Extraction of the six leading-order proton polarizabilities

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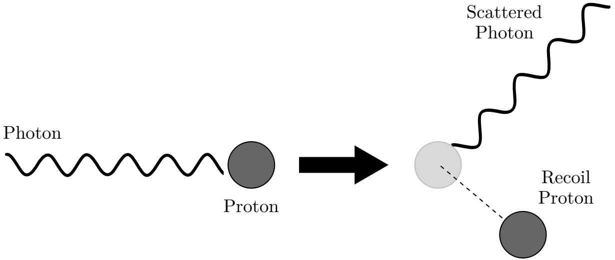
Leading-order proton polarizabilities

Our goal!

Extract the six leading-order proton polarizabilities (α_{E1}, β_{M1}, γ_{E1E1}, γ_{M1M1}, γ_{E1M2}, γ_{M1E2}) from Compton scattering data

- Fundamental properties related to the proton internal structure
- They limit the precision to different area of physics (atomic physics, astrophysics, ...)
- Up to now, no global extraction of all six polarizabilities were done



$$\gamma(k) + P(q) \rightarrow \gamma(k') + P(q')$$

The internal structure of the proton can be accessed by measuring unpolarized cross-section and polarization observables for Compton scattering

Compton scattering - Hamiltonian

Born term

Under the assumption of NO proton internal structure, the effective Hamiltonian can be written in terms of mass, electric charge and anomalous magnetic moment

- Zeroth order: mass and electric charge

$$H_{\text{eff}}^{(0)} = \frac{\vec{\pi}^2}{2m} + e\phi \qquad (\text{where } \vec{\pi} = \vec{p} - e\vec{A})$$

- First order: anomalous magnetic moment

$$H_{\text{eff}}^{(1)} = -\frac{e(1+k)}{2m}\vec{\sigma}\cdot\vec{H} - \frac{e(1+2k)}{8m^2}\vec{\sigma}\cdot\left[\vec{E}\times\vec{\pi} - \vec{\pi}\times\vec{E}\right]$$

Compton scattering - Hamiltonian

Scalar polarizabilities

Effective Hamiltonian at the second order includes scalar polarizabilities, which are related to the proton internal structure

- Second order: scalar polarizabilities α_{E1} and β_{M1}

$$H_{\text{eff}}^{(2)} = -4\pi \left[\frac{1}{2} \alpha_{E1} \vec{E}^2 + \frac{1}{2} \beta_{M1} \vec{H}^2 \right]$$

Spin polarizabilities

Effective Hamiltonian at the third order includes spin polarizabilities, which describe the response of the proton spin to an applied electric or magnetic field

- Third order: spin polarizabilities γ_{E1E1}, γ_{M1M1}, γ_{M1E2} and γ_{E1M2}

$$H^{(3)}_{\text{eff}} = -4\pi\left[\frac{1}{2}\gamma_{E1E1}\vec{\sigma}\cdot(\vec{E}\times\dot{\vec{E}}) + \frac{1}{2}\gamma_{M1M1}\vec{\sigma}\cdot(\vec{H}\times\dot{\vec{H}}) - \gamma_{M1E2}E_{ij}\sigma_i H_j + \gamma_{E1M2}H_{ij}\sigma_i E_j\right]$$

Polarizabilities extractions from data

Fit of the data using a theory model (Dispersion Relations (DRs) or χPT based) with the polarizabilities as free parameters

- Existing extractions
  - Some of the polarizabilities were fixed to given values to reduce uncertainties and correlations
  - Inclusion of a selected part of the entire database
  - Standard gradient, χ^2 minimization using first and second derivative
    - Experimental points assumed Guassian distributed around the model prediction ($\hat{\mathcal{T}}$)

$$e_i \in G[\hat{\mathcal{T}}_i, \sigma_i^2], \qquad \chi^2 = \sum_i \left(\frac{e_i - \hat{\mathcal{T}}_i}{\sigma_i} \right)^2$$

    - Adding systematic uncertainties

$$e_{i,j} \notin G[\hat{\mathcal{T}}_i, \sigma_i^2], \qquad \chi^2 = \sum_{i,j} \left(\frac{\delta_j e_i - \hat{\mathcal{T}}_i}{\delta_j \sigma_i} \right)^2 + \left(\frac{\delta_j - 1}{\sigma_j^{sys}} \right)^2$$

Polarizabilities extractions from data

- Intrinsic problems
  - Systematic errors induce correlations among points making fit parameter distribution not gaussian
  - The error propagation for uncertainties on unfitted model parameters is not always trivial
  - An additional fitting parameter is needed for each dataset
  - The “modified” χ^2 is not a “proper” χ^2 anymore. Which probability distribution should be used?

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  - Include as many data as possible!
  - Datasets ranging over 50 years, and including six different observables
  - Every dataset has its own systematic error

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✓ Parametric bootstrap

Parametric bootstrap

- Independently on the probability distribution (P) of the the experimental points, one can sample from there

$$b_{i,j,k} \in P\left[e_i, \sigma_i^2, \Delta_j\right], \qquad \chi_k^2 = \sum_i \left(\frac{e_{i,k} - \hat{\mathcal{T}}_{i,k}}{\sigma_{i,k}} \right)^2$$

- The obtained distribution can be fitted using Simplex (or gradient) method, and the resulting set of best-value parameters saved
- The probability distribution of the fit parameters can be reconstructed from the fit
- Common systematic uncertainties can be included without any *a priori* distribution assumption
- Uncertainties on nuisance model parameters are taken into account in the sampling procedure
- Fit p-value is provided if goodness-of-fit distribution is not given by the χ^2

P. Pedroni and S. Sconfietti, J. Phys. G **47**, *054001 (2020)*

Our fit

- Systematic errors are assumed Gaussian distributed
- Point-to-point systematic errors are added in quadrature to statistical ones
- Common systematic errors are assumed to be uniform distributed (unless otherwise specified)

$$e_{i,j} \to b_{i,j,k} = (1 + \delta_{j,k})(e_{i,j} + r_{i,j,k}\sigma_{i,j}), \qquad b_{i,j,k} \in G\left[e_{i,j}, \sigma^2_{i,j}\right] \otimes U\left[-\Delta_k, +\Delta_k\right]$$

- $k = 1, \ldots, 10^4$ bootstrap cycles
- Six free parameters
  - $\alpha_{E1} + \beta_{M1}$, $\alpha_{E1} - \beta_{M1}$,
  - γ_{E1E1}, γ_{M1M1},
  - $\gamma_0 = -\gamma_{E1E1} - \gamma_{M1M1} - \gamma_{E1M2} - \gamma_{M1E2}$, and $\gamma_\pi = -\gamma_{E1E1} + \gamma_{M1M1} - \gamma_{E1M2} + \gamma_{M1E2}$
- Three different PWA solutions used: MAID-2021, SAID-MA19, BnGA-2019
- Polarizability best-values are taken as the mathematical averages of the three results using the three different PWAs

Datasets

As many data points as possible were included in the fit!

The final database included

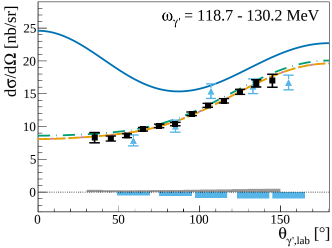
- All existing unpolarized low-energy data ($E_\gamma < 150$ MeV)
    - 14 datasets, 218 points[1]
- New-generation (a.k.a. photon-tagged) unpolarized high-energy data ($E_\gamma = [150 - 300]$ MeV)
    - 4 datasets, 23 points
- Polarized ($\sigma_{||}$, $\sigma_\perp$, Σ_{2x}, Σ_{2z}, and Σ_3) data
    - 7 datasets, 137 points[2]

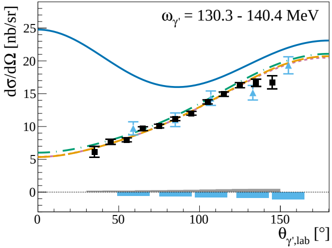
For a total of 388 data points divided in 25 datasets!

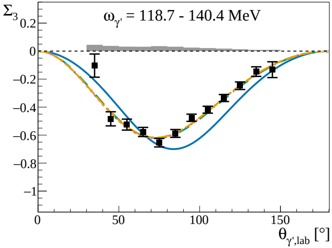
[1] including 10 above-threshold points from TAPS
[2] 65 below- and 72 above-threshold

A2@MAMI ~~measurement~~ advertisement

- Highest precision Compton scattering dataset below π-photoproduction threshold!
    - 1.2 millions Compton scattering events
    - 60 data points of differential unpolarized cross section
    - 36 data points of beam asymmetry
- Crucial to have enough statistics for the concurrent extraction of the six polarizabilities







A2: Phys. Rev. Lett. **128** (2022)

A2 systematic errors

TAPS: Eur Phys J A **10**, 207 (2001)

TAPS systematic errors

Born contribution

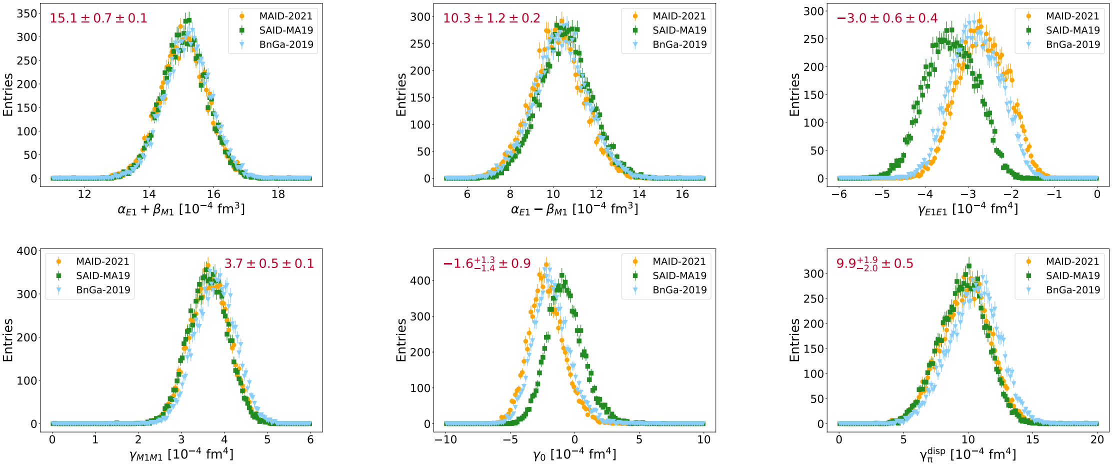
DR: Phys. Rev. C **76**, 015203 (2007)

HBχPT: Eur. Phys. J. A **49**, 12 (2013)

BχPT: Eur. Phys. J. C **65**, 195 (2010)

E. Mornacchi et al. (A2), Phys. Rev. Lett. ***128****, 132503 (2022)*

Parameter distributions

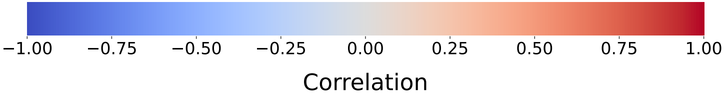


Fit results

	$\alpha+\beta$	$\alpha-\beta$	γ_{E1E1}	γ_{M1M1}	γ_0	γ_π
$\alpha+\beta$	1.0	0.34	0.04	-0.2	-0.17	-0.32
$\alpha-\beta$		1.0	-0.51	0.34	0.36	0.62
γ_{E1E1}			1.0	-0.35	-0.21	-0.67
γ_{M1M1}				1.0	-0.39	0.81
γ_0					1.0	0.13
γ_π						1.0

$\hat{\chi}^2_{\text{red}} = 1.13$

p-value = 24%



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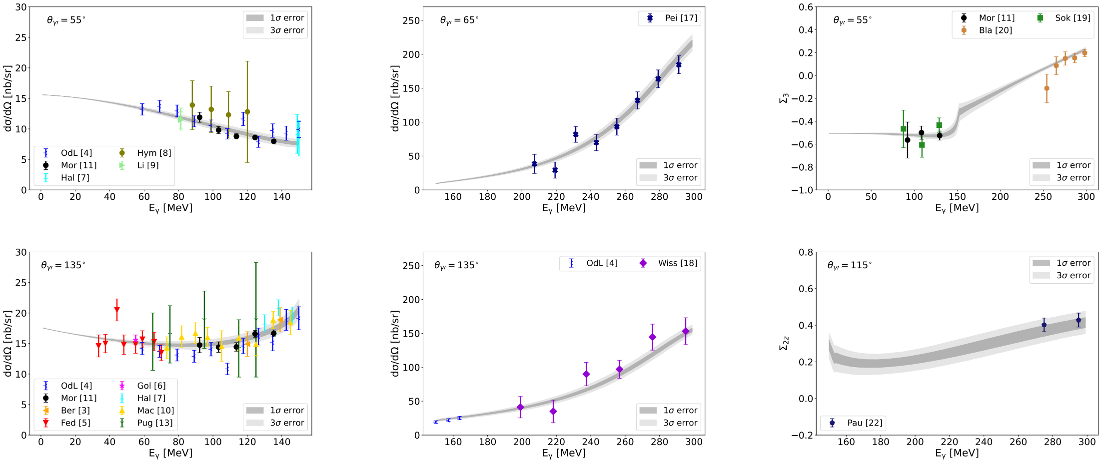
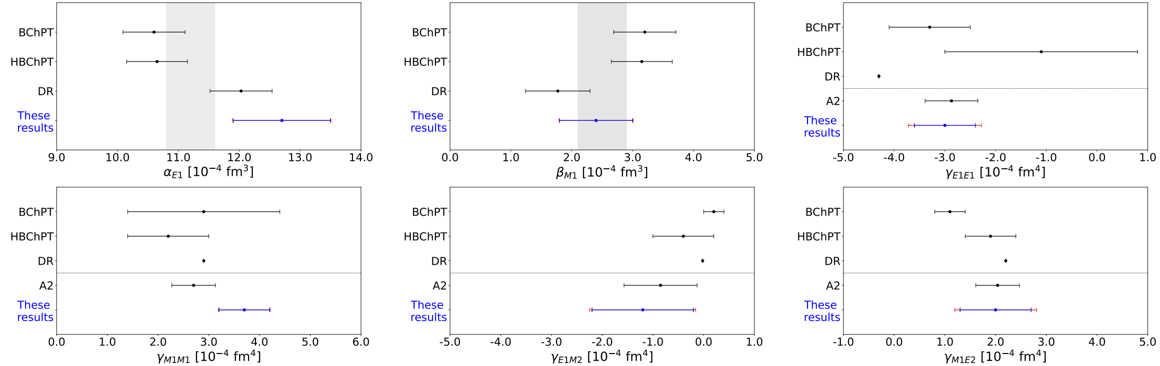


E.M., S. Rodini, B. Pasquini, P. Pedroni, Phys. Rev. Lett. **129**, *102501 (2022)*

Polarizability results



$$\alpha_{E1} = 12.7 \pm 0.8 \pm 0.1; \quad \beta_{M1} = 2.4 \pm 0.6 \pm 0.1; \quad \gamma_{E1E1} = -3.0 \pm 0.6 \pm 0.4$$

$$\gamma_{M1M1} = 3.7 \pm 0.5 \pm 0.1; \quad \gamma_{E1M2} = -1.2 \pm 1.0 \pm 0.3; \quad \gamma_{M1E2} = 2.0 \pm 0.7 \pm 0.4$$

E.M., S. Rodini, B. Pasquini, P. Pedroni, Phys. Rev. Lett. **129**, *102501 (2022)*

Conclusions

- Bootstrap fitting technique successfully used!
  - Extremely useful and versatile technique to be used when gradient method is not possible
  - Straightforward inclusion of different datasets with various systematic error distributions
  - As any bootstrap technique, it relies on large database with possibly small statistical errors (New high statistics A2 data proven to be essential for this extraction!)

- The first concurrent extraction of the six leading-order proton polarizabilities has been performed using fixed-*t* DRs and Bootstrap fitting technique!
  - Polarizability values in agreement with the existing ones
  - Competitive errors with those of the previous extractions performed using constraints
  - High correlation still exists among γ_{M1M1} and γ_π

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Thanks for your attention!