Development of Reaction Models for KY Photo- and Electroproduction

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Motivation

- We aim at understanding the baryon spectrum and production dynamics of particles with strangeness at low energies.
- Constituent Quark Model predicts a lot more N^* states than was observed in pion production experiments $\rightarrow$ "missing" resonance problem.
- Models for the description of elementary hyperon electroproduction are a suitable tool for hypernuclear physics calculations.
- New good-quality photoproduction data from LEPS, GRAAL, MAMI and (particularly) CLAS collaborations allow us to tune free parameters of the models.
- As the α_s increases with decreasing energy, we cannot use perturbative QCD at low energies $\rightarrow$ need for introducing effective theories and models.

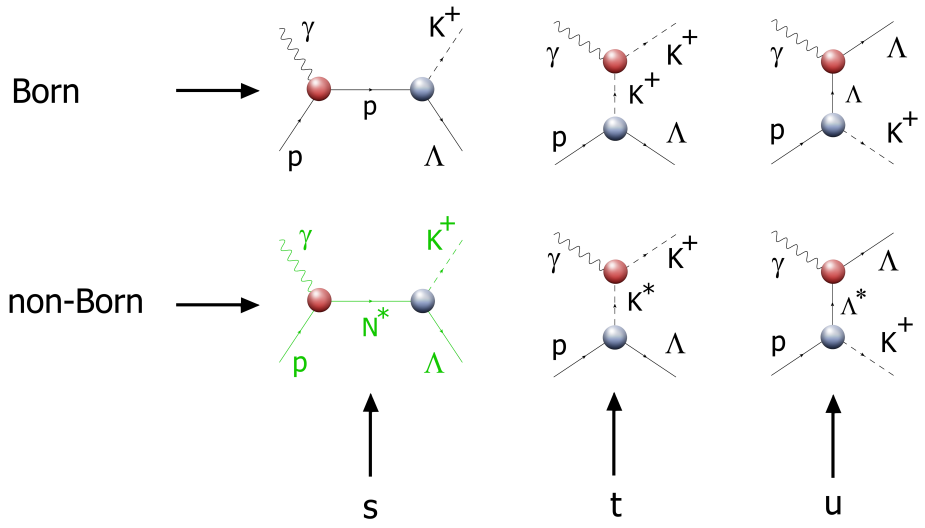
Isobar model

Single-channel approximation

- higher-order contributions (rescattering, FSI) included, to some extent, by means of effective values of the coupling constants

Use of effective hadron Lagrangian

- hadrons either in their ground or excited states
- amplitude constructed as a sum of tree-level Feynman diagrams
  - **background part**: Born terms with an off-shell proton (s channel), kaon (t), and hyperon (u) exchanges; non Born terms with (axial) vector K^* (t) and Y^* (u) exchange
  - **resonance part**: s-channel Feynman diagram with N^* exchanges



Free parameters adjusted to experimental data

Satisfactory agreement with the data in the energy range $W = 1.6 - 2.5\,\mathrm{GeV}$

Isobar model

Novel features of our IM

Exchanges of high-spin resonant states

- non physical lower-spin components removed by appropriate choice of $\mathcal{L}_{int}$

$$V_S^{\mu}\,\mathcal{P}^{(1/2)}_{ij,\mu\nu}\,V_{EM}^{\nu} = 0$$

Energy-dependent decay widths of nucleon resonances $\rightarrow$ restoration of unitarity

$$\Gamma(\vec{q}) = \Gamma_{N^*}\,\frac{\sqrt{s}}{m_{N^*}}\,\sum_i x_i \left(\frac{|\vec{q}_i|}{|\vec{q}_i^{N^*}|}\right)^{2l+1} \frac{D(|\vec{q}_i|)}{D(|\vec{q}_i^{N^*}|)},$$

Extension from photoproduction to electroproduction

- Phenomenological form factors in the electromagnetic vertex
- Longitudinal couplings of N^*'s to γ^* (crucial at small Q^2)

$$V^{EM}(N^*_{1/2}p\gamma) = -\,i\,\frac{g_3^{EM}}{(m_R+m_p)^2}\Gamma_{\mp}\,\gamma_\beta\,\mathcal{F}^\beta,$$

$$V^{EM}_{\mu}(N^*_{3/2}p\gamma) = -\,i\,\frac{g_3^{EM}}{m_R(m_R+m_p)^2}\gamma_5\Gamma_{\mp}\,(\not{q}\,g_{\mu\beta} - q_\beta\gamma_\mu)\,\mathcal{F}^\beta,$$

$$V^{EM}_{\mu\nu}(N^*_{5/2}p\gamma) = -\,i\,\frac{g_3^{EM}}{(2m_p)^5}\Gamma_{\mp}(q_\alpha q_\beta g_{\mu\nu} + q^2 g_{\alpha\mu}g_{\beta\nu} - q_\alpha q_\nu g_{\beta\mu} - q_\beta q_\nu g_{\alpha\mu})\,p^\alpha\,\mathcal{F}^\beta.$$

Fitting procedure

Minimization of χ^2/n.d.f. with help of MINUIT code

Resonance selection

- s channel: spin-1/2, 3/2, and 5/2 N^* with mass $< 2\,\text{GeV}$;
  initial set from the Bayesian analysis (PR C **86** (2012) 015212) and then varied
  - missing resonances $D_{13}(1875)$, $P_{11}(1880)$, $P_{13}(1900)$
- t channel: $K^*(892)$, $K_1(1272)$
- u channel: $Y^*(1/2)$ and $Y^*(3/2)$

Free parameters ($\approx 30 + 10$):

- SU(3)$_f$: $-4.4 \leq g_{K\Lambda N}/\sqrt{4\pi} \leq -3.0$,
  $0.8 \leq g_{K\Sigma N}/\sqrt{4\pi} \leq 1.3$
- K^*'s have vector and tensor couplings
- spin-1/2 resonance $\rightarrow$ 1 parameter;
  spin-3/2 and 5/2 resonance
  $\rightarrow$ 2 parameters
- 2 cut-off parameters for the hff
- 1 longitudinal coupling for each N^*
- 2 cut-off parameters for the emff of K^* and K_1

Experimental data

3383 $p(\gamma, K^+)\Lambda$ data

- cross section for $W < 2.355\,\text{GeV}$
  (CLAS 2005 & 2010; LEPS, Adelseck-Saghai)
- hyperon polarisation for $W < 2.225\,\text{GeV}$
  (CLAS 2010)
- beam asymmetry (LEPS)

171 $p(e, e' K^+)\Lambda$ data

- $\sigma_U, \sigma_T, \sigma_L, \sigma_{LT'}, \sigma_K$

Results of the fitting procedure

Solutions: BS1 and BS2, $\chi^2/\text{n.d.f.} = 1.64$
(constant widths of N^*'s; fit on $p(\gamma, K^+)\Lambda$ data; PR C **93** (2016) 025204),
BS3, $\chi^2/\text{n.d.f.} = 1.74$
(energy-dependent widths of N^*'s; fit on $p(\gamma, K^+)\Lambda$ ($\chi^2/\text{n.d.f.} = 1.51$) and $p(e, e'K^+)\Lambda$ data; PR C **97** (2018) 025202)

- χ^2's, fitted parameter values (smallness) and correspondence with data taken into account
- sets of chosen Y^* differ in all BS models $\rightarrow$ different description of background
- electromagnetic form factors of K^* and K_1: crucial for $Q^2 > 2\,(\text{GeV}/c)^2$

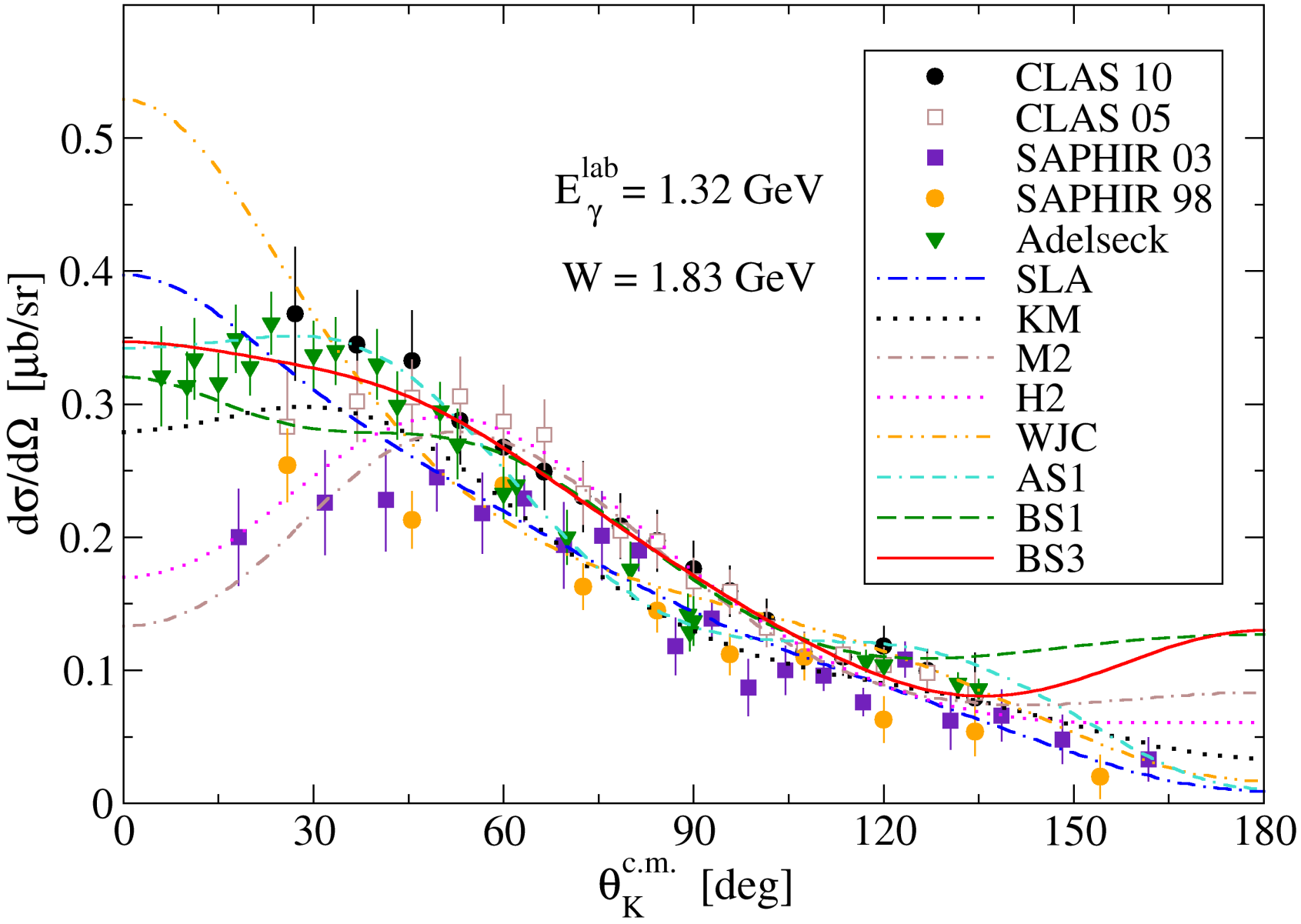
BS1 model

- $S_{11}(1535)$, $S_{11}(1650)$, $F_{15}(1680)$, $P_{13}(1720)$, $F_{15}(1860)$, $D_{13}(1875)$, $F_{15}(2000)$;
- $K^*(892)$, $K_1(1272)$;
- $\Lambda(1520)$, $\Lambda(1800)$, $\Lambda(1890)$, $\Sigma(1660)$, $\Sigma(1750)$, $\Sigma(1940)$;
- multidipole form factor: $\Lambda_{bgr} = 1.88\,\text{GeV}$, $\Lambda_{res} = 2.74\,\text{GeV}$

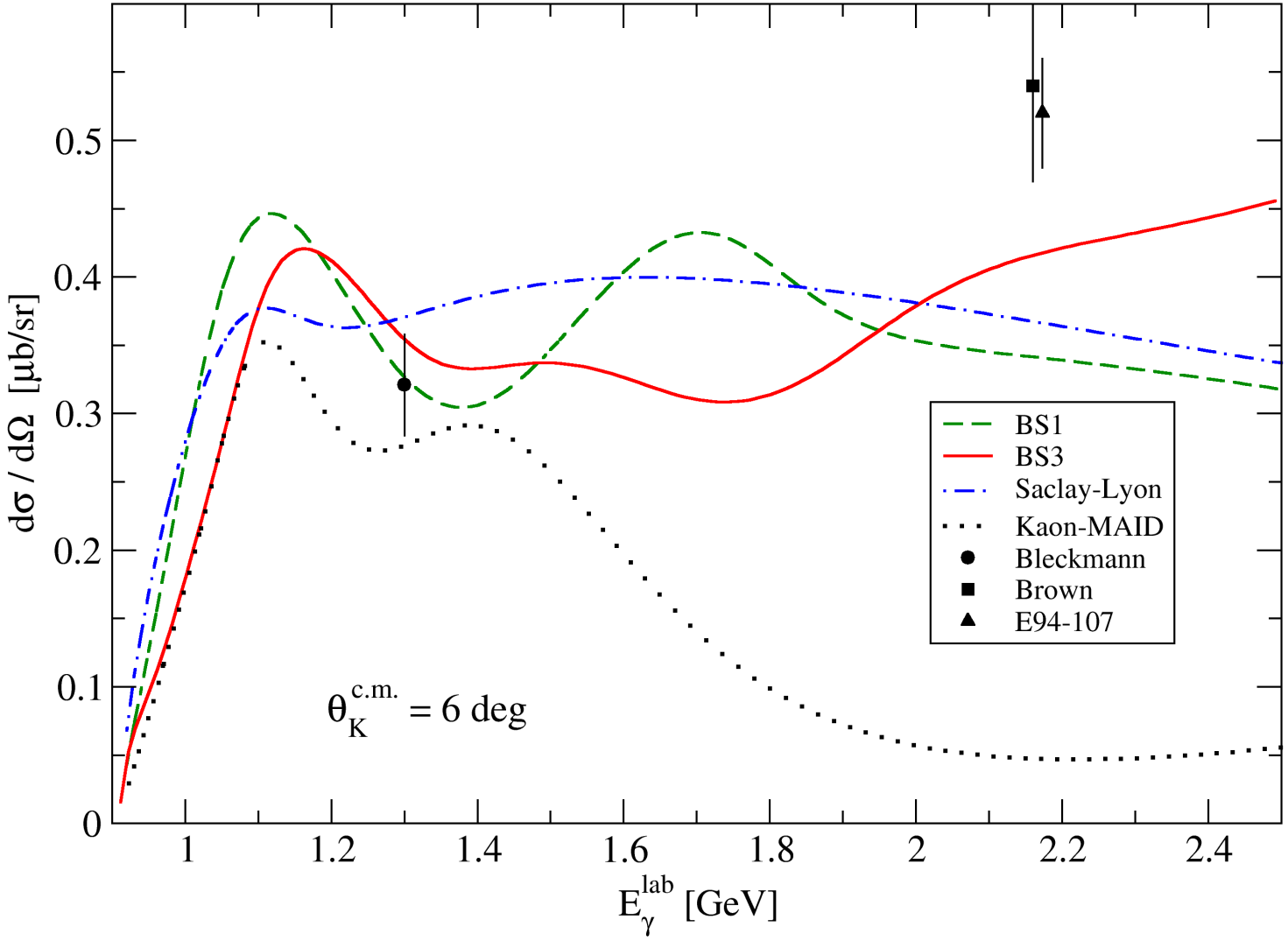
BS3 model

- $S_{11}(1535)$, $S_{11}(1650)$, $F_{15}(1680)$, $P_{11}(1710)$, $P_{13}(1720)$, $F_{15}(1860)$, $D_{13}(1875)$, $P_{13}(1900)$, $F_{15}(2000)$, $D_{13}(2120)$;
- $K^*(892)$, $K_1(1272)$;
- $\Lambda(1405)$, $\Lambda(1600)$, $\Lambda(1890)$, $\Sigma(1670)$;
- dipole form factor: $\Lambda_{bgr} = 1.24\,\text{GeV}$, $\Lambda_{res} = 0.89\,\text{GeV}$

Angular dependence of the cross section for $p(\gamma, K^+)\Lambda$

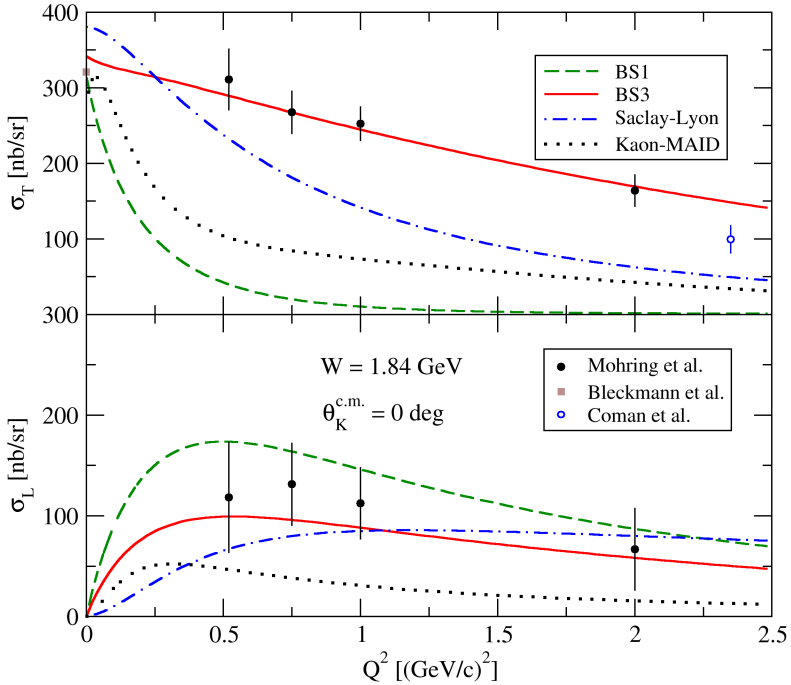


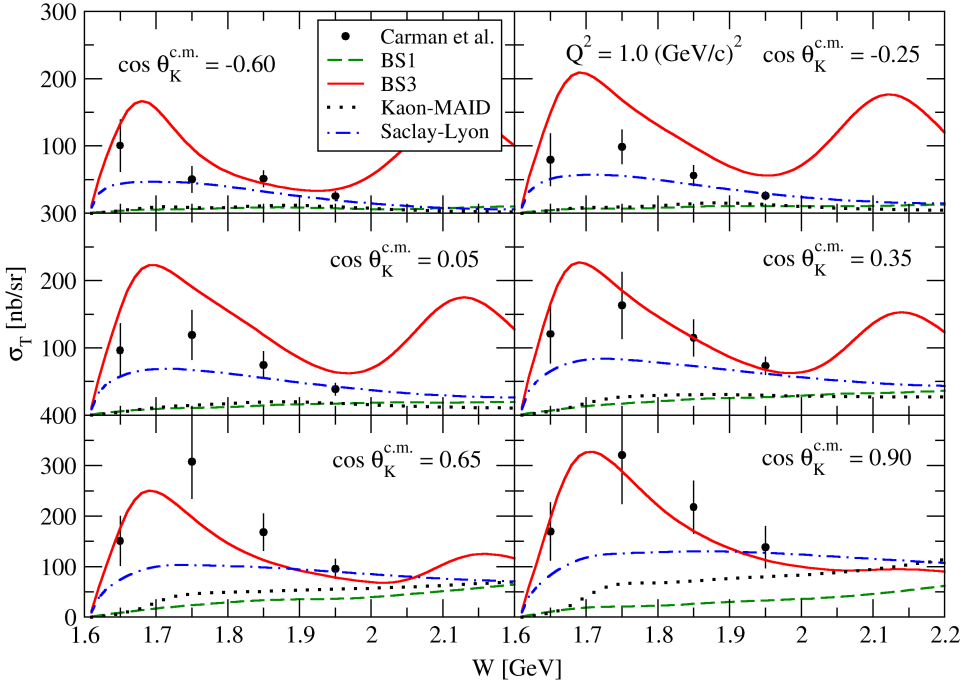
Predictions of $d\sigma/d\Omega$ for $p(\gamma, K^+)\Lambda$ at $\theta_K^{c.m.} = 6^\circ$



- Brown [$Q^2 = 0.18\,(\mathrm{GeV}/c)^2$] & E94-107 [$Q^2 = 0.07\,(\mathrm{GeV}/c)^2$]:
  data for $p(e, e'K^+)\Lambda$ but: $\sigma_L \sim Q^2$, $\sigma_{TT} \sim \sin^2\theta_K^{c.m.}$, and $\sigma_{LT} \sim \sqrt{Q^2}\sin\theta_K^{c.m.} \Rightarrow \sigma \approx \sigma_T$

Transverse, σ_T, and longitudinal, σ_L, cross sections of $p(e, e'K^+)\Lambda$





Extension from photo- to electroproduction

- BS1: naive extension by adding em. form factors only
- BS3: em. form factors and longitudinal couplings of N^*'s to γ^* added

New fits for $K^+\Sigma^-$ channel

Least Absolute Shrinkage Selection Operator (LASSO)

Fitting procedure with MINUIT library: **minimizing the** χ^2

$$\chi^2 = \sum_{i=1}^{N} \frac{[d_i - p_i(c_1, \ldots, c_n)]^2}{\sigma_{d_i}^2},$$

$(c_1, \ldots, c_n)$ - set of free parameters, $(d_1, \ldots, d_N)$ - set of data points, p_i - theory, σ_{d_i} - error

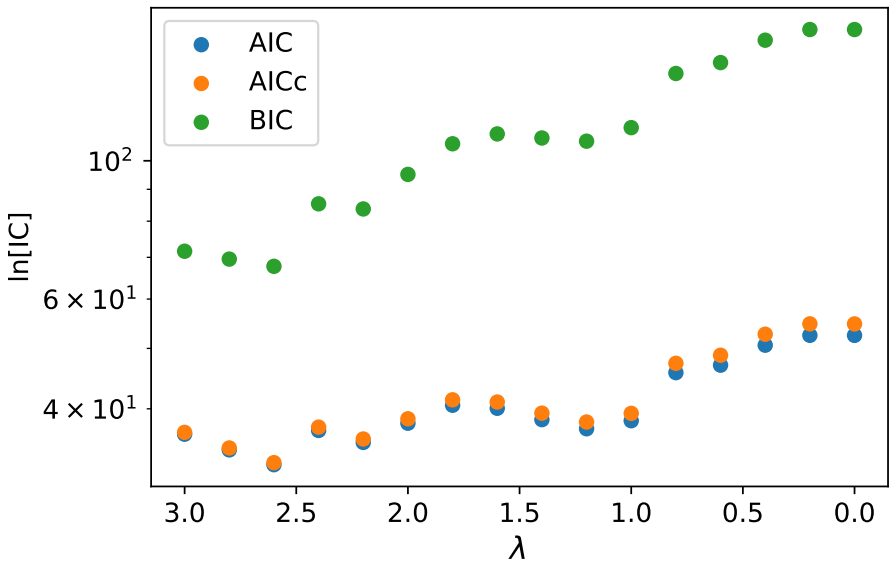
Problem: χ^2 minimization cannot prevent overfitting

Remedy to the overfitting issue: regularization (in this case, it is LASSO)

- penalized χ_T^2: $\chi_T^2 = \chi^2 + P(\lambda)$
- penalty term: $P(\lambda) = \lambda^4 \sum_{i=1}^{N_{res}} |g_i|$
  λ - regularization parameter, g_i - resonances' couplings

Information criteria:

- $AIC = 2n + \chi_T^2$
- $AICc = AIC + \frac{2n(n+1)}{N-n-1}$
- $BIC = n\ln(N) + \chi_T^2$



New fits for $K^+\Sigma^-$ channel

P. Bydžovský, A. Cieplý, D. Petrellis, D. Skoupil, and N. Zachariou, PR C **104**, 065202 (2021)

Fitting procedure

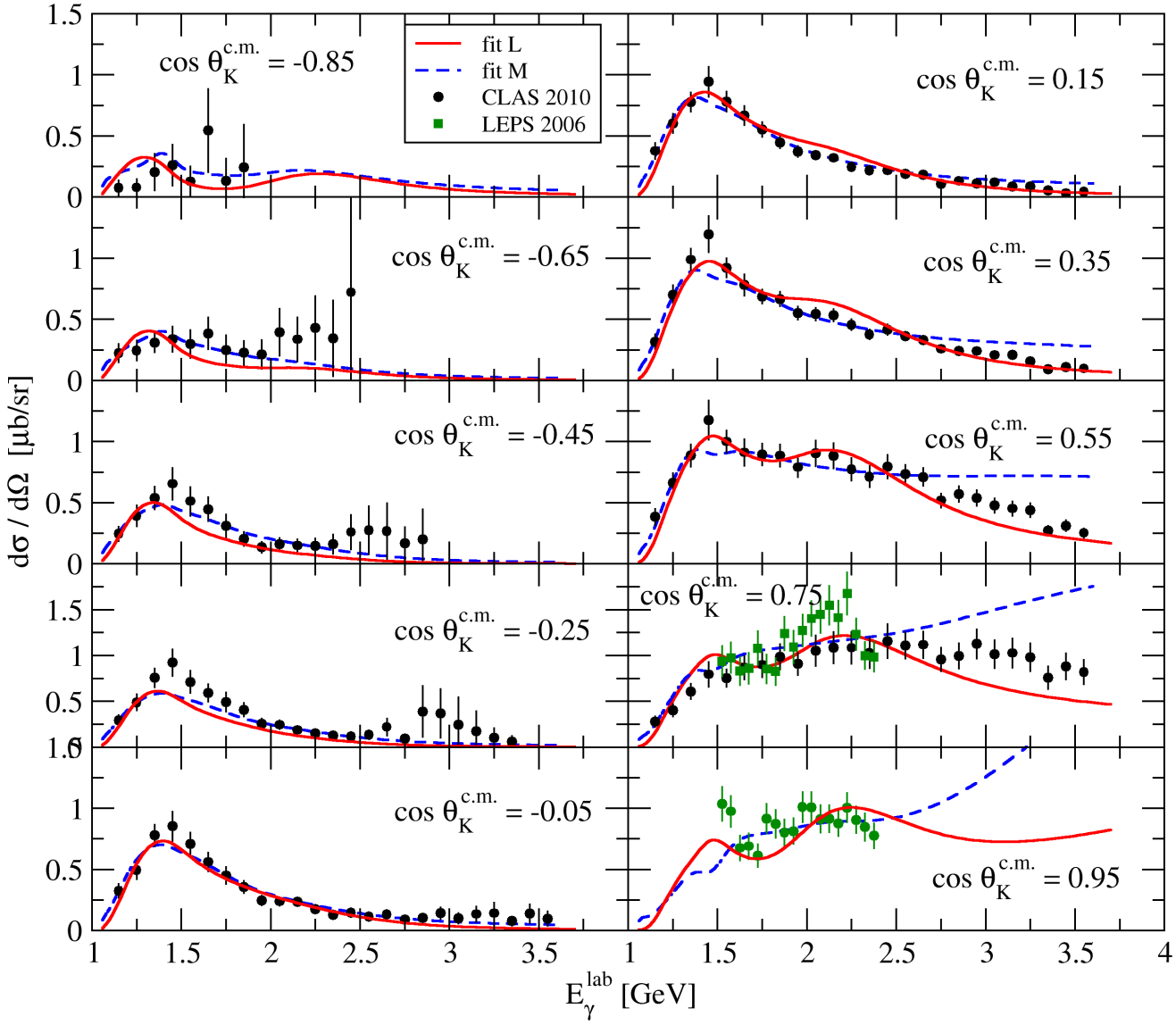
- non resonant part: Born terms and exchanges of K^* and K_1 (t channel) and Σ^* (u channel)
- resonant part: exchanges of N^*'s and Δ's in the s channel
- around 600 data utilized to fit $\approx$ 25 parameters
- the main coupling, $g_{K^+\Sigma^- n} = \sqrt{2} g_{K^+\Sigma^0 p} = 1.568$, taken from $K^+\Lambda$ channel
- result with the smallest $\chi^2/\mathrm{ndf} = 2.3 \rightarrow$ **fit M** (25 parameters, 14 resonances)
- **LASSO method** used: $\chi^2_T/\mathrm{ndf} = 3.4 \rightarrow$ **fit L** (17 parameters, 9 resonances)

Characteristics of models

- only one Δ resonance introduced
- no hyperon resonances needed for reliable data description
- results in very good agreement with the cross-section and beam-asymmetry data
- fit L: a very economical fit

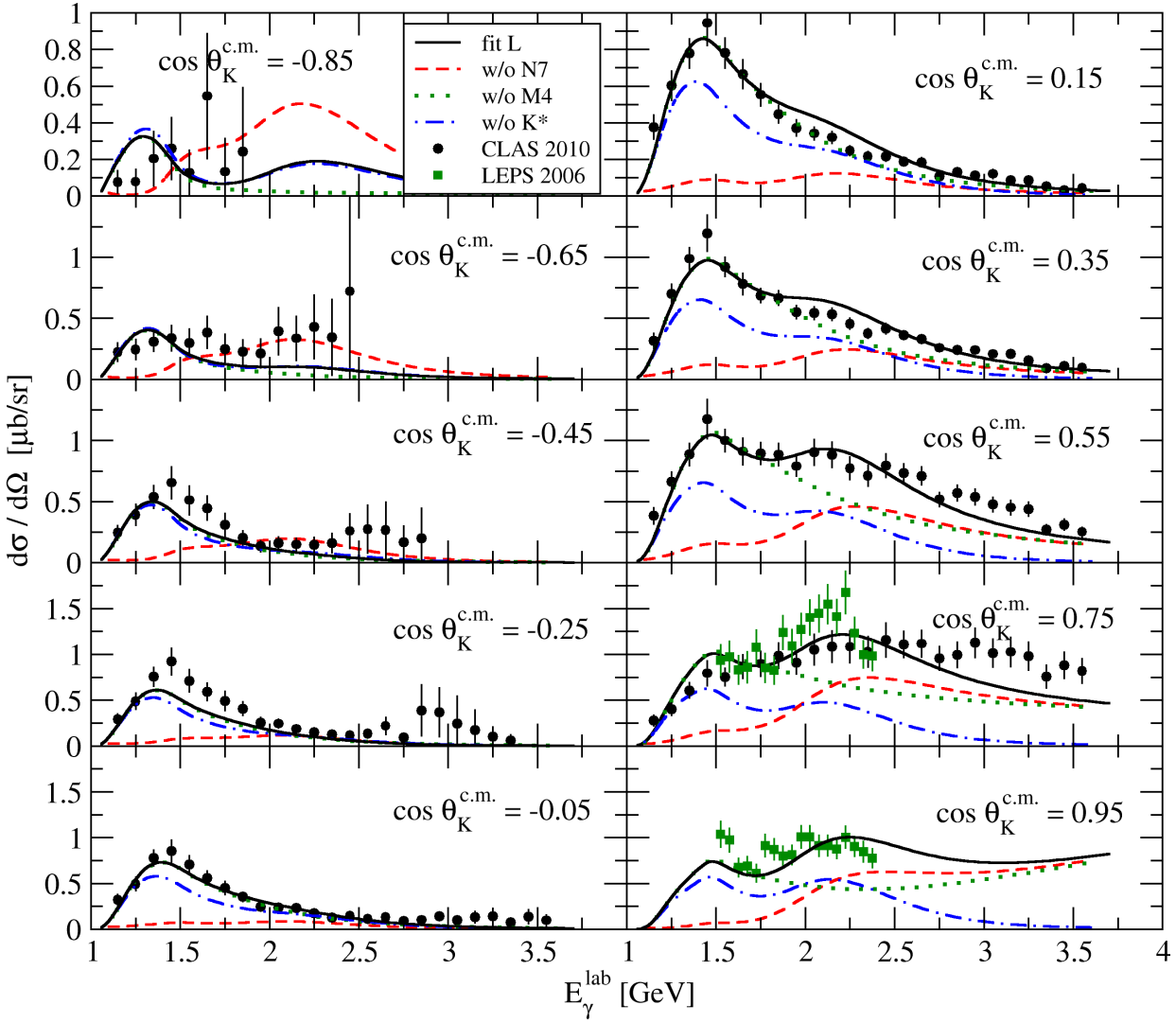
New fits for $K^+\Sigma^-$ channel

Differential cross section in dependence on the photon lab energy



New fits for $K^+\Sigma^-$ channel

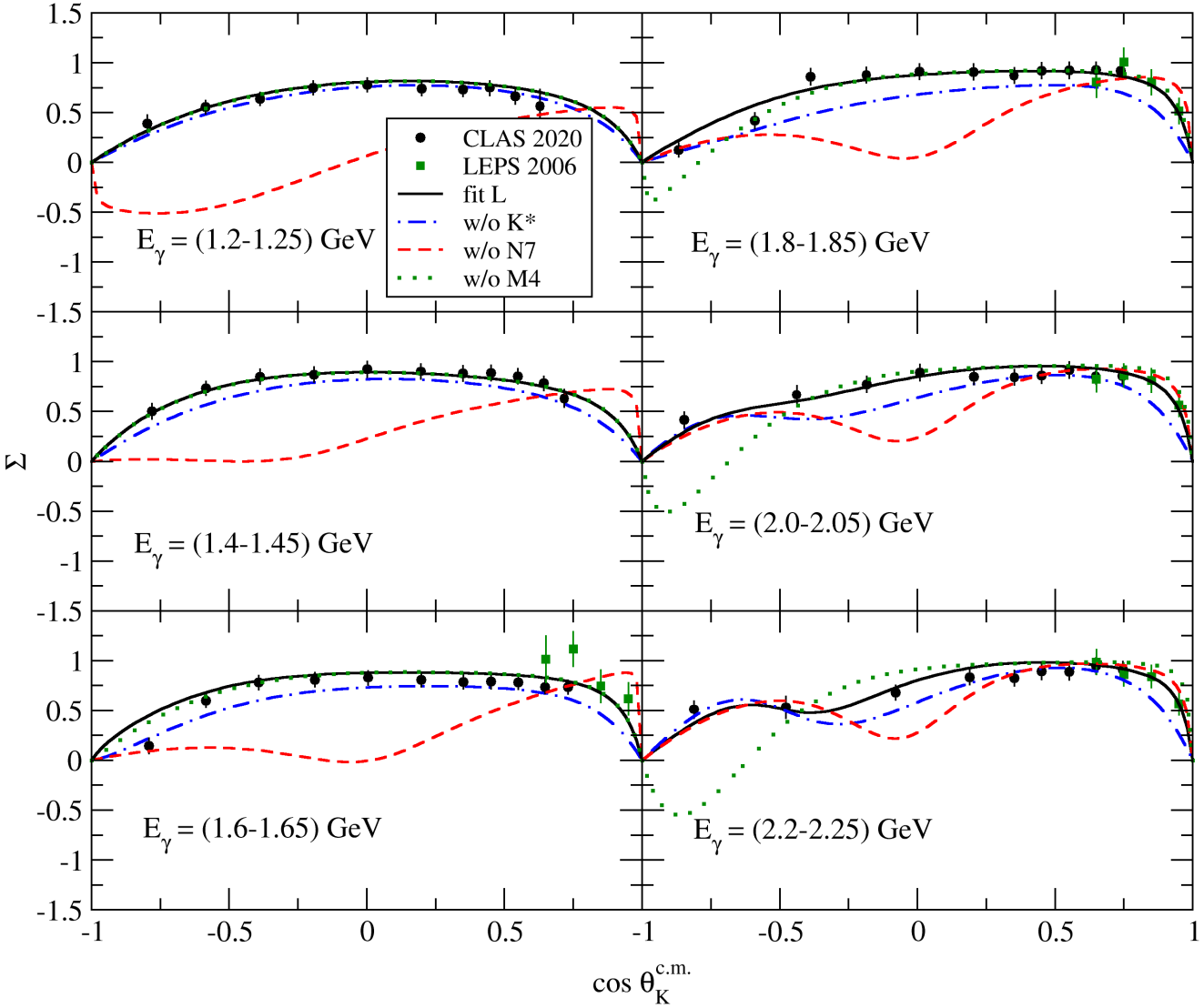
Differential cross section in dependence on the photon lab energy - fit L w/o individual resonances



N7: N(1720)$3/2^+$, M4: N(2060)$5/2^-$

New fits for $K^+\Sigma^-$ channel

Beam asymmetry in dependence on the kaon center-of-mass angle - fit L w/o individual resonances



Refitting the model's parameters in the $K^+\Lambda$ channel

Ridge regression and cross validation for suppressing hyperon couplings

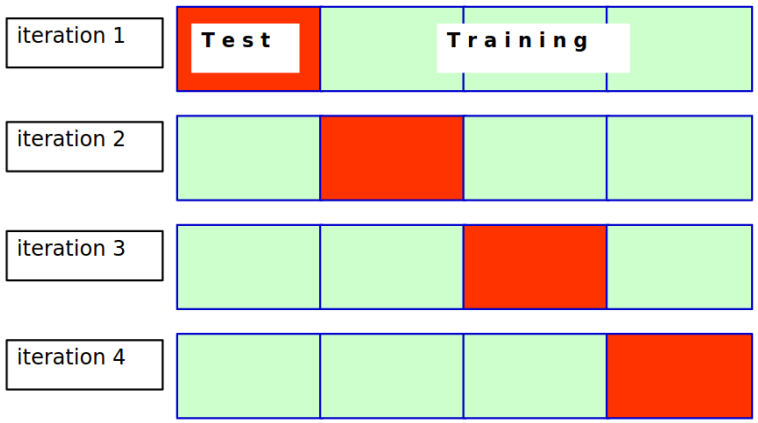
Why refit?

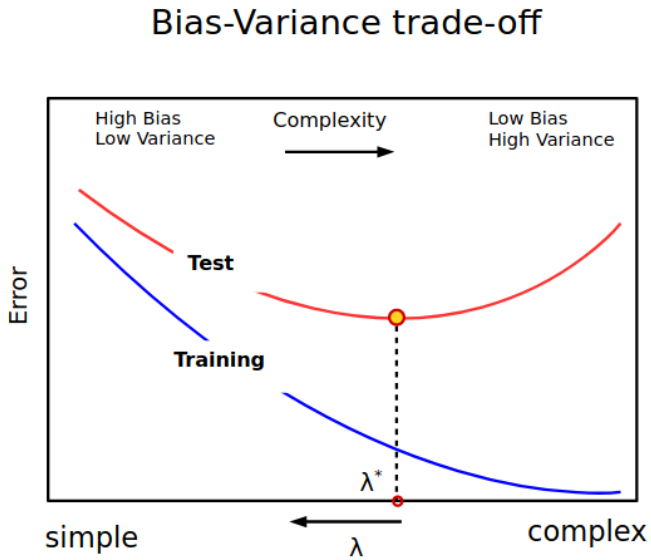
- include recent measurements of polarization observables
- need to investigate more the role of hyperon resonances in KY photoproduction
- large values of hyperon couplings: ridge regression to suppress them during the fitting procedure

Ridge regularization

- penalized χ^2_T: $\chi^2_T = \chi^2 + \lambda^4 \sum_{i=1}^{K_\Lambda} g_i^2$, ($K_\Lambda$ = no. of Y couplings)
- parameter values reduced but they are *not* reduced to zero

Cross validation





Effect on the couplings of hyperon resonacnes

BS2

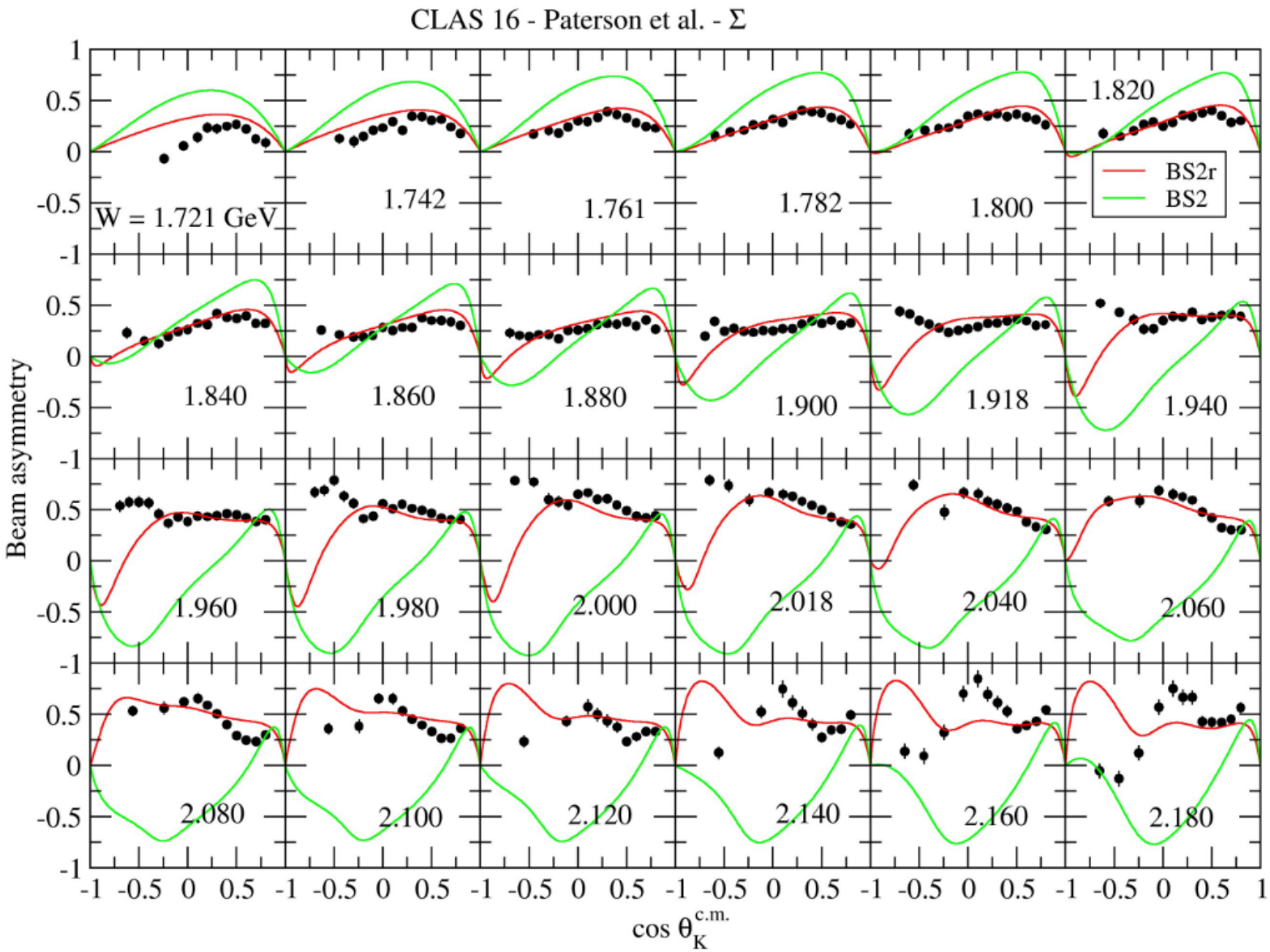
Tag	Resonance	g_1	g_2
L1	$\Lambda(1405)\ 1/2$	9.67	–
S1	$\Sigma(1660)\ 1/2^+$	-8.09	–
L4	$\Lambda(1800)\ 1/2^-$	-11.55	–
S4	$\Sigma(1940)\ 3/2^-$	-0.86	0.18

BS2 with Ridge

Tag	Resonance	Mass [MeV]	Width [MeV]	g_1	g_2
K*	$K^*(892)$	891.7	50.8	-0.176	0.011
K1	$K_(1272)$	1272	90	0.321	-1.136
N3	$N(1535)\ 1/2^-$	1530	150	-0.012	–
N4	$N(1650)\ 1/2^-$	1650	125	-0.075	–
P5	$N(1860)\ 5/2^+$	1860	270	-0.019	0.009
N7	$N(1720)\ 3/2^+$	1720	250	0.157	0.009
P4	$N(1875)\ 3/2^-$	1875	200	0.141	0.135
P2	$N(1900)\ 3/2^+$	1920	200	-0.045	-0.010
P3	$N(2050)\ 5/2^+$	2050	220	-0.012	0.013
N9	$N(1685)\ 5/2^+$	1685	130	0.048	-0.041
N6	$N(1710)\ 1/2^+$	1710	140	-0.172	–
L1	$\Lambda(1405)\ 1/2^-$	1405	51	1.308	–
S1	$\Sigma(1660)\ 1/2^+$	1660	100	-1.938	–
L4	$\Lambda(1800)\ 1/2^-$	1800	300	-0.342	–
S4	$\Sigma(1940)\ 3/2^-$	1940	220	-0.567	-0.025

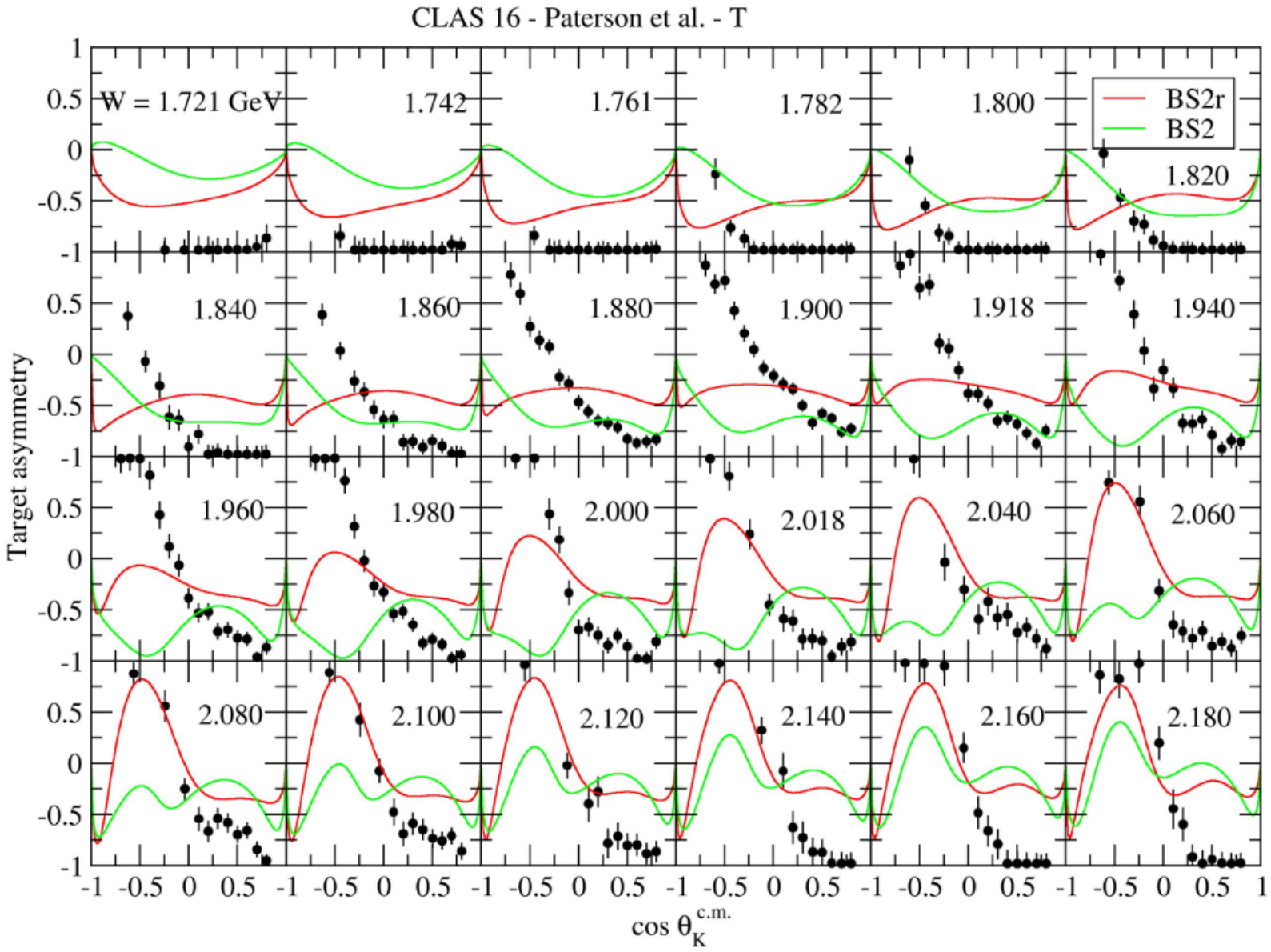
$K^+\Lambda$ channel: beam asymmetry Σ

(results are still preliminary!)



$K^+\Lambda$ channel: target asymmetry T

(results are still preliminary!)



Summary

New version of isobar model for the $K^+\Lambda$ channel

- consistent formalism for high-spin resonances
- energy-dependent widths of N^*'s impemented
- longitudinal couplings for extension towards electroproduction of $K^+\Lambda$
- available for calculations online at:
  `http://www.ujf.cas.cz/en/departments/department-of-theoretical-physics/isobar-model.html`

Description extended from the $K^+\Lambda$ channel to the $K^+\Sigma^-$ channel

Regularization methods (LASSO, ridge) introduced as a remedy for overfitting

Outlook

- testing the models in the DWIA calculations for hypernucleus production
- performing a multi-channel analysis of all Σ photoproduction channels
- extending our analysis of electroproduction beyond $Q^2 = 1\,\mathrm{GeV}^2$
- studying the production of Ξ hypernuclei

Thank you for your attention!