Challenges and prospects for baryonic resonances from lattice QCD

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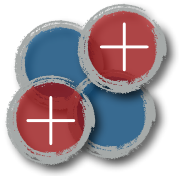
THE UNIVERSITY of EDINBURGH

Lattice QCD: *Recipe for strong force predictions*

1. Lagrangian defining QCD
2. Formal / numerical machinery (lattice QCD)
3. A few experimental inputs (e.g. M_π, M_K, M_Ω)

$$\mathcal{L}_{\rm QCD} = \sum_f \overline{\Psi}_f (i\not{D} - m_f)\Psi_f - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$



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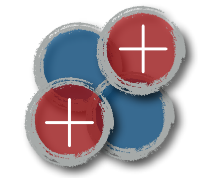
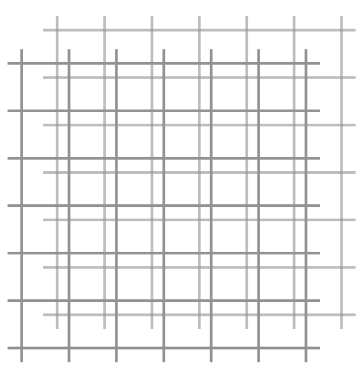


Wide range of precision pre-/post-dictions



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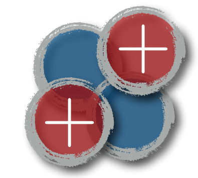

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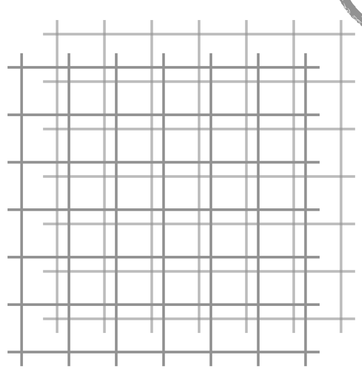
$m_q^{\overline{\rm MS}}(\mu)$ m_p $\alpha_{\overline{\rm MS}}(\mu)$ m_n f_π g_A f_K $(g-2)_\mu^{\rm HVP}$ f_D f_B



FLAG 2019 Flavour Lattice Averaging Group

$\alpha^{(5)}_{\overline{\rm MS}}(M_Z) = 0.1182(8)$ lattice average

$\alpha^{(5)}_{\overline{\rm MS}}(M_Z) = 0.1174(16)$ PDG 18 (non-lattice)

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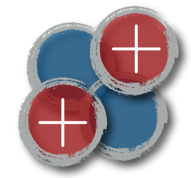

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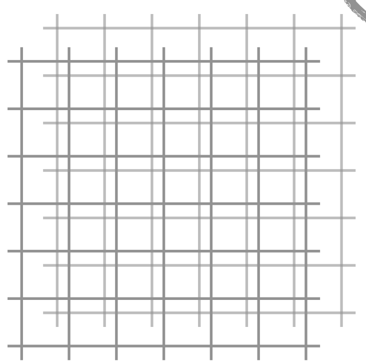

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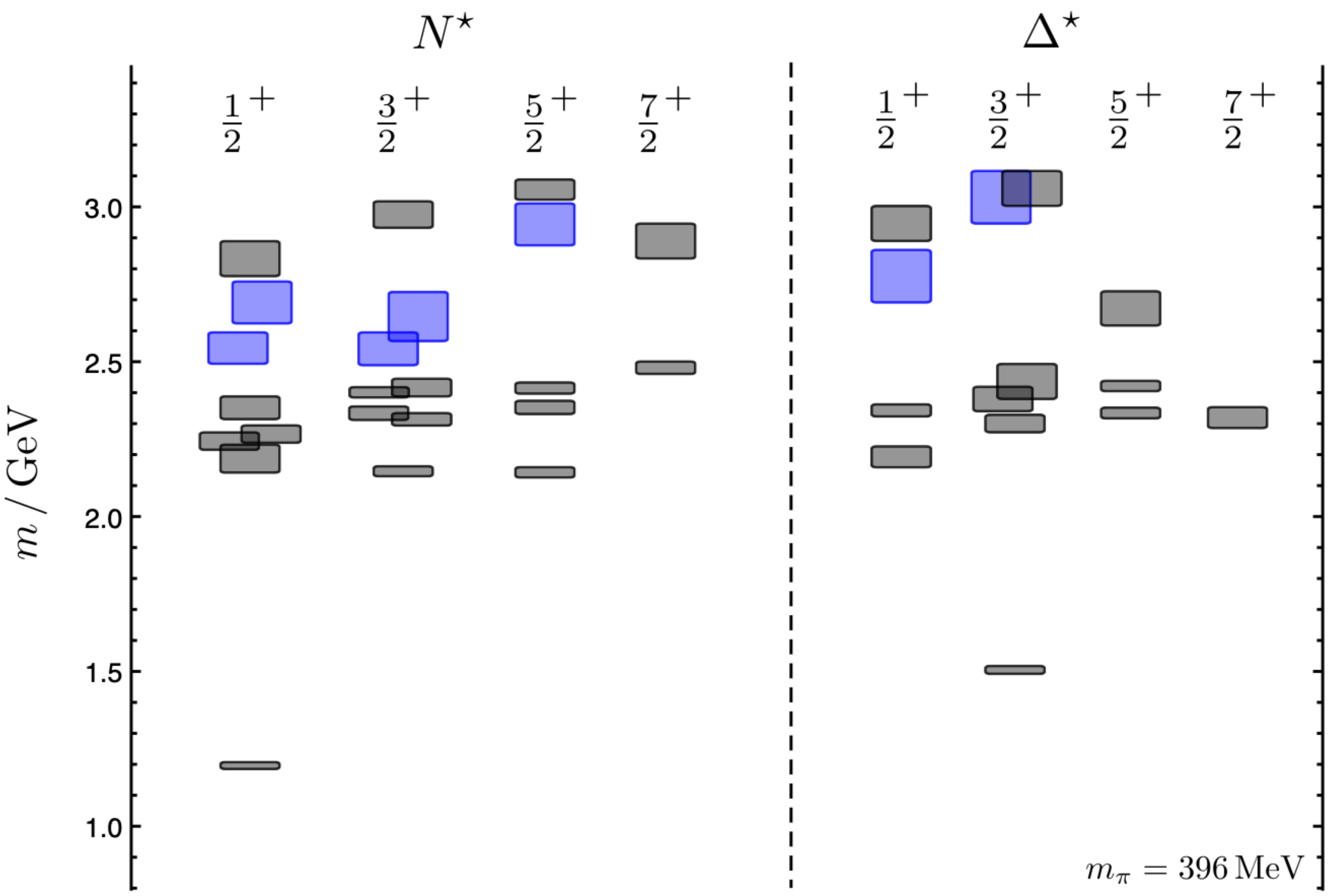
Overwhelming evidence for QCD ✓ Lattice QCD as a reliable tool ✓

More challenging observables? 😮

Baryonic resonances



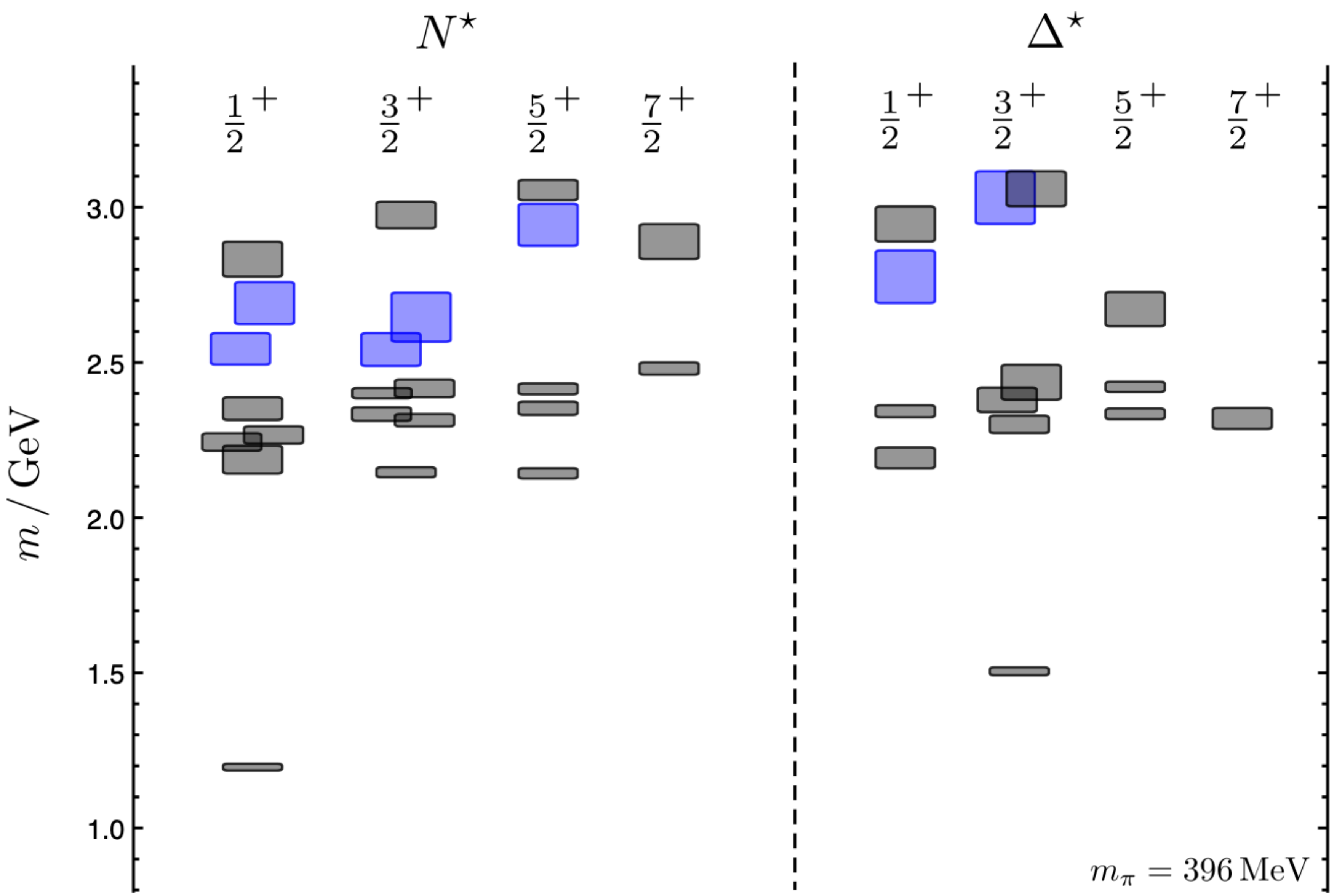
heavier than physical M_π

Height of rectangle = statistical uncertainty

States created using local operators

- Dudek and Edwards, *Hybrid Baryons in QCD*, PRD, 2012

Baryonic resonances



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States created using local operators

Not quite right to say "*States cannot decay*" (more later)

Remarkable progress… but not the complete picture!

• Dudek and Edwards, *Hybrid Baryons in QCD*, PRD, 2012 •

QCD Fock space

- At low-energies QCD = hadronic degrees of freedom $\pi \sim \bar{u}d,\ K \sim \bar{s}u,\ p \sim uud$
- Overlaps of multi-hadron ***asymptotic states*** → S matrix

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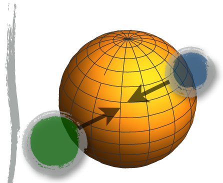
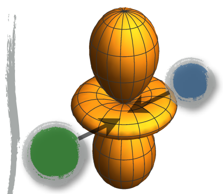
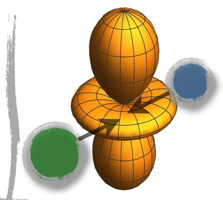
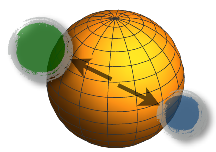
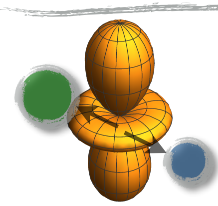
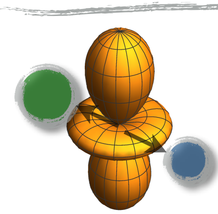
$$S(s) \equiv \langle N\pi, \text{out}| \quad |N\pi, \text{in}\rangle$$

			
	$e^{2i\delta_{1/2,0}(s)}$	0	0
	0	$e^{2i\delta_{1/2,1}(s)}$	0
	0	0	$e^{2i\delta_{3/2,1}(s)}$

depends on $s = E_{\text{cm}}^2$
and angular variables

diagonal in total angular momentum

$$\mathcal{M}_{J,\ell}(s) \propto e^{2i\delta_{J,\ell}(s)} - 1$$

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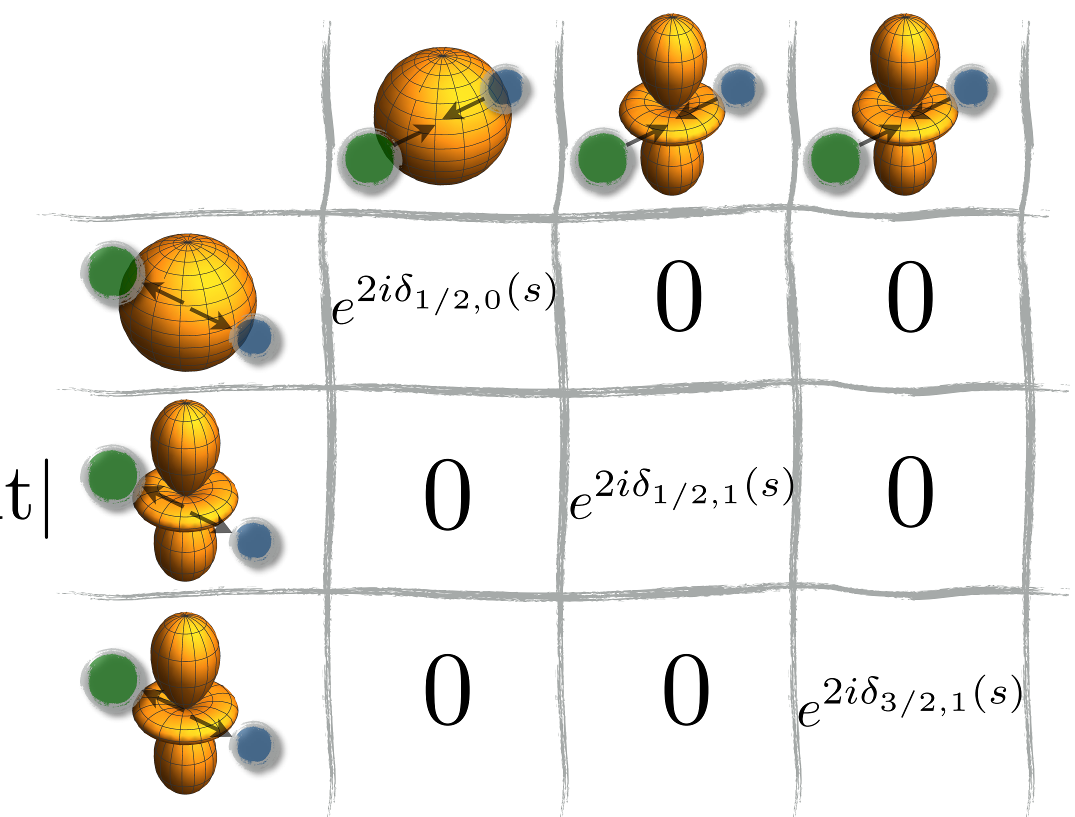

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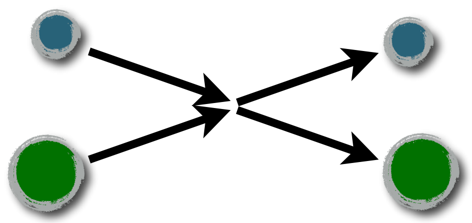
- Poles on the second Riemann sheet give resonances

Full QCD demands this description… lattice QCD cannot escape it

Unitarity and analyticity



- For $s < (M_N + 2M_\pi)^2$, the optical theorem tells us…

$$\rho_{N\pi}(s)|\mathcal{M}_{J,\ell}(s)|^2 = \mathrm{Im}\,\mathcal{M}_{J,\ell}(s)$$

where $\rho_{N\pi}(s) = \sqrt{p^2(s, M_\pi, M_N)/s}$ is the phase space… continuum of $N\pi$ states

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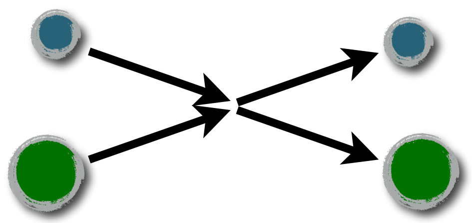

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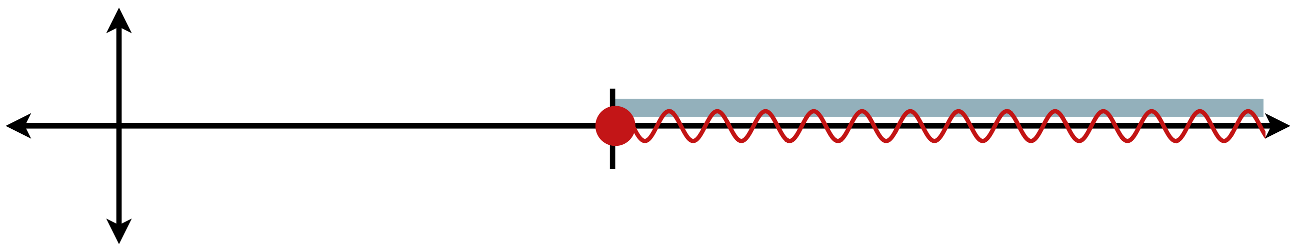
- Unique solution is…

$$\mathcal{M}_{J,\ell}(s) = \frac{1}{\mathcal{K}_{J,\ell}(s)^{-1} - i\rho_{N\pi}(s)}$$

K matrix (short distance)

phase-space cut (long distance)

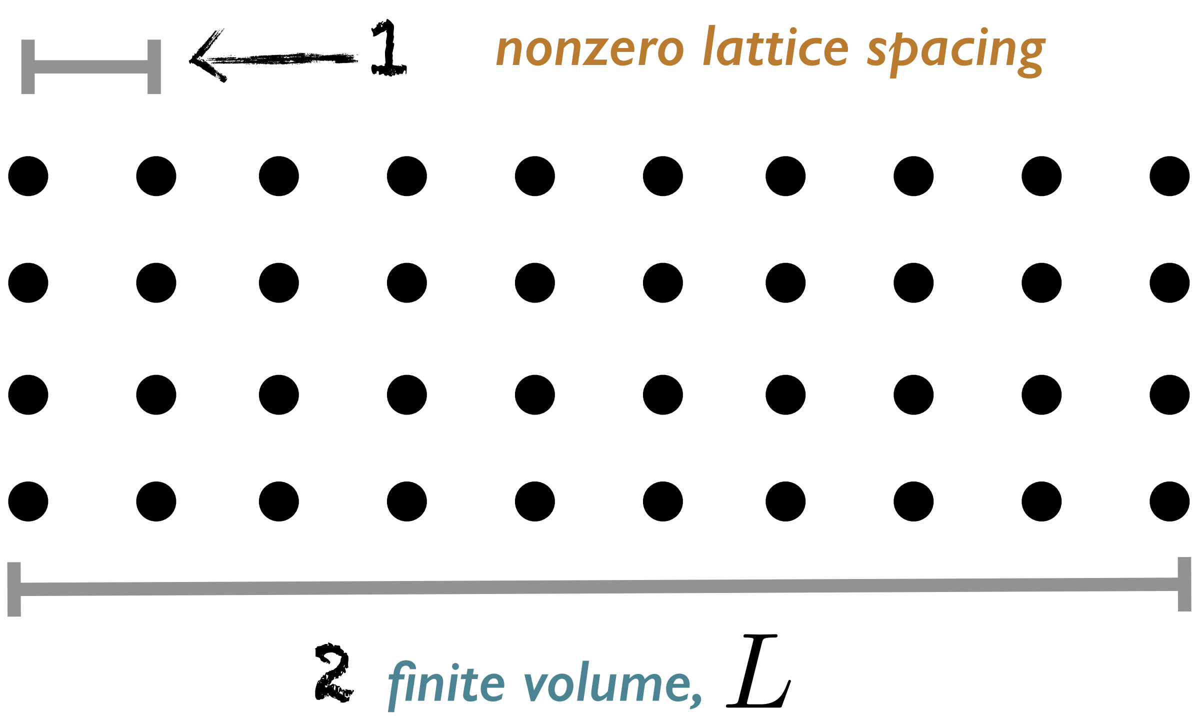


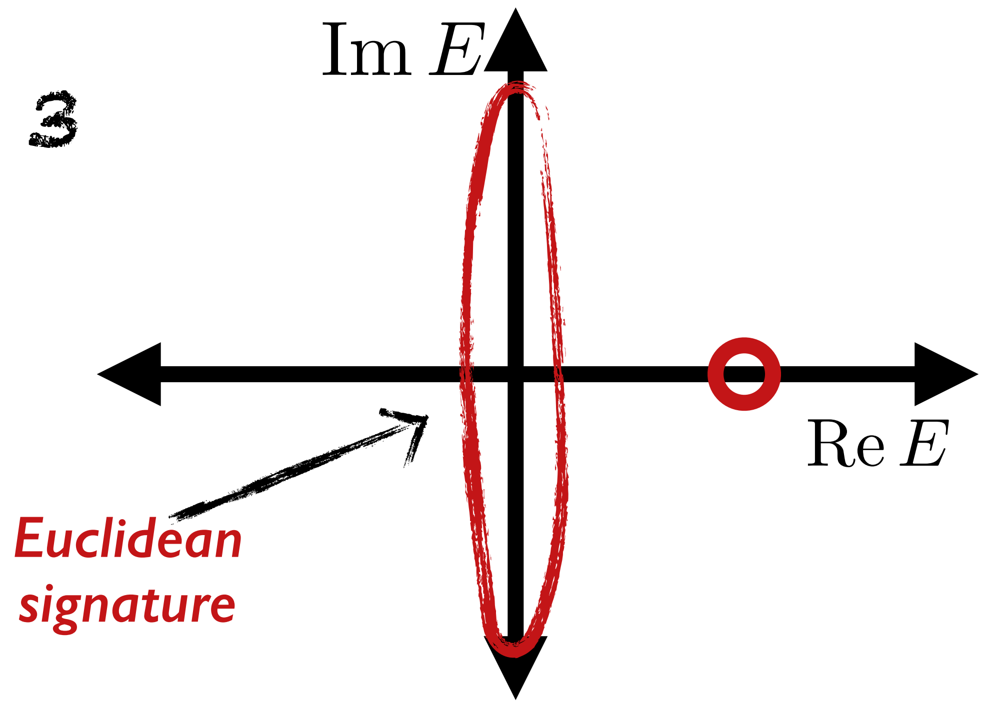
Amplitude has a branch cut ✓ K-matrix is useful for parametrizing ✓

Lattice QCD

$$\text{observable?} = \int d^N\phi \, e^{-S} \left[\begin{array}{c} \textbf{interpolator} \\ \textbf{for observable} \end{array} \right]$$

To proceed we have to make *three modifications*



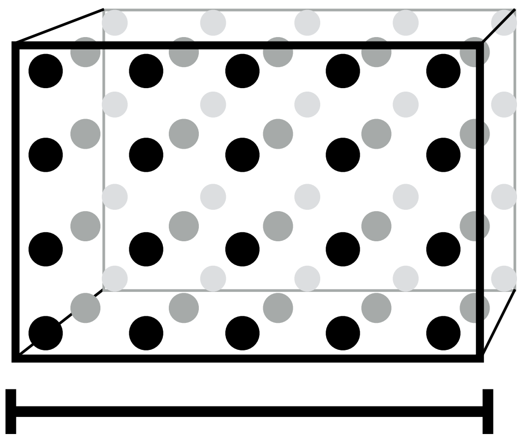


Also… $M_{\pi,\text{lattice}} > M_{\pi,\text{our universe}}$

(but physical masses → increasingly common)

Difficulties for multi-hadron observables

- The ***Euclidean signature***…
  - ***Prevents*** usual on-shell approach (want $p_4^2 = -E(p)^2$, but have only $p_4^2 > 0$)



- The ***finite volume***…
  - *Discretizes* the spectrum
  - *Eliminates* the branch cuts and extra sheets
  - *Hides* the resonance poles

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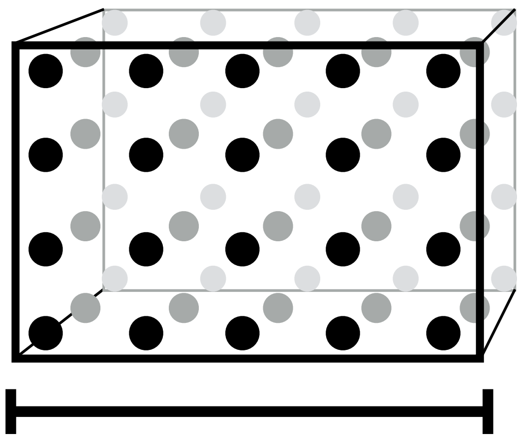

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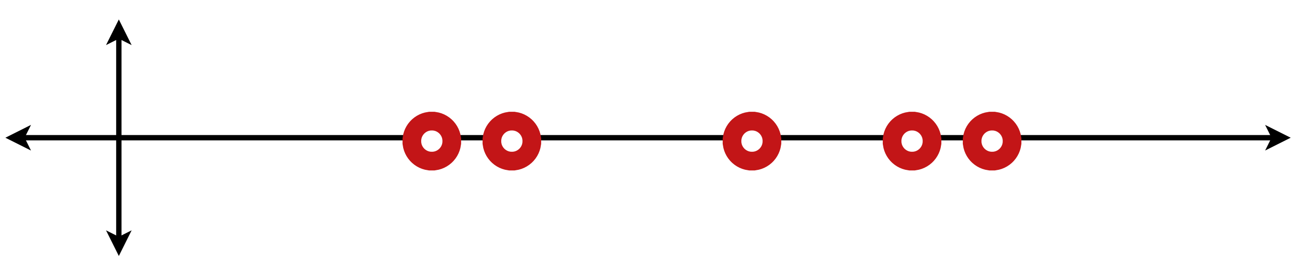
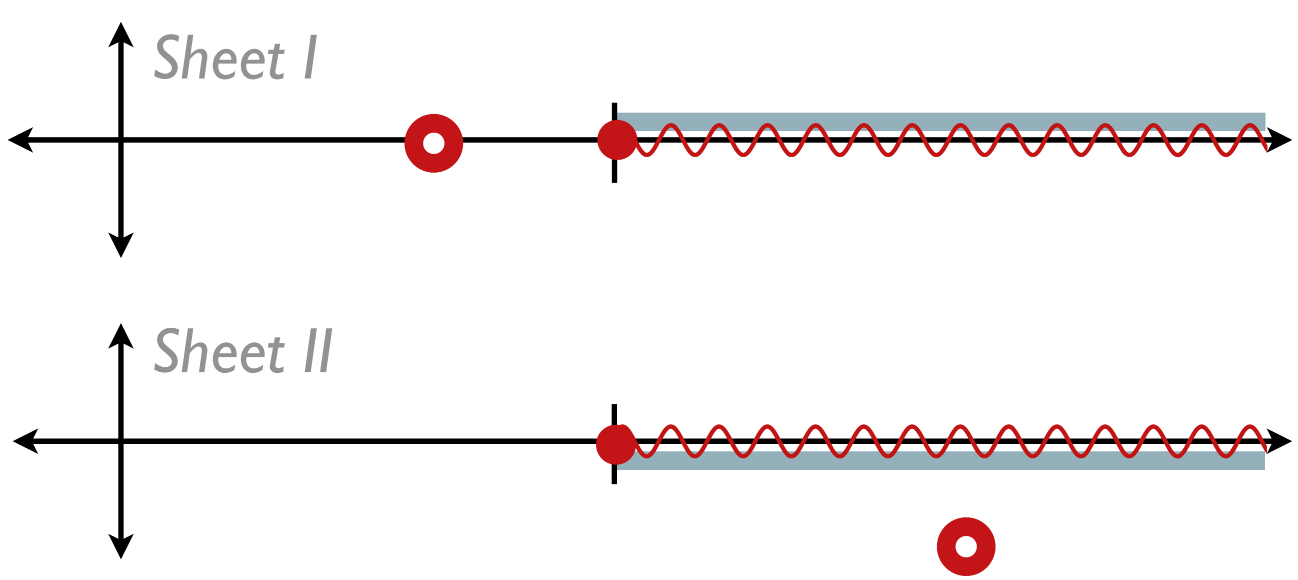
Finite-volume analytic structure



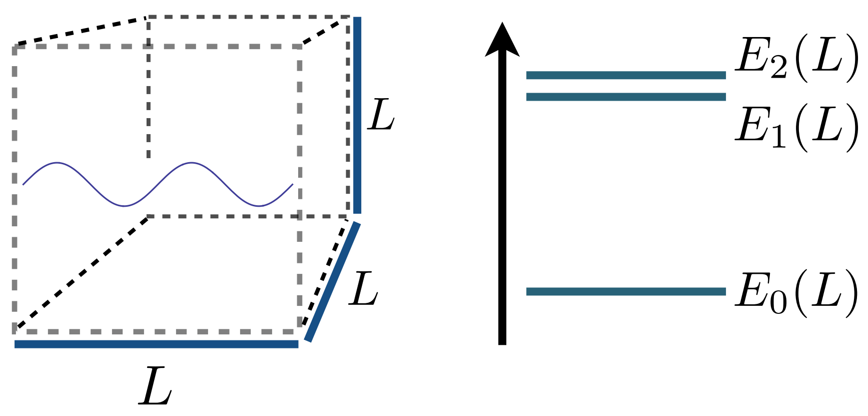
Note: cannot count finite-volume energies to count resonance poles!

Infinite-volume analytic structure



The finite-volume as a tool

- Finite-volume set-up



- **cubic**, spatial volume (extent *L*)
- **periodic**
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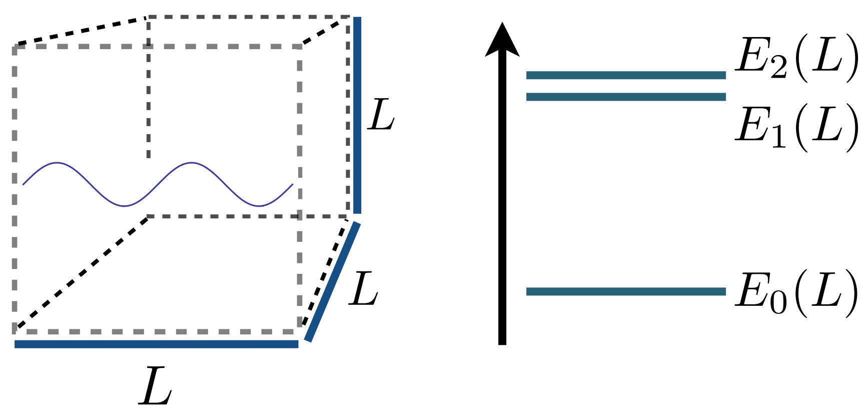

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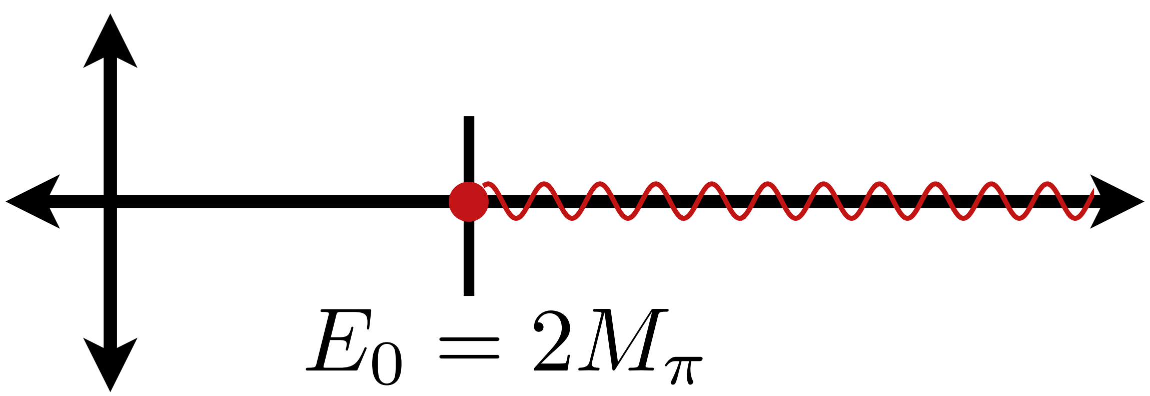
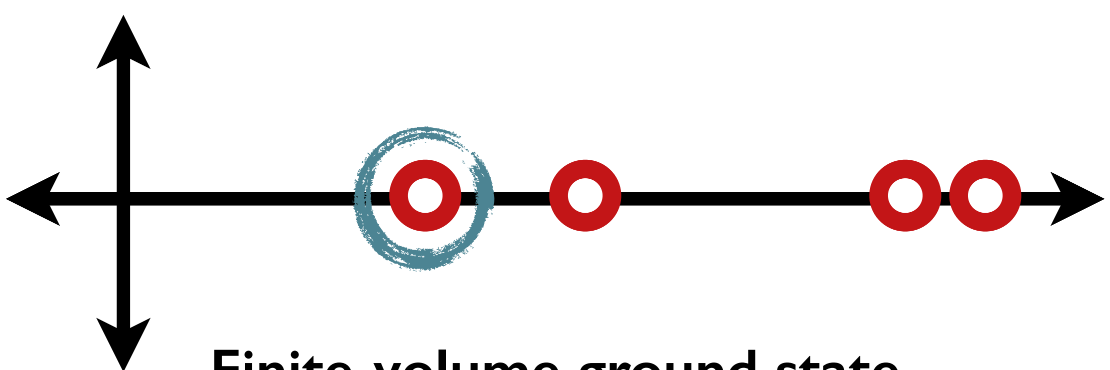
- Scattering leaves an *imprint* on finite-volume quantities



Infinite-volume threshold



Finite-volume ground state

$$\mathcal{M}_{\ell=0}(2M_\pi) = -32\pi M_\pi a$$

$$E_0(L) = 2M_\pi + \frac{4\pi a}{M_\pi L^3} + \mathcal{O}(1/L^4)$$

• Huang, Yang (1958) •

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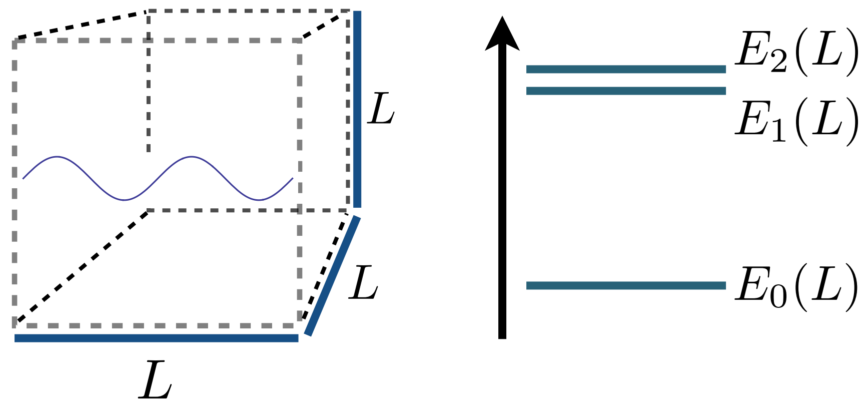

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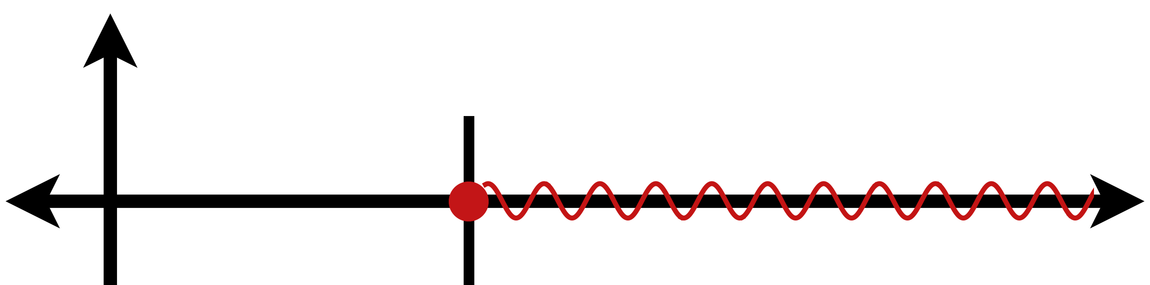

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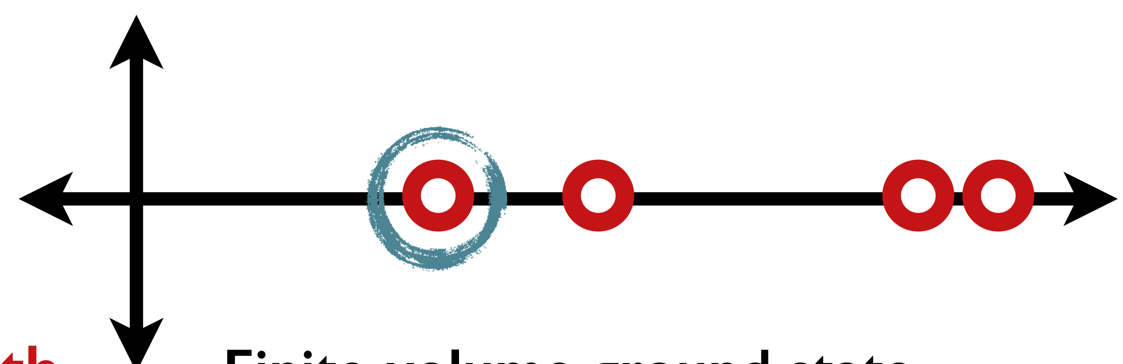

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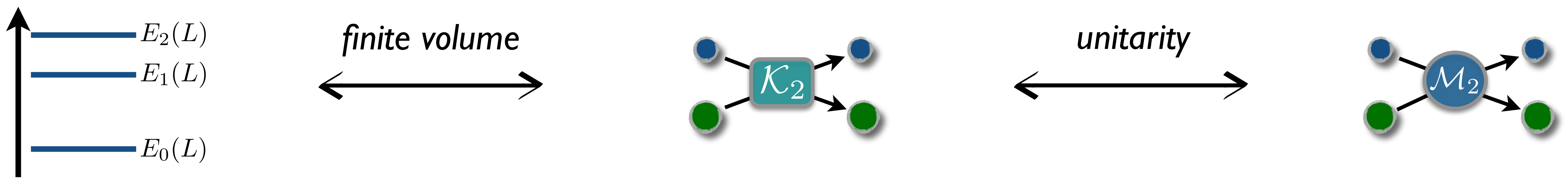
General method

$$\det[\mathcal{K}^{-1}(s) + F(P,L)] = 0$$

$F(P,L) \equiv$ Matrix of known geometric functions



Holds only for two-particle energies $s < (M_N + 2M_\pi)^2$ Neglects $e^{-M_\pi L}$

Generalized to *non-degenerate masses, multiple channels, spinning particles*

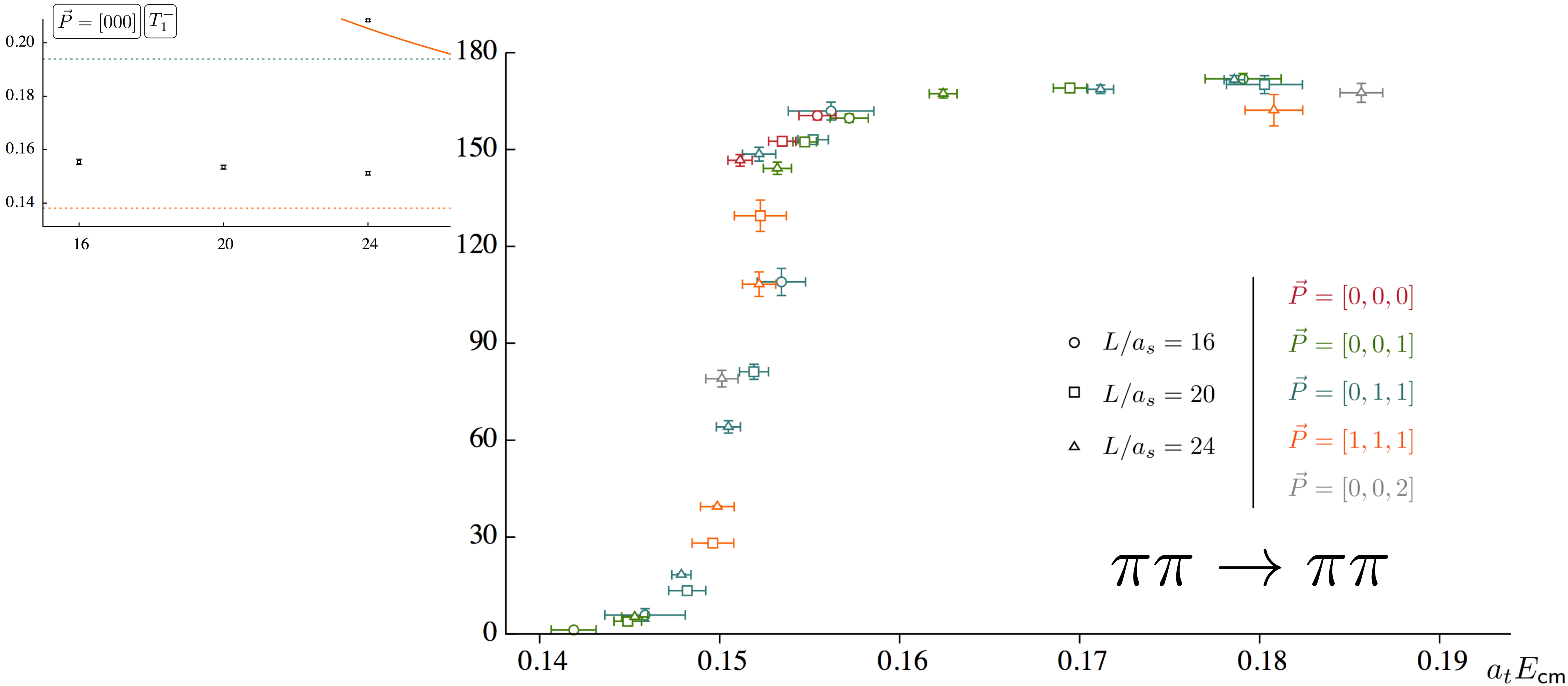
Encodes angular momentum mixing

Huang, Yang (1958) • Lüscher (1986, 1991) • Rummukainen, Gottlieb (1995)
Kim, Sachrajda, Sharpe (2005) • Christ, Kim, Yamazaki (2005) • He, Feng, Liu (2005)
Leskovec, Prelovsek (2012) • Bernard *et. al.* (2012) • MTH, Sharpe (2012) • Briceño, Davoudi (2012)
Li, Liu (2013) • Briceño (2014)

Using the result

- Single-channel case *(pions in a p-wave)*

$$\mathcal{K}(s_n)^{-1} = \rho \cot \delta(s_n) = -F(E_n, \vec{P}, L)$$



• Dudek, Edwards, Thomas in *Phys.Rev.* D87 (2013) 034505 •

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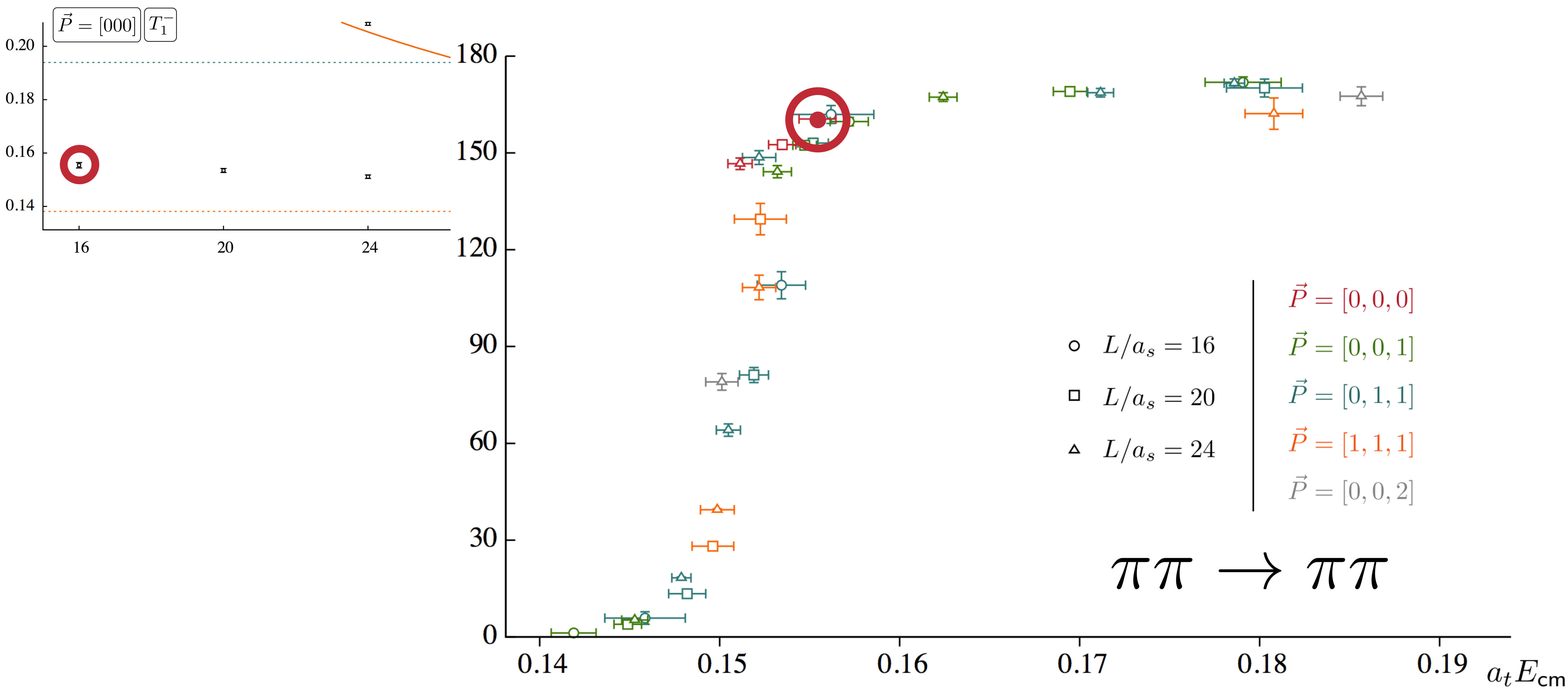

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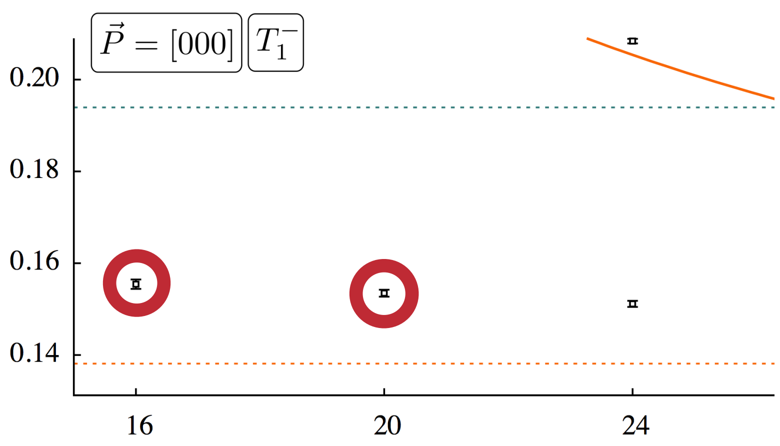

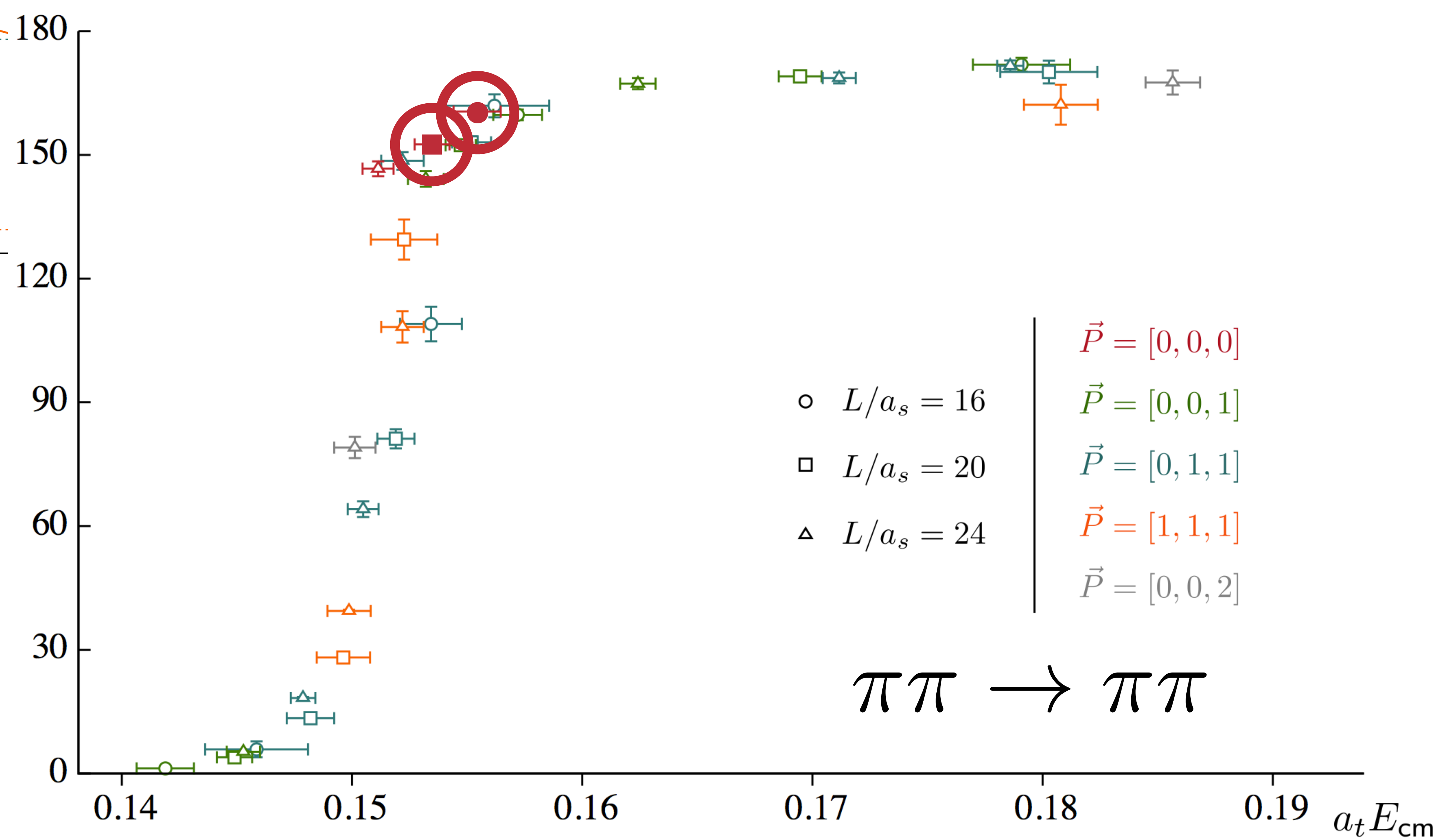

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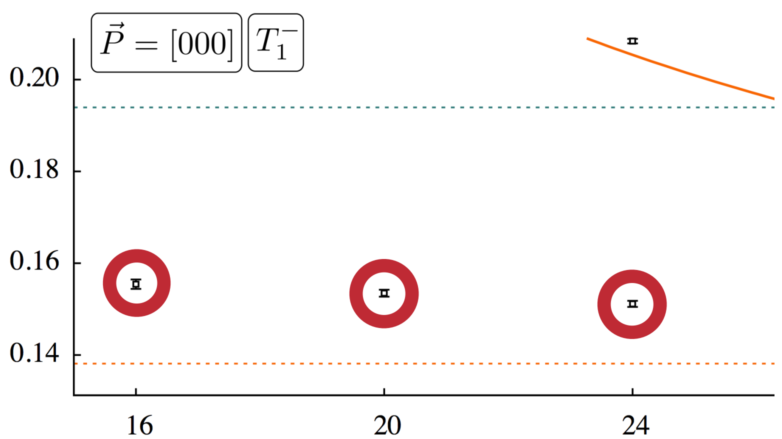

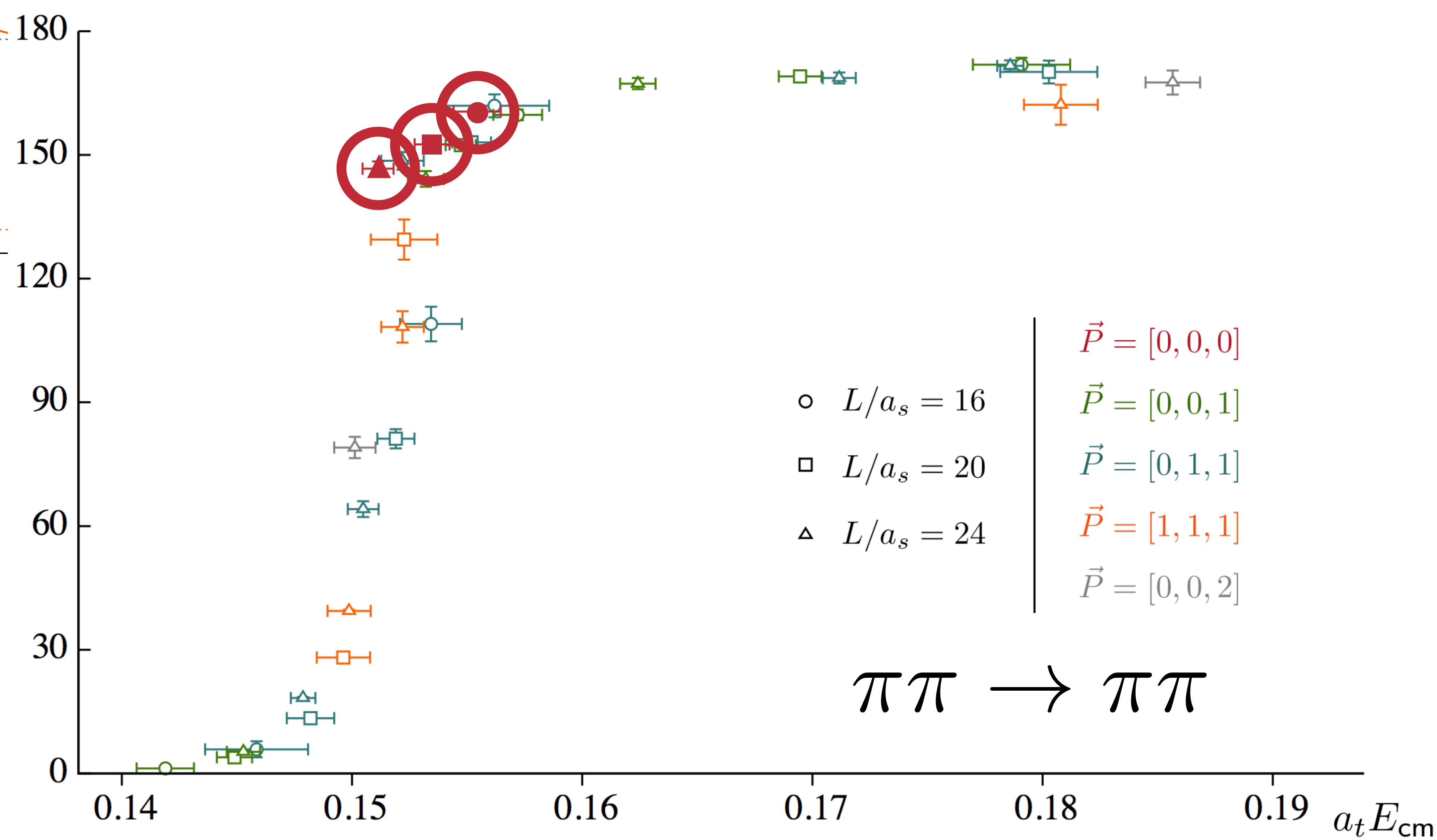

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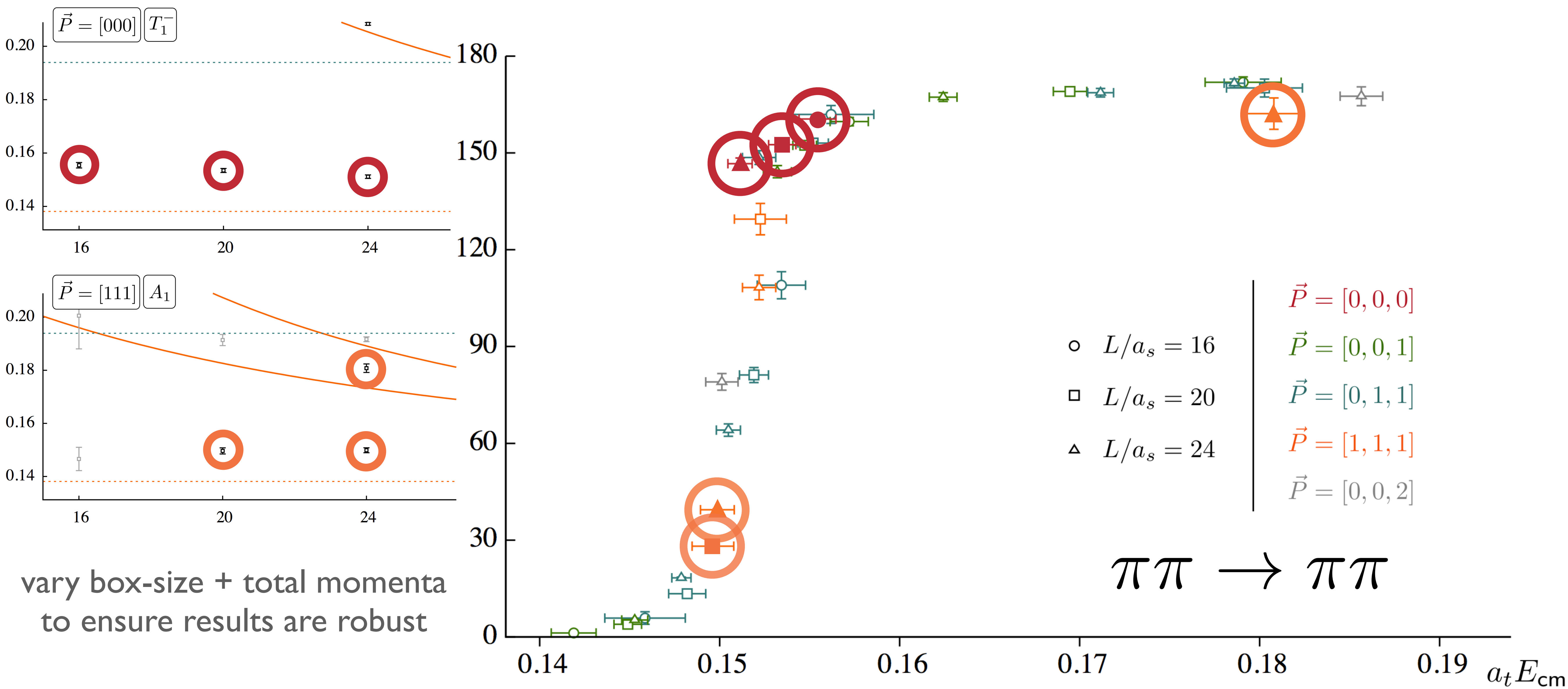


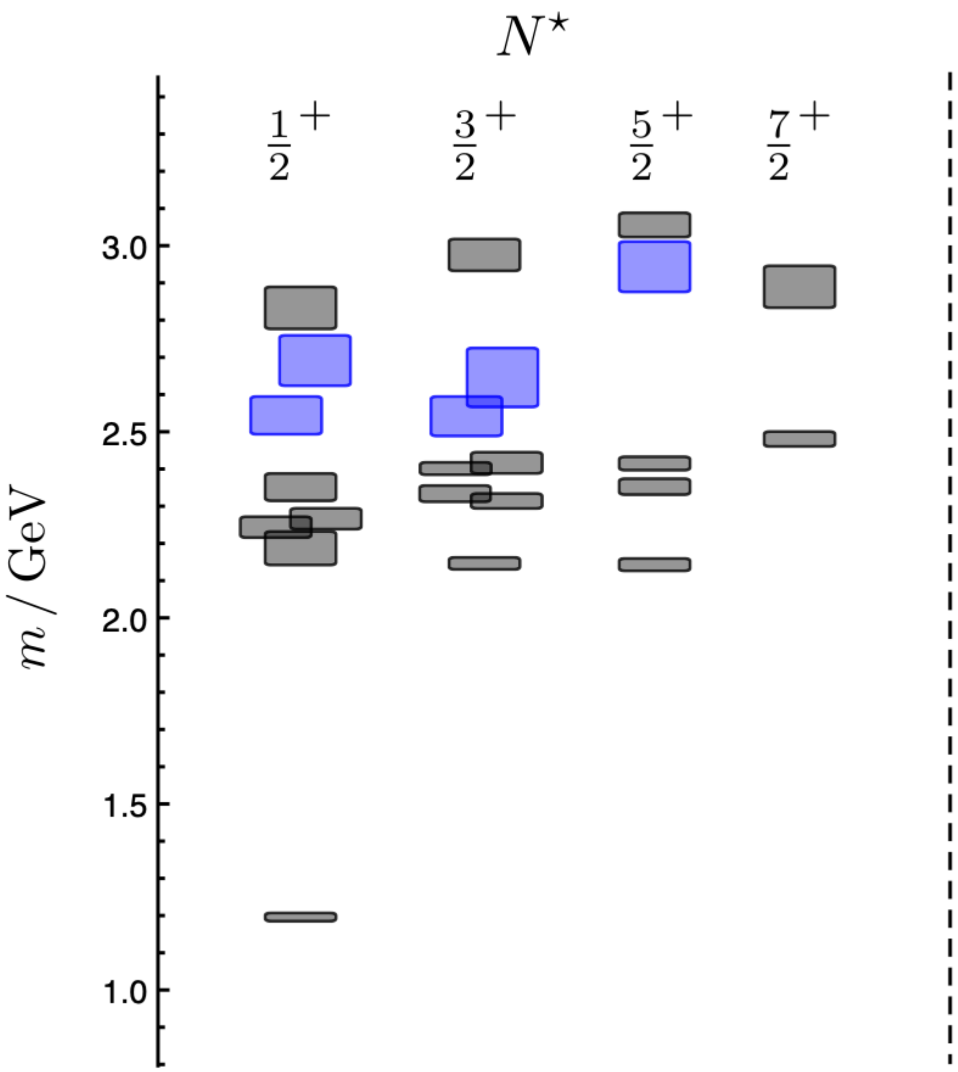
vary box-size + total momenta to ensure results are robust

• Dudek, Edwards, Thomas in *Phys.Rev.* D87 (2013) 034505 •

Two types of spectroscopy

Explore the spectrum of compact QCD excited states

(via quark-model inspired local operators)



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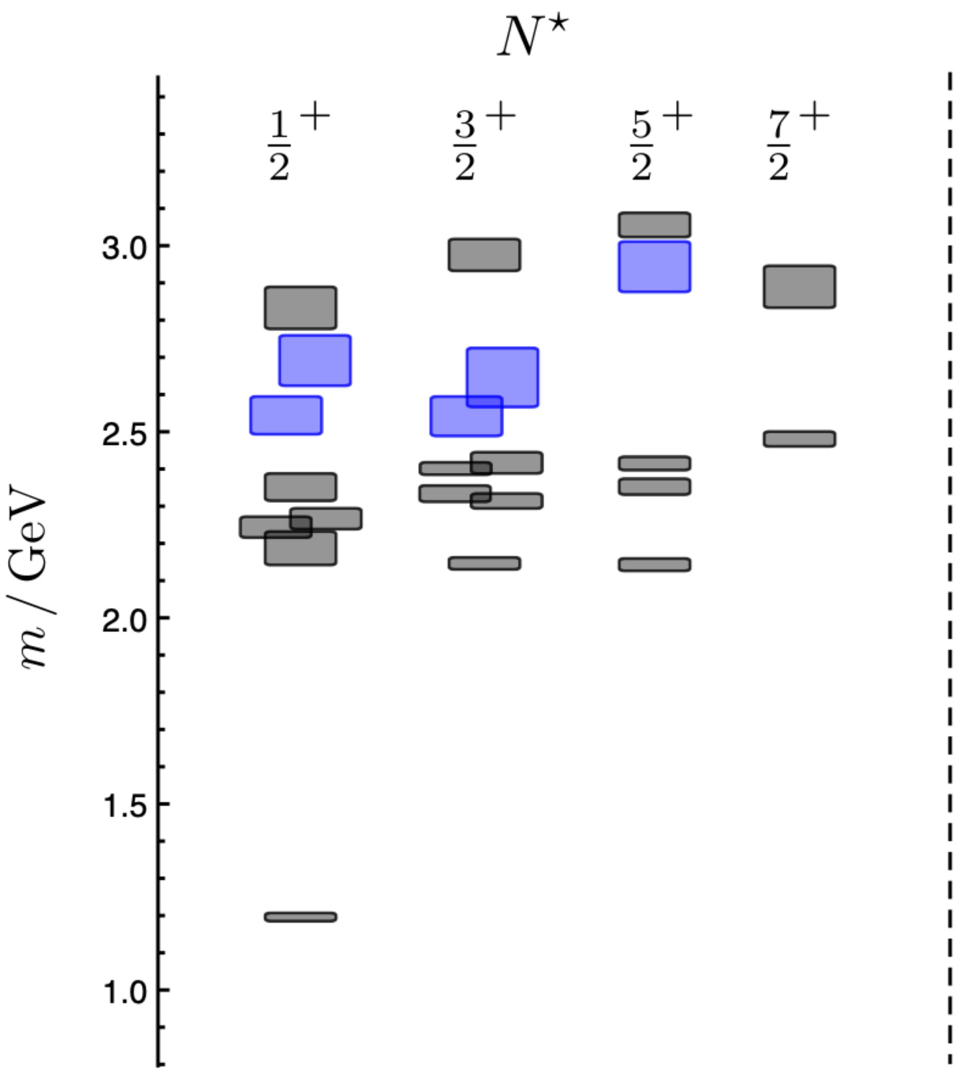


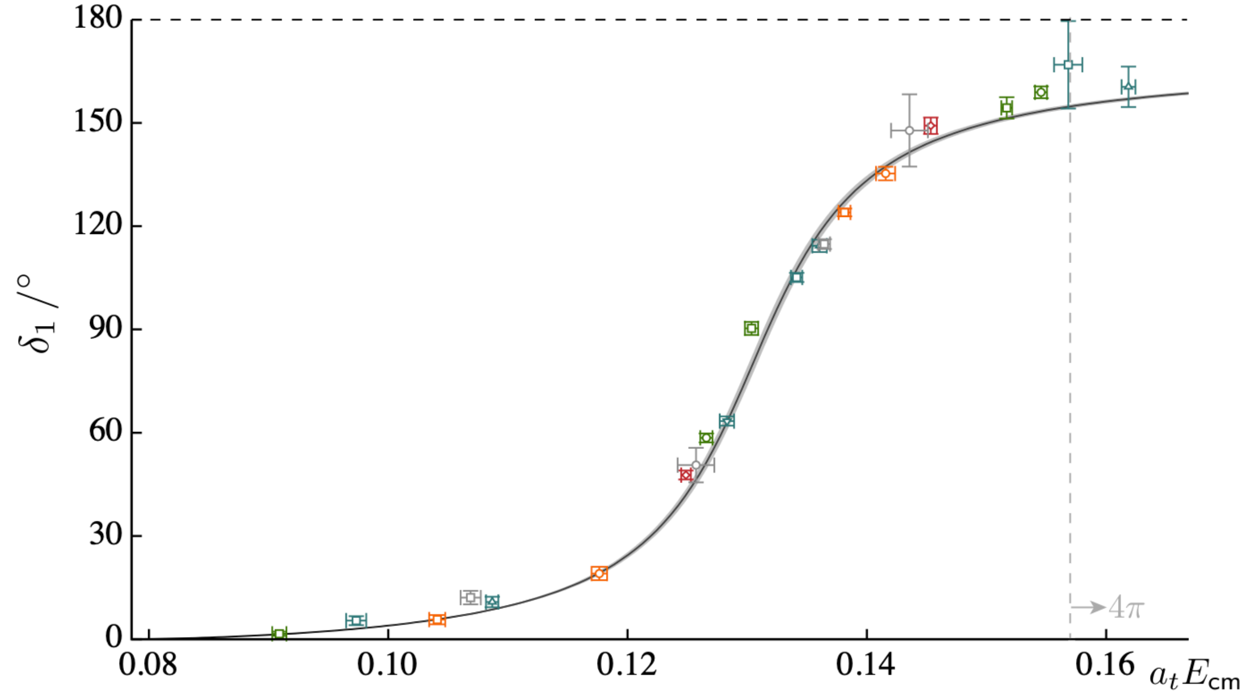
Dudek, Edwards (2012)

Extract the full finite-volume energy spectrum

local operators

+ many multi-hadron operators

$$\pi\pi \to \rho \to \pi\pi$$



Wilson, Briceño, Dudek, Edwards, Thomas (2015)

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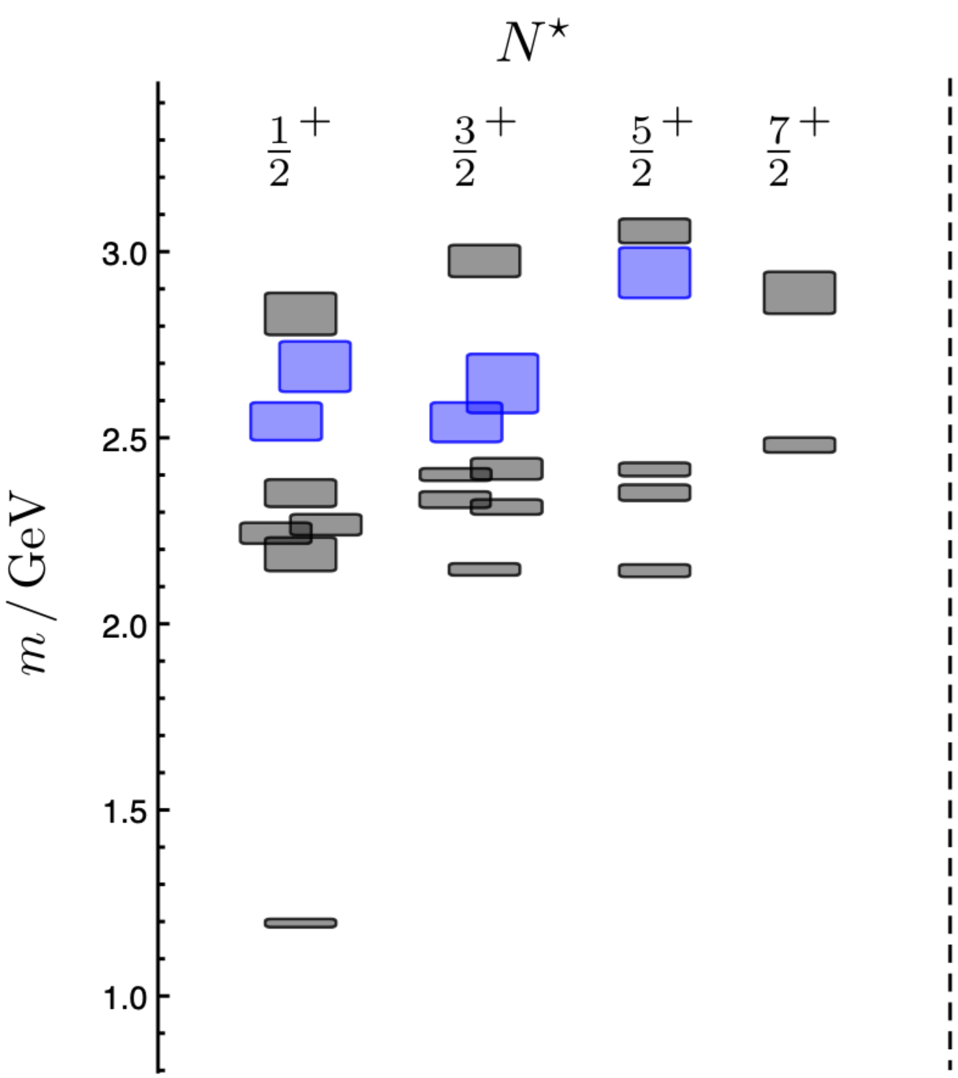
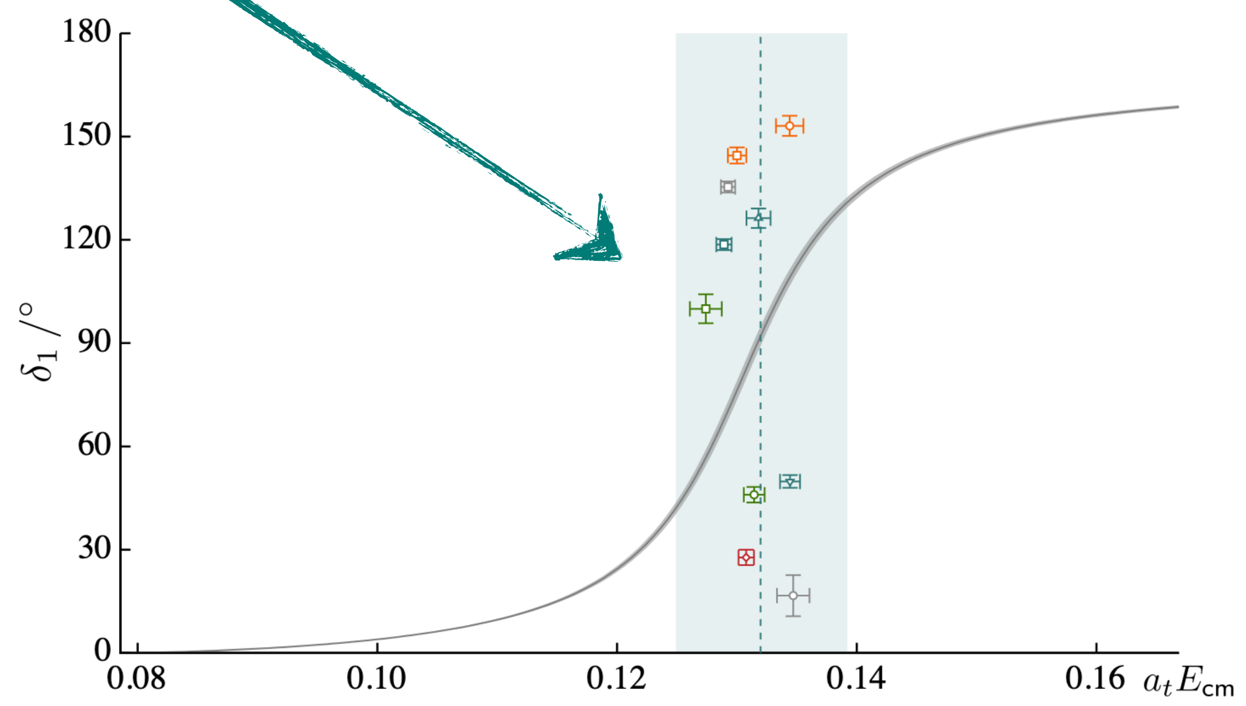


Dudek, Edwards (2012)

local operator spectrum =
not suitable for phase shift extraction

$\pi\pi \to \rho \to \pi\pi$



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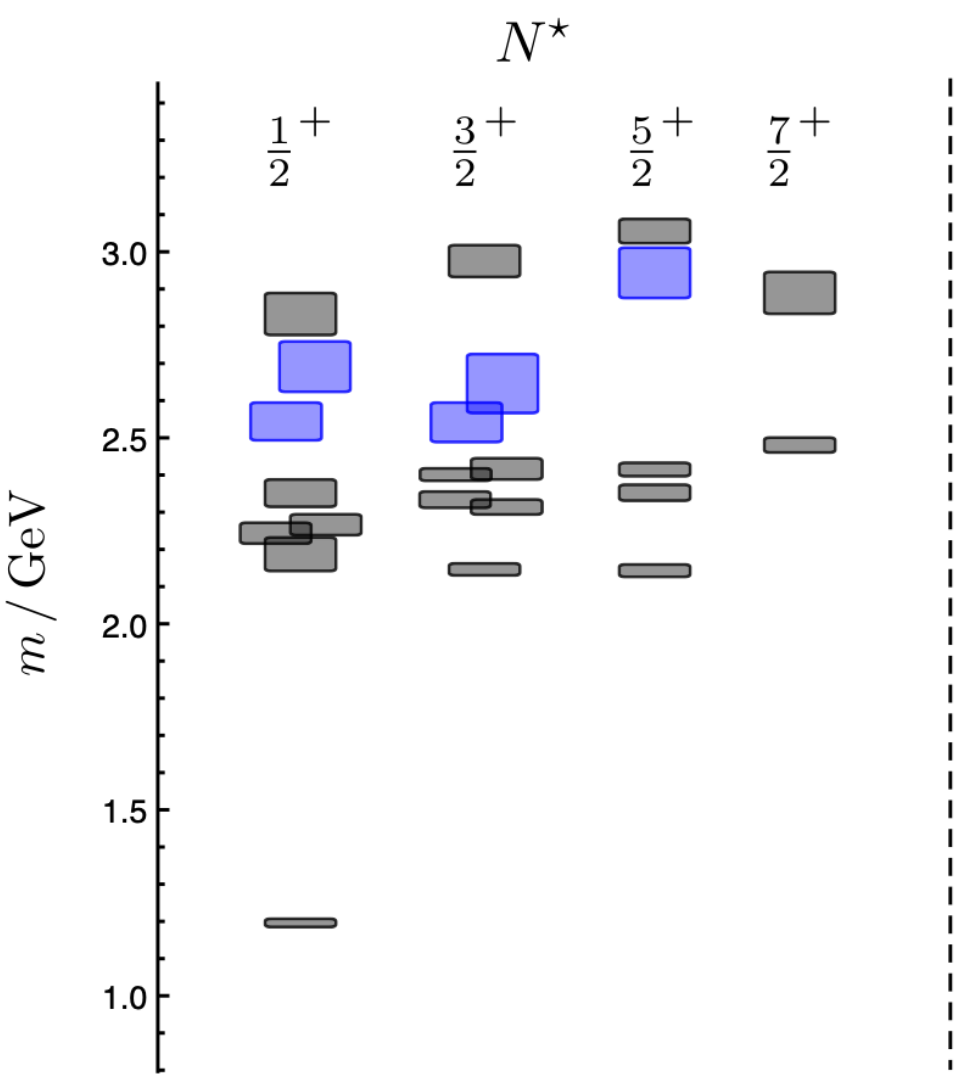
Explore the spectrum of compact QCD excited states

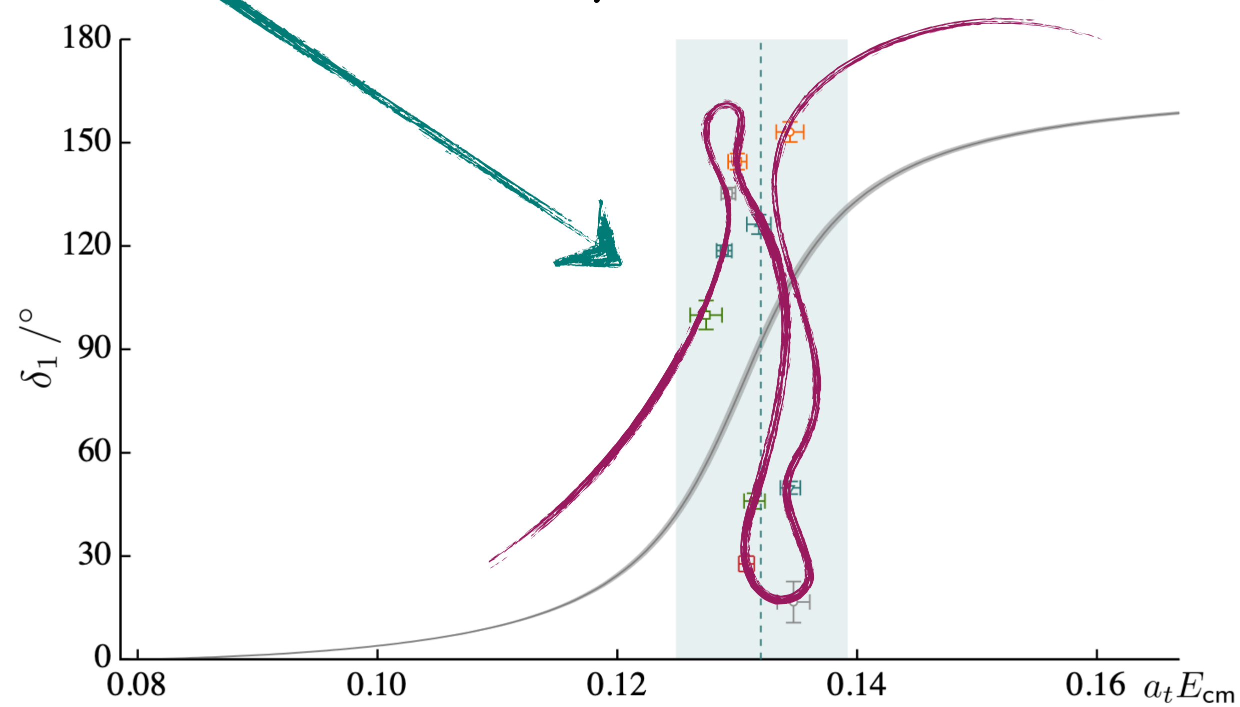
(via quark-model inspired local operators)



local operator spectrum =

not suitable for phase shift extraction

$$\pi\pi \to \rho \to \pi\pi$$



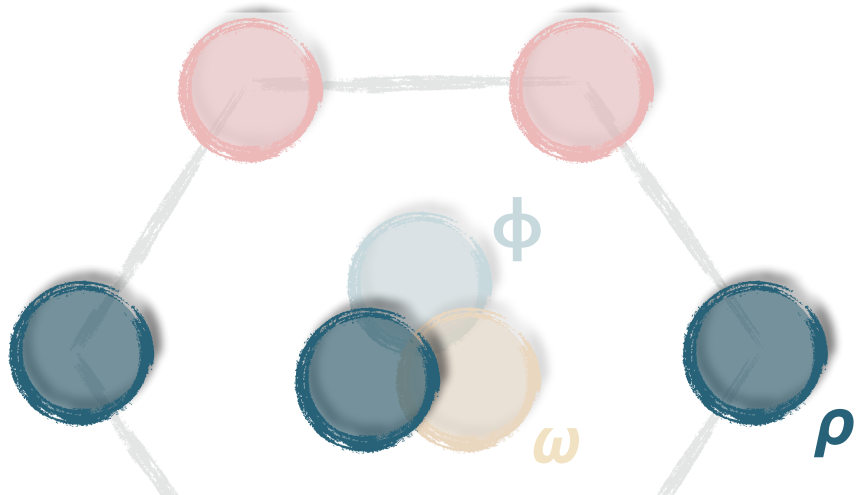
Note: cannot count finite-volume energies to count resonance poles!

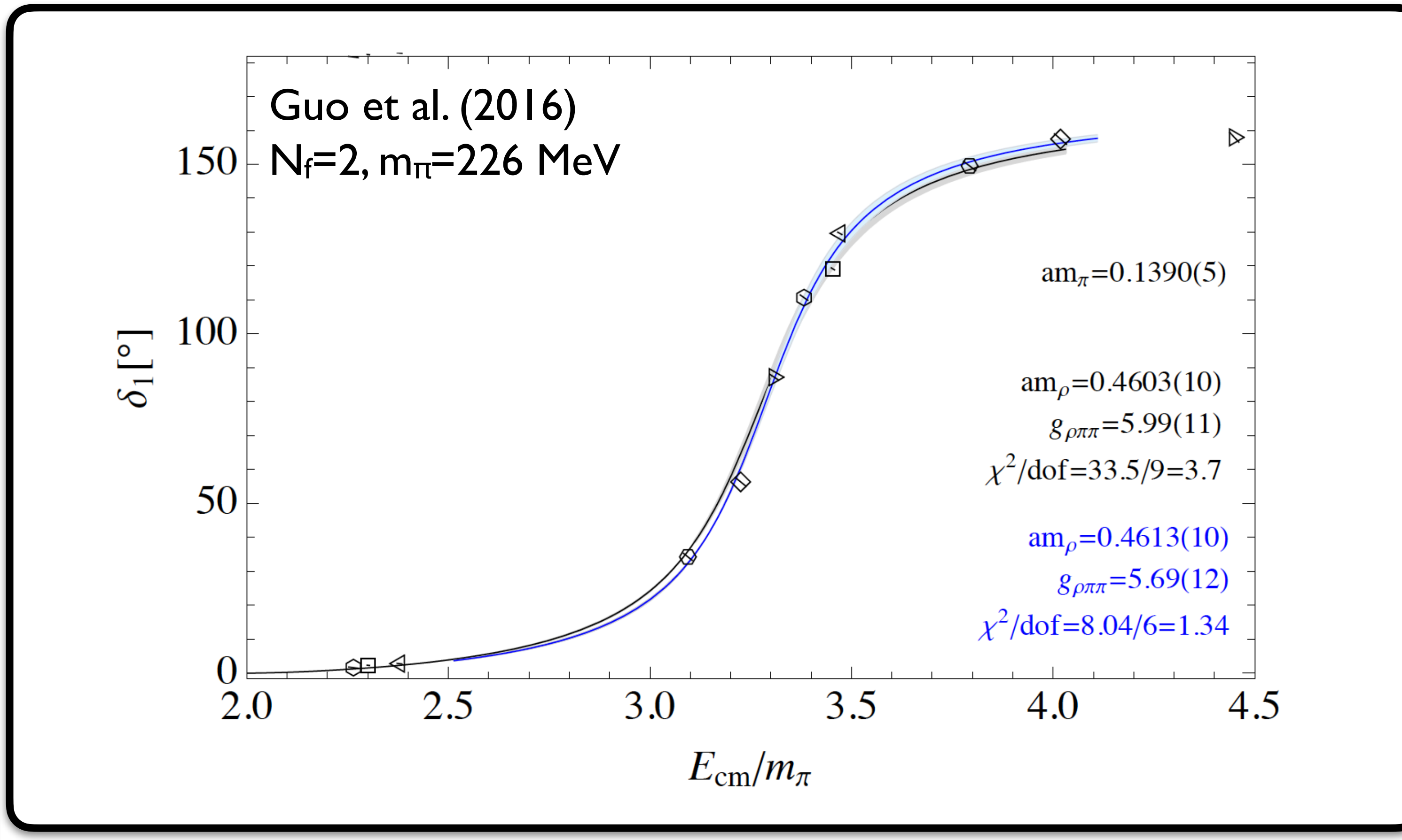
Dudek, Edwards (2012)

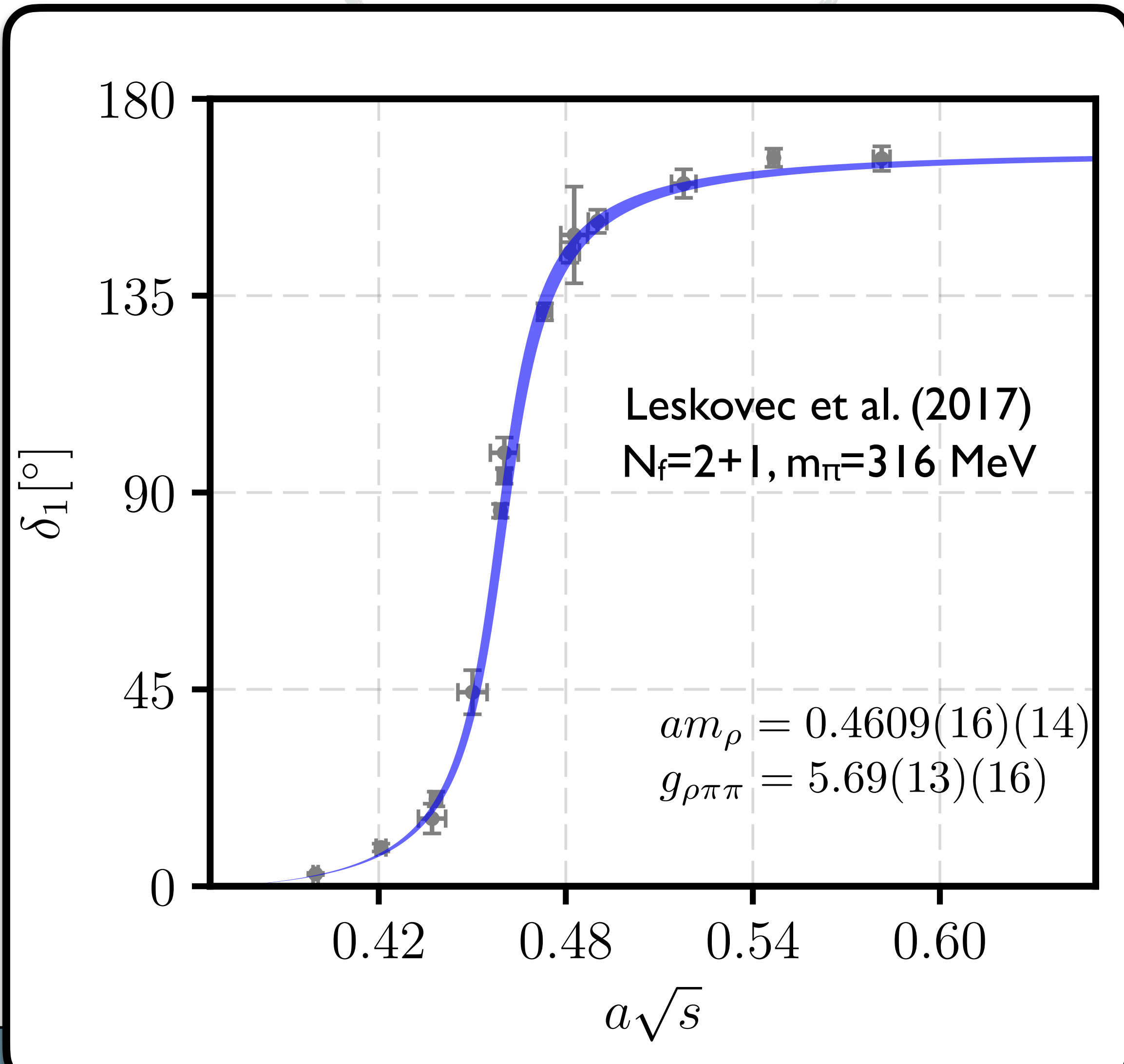
Wilson, Briceño, Dudek, Edwards, Thomas (2015)

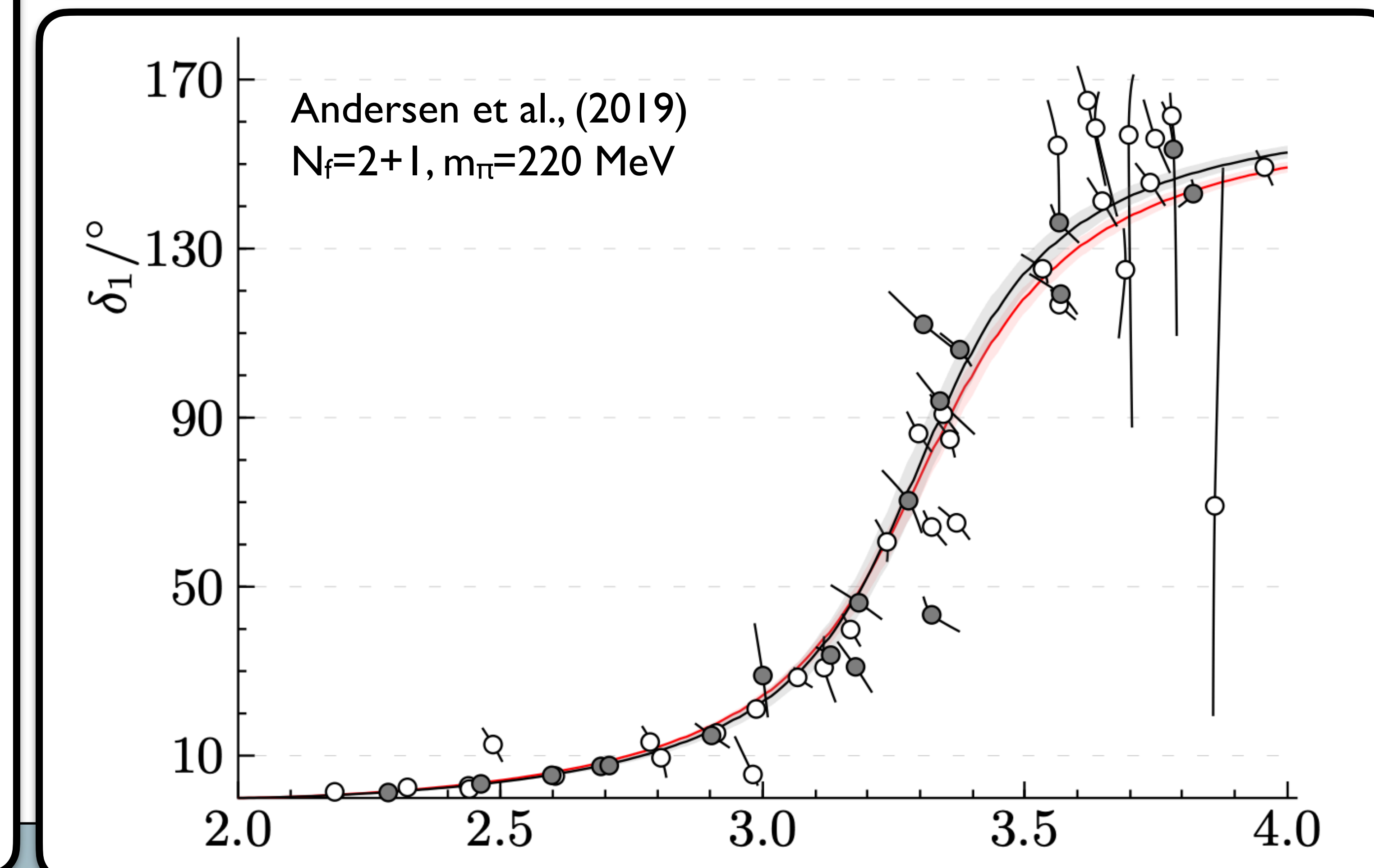
$$\rho \to \pi\pi$$

$$I^G(J^{PC}) = 1^+(1^{--})$$







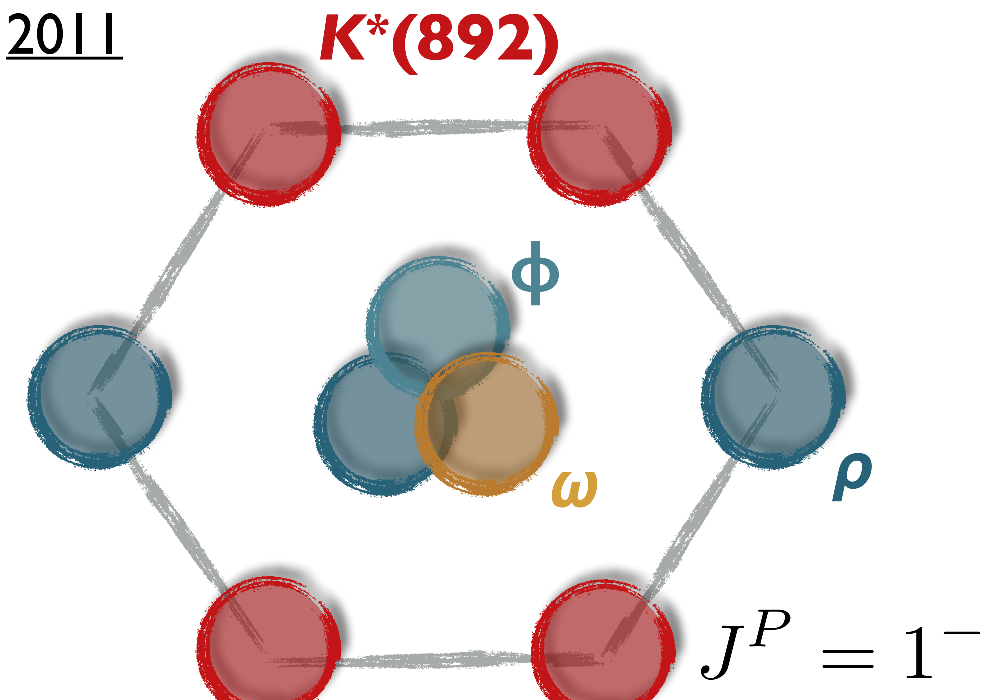


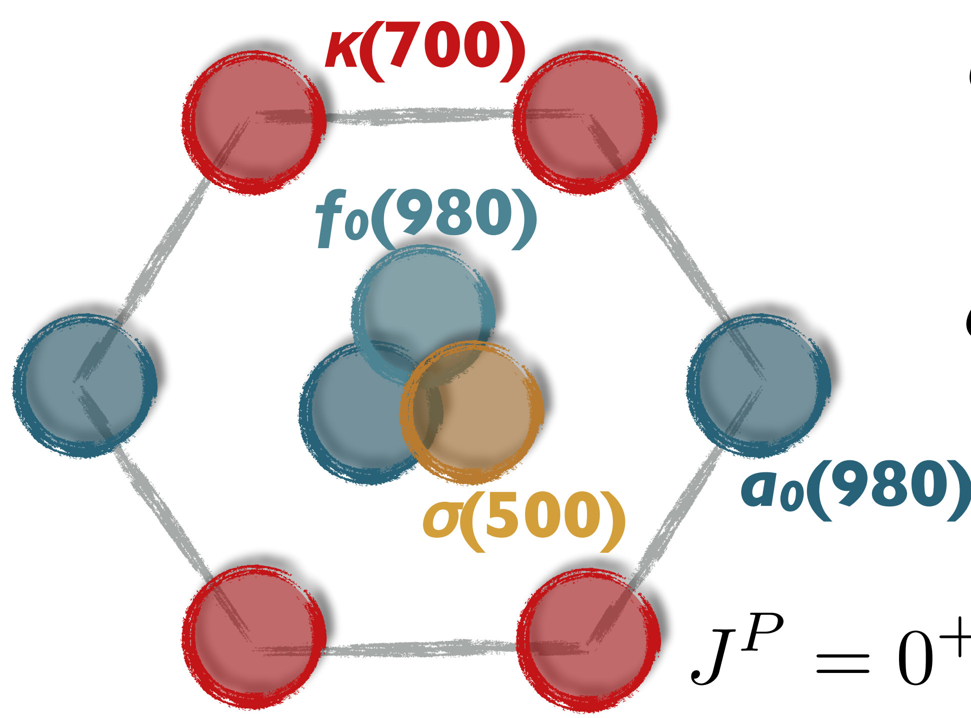
$\rho \to \pi\pi$

- CP-PACS/PACS-CS 2007, 2011
- ETMC 2010
- Lang et al. 2011
- HadSpec 2012, *2016*
- Pellisier 2012
- RQCD 2015
- Guo et al. 2016
- Fu et al. 2016
- Bulava et al. 2016
- Alexandrou et al. 2017
- Andersen et al. 2018
- Fischer et al. 2020
- Erben et al. 2020

$\sigma \to \pi\pi$

- Prelovsek et al. 2010
- Fu 2013
- Wakayama 2015
- Howarth and Giedt 2017
- Briceño et al. 2017
- Guo et al. 2018





$\kappa \to K\pi$

$K^* \to K\pi$

- Lang et al. 2012
- Prelovsek et al. 2013
- *Wilson et al. 2015*
- RQCD 2015
- *Brett et al. 2018*
- Wilson et al. 2019
- Rendon et al. 2020

$b_1 \to \pi\omega, \pi\phi$

- *Woss et al. 2019*

$a_0(980) \to \pi\eta, K\overline{K}$

- *Dudek et al. 2016*

$\sigma, f_0, f_2 \to \pi\pi, K\overline{K}, \eta\eta$

- *Briceño et al. 2017*

See the recent review by Briceño, Dudek and Young

$$\Delta \to N\pi$$

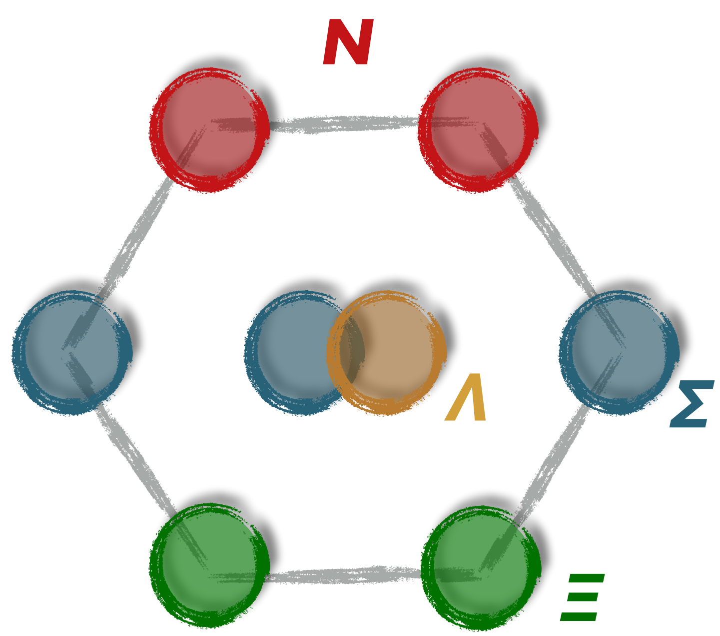
- ❑ Andersen et al. 2018
- ❑ Andersen et al. 2019
- ❑ Silvi et al. 2021
- ❑ Pittler et al. 2021
- ❑ Bulava et al. 2022

$$N^* \to N\pi$$

- ❑ Lang et al. 2017
- ❑ Wu et al. 2017
- ❑ Kiratidis et al. 2017

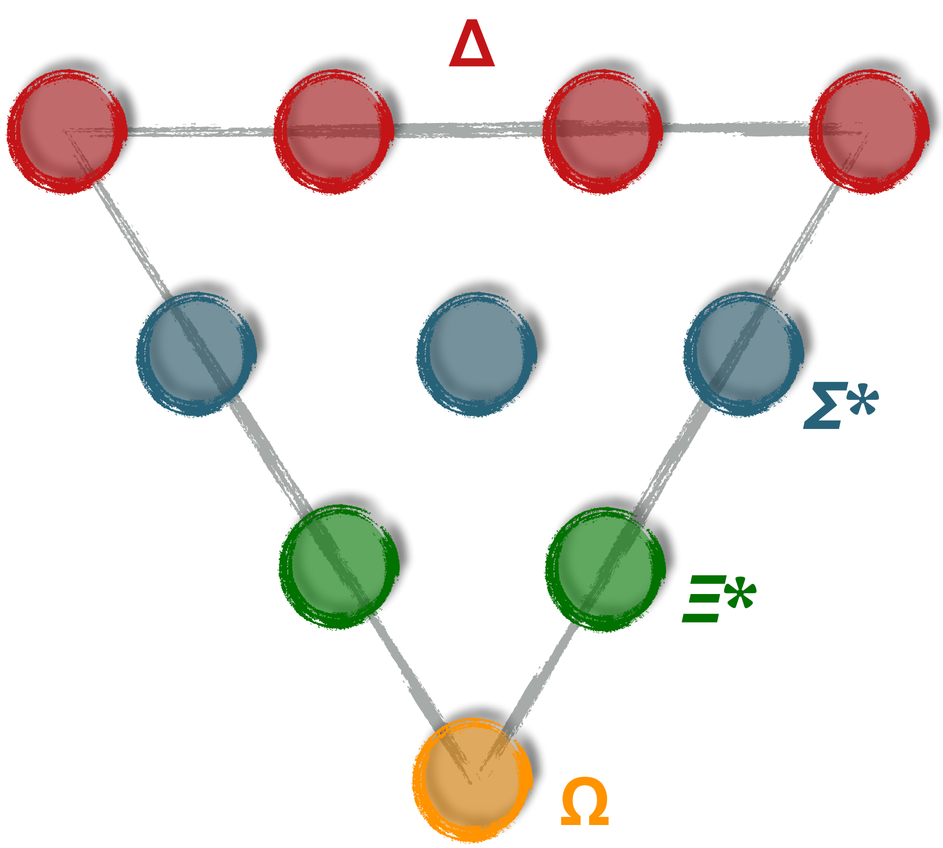
$$\Lambda \to \overline{K}N$$

- ❑ Hall et al. 2015



(focusing here on studies with scattering states)

Baryons are difficult!



See also…

- ❑ Detmold and Nicholson 2015
- ❑ Wu et al. 2018
- ❑ Xing & Liu, LATT2022 (in prep)

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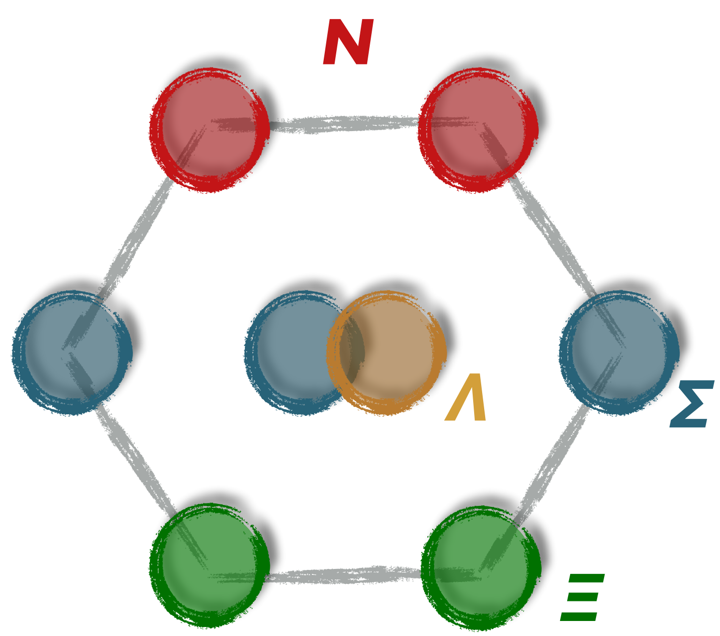

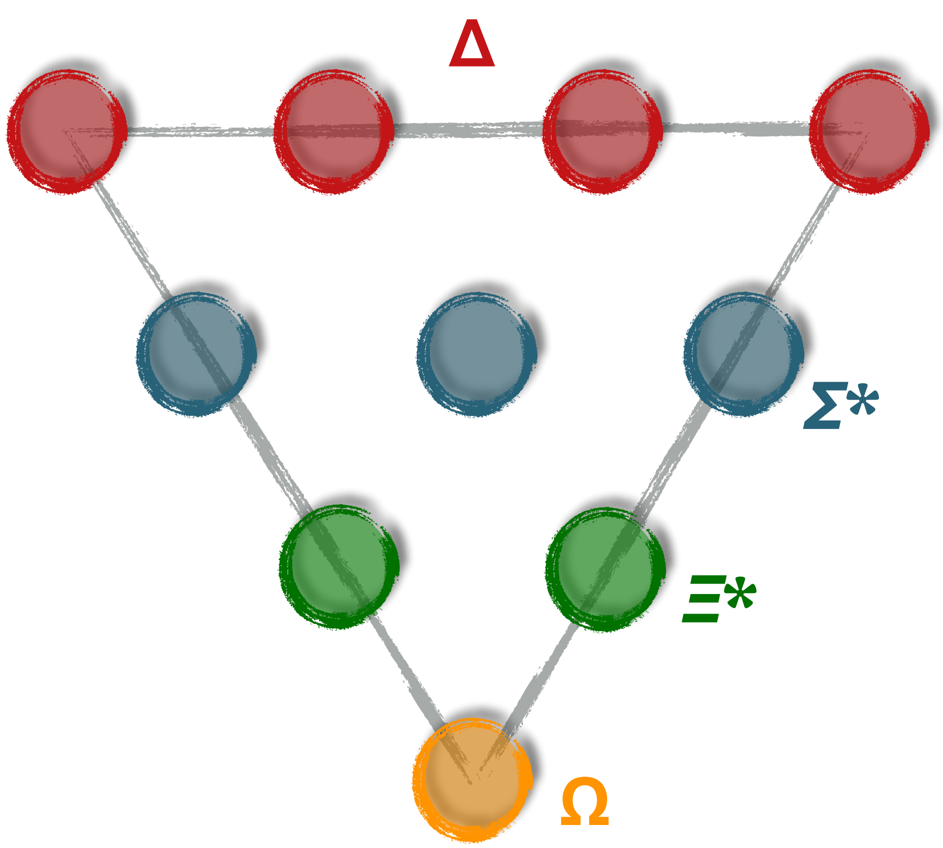

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$N\pi$ elastic scattering ($M_\pi = 255$ MeV)

PHYSICAL REVIEW D
covering particles, fields, gravitation, and cosmology

P-wave nucleon-pion scattering amplitude in the $\Delta(1232)$ channel from lattice QCD

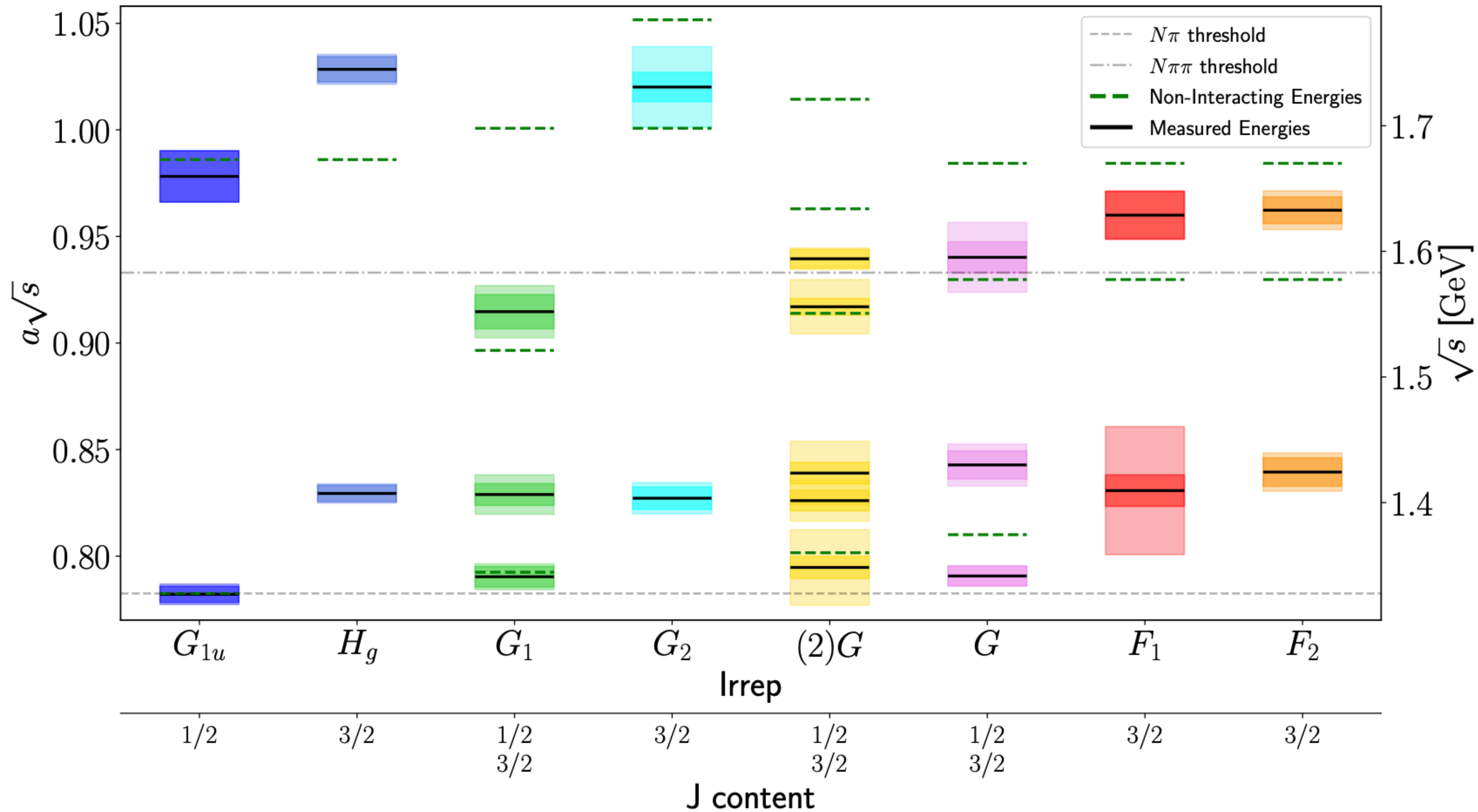
Giorgio Silvi, Srijit Paul, Constantia Alexandrou, Stefan Krieg, Luka Leskovec, Stefan Meinel, John Negele, Marcus Petschlies, Andrew Pochinsky, Gumaro Rendon, Sergey Syritsyn, and Antonino Todaro

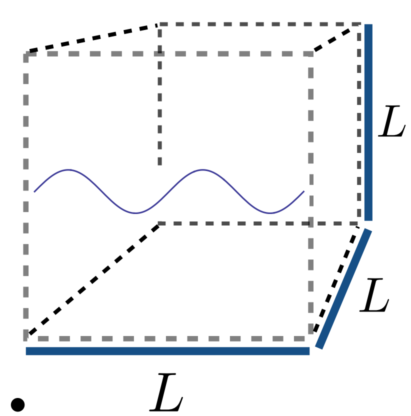
- ❑ $M_\pi = 255$ MeV
- ❑ Studied scattering-lengths and the Δ channel

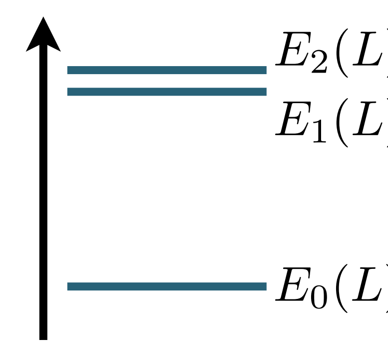
  $I = 3/2: \quad J^P = 1/2^- \ (S), \ \ 3/2^+ \ (P) \qquad [1/2^+ \ (P), \ \ 3/2^- (D), \ \ 5/2^- (D)]$
- ❑ Local Δ-like operators + $N(\boldsymbol{p}_1)\pi(\boldsymbol{p}_2)$ operators

• Silvi et al., PRD 2021, 2101.00689 •

$N\pi$ finite-volume energies $\quad (M_\pi = 255\ \mathrm{MeV})$

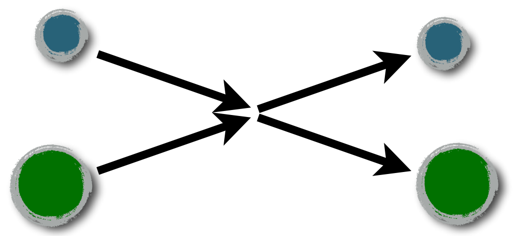


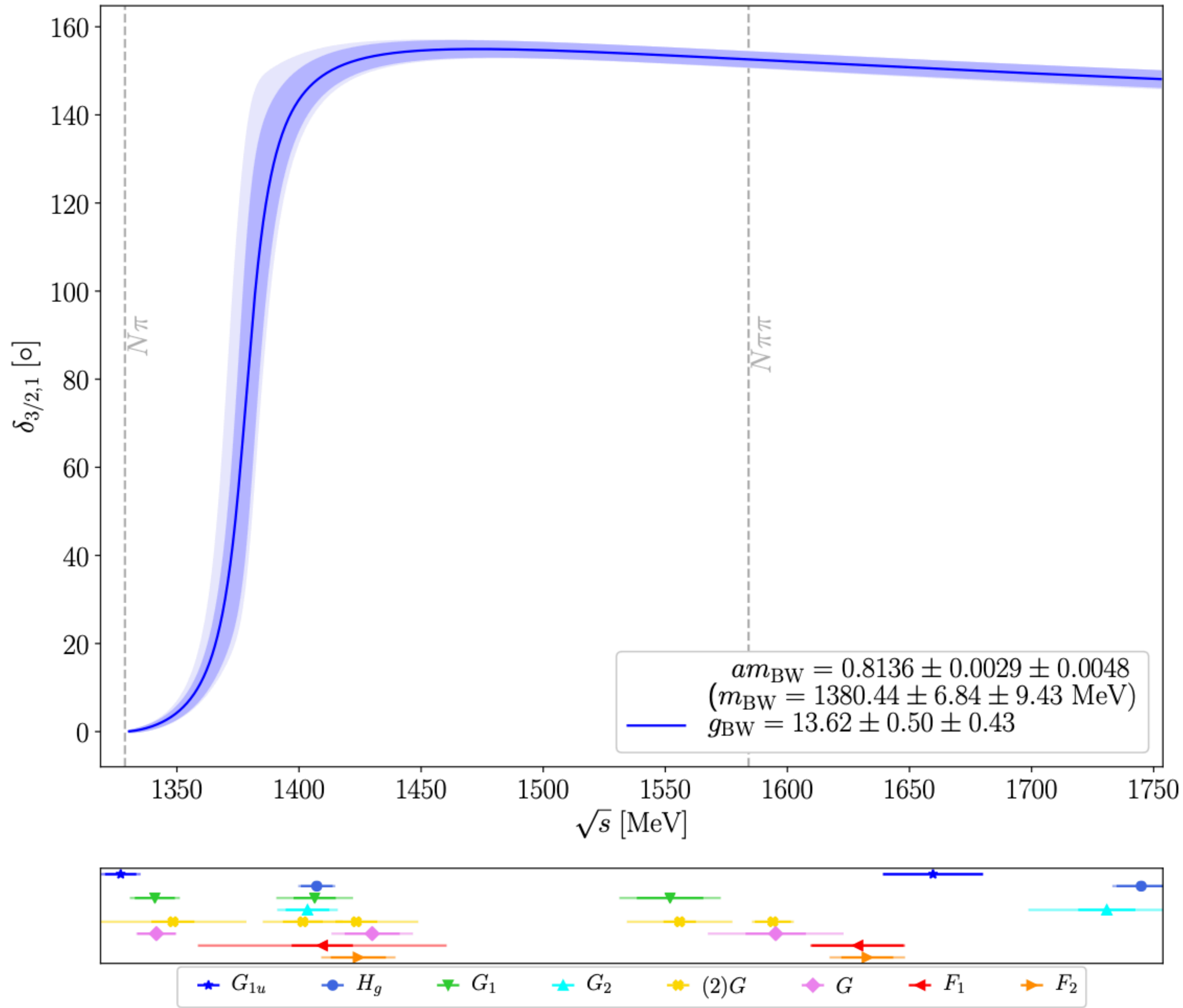




- Silvi et al., PRD 2021, 2101.00689
-

$N\pi \rightarrow \Delta \rightarrow N\pi \quad (M_\pi = 255 \text{ MeV})$





• Silvi et al., PRD 2021, 2101.00689 •

$$\Delta \to N\pi$$

- ☐ Andersen et al. 2018
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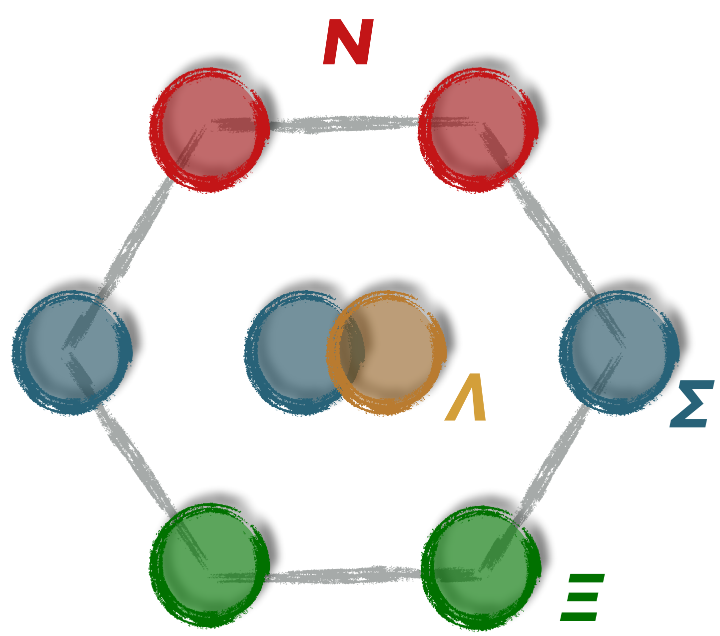

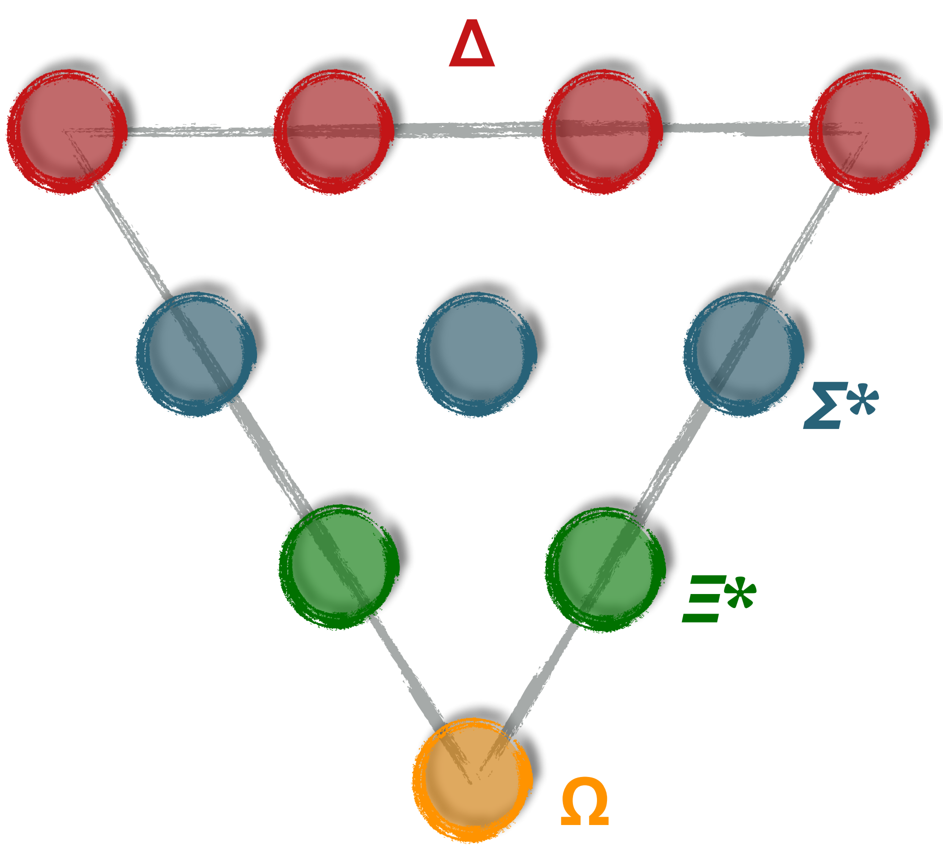

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$N\pi$ elastic scattering ($M_\pi = 200$ MeV)

arXiv > hep-lat > arXiv:2208.03867

High Energy Physics – Lattice

[Submitted on 8 Aug 2022]

Elastic nucleon–pion scattering at $m_\pi \approx 200$ MeV from lattice QCD

John Bulava, Andrew D. Hanlon, Ben Hörz, Colin Morningstar, Amy Nicholson, Fernando Romero–López, Sarah Skinner, Pavlos Vranas, André Walker–Loud

- Studied scattering-lengths and the Δ channel

  $I = 1/2: \quad J^P = 1/2^- \ (S)$

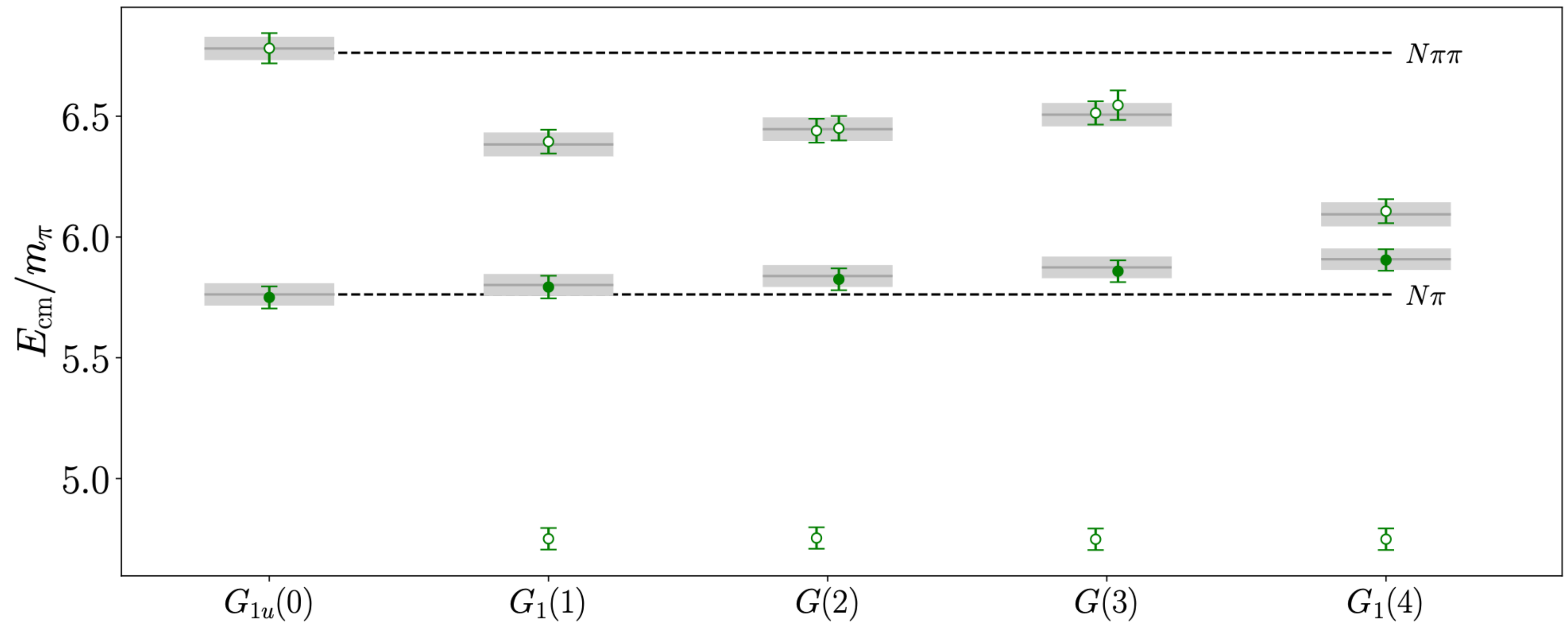
  $I = 3/2: \quad J^P = 1/2^- \ (S), \ \ 3/2^+ \ (P) \qquad [1/2^+ \ (P), \ \ 3/2^- \ (D), \ \ 5/2^- \ (D)]$

- Advanced operators + fits to extract finite-volume energies

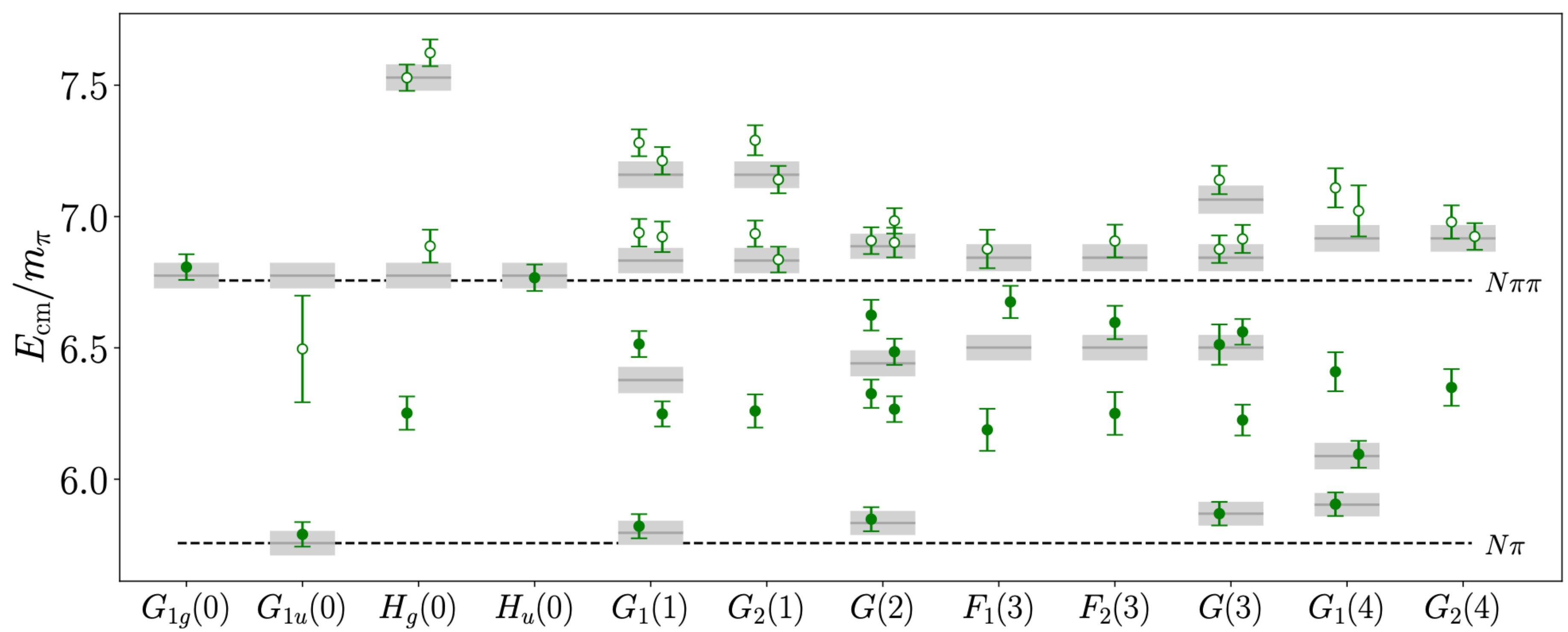
• Bulava et al. (2022) 2208.03867 •

$N\pi$ finite-volume energies ($M_\pi = 200$ MeV)

$I = 1/2$



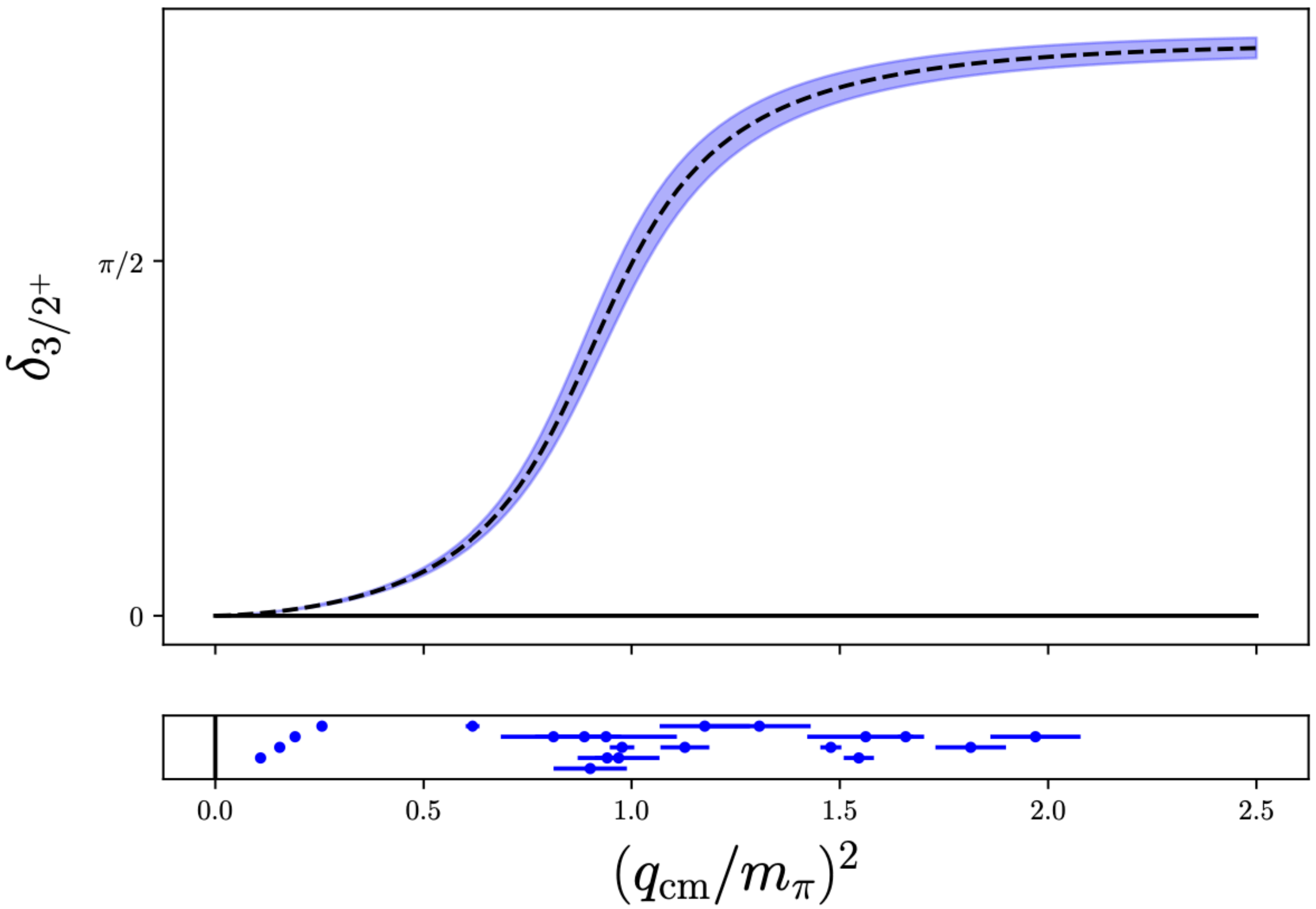
$I = 3/2$



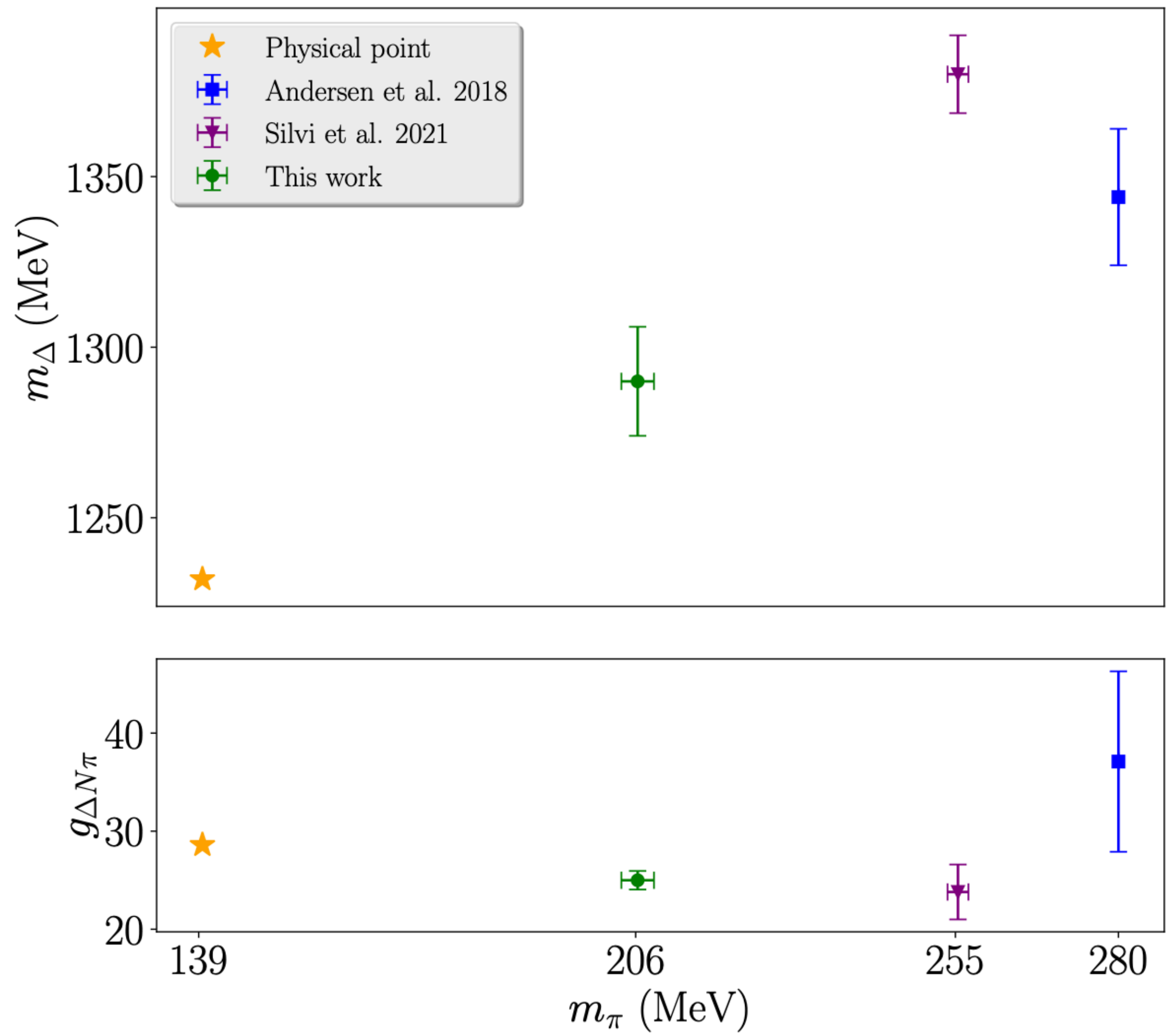
- Bulava et al. (2022) 2208.03867 •

$N\pi \to \Delta \to N\pi$

Scattering phase shift



Δ summary plot



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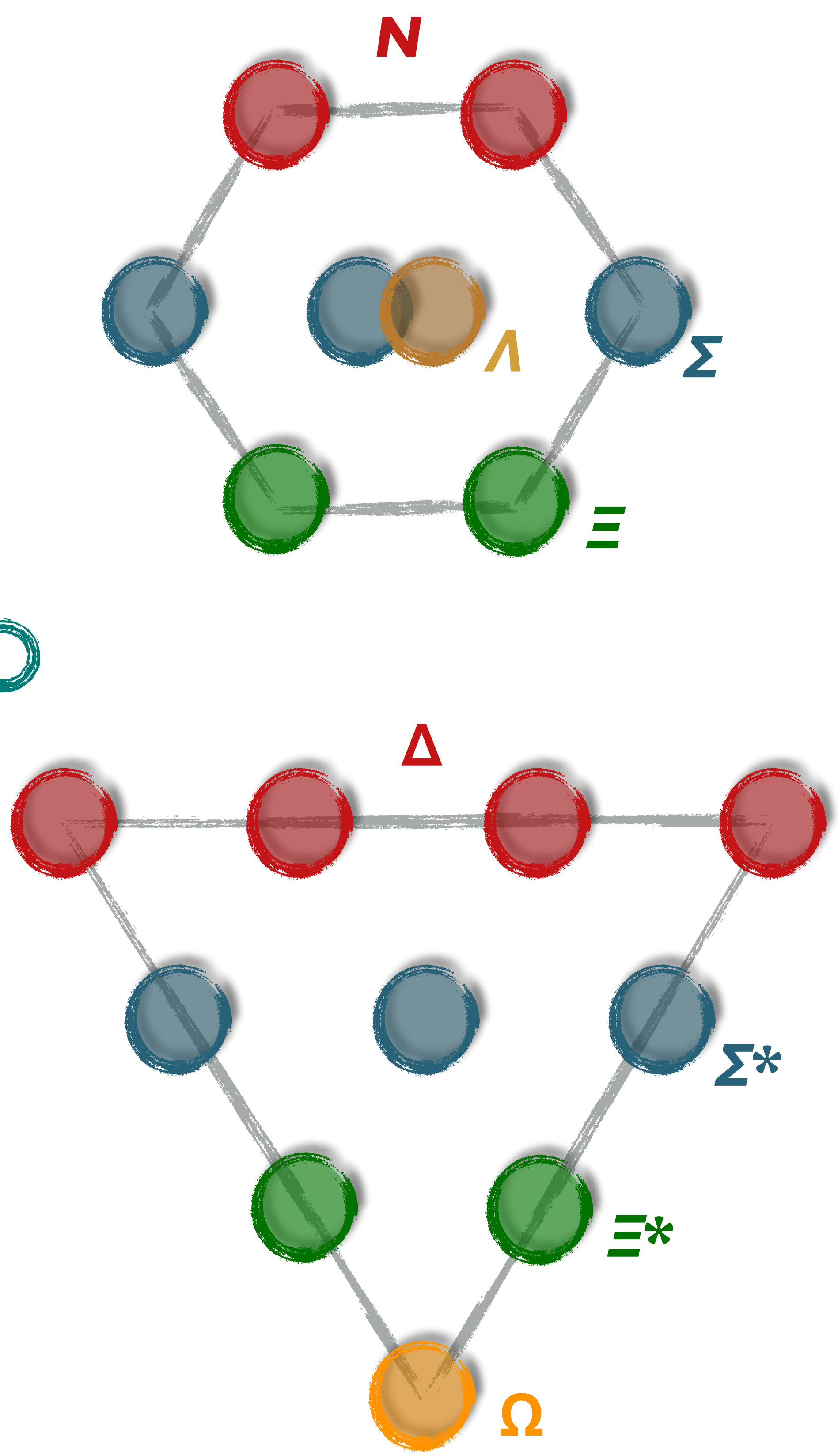

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Baryons are difficult!

See also…

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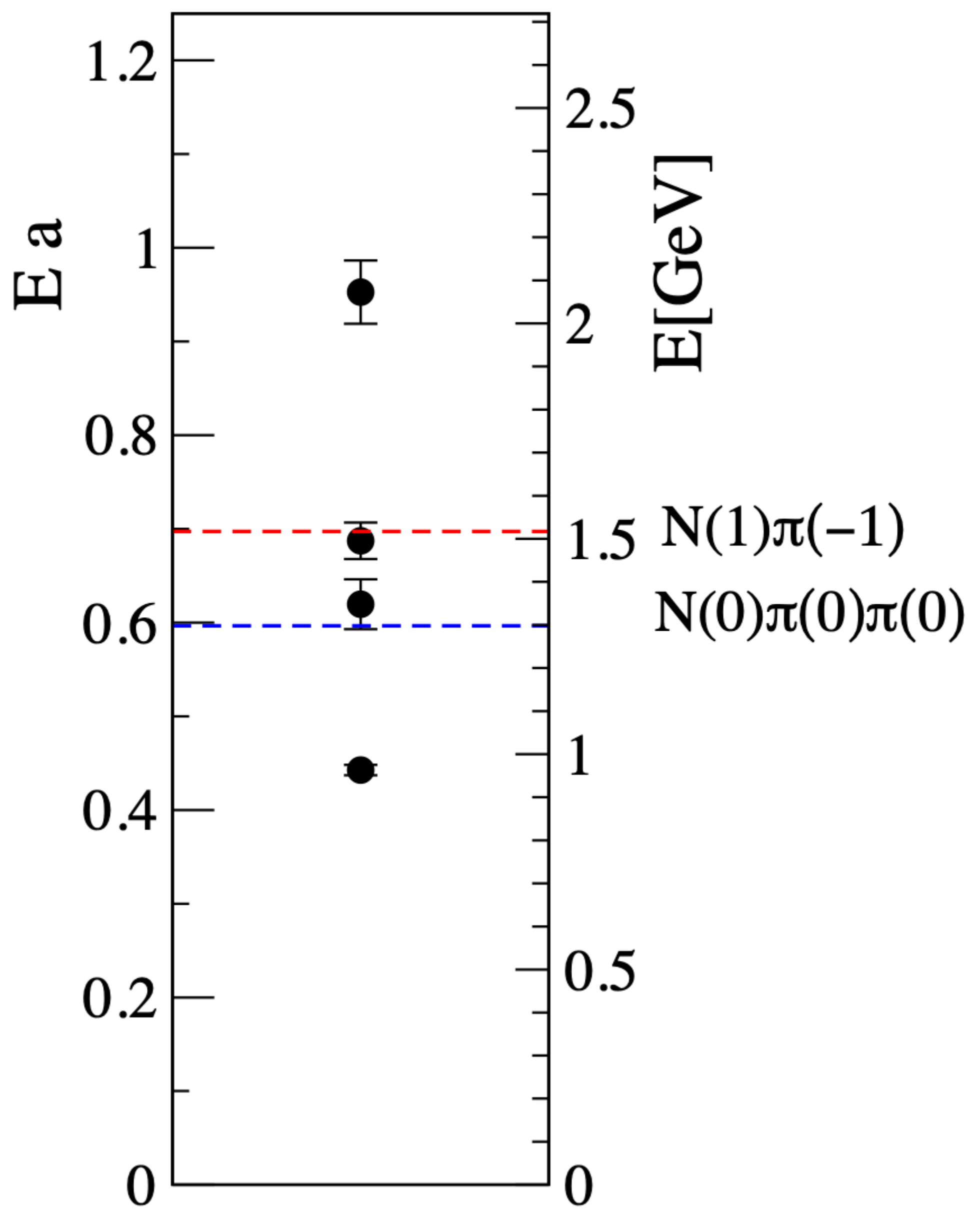
- ☐ Xing & Liu, LATT2022 (in prep)

Roper resonance

- Results with impressive operators from Lang et al., $M_\pi = 156\ \text{MeV}$
- Interpretation presented by D. Leinweber this morning



• Lang, Leskovec, Padmanath, Prelovsek (2017) •

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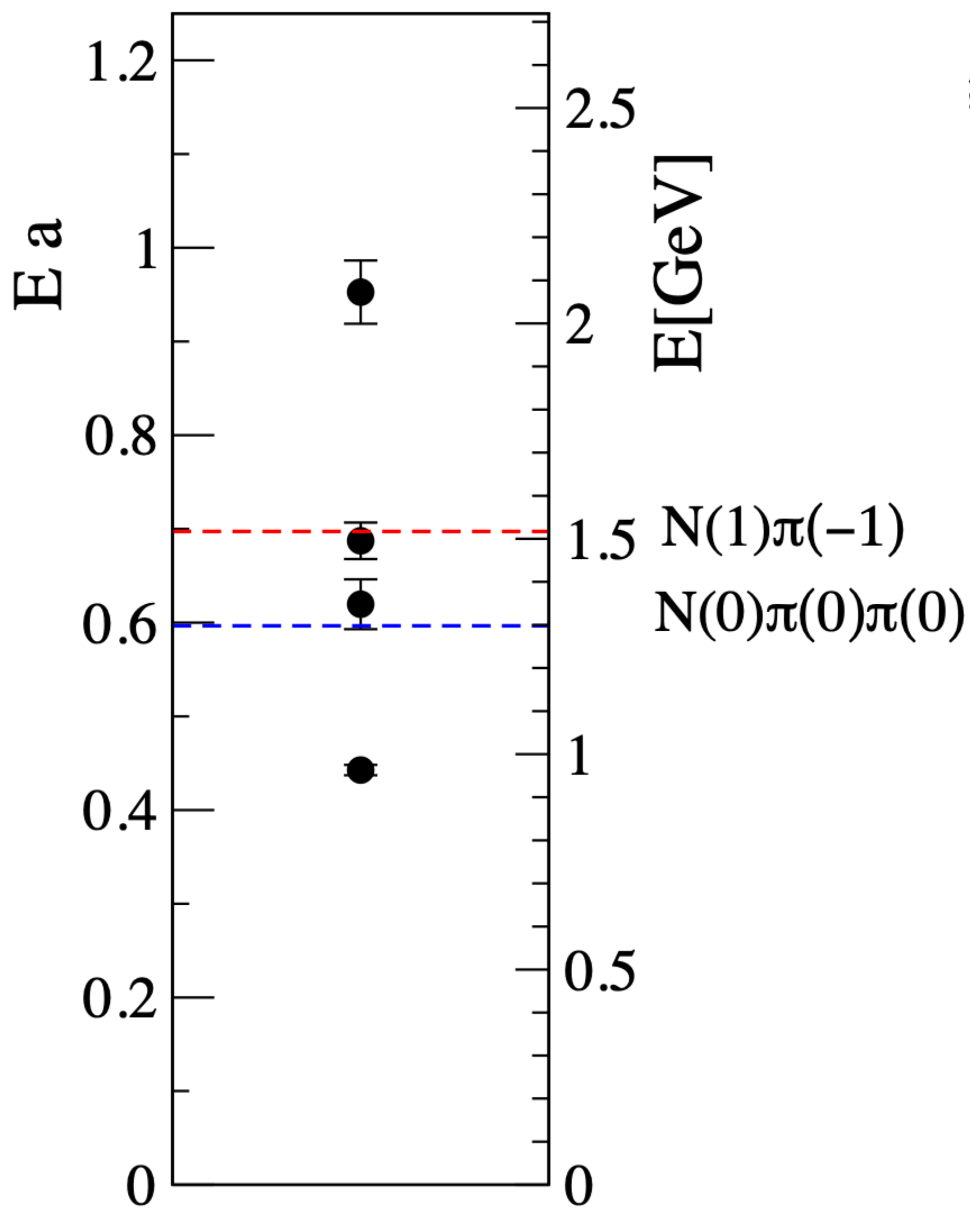
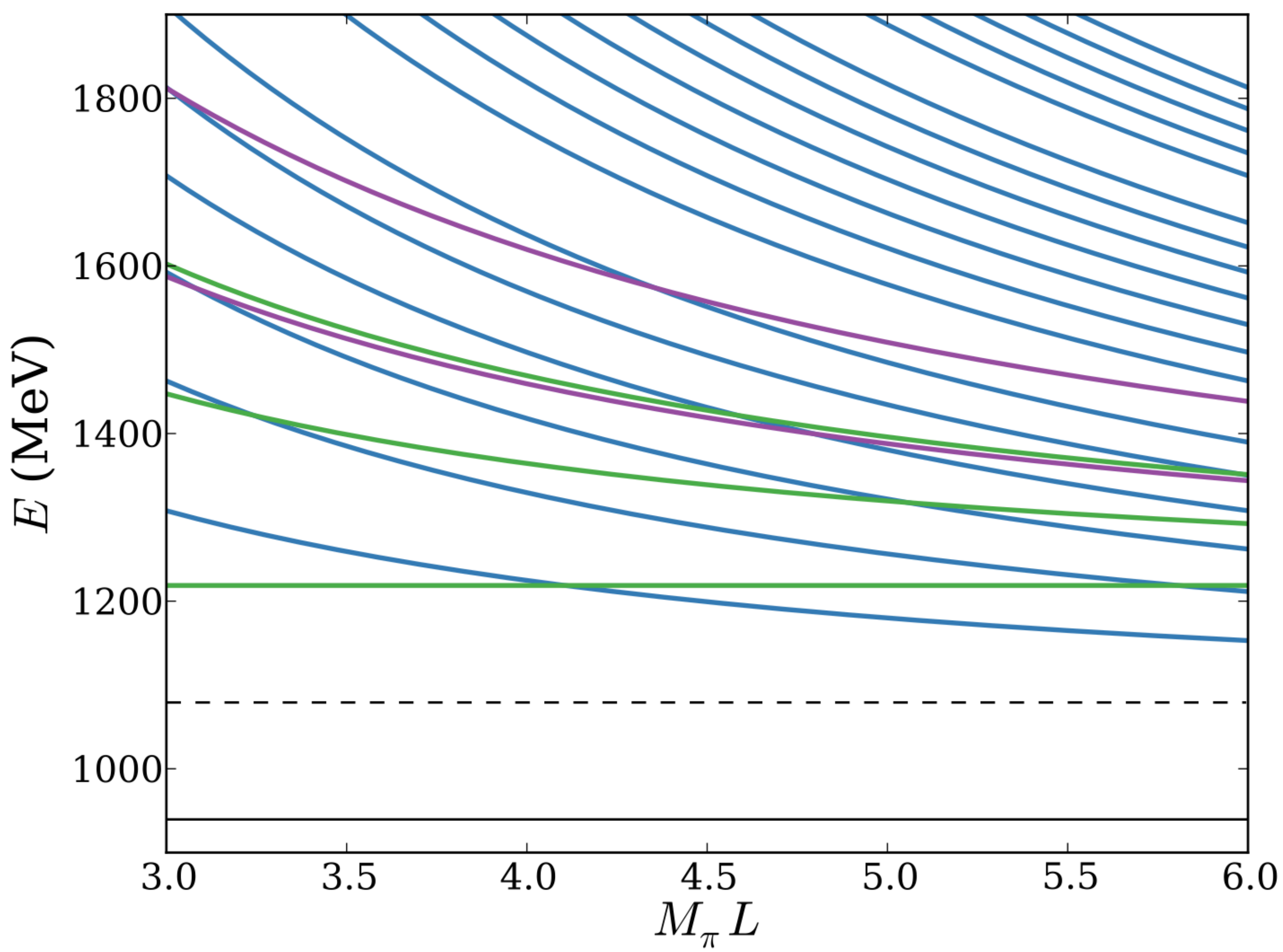


- My personal take, need more data…
  - Lattice "rule-of-thumb" is to take $M_\pi L \sim 4$ or larger
  - Here $M_\pi L \sim 2.2$
  - First three states are statistically consistent with non-interacting nucleons and pions
  - Careful with operator overlaps… discretisation dependent, no guarantee of continuum limit

• Lang, Leskovec, Padmanath, Prelovsek (2017) •

Roper resonance

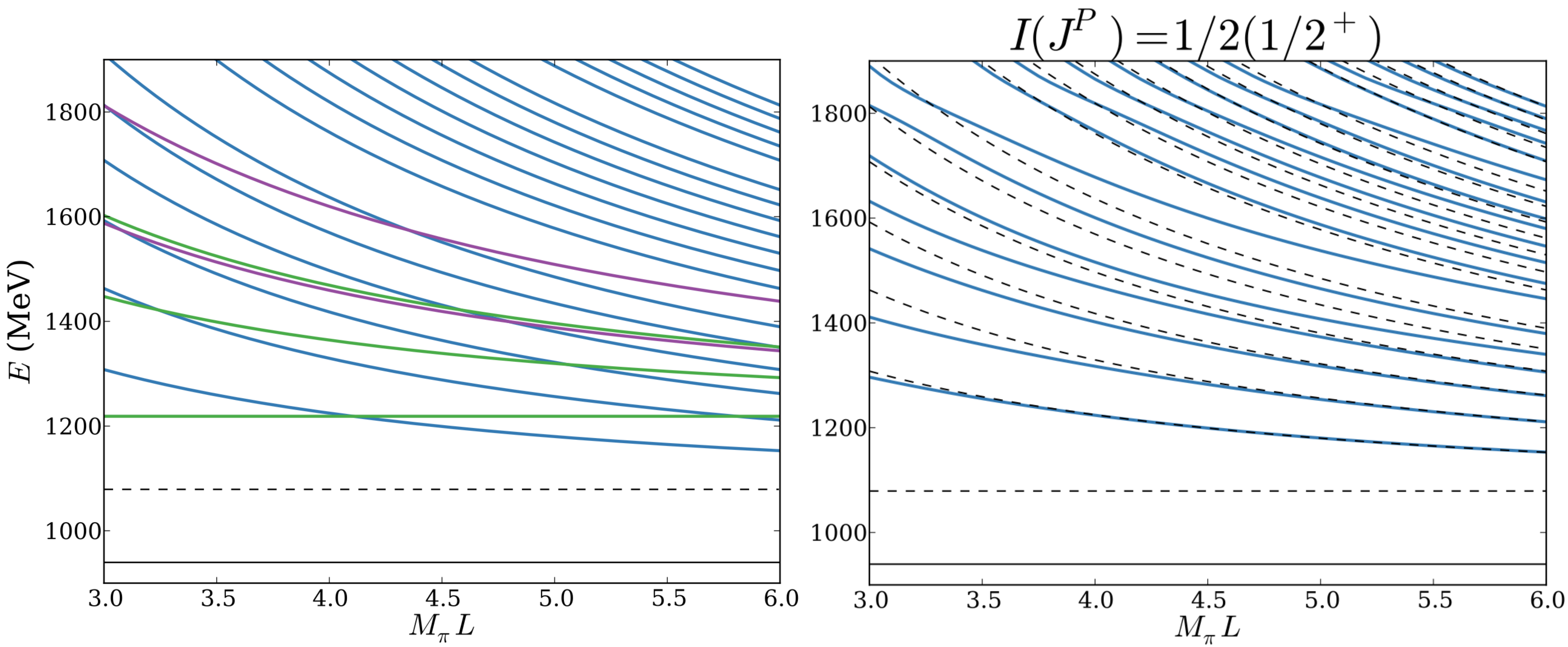
- Naive spectra from MTH and Meyer 2016
  - Non-interacting



• MTH and Meyer (2016) • see also Döring et al. (2013) • Daniel Severt LATT2022 •

Roper resonance

- Naive spectra from MTH and Meyer 2016
  - Non-interacting
  - GWU WI08 + Lüscher



- But, right panel ignores three-body effects!… I would say work is needed

• MTH and Meyer (2016) • see also Döring et al. (2013) • Daniel Severt LATT2022 •

$$\Delta \to N\pi$$

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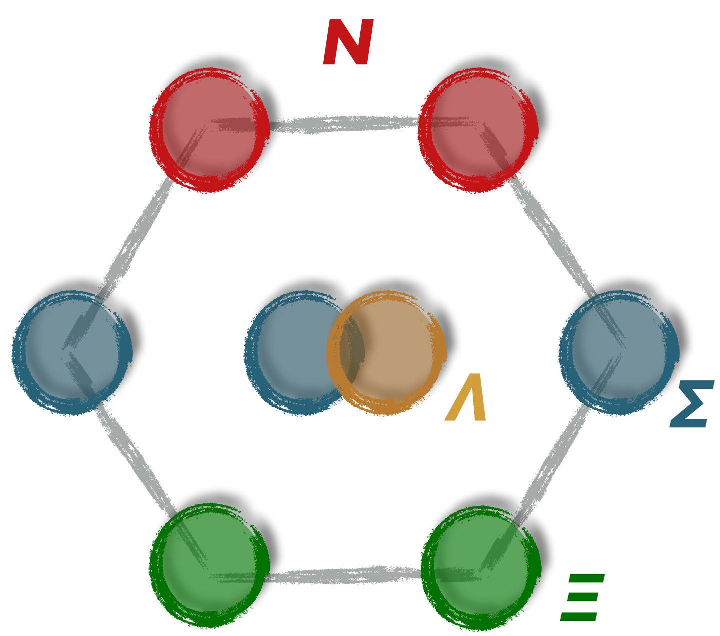
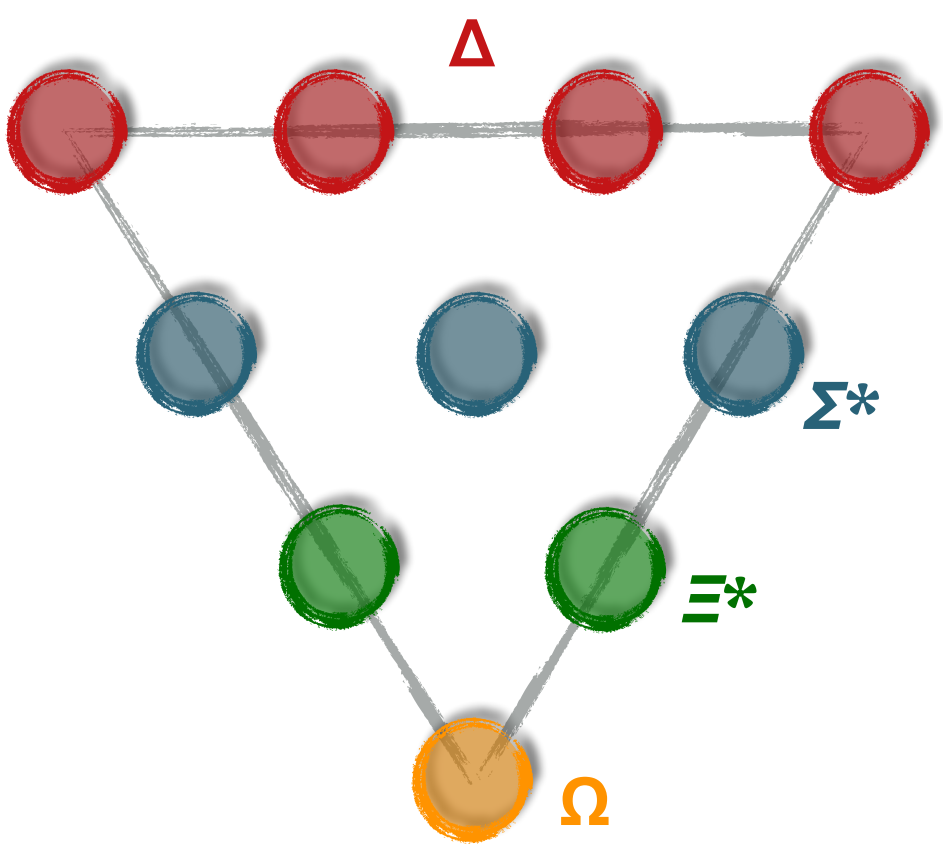


(focusing here on studies with scattering states)

Baryons are difficult…

Stay tuned!



See also…

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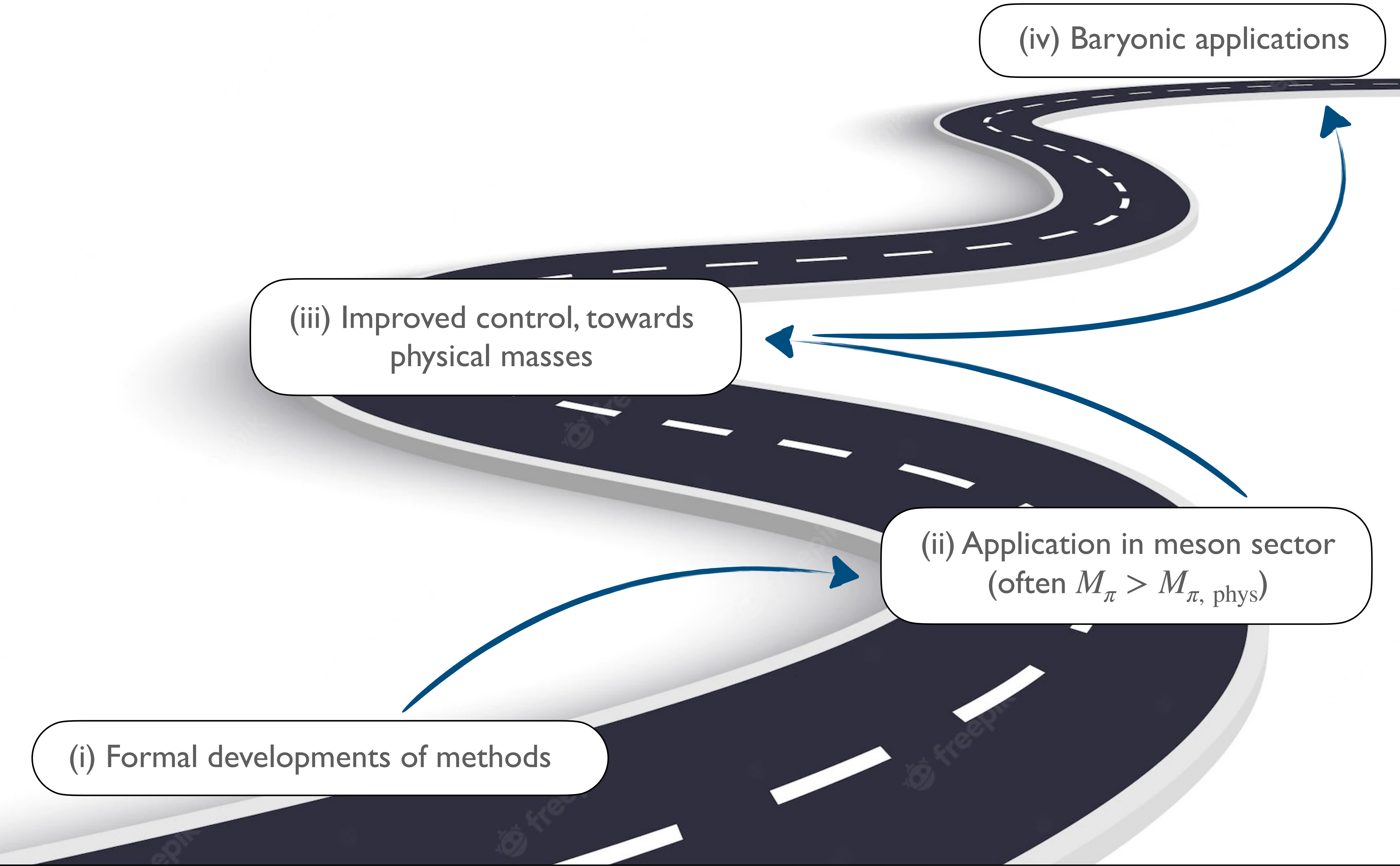
Journey of a lattice calculation

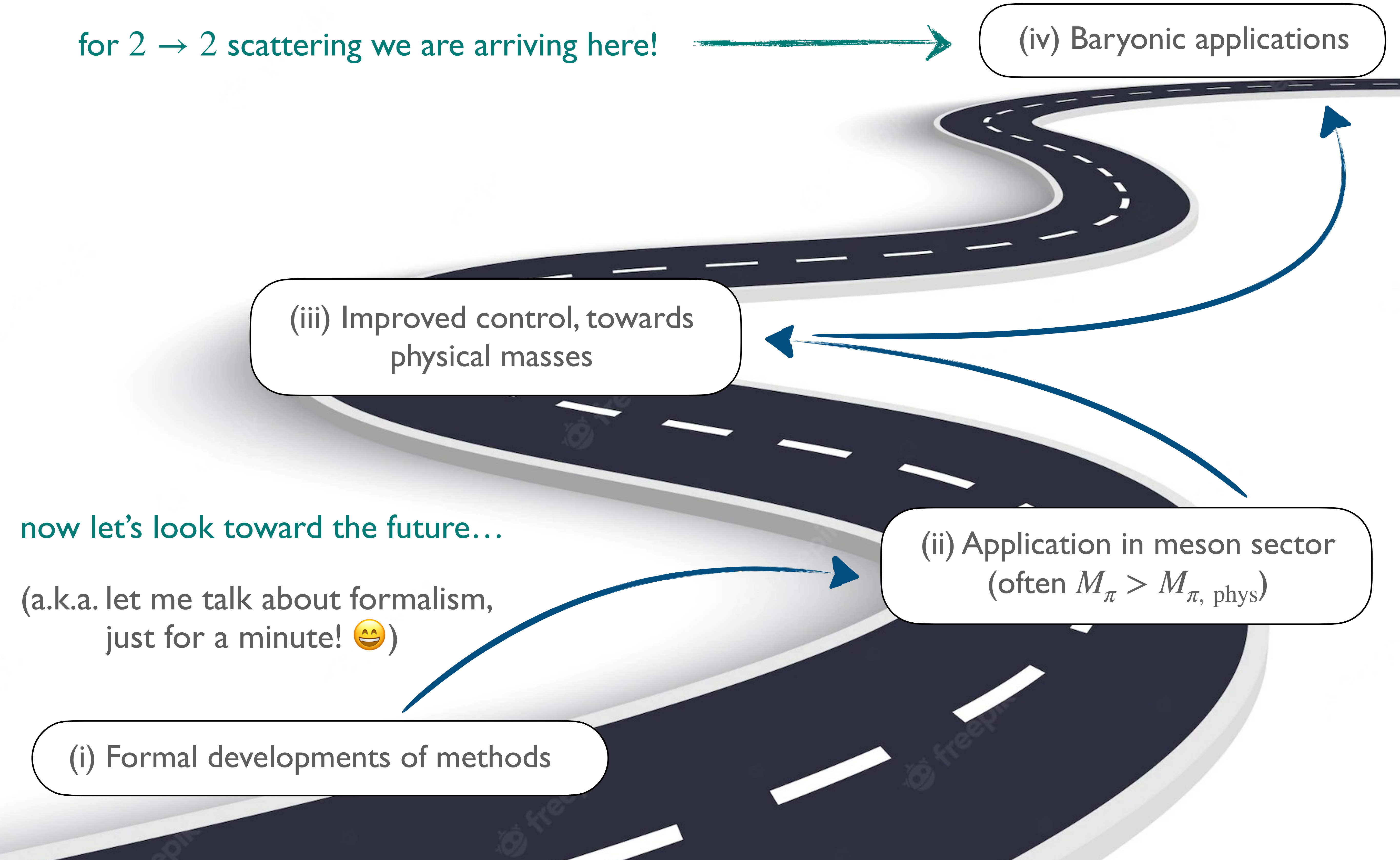


Journey of a lattice calculation

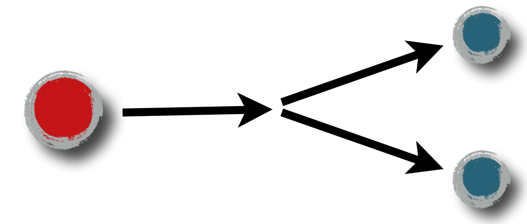
for $2 \to 2$ scattering we are arriving here!

now let's look toward the future…

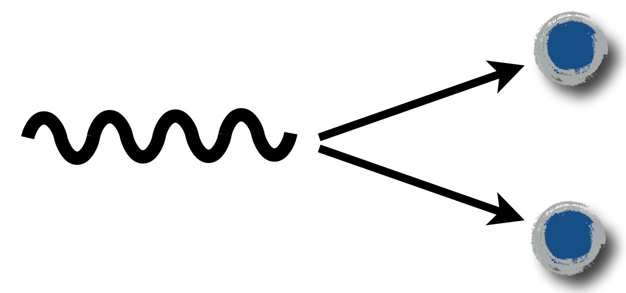
(a.k.a. let me talk about formalism, just for a minute! 😄)



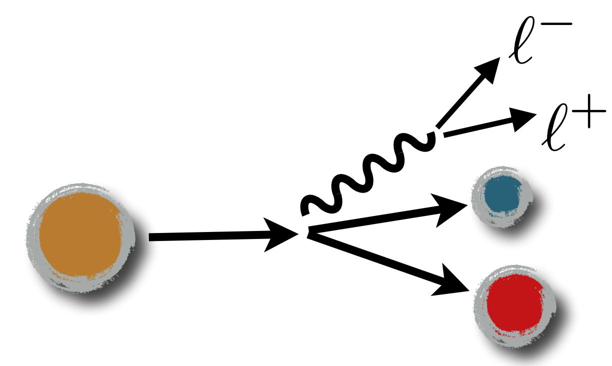
Formal progress: Transition amplitudes

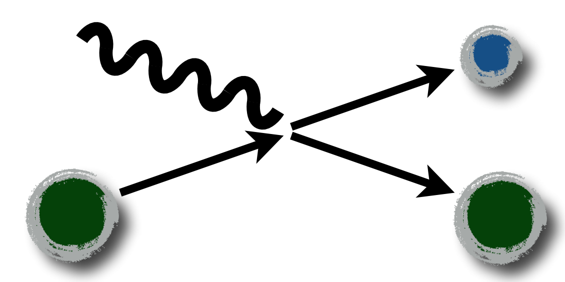
- ☐ Weak decay $\langle \pi\pi, \mathrm{out} | \mathcal{H} | K \rangle \equiv$ 

Lellouch, Lüscher *(2001)* • Kim, Sachrajda, Sharpe *(2005)* • Christ, Kim, Yamazaki *(2005)* • MTH, Sharpe *(2012)*

- ☐ Time-like form factors $\langle \pi\pi, \ \mathrm{out} | \mathcal{J}_\mu | 0 \rangle \equiv$ 

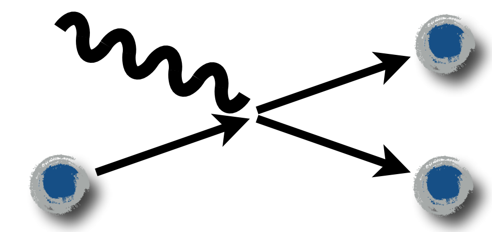
Meyer *(2011)*

- ☐ Resonance form factors $\langle K\pi, \mathrm{out} | \mathcal{J}_{\alpha\beta} | B \rangle \equiv$ 

- ☐ Particles with spin $\langle N\pi, \mathrm{out} | \mathcal{J}_\mu | N \rangle \equiv$ 

Agadjanov *et al. (2014)* • Briceño, MTH, Walker-Loud *(2015)* • Briceño, MTH *(2016)*

Pion photo-production

$\langle \pi\pi, \mathrm{out} | \mathcal{J}_\mu | \pi \rangle \equiv$ 

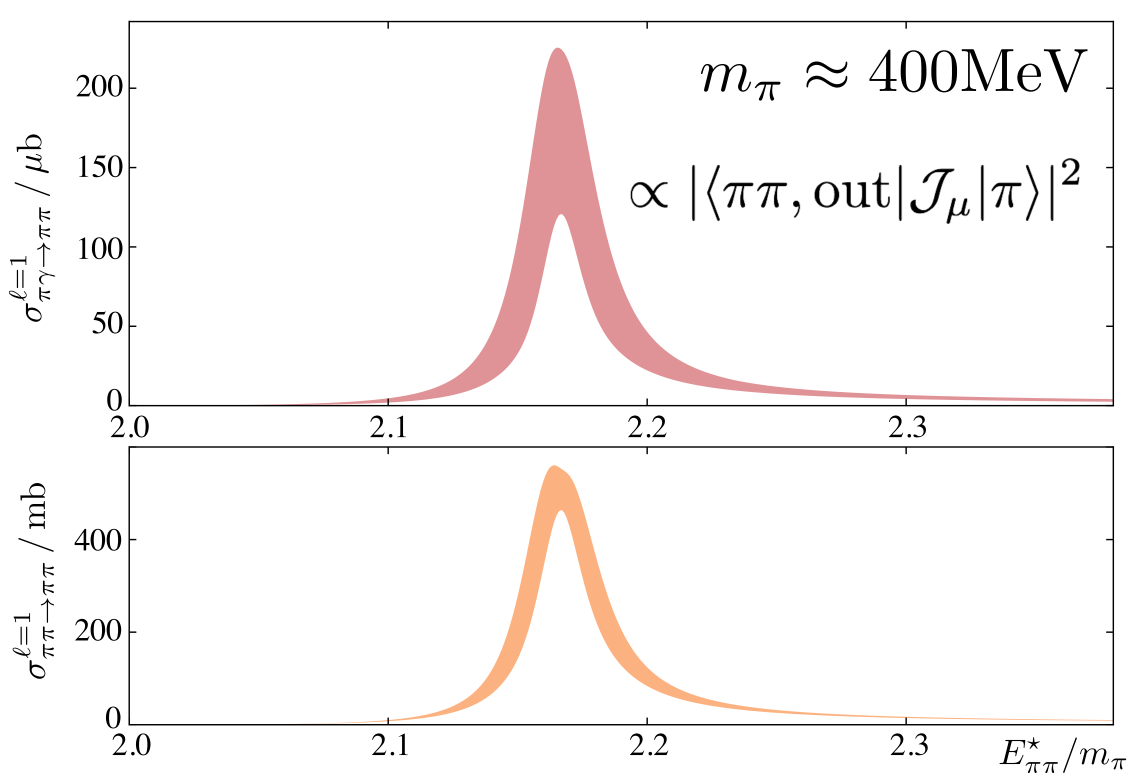
- Formal relation

get this from the lattice — experimental observable

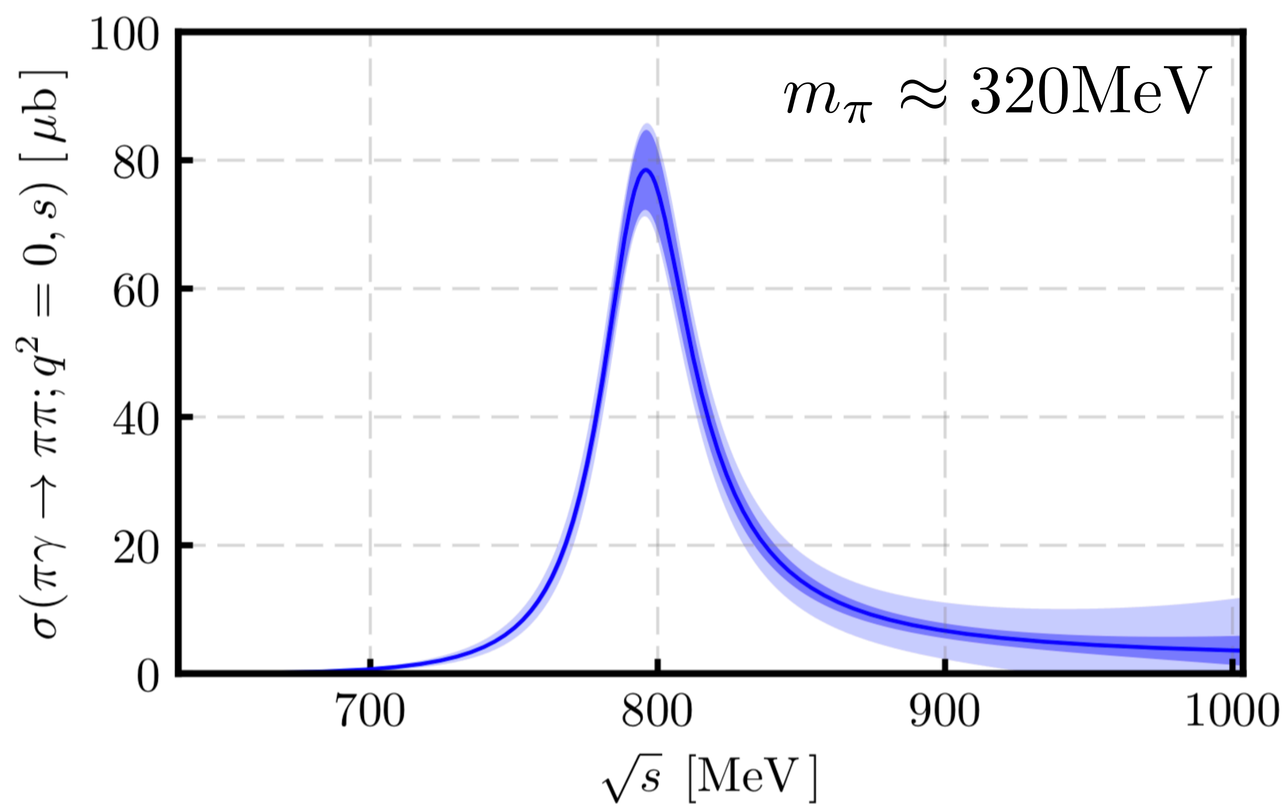
$$|\langle n, L | \mathcal{J}_\mu | \pi \rangle|^2 = \langle \pi | \mathcal{J}_\mu | \pi\pi, \mathrm{in} \rangle \mathcal{R}(E_n, L) \langle \pi\pi, \mathrm{out} | \mathcal{J}_\mu | \pi \rangle$$

Briceño, MTH, Walker-Loud (2015)

- Numerical implementation



Briceño et. al., Phys. Rev. D93, 114508 (2016)



Alexandrou et. al., Phys. Rev. D98, 074502 (2018)

Formal progress: Towards $N\pi\pi$

- Multiple three-particle finite-volume formalisms developed (so far only spin zero)

MTH, Sharpe (2014-2016) *See also Döring, Mai, Hammer, Pang, Rusetsky*

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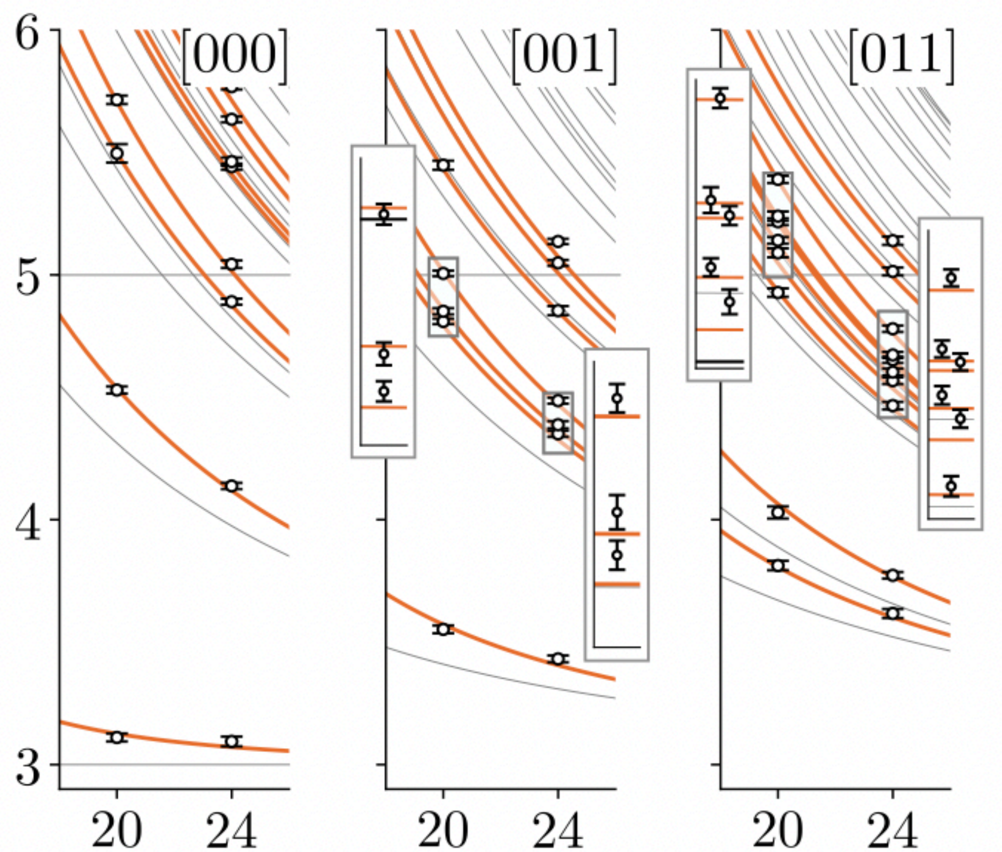
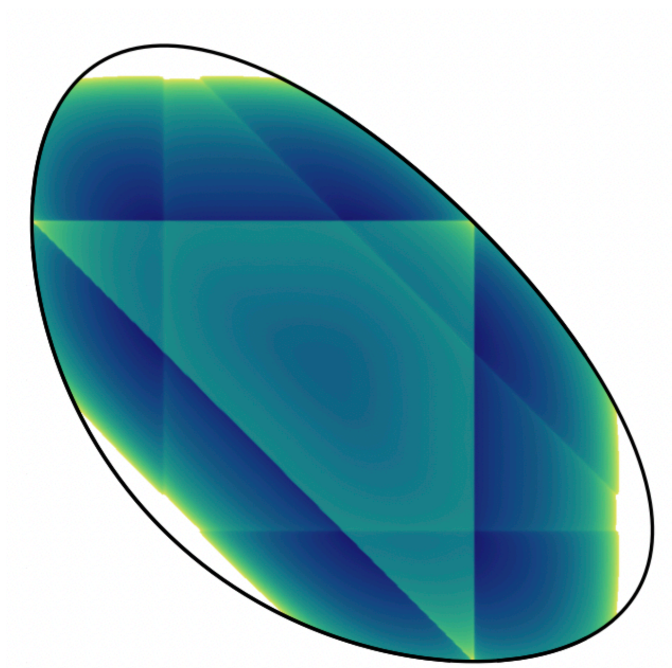
- First lattice calculations appearing… e.g. $\pi^+\pi^+\pi^+ \to \pi^+\pi^+\pi^+$





- Extract reliable spectrum
- Use formalism to fit scheme-dependent K-matrix
- Solve integral equations to reach physical amplitude

MTH, Briceño, Edwards, Thomas, Wilson, *Phys.Rev.Lett.* 126 (2021) 012001

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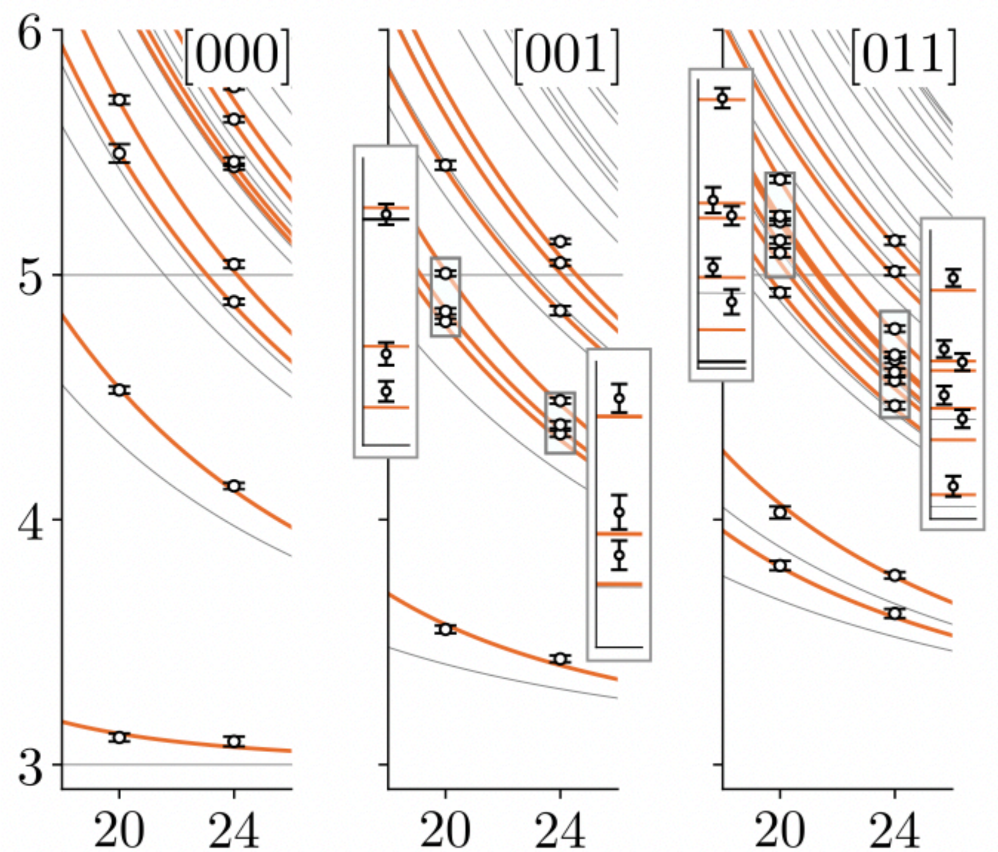

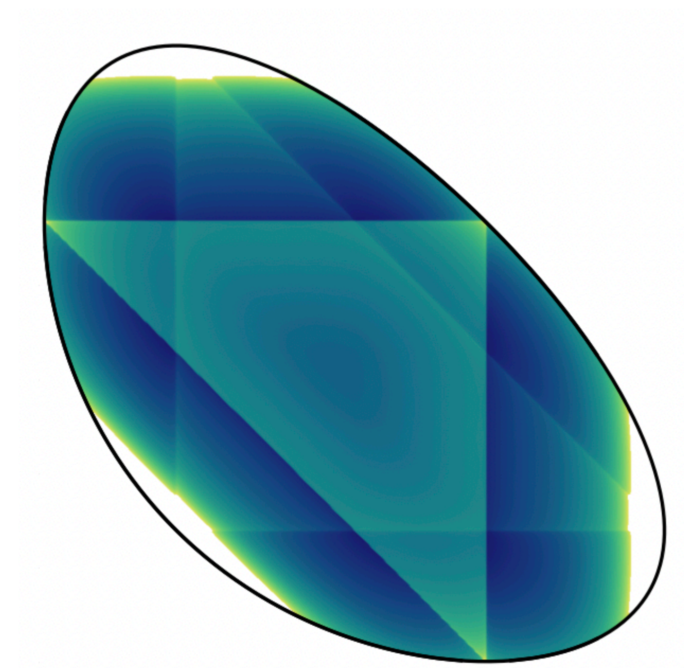

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MTH, Briceño, Edwards, Thomas, Wilson, *Phys.Rev.Lett.* 126 (2021) 012001

- See Blanton et al. for pion and kaon results
- See Sadasivan et al. for application to $a_1(1260)$

Not discussed here…

- Hamiltonian Effective Theory (*D. Leinweber, morning talk*)

  My two cents: Lüscher formalism (+ extensions) uniquely predict spectrum…

  - For given amplitudes
  - For any interactions (same for $\pi^+\pi^+$ and $N\pi \to \Delta \to N\pi$)
  - Up to $e^{-M_\pi L}$ + must include all channels

HET gives (i) amplitude model, (ii) pion mass dependence, (iii) quark-model interpretation

Not discussed here…

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  - For given amplitudes
  - For any interactions (same for $\pi^+\pi^+$ and $N\pi \to \Delta \to N\pi$)
  - Up to $e^{-M_\pi L}$ + must include all channels

HET gives (i) amplitude model, (ii) pion mass dependence, (iii) quark-model interpretation

- HAL-QCD potential method
  - Extract effective potential from lattice calculation
  - Requires derivative expansion

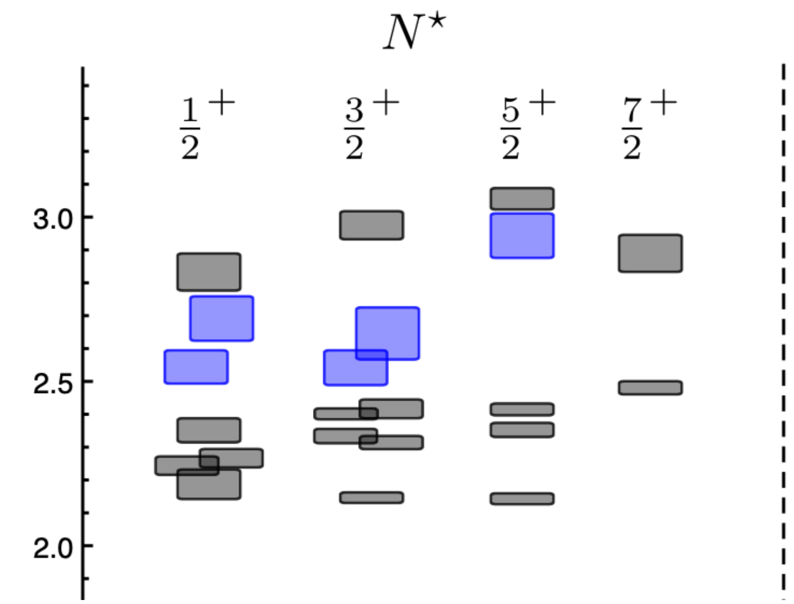
See… Murakami et al.,
Lattice QCD studies on decuplet baryons as meson-baryon bound states in the HAL QCD method

- Excited states in structure calculations
  - Next talks from Marcus and Lorenzo

Summary and outlook

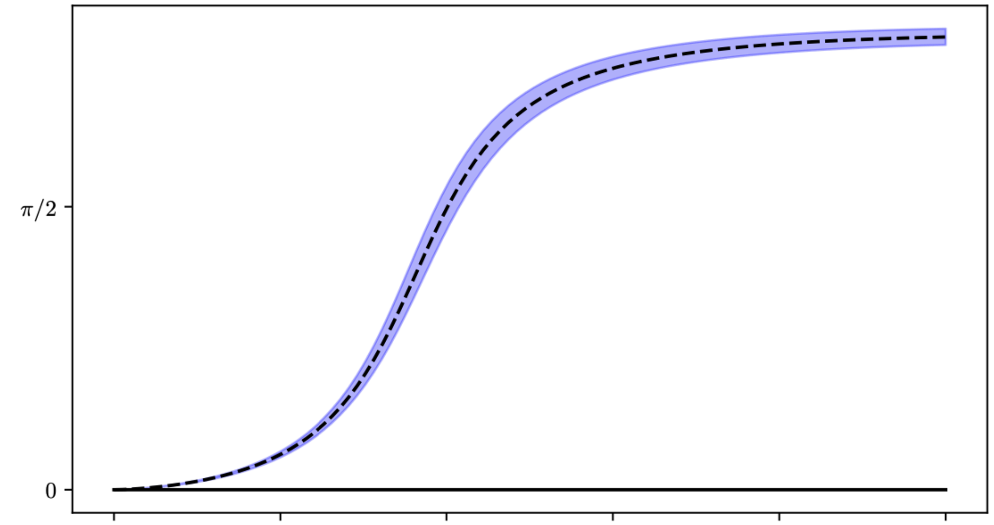
- ❑ Local-operator spectroscopy = useful guide

  result of correlator fit, indicating region where finite-volume states strongly overlap local operators



- ❑ Robust baryonic scattering studies now appearing

  complete finite-volume spectrum, field theoretically mapped to amplitudes



- ❑ Formal developments are ahead of numerical lattice QCD calculations

- ❑ Many scalar resonances + the Δ are very well controlled

Thanks for listening!… questions?

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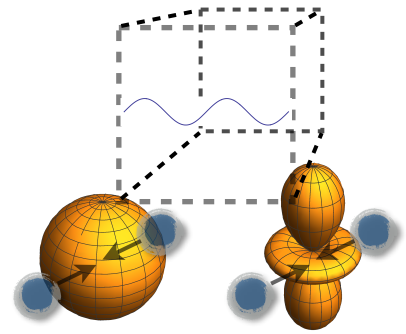
Coupled channels

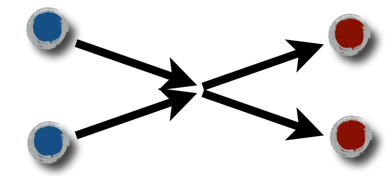
- The cubic volume mixes different partial waves…

e.g. $K\pi \to K\pi$, $\vec{P} \neq 0$ $\longrightarrow$ $\det\left[\begin{pmatrix} \mathcal{K}_s^{-1} & 0 \\ 0 & \mathcal{K}_p^{-1} \end{pmatrix} + \begin{pmatrix} F_{ss} & F_{sp} \\ F_{ps} & F_{pp} \end{pmatrix}\right] = 0$



…as well as different flavor channels…

e.g. $a = \pi\pi$, $b = K\overline{K}$ $\longrightarrow$ $\det\left[\begin{pmatrix} \mathcal{K}_{a\to a} & \mathcal{K}_{a\to b} \\ \mathcal{K}_{b\to a} & \mathcal{K}_{b\to b} \end{pmatrix}^{-1} + \begin{pmatrix} F_a & 0 \\ 0 & F_b \end{pmatrix}\right] = 0$

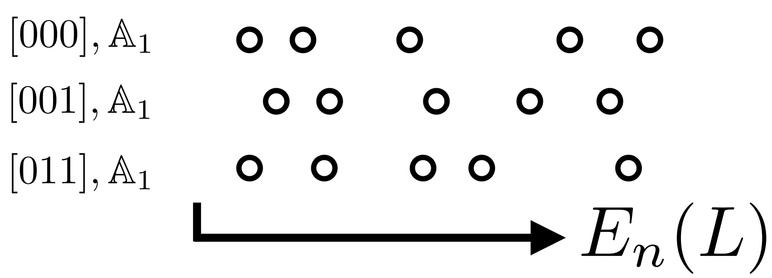


- Workflow…

Correlators with a large operator basis
$\langle \mathcal{O}_a(\tau)\mathcal{O}_b^\dagger(0)\rangle$

→ Reliably extract finite-volume energies
$\langle \Omega_m(\tau)\Omega_m^\dagger(0)\rangle \sim e^{-E_m(L)\tau}$

→ Vary L and P to recover a dense set of energies



hadspec

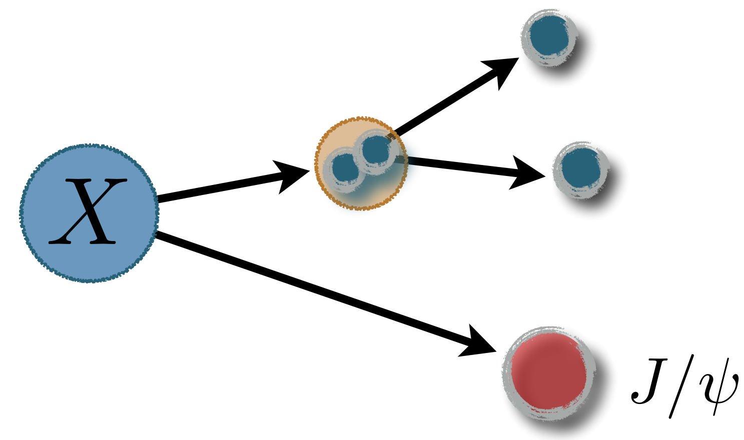
Identify a broad list of K-matrix parametrizations
polynomials and poles
EFT based
dispersion theory based

→ **Perform global fits to the finite-volume spectrum**

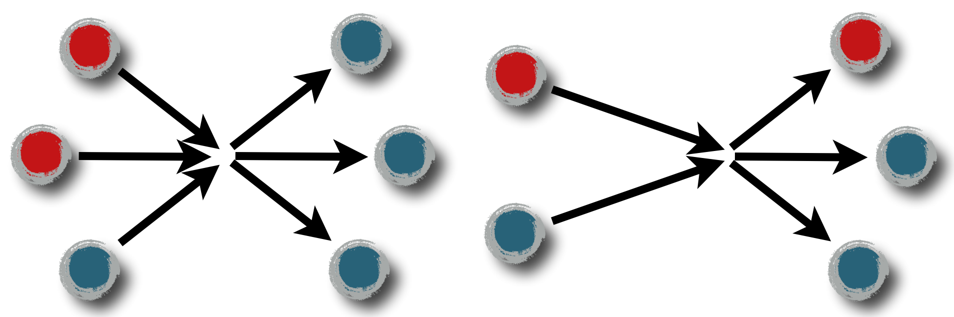
3-particle amplitudes

2-to-2 only samples J^P 0^+ 1^- $2^+ \cdots$

many interesting resonances have significant 3-body decays



Goal: ***finite-volume + unitarity formalism*** for generic two- and three-particle systems



Applications…

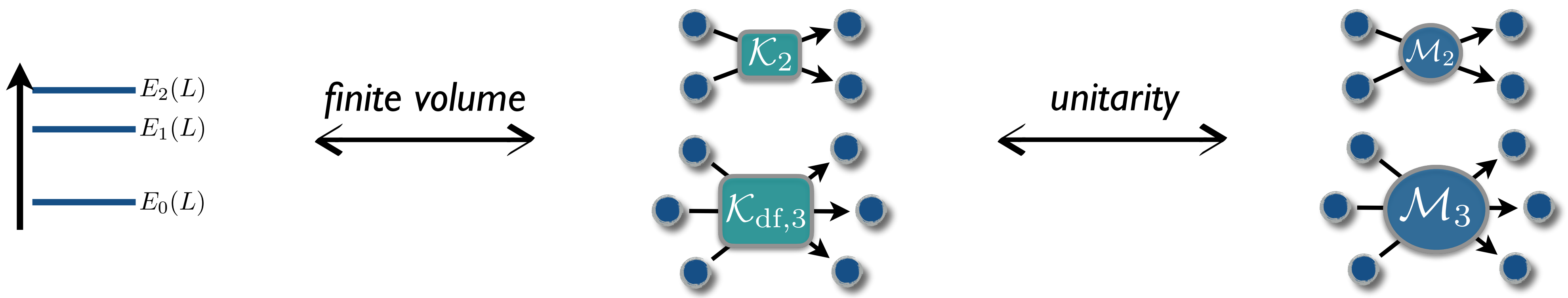
exotic resonance pole positions, couplings, quantum numbers

$\omega(782),\ a_1(1420) \to \pi\pi\pi$ $\quad X(3872) \to J/\psi\pi\pi$ $\quad X(3915)[Y(3940)] \to J/\psi\pi\pi$

form factors and transitions

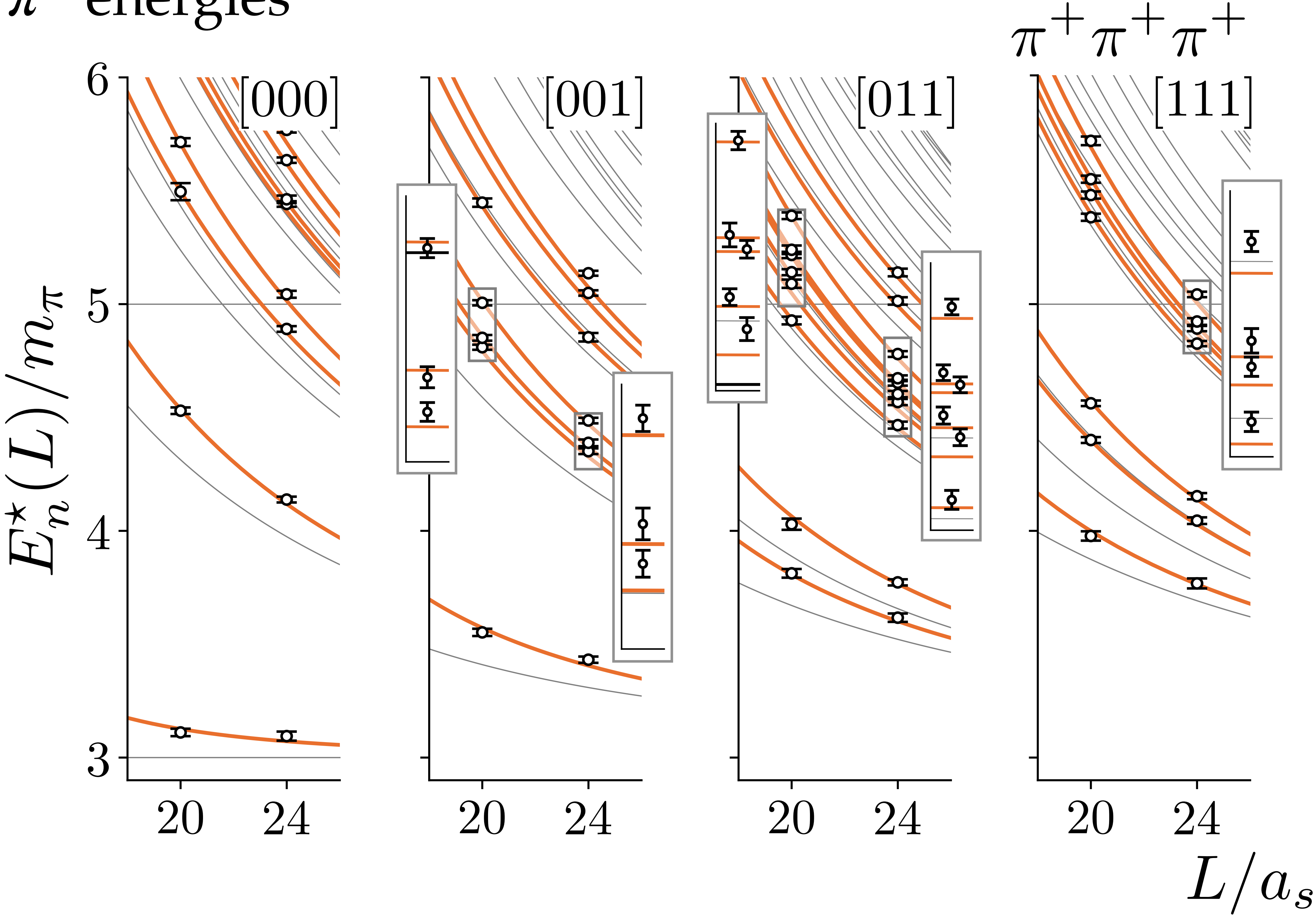
and much more!… *(3-body forces, weak transitions, gluons content)*

Status…



- ☑ Identical spin-zero, no 2-to-3, no K2 poles • MTH, Sharpe (2014, 2015) •
- ☑ as above… but including 2-to-3 • Briceño, MTH, Sharpe (2017) •
- ☑ as above… but including K2 poles • Briceño, MTH, Sharpe (2018) •
- ☑ Non-identical, non-degenerate spin-zero $\pi\pi\pi \to \rho\pi \to \omega \to \rho\pi \to \pi\pi\pi$
  • MTH, Romero-López, Sharpe (2020) • Blanton, Sharpe (2020, 2021)
- ☐ Multiple three-particle channels… Spin!

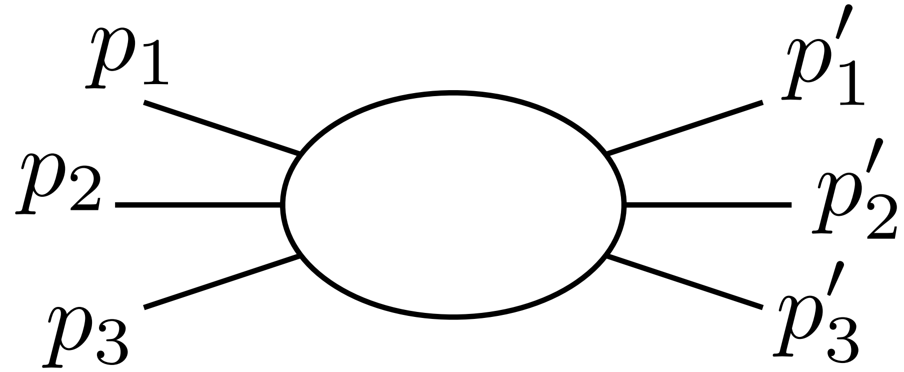
$\pi^+\pi^+\pi^+$ energies

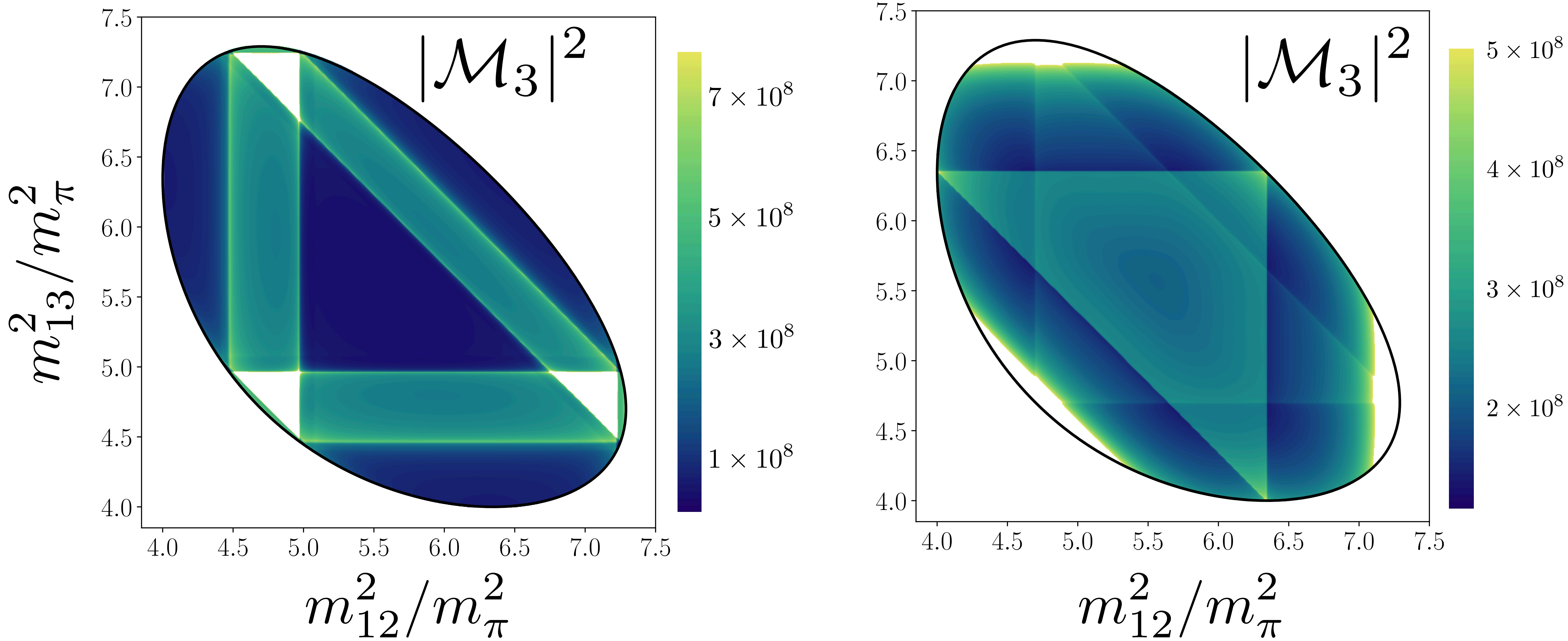


MTH, Briceño, Edwards, Thomas, Wilson, *Phys.Rev.Lett.* 126 (2021) 012001,

see also work by… Culver, Döring, Hanlon, Hörz, Mai, Morningstar, Romero-Lopez, Sharpe + ETMC

$$\mathcal{M}_3 = \sum_{i,j\in\{1,2,3\}} \mathcal{M}_3^{\mathsf{un}}(p'_i, p_j)$$





MTH, Briceño, Edwards, Thomas, Wilson, *Phys.Rev.Lett.* 126 (2021) 012001