

# **Amplitude analysis of photo-/electroproduction data in the resonance region**



Petersburg  
Nuclear  
Physics  
Institute

**A. Sarantsev**

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## Bonn-Gatchina partial wave analysis group:

A. Anisovich, E. Klempt, K. Nikonov, A. Sarantsev, U. Thoma.

<http://pwa.hiskp.uni-bonn.de/>



### Bonn-Gatchina Partial Wave Analysis



Address: Nussallee 14-16, D-53115 Bonn      Fax: (+49) 228 / 73-2505

| <a href="#"><u>Data Base</u></a>   | <a href="#"><u>Meson Spectroscopy</u></a>  | <a href="#"><u>Baryon Spectroscopy</u></a> | <a href="#"><u>NN-interaction</u></a>   | <a href="#"><u>Formalism</u></a> |
|--|--|--|---|----------------------------------|
| Analysis of Other Groups <ul style="list-style-type: none"><li>• <a href="#">SAID</a></li><li>• <a href="#">MAID</a></li><li>• <a href="#">Giessen Uni</a></li></ul> | BG PWA <ul style="list-style-type: none"><li>• <a href="#">Publications</a></li><li>• <a href="#">Talks</a></li><li>• <a href="#">Contacts</a></li></ul> |  | Useful Links <ul style="list-style-type: none"><li>• <a href="#">SPIRES</a></li><li>• <a href="#">PDG Homepage</a></li><li>• <a href="#">Durham Data Base</a></li><li>• <a href="#">Bonn Homepage</a></li></ul> |                                  |
| <a href="#"><u>CB-ELSA Homepage</u></a>  |  |  |   |                                  |

Responsible: Dr. V. Nikonov, E-mail: [nikonov@hiskp.uni-bonn.de](mailto:nikonov@hiskp.uni-bonn.de)

Last changes: January 26<sup>th</sup>, 2010.

## Energy dependent fully covariant approach

In many cases an unambiguous partial wave decomposition at fixed energies is impossible. Then the energy and angular parts should be analyzed together:

$$A(s, t) = \sum_{\beta\beta'n} A_n^{\beta\beta'}(s) Q_{\mu_1\dots\mu_n}^{(\beta)+} F_{\nu_1\dots\nu_n}^{\mu_1\dots\mu_n} Q_{\nu_1\dots\nu_n}^{(\beta')}$$

$\pi N$  interaction:

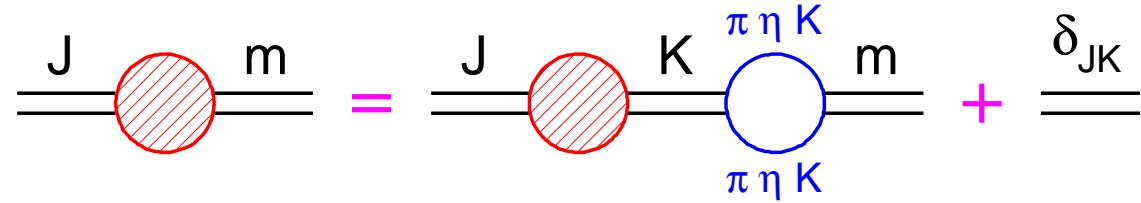
$$Q_{\mu_1\dots\mu_n}^{(+n)} = X_{\mu_1\dots\mu_n}^{(n)} \quad Q_{\mu_1\dots\mu_n}^{(-n)} = i\gamma_\nu\gamma_5 X_{\nu\mu_1\dots\mu_n}^{(n+1)}$$

$$X^0 = 1 ; \quad X_\mu^1 = k_\mu^\perp ; \quad X_{\mu\nu}^2 = \frac{3}{2} \left( k_\mu^\perp k_\nu^\perp - \frac{1}{3} k_\perp^2 g_{\mu\nu}^\perp \right) ;$$

$$X_{\mu\nu\alpha}^3 = \frac{5}{2} \left[ k_\mu^\perp k_\nu^\perp k_\alpha^\perp - \frac{k_\perp^2}{5} (g_{\mu\nu}^\perp k_\alpha^\perp + g_{\mu\alpha}^\perp k_\nu^\perp + g_{\nu\alpha}^\perp k_\mu^\perp) \right] ,$$

1. C. Zemach, Phys. Rev. 140, B97 (1965); 140, B109 (1965).
2. S.U.Chung, Phys. Rev. D 57, 431 (1998).
- A. V. Anisovich, V. V. Anisovich, V. N. Markov, M. A. Matveev and A. V. Sarantsev, J. Phys. G 28, 15 (2002)
3. B. S. Zou and D. V. Bugg, Eur. Phys. J. A 16, 537 (2003)

# N/D based (D-matrix) analysis of the data



$$D_{jm} = D_{jk} \sum_{\alpha} B_{\alpha}^{km}(s) \frac{1}{M_m - s} + \frac{\delta_{jm}}{M_j^2 - s} \quad \hat{D} = \hat{\kappa} (I - \hat{B} \hat{\kappa})^{-1}$$

$$\hat{\kappa} = diag \left( \frac{1}{M_1^2 - s}, \frac{1}{M_2^2 - s}, \dots, \frac{1}{M_N^2 - s}, R_1, R_2 \dots \right)$$

$$\hat{B}_{ij} = \sum_{\alpha} B_{\alpha}^{ij} = \sum_{\alpha} \int \frac{ds'}{\pi} \frac{g_{\alpha}^{(R)i} \rho_{\alpha}(s', m_{1\alpha}, m_{2\alpha}) g_{\alpha}^{(L)j}}{s' - s - i0}$$

**Channels included in D-matrix:**  $\pi N, \eta N, K\Lambda, K\Sigma, \Delta\pi, N\sigma, N\rho(770), N(1520)\pi, N(1535)\pi, N\omega, \text{Black Box}$

## Minimization methods

1. The two body final states  $\pi N, \gamma N \rightarrow \pi N, \eta N, K\Lambda, K\Sigma, \omega N, K^*\Lambda$ :  $\chi^2$  method.

For  $n$  measured bins we minimize

$$\chi^2 = \sum_j^n \frac{(\sigma_j(PWA) - \sigma_j(exp))^2}{(\Delta\sigma_j(exp))^2}$$

Present solution for  $\gamma p$  reaction  $\chi^2 = 69435$  for 46644 points.  $\chi^2/N_F = 1.49$

2. Reactions with three or more final states are analyzed with logarithm likelihood method.  $\pi N, \gamma N \rightarrow \pi\pi N, \pi\eta N$ . The minimization function:

$$f = - \sum_j^{N(data)} \ln \frac{\sigma_j(PWA)}{\sum_m^{N(recMC)} \sigma_m(PWA)}$$

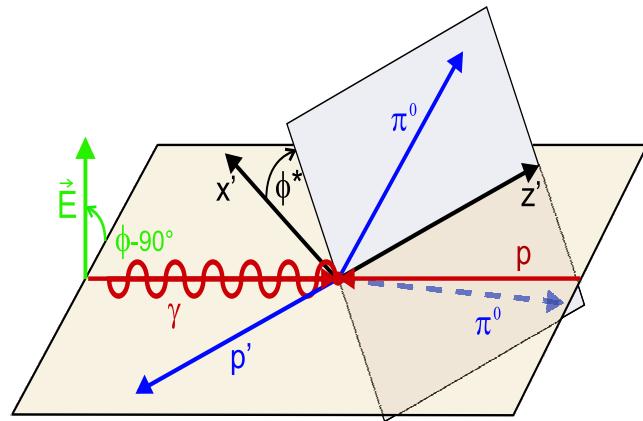
This method allows us to take into account all correlations in many dimensional phase space. Above 1 000 000 data events are taken in the fit.

## The included meson photoproduction data

| DATA   | 2011-2019  | added in 2019-2022                                   |
|--|--|--|
| $\pi N \rightarrow \pi N$ ampl.              | <b>SAID</b>  | <b>Hoehler (energy fixed)</b>                        |
| $\pi^- p \rightarrow \pi\pi N$               | $d\sigma/d\Omega (\pi^0\pi^0 n, \pi^+\pi^- n, \pi^-\pi^0 p)$   |  |
| $\pi^- p \rightarrow \eta n$                 | $d\sigma/d\Omega$  |  |
| $\pi p \rightarrow K\Lambda, K\Sigma$        | $d\sigma/d\Omega, P, \beta$  |  |
| $\pi p \rightarrow \omega n$                 |  | $d\sigma/d\Omega$                                    |
| $\gamma p \rightarrow \pi N$                 | $d\sigma/d\Omega, \Sigma, T, P, E, G, H (\pi^0 p, \pi^+ n)$  |  |
| $\gamma p \rightarrow \eta p$                | $d\sigma/d\Omega, \Sigma, F, T, P, H, G, E$  |  |
| $\gamma p \rightarrow \eta' p$               | $d\sigma/d\Omega, \Sigma$  |  |
| $\gamma p \rightarrow K\Lambda, K\Sigma$     | $d\sigma/d\Omega, \Sigma, P, T, C_x, C_z, O_{x'}, O_{z'}, T_x, T_z$  |  |
| $\gamma p \rightarrow \pi^0\pi^0 p$          | $d\sigma/d\Omega, \Sigma, E, I_c, I_s$   | $\Sigma, E, T, P, H, F, P_x, P_y$ ( <b>CB-ELSA</b> ) |
| $\gamma p \rightarrow \pi^+\pi^- p$          | $d\sigma/d\Omega$  | $I_c, I_s$ ( <b>CLAS</b> )                           |
| $\gamma p \rightarrow \omega p$              | $d\sigma/d\Omega, \Sigma, \rho_{ij}^k, E, G$ ( <b>CB-ELSA</b> ), $\Sigma, \text{P,T,F,H}$ ( <b>CLAS</b> )  | Taken explicitly                                     |
| $\gamma n \rightarrow \Lambda K, \Sigma^- K$ | $d\sigma/d\Omega$ ( <b>CLAS</b> ), <b>E</b> ( <b>CLAS</b> )  | $\Sigma, G$ ( <b>CLAS</b> )                          |
| $\gamma n \rightarrow \pi^- p$               | $d\sigma/d\Omega, \Sigma, P, E, \Sigma$ ( <b>CLAS</b> )  |  |
| $\gamma n \rightarrow \eta n$                | $d\sigma/d\Omega$ ( <b>CB-ELSA, MAMI</b> ), $\Sigma, d\sigma/d\Omega (h = \frac{1}{2})$ ( <b>CB-ELSA</b> ) |  |

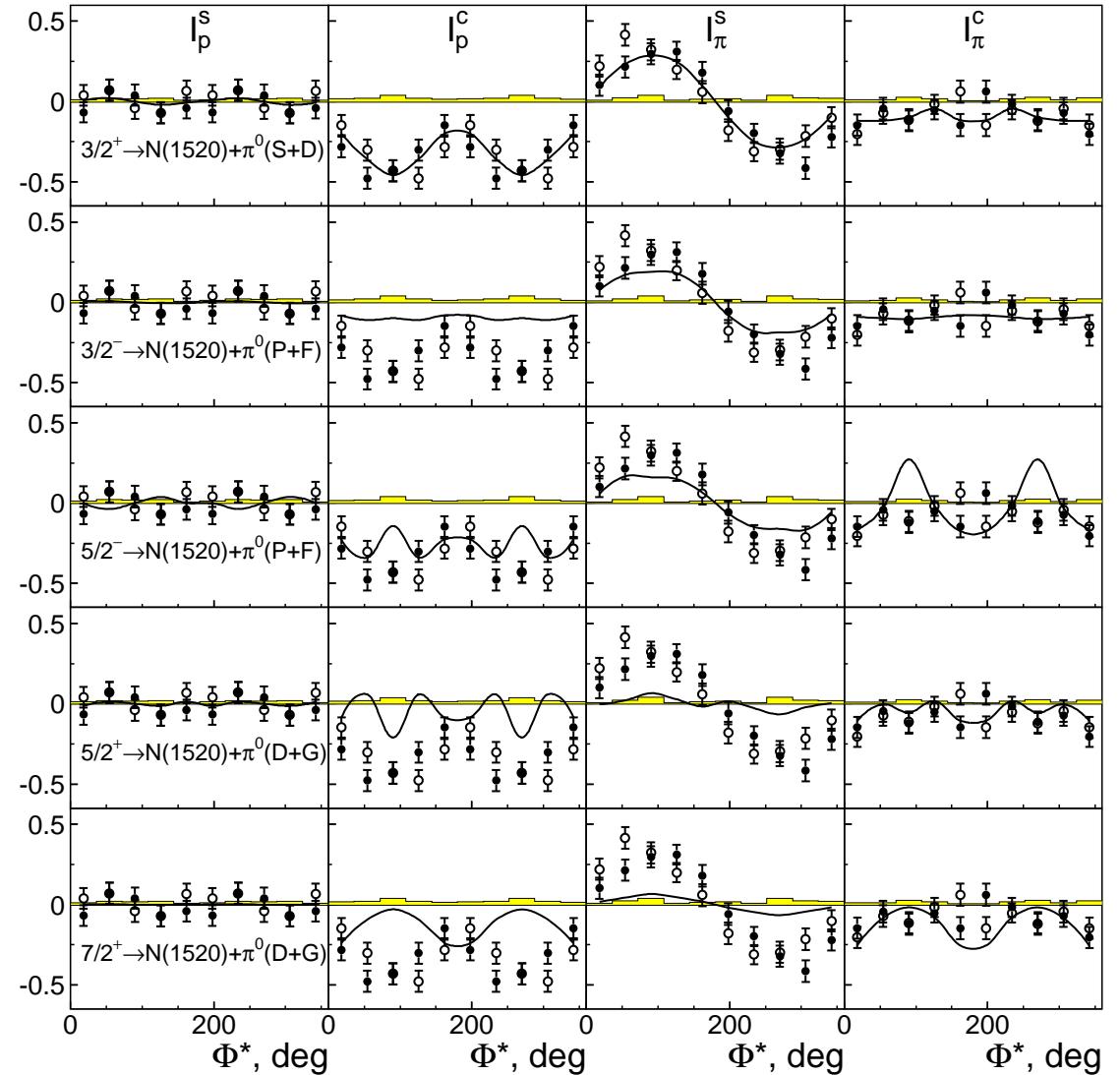
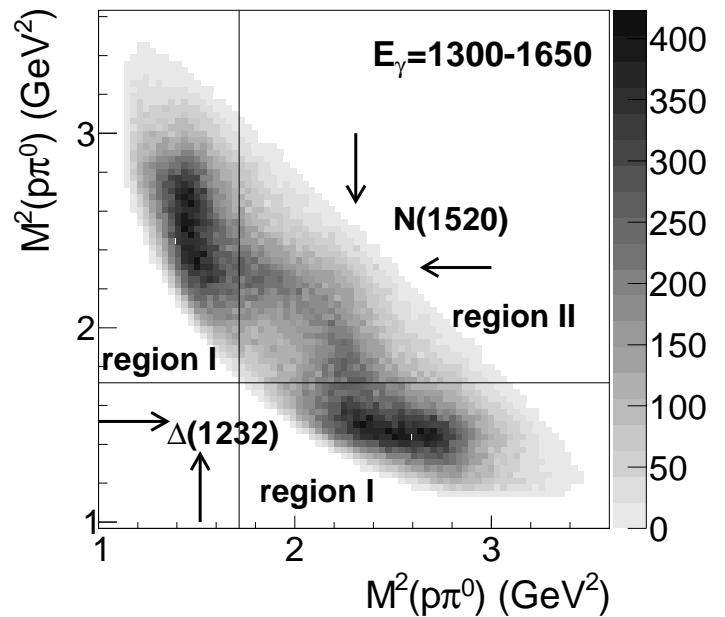
# $\gamma p \rightarrow \pi^0 \pi^0 p$ **Polarization observables**

$$\frac{d\sigma}{d\Omega}(\Theta, \varphi) = \frac{d\sigma_0}{d\Omega}(\Theta) [1 - \Sigma(\Theta) \cos(2\varphi) - \Lambda_x H(\Theta) \sin(2\phi) - \Lambda_y P(\Theta) \cos(2\varphi) + \Lambda_y T(\Theta)]$$

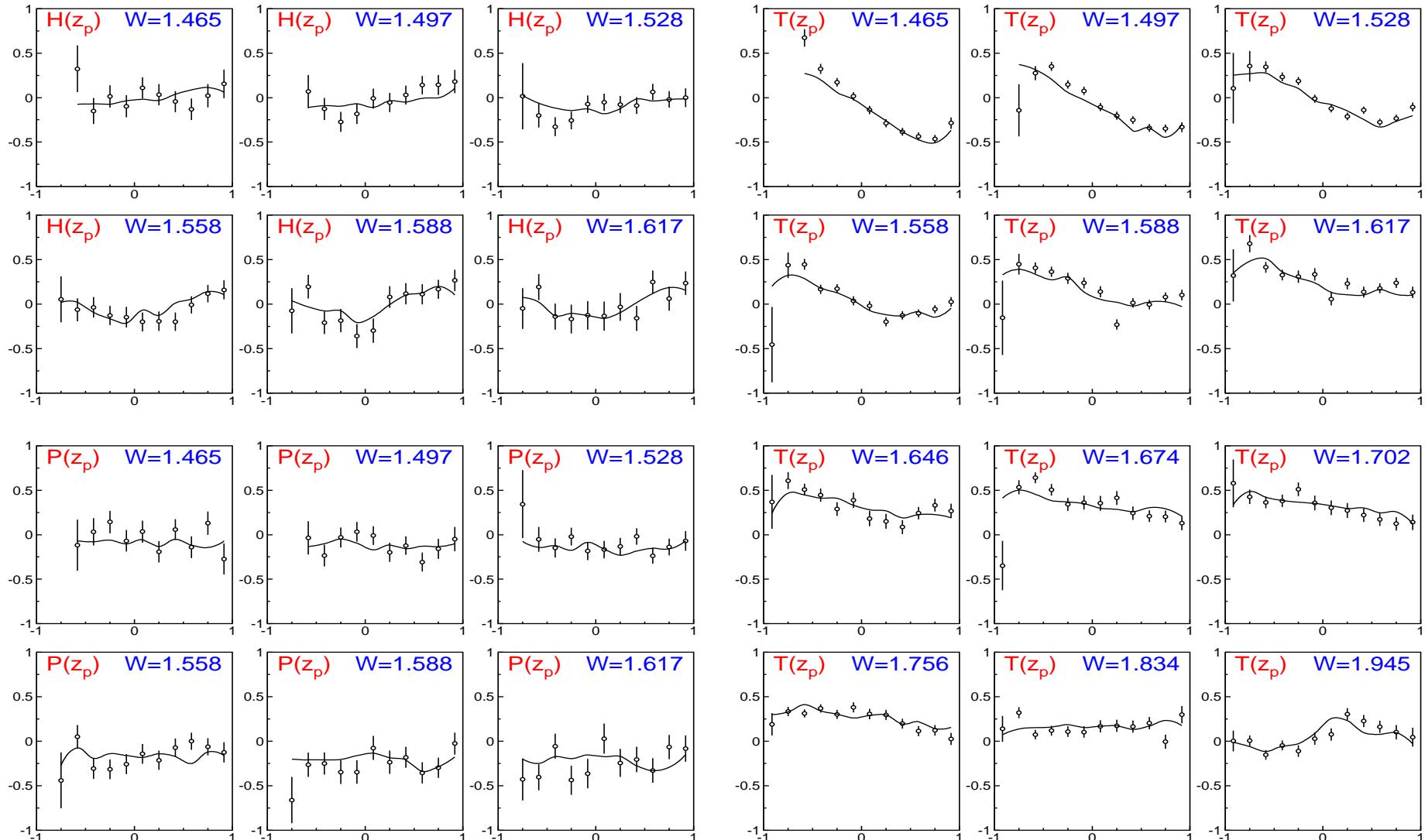


$$\begin{aligned} \frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega}(\Theta) & \left[ 1 + \Lambda_x P_x + \Lambda_y P_y \quad + \sin(2\varphi) (I^s + \Lambda_x P_x^s + \Lambda_y P_y^s) \right. \\ & \left. + \cos(2\varphi) (I^c + \Lambda_x P_x^c + \Lambda_y P_y^c) \right] \end{aligned}$$

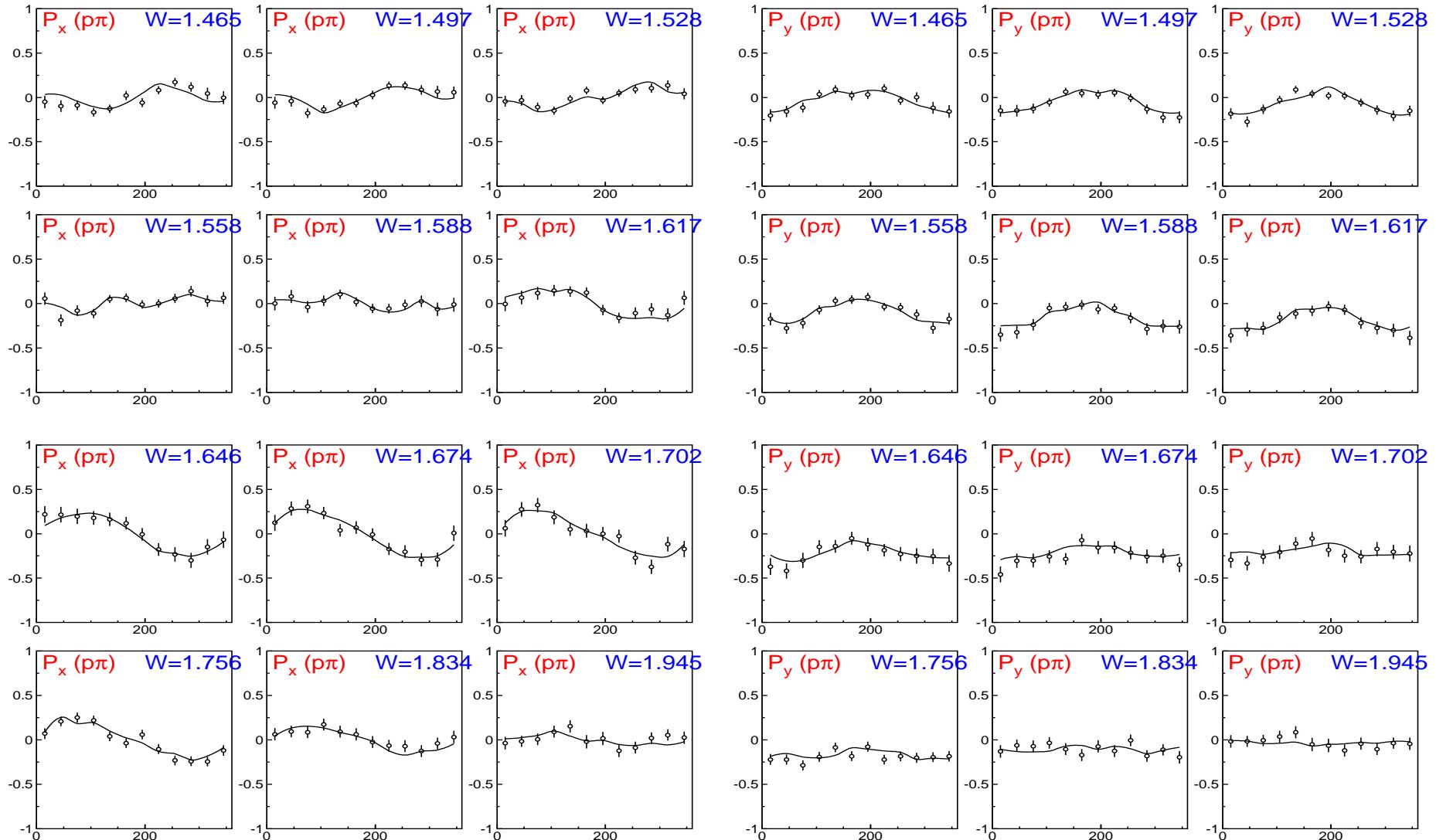
$I_c$  and  $I_s$  polarization data are very important for the partial wave analysis



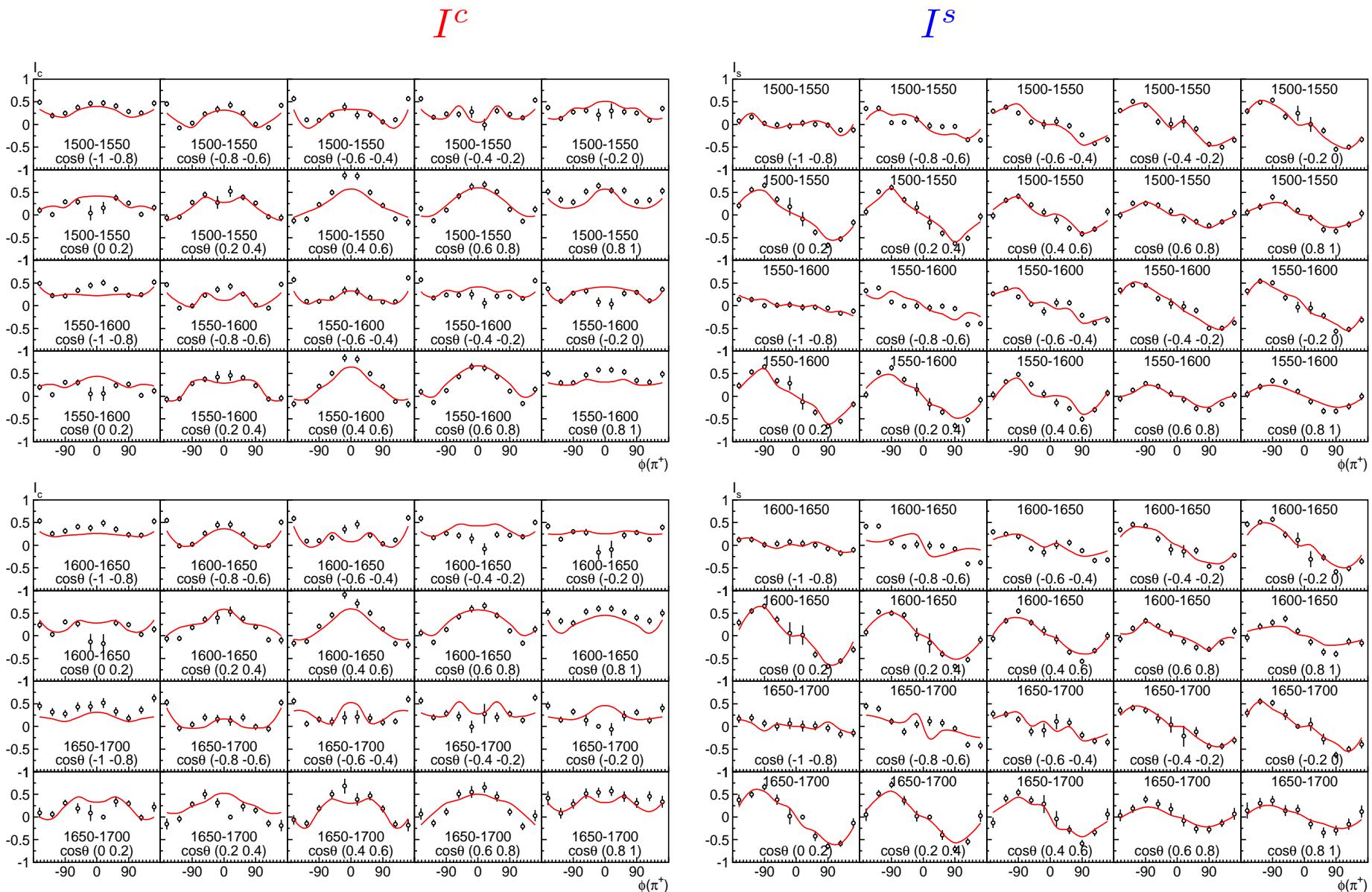
$\gamma p \rightarrow \pi^0 \pi^0 p$  polarization observables from CB-ELSA (T.Seifen)



$\gamma p \rightarrow \pi^0 \pi^0 p$  polarization observables from CB-ELSA (T.Seifen)

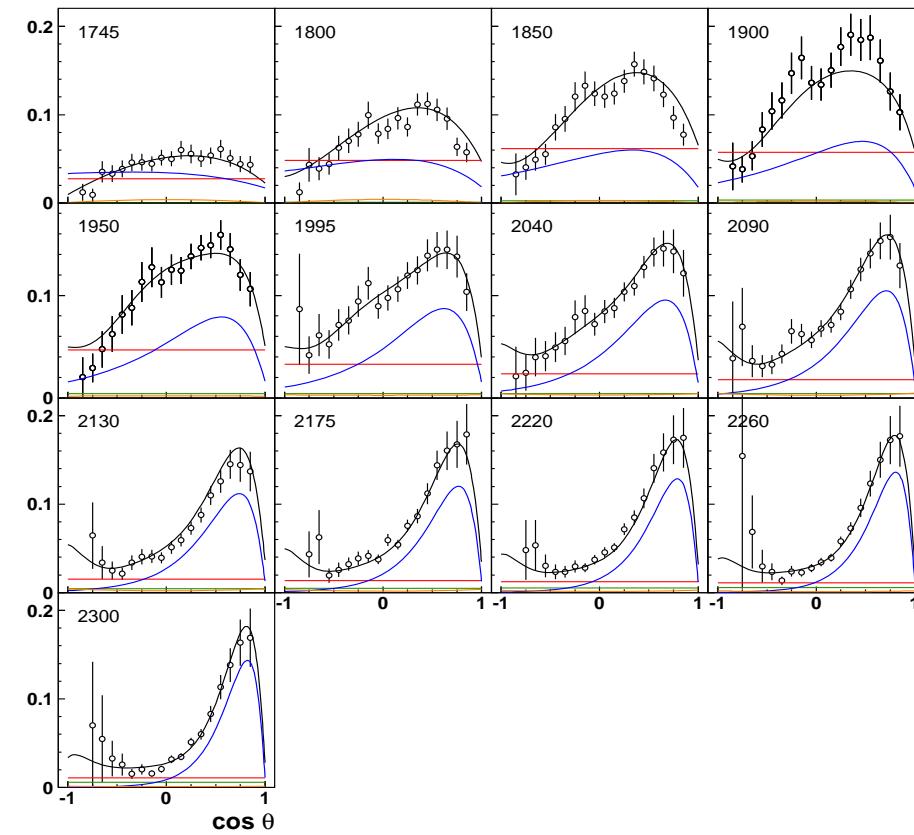
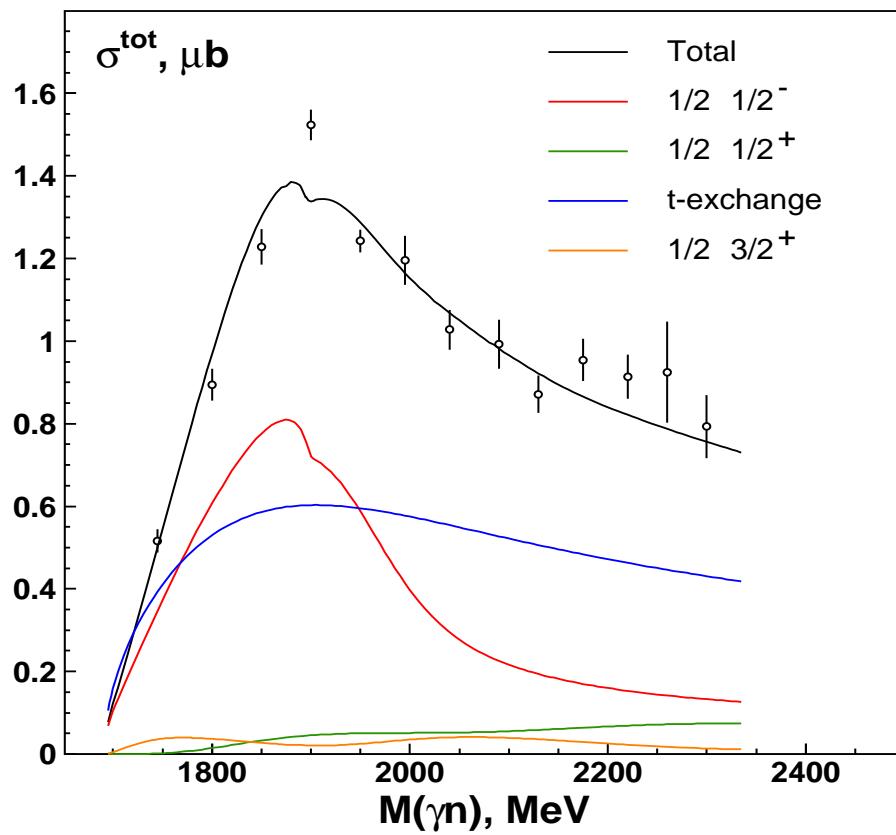


# $\gamma p \rightarrow \pi^+ \pi^- p$ : $I^c$ and $I^s$ polarization observables from CLAS (V.Crede)



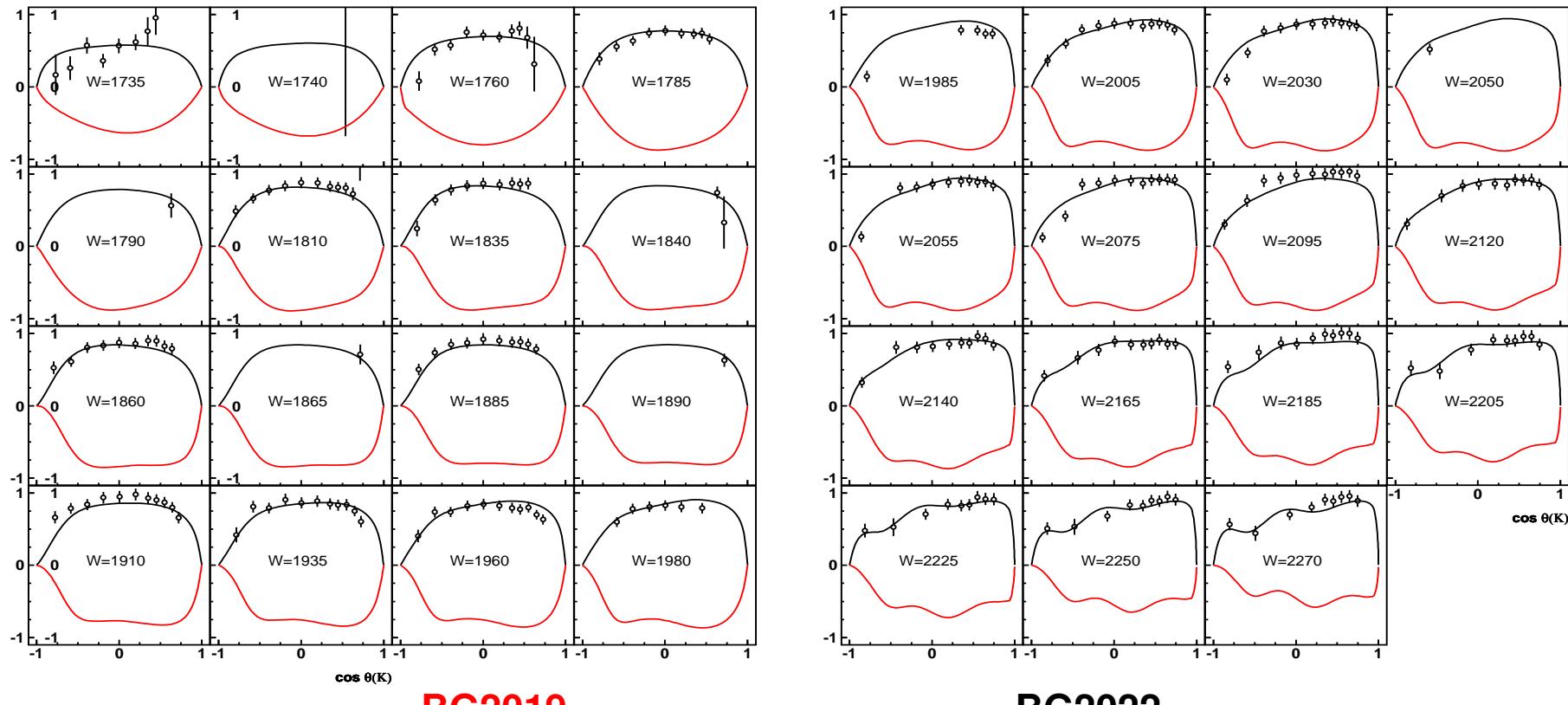
|                      | $N\pi$                   | $\Delta\pi$<br>( $L < J$ ) | $\Delta\pi$<br>( $L > J$ ) | $N(1440)\pi$             | $N(1520)\pi$           | $N(1535)\pi$           | $N\sigma$                  |
|----------------------|--------------------------|----------------------------|----------------------------|--------------------------|------------------------|------------------------|----------------------------|
| $N(1535) 1/2^-$      | $46 \pm 5$<br>$52 \pm 5$ | x                          | $5 \pm 3$<br>$2.5 \pm 1.5$ | $6 \pm 5$<br>$12 \pm 8$  | -                      | -                      | $4 \pm 2$<br>$6 \pm 4$     |
| $N(1520) 3/2^-$      | $61 \pm 3$<br>$61 \pm 2$ | $10 \pm 4$<br>$19 \pm 4$   | $10 \pm 3$<br>$9 \pm 2$    | $\leq 1$<br>$\leq 1$     | -                      | -                      | $\leq 2$<br>$\leq 2$       |
| $N(1650) 1/2^-$      | $48 \pm 4$<br>$51 \pm 4$ | x                          | $6 \pm 3$<br>$12 \pm 6$    | $5 \pm 3$<br>$16 \pm 10$ | -                      | -                      | $3 \pm 2$<br>$10 \pm 8$    |
| $N(1700) 3/2^-$      | $20 \pm 8$<br>$15 \pm 6$ | $66 \pm 17$<br>$65 \pm 15$ | $7 \pm 4$<br>$9 \pm 5$     | $9 \pm 5$<br>$7 \pm 4$   | $\leq 2$<br>$\leq 4$   | $\leq 1$<br>$\leq 1$   | $6 \pm 4$<br>$8 \pm 6$     |
| $N(1675) 5/2^-$      | $40 \pm 1$<br>$41 \pm 2$ | $19 \pm 3$<br>$30 \pm 7$   | -                          | -                        | -                      | -                      | $1 \pm 1$<br>$5 \pm 2$     |
| $\Delta(1620) 1/2^-$ | $30 \pm 5$<br>$28 \pm 3$ | x                          | $28 \pm 15$<br>$62 \pm 10$ | $15 \pm 8$<br>$6 \pm 3$  | -                      | -                      | x                          |
| $\Delta(1700) 3/2^-$ | $22 \pm 6$<br>$22 \pm 4$ | $16 \pm 15$<br>$20 \pm 15$ | $8 \pm 6$<br>$10 \pm 6$    | $3 \pm 2$<br>$\leq 1$    | $\leq 1$<br>$3 \pm 2$  | $\leq 1$               | x                          |
| $\Delta(1600) 3/2^+$ | $17 \pm 4$<br>$14 \pm 4$ | $70 \pm 6$<br>$77 \pm 5$   | $\leq 2$<br>$\leq 2$       | $\leq 1$<br>$22 \pm 5$   | -                      | -                      | x                          |
| $N(1720) 3/2^+$      | $13 \pm 5$<br>$11 \pm 4$ | $15 \pm 7$<br>$62 \pm 15$  | $6 \pm 6$<br>$6 \pm 6$     | $6 \pm 5$<br>$\leq 2$    | $7 \pm 3$<br>$3 \pm 2$ | $4 \pm 2$<br>$\leq 2$  | $20 \pm 10$ 2<br>$8 \pm 6$ |
| $N(1680) 5/2^+$      | $68 \pm 8$<br>$62 \pm 4$ | $8 \pm 4$<br>$7 \pm 3$     | $8 \pm 4$<br>$10 \pm 3$    | -                        | $\leq 1$<br>$\leq 1$   | -                      | $8 \pm 4$ 2<br>$14 \pm 5$  |
| $\Delta(1910) 1/2^+$ | $16 \pm 6$<br>$12 \pm 3$ | x                          | $17 \pm 9$<br>$50 \pm 16$  | $50 \pm 18$<br>$6 \pm 3$ | -                      | $4 \pm 2$<br>$5 \pm 3$ | x                          |
| $\Delta(1920) 3/2^+$ | $12 \pm 6$<br>$8 \pm 4$  | $5 \pm 4$<br>$18 \pm 10$   | $40 \pm 20$<br>$58 \pm 14$ | $9 \pm 6$<br>$\leq 4$    | $10 \pm 8$<br>$\leq 5$ | $5 \pm 5$<br>$\leq 2$  | x                          |
| $\Delta(1905) 5/2^+$ | $13 \pm 4$<br>$13 \pm 2$ | $20 \pm 12$<br>$33 \pm 10$ | -                          | -                        | -                      | $\leq 1$               | x                          |
| $\Delta(1950) 7/2^+$ | $46 \pm 4$<br>$46 \pm 2$ | $5 \pm 4$<br>$5 \pm 4$     | -                          | -                        | -                      | -                      | x                          |

# The $\gamma n \rightarrow K^+ \Sigma^-$ photoproduction data (included in BG2019)



# The beam asymmetry for the $\gamma n \rightarrow K^+ \Sigma^-$ data

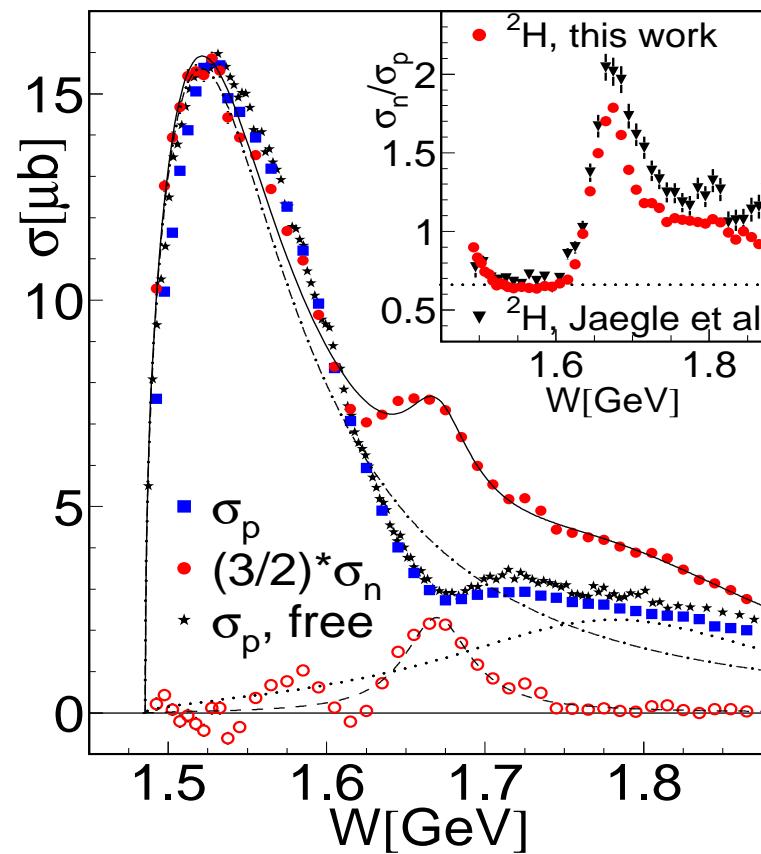
N. Zachariou *et al.* [CLAS], Phys. Lett. B 827, 136985 (2022).



The data fix the  $\gamma n$  couplings for the  $N^*(3/2^+)$  states. For  $N(1900)3/2^+$  the helicity couplings changed the sign.

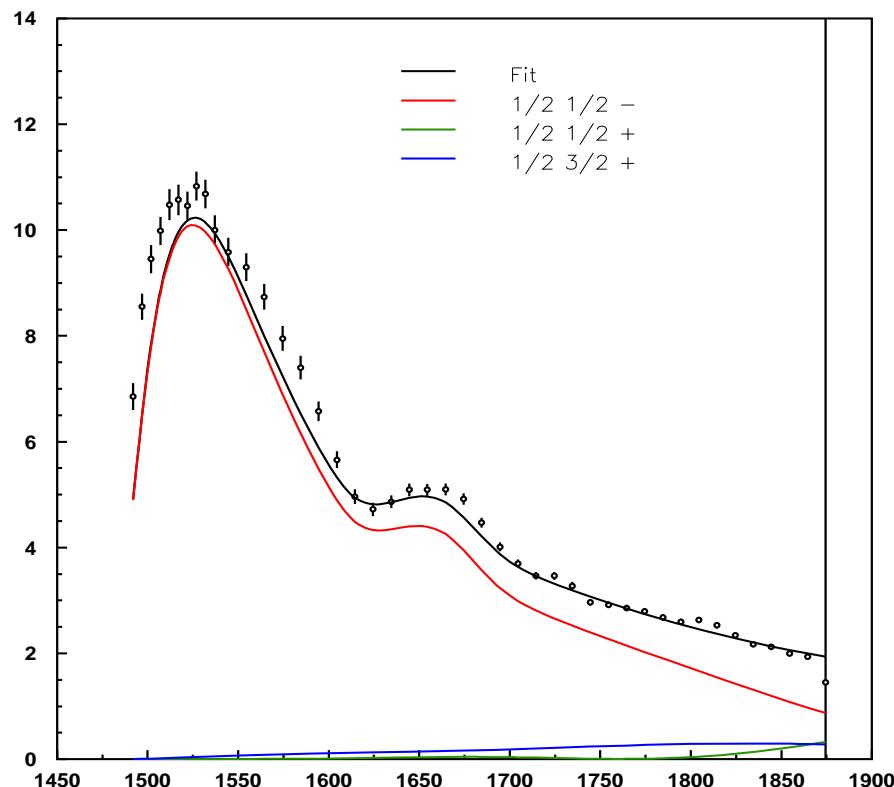
# The search for the pentaquark $P_{11}$ state

In memory of Bernd Krusche

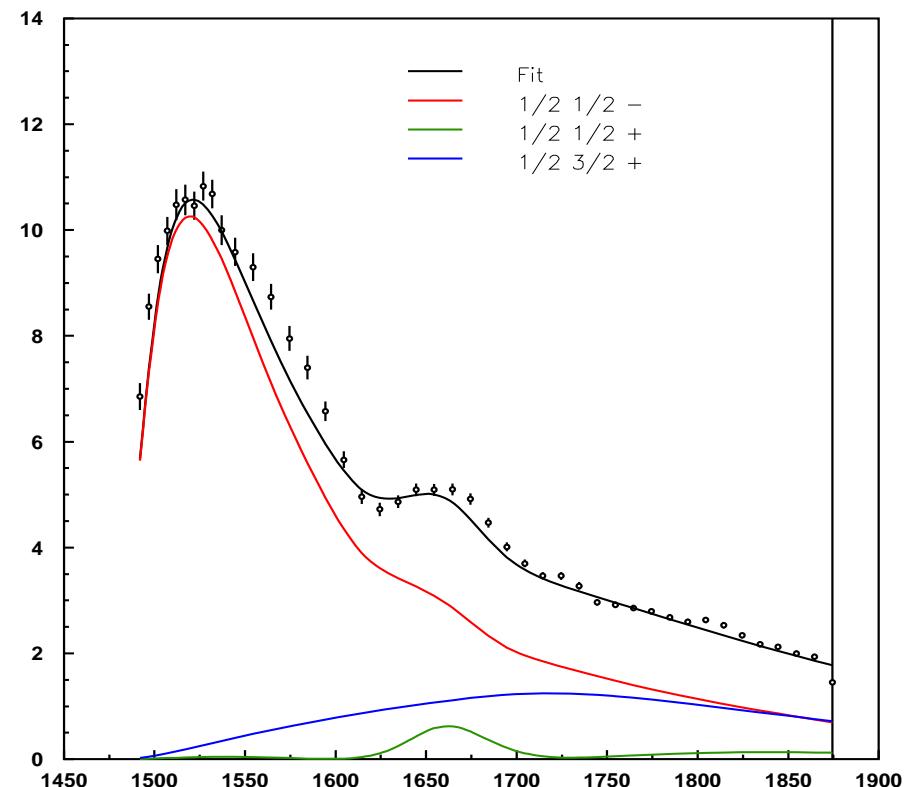


# The fit of the $\gamma n \rightarrow \eta n$ data with BG2022

D. Werthmüller *et al.* [A2 Collaboration], Phys. Rev. Lett. 111, 232001 (2013)



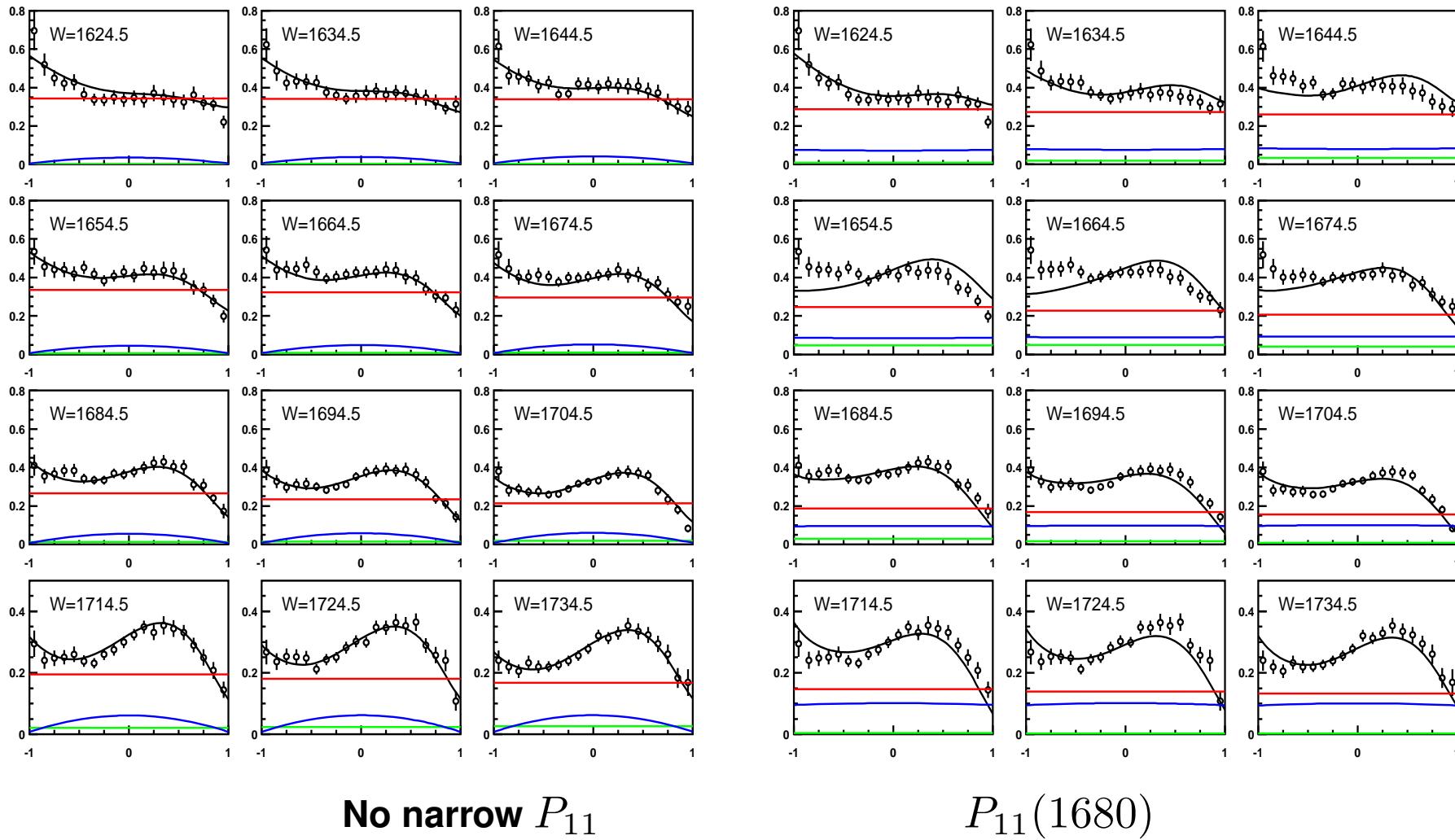
No narrow  $P_{11}$



$P_{11}(1680)$

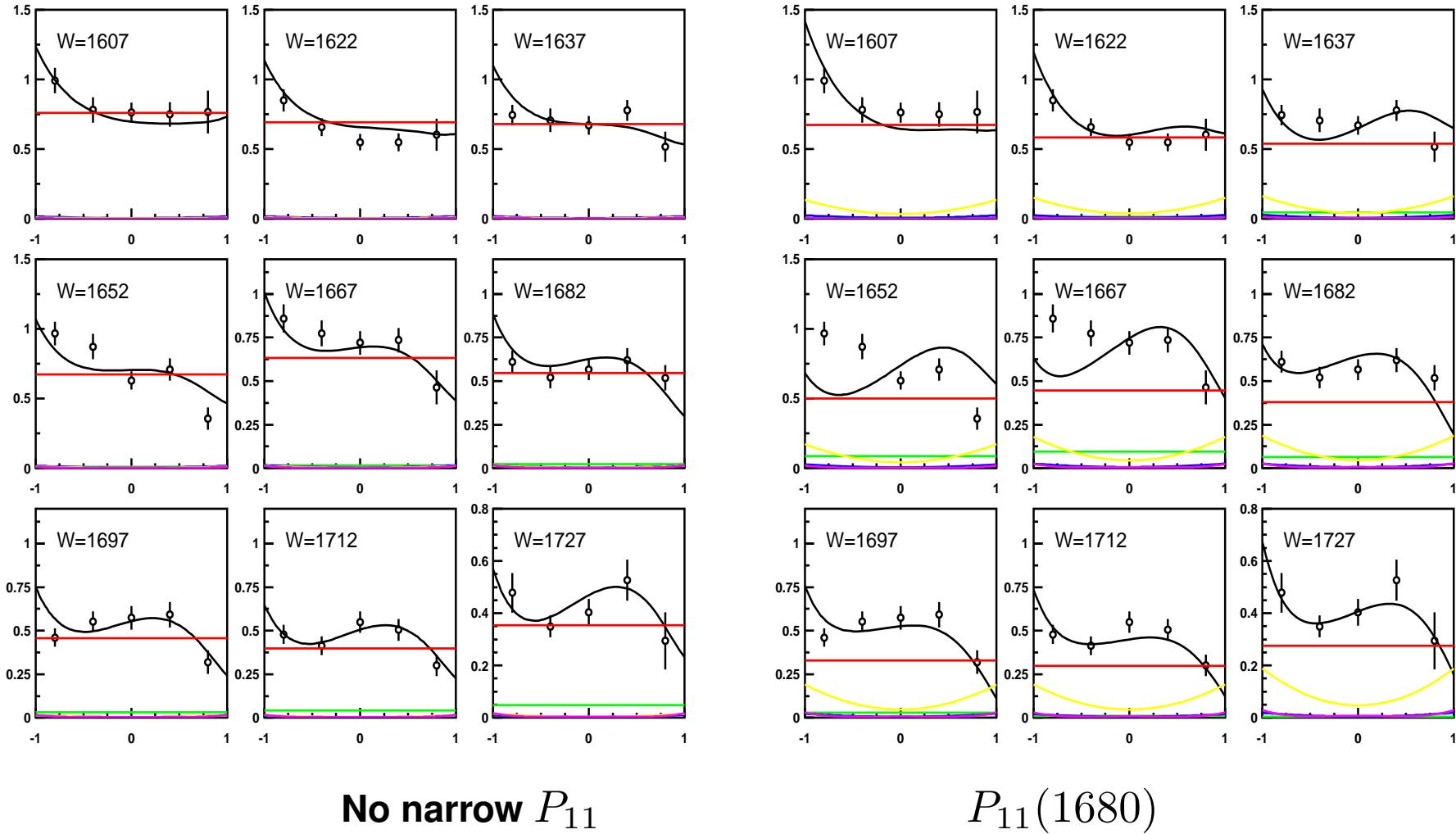
# The fit of the $\gamma n \rightarrow \eta n$ data with BG2022

D. Werthmüller *et al.* [A2 Collaboration], Phys. Rev. Lett. 111, 232001 (2013)

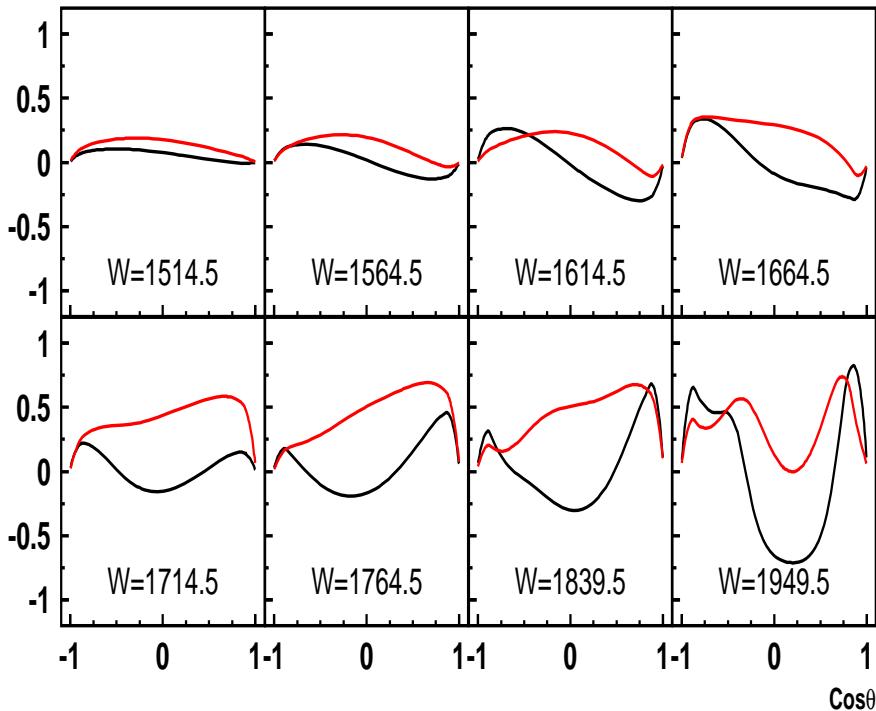


# The helicity 1/2 $\gamma n \rightarrow \eta n$ data with BG2022

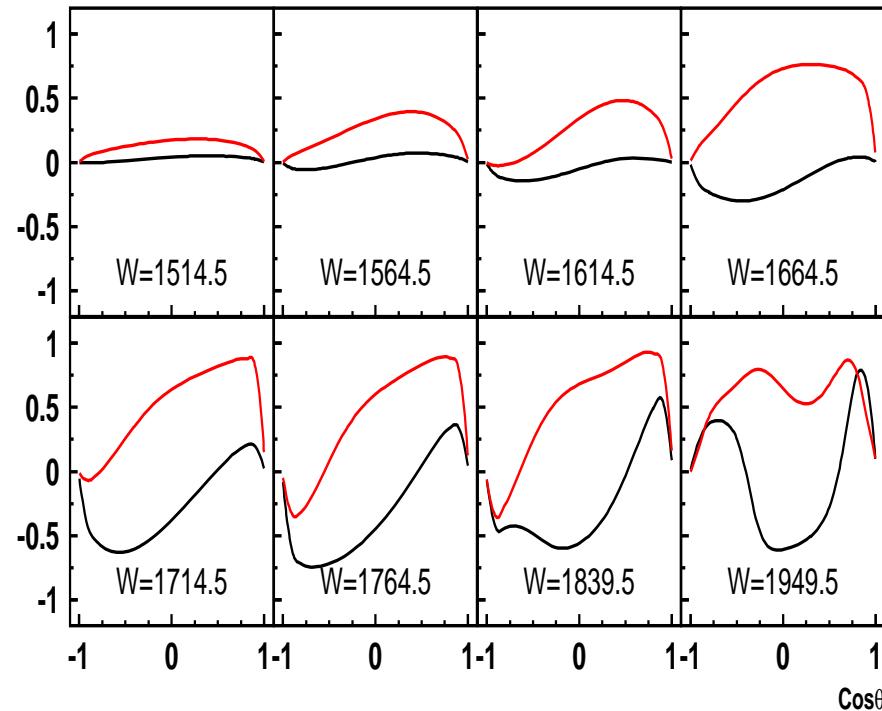
L. Witthauer *et al.*, Phys. Rev. Lett. 117, no. 13, 132502 (2016)



## Prediction for the T and P observables

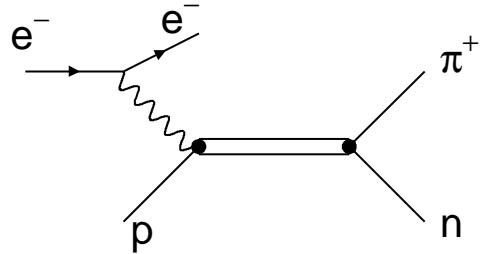


Interference  $S_{11}$  states



$P_{11}(1680)$

## Electro-production of pseudoscalar mesons



$$A = \omega^* J_\mu \omega' \bar{u}(k_f) \gamma_\mu u(k_i) \frac{e}{q^2}$$

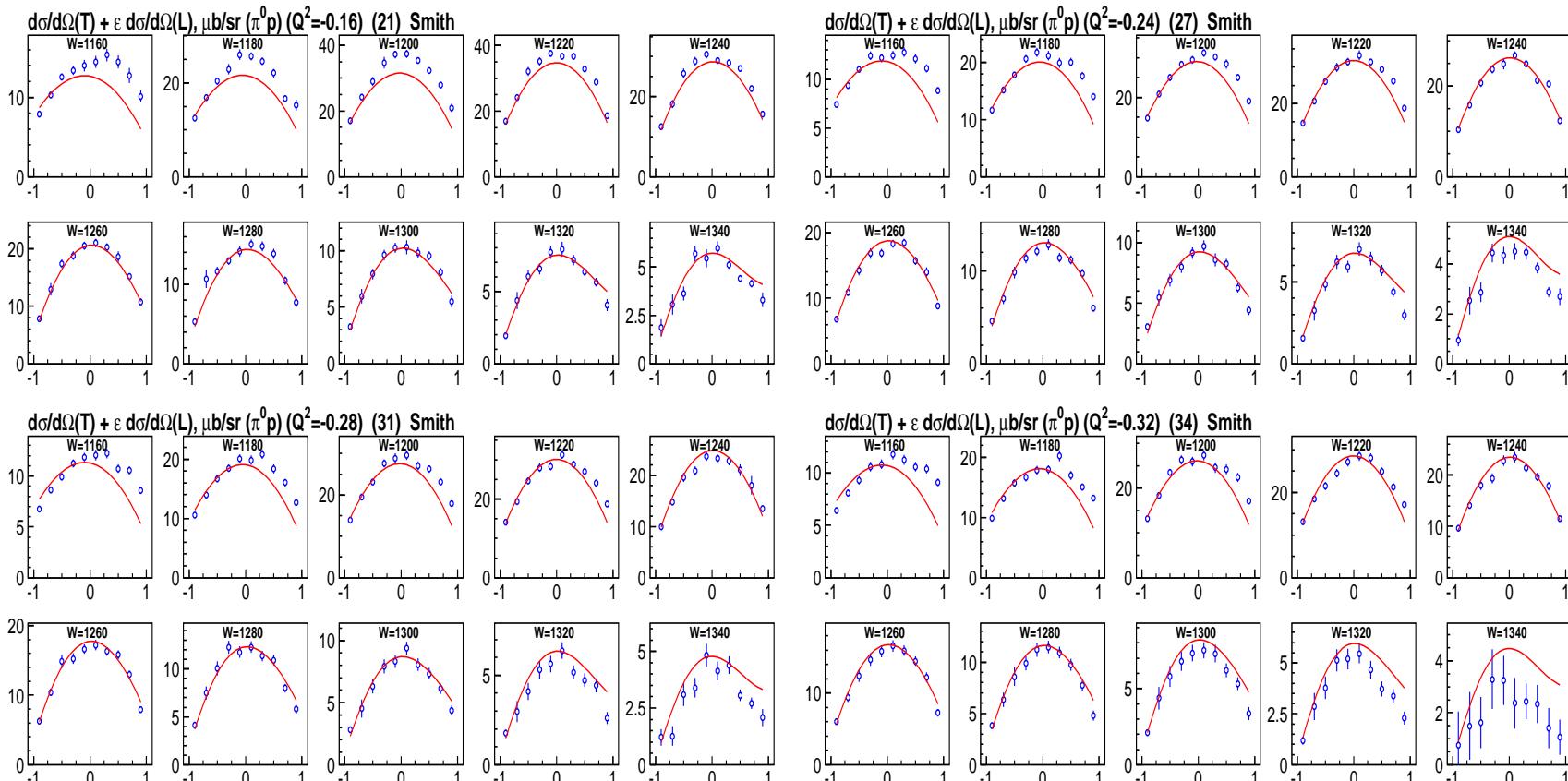
$$|A|^2 = J_\mu J_\nu^* \frac{e^2}{2Q^4} \left( 2K_\mu K_\nu + \frac{q^2}{2} g_{\mu\nu} - \frac{1}{2} q_\mu q_\nu + i h \varepsilon_{\mu\nu\alpha\beta} q_\alpha K_\beta \right)$$

$$\frac{d\sigma}{d\Omega_f d\varepsilon_f d\Omega_\pi} = \Gamma \frac{d\sigma_v}{d\Omega_\pi} \quad \Gamma = \frac{\alpha}{2\pi^2} \frac{\varepsilon_f}{\varepsilon_i} \frac{|k_\gamma|}{Q^2} \frac{1}{1-\varepsilon}$$

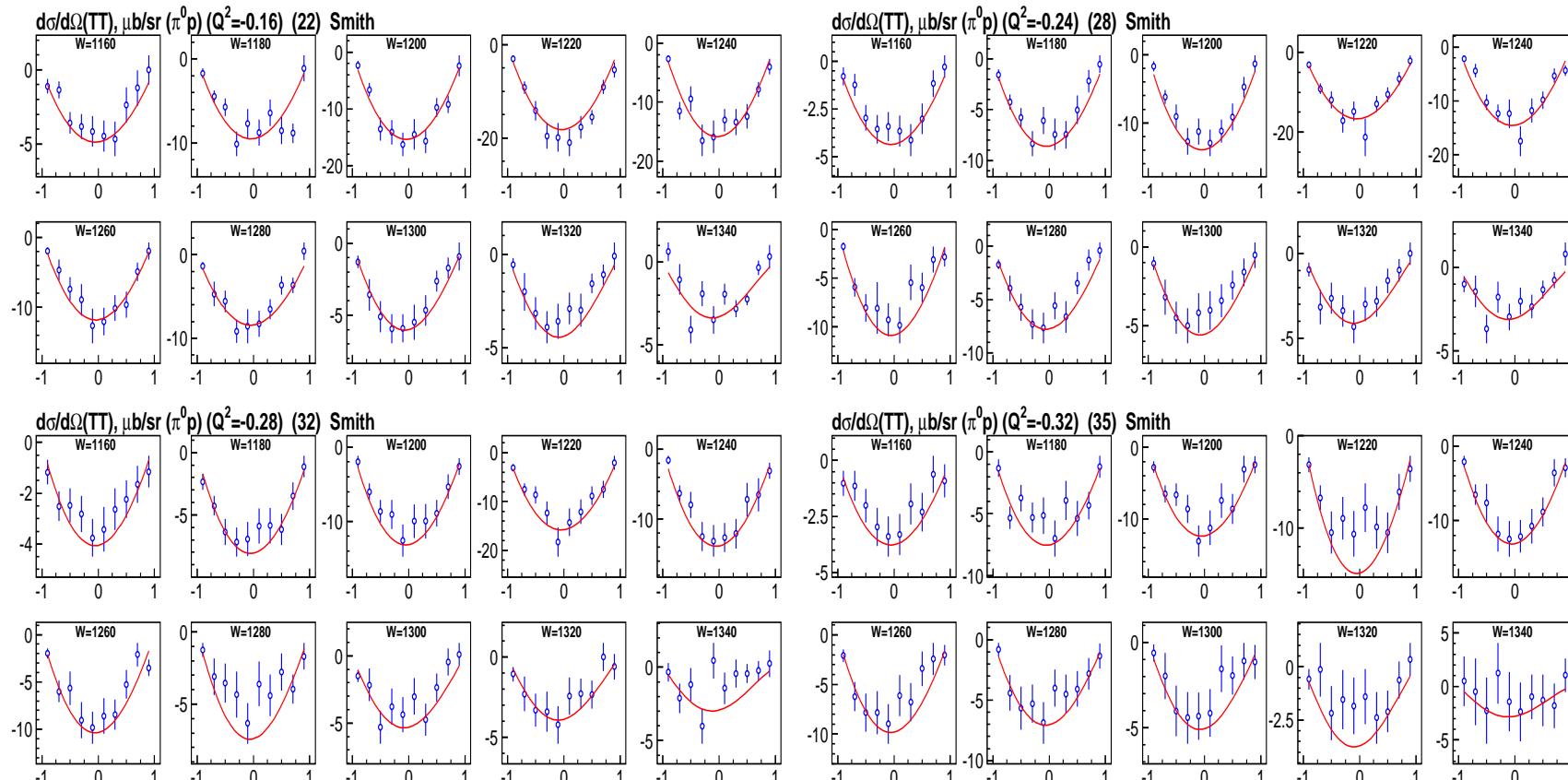
$$\begin{aligned} \frac{d\sigma_v}{d\Omega_\pi} &= \frac{d\sigma_T}{d\Omega_\pi} + \varepsilon_L \frac{d\sigma_L}{d\Omega_\pi} + [2\varepsilon_L(1+\varepsilon)]^{\frac{1}{2}} \frac{d\sigma_{TL}}{d\Omega_\pi} \cos \Phi_\pi + \varepsilon \frac{d\sigma_{TT}}{d\Omega_\pi} \cos 2\Phi_\pi \\ &+ h[2\varepsilon_L(1-\varepsilon)]^{\frac{1}{2}} \frac{d\sigma_{TL'}}{d\Omega_\pi} + h(1-\varepsilon^2)^{\frac{1}{2}} \frac{d\sigma_{TT'}}{d\Omega_\pi} \end{aligned}$$

$\varepsilon_i, k_i, \varepsilon_f, k_f$ - momenta of the initial and final electrons ( $K = \frac{1}{2}(k_i + k_f)$ ).  $\vec{q}$  and  $\Theta_e$  are evaluated in the lab. frame.  $h$  is the helicity of the incoming electron. [Amaldi et al 1979](#), [Donnachie and Shaw 1978](#)

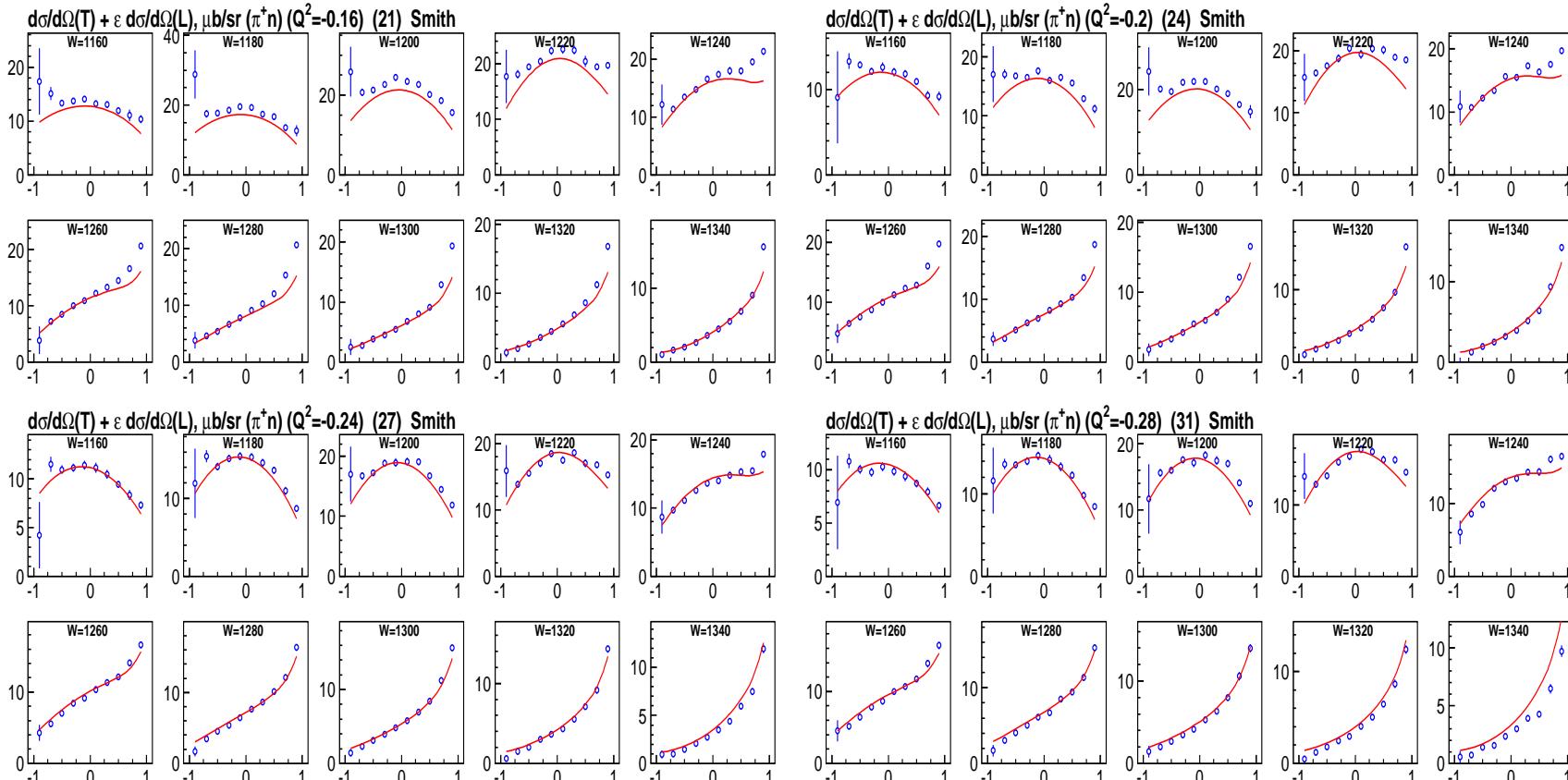
## The description of the $\gamma^* p \rightarrow \pi^0 p$ data



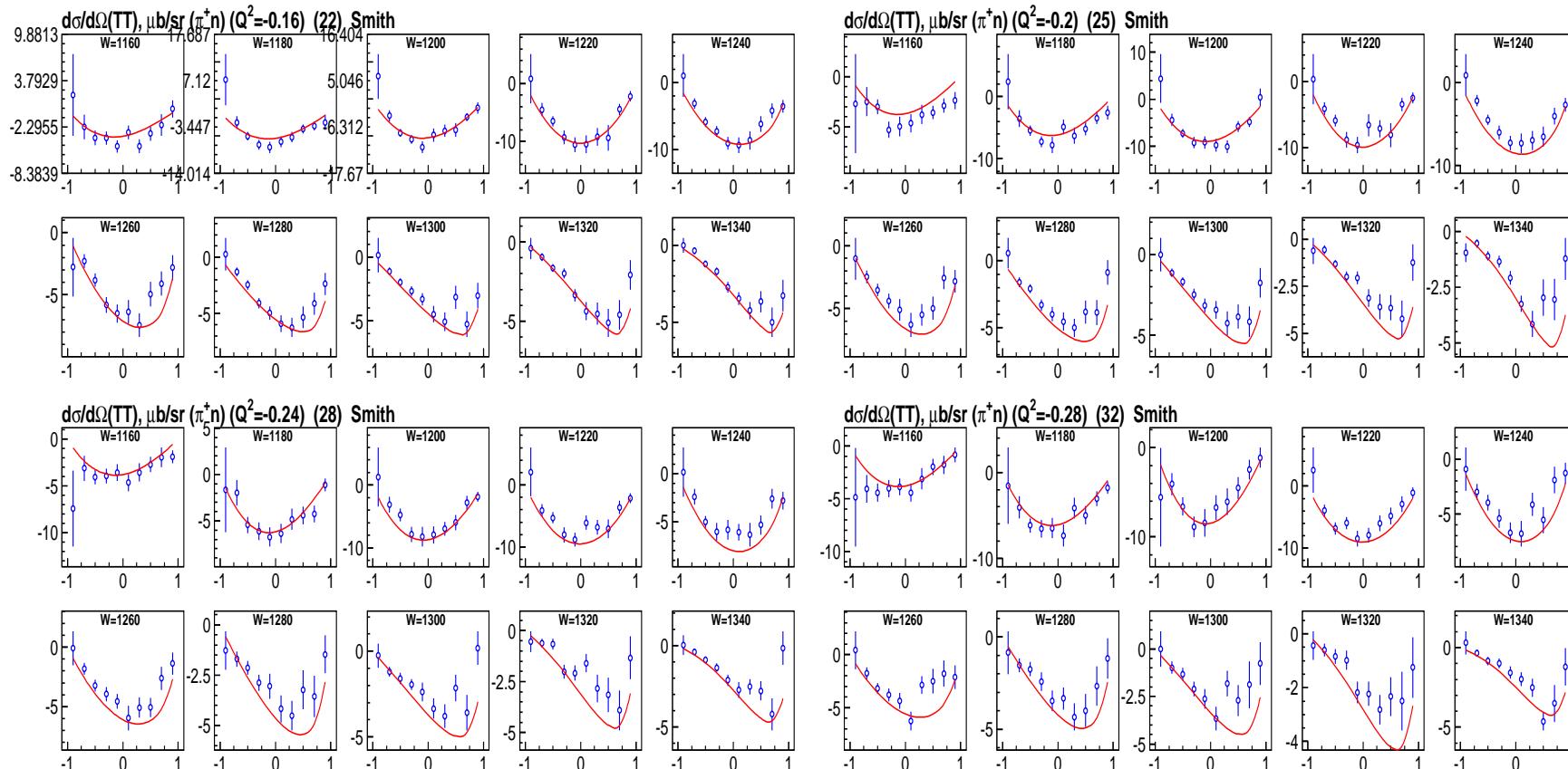
## The description of the $\gamma^* p \rightarrow \pi^0 p$ data



## The description of the $\gamma^* p \rightarrow \pi^+ n$ data

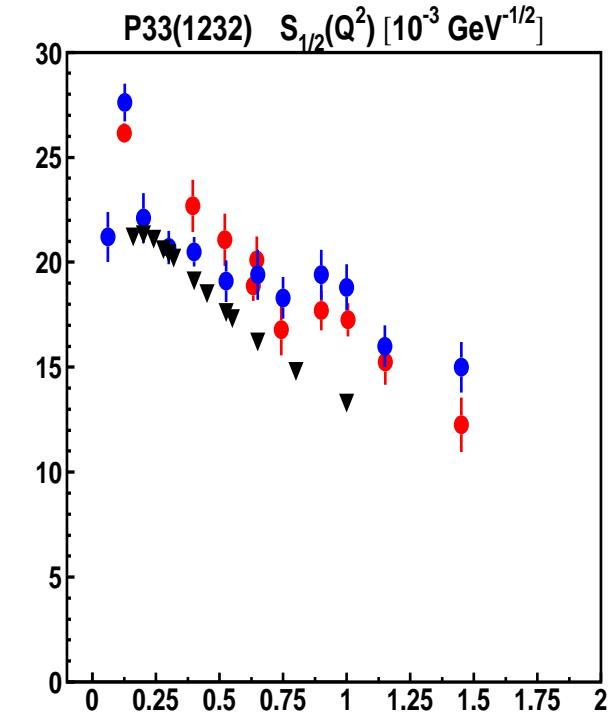
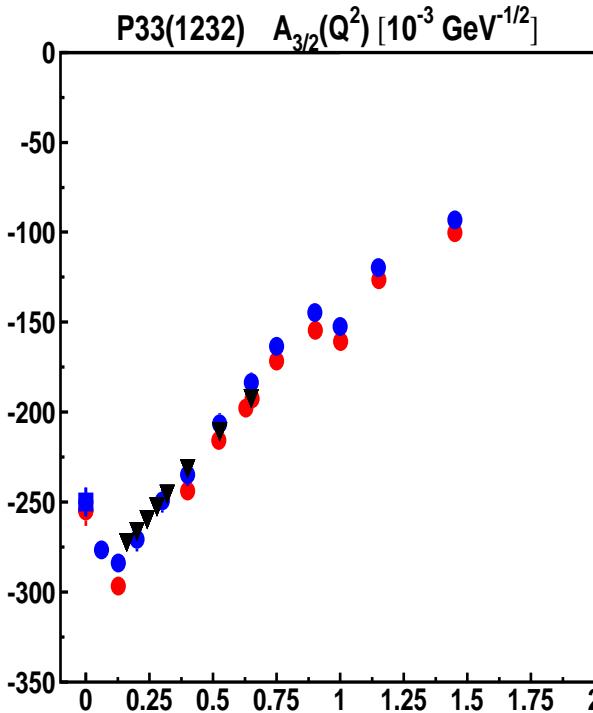
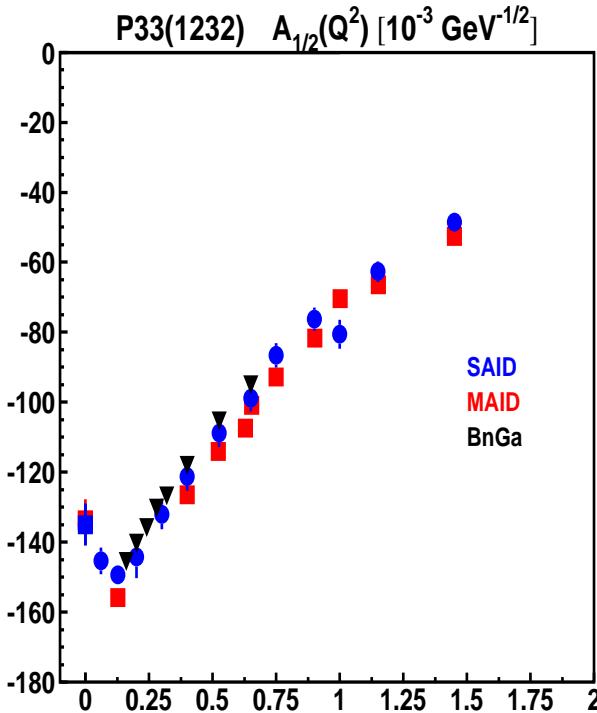


## The description of the $\gamma^* p \rightarrow \pi^+ n$ data



## The form factors for the $P_{33}(1232)$ state

(Very Preliminary)



## SUMMARY

- The new BG2022-02 solution, which describes 198 data sets is obtained.
- The new polarization data on the double pion photoproduction provide an important constrain for the data analysis.
- The branching ratios of the baryon states into  $\Delta\pi$ ,  $N\sigma$ ,  $N(1440)\pi$ ,  $N(1520)\pi$ ,  $N(1535)\pi$  and  $N(1680)\pi$  decay channels are determined with a good precision.
- The new data on the beam asymmetry on the  $\gamma n \rightarrow K^+ \Sigma^-$  fix the  $\gamma n$  couplings of the  $N(3/2^+)$  states.
- The new solution confirms that the bump in the mass region of 1620-1720 MeV is due to interference of the  $S_{11}$  states.
- The combined analysis of the all single meson electro-production data in the progress. The first results are obtained. But the description of the data should be improved.

| Resonance            | Rating | $N_{pp}$ | Resonance           | Rating | $N_{pp}$ | Resonance            | Rating | $N_{pp}$ |
|----------------------|--------|----------|---------------------|--------|----------|----------------------|--------|----------|
| $N(1440)1/2^+$       | ****   | 13       | $N(1520)3/2^-$      | ****   | 17       | $N(1535)1/2^-$       | ****   | 15       |
| $N(1650)1/2^-$       | ****   | 18       | $N(1675)5/2^-$      | ****   | 14       | $N(1680)5/2^+$       | ****   | 17       |
| $N(1685)$            | *      |          | $N(1700)3/2^-$      | ***    | 15       | $N(1710)1/2^+$       | ***    | 14       |
| $N(1720)3/2^+$       | ****   | 17       | $N(1860)5/2^+$      | **     | 9        | $N(1875)3/2^-$       | ***    | 16       |
| $N(1880)1/2^+$       | ***    | 20       | $N(1895)1/2^-$      | ****   | 17       | $N(1900)3/2^+$       | ****   | 18       |
| $N(1990)7/2^+$       | **     | 9        | $N(2000)5/2^+$      | **     | 11       | $N(2040)3/2^+$       | *      |          |
| $N(2060)5/2^-$       | **     | 13       | $N(2100)1/2^+$      | *      |          | $N(2150)3/2^-$       | **     | 11       |
| $N(2190)7/2^-$       | ****   | 11       | $N(2220)7/2^-$      | ****   | 7        | $N(2250)9/2^-$       | ****   |          |
| $N(2600)11/2^-$      | ***    |          | $N(2700)13/2^+$     | **     |          |                      |        |          |
| $\Delta(1232)$       | ****   | 8        | $\Delta(1600)3/2^+$ | ***    | 12       | $\Delta(1620)1/2^-$  | ****   | 10       |
| $\Delta(1700)3/2^-$  | ****   | 11       | $\Delta(1750)1/2^+$ | *      |          | $\Delta(1900)1/2^-$  | **     | 13       |
| $\Delta(1905)5/2^+$  | ****   | 11       | $\Delta(1910)1/2^+$ | ****   | 13       | $\Delta(1920)3/2^+$  | ***    | 21       |
| $\Delta(1930)5/2^-$  | ***    |          | $\Delta(1940)3/2^-$ | *      | 5        | $\Delta(1950)7/2^+$  | ****   | 13       |
| $\Delta(2000)5/2^+$  | **     |          | $\Delta(2150)1/2^-$ | *      |          | $\Delta(2200)7/2^-$  | *      |          |
| $\Delta(2300)9/2^+$  | **     |          | $\Delta(2350)3/2^-$ | *      |          | $\Delta(2390)7/2^+$  | *      |          |
| $\Delta(2420)11/2^+$ | ****   |          | $\Delta(2400)9/2^-$ | ****   |          | $\Delta(2750)13/2^-$ | **     |          |
| $\Delta(2950)15/2^+$ | **     |          |                     |        |          |                      |        |          |