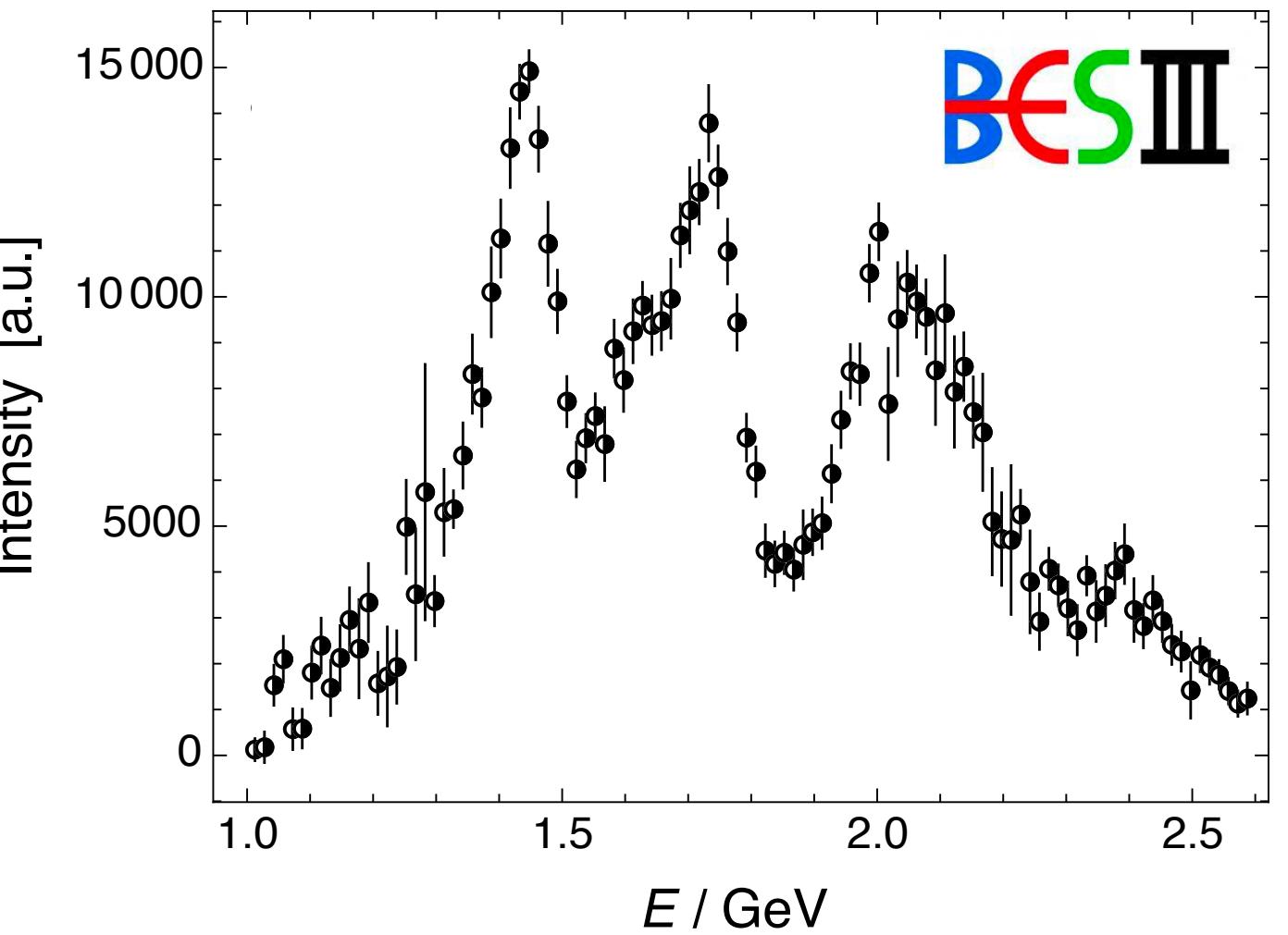


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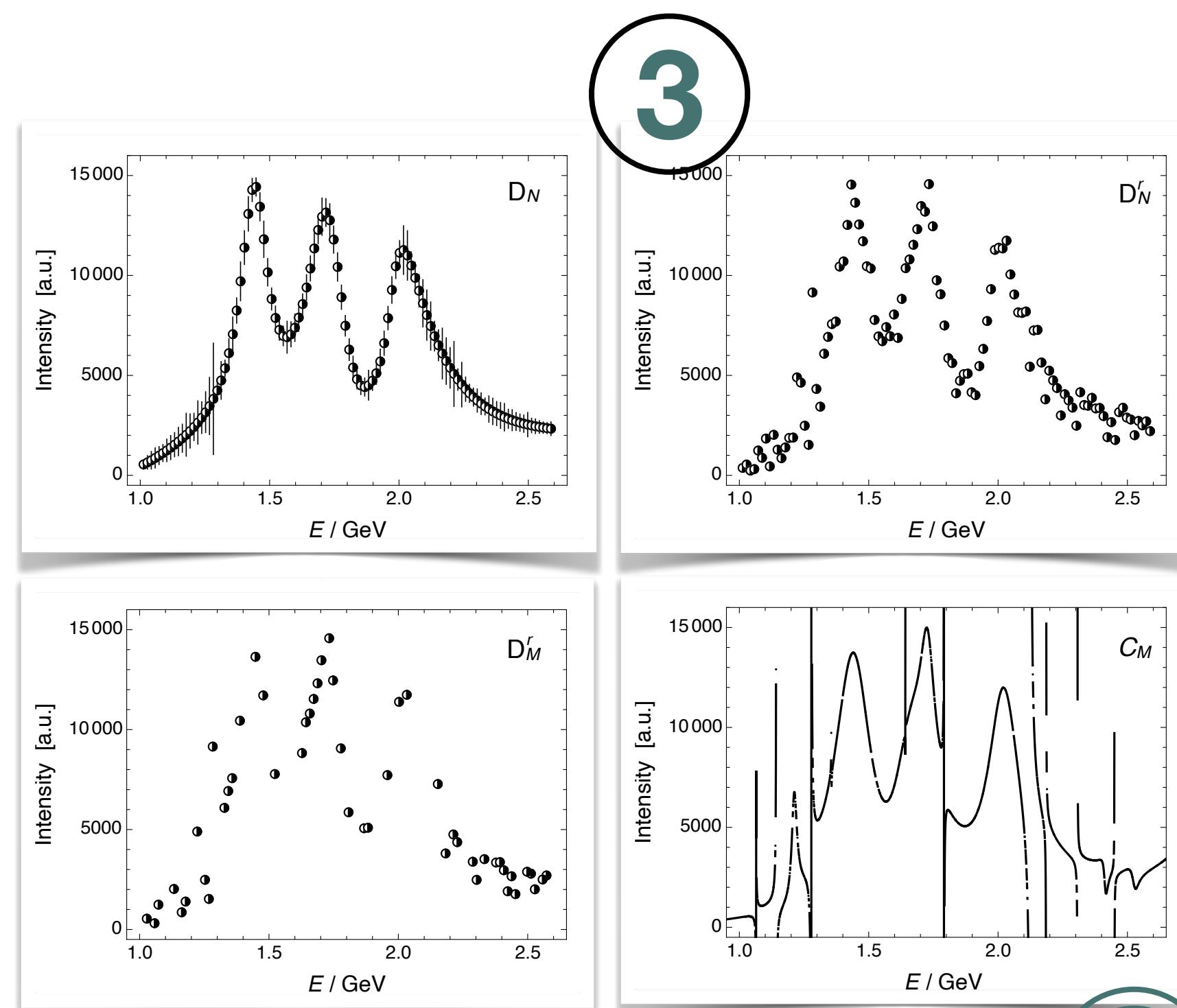
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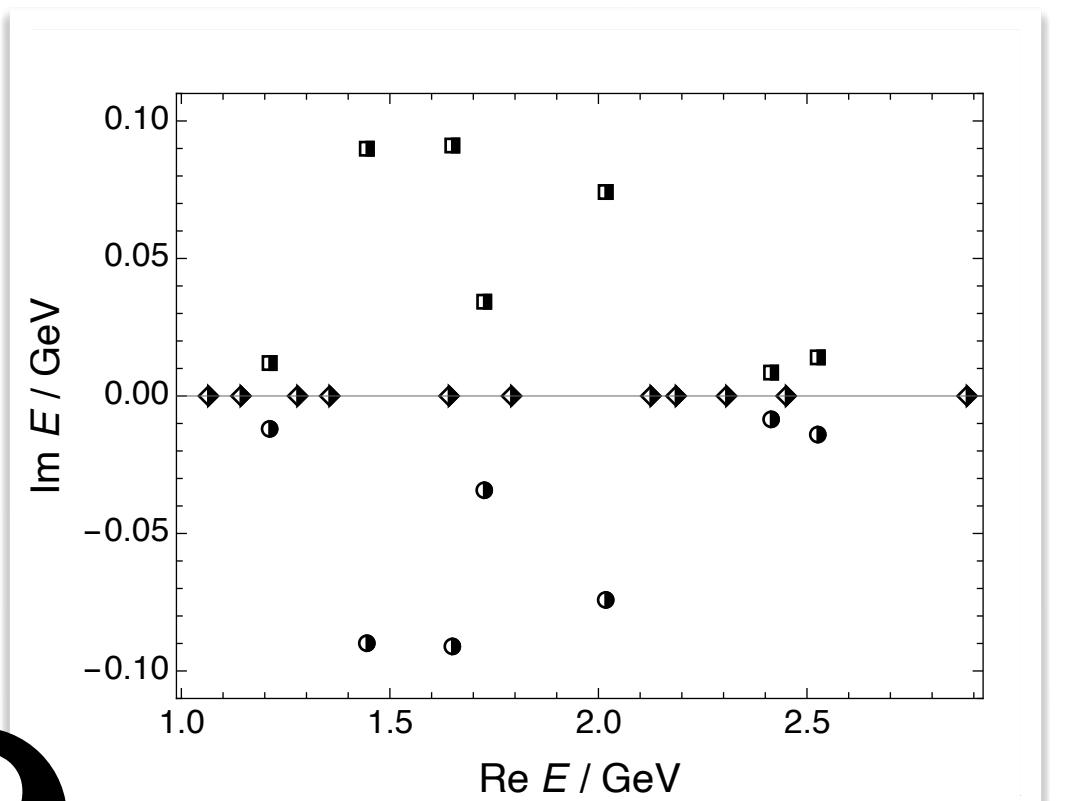
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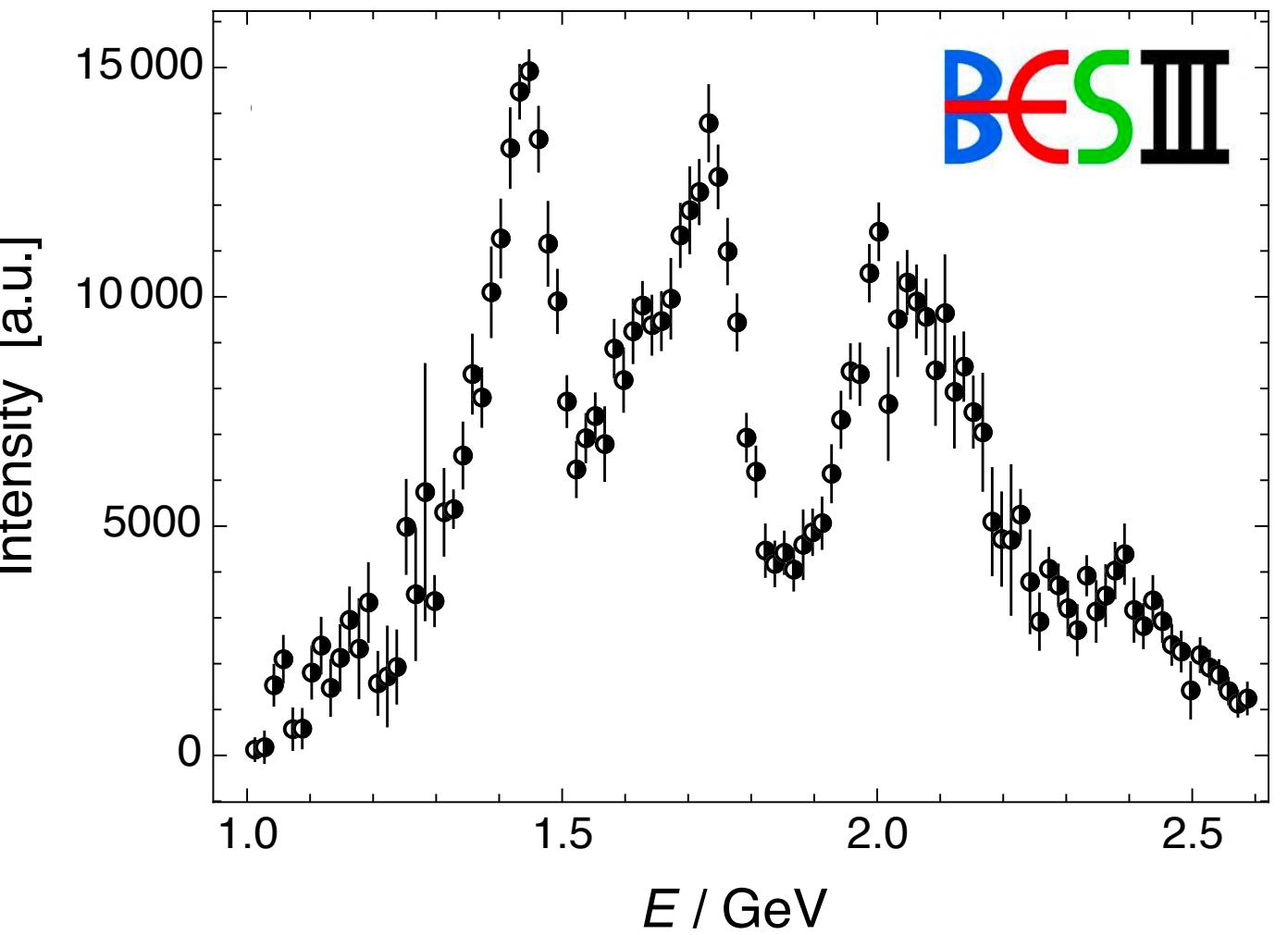
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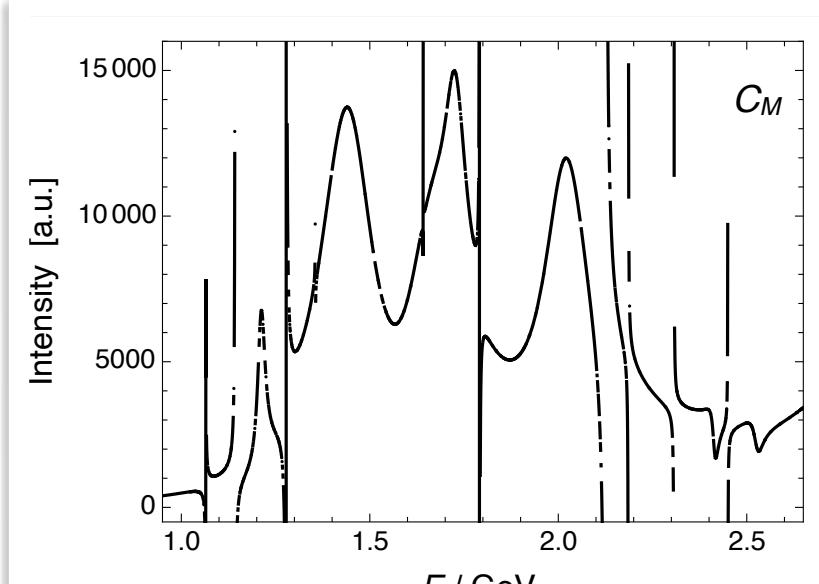
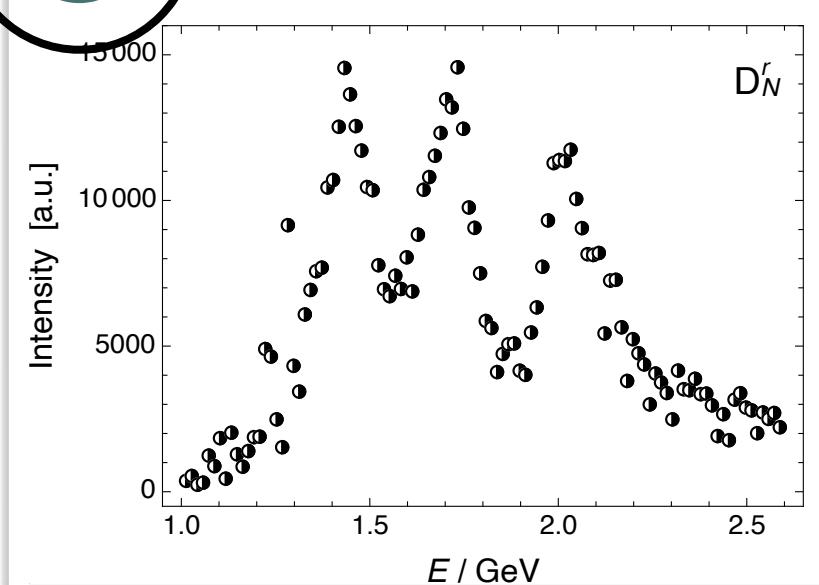
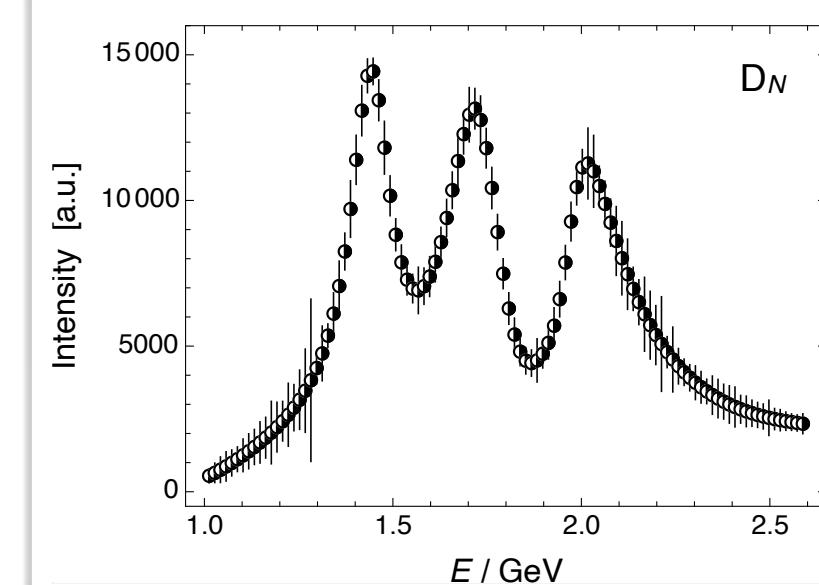
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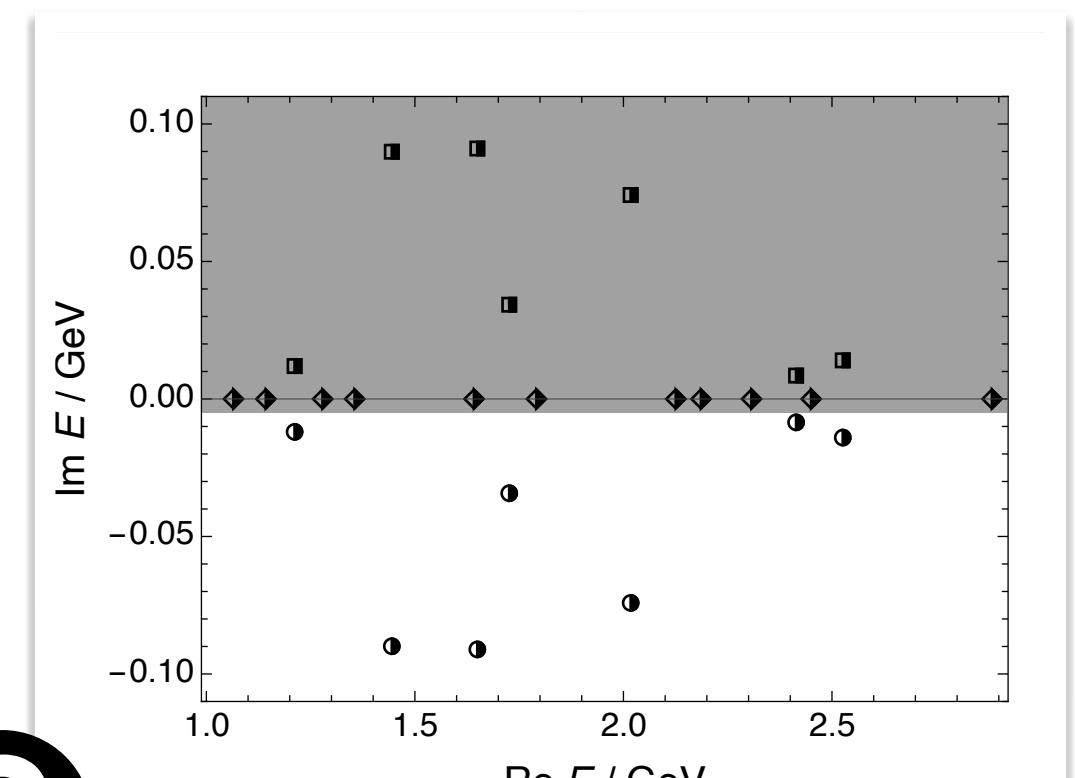
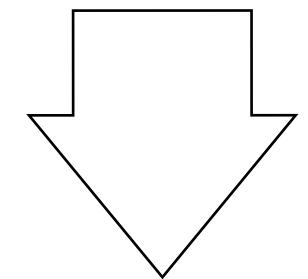
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Propagate **uncertainties** through **resampling**

3



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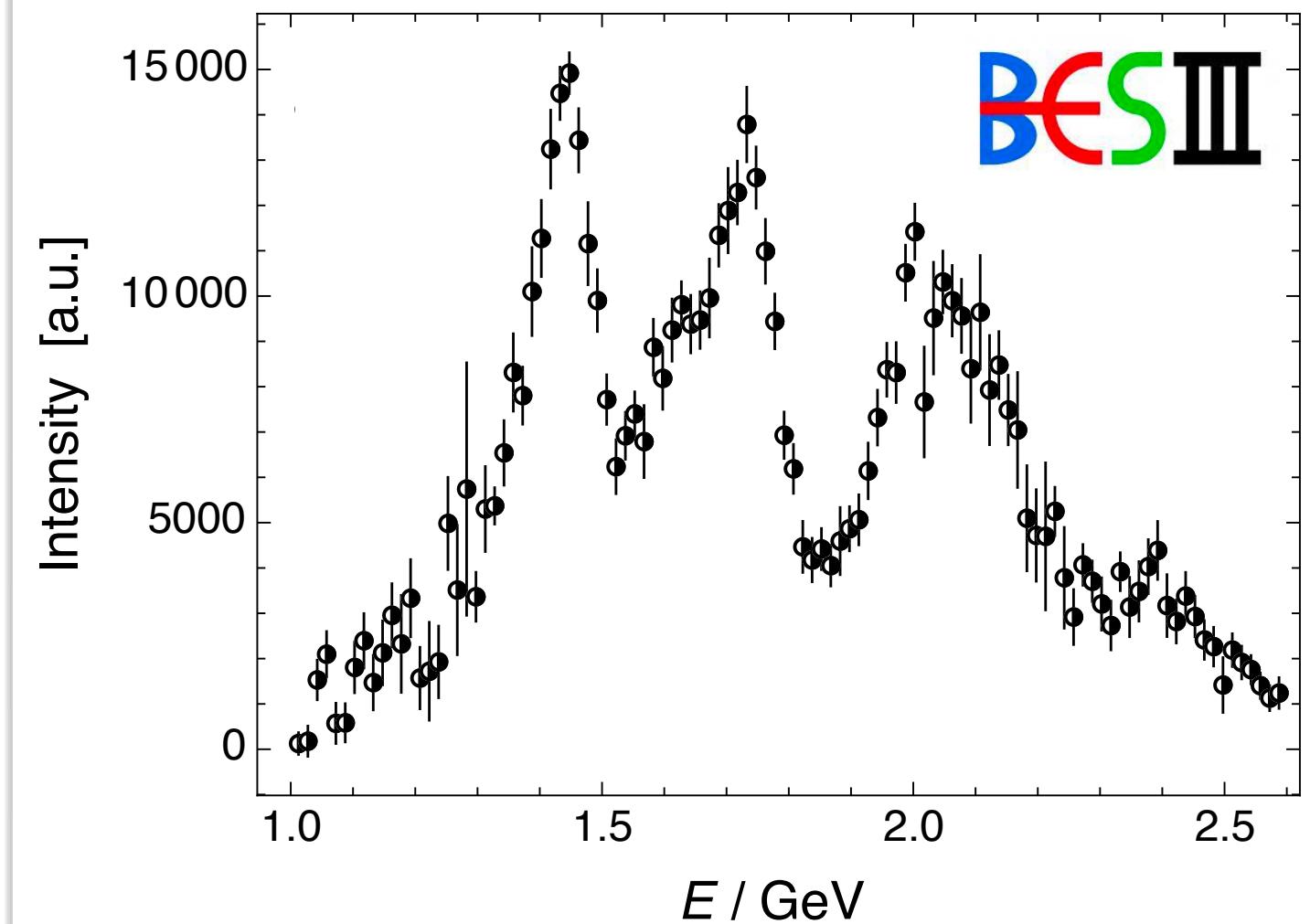


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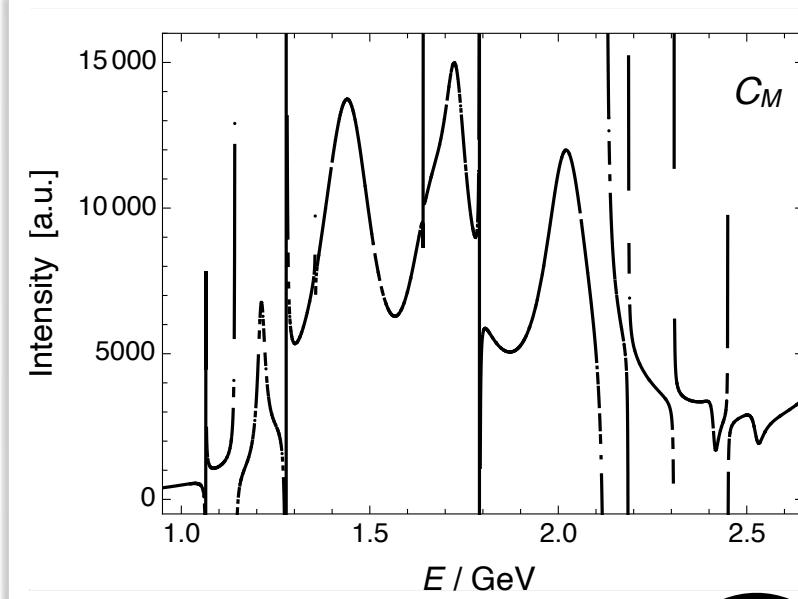
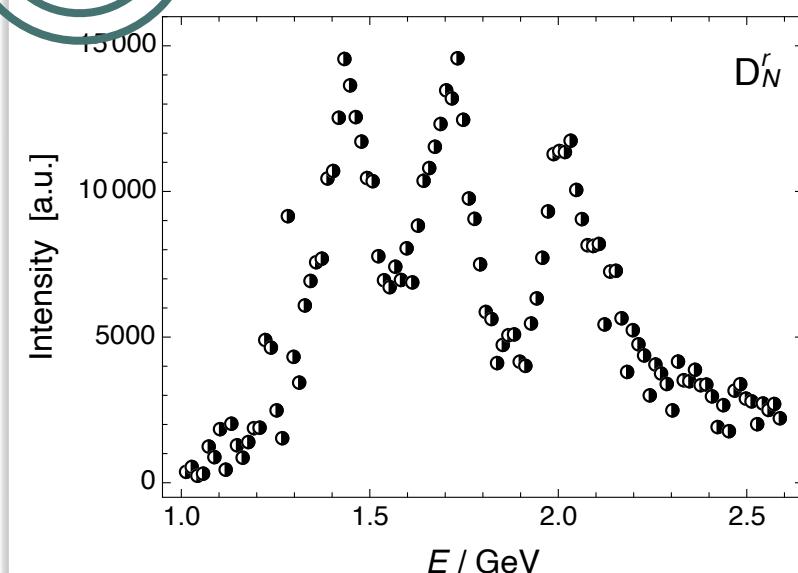
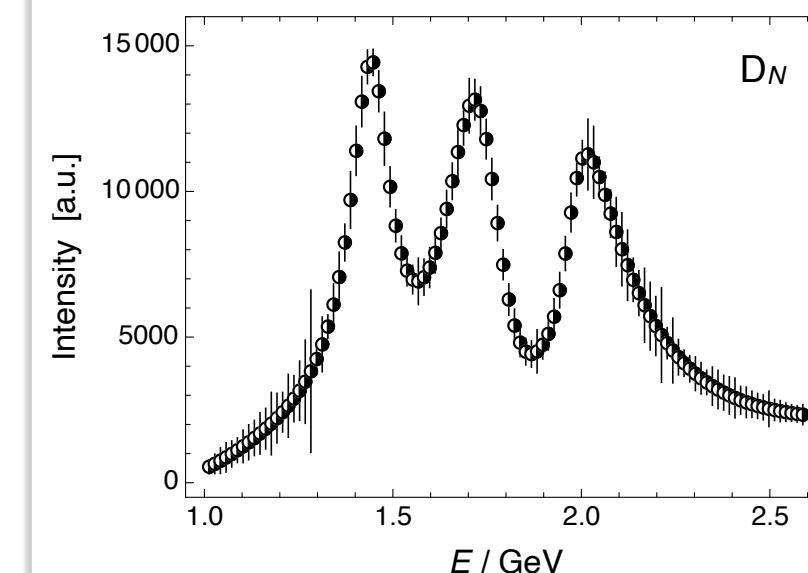
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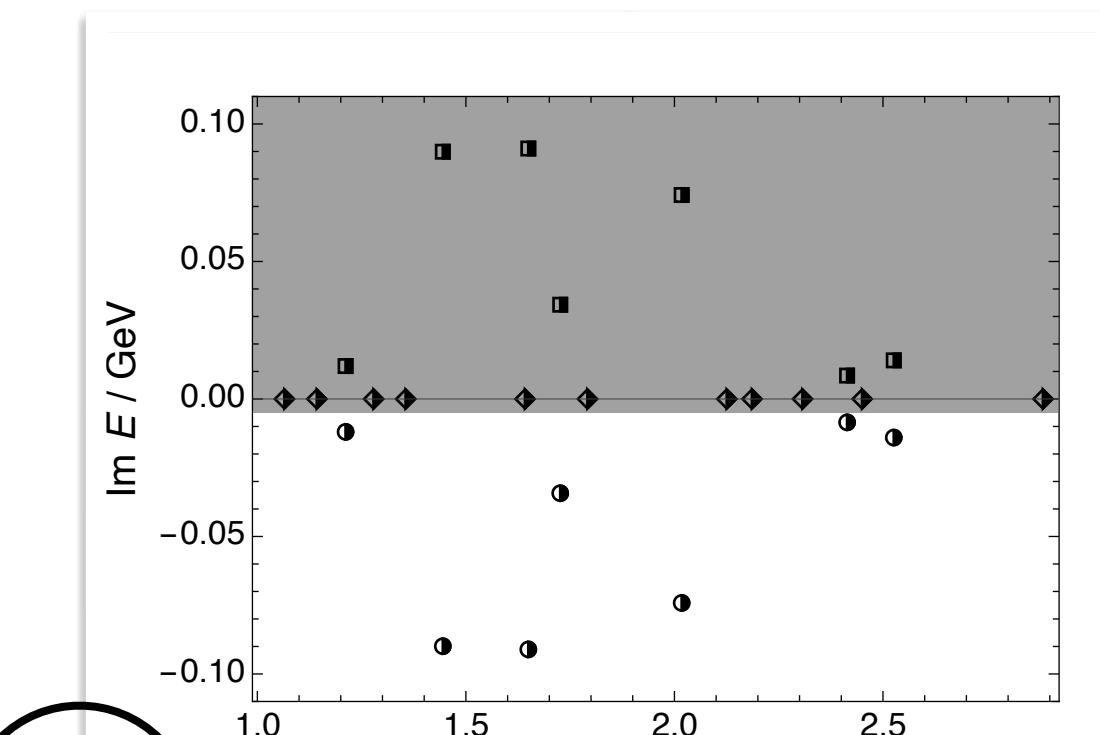
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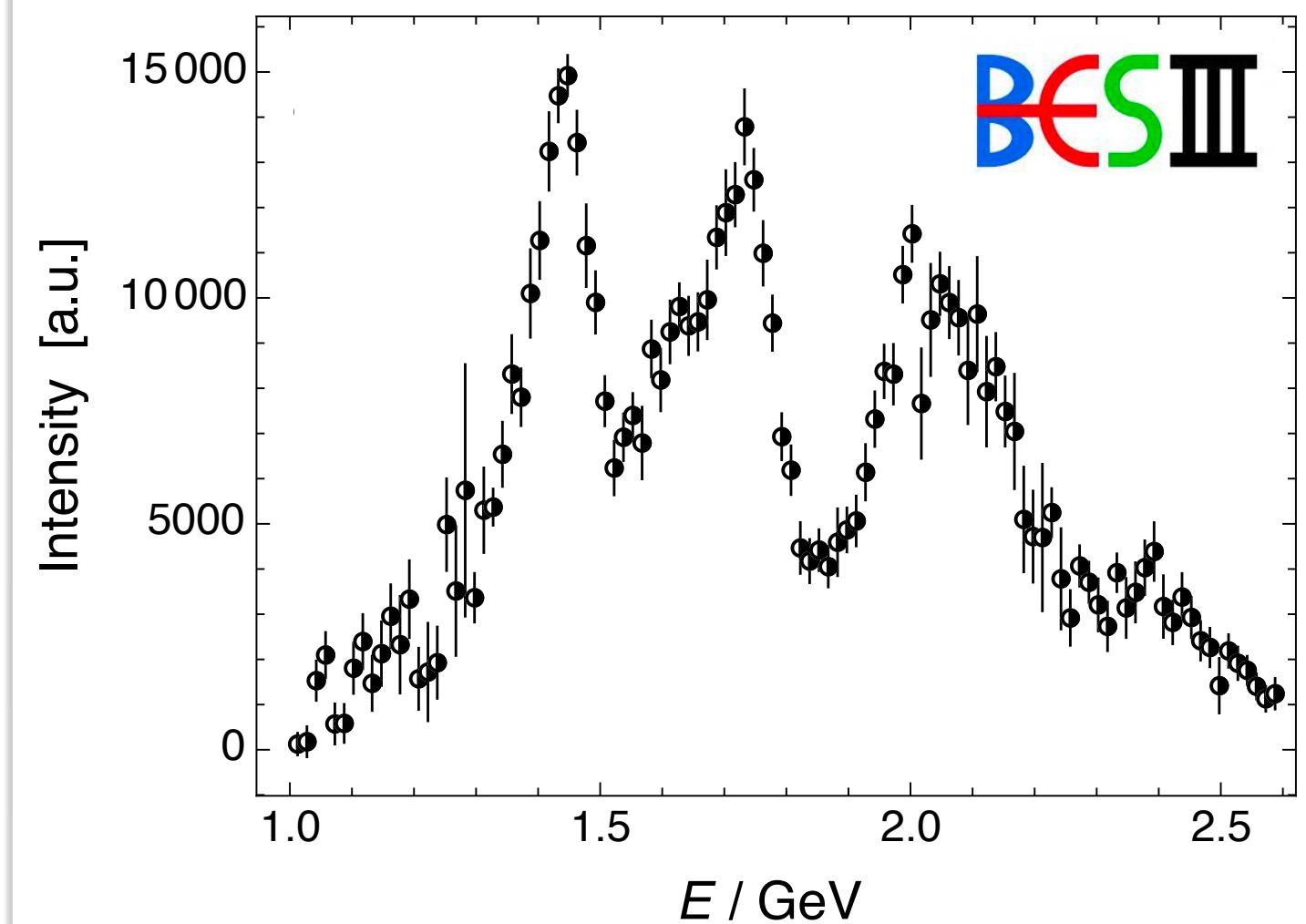


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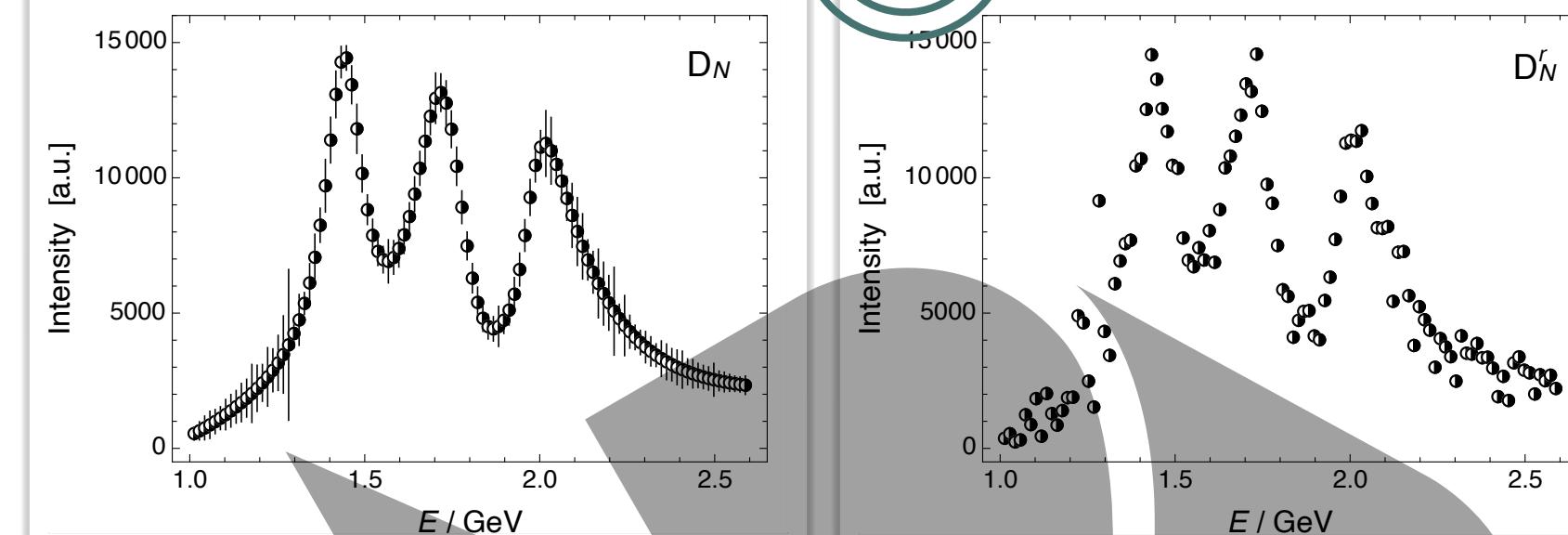
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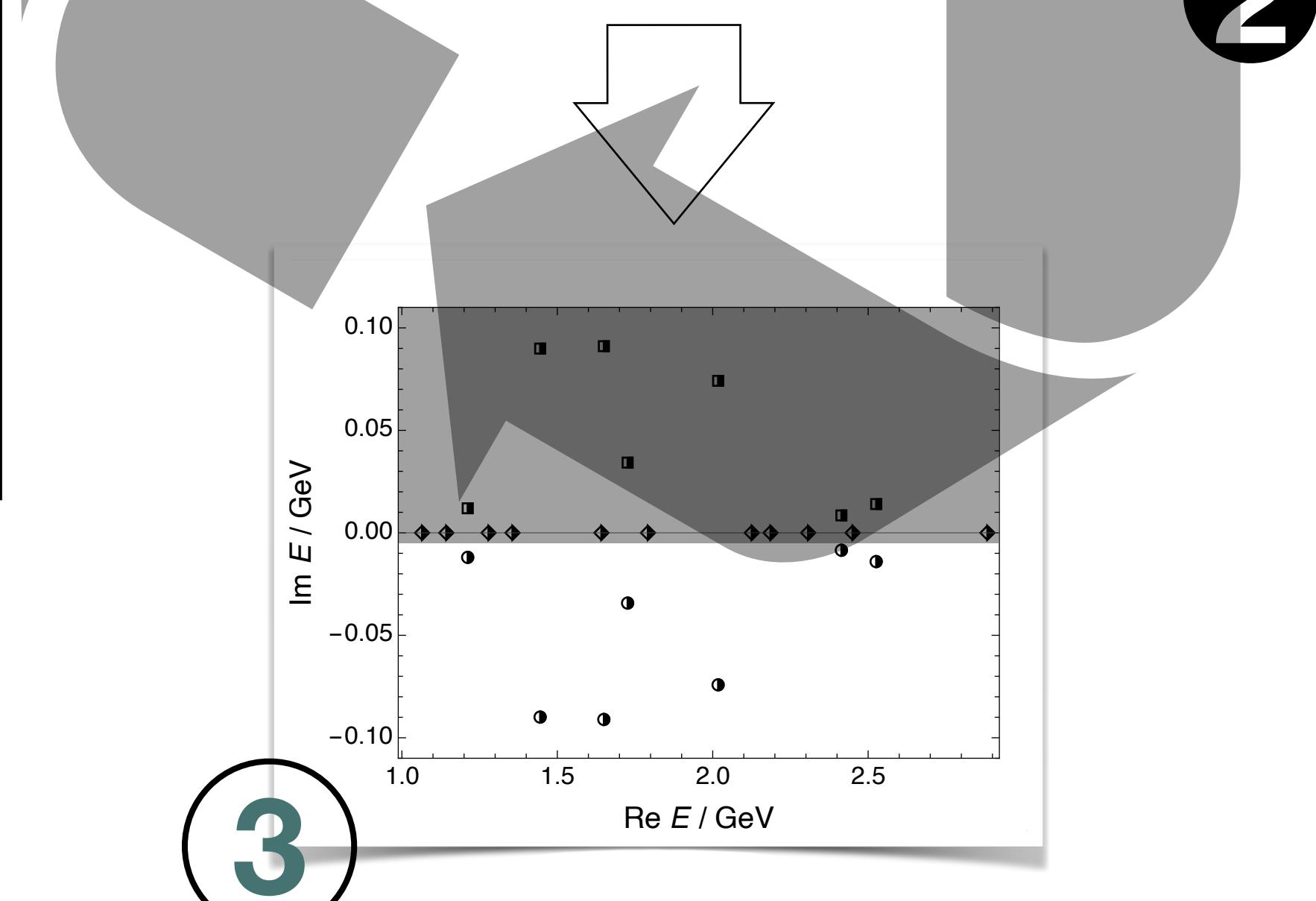
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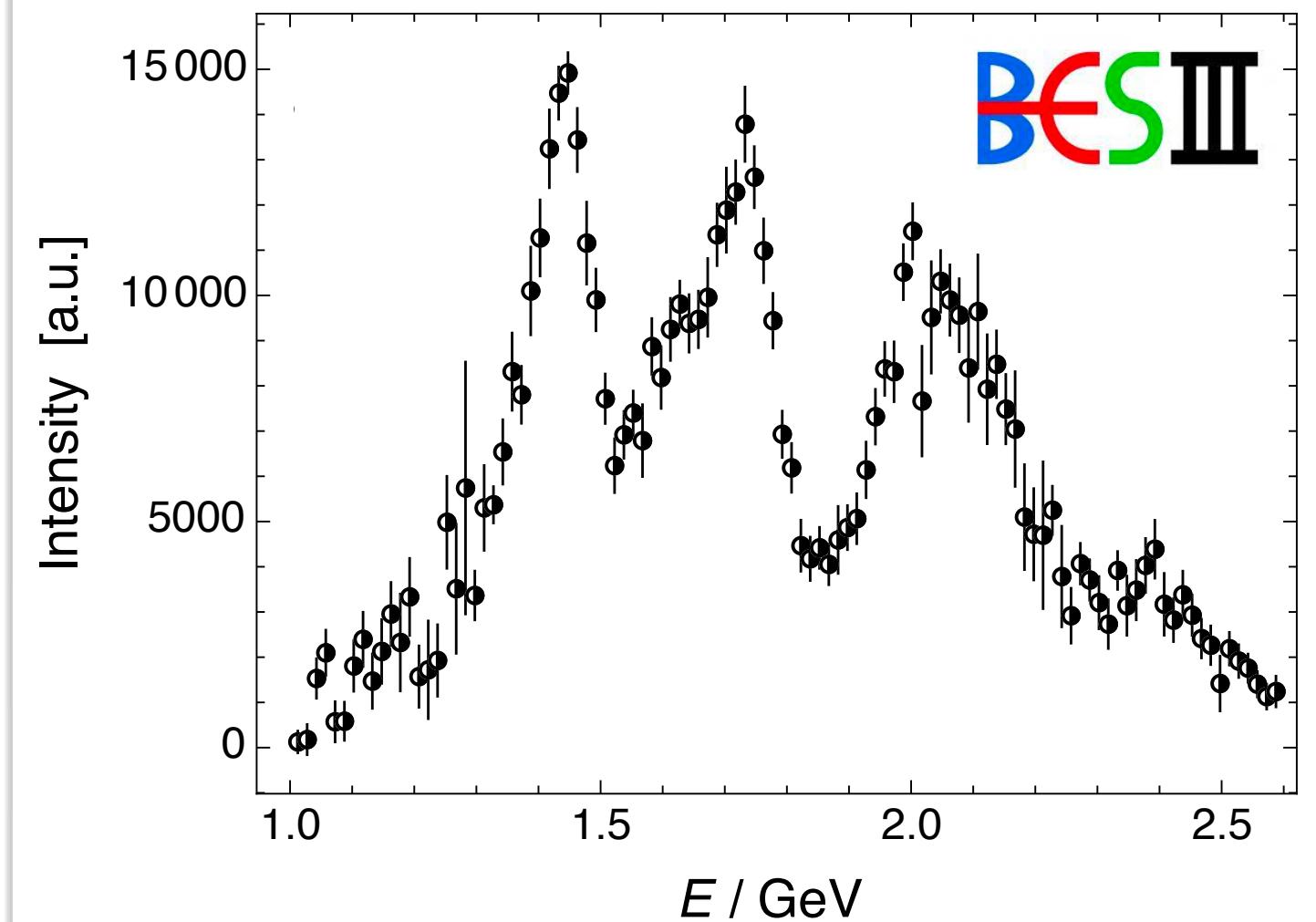


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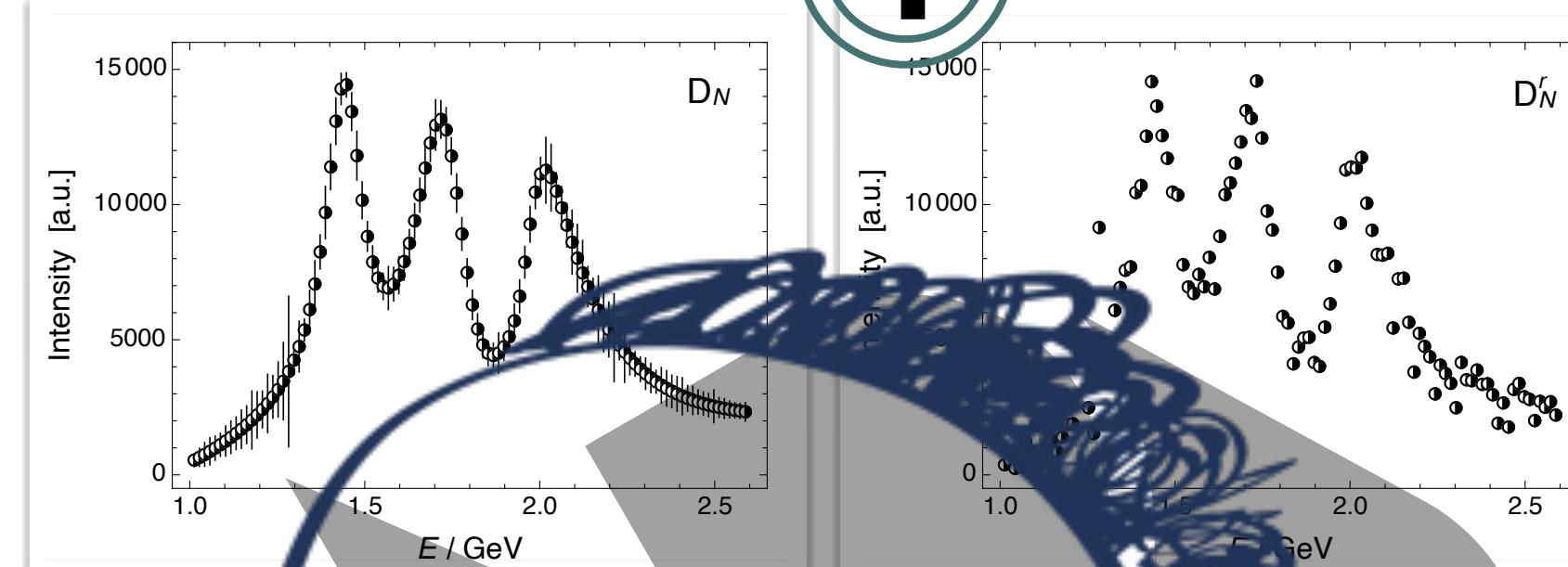
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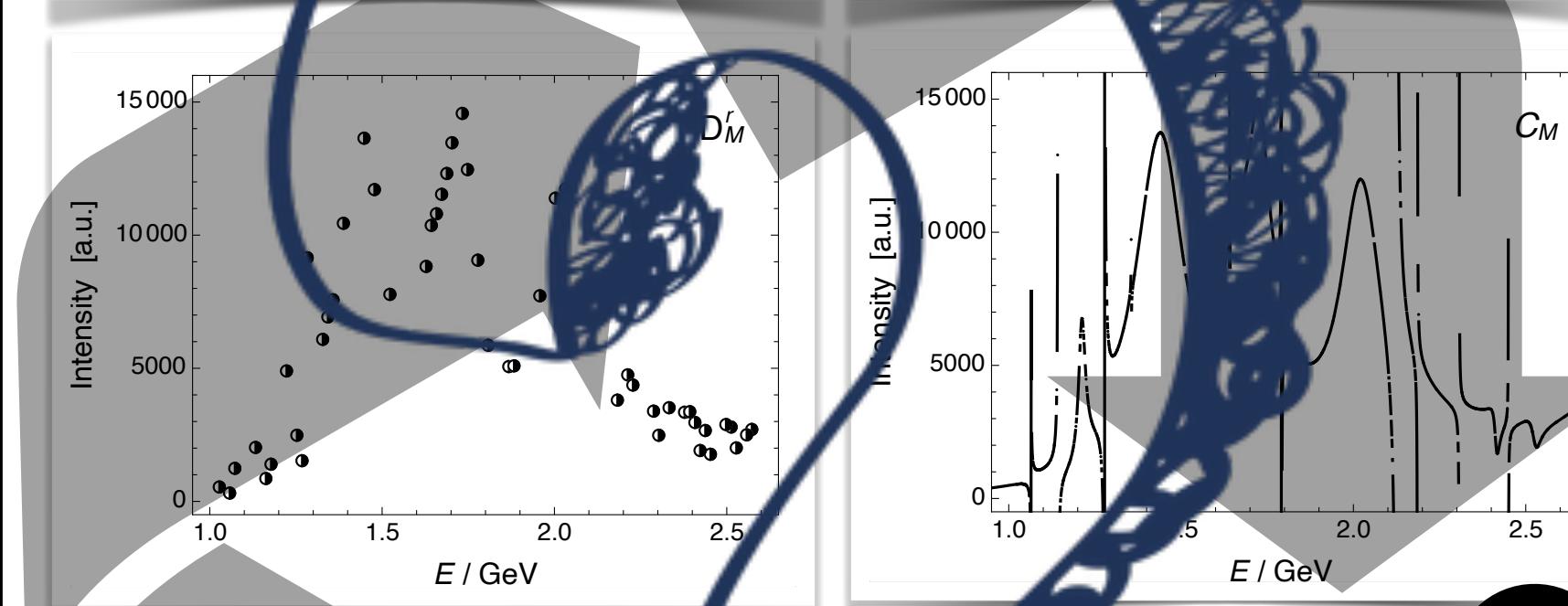
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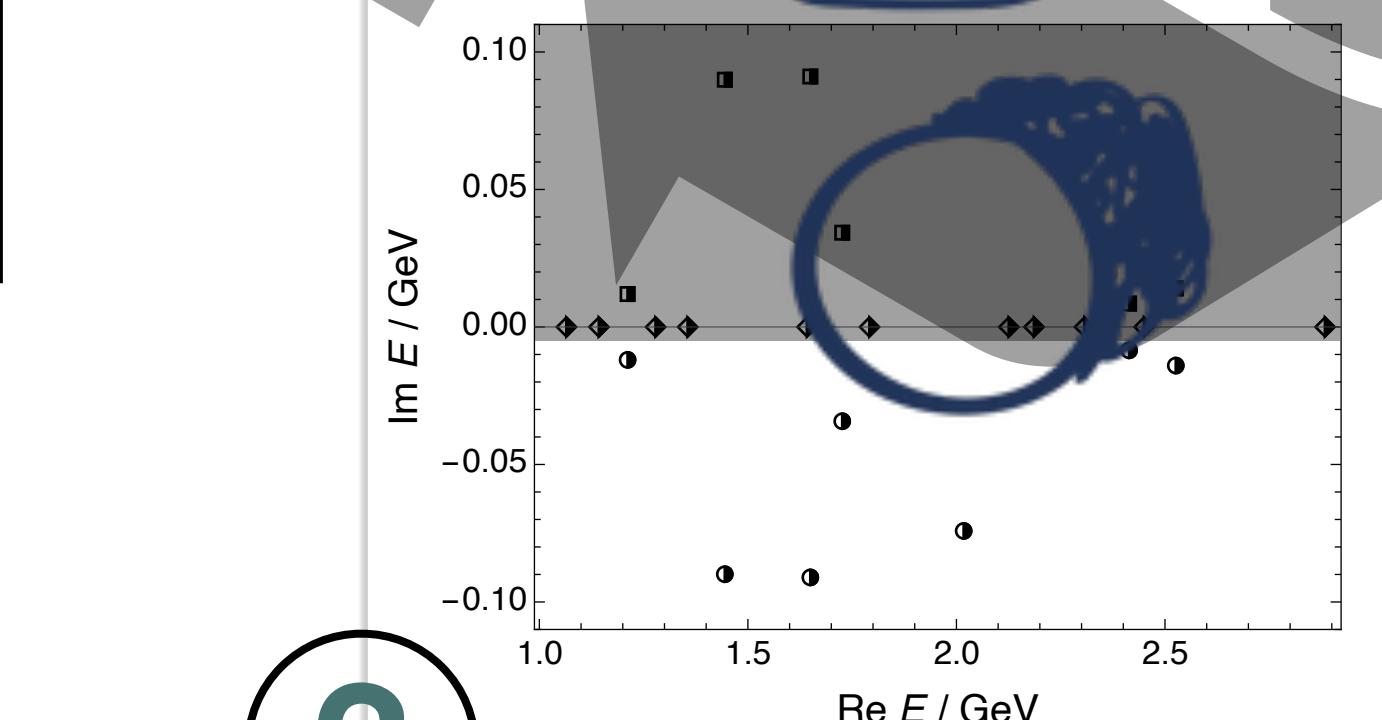
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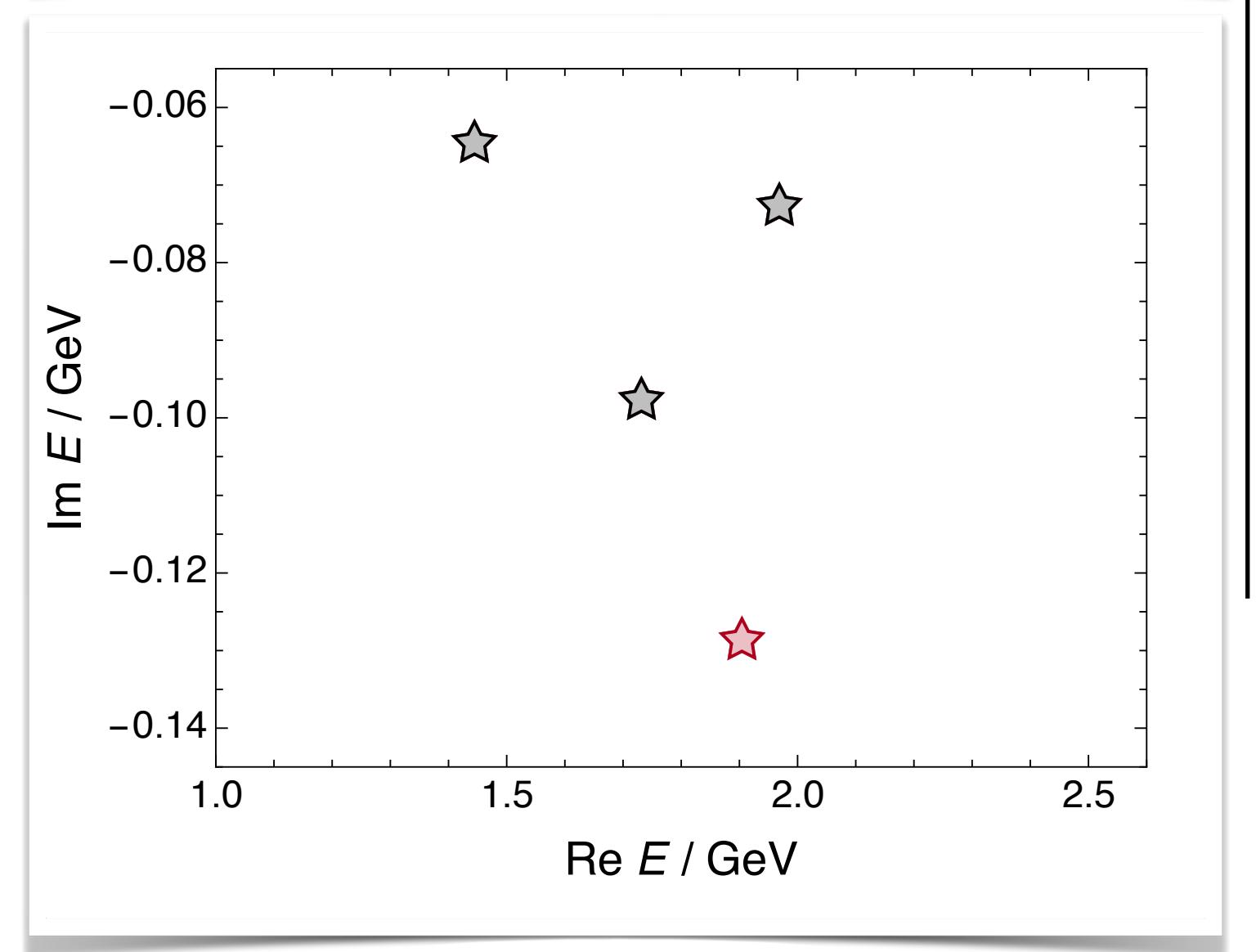
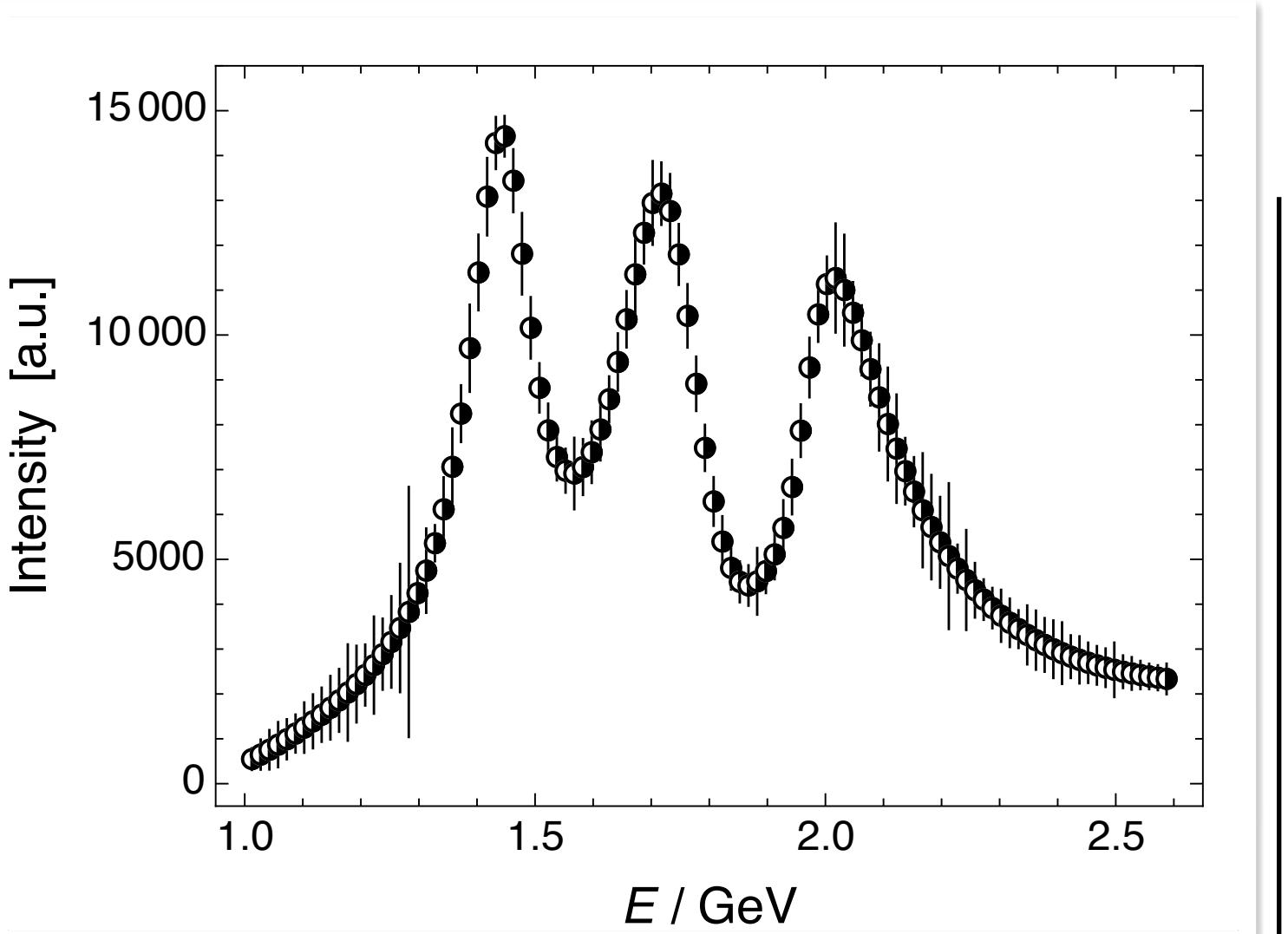
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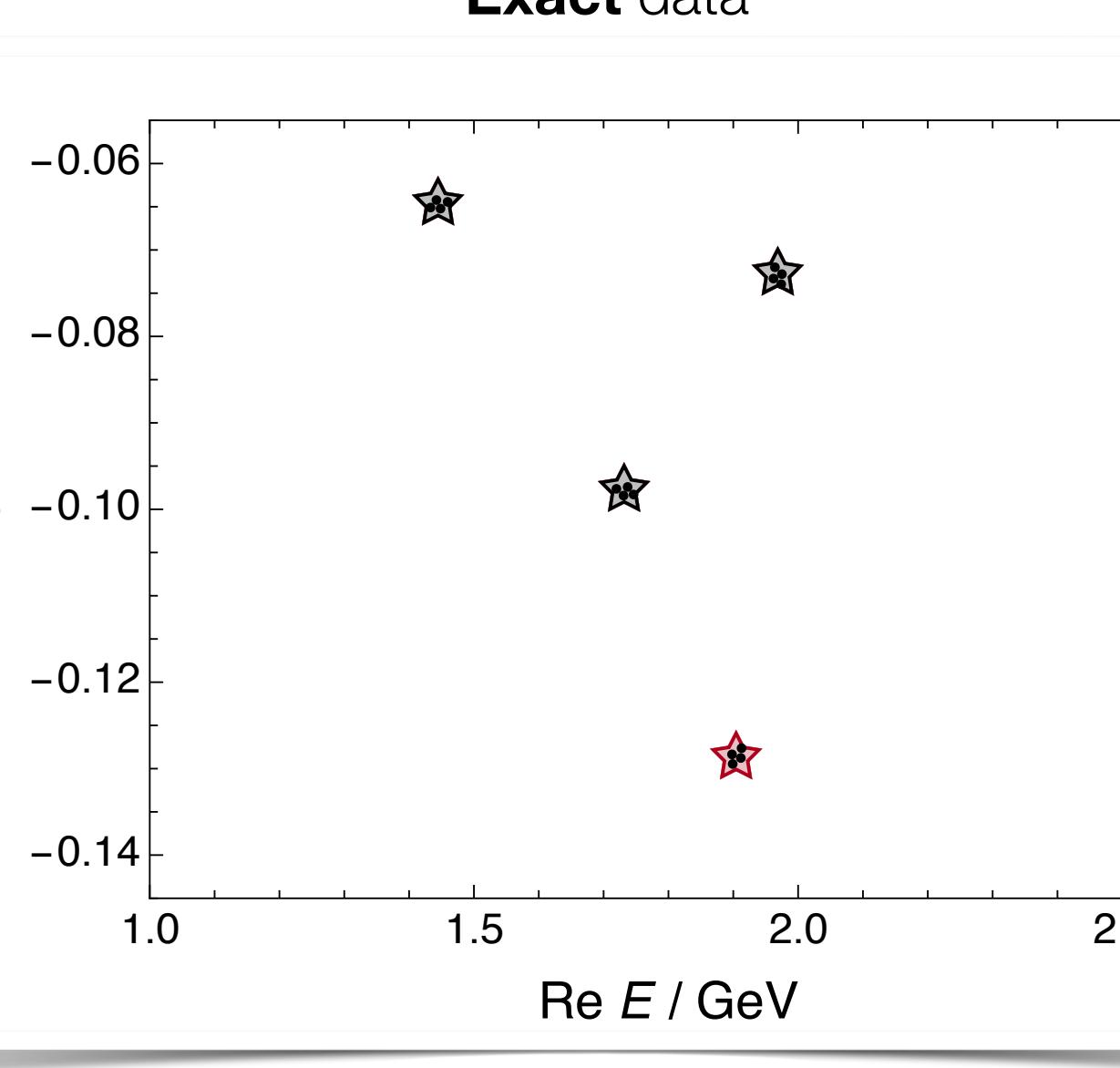
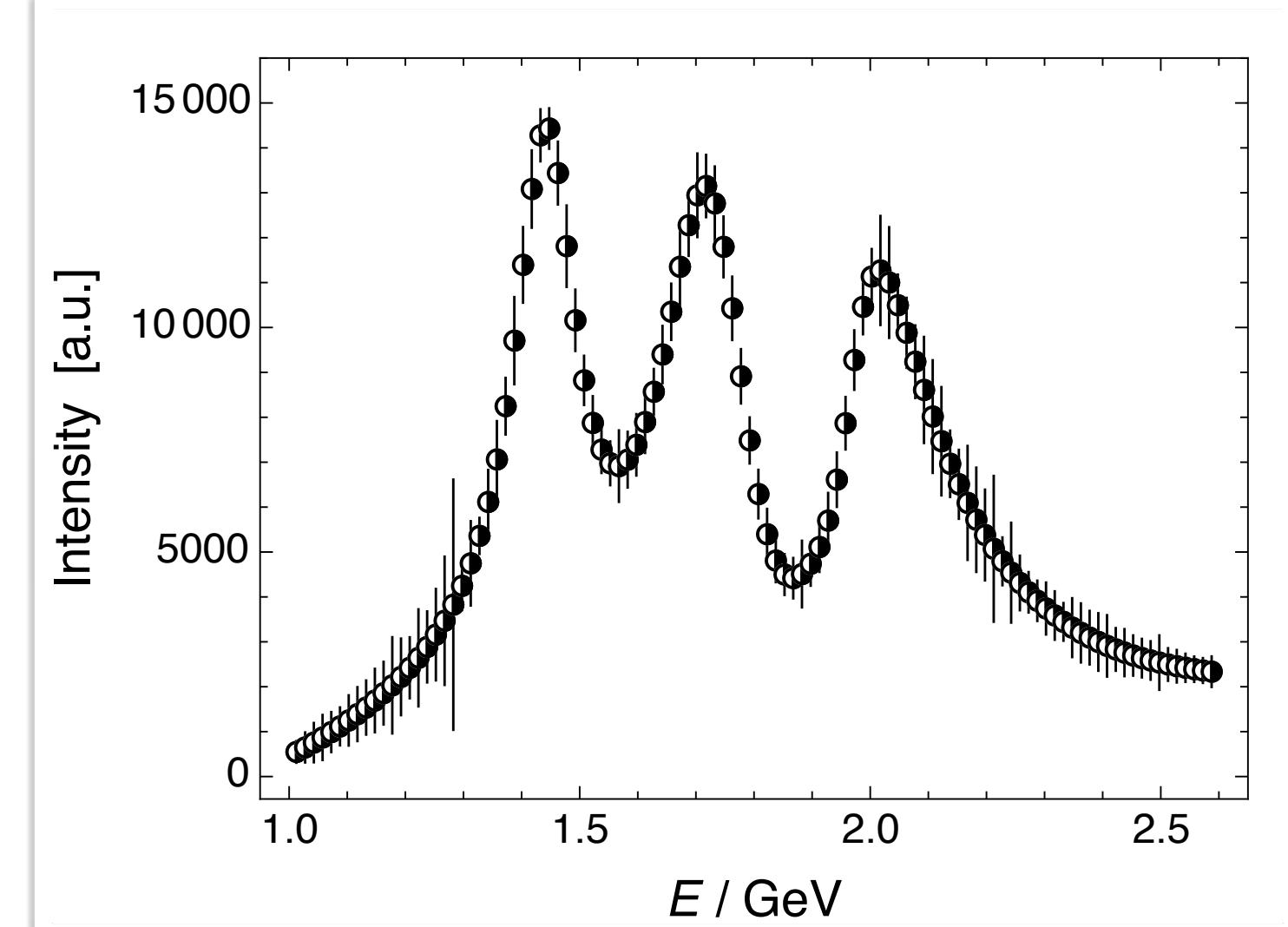
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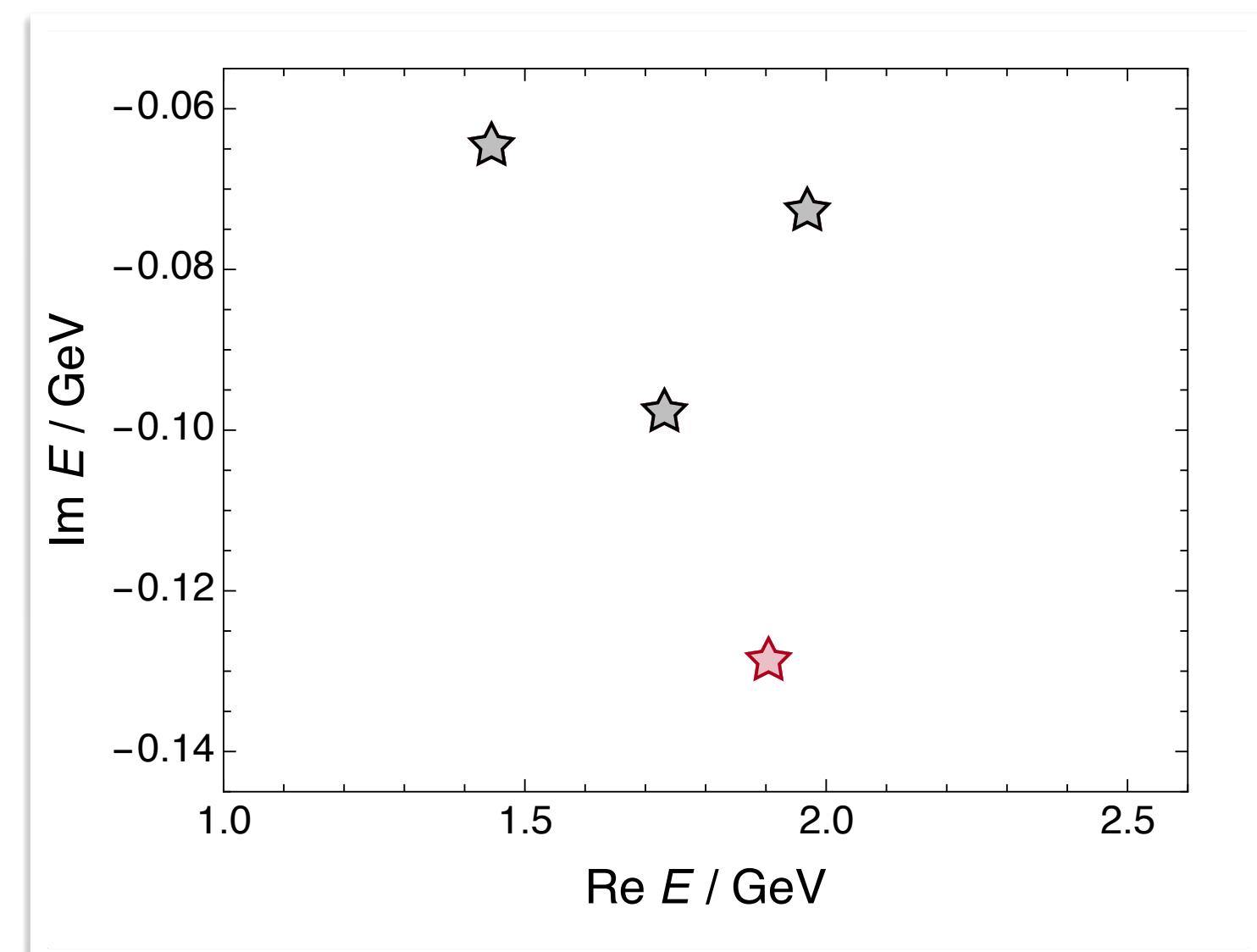
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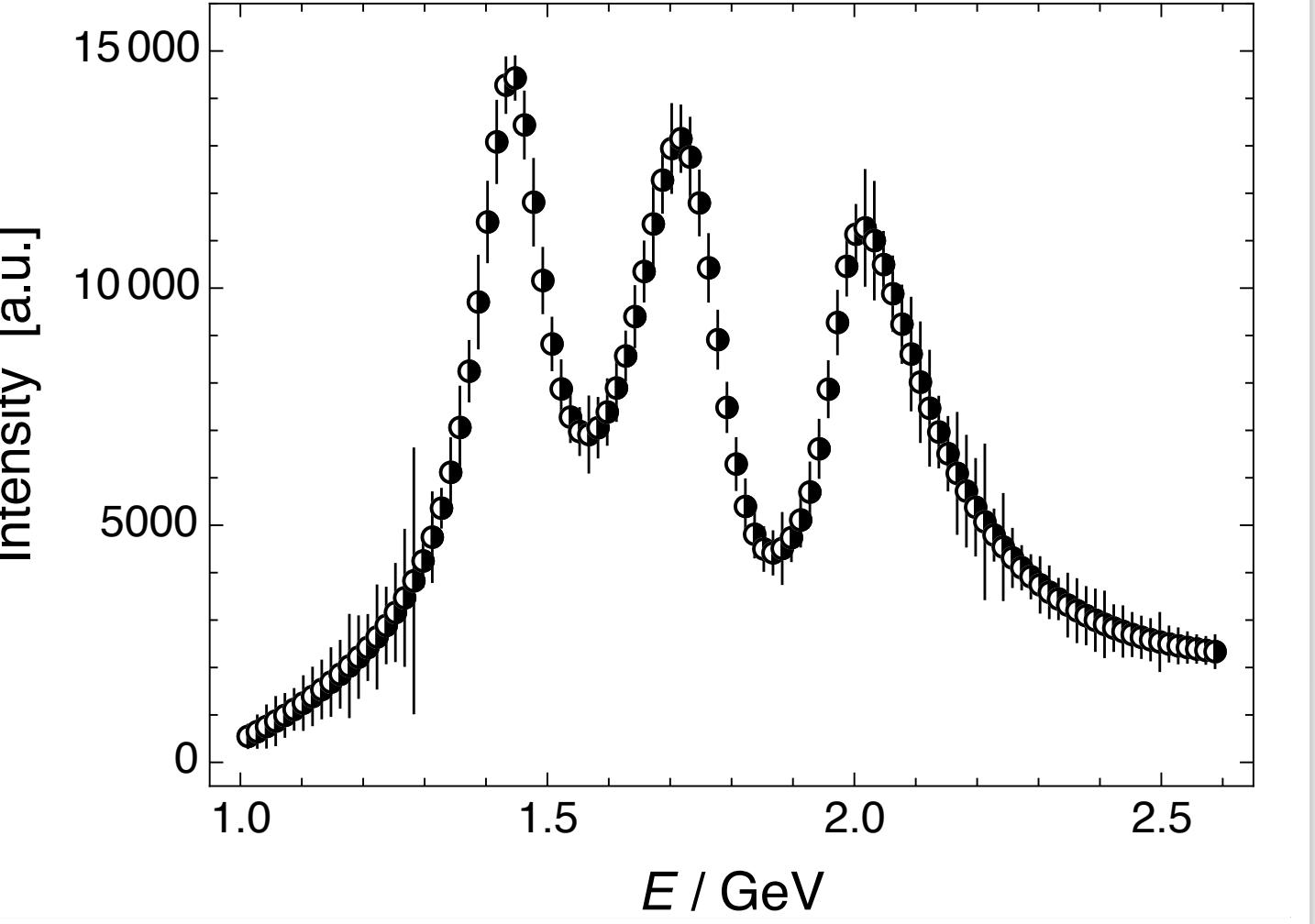
**1**



Nearly **exact cancellation** between **poles** and **zeros**



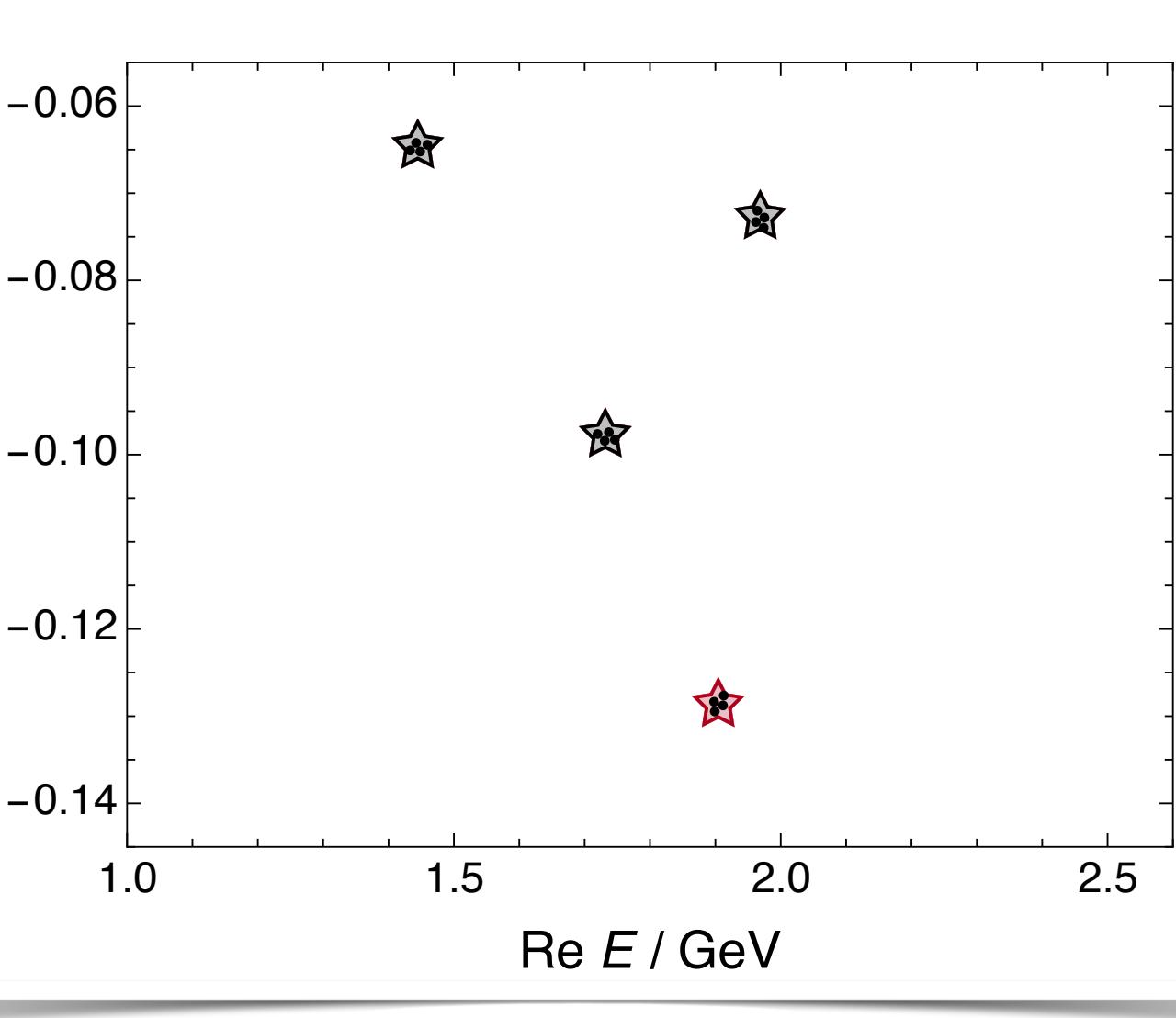
# SPM VALIDATION



$$\left| \begin{array}{l} D_N = \{(E_i, I_i = I(E_i)), i = 1, \dots, N\} \\ D_M \subseteq D_N \\ C_M(E) = \frac{P(E)}{Q(E)} \\ C_M(E_i) = I_i \forall E_i \in D_M \end{array} \right.$$

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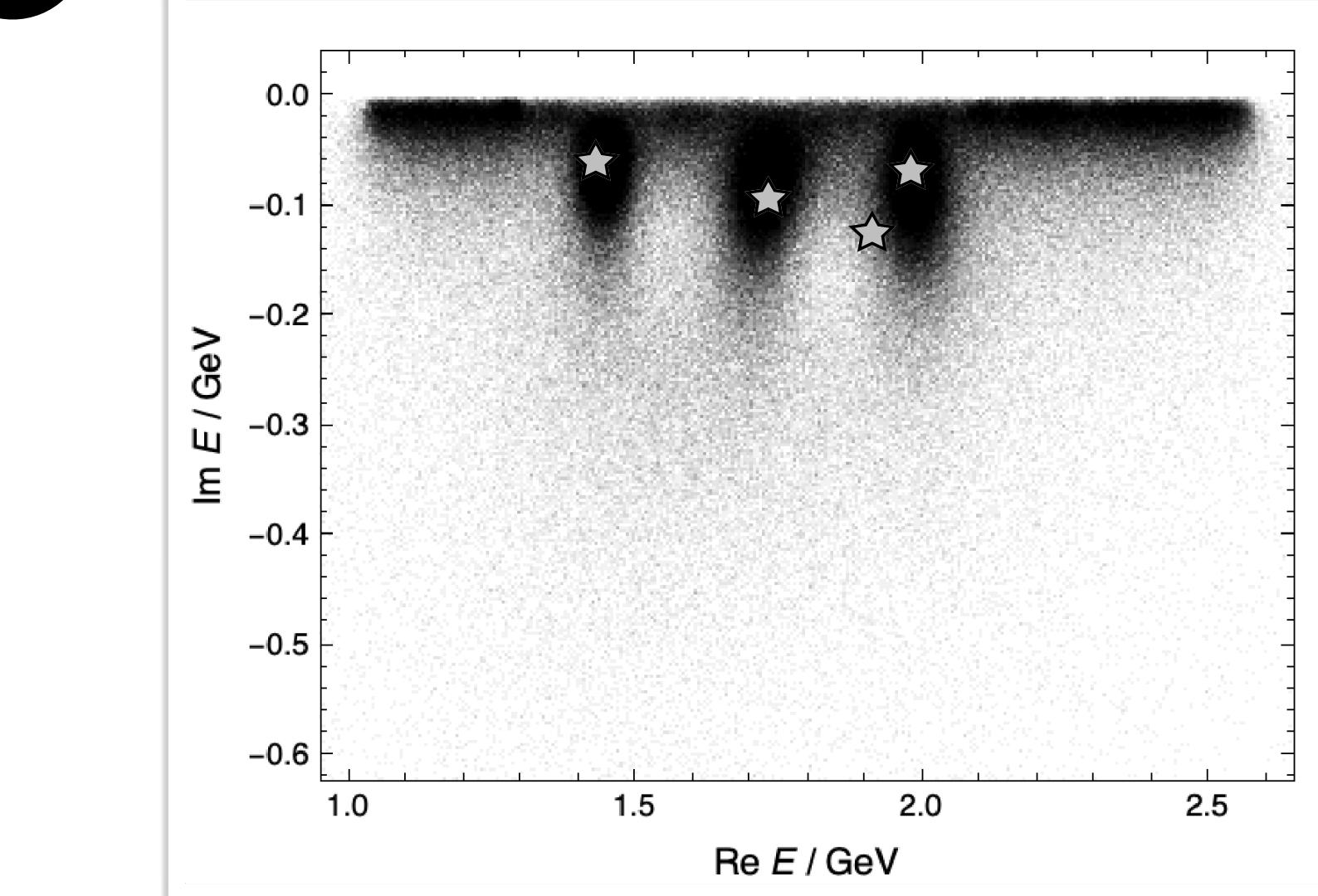
Exact data



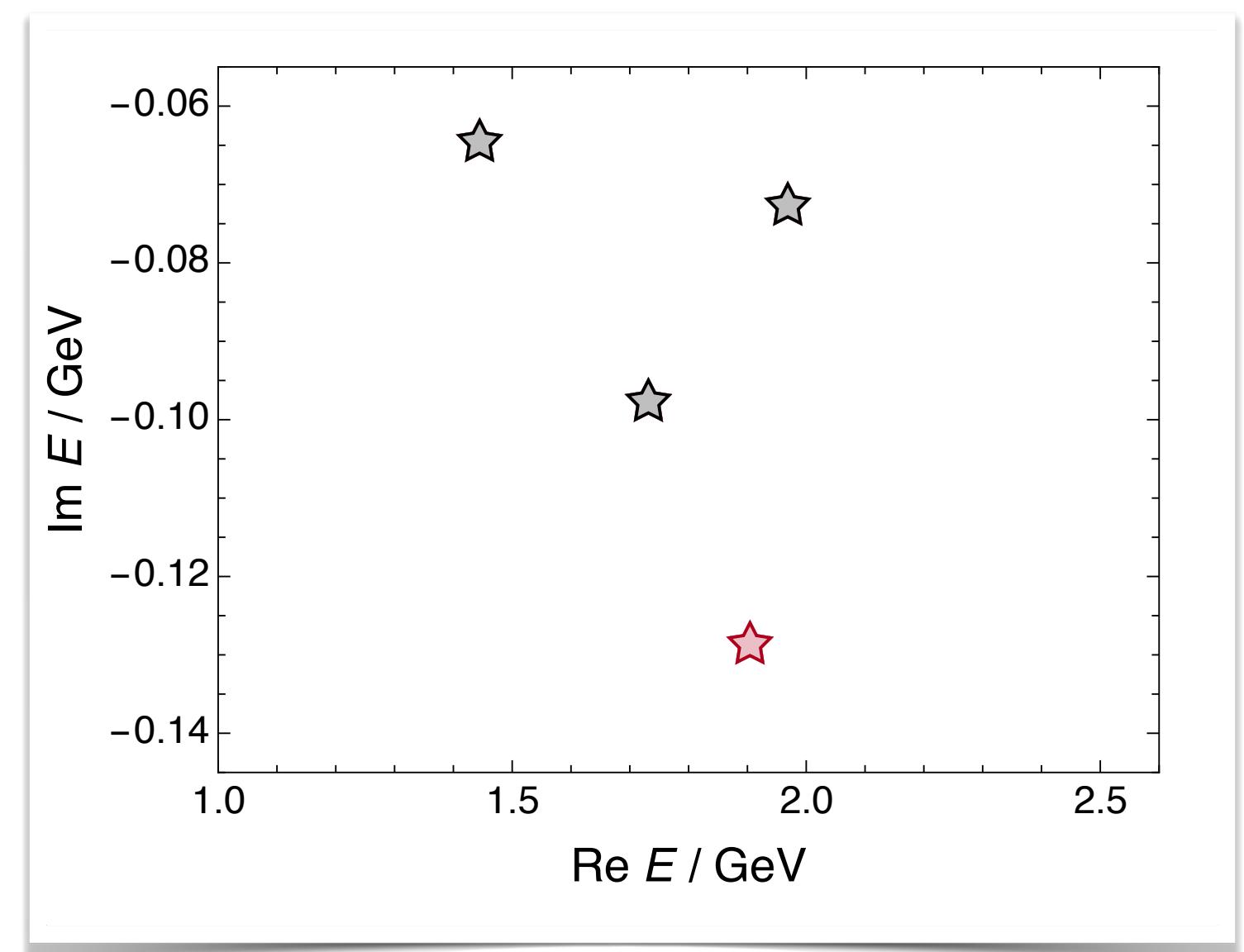
Nearly **exact cancellation** between **poles** and **zeros**

**2**

Adding **uncertainties**



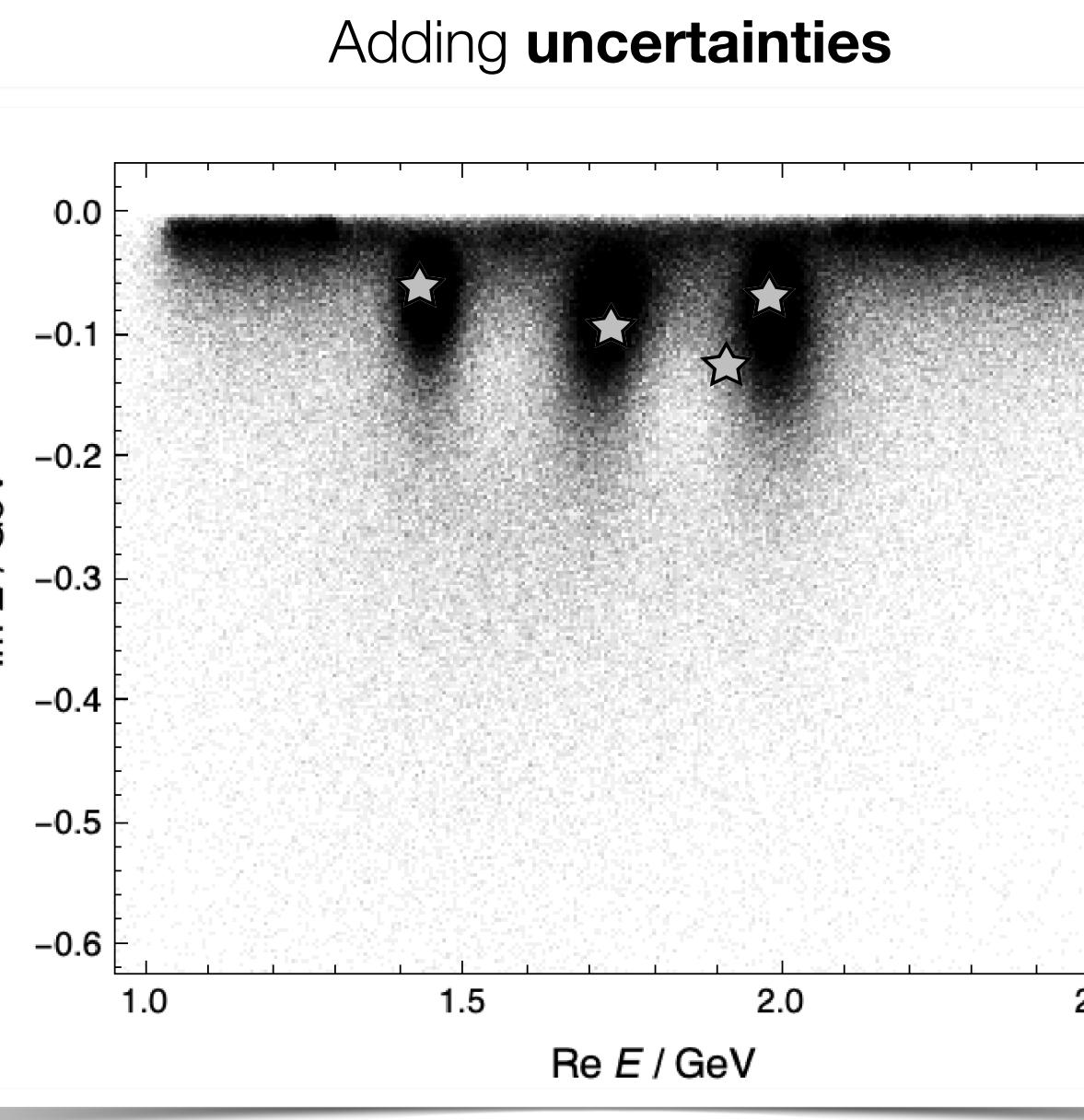
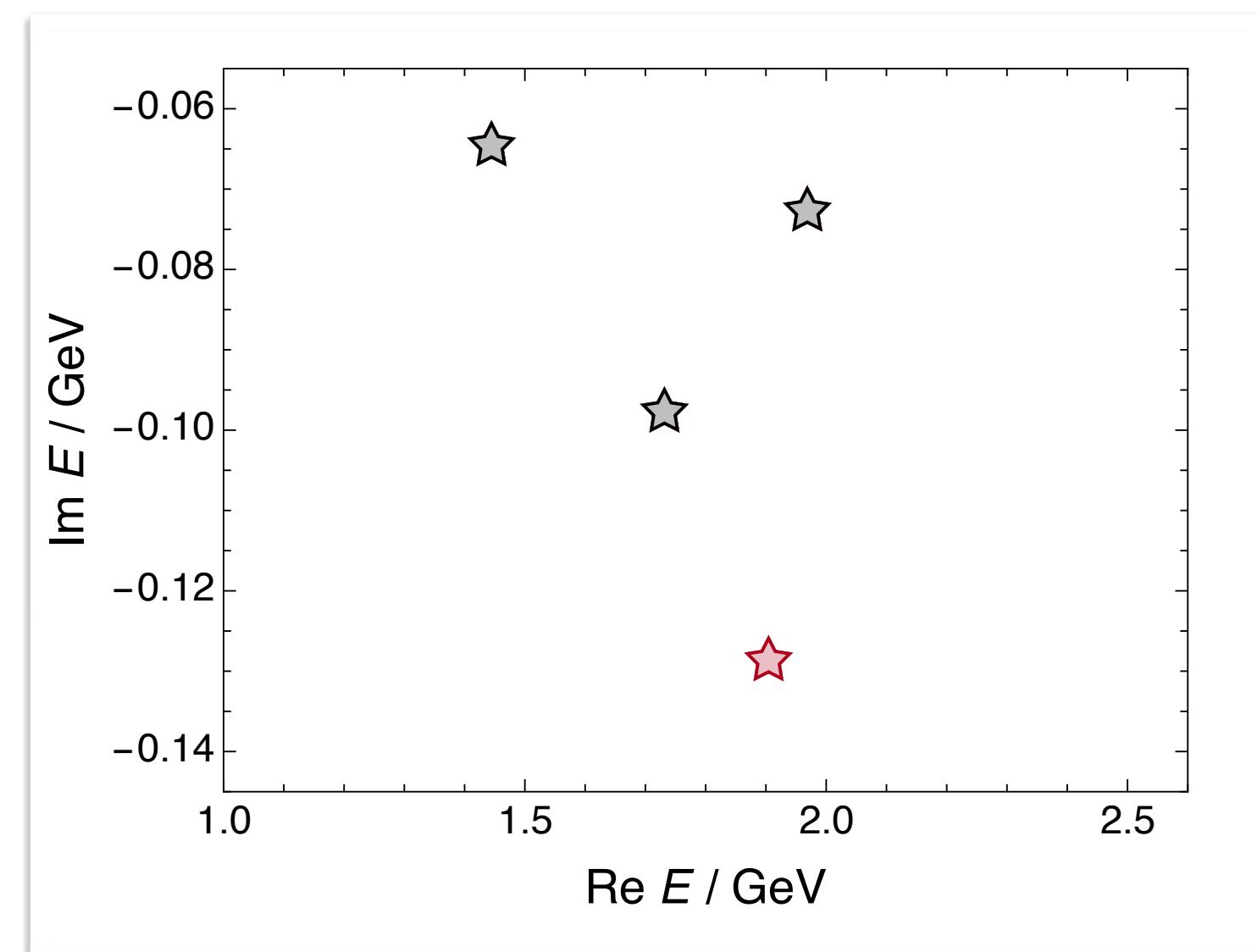
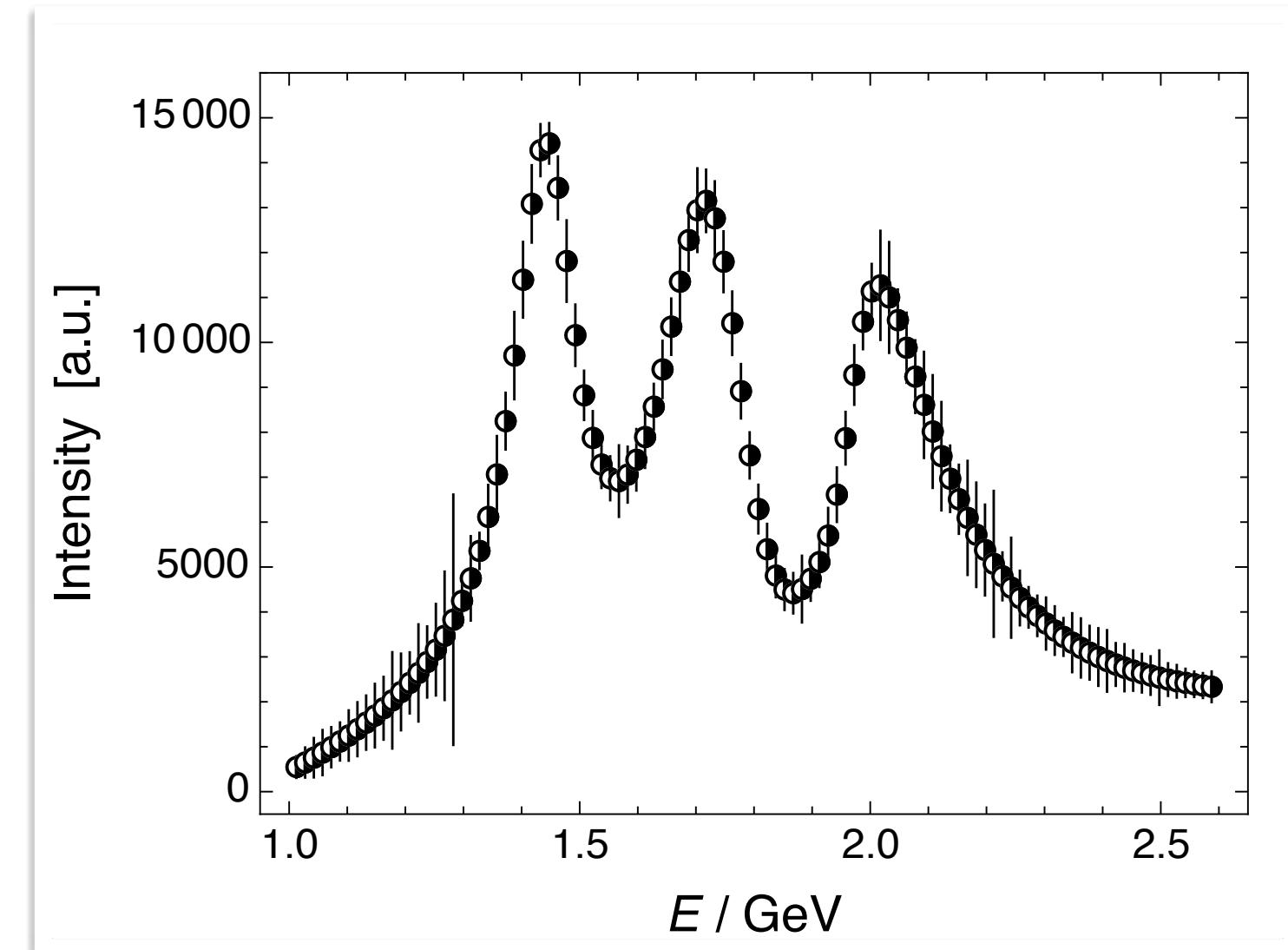
Need to separate **noise poles** from **signal poles**



# SPM VALIDATION

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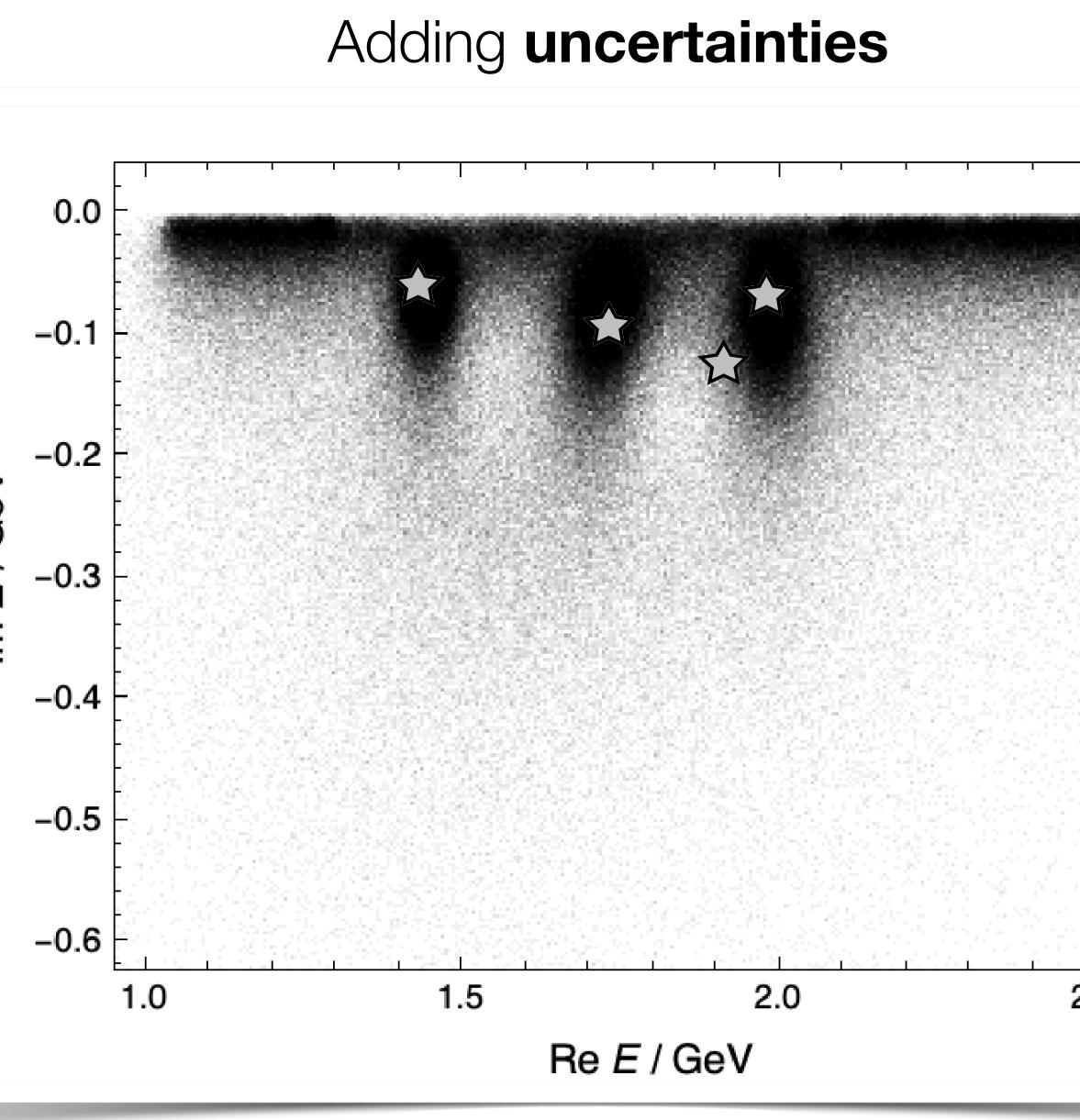
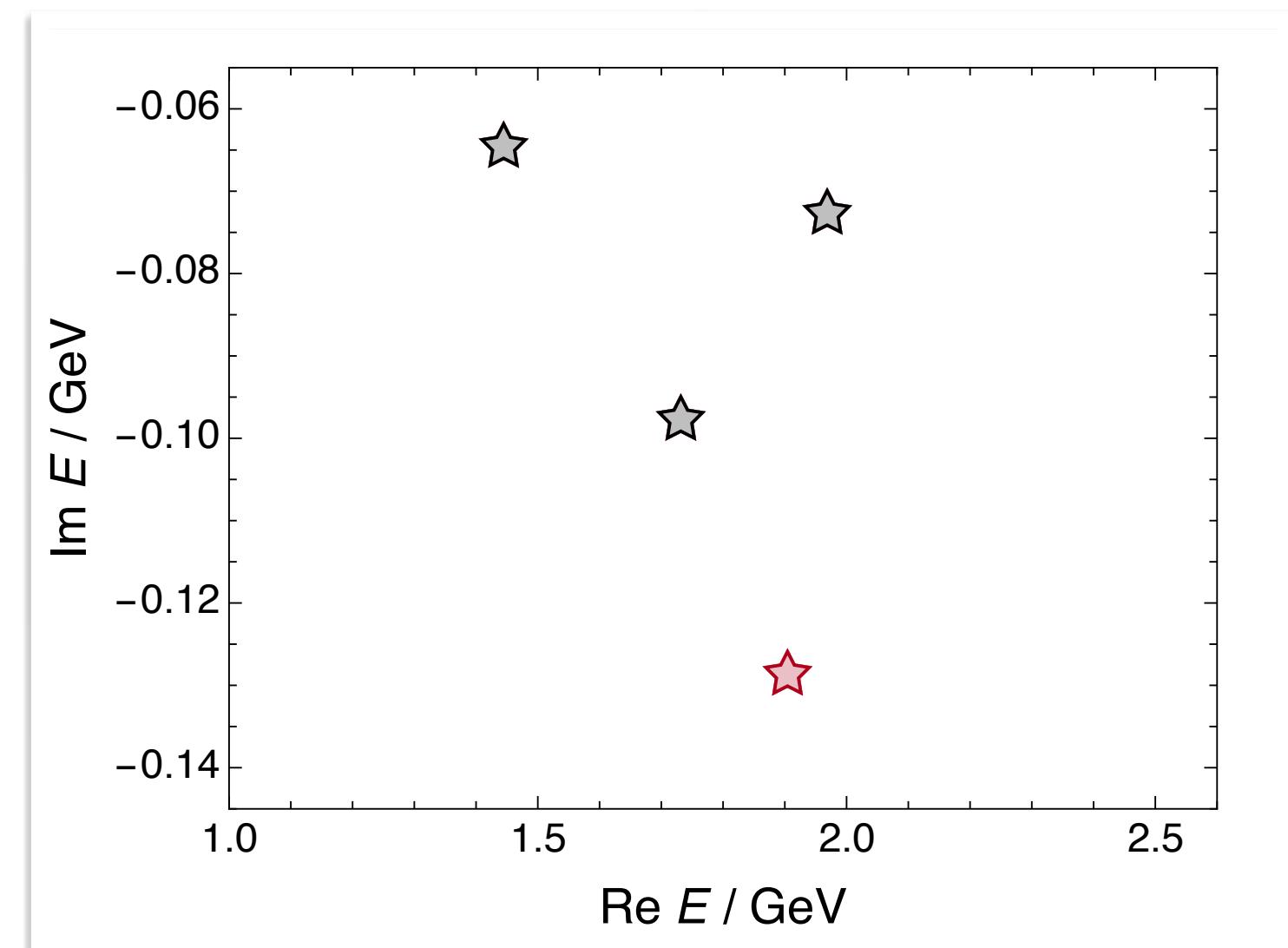
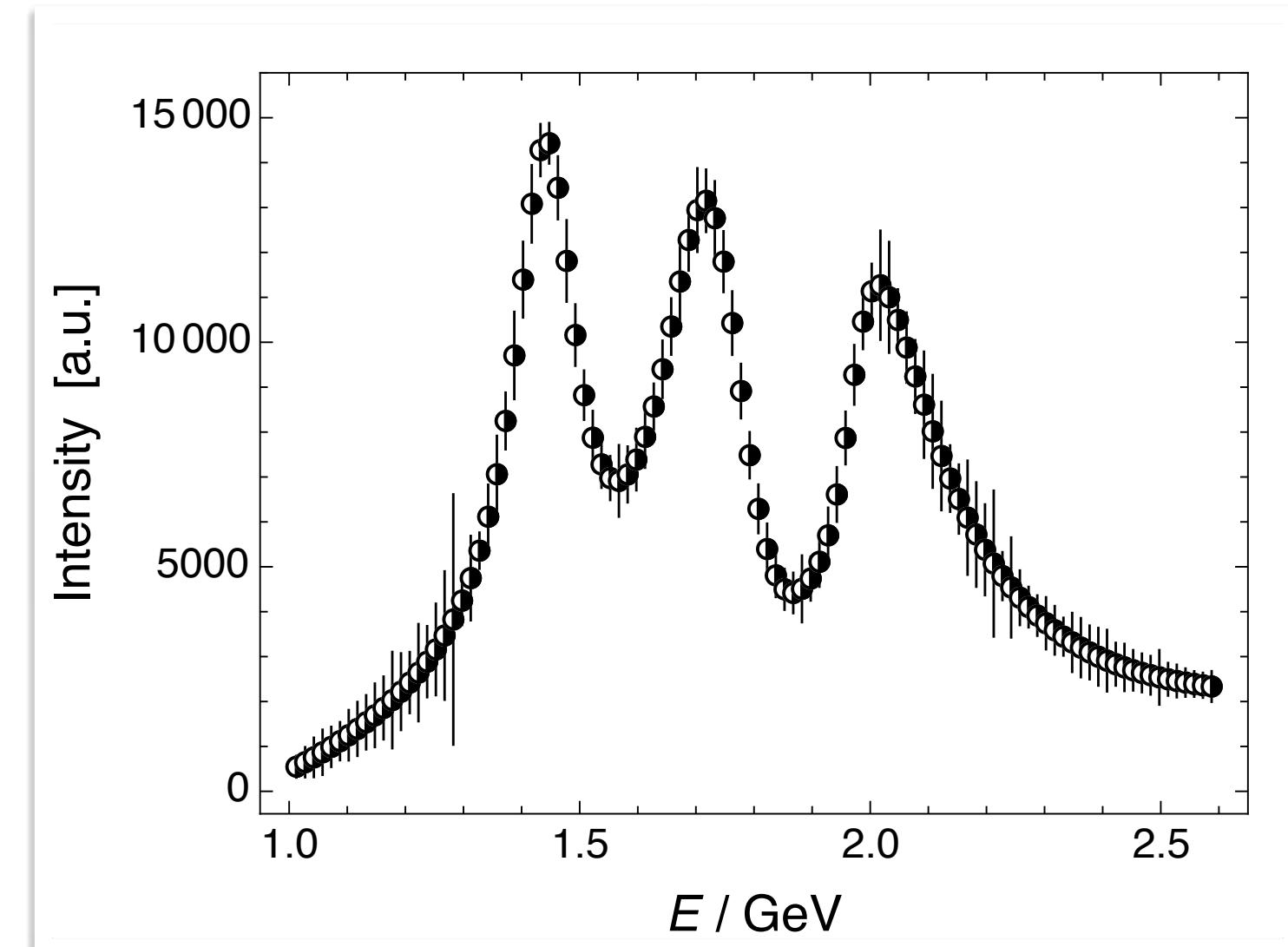


Need to separate **noise poles** from **signal poles**  
 ► Residue filtering

# SPM VALIDATION

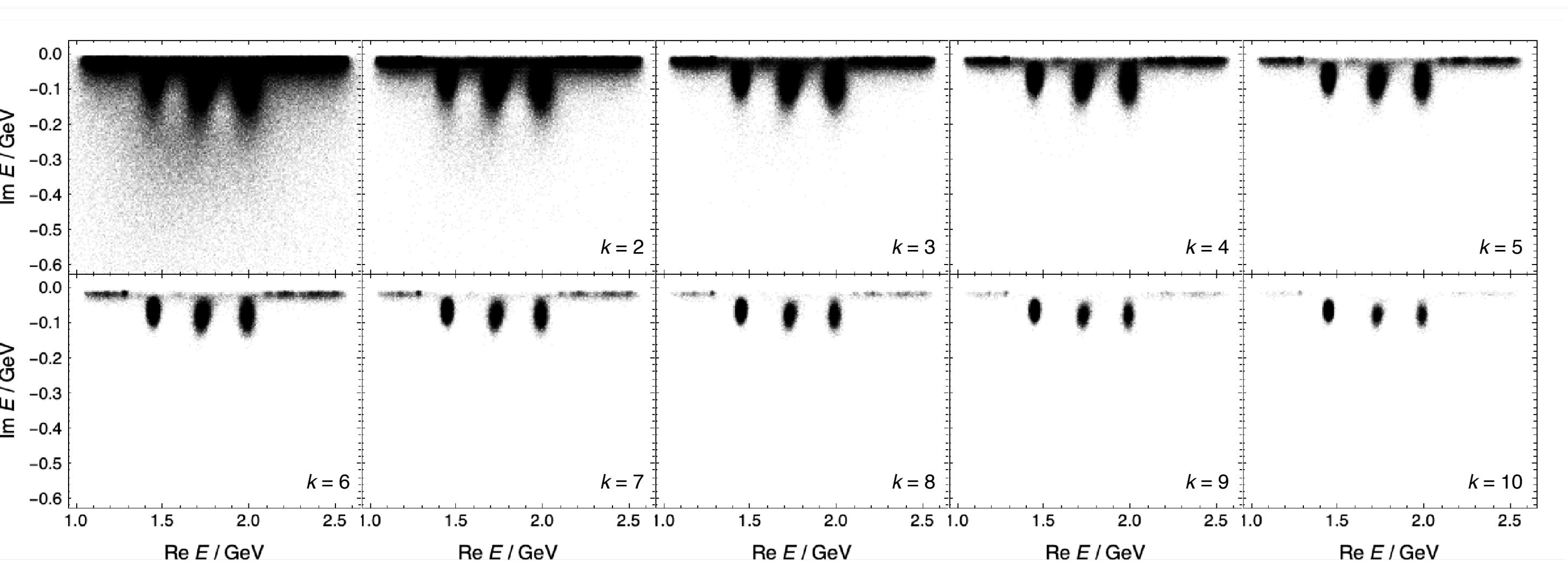
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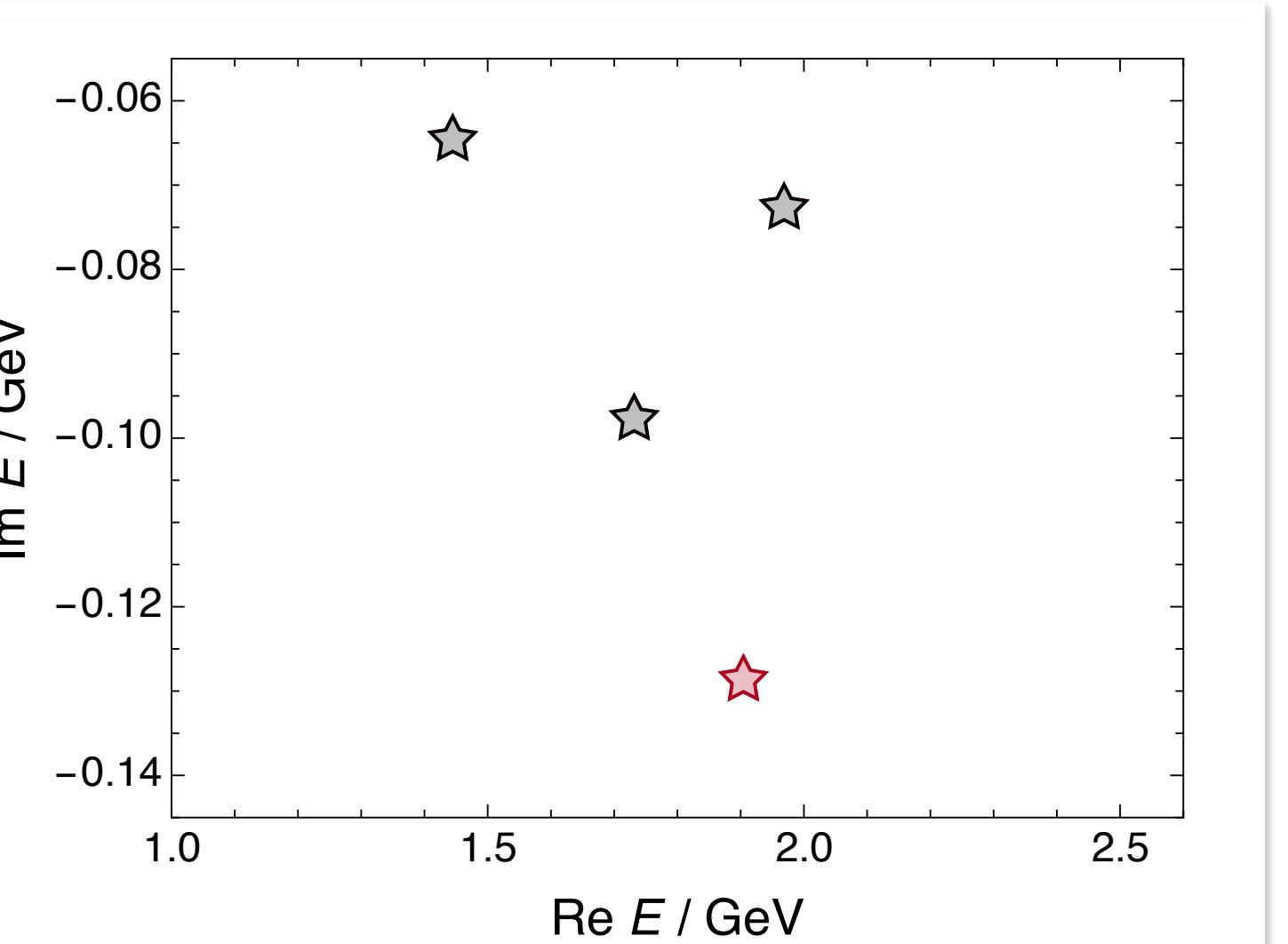
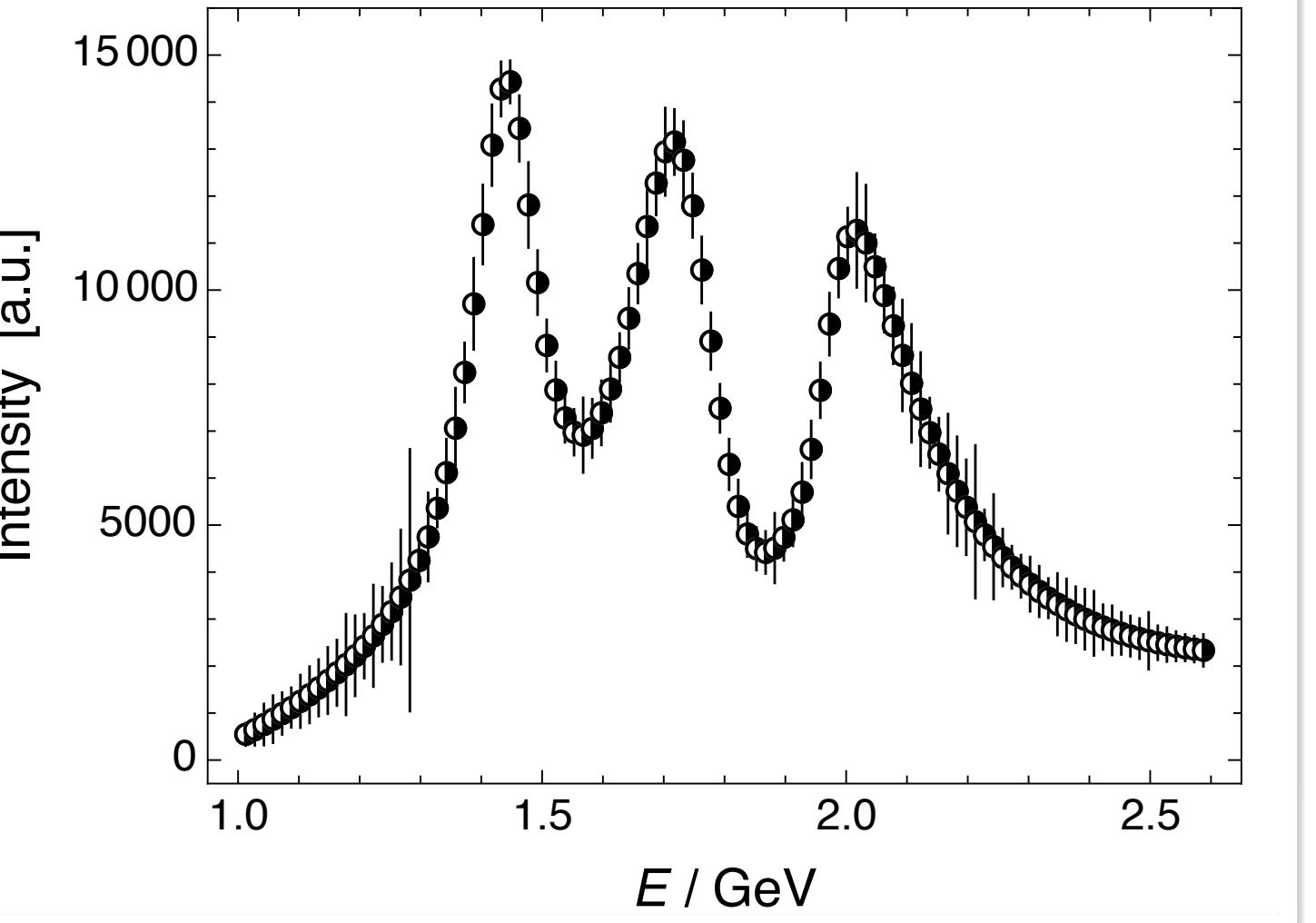


Need to separate **noise poles** from **signal poles**

- ▶ Residue filtering X
- ▶ Image differencing



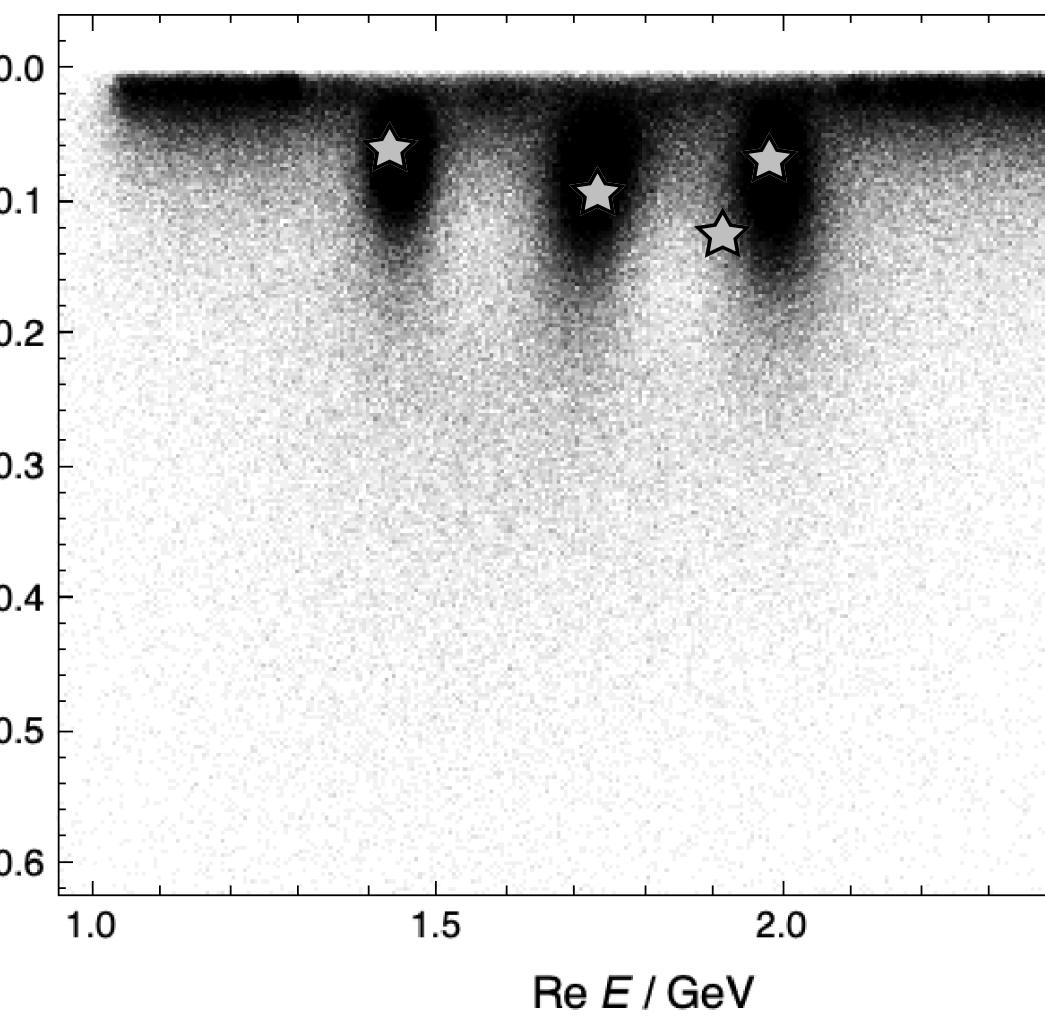
# SPM VALIDATION



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**2**

Adding **uncertainties**

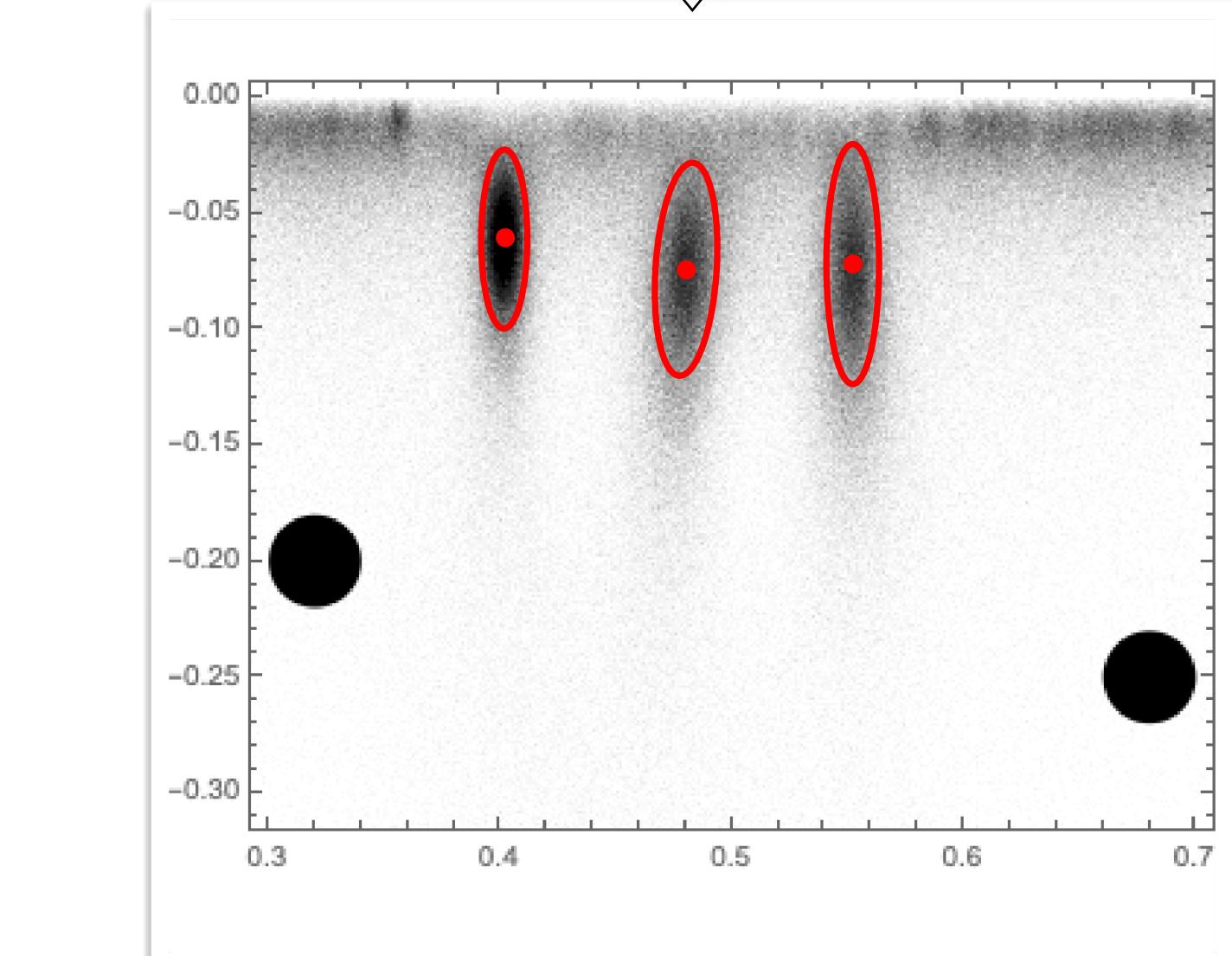
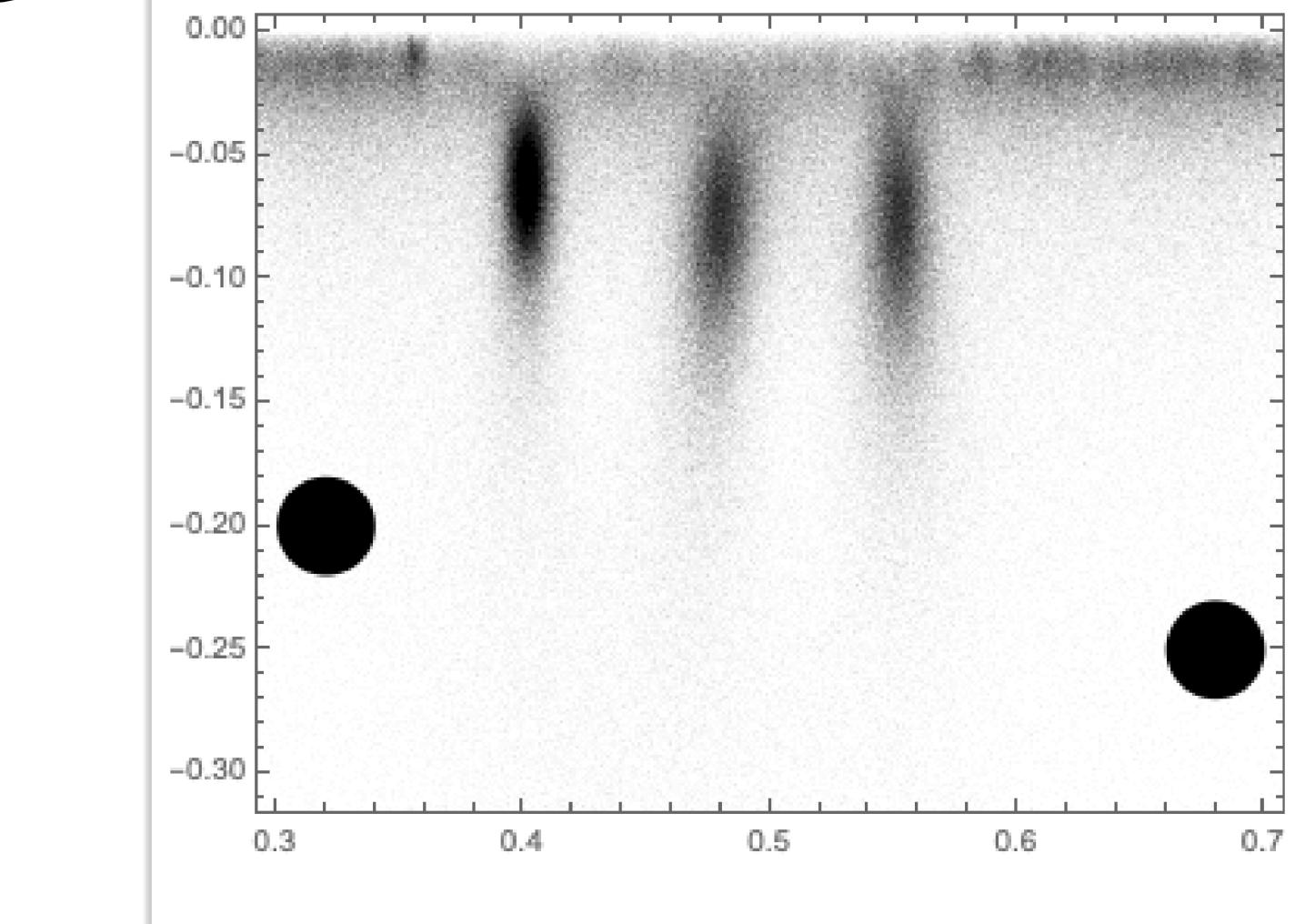


Need to separate **noise poles** from **signal poles**

- ▶ Residue filtering ✗
- ▶ Image differencing ✗
- ▶ Image filtering + pattern recognition ✓
- ✓ Apply filtering to enhance signal/noise
- ✓ Apply pattern recognition
- ✓ Signal is ‘vertical’; noise (mostly) ‘horizontal’
- ✓ Determine points inside the reconstructed regions

**3**

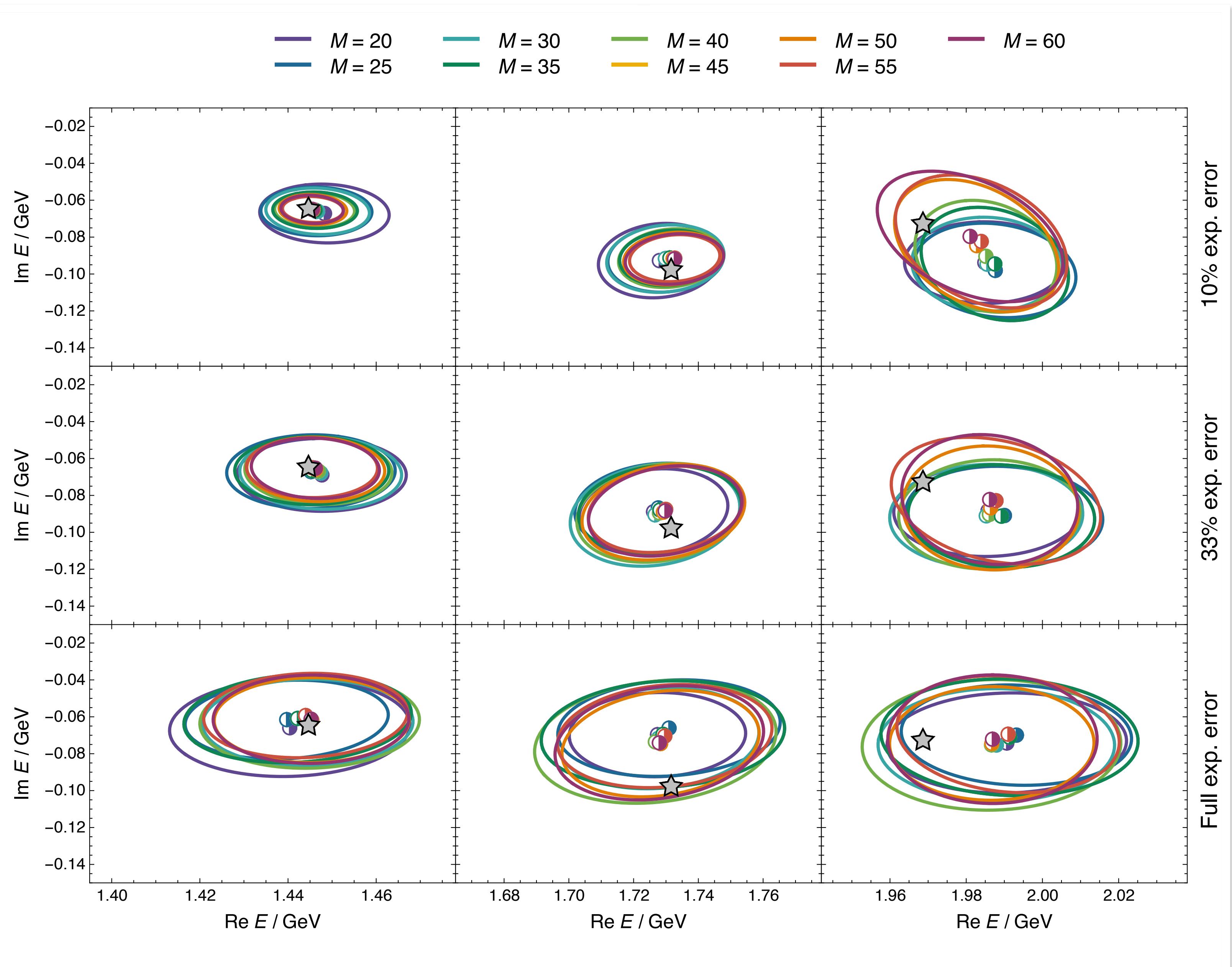
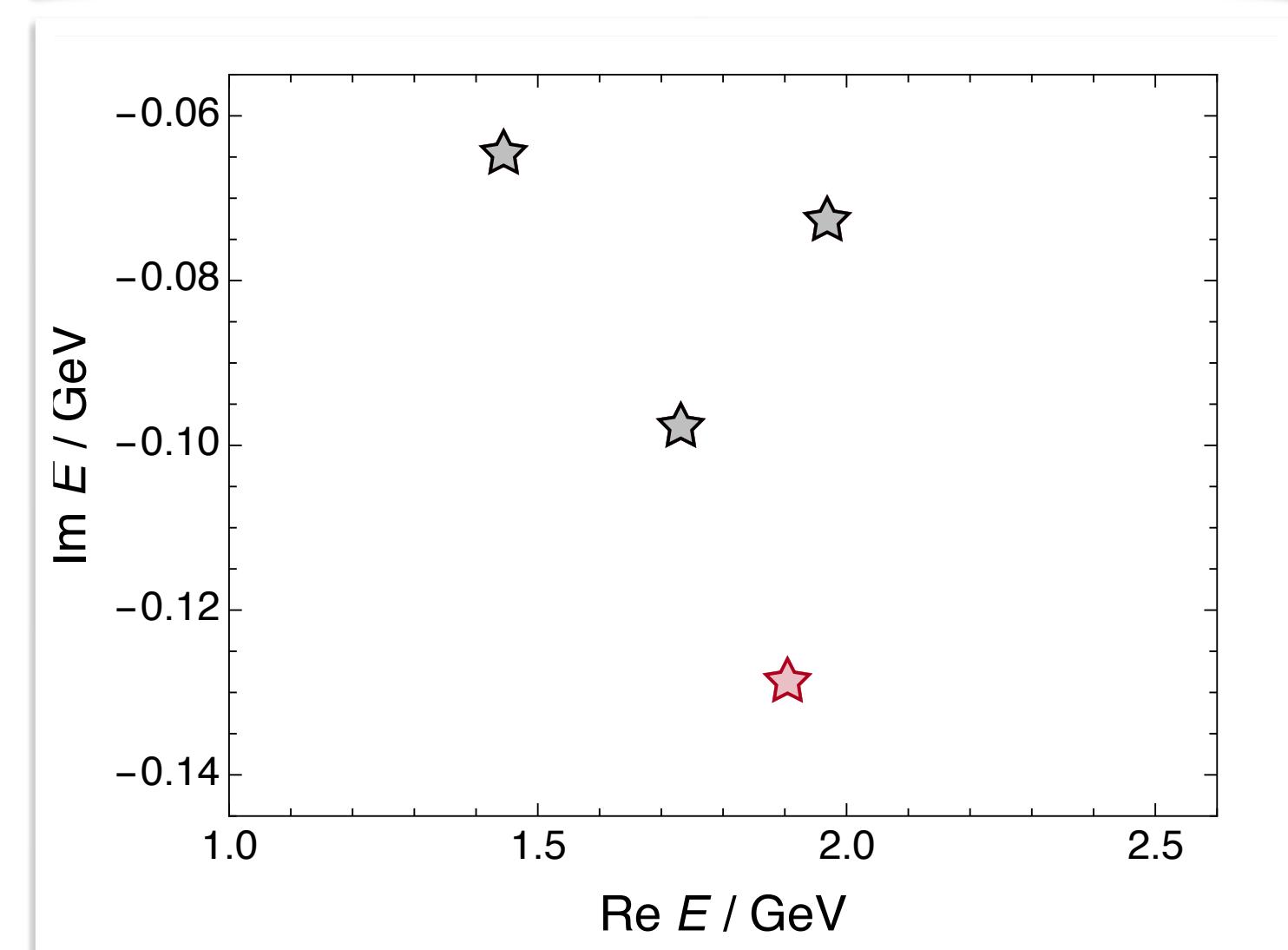
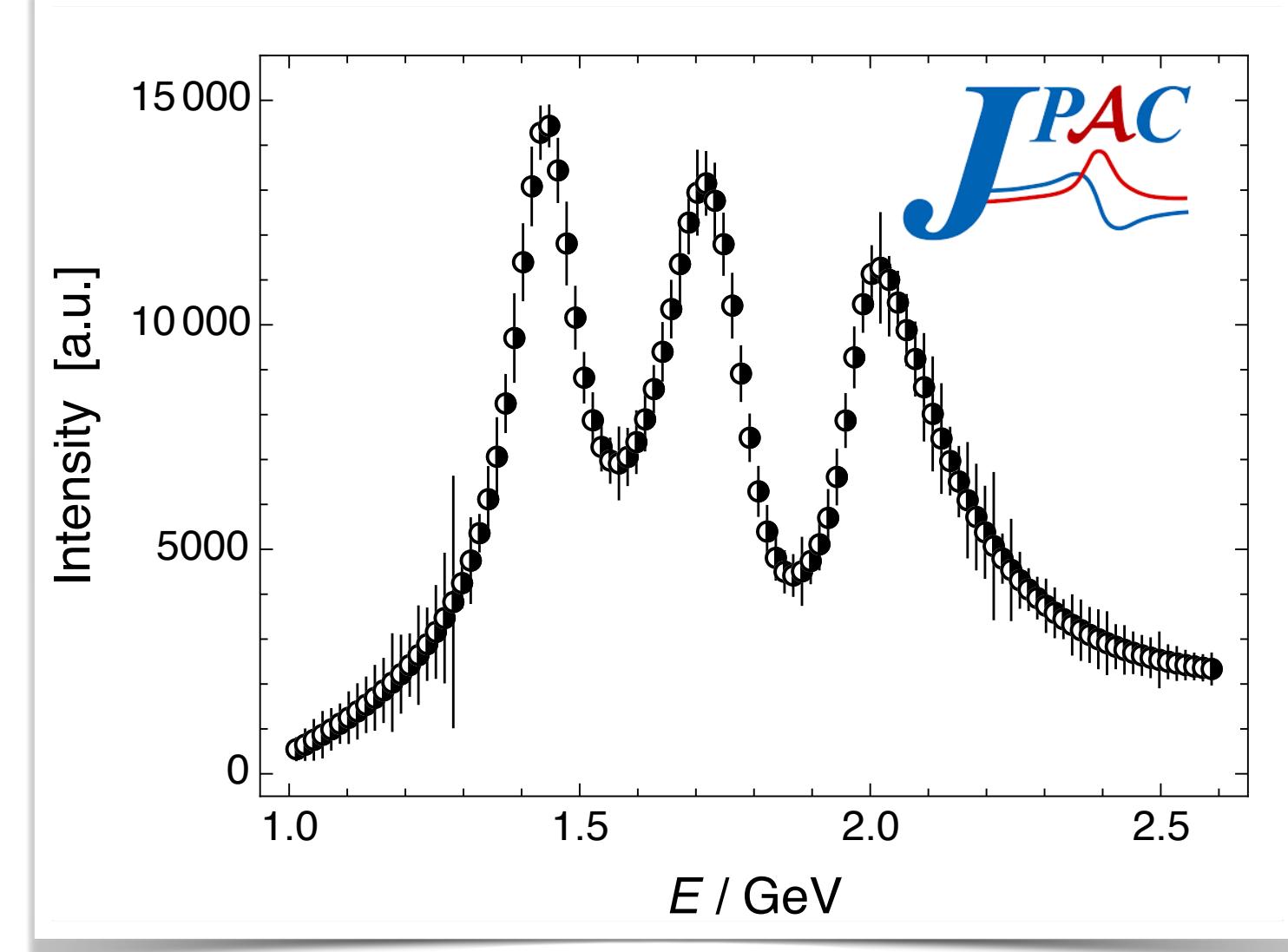
Image filtering + pattern recognition



# SPM VALIDATION

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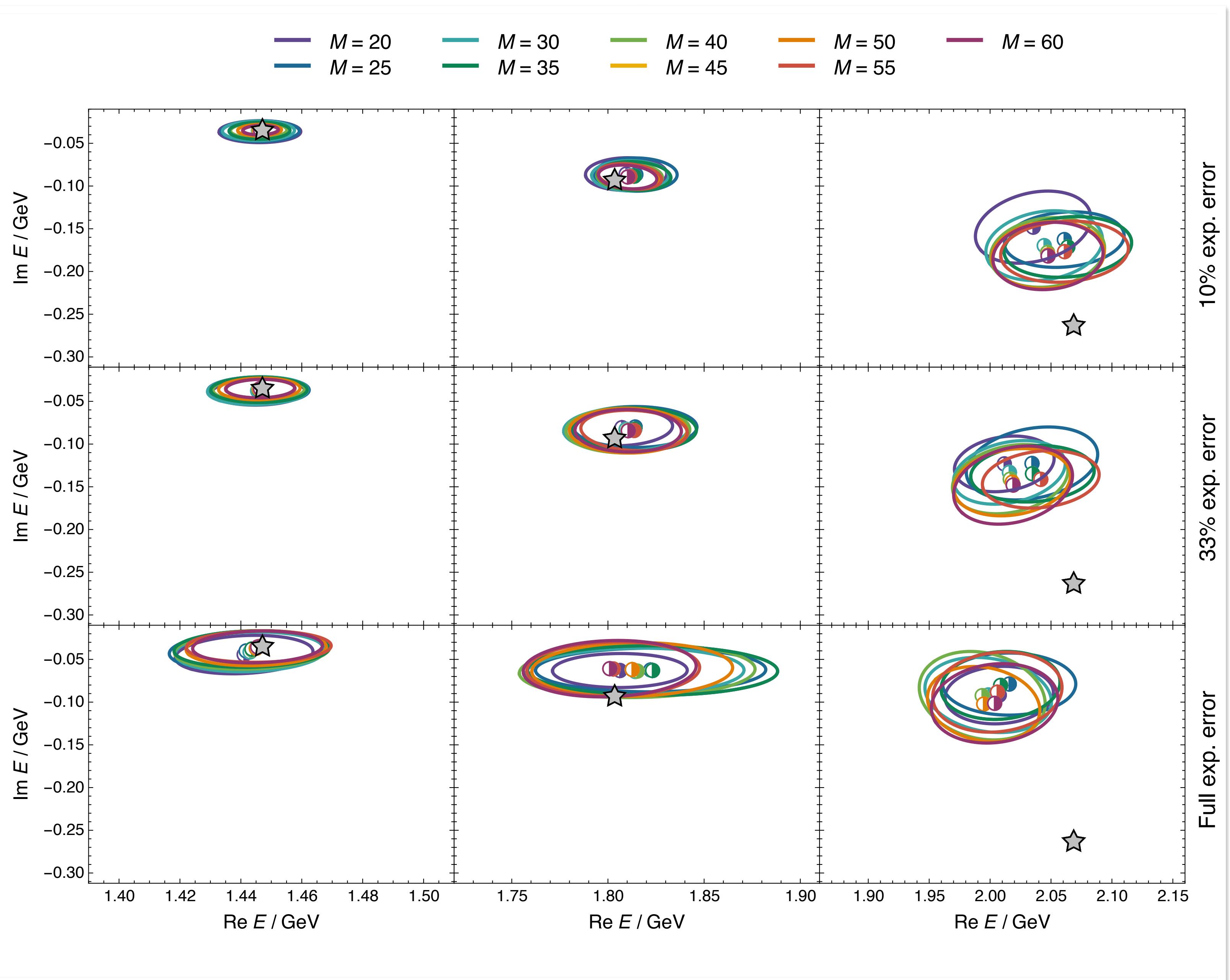
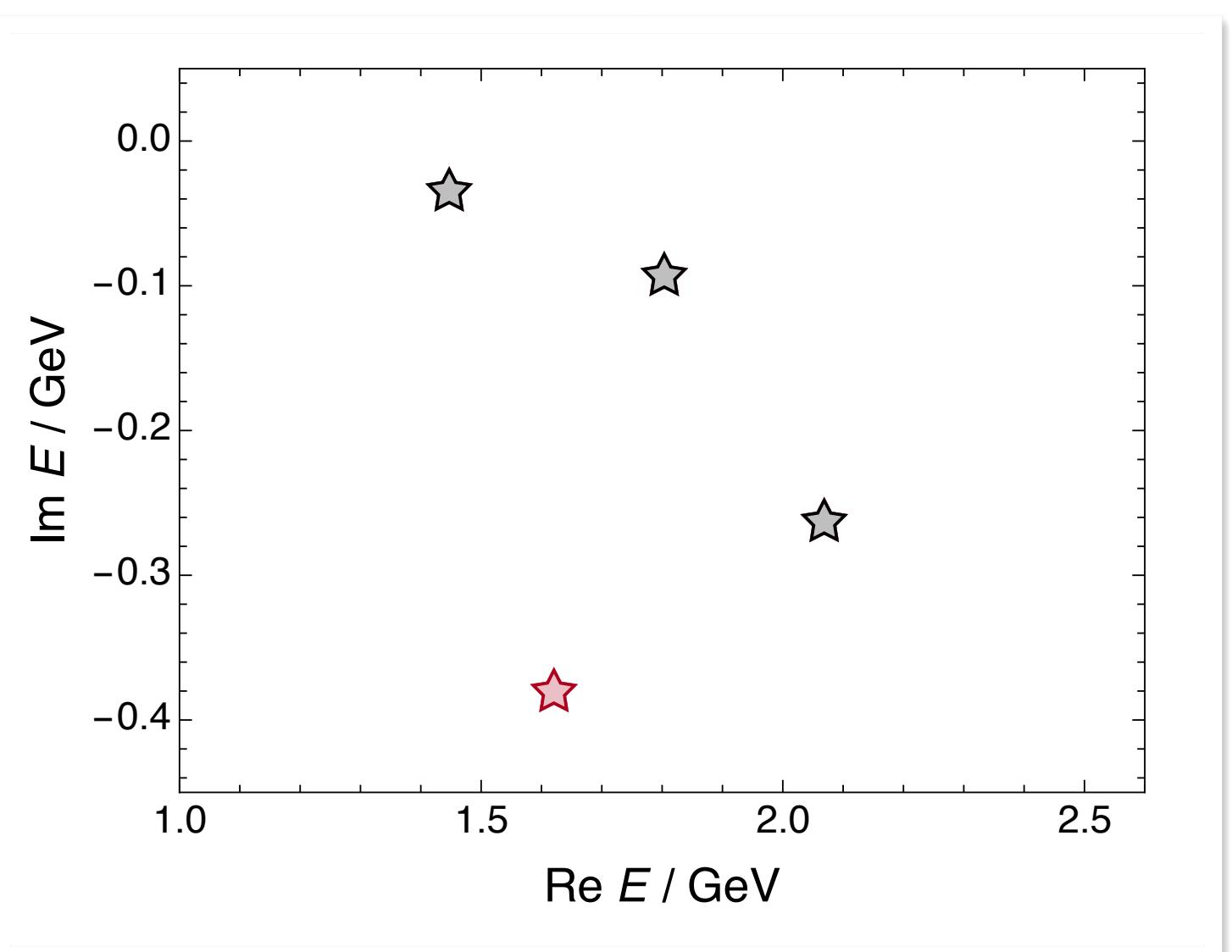
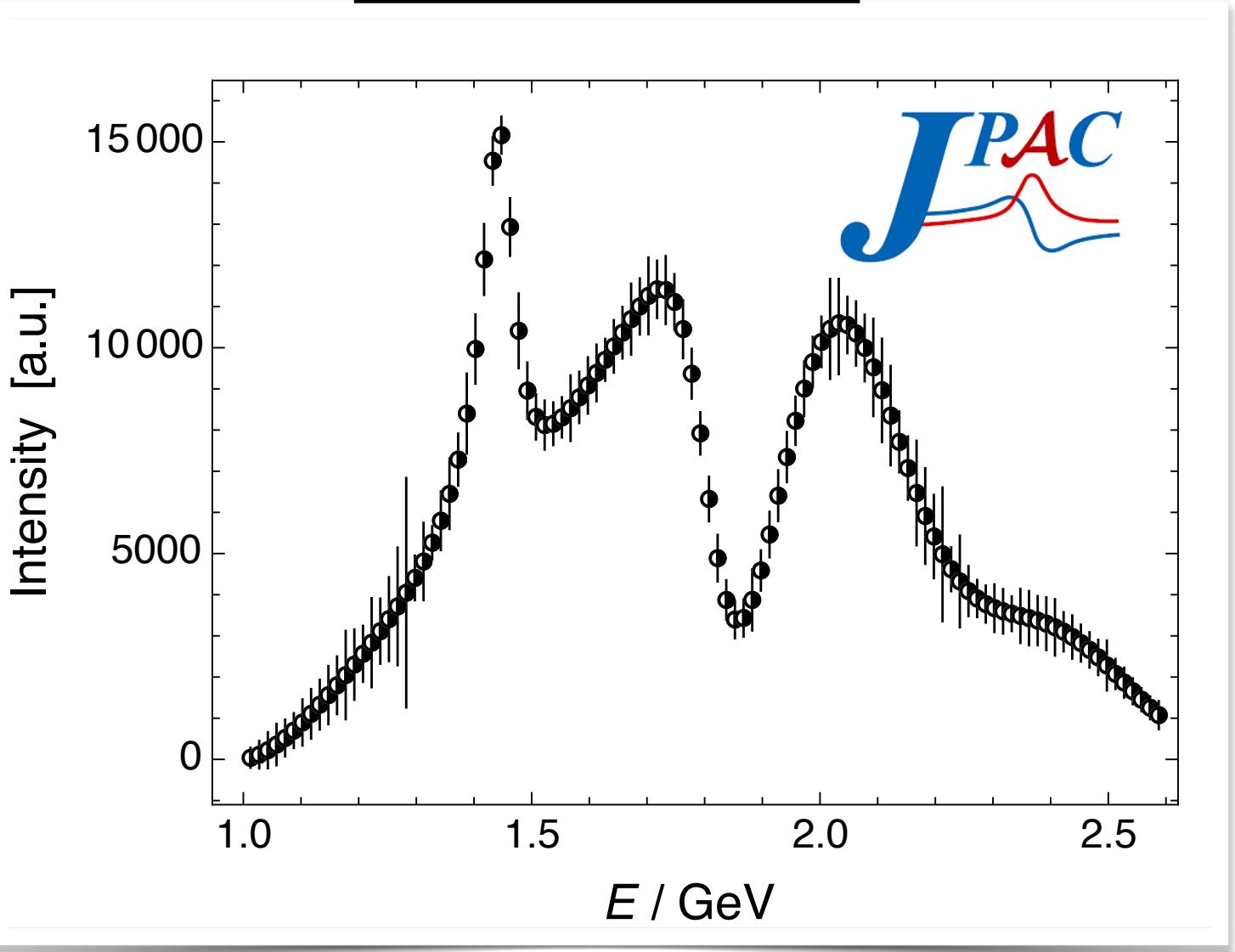
A. Rodas et al., JPAC, arXiv:2110.00027 [hep-ph]



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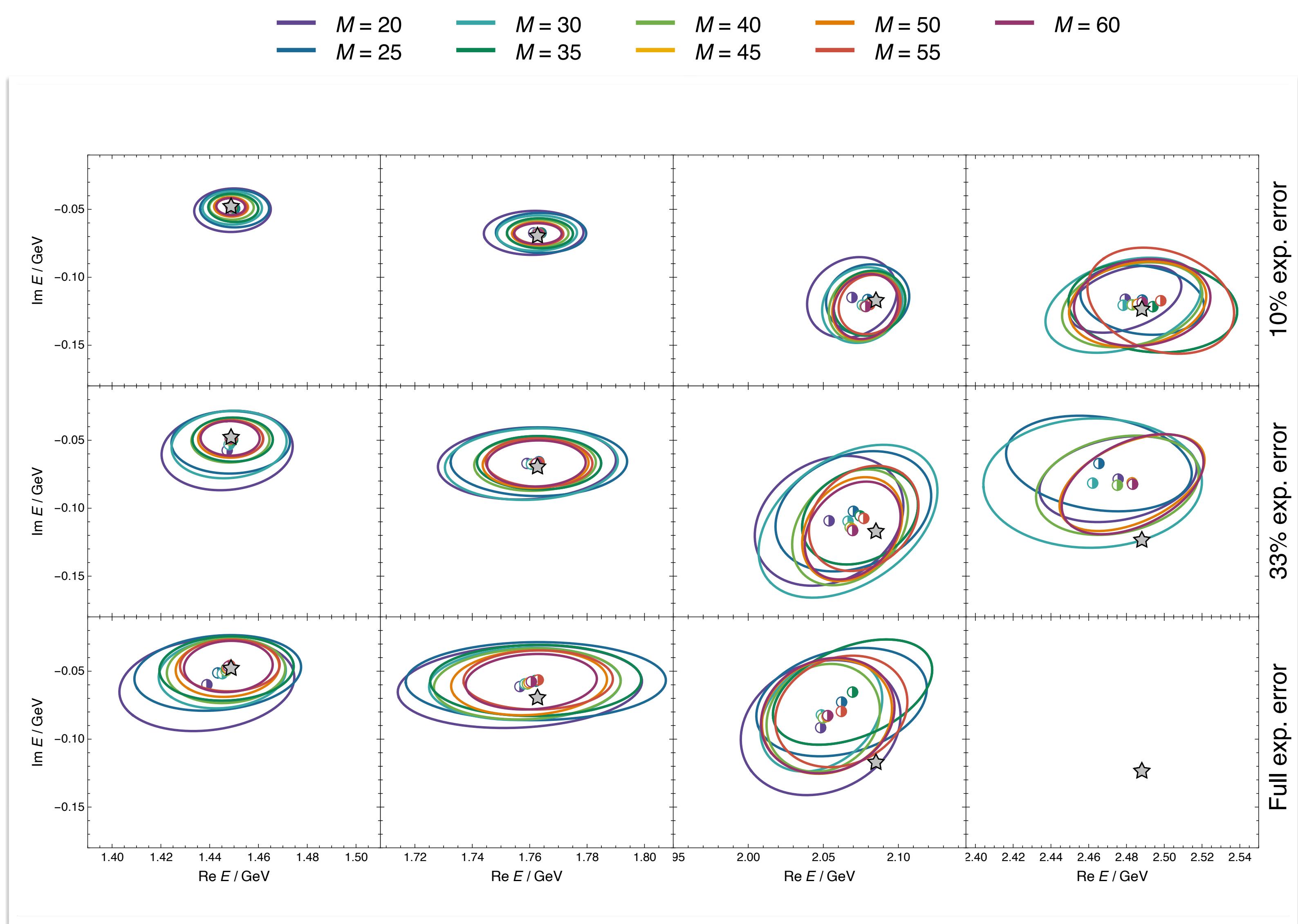
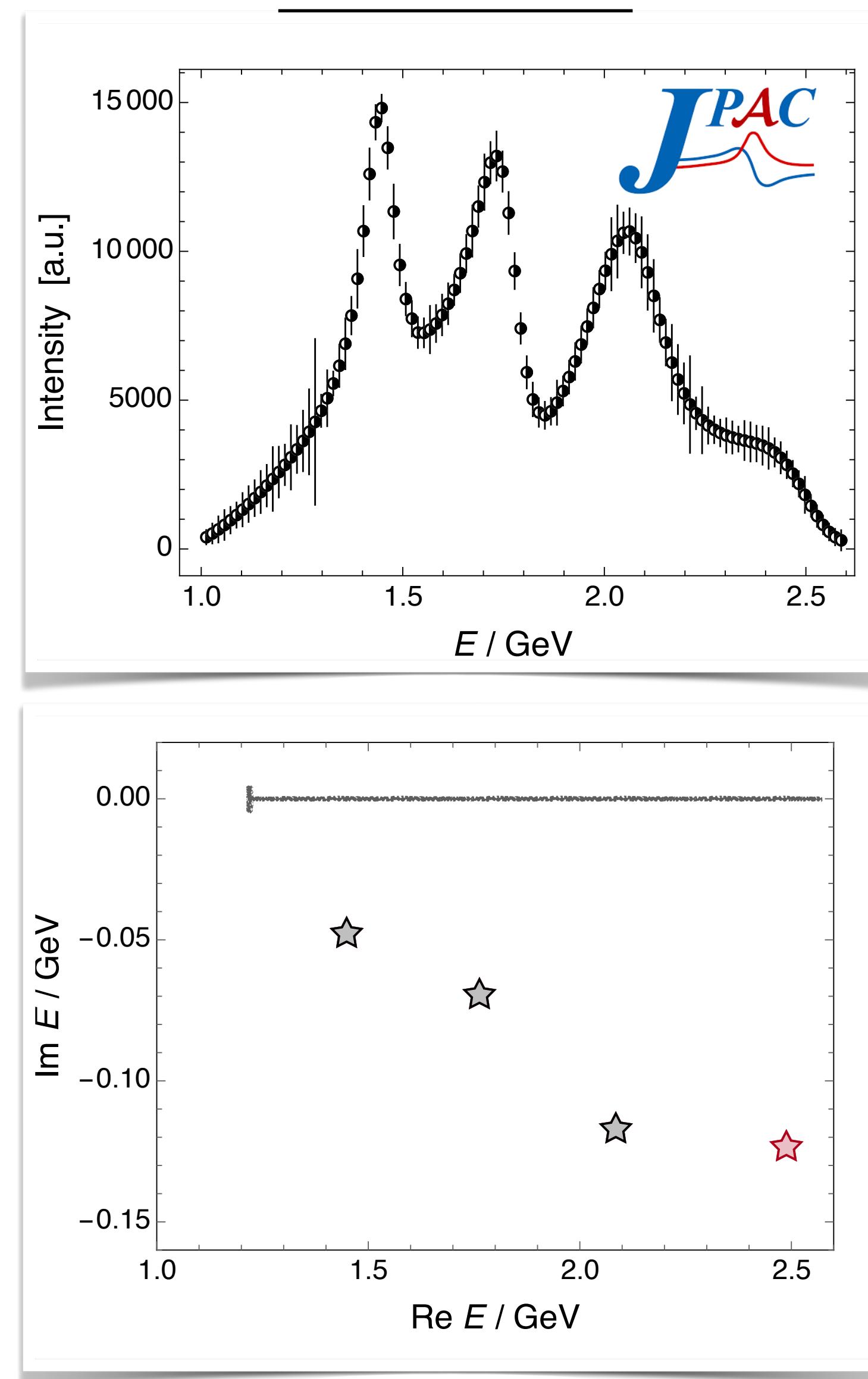
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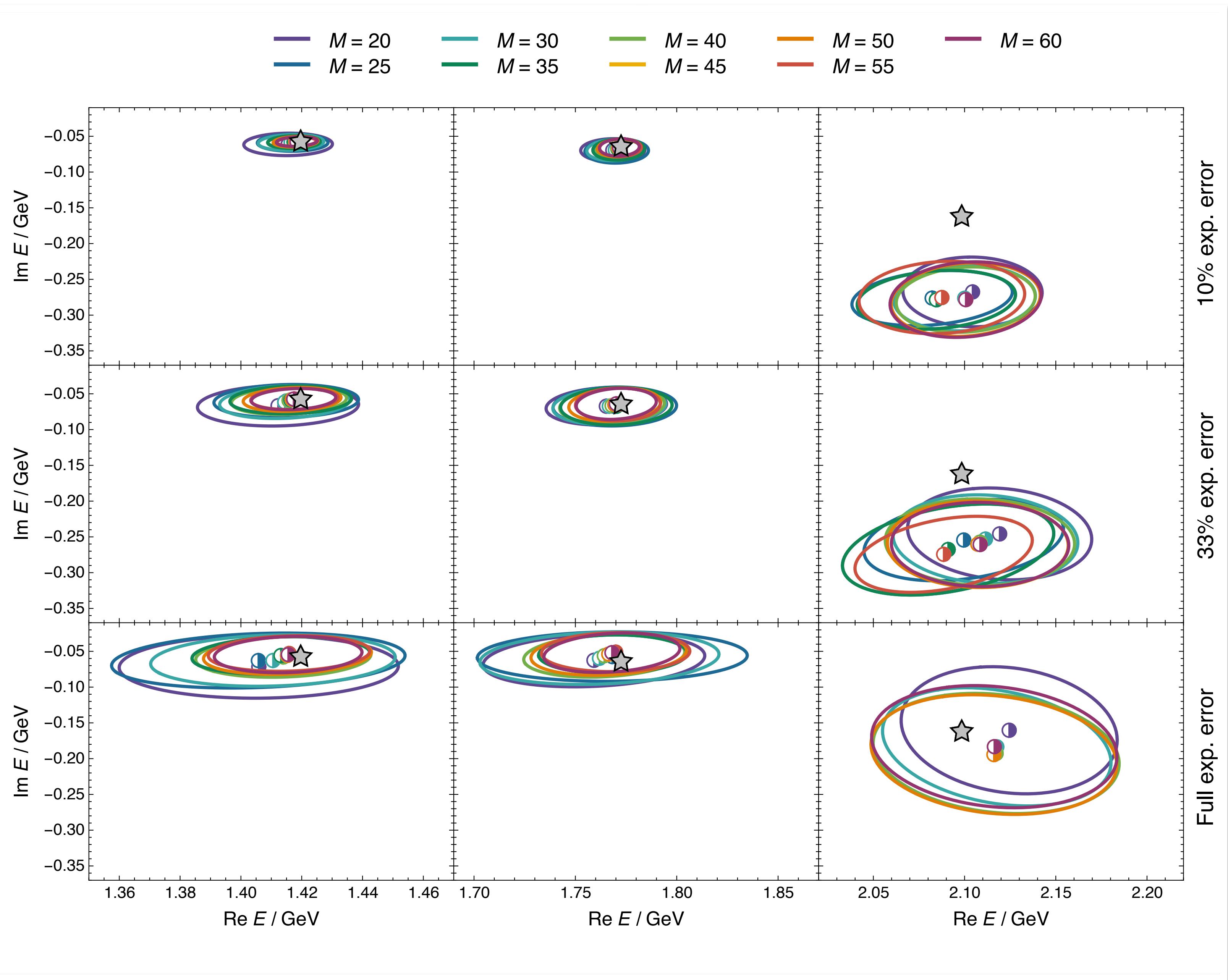
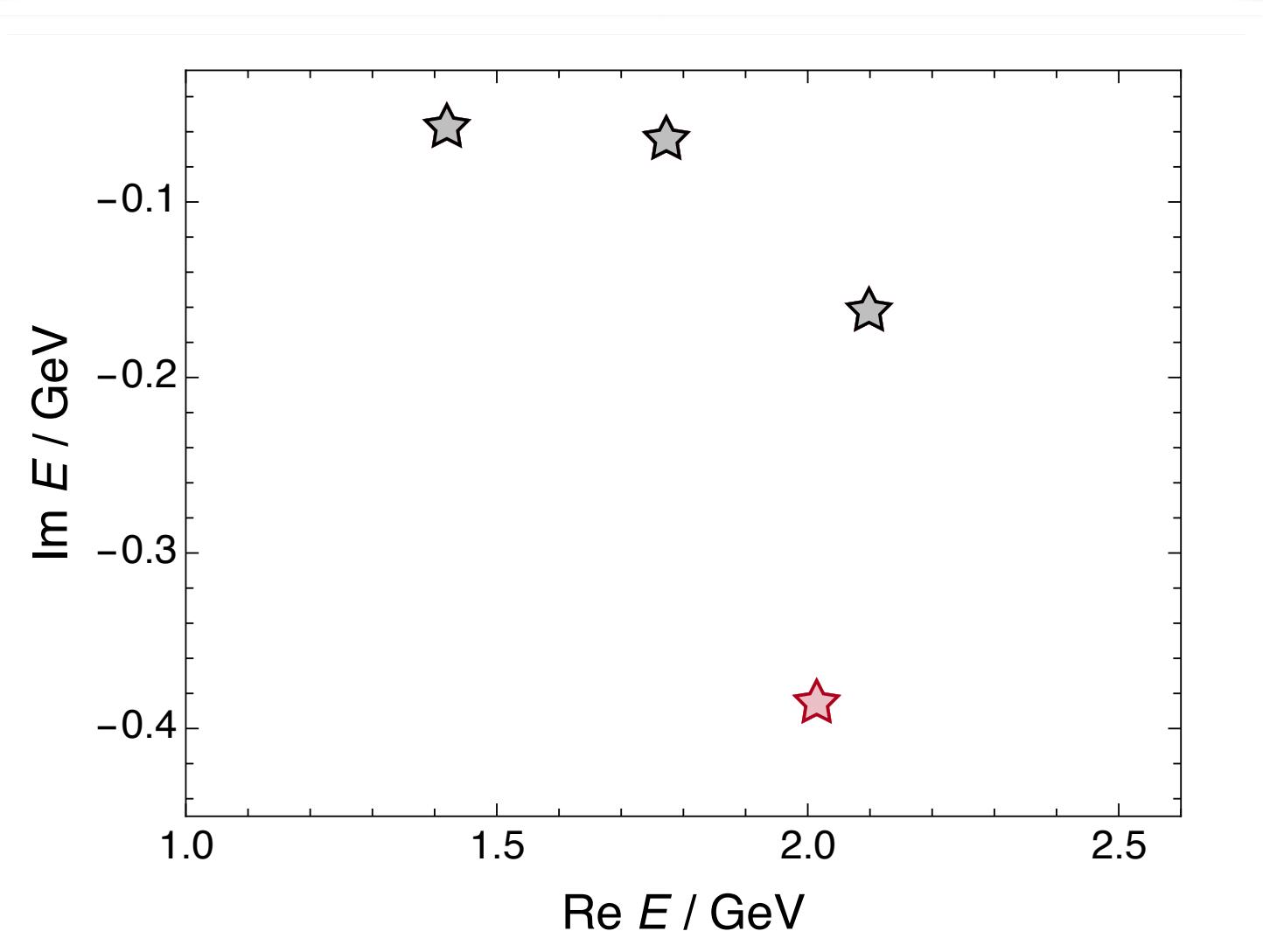
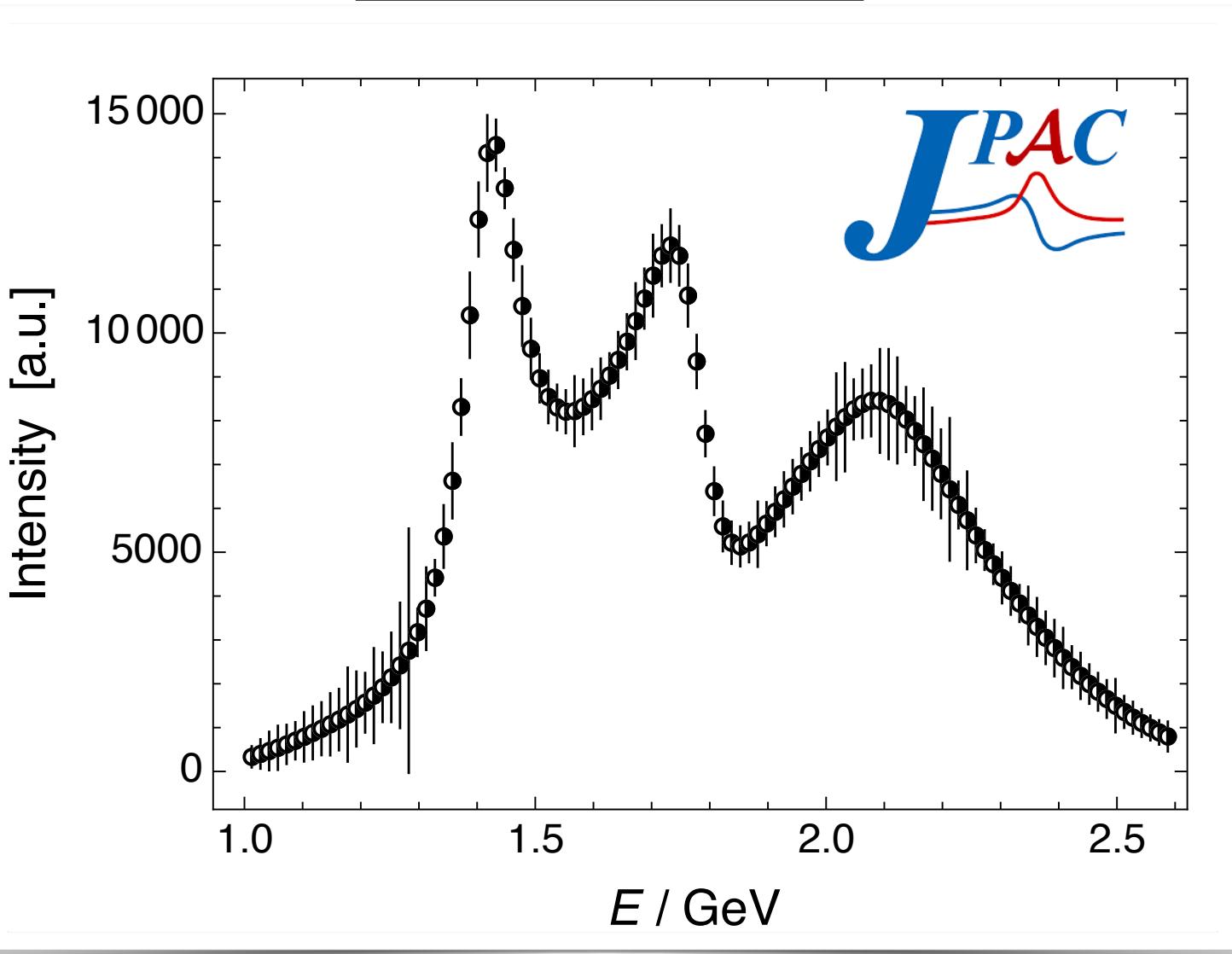
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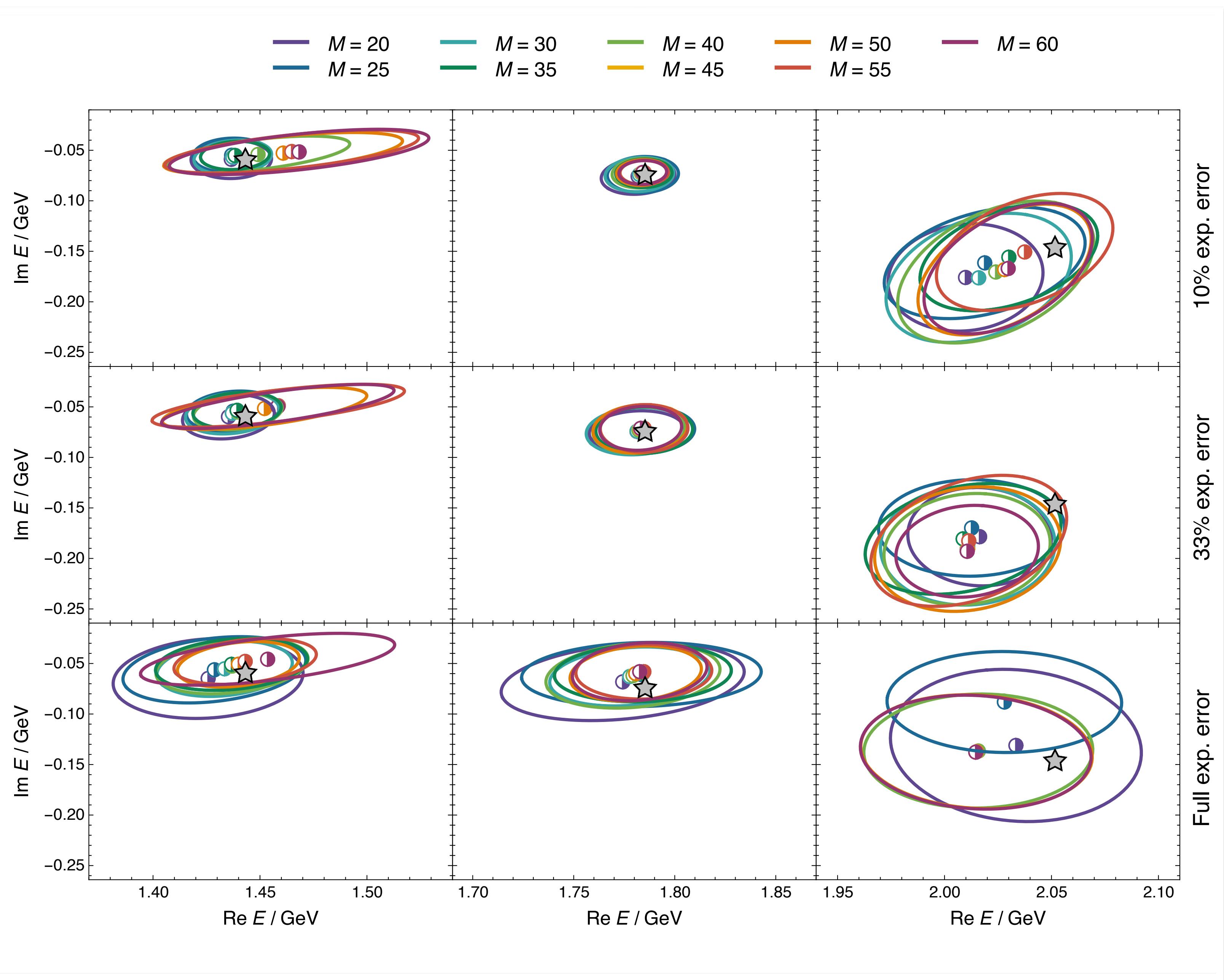
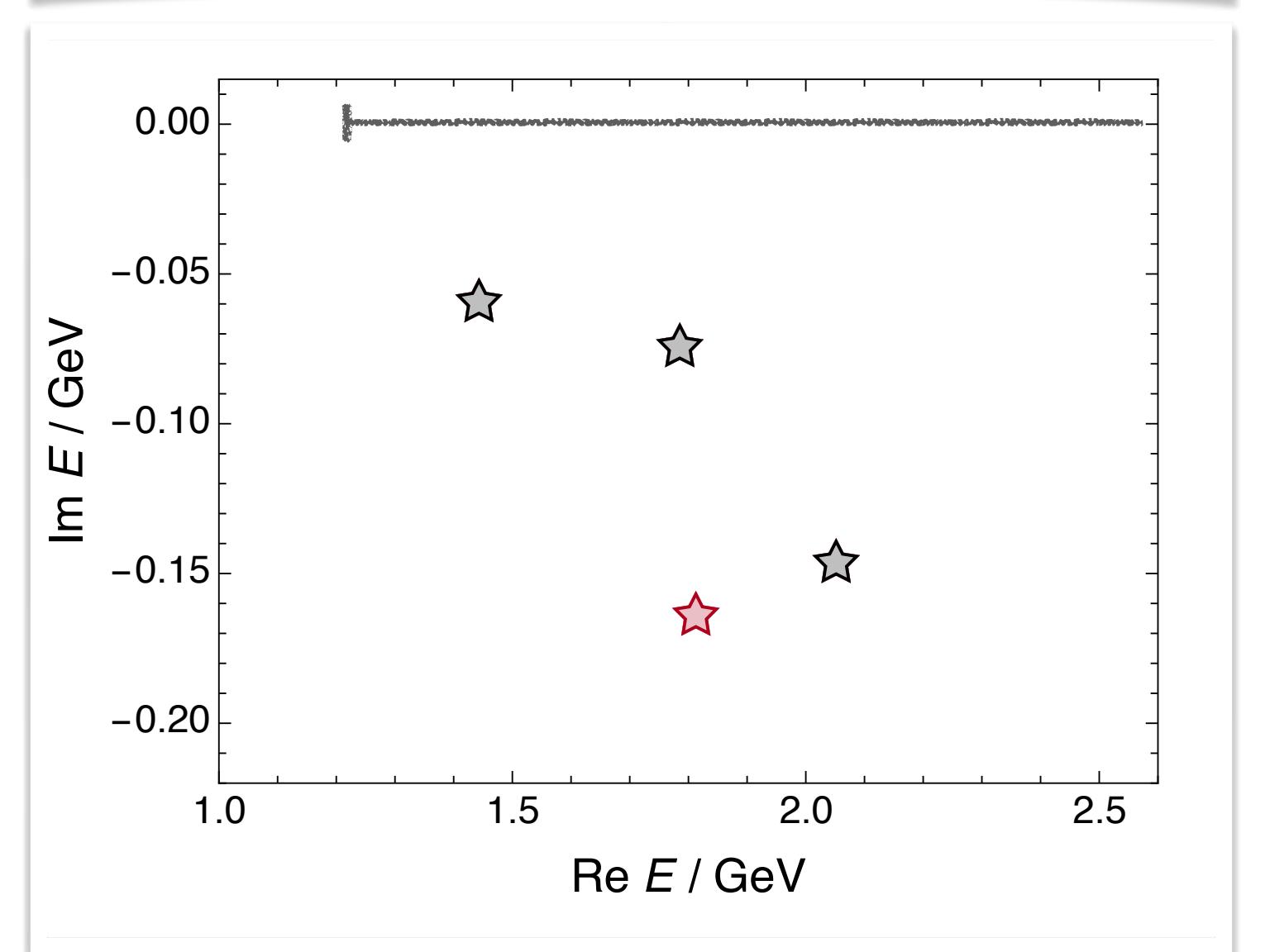
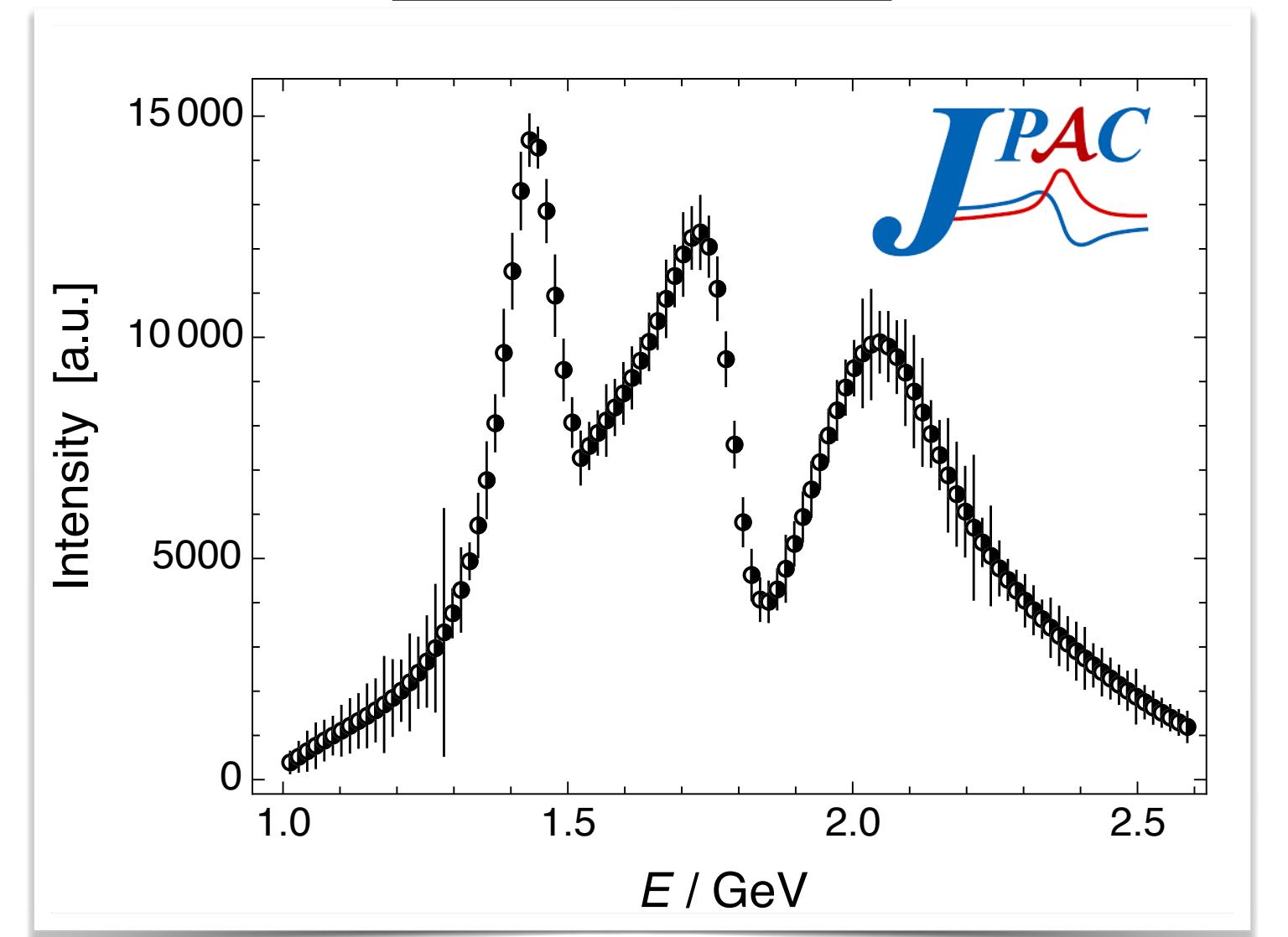
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$J/\psi \rightarrow \gamma\pi^0\pi^0$

**SPM  
RESULTS**

JPAC



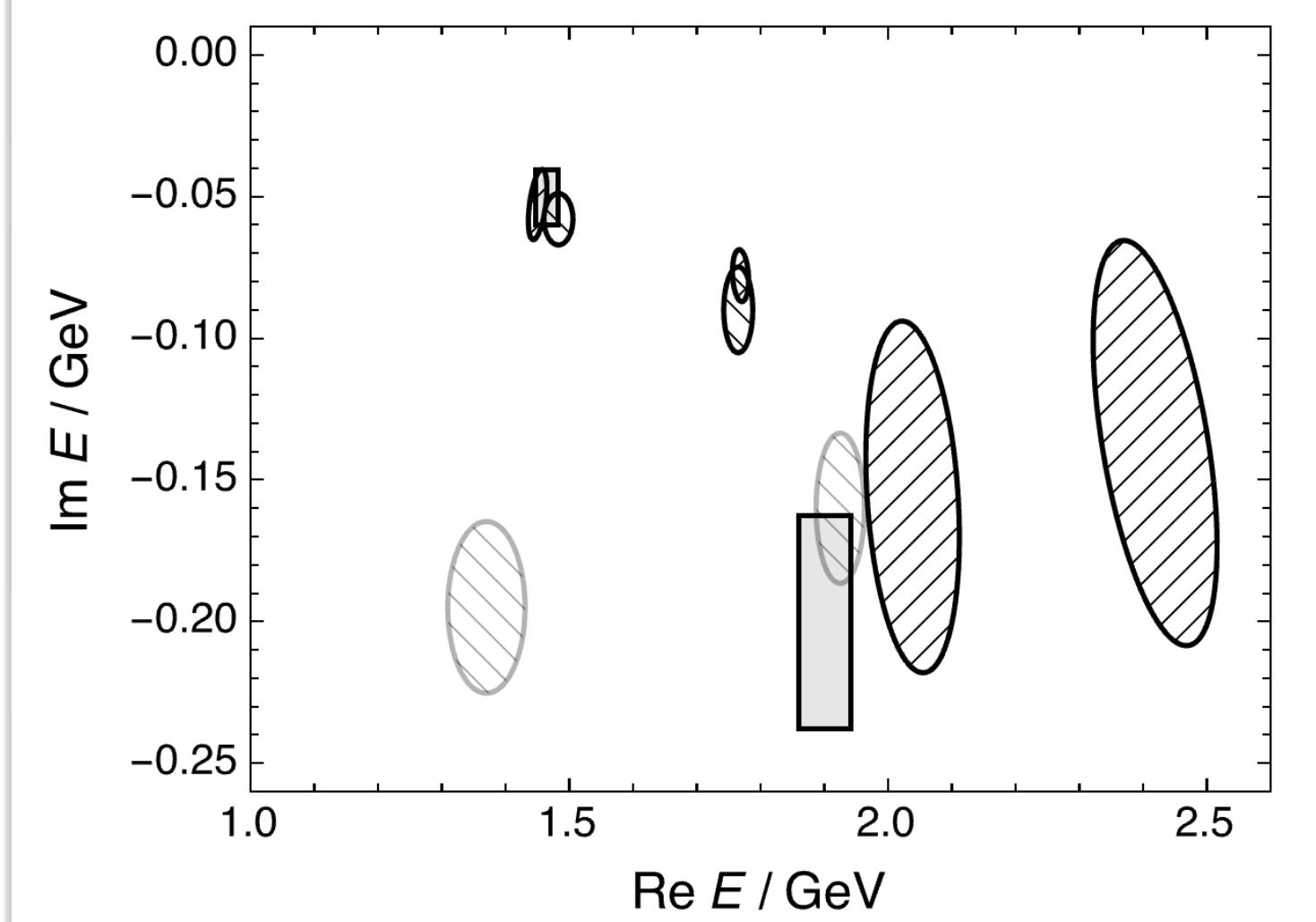
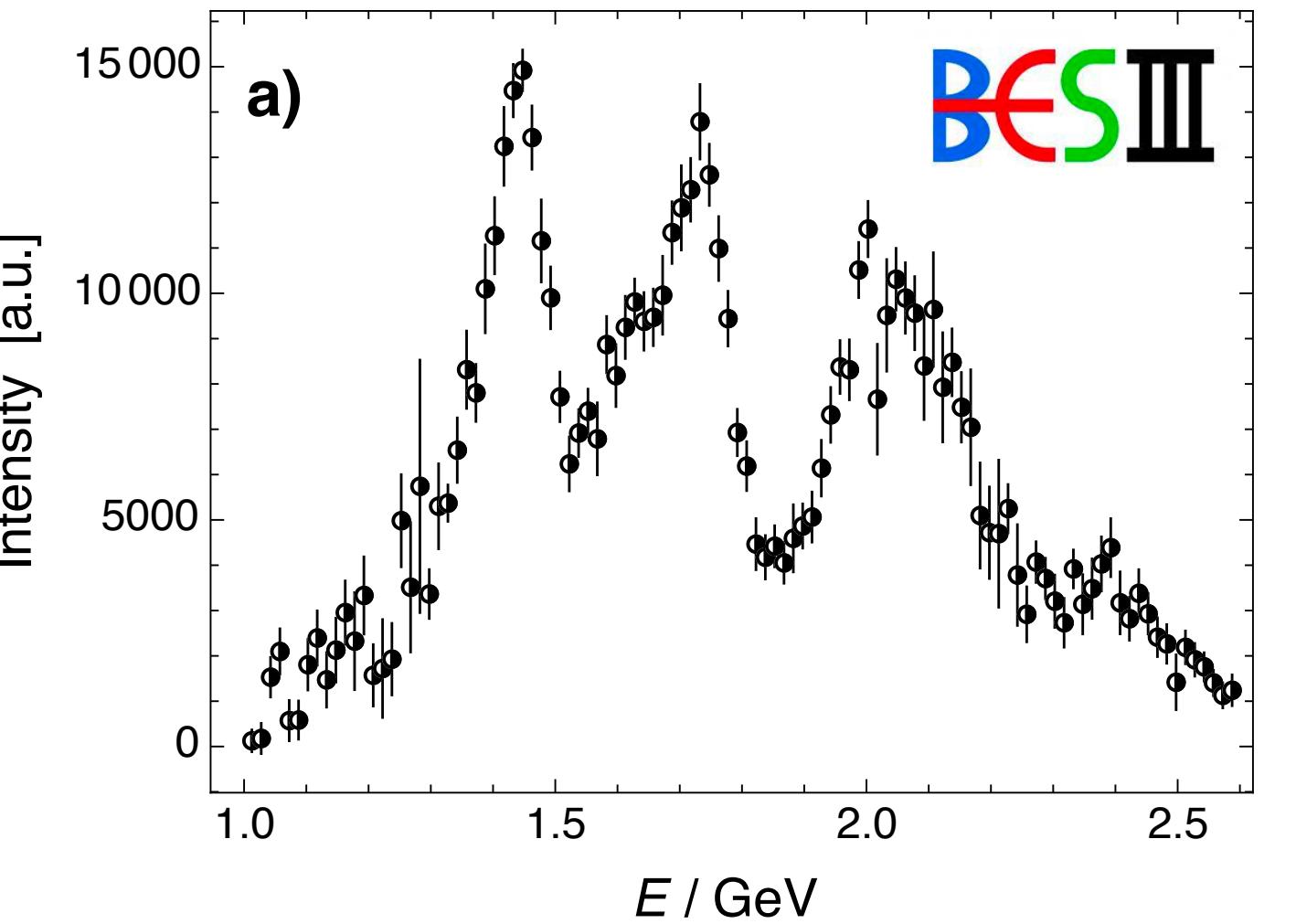
BoGa



Ropertz, et al.



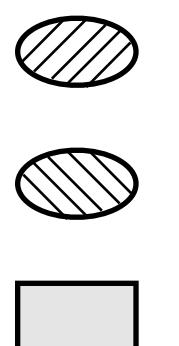
**BESIII data**



$J/\psi \rightarrow \gamma\pi^0\pi^0$

**SPM  
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JPAC



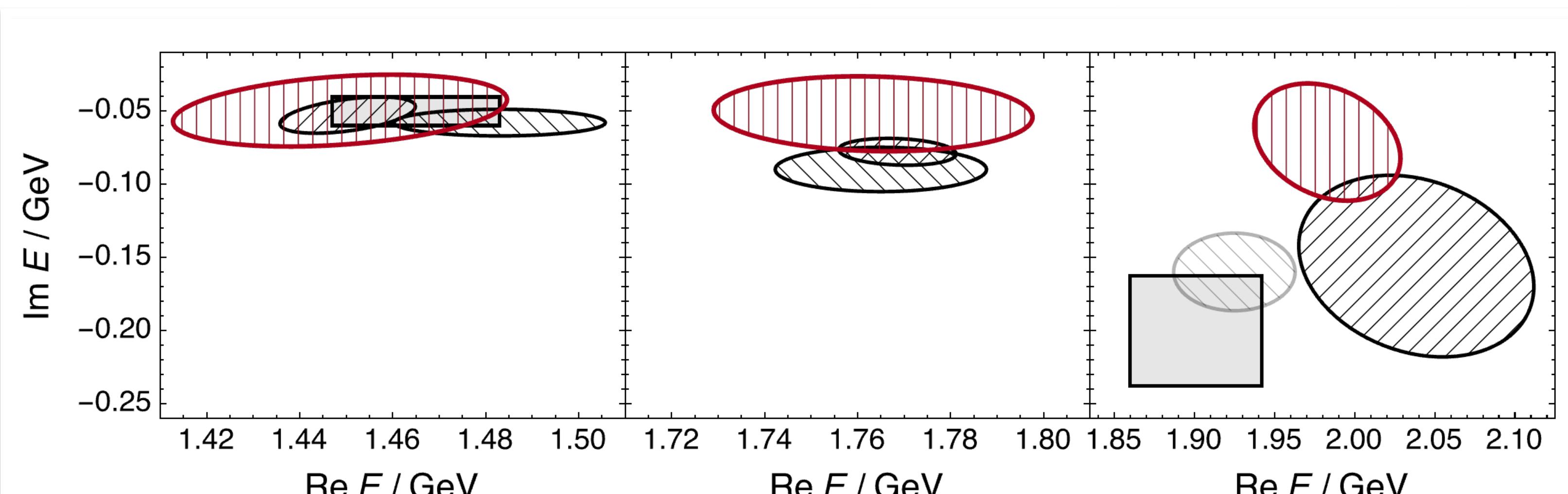
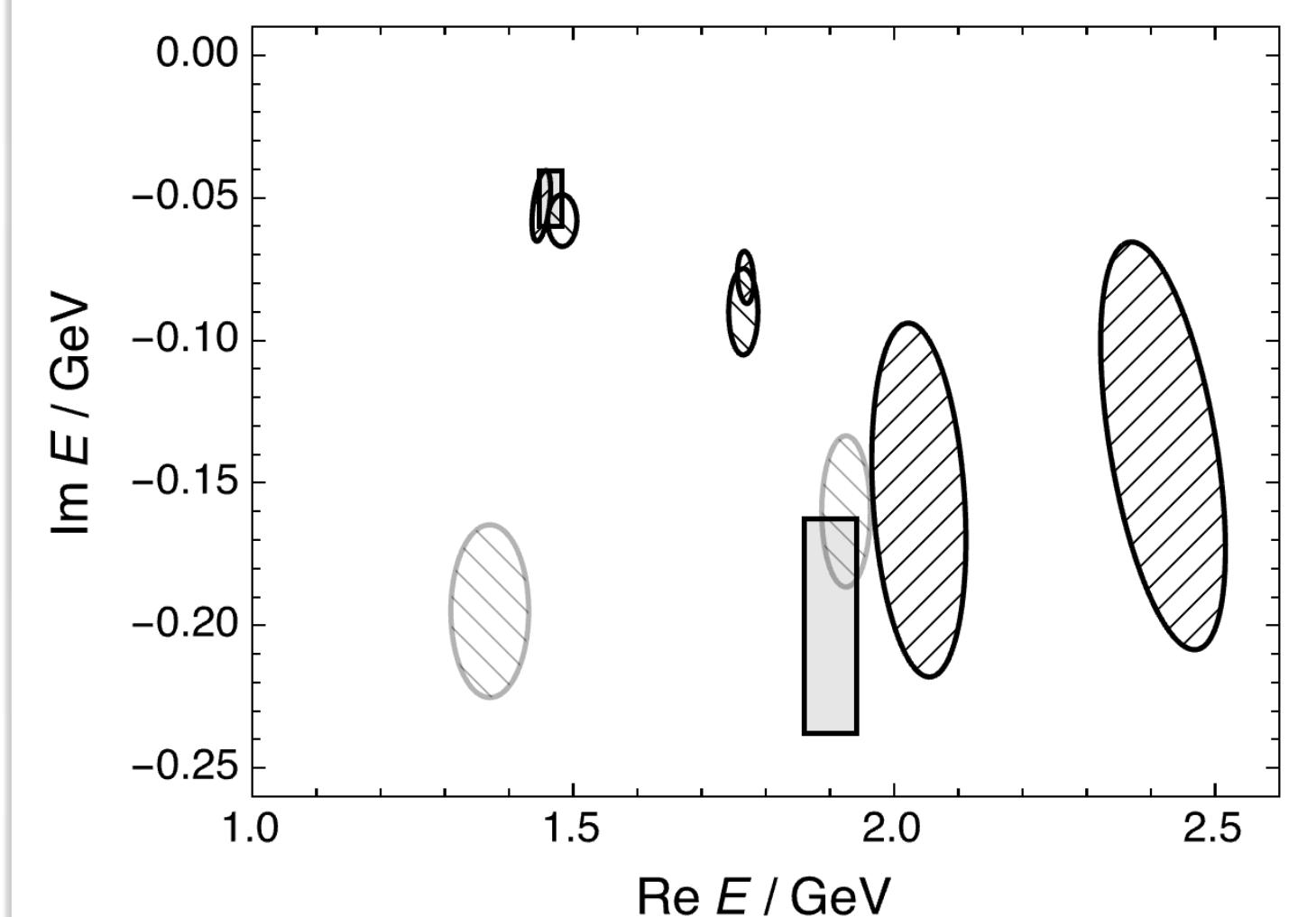
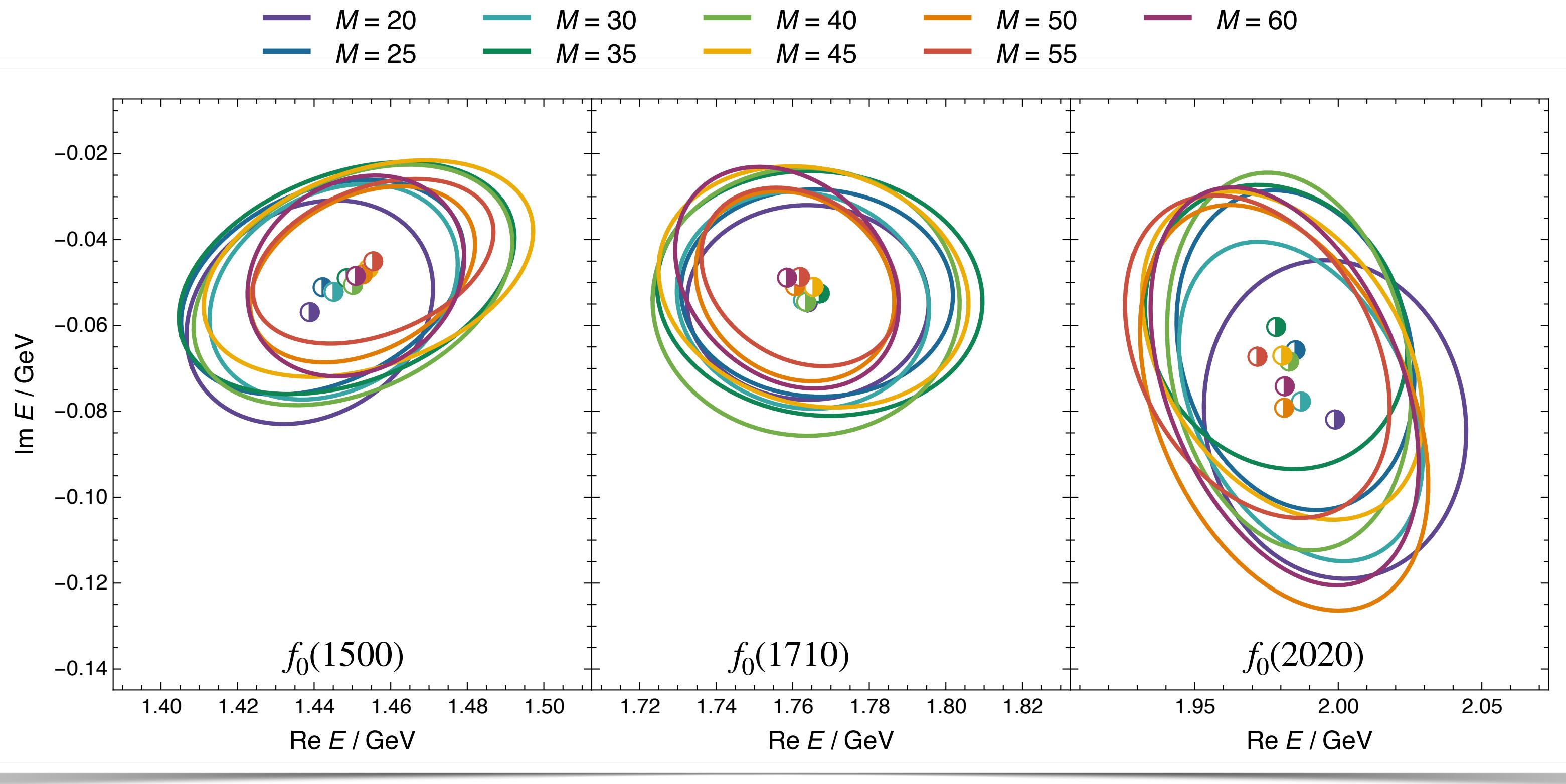
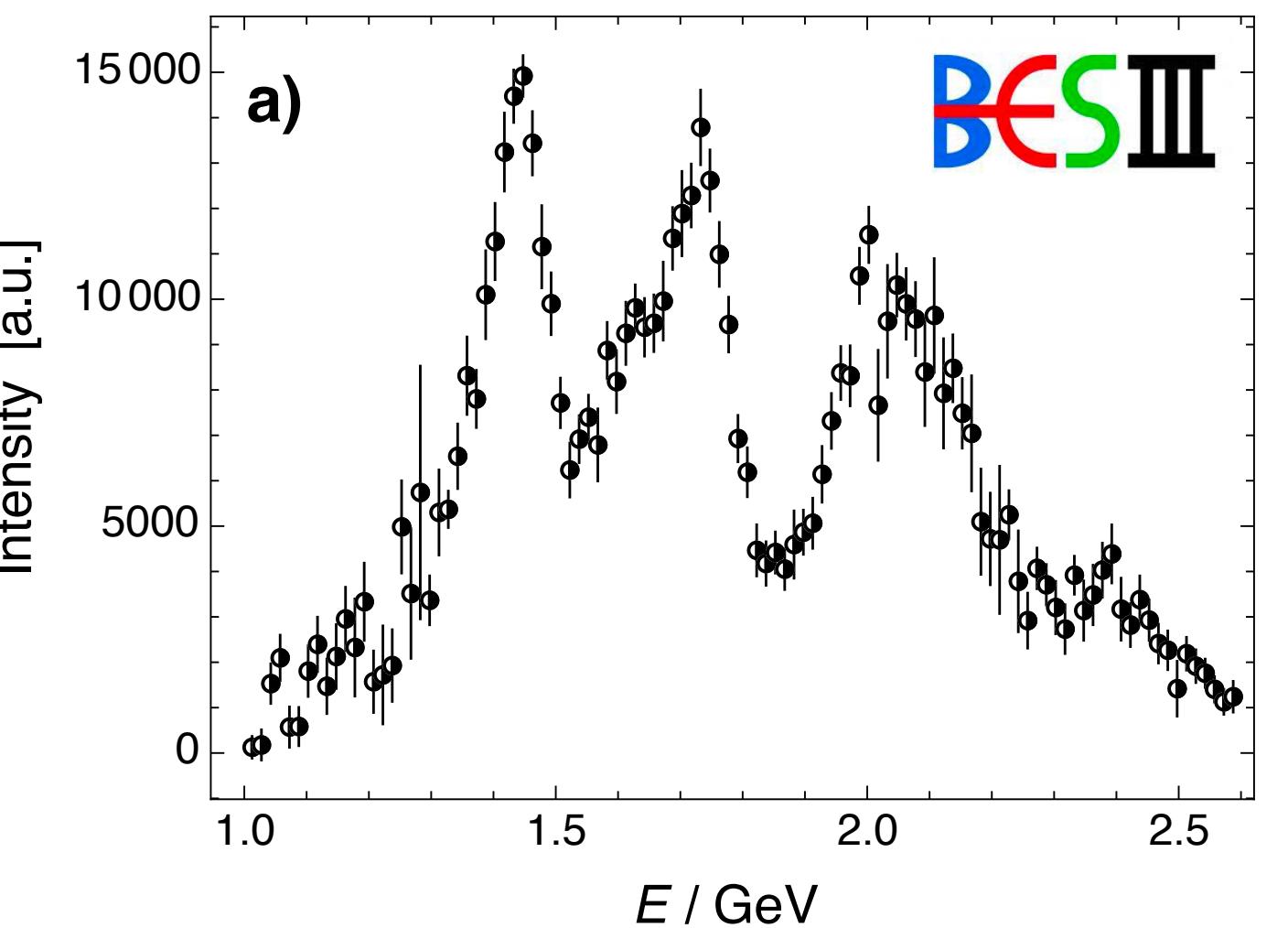
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Ropertz, et al.



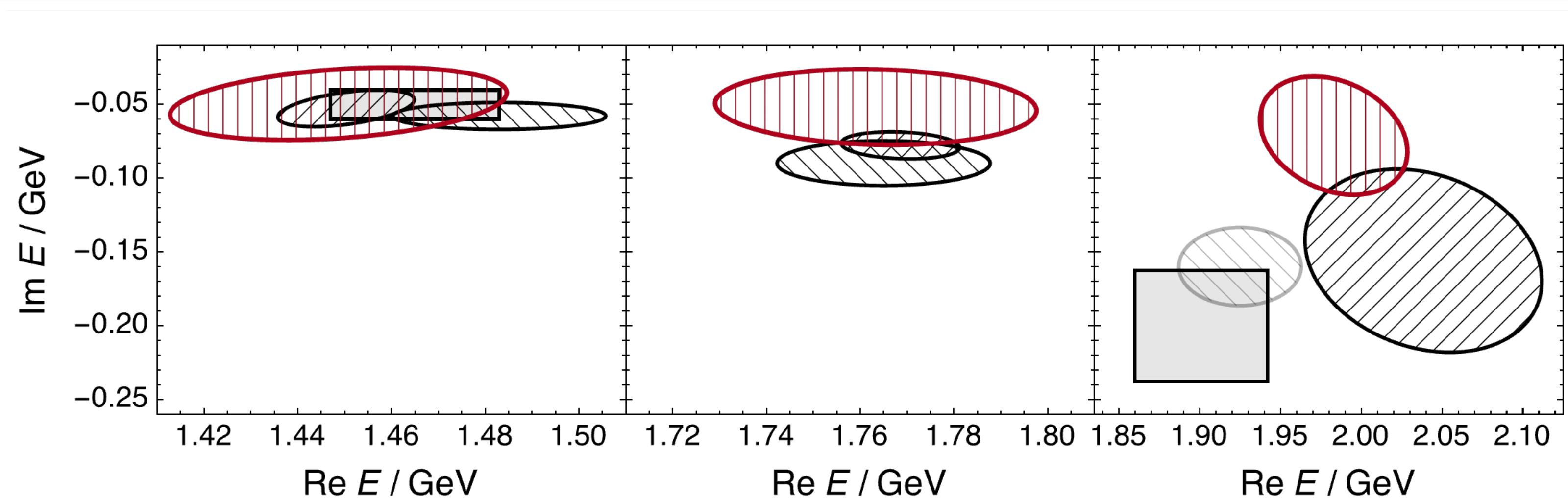
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**SPM  
RESULTS**

JPAC	
BoGa	
Ropertz, et al.	



	This work	JPAC	Bonn-Gatchina	Ropertz <i>et al.</i>
$f_0(1500)$	$(1449 \pm 24) - i(100 \pm 32)/2$	$(1450 \pm 10) - i(106 \pm 16)/2$	$(1370 \pm 40) - i(390 \pm 40)/2$ $(1483 \pm 15) - i(116 \pm 12)/2$	$(1465 \pm 18) - i(101 \pm 20)/2$
$f_0(1710)$	$(1763 \pm 23) - i(104 \pm 34)/2$	$(1769 \pm 8) - i(156 \pm 12)/2$	$(1765 \pm 15) - i(180 \pm 20)/2$	/
$f_0(2020)$	$(1983 \pm 31) - i(143 \pm 54)/2$	$(2038 \pm 48) - i(312 \pm 82)/2$	$(1925 \pm 25) - i(320 \pm 35)/2$ $(2075 \pm 20) - i(260 \pm 25)/2$	$(1901 \pm 41) - i(401 \pm 76)/2$