

Overview of partial-wave analyses for baryon spectroscopy

Yannick Wunderlich

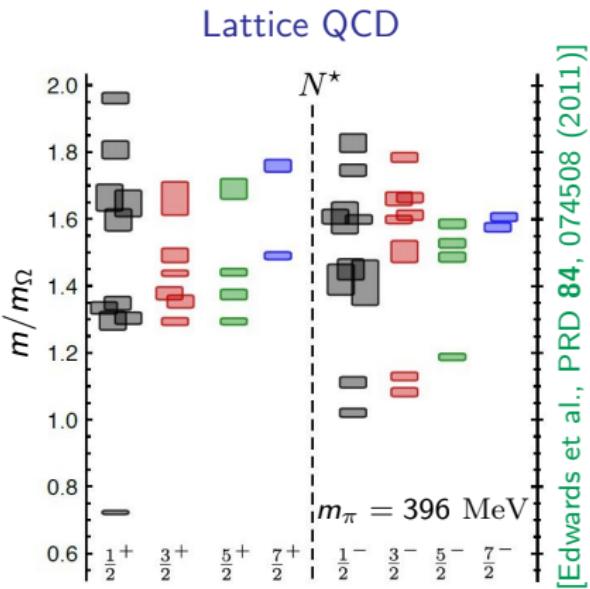
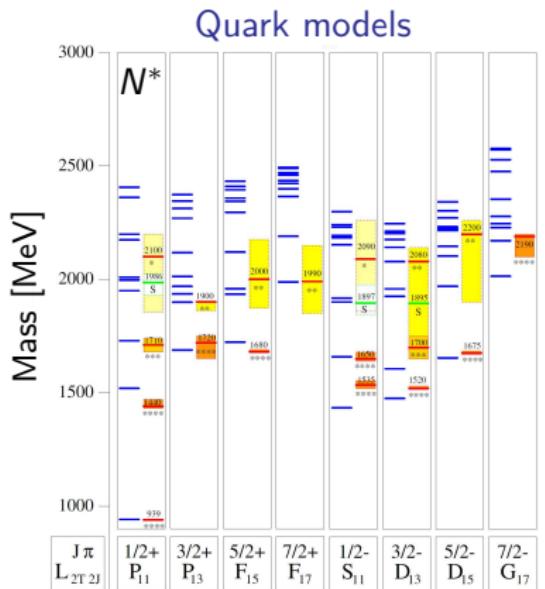
HISKP, University of Bonn

October 20, 2022



[Some slide-layouts by M. Döring, D. Rönchen & A. Sarantsev used]

Introduction: Why baryon spectroscopy?

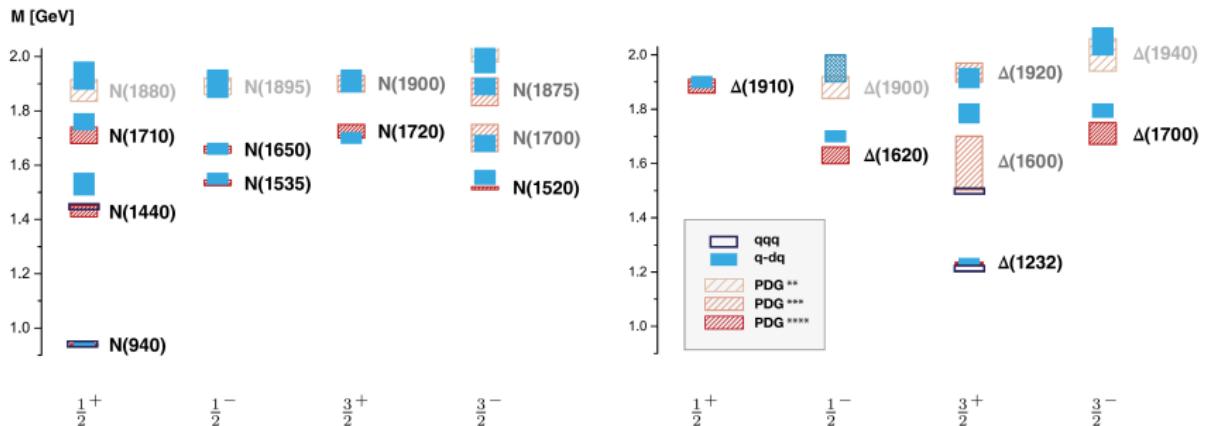


Further study baryon spectroscopy in order to:

- Solve the *missing resonance problem*
- Extract a precision-spectrum as a better basis for comparison for non-perturb. theoretical calculations (quark models, lattice QCD, $U\chi$ PT, ...)
- (light) baryon spectrum is valuable input for modelling *final-state interactions*

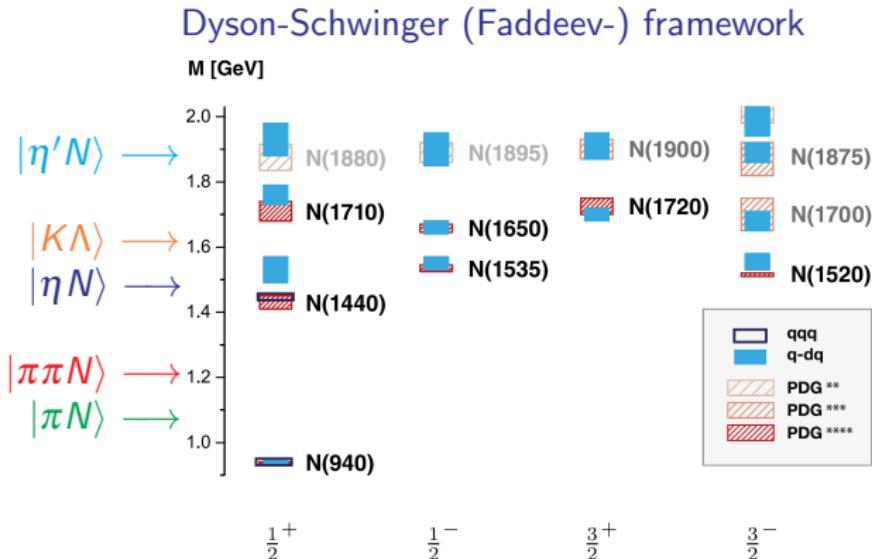
Complications: Spin & Coupled Channels

Dyson-Schwinger (Faddeev-) framework



[G. Eichmann and Ch. S. Fischer, Few Body Syst. 60 (2019) 1, 2]

Complications: Spin & Coupled Channels



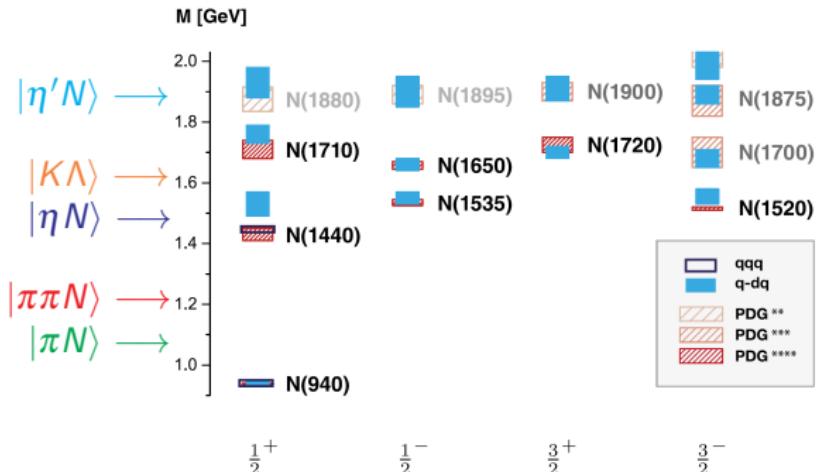
[Few Body Syst. 60 (2019) 1, 2]

Partial-wave analysis (amplitude analysis) for baryons difficult, because:

- Dense population of resonances in one partial wave $J^P \rightarrow$ Breit-Wigner techniques not useful
- Need spin to excite baryonic states (\rightarrow pol.-measurements, complete experiments, ...)
- Inherent coupled-channels problem (in particular, \exists light 3-body state: $|\pi\pi N\rangle$)

Complications: Spin & Coupled Channels

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[Few Body Syst. 60 (2019) 1, 2]

Solution: Construct and fit so-called energy-dependent ('ED') partial-wave analysis ('PWA') models, utilizing (as good as possible) the S -Matrix Constraints:

- Unitarity,
- Analyticity,
- Crossing Symmetry.

The Analytic
 S -Matrix

R.J. EDEN
P.V. LANDSHOFF
D.I. OLIVE
J.C. POLKINGHORNE

S-Matrix Constraints I: Unitarity

Transitions $a \rightarrow b$ ($a, b \in \text{'channel-space'} = \{|\pi N\rangle, |\pi\pi N\rangle, \dots\}$) described by matrix-element $\langle b | \hat{S} | a \rangle$ of an abstract operator ('S-Matrix'):

$$\hat{S} = \mathbb{1} + 2i\hat{T} \quad (\text{amplitude: } T_{fi} \propto \langle f | \hat{T} | i \rangle).$$

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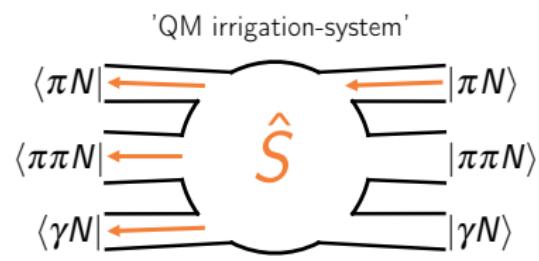
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Conservation of probabilities, i.e. $\sum_b P_{a \rightarrow b} = 1$,

with $P_{a \rightarrow b} = \left| \langle b| \hat{S} |a\rangle \right|^2$, leads to:

$$\begin{aligned} 1 &\stackrel{!}{=} \sum_b \left| \langle b| \hat{S} |a\rangle \right|^2 = \sum_b \langle a| \hat{S}^\dagger |b\rangle \langle b| \hat{S} |a\rangle \\ &= \langle a| \hat{S}^\dagger \hat{S} |a\rangle, \end{aligned}$$

i.e., is equivalent to the *unitarity* of S : $\hat{S}^\dagger \hat{S} = \mathbb{1}$.



[cf. Taylor, scatt. theory, Fig. 16.1]

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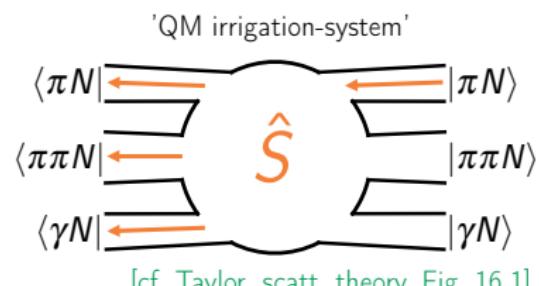
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*) Unitarity-condition for the T -matrix:

$$\hat{T} - \hat{T}^\dagger = 2i\hat{T}^\dagger \hat{T} \quad (\text{'optical theorem in operator-notation'}). \quad \text{(*)}$$

*) Unitarity-condition for matrix-elements:

$$\frac{1}{2i} \left(\langle b| \hat{T} |a\rangle - \langle b| \hat{T}^\dagger |a\rangle \right) = \sum_n \langle b| \hat{T}^\dagger |n\rangle \langle n| \hat{T} |a\rangle.$$

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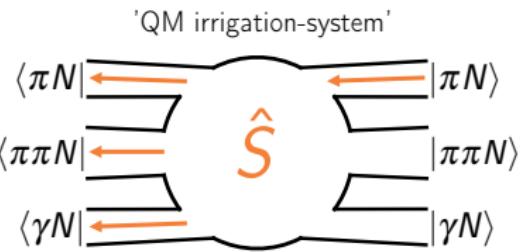
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- Unitarization techniques facilitate *dynamical generation of resonances*,
- Unitarity alone does *not* (!) fix the amplitude uniquely,
- In case: unitarity \oplus 'complete experiment' \rightarrow different story ... (see slide 18).

S-Matrix Constraints II: Analyticity

Principle: Scattering amplitude $\mathcal{T}_{ba} = \langle b | \hat{\mathcal{T}} | a \rangle$ has to be *analytic* (i.e. *complex differentiable*) function of the external Lorentz-invariants

↔ connected to *microcausality* [cf. Gell-Mann et al., Phys. Rev. 95, 1612 (1954)]

- In particular: 2-body part.-wave $\mathcal{T}_{ba}(W)$ analytic in the complex energy $z = W$.

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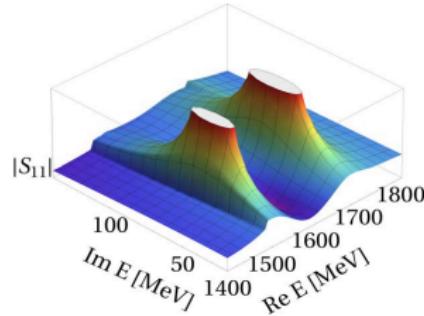
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Singularities: dictated by **Physics**, i.e.:

*) Poles \leftrightarrow (meta-stable) bound states

- Resonances are T -matrix poles on the 2nd Riemann-sheet, above threshold:
 $\text{Re}[\text{'pole'}] \leftrightarrow$ mass, $2\text{Im}[\text{'pole'}] \leftrightarrow$ width,
Residues \leftrightarrow branching fractions.
- Bound states, virtual states, ...



[Fig. courtesy of M. Döring]

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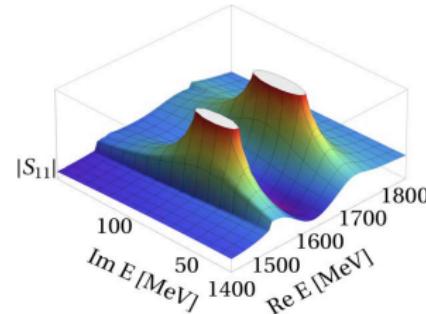
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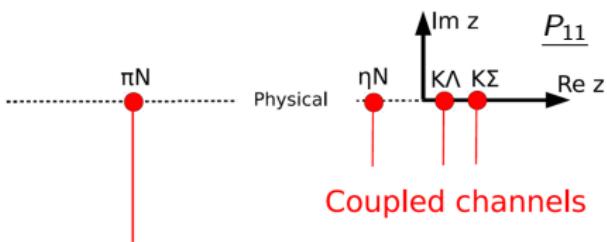
[Fig. courtesy of M. Döring]

*) Branch-points ↔ thresholds

- Matrix-el.'s $\langle b | \hat{T} | a \rangle$
($\langle b | \hat{T}^\dagger | a \rangle = \langle a | \hat{T} | b \rangle^*$)
≡ \mathcal{T} above (below) cut

↪ Unitarity:

$$\text{Disc} [\mathcal{T}_{ba}] = \sum_n \mathcal{T}_{nb}^* \mathcal{T}_{na}.$$



[Döring, Baryons School (2021)]

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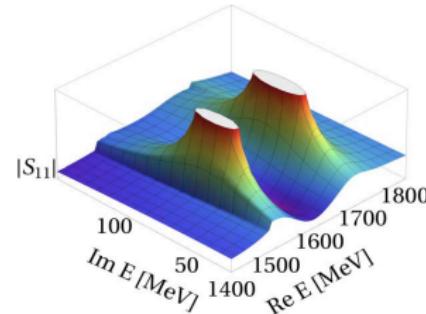
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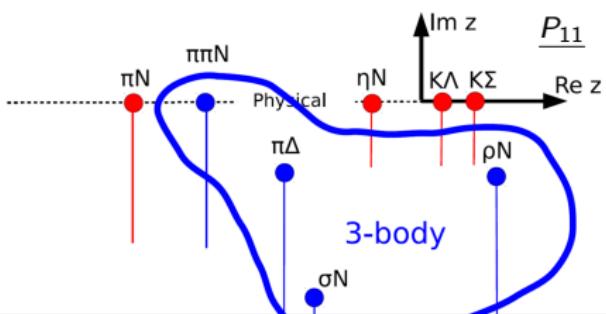


[Fig. courtesy of M. Döring]

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- $\pi\pi N$ -system parametr. effectively as $\pi\Delta$ -, σN - and ρN -contributions
- branch-points pushed into complex plane

[S. Ceci et al., PRC 84, 015205 (2011)]



[Döring, Baryons School (2021)]

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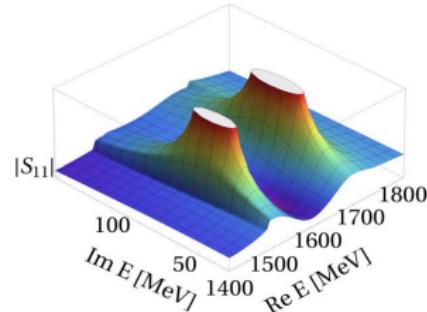
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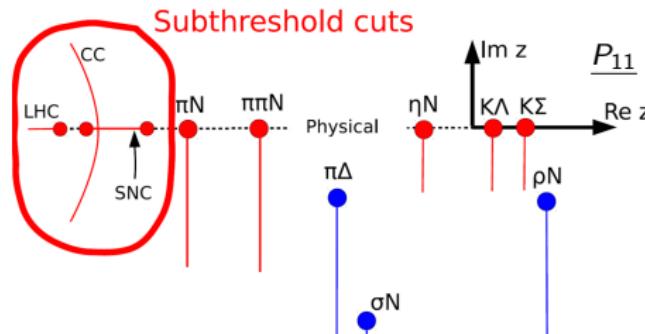
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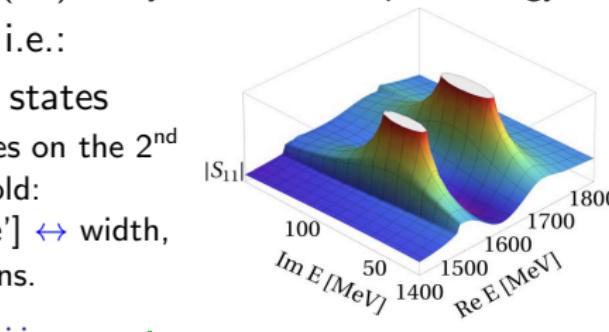
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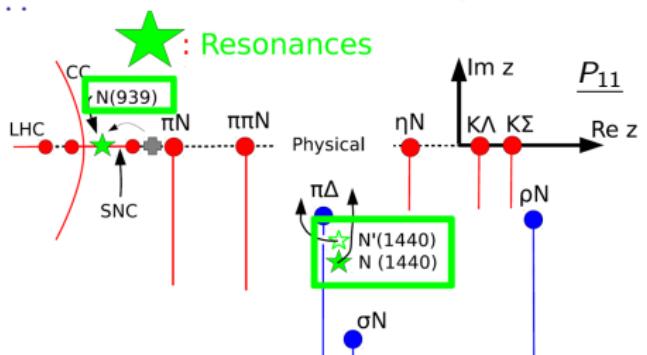
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Parametrization-methods for models I

K-matrix approach:

Lippmann-Schwinger type approach:

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$$\hat{T} = \hat{K} \left(\mathbb{1} - i\hat{K} \right)^{-1}, \text{ with:}$$

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*) 'Punchline': \hat{T} is unitary whenever \hat{K} is hermitean (in most cases: real symm.).

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*) For $\text{Re}\hat{G} \rightarrow 0$: recover K-matrix approx.

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[Anisovich et al., EPJ A **48**, 15 (2012)]
- SAID (CM12 photoprod.-fit)
[Workman et al., PRC **86**, 015202 (2012)]
- Kent-State University ('KSU')
[Shrestha & Manley, PRC **86**, 055203 (2012)]
- Giessen
[Shklyar et al., PRC **87.1**, 015201 (2013)]

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[Rönchen et al., arXiv:2208.00089]
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Alternative approaches:

- Isobar models (MAID, JLab JM15, ...)
- L+P pole-extraction method [Švarc et al. PRC **88**, 035206 (2013); see Alfreds Talk]

Parametrization-methods for models II

K-matrix approach:

*) Parametrization:

$$\hat{T} = \hat{K} \left(\mathbb{1} - i\hat{K} \right)^{-1},$$

*) Models:

- **Bonn-Gatchina ('BoGa')**
[Anisovich et al., EPJ A **48**, 15 (2012)]
- SAID (CM12 photoprod.-fit)
[Workman et al., PRC **86**, 015202 (2012)]
- Kent-State University ('KSU')
[Shrestha & Manley, PRC **86**, 055203 (2012)]
- Giessen
[Shklyar et al., PRC **87.1**, 015201 (2013)]

Lippmann-Schwinger type approach:

*) Parametrization:

$$\begin{aligned}\hat{T} &= \hat{V} + \hat{V}\hat{G}\hat{T} = \hat{V} + \hat{T}\hat{G}\hat{V} \\ &= \hat{V} + \hat{V}\hat{G}\hat{V} + \hat{V}\hat{G}\hat{V}\hat{G}\hat{V} + \dots,\end{aligned}$$

*) Models:

- **Jülich-Bonn (-Washington), 'JüBo' ('JüBoW')**
[Rönchen et al., arXiv:2208.00089]
- [Mai et al., PRC **106**, no.1, 015201 (2022)]
- ANL-Osaka
[Kamano et al., arXiv:1909.11935]

Now: Discuss **two exemplary models** in more detail ...

The Bonn-Gatchina Model - I

Bonn-Gatchina 'I' ('pure' K -matrix, up to ≈ 2014) [cf. Anisovich et al., EPJ A 48, 15 (2012)]

- Ansatz: $A_{ab} = K_{ac} \left(\mathbb{1} - i\hat{\rho}\hat{K} \right)_{cb}^{-1} \{A \equiv \mathcal{T}\}$, where $K_{ab}(s) = \sum_{\alpha} \frac{g_a^{\alpha} g_b^{\alpha}}{M_{\alpha}^2 - s} + f_{ab}(s)$.
 - 'Background': $f_{ab}(s) = \frac{f_{ab}^{(0)} + f_{ab}^{(1)} \sqrt{s}}{s - s_{ab}^0}$ (S_{11} and S_{31} waves), $f_{ab}(s) = \text{const.}$ otherwise.
 - Production-reactions in ' P -vector approach': $a_b^h = P_a^h \left(\mathbb{1} - i\hat{\rho}\hat{K} \right)_{ab}^{-1}$.
- Dedicated *pole-search in A_{ab}* , once (coupled-channels) fit has converged.

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Bonn-Gatchina 'II' (' N/D -based' approach, from ≈ 2015 onwards)

Use ' D -matrix' formalism:

[cf. Anisovich et al., EPJ A 52.9, 284 (2016)]

$$D_{jm} = D_{jk} \sum_{\alpha} \frac{B_{\alpha}^{km}(s)}{M_m^2 - s} + \frac{\delta_{jm}}{M_j^2 - s} \longleftrightarrow \begin{array}{c} J \\ \parallel \end{array} \circledcirc_m = \begin{array}{c} J \\ \parallel \end{array} \circledcirc_K \begin{array}{c} \pi \eta K \\ \parallel \\ \pi \eta K \end{array} m + \begin{array}{c} \delta_{JK} \\ \parallel \end{array}$$

[cf. A. Sarantsev, Talks at HADRON2017 and NSTAR2019]

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[cf. A. Sarantsev, Talks at HADRON2017 and NSTAR2019]

- Write 'loop-functions' in a once-subtracted dispersive representation:

$$B_{\alpha}^{ij}(s) = g_{\alpha}^{(R)i} \left(b^{\alpha} + (s - M_{\alpha}^2) \int_{M_{\alpha}^2}^{\infty} \frac{ds'}{\pi} \frac{\rho_{\alpha}(s', m_{1\alpha}, m_{2\alpha})}{(s' - s - i0)(s' - M_{\alpha}^2)} \right) g_{\alpha}^{(L)j} = g_{\alpha}^{(R)i} B_{\alpha} g_{\alpha}^{(L)j}.$$

- Reduction to: $\hat{A} = \hat{K} \left(\mathbb{1} - \hat{B}\hat{K} \right)^{-1}$, with $B_{\alpha\beta} = \delta_{\alpha\beta} B_{\alpha}$ and $\text{Re}[\hat{B}] \neq 0$.
- Analytic properties of the model have been improved!

The Bonn-Gatchina Model - II

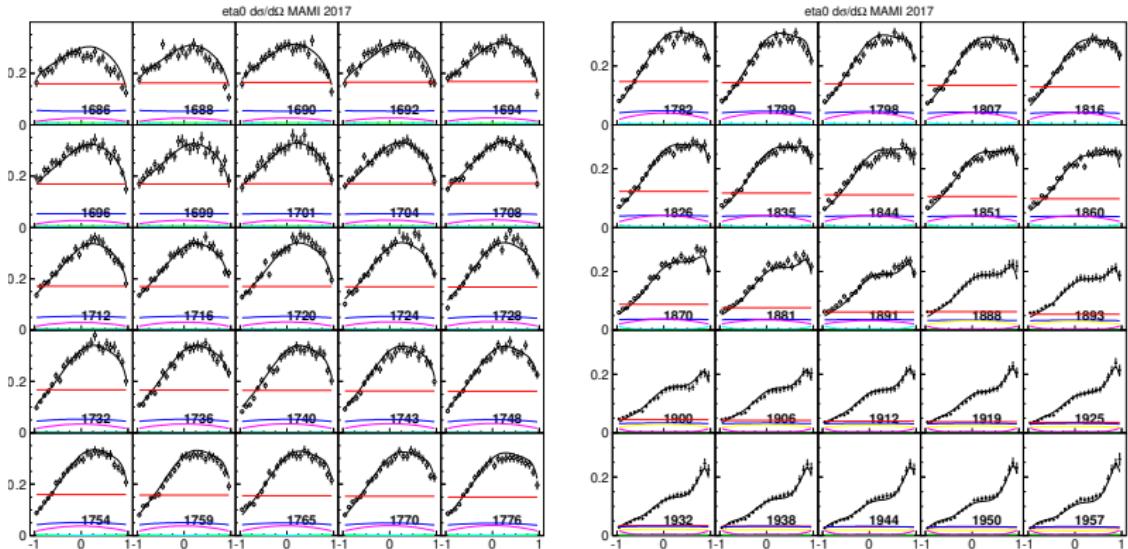
Recently included meson photoproduction data

DATA	2011-2016	added in 2016-2018
$\gamma n \rightarrow \Lambda K, \Sigma^- K$		$\frac{d\sigma}{d\Omega}$ (CLAS), E (CLAS)
$\gamma n \rightarrow \pi^- p$	$\frac{d\sigma}{d\Omega}, \Sigma, P$	E, Σ (CLAS)
$\gamma n \rightarrow \eta n$	$\frac{d\sigma}{d\Omega}, \Sigma$	$\frac{d\sigma}{d\Omega}$ (MAMI) $\frac{d\sigma}{d\Omega} (h = \frac{1}{2})$ (CB-ELSA)
$\gamma p \rightarrow \eta p$	$\frac{d\sigma}{d\Omega}, \Sigma$ (GRAAL)	$\frac{d\sigma}{d\Omega}, F, T$ (MAMI) T, P, H, G , (CB-ELSA) E, Σ (CB-ELSA, CLAS)
$\gamma p \rightarrow \eta' p$		$\frac{d\sigma}{d\Omega}, \Sigma$
$\gamma p \rightarrow K^+ \Lambda$	$\frac{d\sigma}{d\Omega}, \Sigma, P, T, C_x, C_z, O_{x'}, O_{z'}$	Σ, P, T, O_x, O_z (CLAS)
$\gamma p \rightarrow K^+ \Sigma^0$	$\frac{d\sigma}{d\Omega}, \Sigma, P, C_x, C_z$	Σ, P, T, O_x, O_z (CLAS)
$\pi^- p \rightarrow \pi^+ \pi^- n$		$d\sigma/d\Omega$ (HADES)
$\pi^- p \rightarrow \pi^- \pi^0 p$		$d\sigma/d\Omega$ (HADES)
$\gamma p \rightarrow \pi^0 \pi^0 p$	$d\sigma/d\Omega, \Sigma, E, I_c, I_s$	T, P, H, F, P_x, P_y (CB-ELSA)
$\gamma p \rightarrow \pi^+ \pi^- p$		$d\sigma/d\Omega, I_c, I_s$ (CLAS)
$\gamma p \rightarrow \omega p$	$d\sigma/d\Omega, \Sigma, \rho_{ij}^k, E, G$ (CB-ELSA)	Σ (CLAS) P,T,F,H (CLAS)
$\gamma p \rightarrow K^* \Lambda$		$d\sigma/d\Omega, \rho_{ij}$

[cf. A. Sarantsev, Talk at NSTAR2019]

The Bonn-Gatchina Model - III

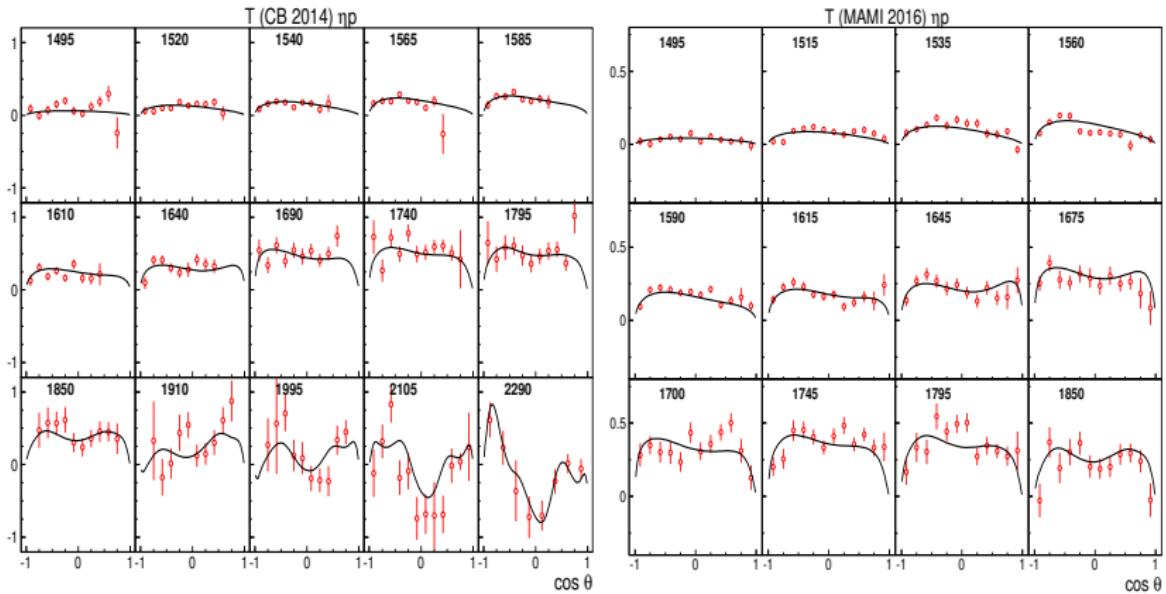
The analysis of the new $\gamma p \rightarrow \eta p$ data. $d\sigma/d\Omega$ (MAMI)



[cf. A. Sarantsev, Talk at NSTAR2019]

The Bonn-Gatchina Model - III

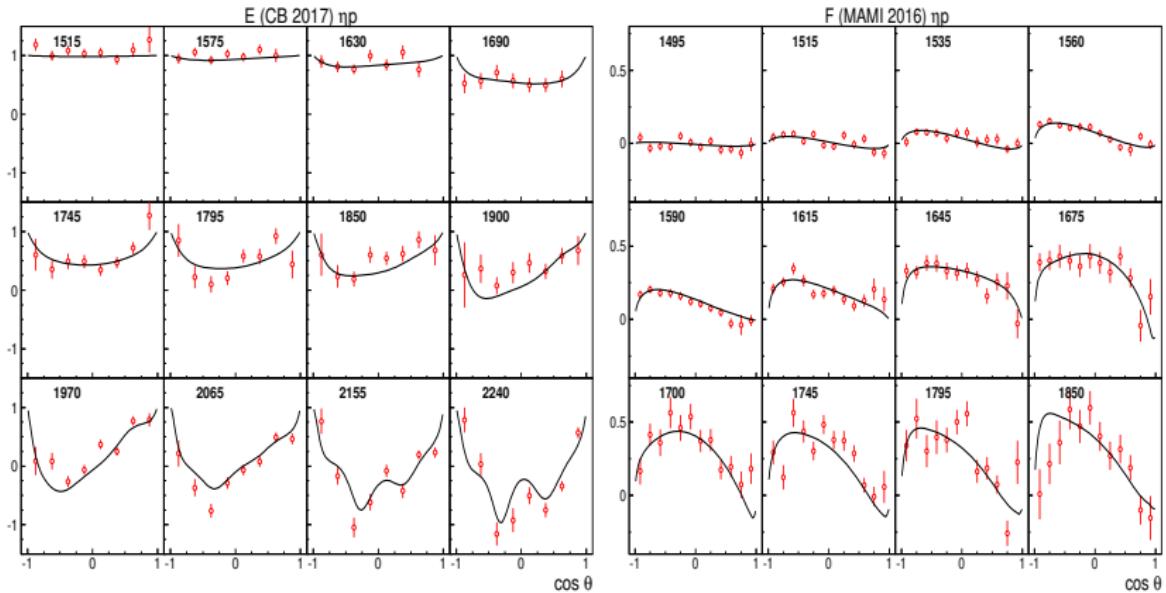
The analysis of the new $\gamma p \rightarrow np$ data. T (CB-ELSA, Preliminary), (MAMI scale 1.4)



[cf. A. Sarantsev, Talk at NSTAR2019]

The Bonn-Gatchina Model - III

The analysis of the new $\gamma p \rightarrow \eta p$ data. E (CB-ELSA, Preliminary), F (MAMI) (scale 1.4)



[cf. A. Sarantsev, Talk at NSTAR2019]

The Bonn-Gatchina Model - IV

Resonance content:

[cf. J. Müller et al., Phys. Lett. B 803, 135323 (2020)]

- *) All PDG **** and ***, N - as well as Δ -resonances are included in the PWA (solution BnGa 2019), as well as most ** resonances.
Noteable exceptions: *** resonance $N(2600)11/2^-$ and
**** resonance $\Delta(2420)11/2^+$.
- *) Noteable newly included state: $N(1895)1/2^-$.

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Recent developments:

- *) Big coupled-channels fit including recent polarization measurements for $p\eta$ -photoproduction
[J. Müller et al., Phys. Lett. B **803**, 135323 (2020)]
- *) Elaborate analyses on hyperon resonances
[M. Matveev, et al., Eur. Phys. J. A **55**, no.10, 179 (2019)]
[A. Sarantsev, et al., Eur. Phys. J. A **55**, no.10, 180 (2019)]

The Jülich-Bonn-Washington Model - I [cf. D. Rönchens Talk]

Hadronic model: dynamical coupled-channels (DCC) fit of various reactions

Lippmann-Schwinger eq. ('scattering eq.') in the partial-wave basis:

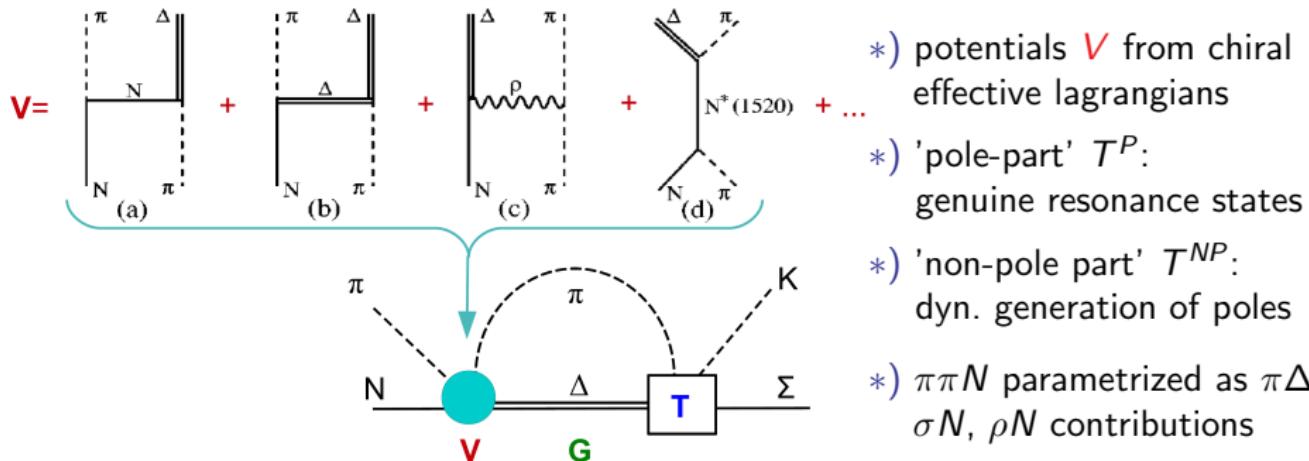
$$\langle L'S'p' | \textcolor{blue}{T}_{\mu\nu}^{IJ} | LSp \rangle = \langle L'S'p' | \textcolor{red}{V}_{\mu\nu}^{IJ} | LSp \rangle + \\ \sum_{\gamma, L''S''} \int_0^{\infty} dq q^2 \langle L'S'p' | \textcolor{red}{V}_{\mu\gamma}^{IJ} | L''S''q \rangle \frac{1}{E - E_{\gamma}(q) + i\epsilon} \langle L''S''q | \textcolor{blue}{T}_{\gamma\nu}^{IJ} | LSp \rangle .$$

The Jülich-Bonn-Washington Model - I [cf. D. Rönchens Talk]

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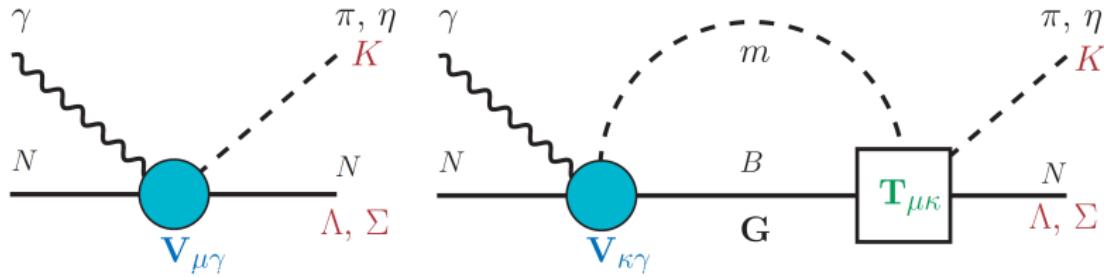
The Jülich-Bonn-Washington Model - I [cf. D. Rönchens Talk]

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Photoproduction model: phenomenological potential $\textcolor{teal}{V}_{\mu\gamma}$ approximated by energy-dependent polynomials [see D. Rönchens Talk]



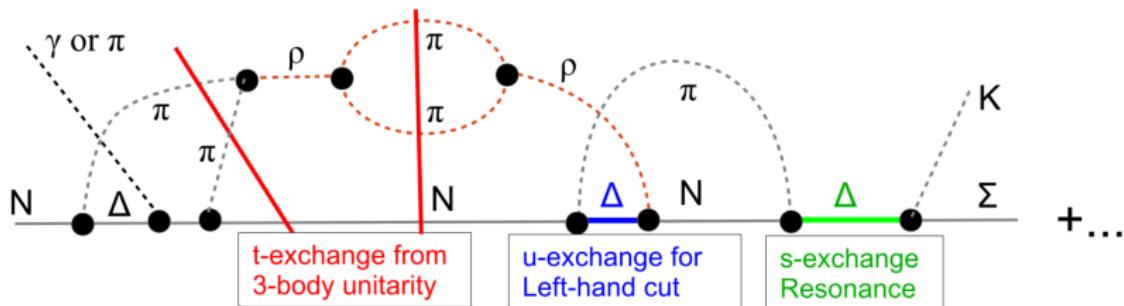
The Jülich-Bonn-Washington Model - I [cf. D. Rönchens Talk]

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A different type of visualization for one rescattering contribution:



[cf. M. Döring, Baryons School (2021)]

The Jülich-Bonn-Washington Model - II

*) $\pi N \rightarrow X$: $\gtrsim 7000$ data points

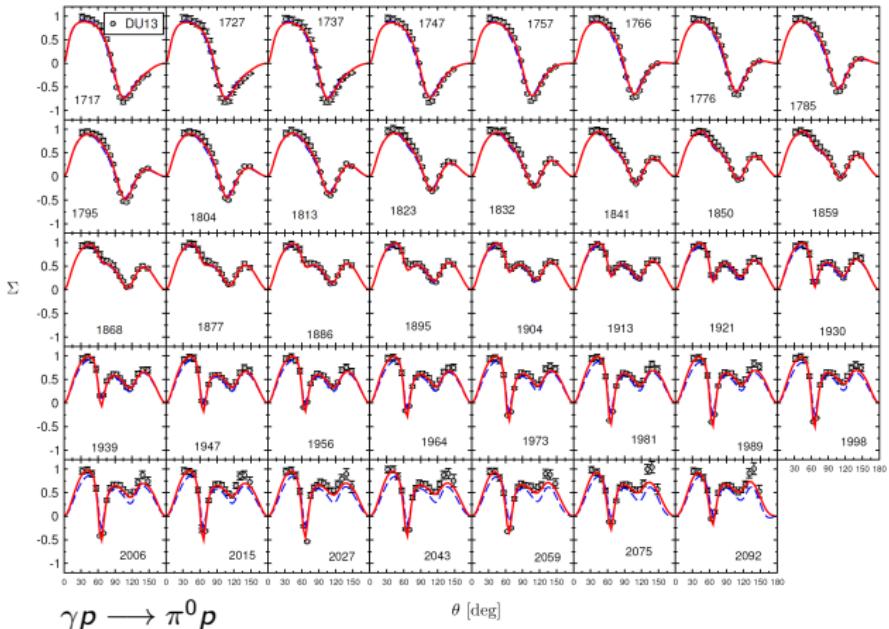
*) $\gamma N \rightarrow X$:

Reaction	Observables (# data points)	p./channel
$\gamma p \rightarrow \pi^0 p$	$d\sigma/d\Omega$ (18721), Σ (2927), P (768), T (1404), $\Delta\sigma_{31}$ (140), G (393), H (225), E (467), F (397), $C_{x_L'}$ (74), $C_{z_L'}$ (26)	25,542
$\gamma p \rightarrow \pi^+ n$	$d\sigma/d\Omega$ (5961), Σ (1456), P (265), T (718), $\Delta\sigma_{31}$ (231), G (86), H (128), E (903)	9,748
$\gamma p \rightarrow \eta p$	$d\sigma/d\Omega$ (9112), Σ (403), P (7), T (144), F (144), E (129)	9,939
$\gamma p \rightarrow K^+ \Lambda$	$d\sigma/d\Omega$ (2478), P (1612), Σ (459), T (383), $C_{x'}$ (121), $C_{z'}$ (123), $O_{x'}$ (66), $O_{z'}$ (66), O_x (314), O_z (314),	5,936
$\gamma p \rightarrow K^+ \Sigma^0$	$d\sigma/d\Omega$ (4271), P (422), Σ (280), T (127), $C_{x',z'}$ (188), $O_{x,z}$ (254)	5,542
$\gamma p \rightarrow K^0 \Sigma^+$	$d\sigma/d\Omega$ (242), P (78)	320
in total		57,027

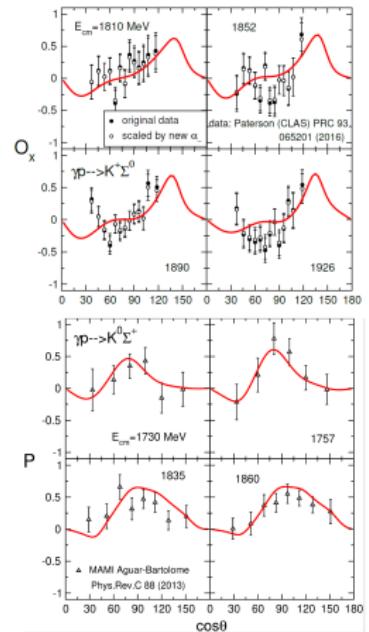
[M. Döring, Talk at APCTP meeting, Jeju-do (2022)]

*) More (steadily updated) details: <https://jbw.phys.gwu.edu/>

The Jülich-Bonn-Washington Model - III



[D. Rönchen et al., Eur. Phys. J. A 50, no.6, 101 (2014)]



[D. Rönchen et al., arXiv:2208.00089 [nucl-th]]

The Jülich-Bonn-Washington Model - IV

Resonance content: (including newest $K\Sigma$ -analyses) [cf. D. Rönchen et al., arXiv:2208.00089]

- *) All *** N - and Δ -resonances with $J \leq \frac{9}{2}$ are included, with the exception of $N(1895)1/2^-$ (\oplus some states with a rating of $\leq ***$)
- *) Indications for *new* dynamically generated poles, e.g. $\Delta(1920)3/2^+$

The Jülich-Bonn-Washington Model - IV

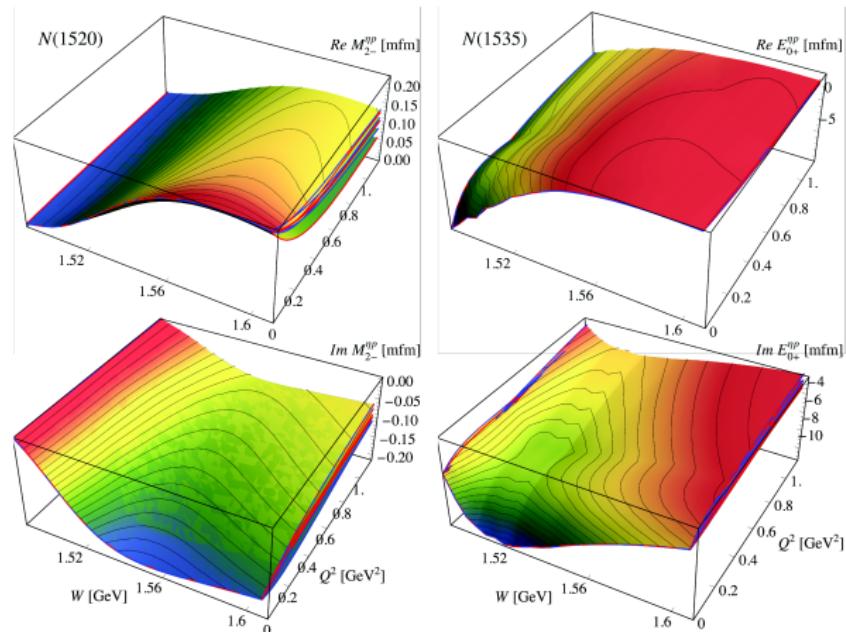
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Recent development:
analysis of
 e^- -production in
JüBoW-approach:

- *) JüBo-model is boundary-cond. at $Q^2 = 0$,
- *) Consistency requirements at pseudothreshold $q = 0$ ('Siegrist's theorem').

[cf. Talk by M. Mai]



A Compendium of Models

Model(s)	BoGa I/II	SAID (CM12)	MAID	KSU	Giessen	Jülich-Bonn ANL-Osaka
Constraints from chiral lagrangians?	No	(Yes)	(Yes)	No	Yes	Yes
Explicit resonance terms?	Yes	No	Yes	Yes	Yes	Yes/No
Analyticity (phys.; disp.)	No/Yes	Yes		No	No	Yes
Effective $ \pi\pi N\rangle$?	Yes	Yes	Yes	Yes	Yes	Yes
3-body unitarity?	No	No	No	No	No	Yes

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Effective $ \pi\pi N\rangle$?	Yes	Yes	Yes	Yes	Yes	Yes
3-body unitarity?	No	No	No	No	No	Yes

- 
- ▷ ED PWA groups perform and publish fits of large collections of datasets on (polarization) measurements
 - ▷ The PDG Baryon-group filters publications and assigns *star-ratings* to states:
 - { * * ** or * * *: established states,
 - ** or *: controversial/claimed states; need confirmation ...

Progress in terms of '*'-ratings

PDG 2002 vs. PDG 2020 (changes in red): [A. Thiel, F. Afzal & YW, PPNP 125, 103949 (2022), Tab. 16]

Particle	J^P	overall	PWA	$N\gamma$	$N\pi$	$\Delta\pi$	$N\sigma$	$N\eta$	ΛK	ΣK	$N\rho$	$N\omega$
N	$1/2^+$	****										
$N(1440)$	$1/2^+$	****	○ ◇ _g ★ ▷	*****	****	****	***	-			-	
$N(1520)$	$3/2^-$	****	○ ◇ ★ ▷	****	****	****	**	****			- - -	
$N(1535)$	$1/2^-$	****	○ ◇ ★ ▷	*****	****	***	*	****			- -	
$N(1650)$	$1/2^-$	****	○ ◇ ★ ▷	*****	****	***	*	****	* - -	- -	- -	
$N(1675)$	$5/2^-$	****	○ ◇ ★ ▷	****	****	****	***	*	*	*	*	-
$N(1680)$	$5/2^+$	****	○ ◇ ★ ▷	****	****	****	***	*	*	*	*	- - -
$N(1700)$	$3/2^-$	***	○ ▷	**	***	***	*	*	- -	-	-	
$N(1710)$	$1/2^+$	****	○ ◇ ▷	*****	****	* -	***	**	**	*	*	*
$N(1720)$	$3/2^+$	****	○ ◇ ★ ▷	*****	****	***	*	*	*****	*	*	*
$N(1860)$	$5/2^+$	**	▷	*	**		*	*				
$N(1875)$	$3/2^-$	***	○ ▷	**	**	*	**	*	*	*	*	*
$N(1880)$	$1/2^+$	***	○ ▷	**	*	**	*	*	**	**	**	
$N(1895)$	$1/2^-$	****	○ ▷	****	*	*	*	****	**	**	*	*
$N(1900)$	$3/2^+$	****	○ ◇ ▷	****	**	**	*	*	**	**	-	*
$N(1990)$	$7/2^+$	**	○ ◇ ▷	**	**			*	*	*	*	
$N(2000)$	$5/2^+$	**	○ ★	**	* -	**	*	*	-	-	- -	*
$N(2040)$	$3/2^+$	*	▷									
$N(2060)$	$5/2^-$	***	○ ◇ _g ▷	***	**	*	*	*	*	*	*	*
$N(2100)$	$1/2^+$	***	○ ▷	**	***	**	**	*	*	*	*	*
$N(2120)$	$3/2^-$	***	○ ▷	***	**	**	**		**	*	*	*
$N(2190)$	$7/2^-$	****	○ ◇ ★ ▷	****	****	****	**	*	**	*	*	*
$N(2220)$	$9/2^+$	****	○ ◇ ★	**	****			*	*	*	*	
$N(2250)$	$9/2^-$	****	○ ◇ ★ ▷	**	****			*	*	*	*	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

○: BnGa-2019, ◇: JüBo-2017 ('g' for dynamically generated), ★: SAID-MA19, ▷: KSU PWA

Progress in terms of 'multiplets'

↪ Work with spin-flavor $SU(6)$ symmetry and assign quarks to fundamental rep.

$$\mathbf{6} = (u \uparrow, d \uparrow, s \uparrow, u \downarrow, d \downarrow, s \downarrow)^T.$$

⇒ Baryon (super-) multiplet structure arises from decomposition into irrep.'s:

$$\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} = \mathbf{56}_S \oplus \mathbf{70}_M \oplus \mathbf{70}_M \oplus \mathbf{20}_A.$$

Progress in terms of 'multiplets'

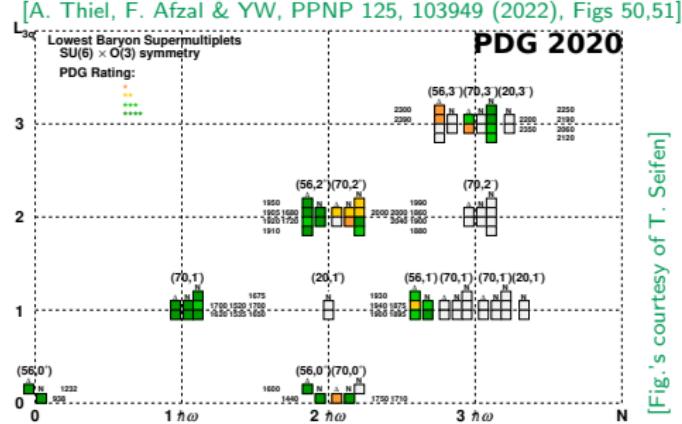
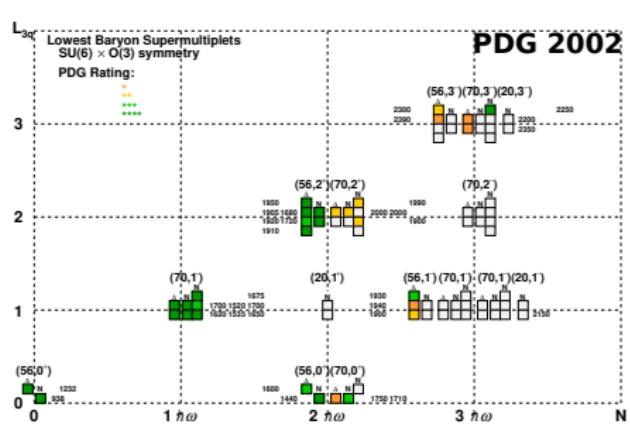
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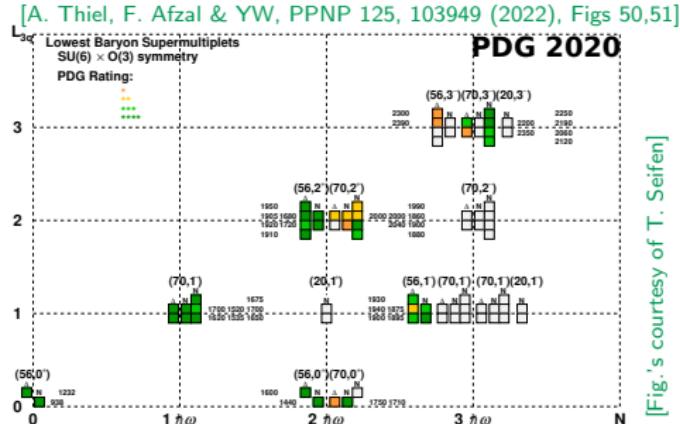
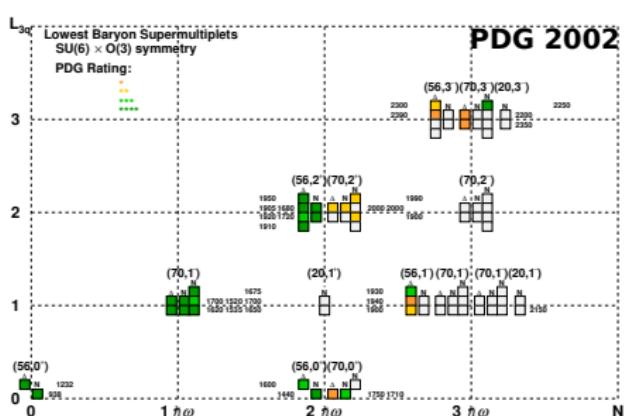
⇒ Assign PDG-resonances to multiplets: [cf. E. Klemt & B. Metsch, EPJ A 48, p. 127 (2012)]



[Fig's courtesy of T. Seifert]

Progress in terms of 'multiplets'

- Work with spin-flavor $SU(6)$ symmetry and assign quarks to fundamental rep.
 $\mathbf{6} = (u \uparrow, d \uparrow, s \uparrow, u \downarrow, d \downarrow, s \downarrow)^T$.
- ⇒ Baryon (super-) multiplet structure arises from decomposition into irrep.'s:
 $\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} = \mathbf{56}_S \oplus \mathbf{70}_M \oplus \mathbf{70}_M \oplus \mathbf{20}_A$.
- ⇒ Assign PDG-resonances to multiplets: [cf. E. Klemt & B. Metsch, EPJ A 48, p. 127 (2012)]



[Fig.'s courtesy of T. Seifen]

- All this progress is great, but is there a way to tell when we are finished?
- There is: one needs a coupled-channels complete experiment.

Dream: the 'coupled-channels complete experiment'

Consider exemplary *channel-space* $\{|\pi N\rangle, |\gamma N\rangle, |\pi\pi N\rangle\}$, i.e.:

$$(\mathcal{T}_{fi}) = \begin{bmatrix} \mathcal{T}_{\pi N, \pi N} & \mathcal{T}_{\pi N, \gamma N} & \mathcal{T}_{\pi N, \pi\pi N} \\ \mathcal{T}_{\gamma N, \pi N} & \mathcal{T}_{\gamma N, \gamma N} \simeq 0 & \mathcal{T}_{\gamma N, \pi\pi N} \\ \mathcal{T}_{\pi\pi N, \pi N} & \mathcal{T}_{\pi\pi N, \gamma N} & \mathcal{T}_{\pi\pi N, \pi\pi N} \end{bmatrix}.$$

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↪ Measure individual complete experiments with perfect *phase-space coverage and overlap* among individual reactions (complete exp.'s determinable using *graphs*):

Reaction	Example complete experiment (yields $ b_i $ & ϕ_{ij})
$\pi N \rightarrow \pi N (N_A = 2)$	$\sigma_0, \hat{P}, \hat{R}, \hat{A}$
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[YW et al., PRC 102, no.3, 034605 (2020)] & [YW, PRC 104, no.4, 045203 (2021)]

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↪ Fit at least two (or more) complementary ED PWA-models (BnGa, JüBo, ...), which have to have *as good unitarity-constraints as possible*, to this database
⇒ Missing phase-information $e^{i\phi_{fi}}$ fixed and resonance-spectrum (hopefully) unique!

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- Issues:
- Can we assume perfect time-reversal inv., to relate $3 \rightarrow 2$ to $2 \rightarrow 3$ processes?
 - $3 \rightarrow 3$ -process $\pi\pi N \rightarrow \pi\pi N$ unmeasurable. Does this hurt the proposal?

Thank You!

Additional Slides

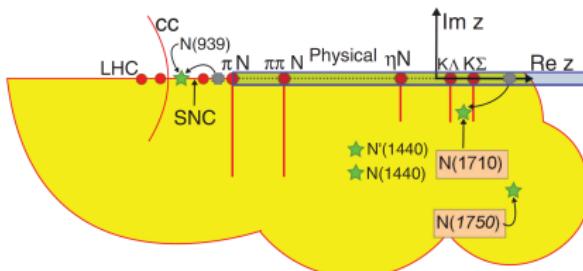
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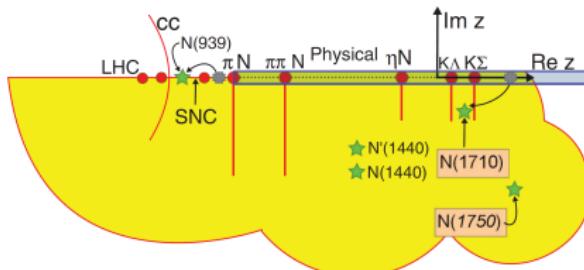
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→ Try to model analytic structure using L+P- ('Laurent+Pietarinen'-) Ansatz:

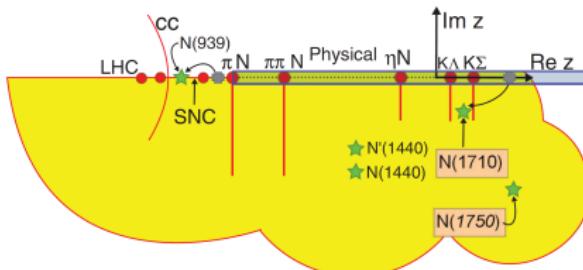
$$\mathcal{M}(W) = \underbrace{\sum_j^{N_{\text{pole}}} \frac{x_j + iy_j}{W_j - W}}_{\text{'Laurent'}} + \underbrace{\sum_{k=0}^{N_1} \mathbf{c}_k X(\alpha, x_P; W)^k + \sum_{l=0}^{N_2} \mathbf{d}_l Y(\beta, x_Q; W)^l}_{\text{'Pietarinen'}} + \dots$$

- Pole-position: $W_j \in \mathbb{C}$; Residue: $a_{-1}^{(j)} = x_j + iy_j$
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→ Method already successfully applied for pole-extraction [→ see Alfreds Talk]