New insights into the quark model from lattice QCD

Derek Leinweber

In collaboration with: Curtis Abell, Liam Hockley, Waseem Kamleh, Yan Li, Zhan-Wei Liu, Finn Stokes, Tony Thomas, Jia-Jun Wu

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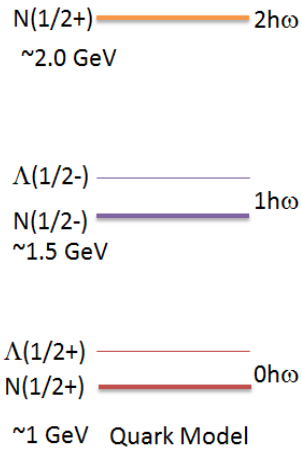
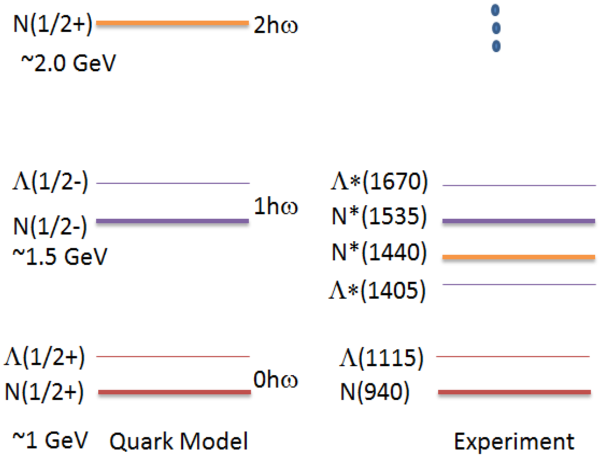
The spectrum of a simple quark model: N and Λ baryons



The challenge of experiment



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- What's new are formalisms able to bring these descriptions to the finite-volume of lattice QCD.
- Lattice QCD calculations of the excitation spectrum provide new constraints.
- It's time to reconsider our early notions about the quark-model and its excitation spectrum.

Outline

- Hamiltonian Effective Field Theory (HEFT)
  - Coupled-channel analysis technique aimed at resonance physics.
  - Incorporates the Lüscher formalism.
  - Connects scattering observables to the finite-volume spectrum of lattice QCD.

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- Conclusions

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- Fitting resonance phase-shift data and inelasticities,
  - Predictions of the finite-volume spectrum are made.
- The eigenvectors of the Hamiltonian provide insight into the composition of the energy eigenstates.
  - Insight is similar to that provided by correlation-matrix eigenvectors in Lattice QCD.

Infinite Volume Model

- The rest-frame Hamiltonian has the form $H = H_0 + H_I$, with

$$H_0 = \sum_{B_0} |B_0\rangle\, m_{B_0}\, \langle B_0| + \sum_{\alpha} \int d^3k\, |\alpha(\boldsymbol{k})\rangle\, \omega_\alpha(\boldsymbol{k})\, \langle \alpha(\boldsymbol{k})|\,,$$

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- $|B_0\rangle$ denotes a quark-model-like basis state with bare mass m_{B_0}.
- $|\alpha(\boldsymbol{k})\rangle$ designates a two-particle non-interacting basis-state channel with energy

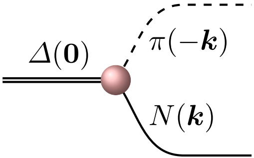
$$\omega_\alpha(\boldsymbol{k}) = \omega_{\alpha_M}(\boldsymbol{k}) + \omega_{\alpha_B}(\boldsymbol{k}) = \sqrt{\boldsymbol{k}^2 + m_{\alpha_M}^2} + \sqrt{\boldsymbol{k}^2 + m_{\alpha_B}^2}\,,$$

for $M =$ Meson, $B =$ Baryon.

Infinite Volume Model

- The interaction Hamiltonian includes two parts, $H_I = g + v$.
- $1 \to 2$ particle vertex

$$g = \sum_{\alpha,\, B_0} \int d^3k \, \left\{ |\alpha(\boldsymbol{k})\rangle \, G^{\dagger}_{\alpha,B_0}(k) \, \langle B_0| + h.c. \right\} \, ,$$



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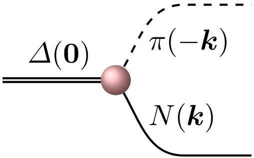
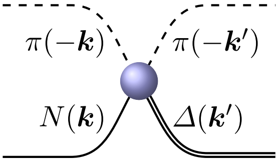


- $2 \to 2$ particle vertex

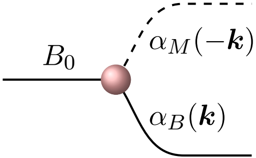
$$v = \sum_{\alpha,\beta} \int d^3k \; d^3k' \, |\alpha(\boldsymbol{k})\rangle \, V^S_{\alpha,\beta}(k,k') \, \langle \beta(\boldsymbol{k}')| \, .$$



S-wave vertex interactions

- S-wave vertex interactions between the one baryon and two-particle meson-baryon channels for e.g. $N^*(1535)$ or $\Lambda^*(1405)$ cases take the form

$$G_{\alpha,B_0}(k) = g_{B_0\alpha}\,\frac{\sqrt{3}}{2\,\pi\,f_\pi}\,\sqrt{\omega_{\alpha_M}(k)}\,u(k,\Lambda)\,,$$



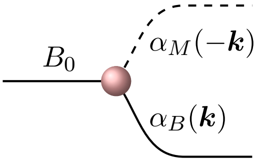
with regulator

$$u(k,\Lambda) = \left(1 + \frac{k^2}{\Lambda^2}\right)^{-2}, \qquad \text{and fixed } \Lambda \sim 0.8 \to 1.0 \text{ GeV.}$$

P-wave and higher vertex interactions

- P-wave and higher vertex interactions for the $\Delta(1232)$ or $N^*(1440)$ take the form

$$G_{\alpha,B_0}(k) = g_{B_0\alpha}\,\frac{1}{4\pi^2}\left(\frac{k}{f_\pi}\right)^{l_\alpha}\frac{u(k,\Lambda)}{\sqrt{\omega_{\alpha_M}(k)}}\,,$$



where l_α is the orbital angular momentum in channel α.

Two-to-two particle interactions

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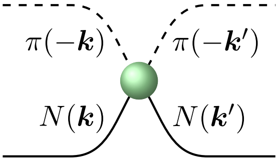
- For the S_{11} πN channel

$$V^S_{\pi N,\pi N}\,(\,k,\;k'\,) = v_{\pi N,\pi N}\,\frac{3}{4\pi^2\,f_\pi^2}\,\tilde{u}_{\pi N}(k,\varLambda)\,\tilde{u}_{\pi N}(k',\varLambda)$$



where the scattering potential gains a low energy enhancement via

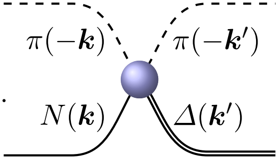
$$\tilde{u}_{\pi N}(k,\varLambda) = u(k,\varLambda)\,\frac{m_\pi^{\text{phys}} + \omega_\pi(k)}{\omega_\pi(k)}$$

and $u(k,\varLambda)$ takes the dipole form.

Two-to-two particle interactions

- For P-wave scattering in the $\Delta(1232)$ or $N^*(1440)$ channels

$$V^S_{\alpha,\beta}\,(\,k,\,k'\,) = v_{\alpha,\beta}\,\frac{1}{4\pi^2\,f_\pi^2}\,\frac{k}{\omega_{\alpha_M}(k)}\,\frac{k'}{\omega_{\beta_M}(k')}\,u(k,\Lambda)\,u(k',\Lambda)\,.$$



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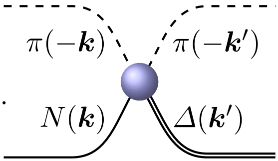

- For the $\Lambda^*(1405)$, the Weinberg-Tomozawa term is considered

$$V^S_{\alpha,\beta}(k,\, k') = g^{\Lambda^*}_{\alpha,\beta}\,\frac{[\,\omega_{\alpha_M}(k) + \omega_{\beta_M}(k')\,]\,u(k,\Lambda)\,u(k',\Lambda)}{16\,\pi^2 f_\pi^2 \sqrt{\omega_{\alpha_M}(k)\,\omega_{\beta_M}(k')}}\,,$$

Infinite-Volume scattering amplitude

- The T-matrices for two particle scattering are obtained by solving the coupled-channel integral equations

$$T_{\alpha,\beta}(k,k';E) = \tilde{V}_{\alpha,\beta}(k,k';E) + \sum_{\gamma} \int q^2 dq \, \frac{\tilde{V}_{\alpha,\gamma}(k,q;E)\, T_{\gamma,\beta}(q,k';E)}{E - \omega_{\gamma}(q) + i\epsilon} \, .$$

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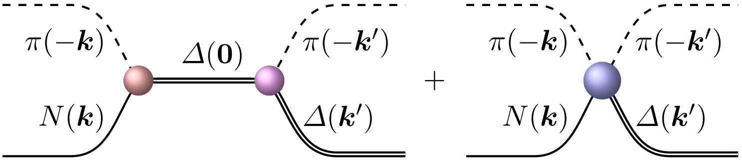
- The coupled-channel potential is readily calculated from the interaction Hamiltonian

$$\tilde{V}_{\alpha,\beta}(k,k') = \sum_{B_0} \frac{G^{\dagger}_{\alpha,B_0}(k)\, G_{\beta,B_0}(k')}{E-m_{B_0}} + V^{S}_{\alpha,\beta}(k,k')\,,$$



Infinite-Volume scattering matrix

- The S-matrix is related to the T-matrix by

$$S_{\alpha,\beta}(E) = 1 - 2i\sqrt{\rho_\alpha(E)\,\rho_\beta(E)}\;T_{\alpha,\beta}(k_{\alpha\,\mathrm{cm}}, k_{\beta\,\mathrm{cm}}; E)\,,$$

with

$$\rho_\alpha(E) = \pi\frac{\omega_{\alpha_M}(k_{\alpha\,\mathrm{cm}})\,\omega_{\alpha_B}(k_{\alpha\,\mathrm{cm}})}{E}\,k_{\alpha\,\mathrm{cm}}\,,$$

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- The cross section $\sigma_{\alpha\,,\beta}$ for the process $\alpha \to \beta$ is

$$\sigma_{\alpha\,,\beta} = \frac{4\pi^3\,k_{\alpha\,\mathrm{cm}}\,\omega_{\alpha_M}(k_{\alpha\,\mathrm{cm}})\,\omega_{\alpha_B}(k_{\alpha\,\mathrm{cm}})\,\omega_{\beta_M}(k_{\alpha\,\mathrm{cm}})\,\omega_{\beta_B}(k_{\alpha\,\mathrm{cm}})}{E^2\,k_{\beta\,\mathrm{cm}}}\;|T_{\alpha,\beta}(k_{\alpha\,\mathrm{cm}}, k_{\beta\,\mathrm{cm}}; E)|^2\,.$$

πN phase shift and inelasticity

- The S-matrix is related to the T-matrix by

$$\begin{aligned} S_{\pi N,\pi N}(E) &= 1 - 2i\pi \, \frac{\omega_\pi(k_{\rm cm})\,\omega_N(k_{\rm cm})}{E} \, k_{\rm cm} \; T_{\pi N,\pi N}(k_{\rm cm}, k_{\rm cm}; E)\,, \\ &= \eta(E)\, e^{2i\,\delta(E)}\,. \end{aligned}$$

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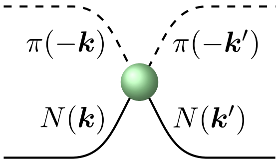

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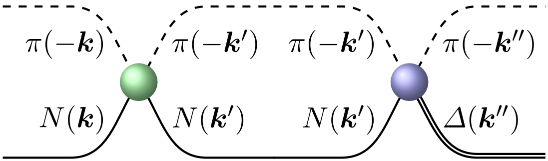

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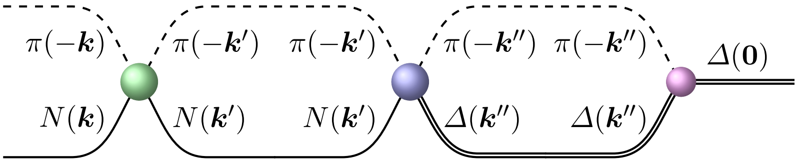

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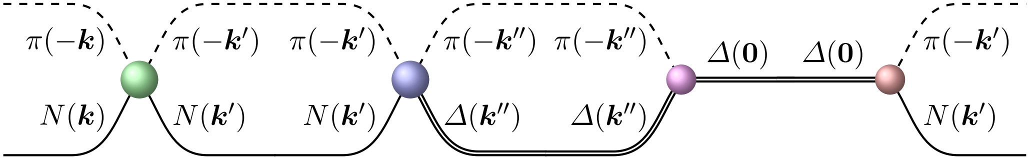
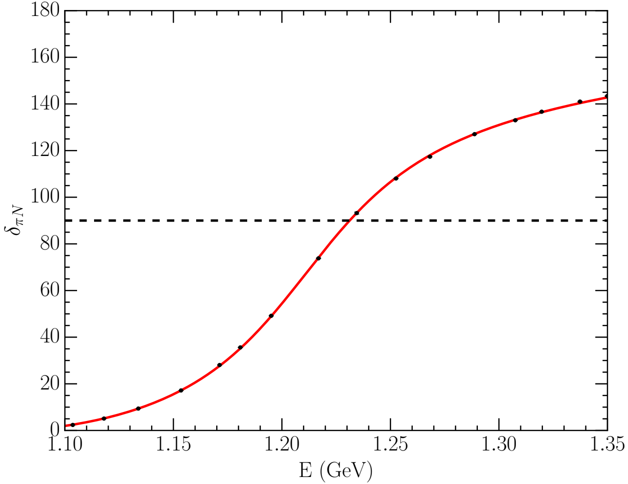


P-wave πN phase shifts in the Δ channel - 1 πN channel



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with $n_i = 0, 1, 2, \ldots$ and integer $n = n_x^2 + n_y^2 + n_z^2$.

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- The degeneracy of each k_n is described by $C_3(n)$, which counts the number of ways the integers n_x, n_y, and n_z, can be squared and summed to n.
- The non-interacting Hamiltonian takes the form

$$H_0 = \mathrm{diag}(m_{B_0},\ \omega_{\pi N}(k_0),\ \omega_{\pi \Delta}(k_0),\ \omega_{\pi N}(k_1),\ \omega_{\pi \Delta}(k_1),\ \ldots)$$

Interaction Hamiltonian Terms

- $1 \to 2$ particle interaction terms sit in the first row and column.

$$H_I = \begin{pmatrix} 0 & \overline{G}_{\pi N, B_0}(k_0) & \cdots & \overline{G}_{\pi \Delta, B_0}(k_0) & \overline{G}_{\pi N, B_0}(k_1) & \cdots & \overline{G}_{\pi \Delta, B_0}(k_1) \cdots \\ \overline{G}^{\dagger}_{\pi N, B_0}(k_0) & 0 & & & & & \\ \vdots & & 0 & & & & \\ \overline{G}^{\dagger}_{\pi \Delta, B_0}(k_0) & & & \ddots & & & \\ \overline{G}^{\dagger}_{\pi N, B_0}(k_1) & & & & & & \\ \vdots & & & & & & \\ \overline{G}^{\dagger}_{\pi \Delta, B_0}(k_1) & & & & & & \\ \vdots & & & & & & \end{pmatrix} .$$

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- $\cdots$ allow for additional channels.
- $2 \to 2$ particle interaction terms, $\overline{V}^{S}_{\alpha,\beta}(k_n, k_{n'})$, fill out the rest of the matrix.

Relation to infinite-volume contributions

- The finite volume Hamiltonian interaction terms are related to the infinite-volume contributions via

$$\int k^2 dk = \frac{1}{4\pi}\int d^3k \to \frac{1}{4\pi}\sum_{\boldsymbol{n}\in\mathbb{Z}^3}\left(\frac{2\pi}{L}\right)^3 = \frac{1}{4\pi}\sum_{n\in\mathbb{Z}} C_3(n)\left(\frac{2\pi}{L}\right)^3 .$$

  such that

$$\bar{G}_{\alpha,B_0}(k_n) = \sqrt{\frac{C_3(n)}{4\pi}}\left(\frac{2\pi}{L}\right)^{\frac{3}{2}} G_{\alpha,B_0}(k_n)\,,$$

$$\bar{V}^S_{\alpha\beta}(k_n,k_m) = \sqrt{\frac{C_3(n)}{4\pi}}\sqrt{\frac{C_3(m)}{4\pi}}\left(\frac{2\pi}{L}\right)^3 V^S_{\alpha\beta}(k_n,k_m)\,.$$

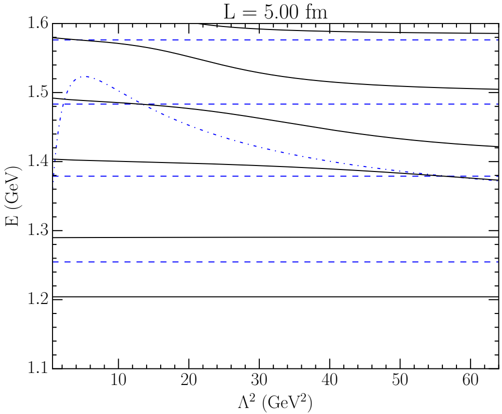
Finite Volume Eigenmode Solution

- Standard Lapack routines provide eigenmode solutions of

$$\langle\, i \,|\, H \,|\, j \,\rangle \, \langle\, j \,|\, E_\alpha \,\rangle = E_\alpha \, \langle\, i \,|\, E_\alpha \,\rangle \,,$$

  - where $|\, i \,\rangle$ and $|\, j \,\rangle$ are the non-interacting basis states,
  - E_α is the energy eigenvalue, and
  - $\langle\, i \,|\, E_\alpha \,\rangle$ is the eigenvector of the
  - Hamiltonian matrix $\langle\, i \,|\, H \,|\, j \,\rangle$.

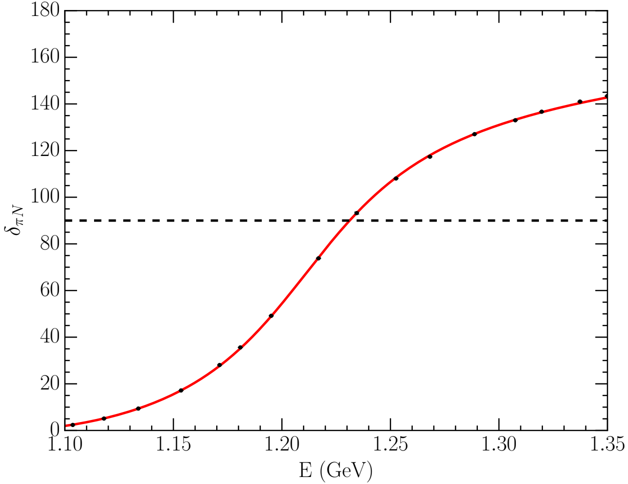
Energy eigenstates on an $L = 5$ fm lattice for different regulators



- dashed lines are the non-interacting πN basis-state energy levels.
- dot-dash line is the bare basis-state mass.
- solid lines are the eigenstate energy levels.

- Incorporation of the Lüscher formalism ensures energy eigenstates below 1.35 GeV are model independent.

P-wave πN phase shifts in the Δ channel - 1 πN channel



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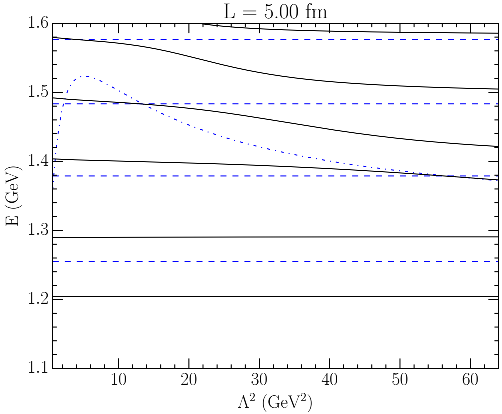

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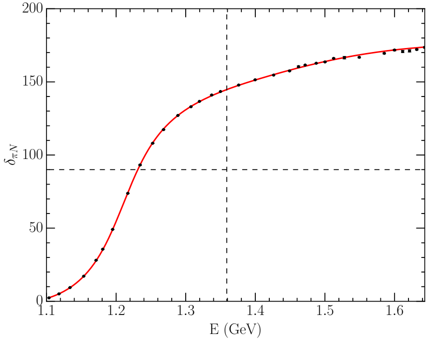

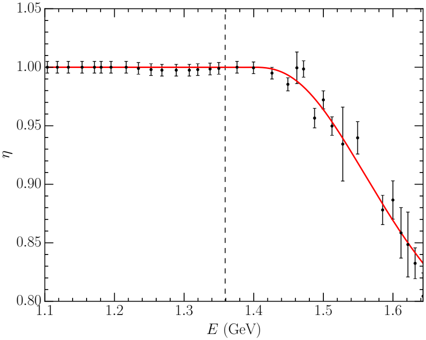

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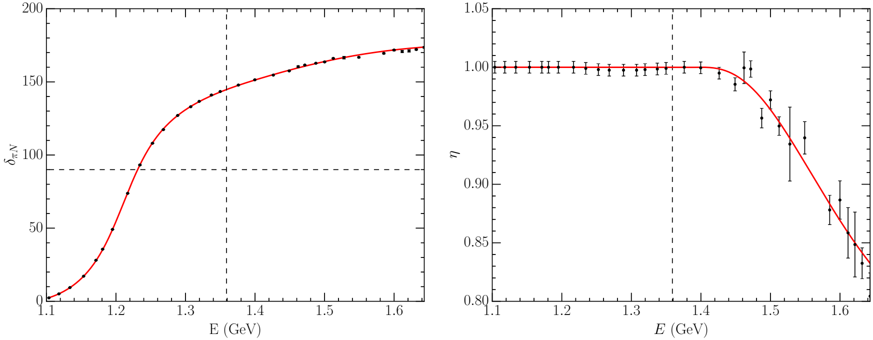
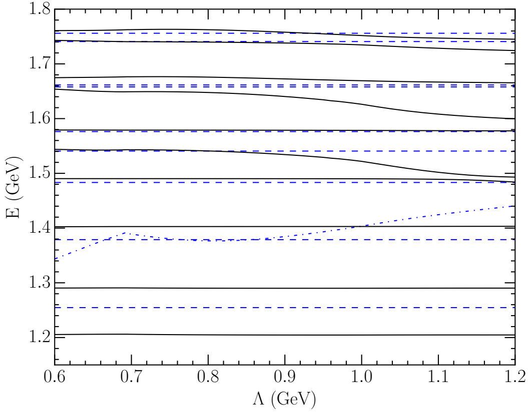


- Anticipate regulator independence to 1.7 GeV.

Energy eigenstates on an $L = 5$ fm lattice for different regulators



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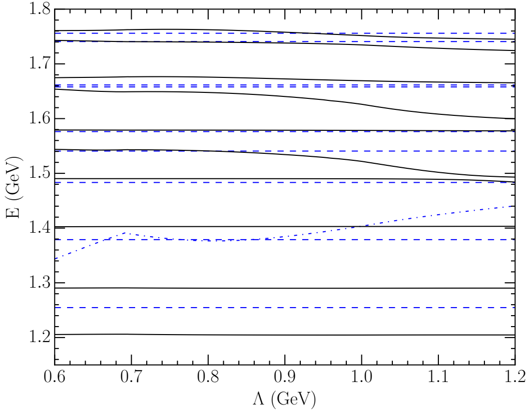
Energy eigenstates on an $L = 5$ fm lattice for different regulators



- dashed lines are the non-interacting πN and $\pi\Delta$ basis-state energy levels.
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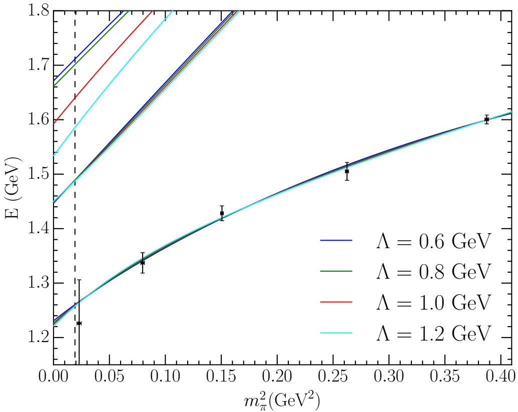
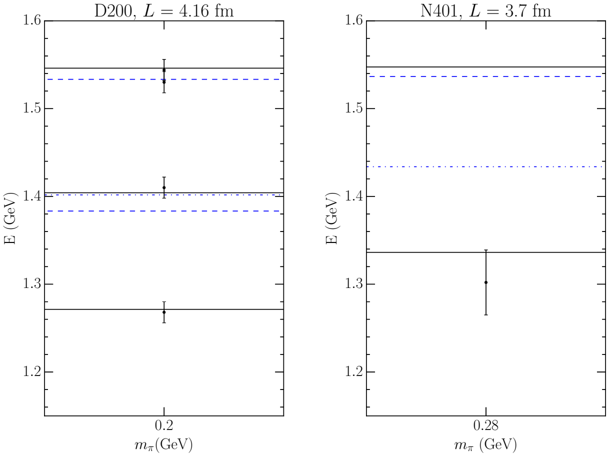
- πN scattering data alone is insufficient to uniquely constrain the Hamiltonian.

Mass dependence of energy eigenstates - Fit to PACS-CS Δ masses



- Lattice QCD results can constrain the Hamiltonian description of experimental data.

CLS Consortium finite-volume lattice energies of Δ-channel excitations



- C. Morningstar, *et al.* PoS **LATTICE2021** (2022), 170 [arXiv:2111.07755 [hep-lat]].
- C. W. Andersen, J. Bulava, B. Hörz and C. Morningstar, Phys. Rev. D **97** (2018) 014506 [arXiv:1710.01557 [hep-lat]].

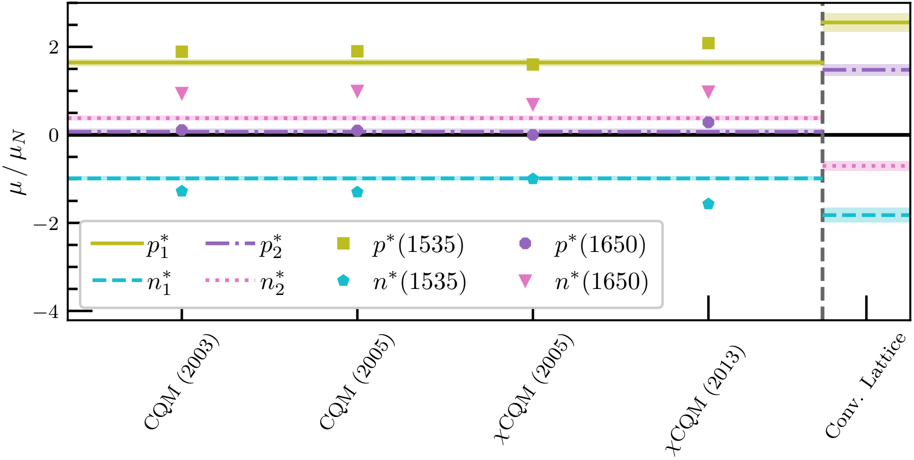
New examination of low-lying odd-parity nucleon resonances

- Motivated by lattice QCD calculations of the electromagnetic form factors of the two low-lying odd-parity states.

  F. M. Stokes, W. Kamleh, DBL, Phys. Rev. D **102** (2020) 014507 [arXiv:1907.00177 [hep-lat]].

- Parity-expanded variational analysis (PEVA) removes opposite-parity contaminants.
- Confirms quark model predictions for N^* magnetic moments.

N^* Magnetic Moments and the constituent quark model



F. M. Stokes, W. Kamleh, DBL, Phys. Rev. D **102** (2020) 014507 [arXiv:1907.00177 [hep-lat]].

Model Calculation References

- CQM (2003)

  W.-T. Chiang, S. N. Yang, M. Vanderhaeghen, and D. Drechsel, Magnetic dipole moment of the S 11 (1535) from the $\gamma p \to \gamma \eta p$ reaction, Nucl. Phys. **A723**, 205 (2003), nucl-th/0211061

- χCQM (2005)

  J. Liu, J. He, and Y. Dong, Magnetic moments of negative-parity low-lying nucleon resonances in quark models, Phys. Rev. **D71**, 094004 (2005).

- χCQM (2013)

  N. Sharma, A. Martinez Torres, K. Khemchandani, and H. Dahiya, Magnetic moments of the low-lying $1/2^-$ octet baryon resonances, Eur. Phys. J. **A49**, 11 (2013), arXiv:1207.3311

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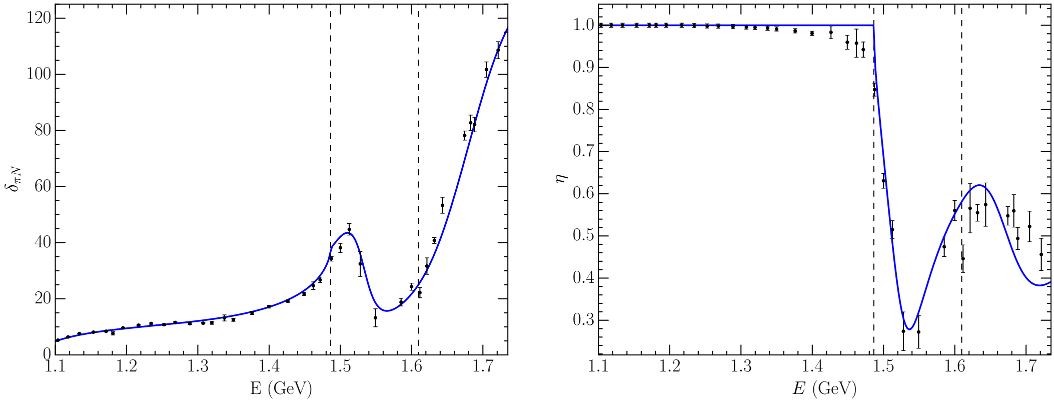
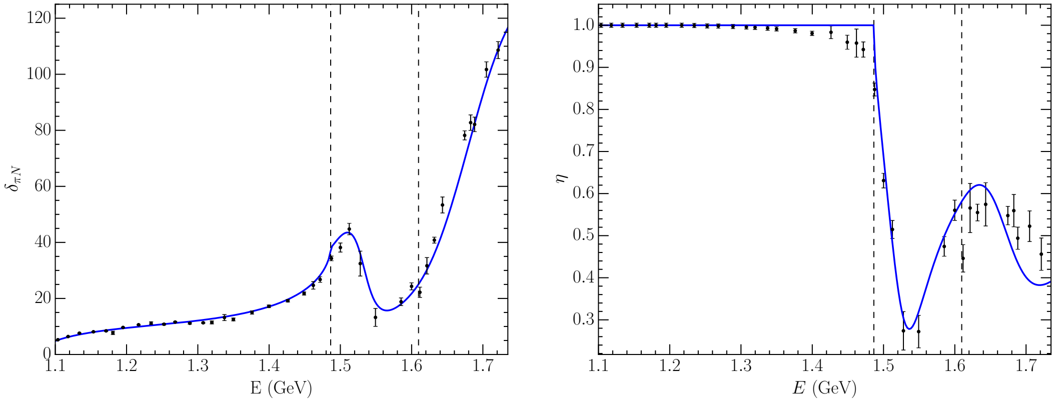
- 21 parameter fit provides an excellent characterisation of the data.
  - Pole positions agree with PDG.

Phase shift and inelasticity for the low-lying odd-parity spin-1/2 nucleon resonances



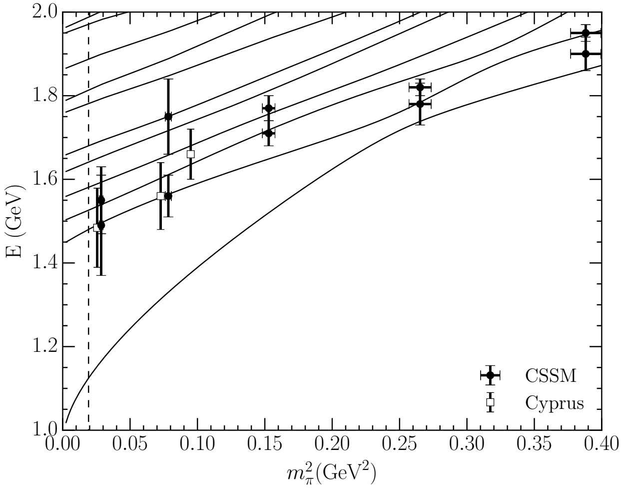
- WI08 single-energy data from SAID.
- Vertical lines indicate the opening of the ηN and $K\Lambda$ thresholds.

Phase shift and inelasticity for the low-lying odd-parity spin-1/2 nucleon resonances

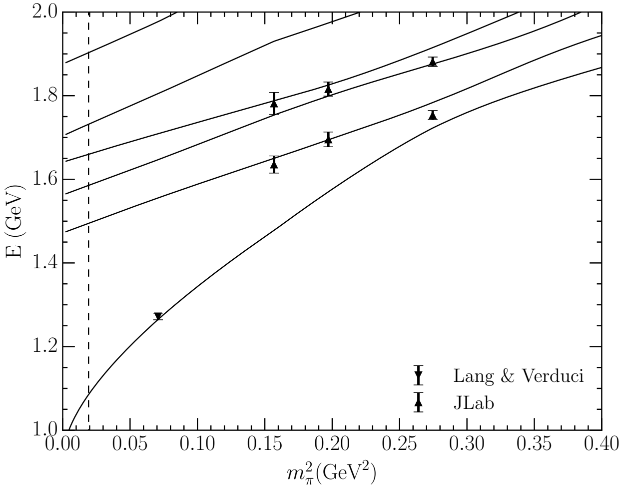


- Note the three-body $\pi\pi N$ threshold at 1.22 GeV.
- See Max Hansen's talk in Parallel Session 1, today at 4:20 pm.

Finite-volume $L = 3$ fm energy levels for low-lying odd-parity spin-1/2 nucleons



Finite-volume $L = 2$ fm energy levels for low-lying odd-parity spin-1/2 nucleons



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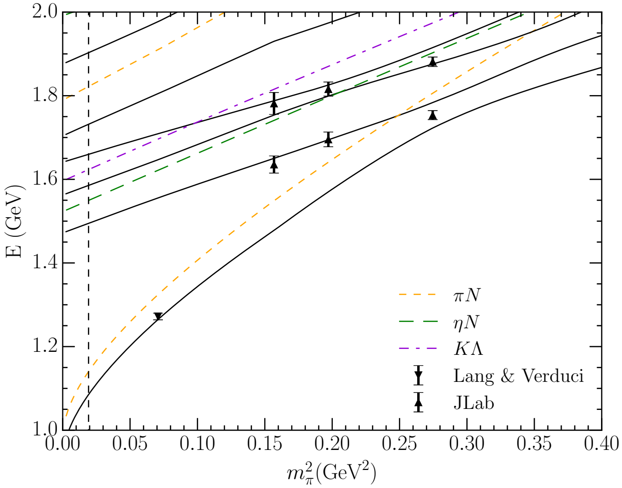

Finite Volume Eigenmode Solution

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- Eigenvector $\langle\, i \,|\, E_\alpha \,\rangle$ describes the composition of the eigenstate $|\, E_\alpha \,\rangle$ in terms of the basis states $|\, i \,\rangle$ with

$$|\, i \,\rangle = |\, B_0 \,\rangle, \quad |\, \pi N, k_0 \,\rangle, \quad |\, \pi N, k_1 \,\rangle, \quad \cdots |\, \eta N, k_0 \,\rangle, \quad |\, \eta N, k_1 \,\rangle, \quad \cdots .$$

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- The overlap of the bare basis state $|\, B_0 \,\rangle$ with eigenstate $|\, E_\alpha \,\rangle$,

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is of particular interest,

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relative to the ground state.

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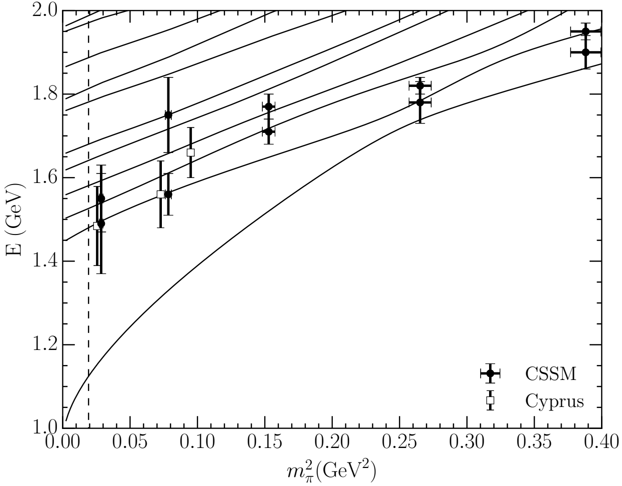
Finite-volume $L = 3$ fm energy levels for low-lying odd-parity spin-1/2 nucleons



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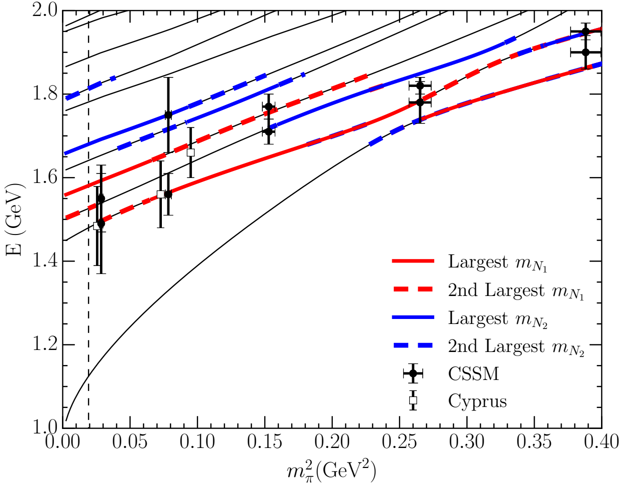

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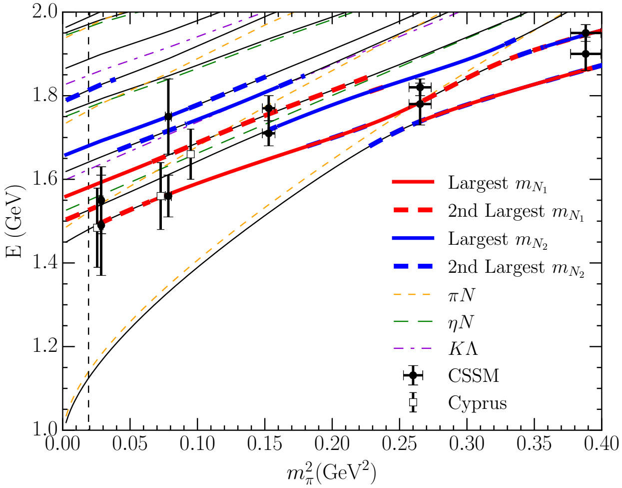
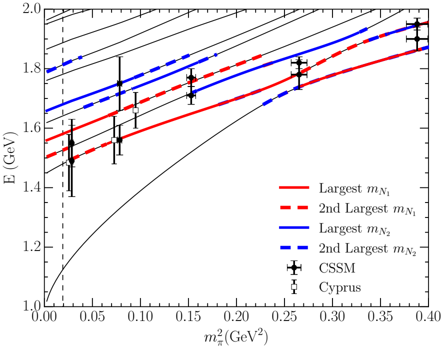
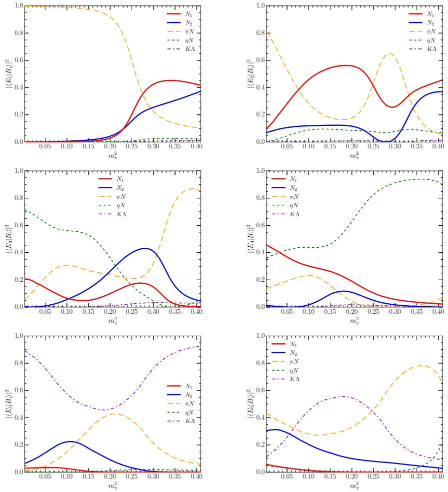
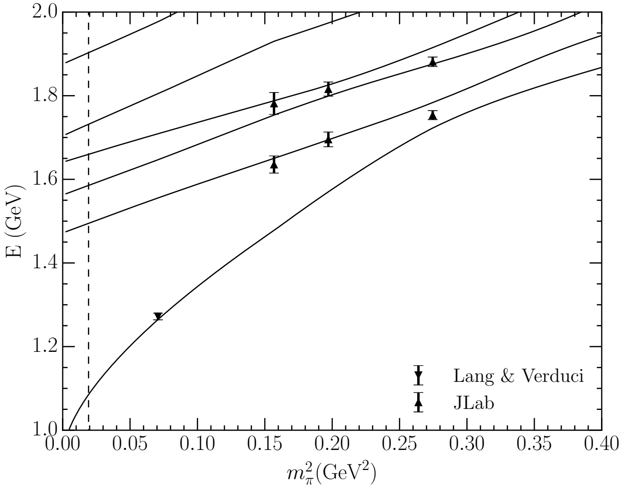


Energy eigenstate composition - 3 fm lattice





Finite-volume $L = 2$ fm energy levels for low-lying odd-parity spin-1/2 nucleons



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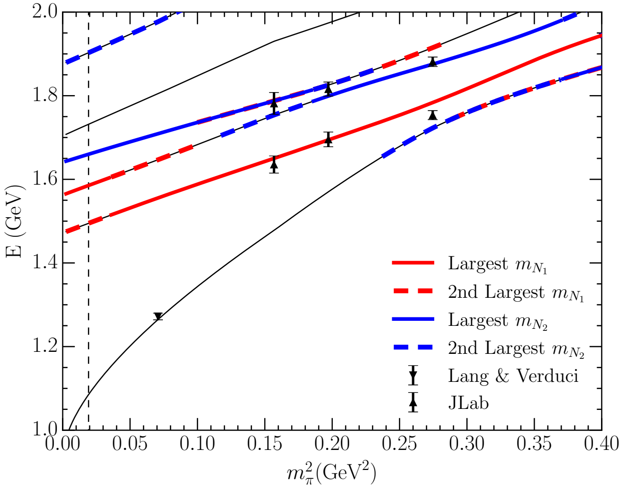

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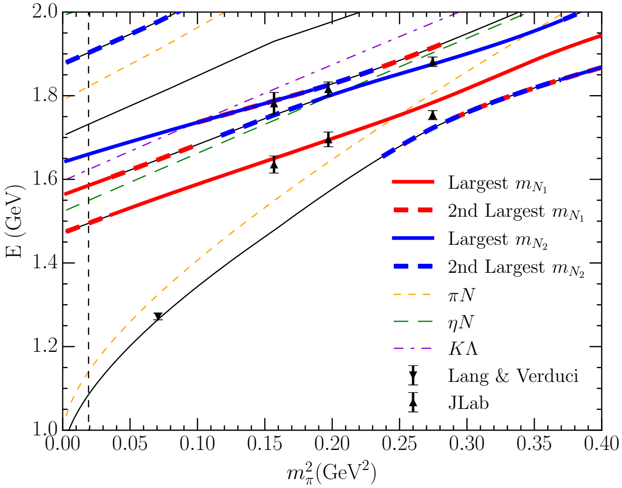
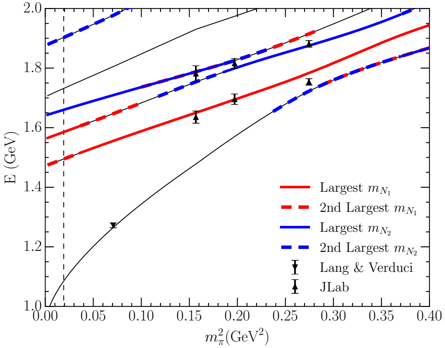
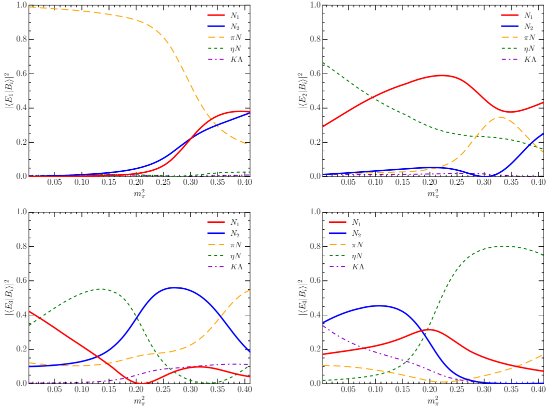


Energy eigenstate composition - 2 fm lattice





Analysis of low-lying odd-parity Λ resonances

Z. W. Liu, *et al.* [CSSM], Phys. Rev. D **95** (2017) 014506 [arXiv:1607.05856 [nucl-th]]

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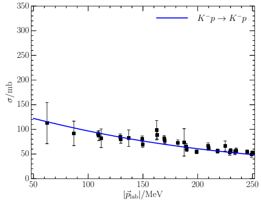
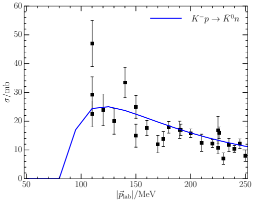
- These 13 parameters are constrained by experimental data.

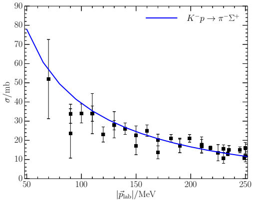
Couplings and m_{B_0} Constrained by Experiment



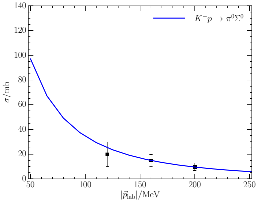
(a) $K^-p \to K^-p$



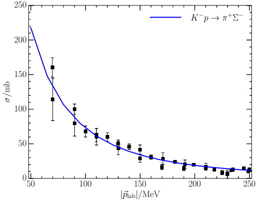
(b) $K^-p \to \bar{K}^0 n$



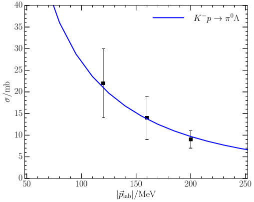
(c) $K^-p \to \pi^-\Sigma^+$



(d) $K^-p \to \pi^0\Sigma^0$

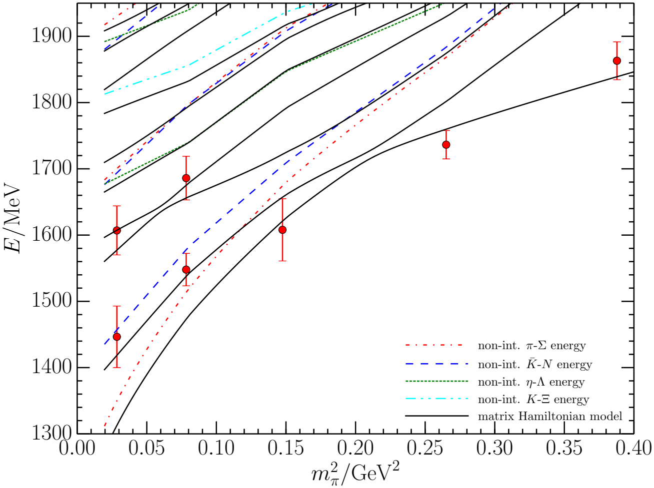


(e) $K^-p \to \pi^+\Sigma^-$



(f) $K^-p \to \pi^0\Lambda$

Finite Volume Λ Spectrum for $L = 3$ fm



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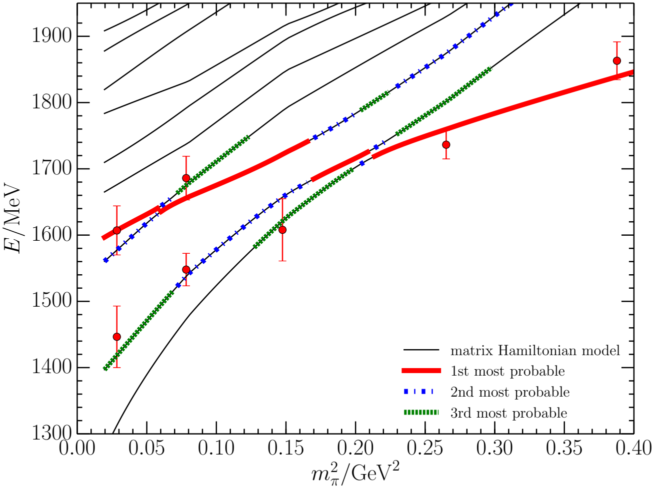
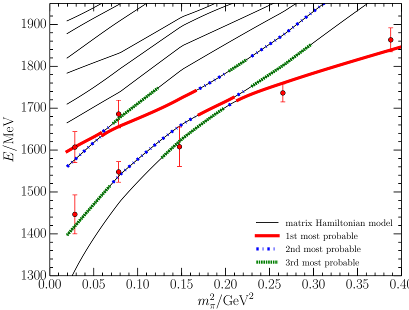
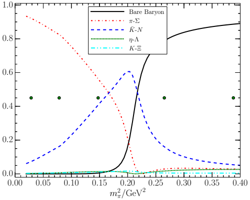
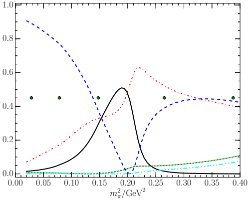


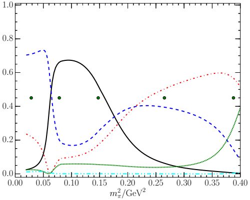




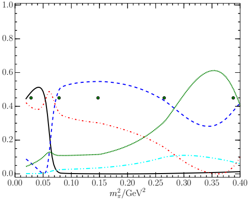
(a) State 1



(b) State 2



(c) State 3



(d) State 4

Strange Magnetic Form Factor of the $\Lambda(1405)$

J. M. M. Hall, *et al.* [CSSM], Phys. Rev. Lett. **114**, 132002 (2015) arXiv:1411.3402 [hep-lat]

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- In forming such a molecular state, the $\Lambda(u,d,s)$ valence quark configuration is complemented by
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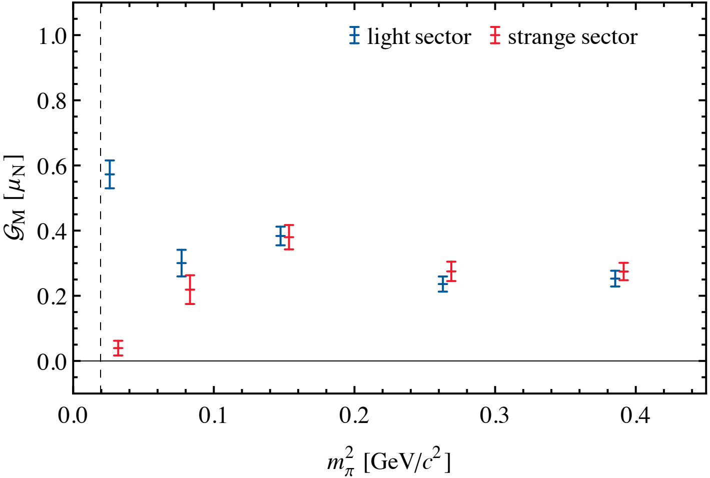
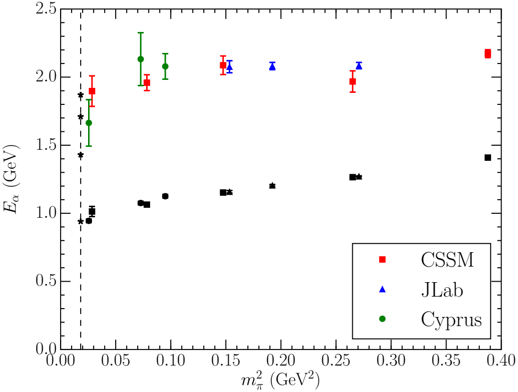


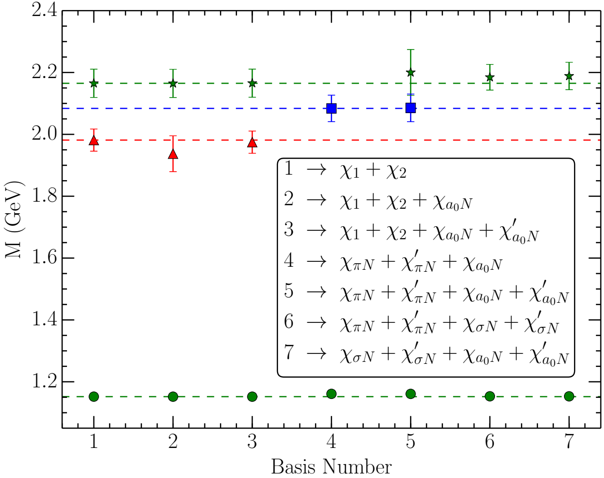
Where is the Roper resonance?



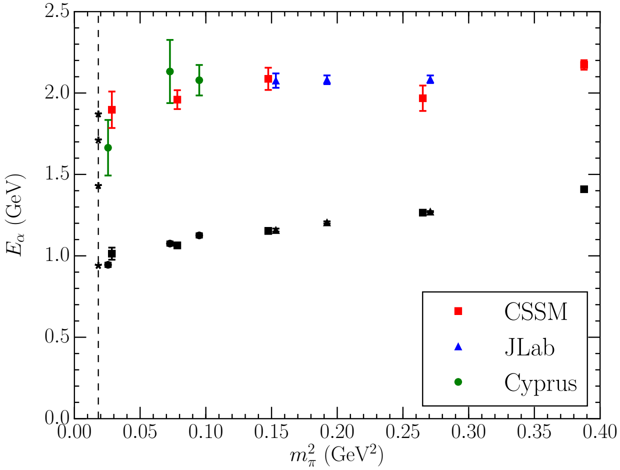
- CSSM: Z. W. Liu, *et al.* [CSSM], Phys. Rev. D **95**, 034034 (2017) arXiv:1607.04536 [nucl-th]
- Cyprus: C. Alexandrou, *et al.* (AMIAS), Phys. Rev. D **91**, 014506 (2015) arXiv:1411.6765 [hep-lat]
- JLab: R. G. Edwards, *et al.* [HSC] Phys. Rev. D **84**, 074508 (2011) [arXiv:1104.5152 [hep-ph]].

Search for low-lying lattice QCD eigenstates in the Roper regime

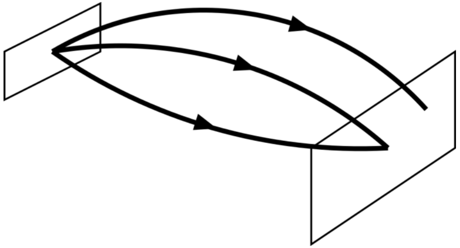
A. L. Kiratidis, *et al.*, [CSSM] Phys. Rev. D **95**, no. 7, 074507 (2017) [arXiv:1608.03051 [hep-lat]].



Have we seen the $2s$ excitation of the quark model?

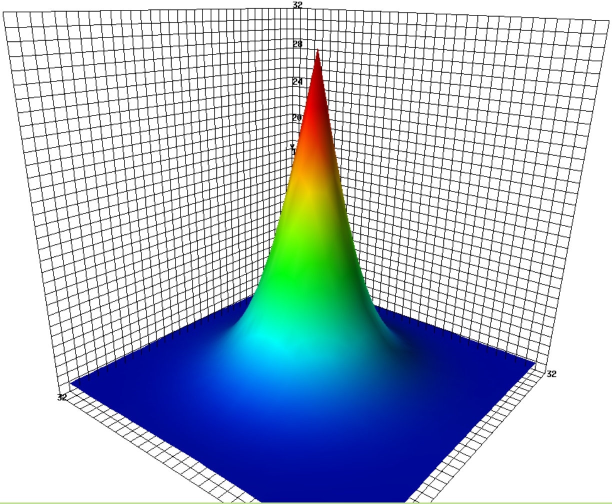


Landau-Gauge Wave functions from the Lattice

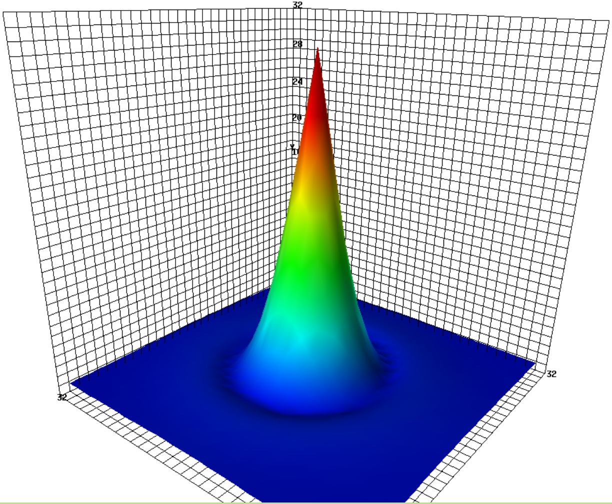


- Measure the *overlap* of the annihilation operator with the state as a function of the quark positions.

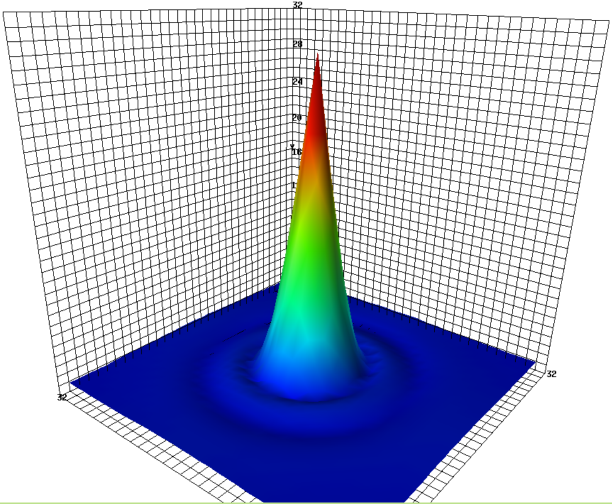
d-quark probability density in ground state proton [CSSM]



d-quark probability density in 1st excited state of proton [CSSM]

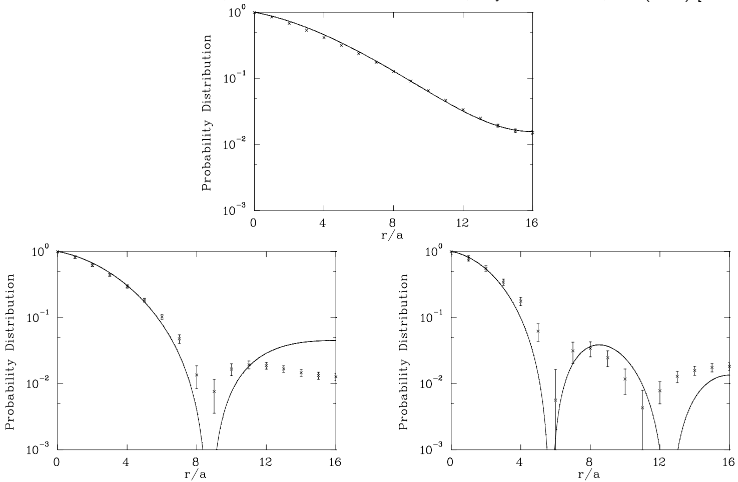


d-quark probability density in $N = 3$ excited state of proton [CSSM]

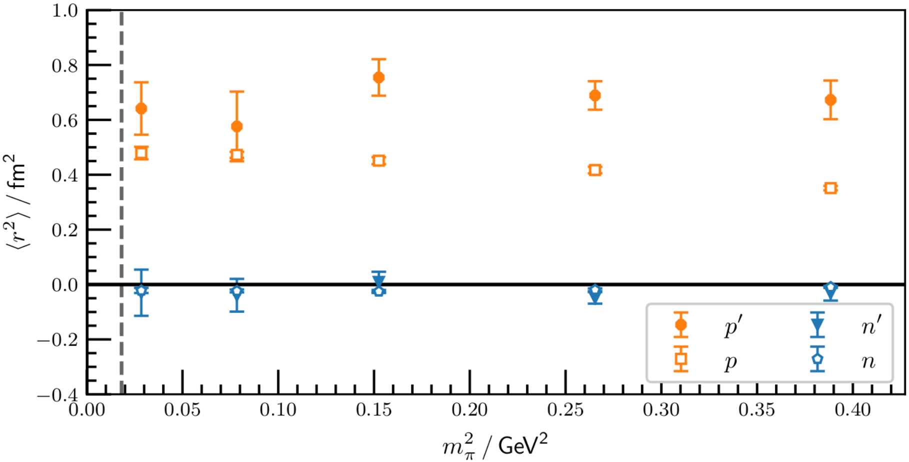


Comparison with the Simple Quark Model [CSSM]

D. S. Roberts, W. Kamleh and D. B. Leinweber, Phys. Lett. B **725**, 164 (2013) [arXiv:1304.0325 [hep-lat]].

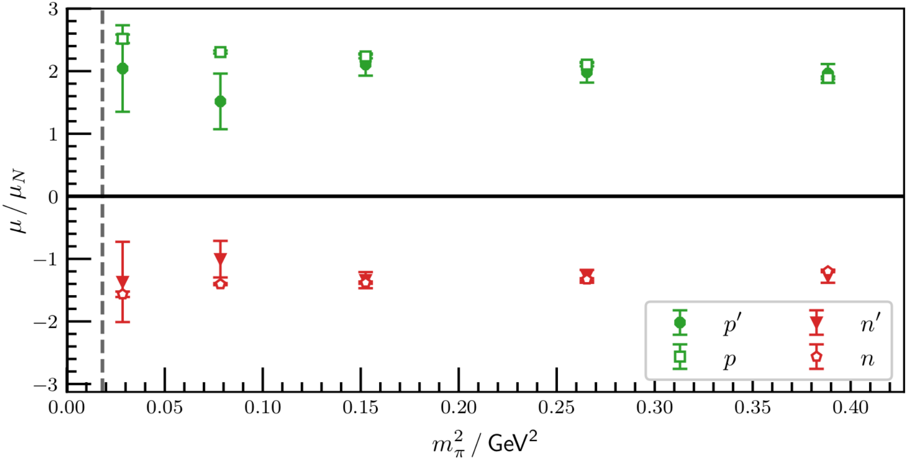


First positive-parity excitation: Charge Radii



F. M. Stokes, W. Kamleh, DBL, Phys. Rev. D **102** (2020) 014507 [arXiv:1907.00177 [hep-lat]].

First positive-parity excitation: Magnetic moments



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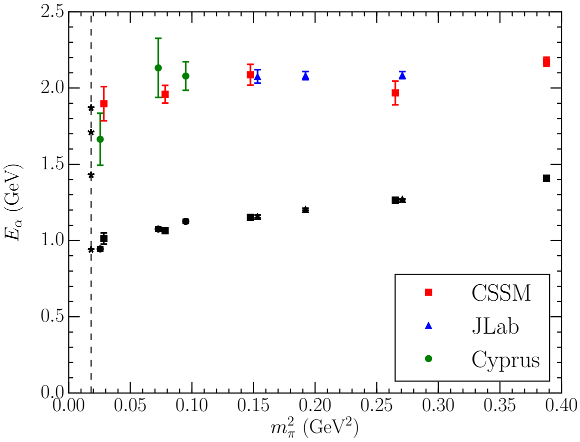

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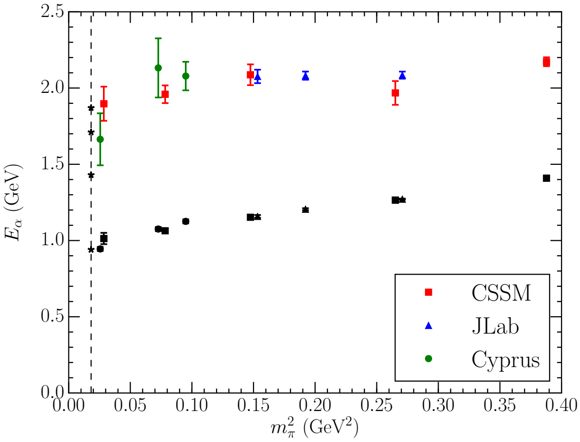

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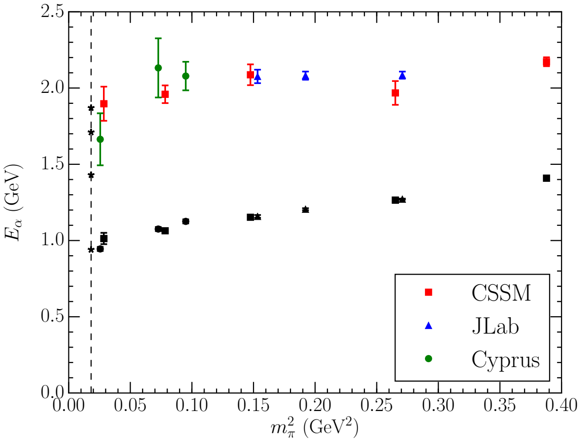

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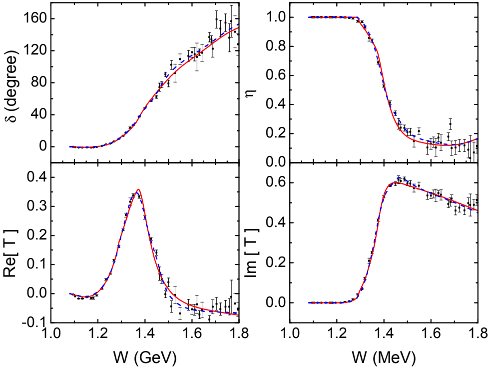
- What about the Roper resonance?

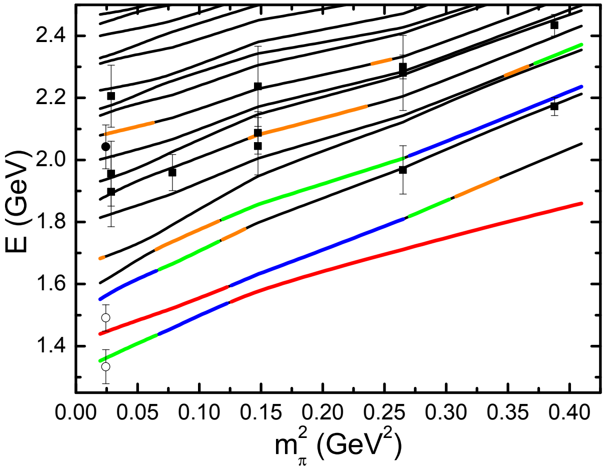
Positive-parity Nucleon Spectrum: Bare Roper Case with $m_0 = 1.7$ GeV

- Consider πN, $\pi\Delta$ and σN channels, dressing a bare state.
- Fit to phase shift and inelasticity. (dashed blue curve)



- Fit yields two poles in the region of the PDG estimate $1365 \pm 15 - i\,95 \pm 15$ MeV.

1.7 GeV Bare Roper: Hamiltonian Model N' Spectrum



Positive-parity Nucleon Spectrum: Bare Roper Case with $m_0 = 2.0$ GeV

J. j. Wu, *et al.* [CSSM], arXiv:1703.10715 [nucl-th]

- Consider πN, $\pi\Delta$ and σN channels, dressing a bare state.
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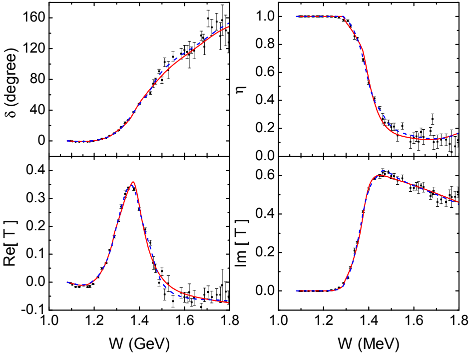
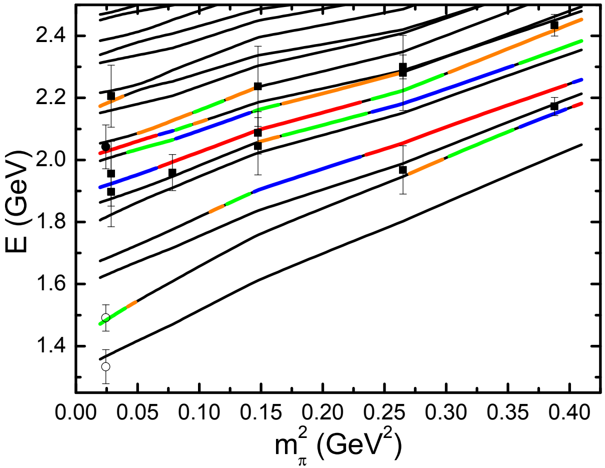


- Fit yields a pole at $1393 - i\,167$ MeV $\sim$ PDG estimate $1365 \pm 15 - i\,95 \pm 15$ MeV.

2.0 GeV Bare Roper: Hamiltonian Model N' Spectrum



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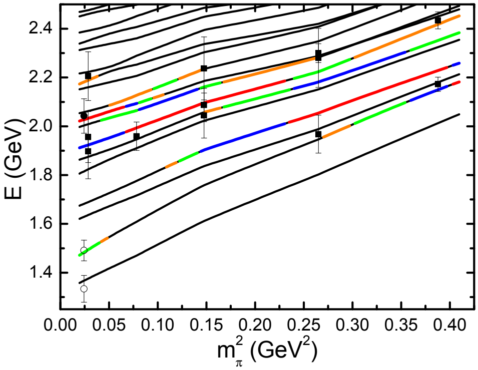

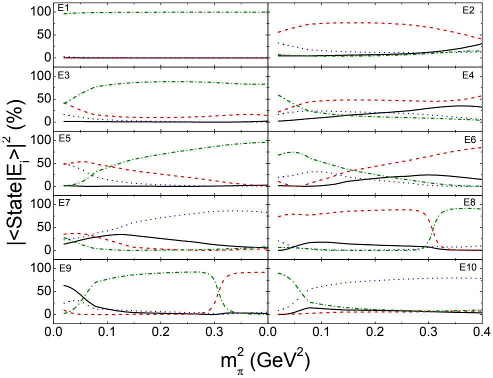

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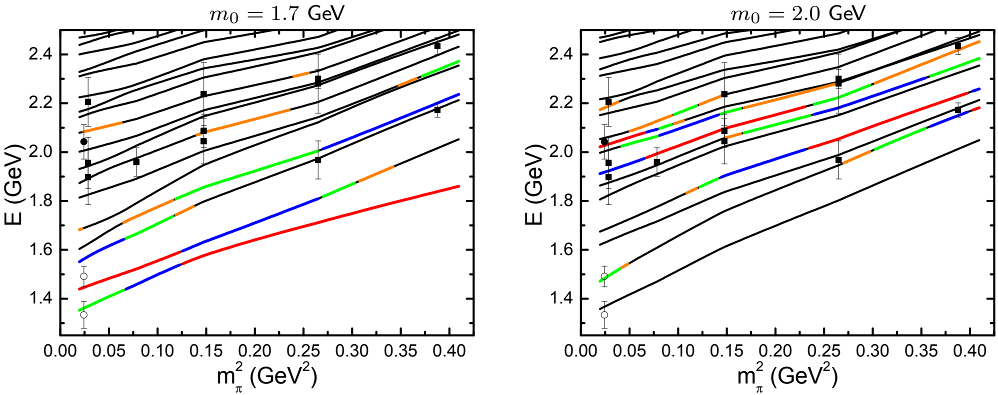
C. B. Lang, L. Leskovec, M. Padmanath and S. Prelovsek, Phys. Rev. D **95**, no. 1, 014510 (2017) [arXiv:1610.01422 [hep-lat]].

Two different descriptions of the Roper resonance



(left) Meson dressings of a quark-model like core.

(right) Resonance generated by strong rescattering in meson-baryon channels.

Score Card

Criteria	$m_0 = 1.7$ GeV	$m_0 = 2.0$ GeV
Describes experimental data well.		

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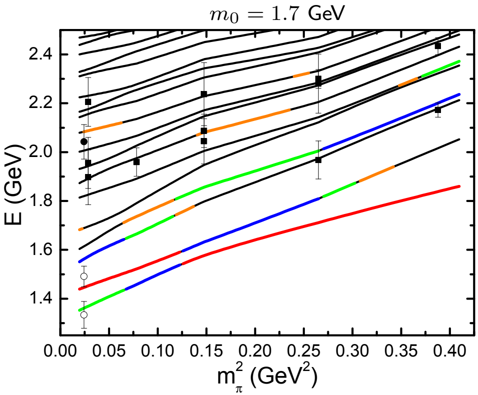

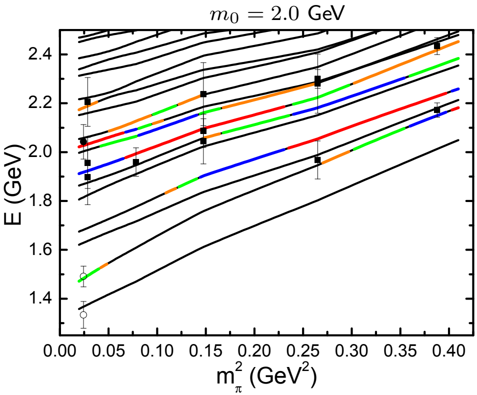

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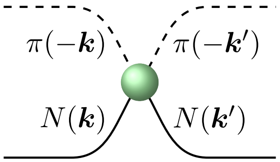

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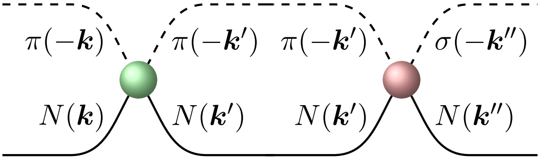

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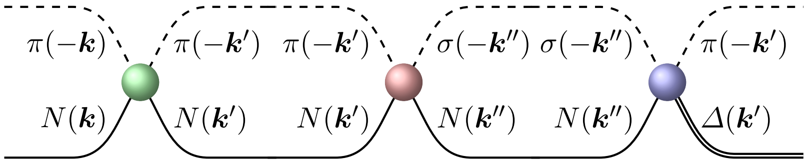

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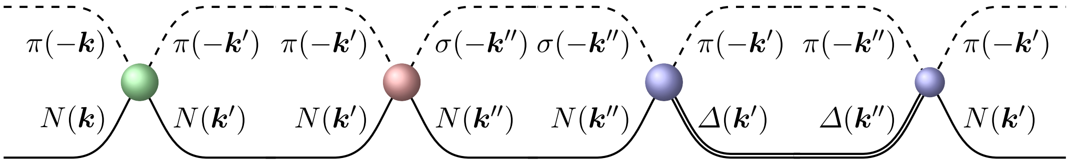

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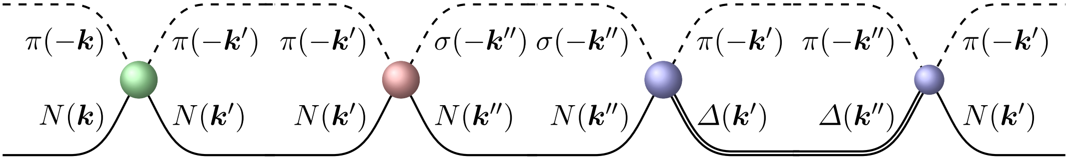

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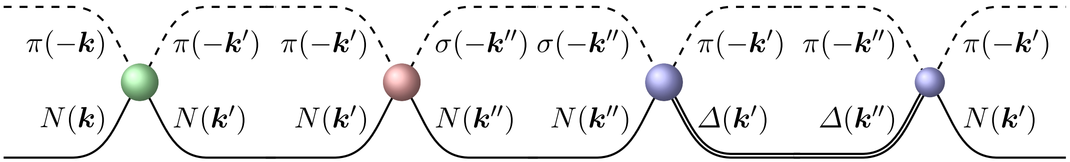

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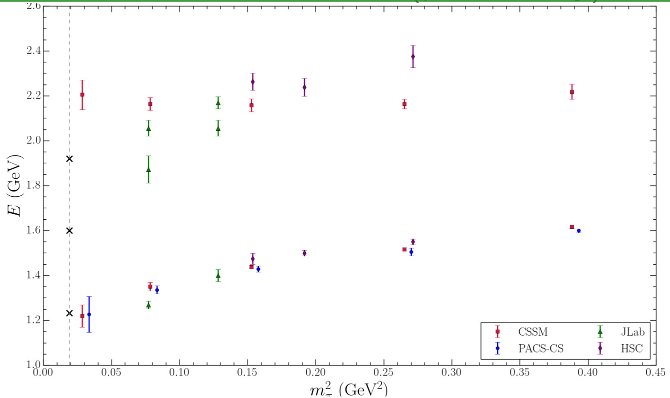
Δ-baryon spectrum from lattice QCD (preliminary)



HSC: J. Bulava, *et al.*, Phys. Rev. D **82** (2010) 014507 [arXiv:1004.5072 [hep-lat]].

JLab: T. Khan, D. Richards and F. Winter, Phys. Rev. D **104** (2021) 034503 [arXiv:2010.03052 [hep-lat]].

PACS-CS: S. Aoki *et al.* [PACS-CS], Phys. Rev. D **79** (2009) 034503 [arXiv:0807.1661 [hep-lat]].

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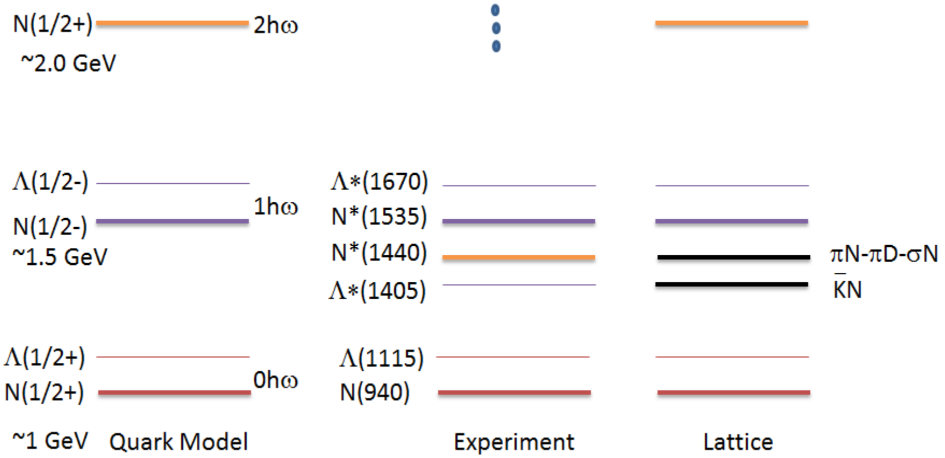
  - Knowledge of eigenstate composition can be used to understand the states observed.
  - Dominated by a quark-core bare state dressed by meson degrees of freedom.
- Roper $N(1440)$ Resonance: Arises from dynamical coupled-channel effects.
  - Lattice QCD results constrain the HEFT description of experimental data.
  - State composition matches when the $2s$ excitation of the quark model sits at ~ 2 GeV.

The spectrum of quark-model-like states is relatively simple



HEFT Extensions

- Formalism for partial-wave mixing in HEFT has been developed in Y. Li, J. J. Wu, C. D. Abell, D. B. L. and A. W. Thomas. Phys. Rev. D **101**, no.11, 114501 (2020) [arXiv:1910.04973 [hep-lat]]

- And extended to moving and elongated finite-volumes in Y. Li, J. J. Wu, D. B. L. and A. W. Thomas Phys. Rev. D **103** no.9, 094518 (2021) [arXiv:2103.12260 [hep-lat]].