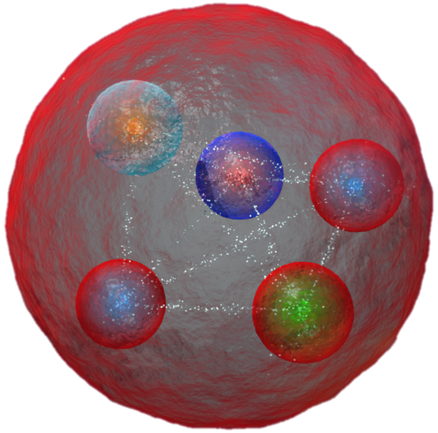
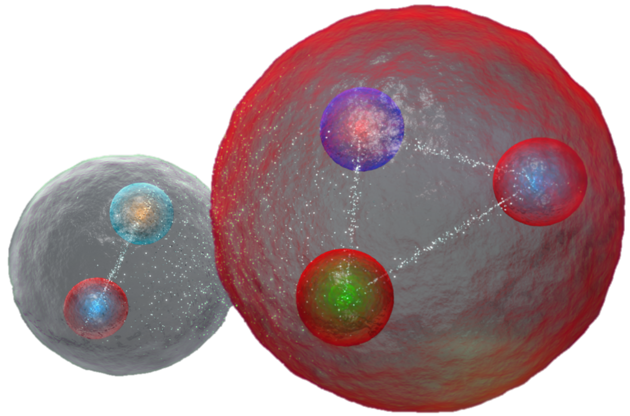
Pentaquarks:
In a Two-Body Bethe-Salpeter Equation





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NSTAR 2022, October 19

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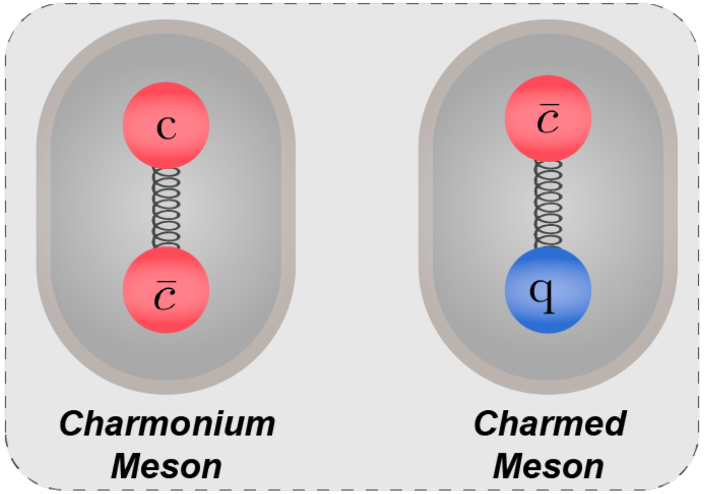
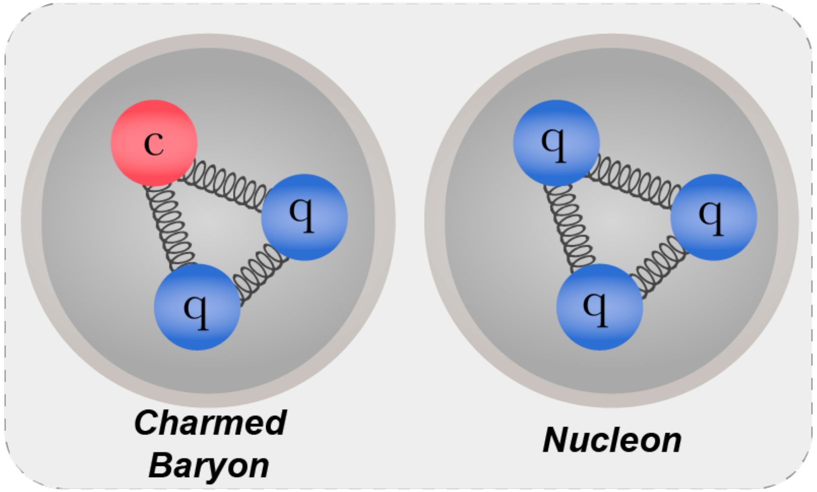
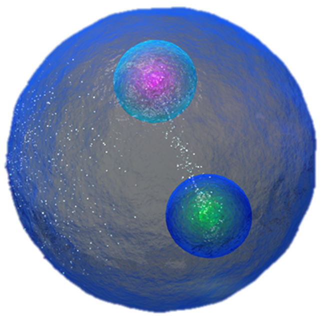
• **1964**: Ordinary hadrons in the quark model.

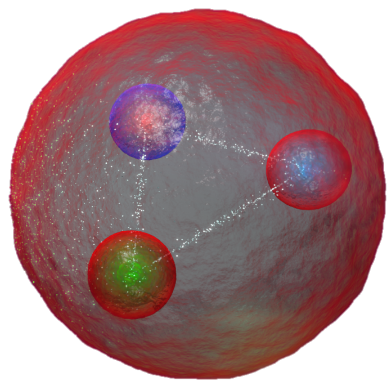
Mesons : $(q\bar{q}), (c\bar{c}), (c\bar{q}), \ldots$



Baryons : $(qqq), (qqc), \ldots$

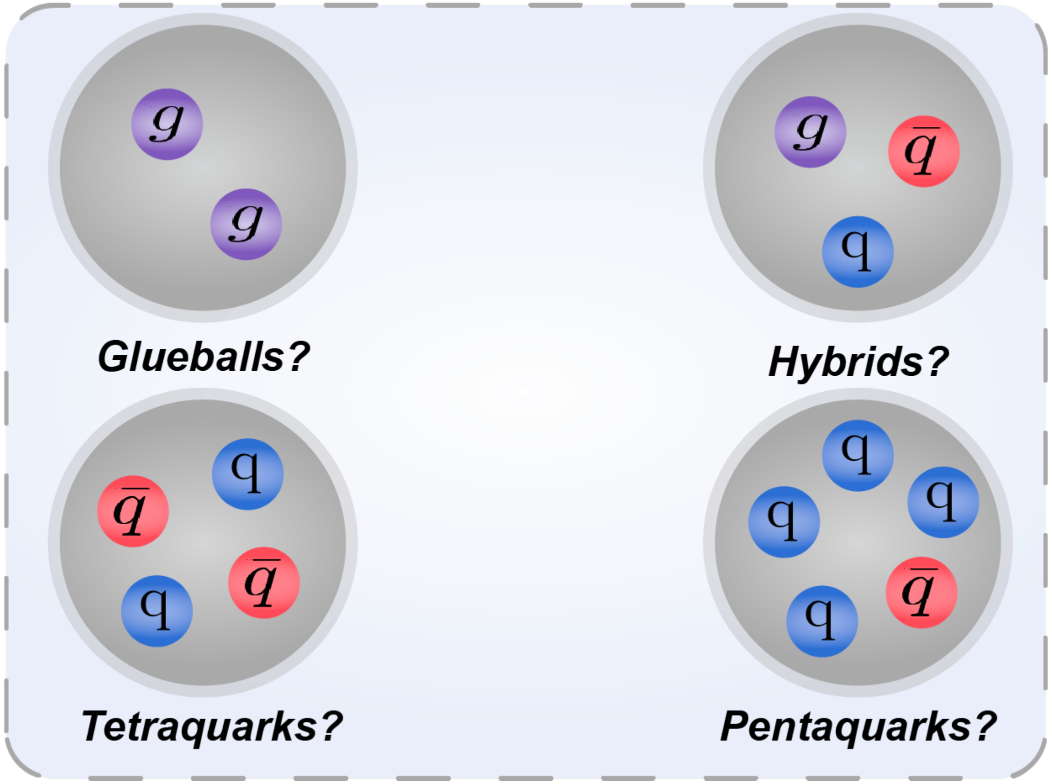






- Gell-Mann, M. (1964). A schematic model of baryons and mesons. *Physics Letters*, *8*(3), 214-215.
- Zweig, G. (1964). *An SU _3 model for strong interaction symmetry and its breaking* (No. CERN-TH-412). CM-P00042884.

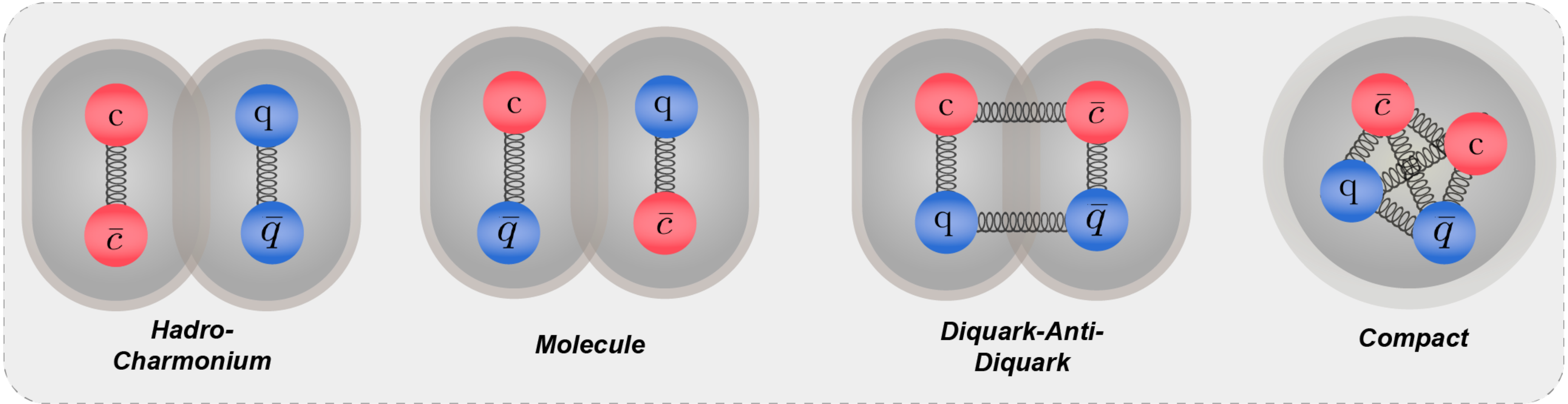
- **Last 20 years:** some discovered states do not fit into the most elementary picture, so they were called exotic states.



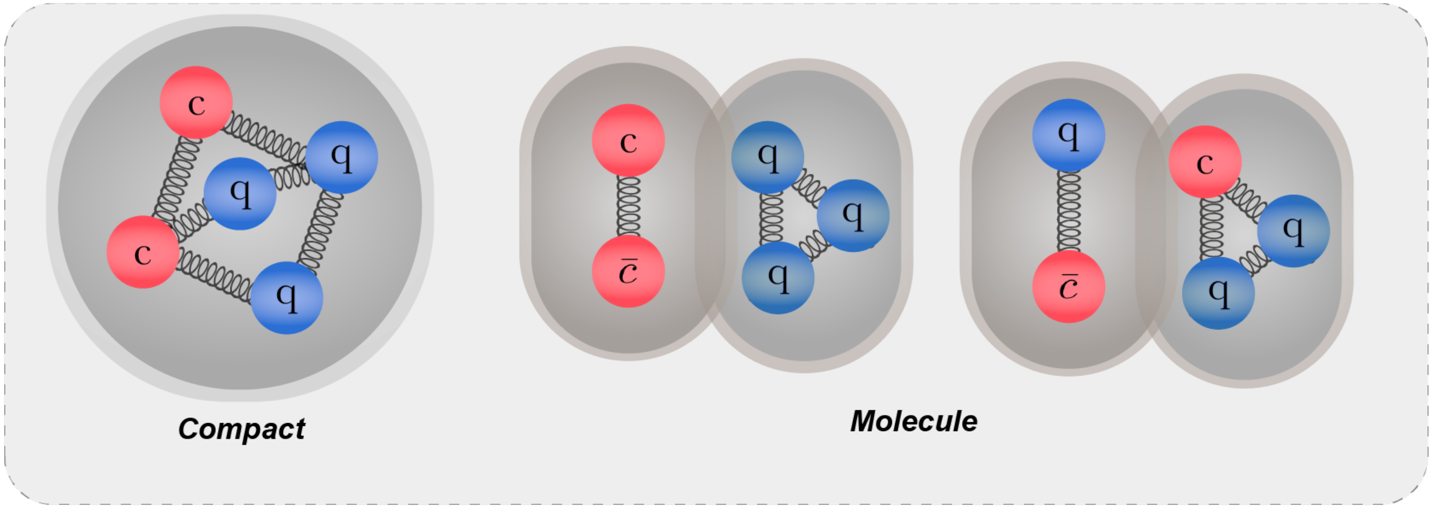
- PLB 590 209 (2004), PRD 77 014029 (2008), PRD 100 011502 (R) (2019),
- PRL 115 12 122001 (2015)

- PRD 71 014028 (2005), PLB 666 344 (2008),
- PLB 662 424(2008), PLB 671 82(2009)

Tetraquarks : $(c\bar{c}q\bar{q}), (c\bar{q}\bar{c}q), (\bar{c}\bar{q}cq), \ldots$

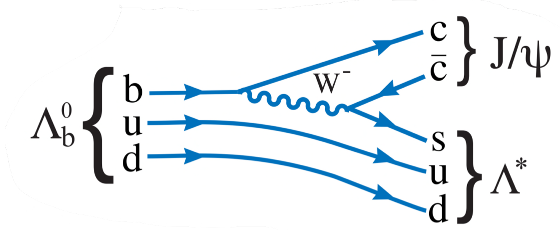


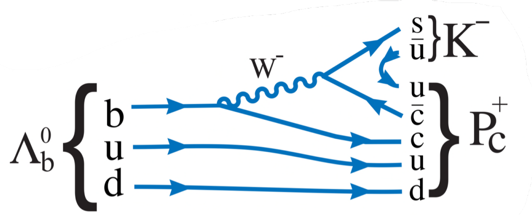
Pentaquarks : $(c\bar{c}qqq), (\bar{c}qcqq), \ldots$

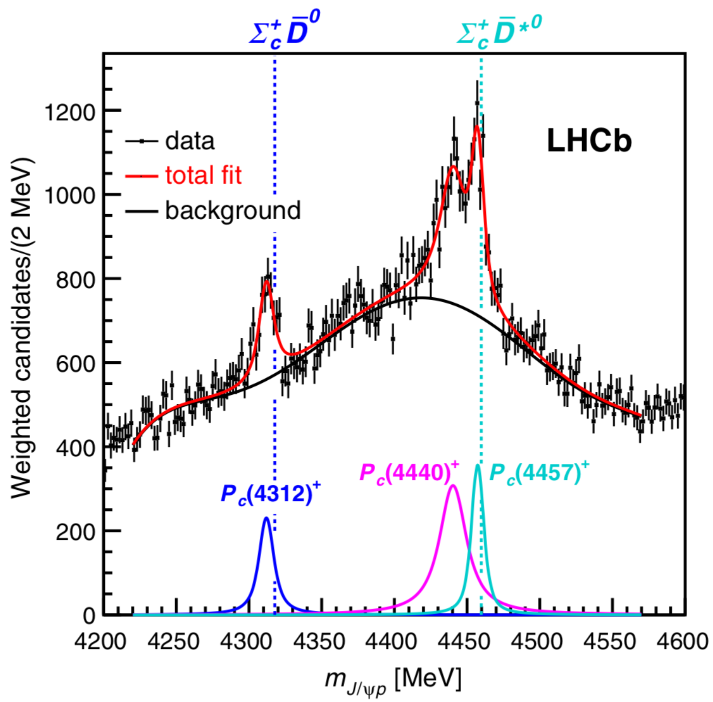


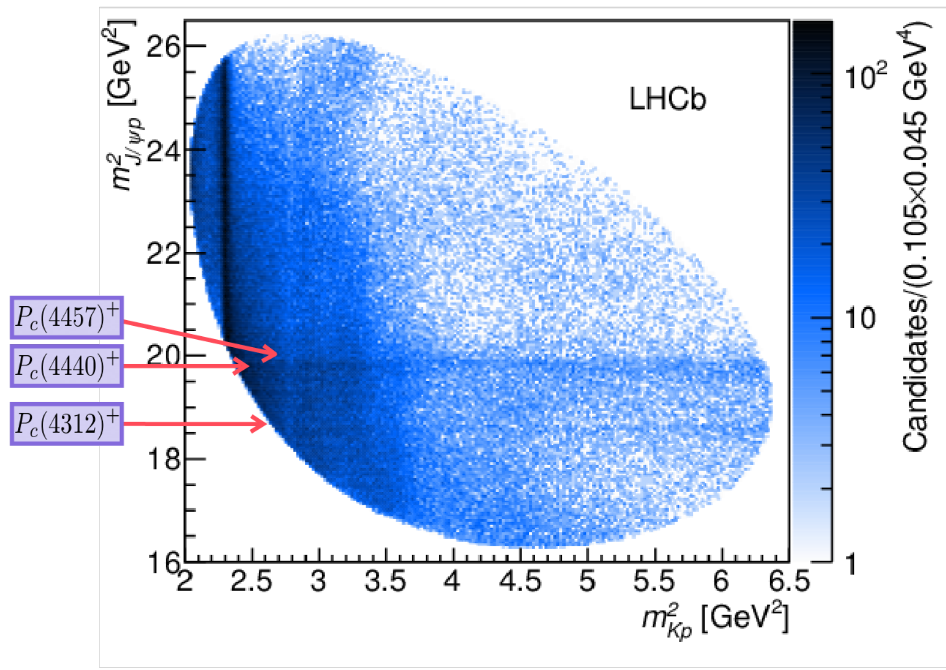
- Kvinikhidze & Khvedelidze,Theor. Math. Phys. 90 (1992) Heupel, Eichman, CF, PLB 718 (2012) 545-549
- Eichman, CF, Heupel, PLB 753 (2016) 282-287
- Wallbott,EichmannandCF, PRD100(2019)014033,[1905.02615]
- Wallbott, Eichmann and CF, PRD102 (2020), 051501, [2003.12407]
- Santowsky, CF, EPJC 82 (2022) 4, 313 [2111.15310]

- **2015:** studying the decay $\Lambda_b^0 \longrightarrow J/\Psi K^- p$, the LHCb collaboration found evidence for <u>two Pentaquark</u> states.
- **2019:** Combining the data from Run 1 and Run 2 and having a better resolution near the peaks, the LHCb collaboration found a <u>third state</u>.



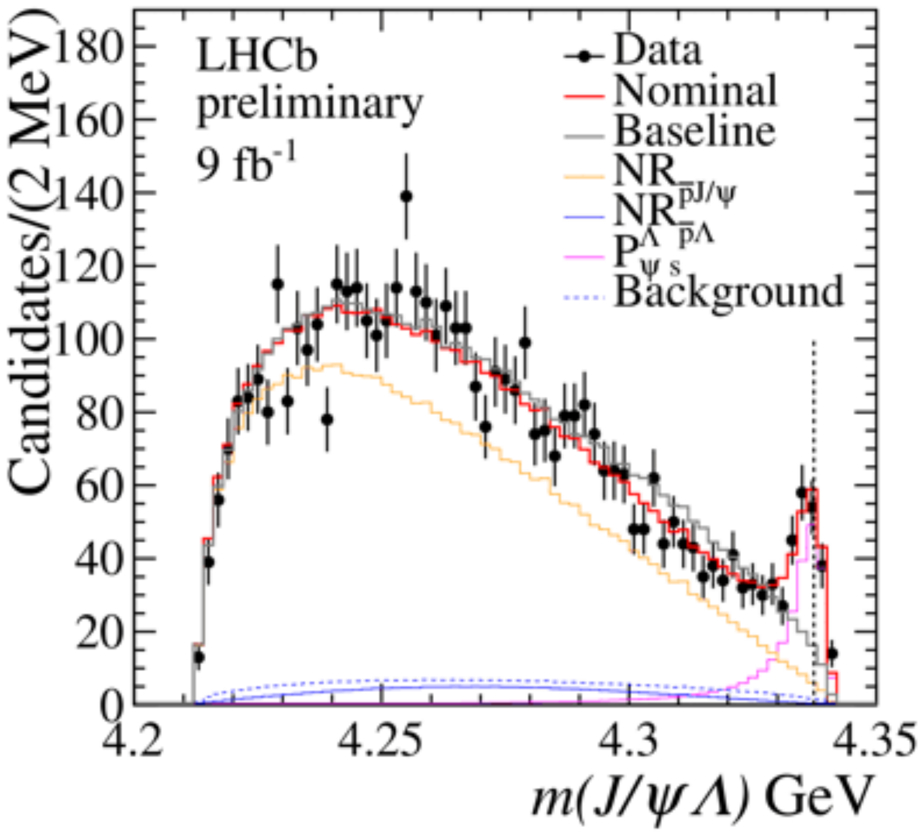


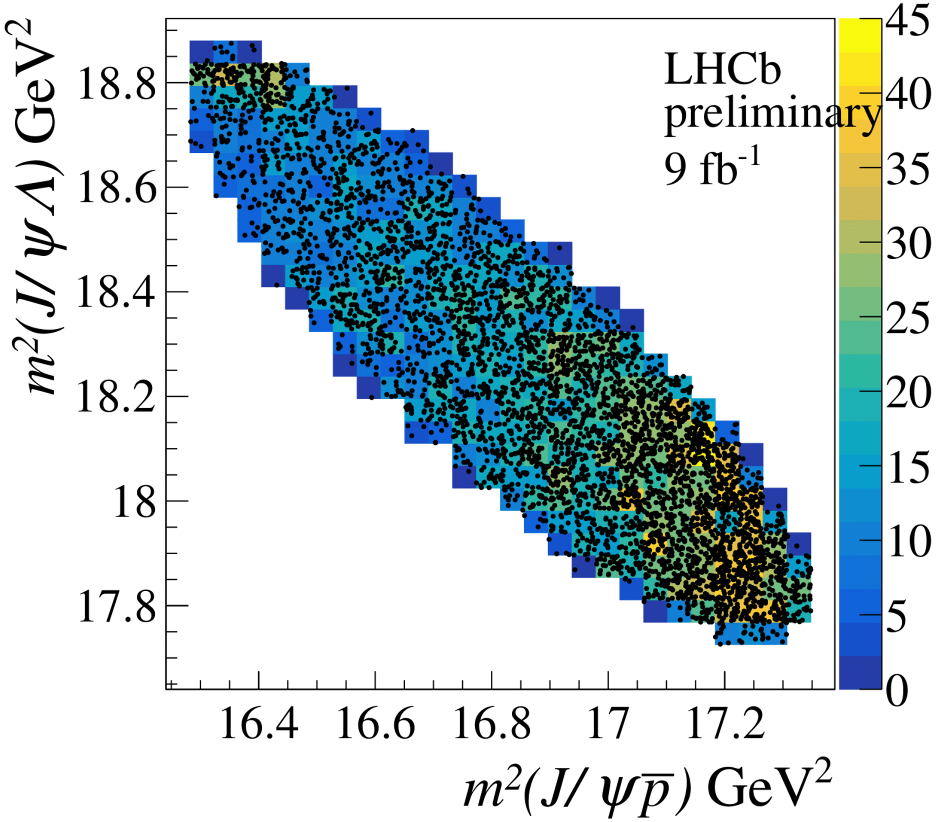




- Aaij et Al., (2015) *Phys. Rev. Lett.*, *115*(7), 072001.
- Aaij et Al., (2019) *Phys. Rev. Letts*, *122*(22), 222001.

- **July 2022:** Observation of a strange pentaquark $P^{\Lambda}_{J/\psi}(4338)^0$ in the $J/\psi\Lambda$ mass in the $B^- \to J/\psi\Lambda p$ decays. Near $\Xi_c^- D^+$ threshold.





- [LHCb-PAPER-2022-031], [LHCb-PAPER-2022-026, LHCb-PAPER-2022-027],
- [LHCb-PAPER-2022-018, LHCb-PAPER-2022-019]

Bethe-Salpeter Equation

- Bound states appear as poles in the n-point Green functions, which encode hadron properties

$$T^{(n)} = K^{(n)} + K^{(n)} G_0^{(n)} T^{(n)} \qquad (1)$$

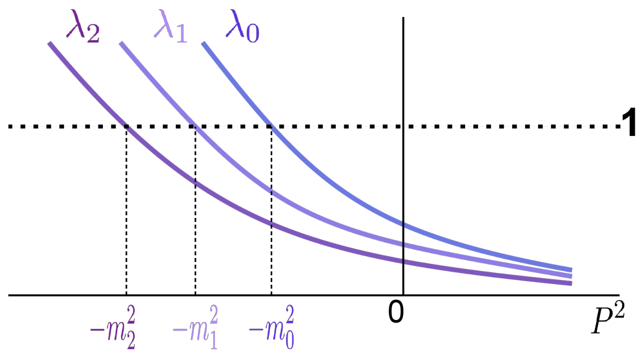
- At the pole, the BS amplitude $\Gamma^{(n)}$ is the residue of the scattering matrix

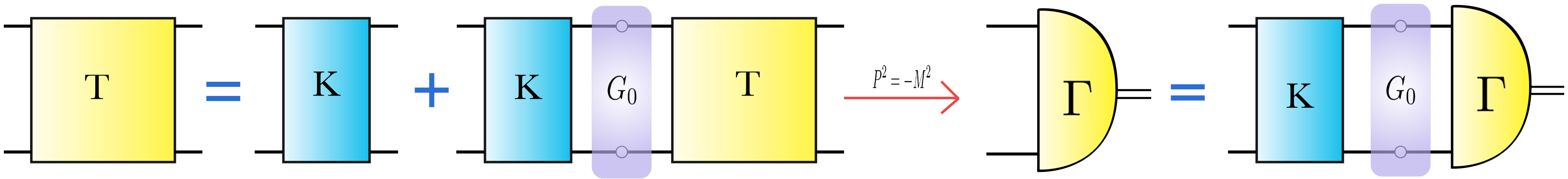
$$T^{(n)} \xrightarrow{P^2 \to -M^2} \mathcal{N} \frac{\Psi \, \overline{\Psi}}{P^2 + M^2} \qquad (2)$$

- We identify the pole and $T^{(n)}$ by comparing the residues yielding the homogeneous equation at the pole:

Homogeneous Bethe Salpeter Equation

$$\Gamma^{(n)} = K^{(n)} G_0^{(n)} \Gamma^{(n)} \qquad (3)$$

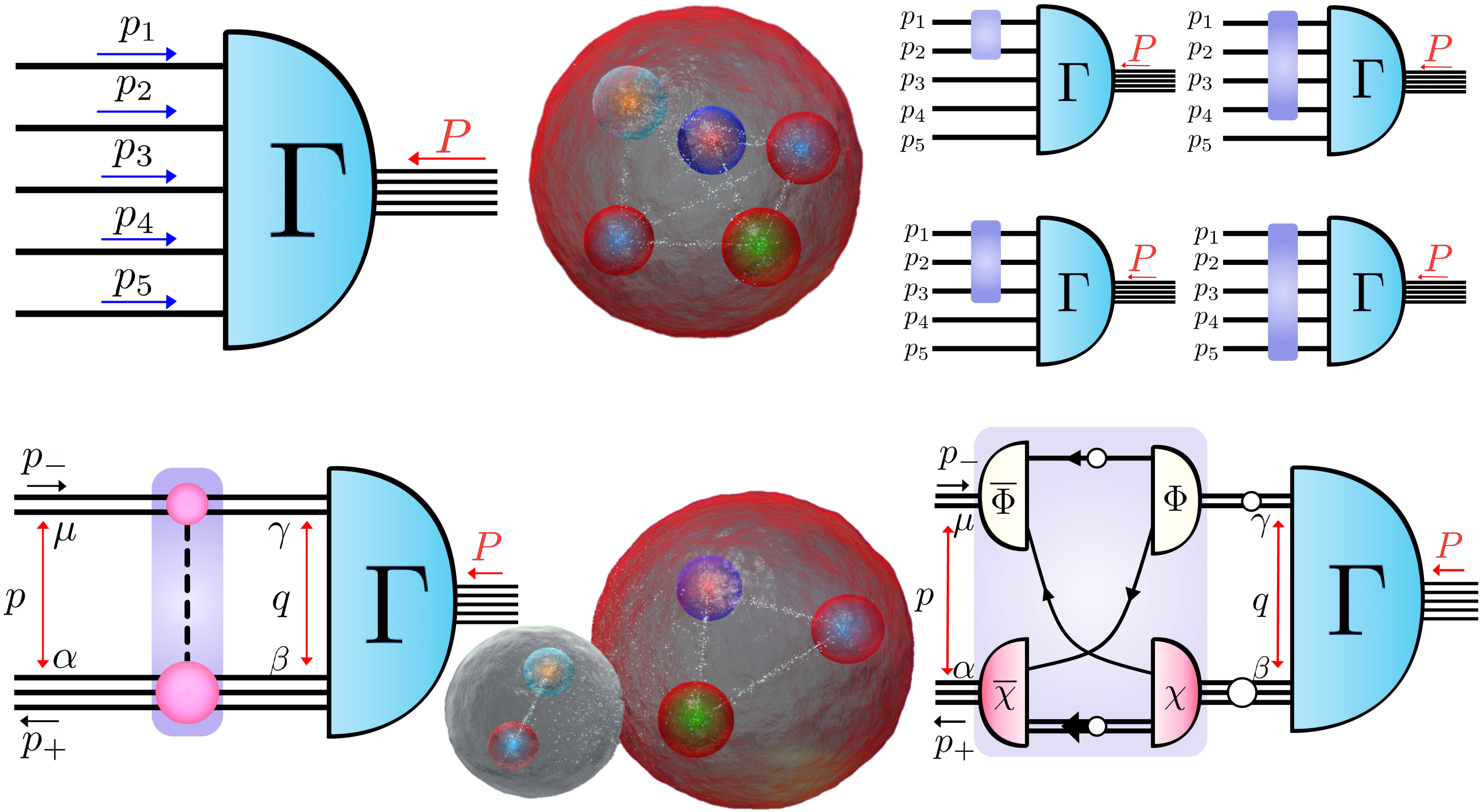




- Phys. Rept. 353 (2001) 281 [hep-ph/0007355]. Phys. E 12 (2003) 297 [nucl-th/0301049].
- Phys.G32(2006)R253 [hep-ph/0605173]. Phys.58(2012)79 [arXiv:1201.3366].

Pentaquarks

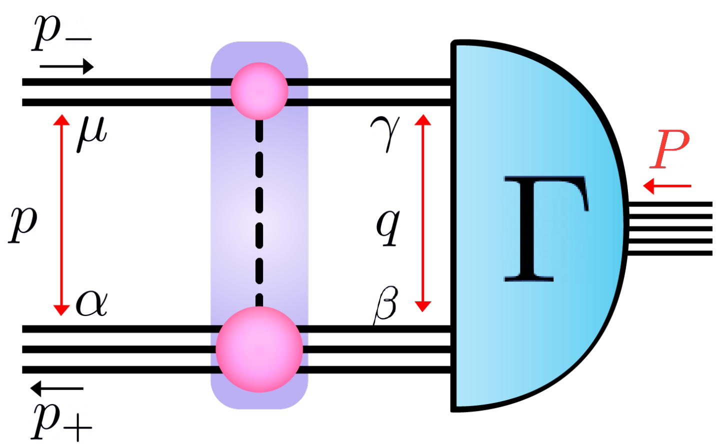
- A Pentaquark system constitutes a five-body problem.
- Or, a two-body system where the pentaquark is described as a molecule made of a baryon and a meson.



- The easiest way to solve the pentaquark system is describing it as a "molecule". (5 body on going...)
- Solve a two body BSE (pentaquark $c\bar{c}uud$)

2-body Bethe-Salpeter Equation

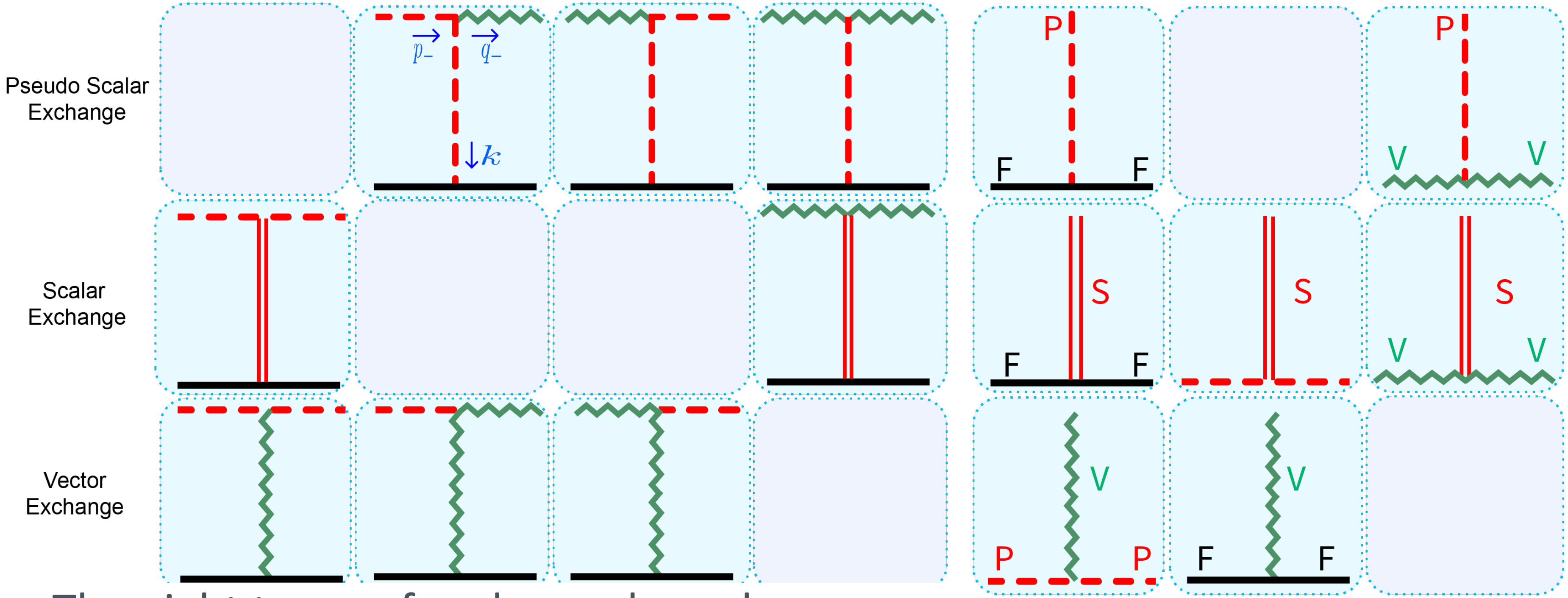
$$\Gamma^{\mu}_{\alpha\beta}(p,P) = \int \frac{d^4q}{(2\pi)^4} \{K^{\mu\nu}(p,q)\, S(q_+)\, \Gamma^{\gamma}(q,P)\}_{\alpha\beta}\, D^{\nu\gamma}(q_-) \quad (4)$$



- The meson and baryon propagators are taken to be free propagators:

Pseudo-Scalar Meson	Vector Meson	Baryon
$D_p(q) = \frac{1}{q^2 + M_P^2}$	$D_V^{\mu\nu}(q) = \frac{1}{q^2 + M_V^2} T_q^{\mu\nu}$	$S(q) = \frac{-i\gamma^{\mu} q^{\mu} + M_b^2}{q^2 + M_b^2}$

(5)



- The eight types of exchange kernels

Pseudo Scalar Exchange

$$
\begin{aligned}
K^{\nu}_{PV} &= i\gamma_5 D(k)(k+p_-)^{\nu} \\
K^{\mu}_{VP} &= i\gamma_5 D(k)(k-q_-)^{\mu} \\
K^{\mu\nu}_{VV} &= i\gamma_5 D(k)\frac{\Pi^{\mu\nu}}{m_V}
\end{aligned}
$$

With

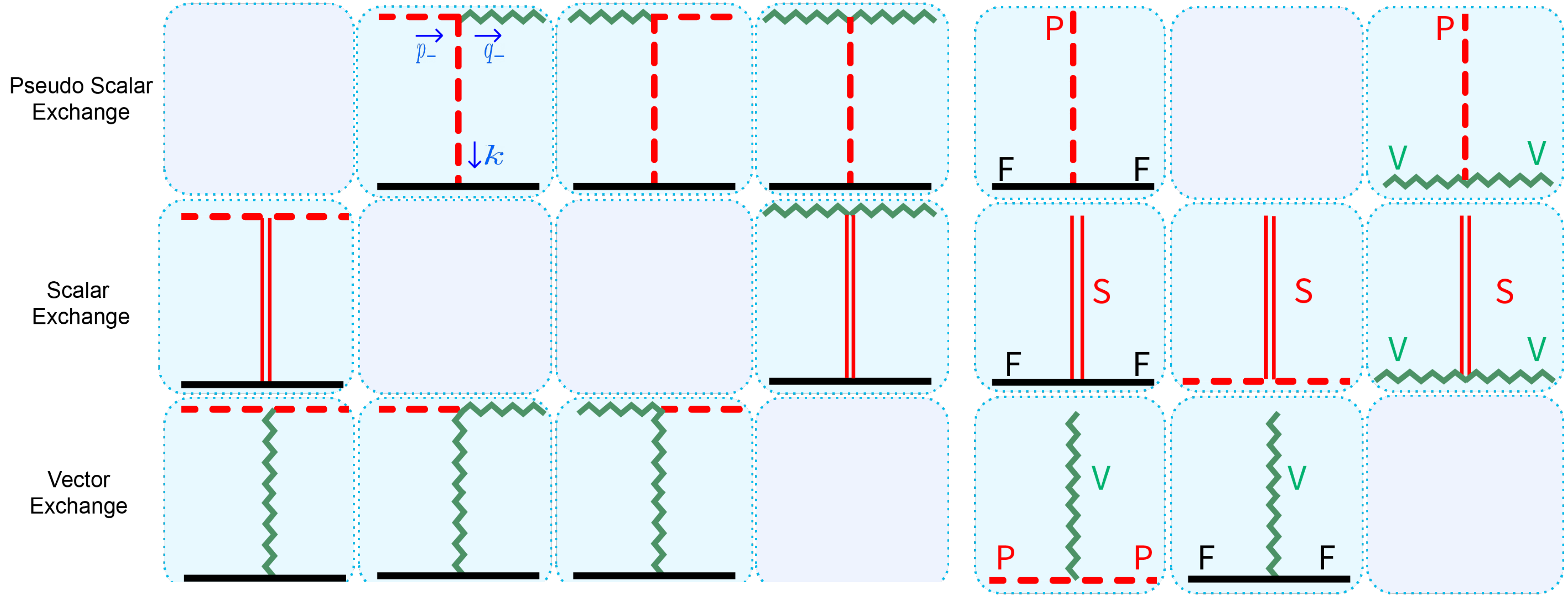
$$
\Pi = \epsilon^{\mu\nu}_{qp} - \frac{\epsilon - 1}{2}\epsilon^{\mu\nu}_{kP}
$$

Scalar Exchange

$$
\begin{aligned}
K_{PP} &= m_P D(k) \\
K^{\mu\nu}_{VV} &= m_V D(k)\delta^{\mu\nu}
\end{aligned}
$$

Vector Exchange

$$
\begin{aligned}
K_{PP} &= 2iD(k)\gamma_{\mu}\left(r^{\mu}_{-} + b(x)k^{\mu}\frac{k\cdot r_-}{\mu^2}\right) \\
K^{\nu}_{PV} &= D(k)i\gamma^{\alpha}\frac{\Pi^{\alpha\nu}}{m_V} \\
K^{\mu}_{VP} &= D(k)\frac{\Pi^{\mu\alpha}}{m_V}i\gamma^{\alpha}
\end{aligned}
\tag{6}
$$



- The seven type of vertices (leading order)

$$\Gamma_{FFP}(k,Q) = g_{FFP}\, i\gamma_5, \qquad \Gamma_{PPS}(k,Q) = g_{PPS}\, m_P$$

$$\Gamma_{FFS}(k,Q) = g_{FFS}, \qquad \Gamma^{\mu}_{PPV}(k,Q) = g_{PPV}\, 2k^{\mu}$$

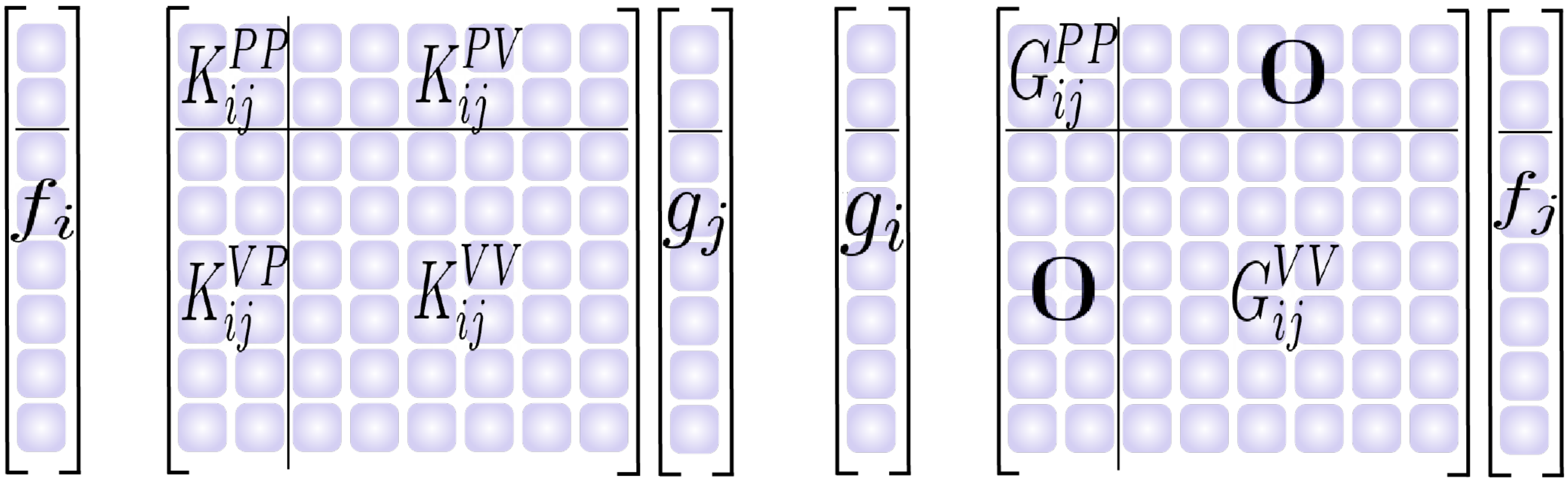
$$\Gamma^{\mu}_{FFV}(k,Q) = g_{FFV}\, i\gamma^{\mu}, \qquad \Gamma^{\mu\nu}_{VVP}(k,Q) = \frac{g_{VVP}}{m_V}\epsilon^{\mu\nu\alpha\beta} Q^{\alpha} k^{\beta}$$

$$\Gamma^{\mu\nu}_{VVS}(k,Q) = g_{VVS}\, m_V\, \delta^{\mu\nu} \tag{7}$$

- Having all the ingredients, one could solve the Bethe-Salpeter equation that couples all relevant channels:

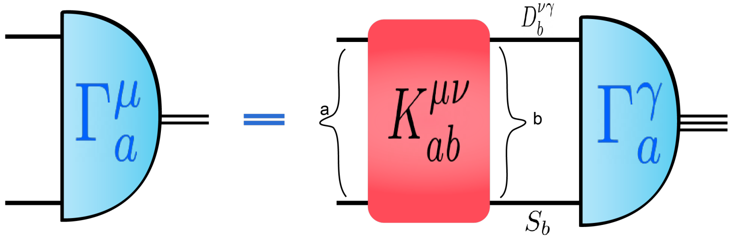
$$K_{ab} G_b \Gamma_b = \Gamma_a \tag{8}$$

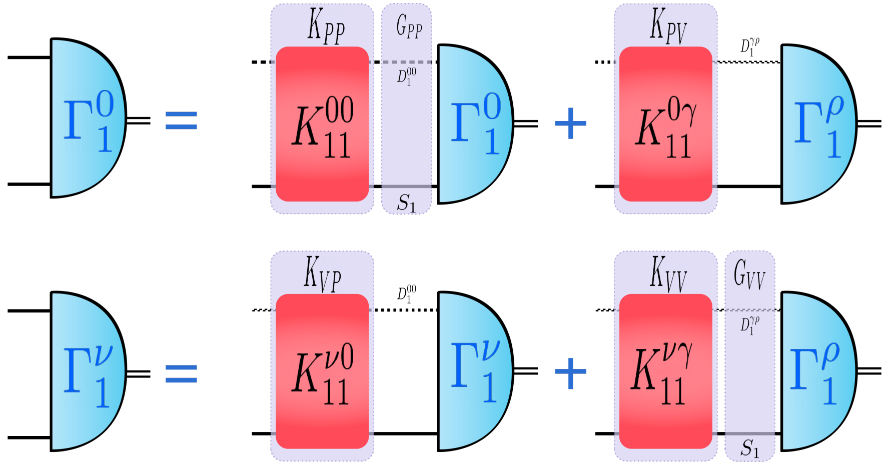
$$\Gamma^{\mu}_{a,\,\alpha\beta} = \int_q \left\{ K^{\mu\nu}_{ab}(p,q)\, S_b(q_+)\, \Gamma^{\gamma}_{b}(q,P) \right\}_{\alpha\beta} D^{\nu\gamma}_{b}(q_-) \tag{9}$$



- Graphically,

$$\Gamma^{\mu}_{a,\,\alpha\beta} = \int_q \left\{K^{\mu\nu}_{ab}(p,q)\, S_b(q_+)\, \Gamma^{\gamma}_{b}(q,P)\right\}_{\alpha\beta} D^{\nu\gamma}_{b}(q_-) \tag{9}$$





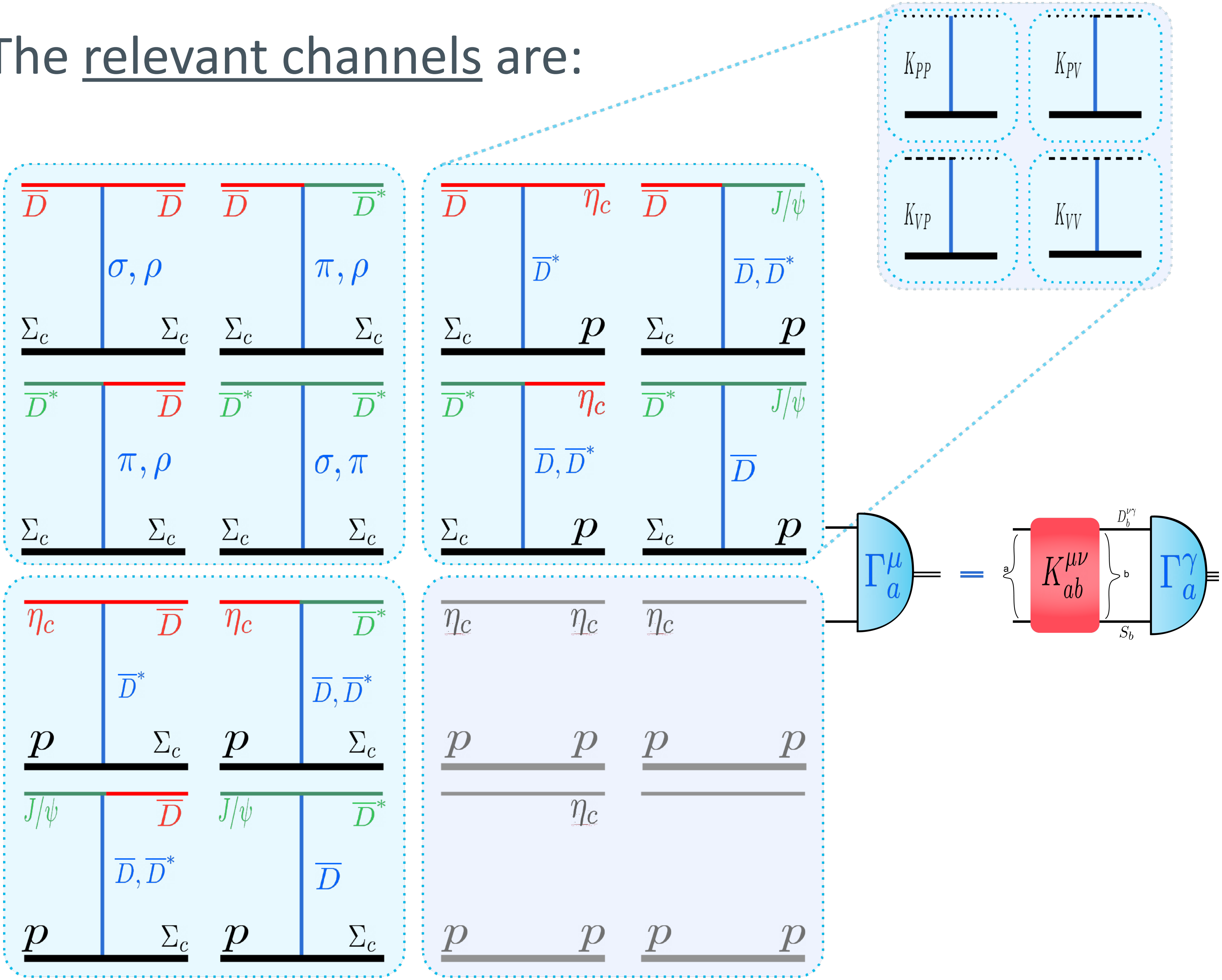
- There are two possible combinations for the initial and final state:

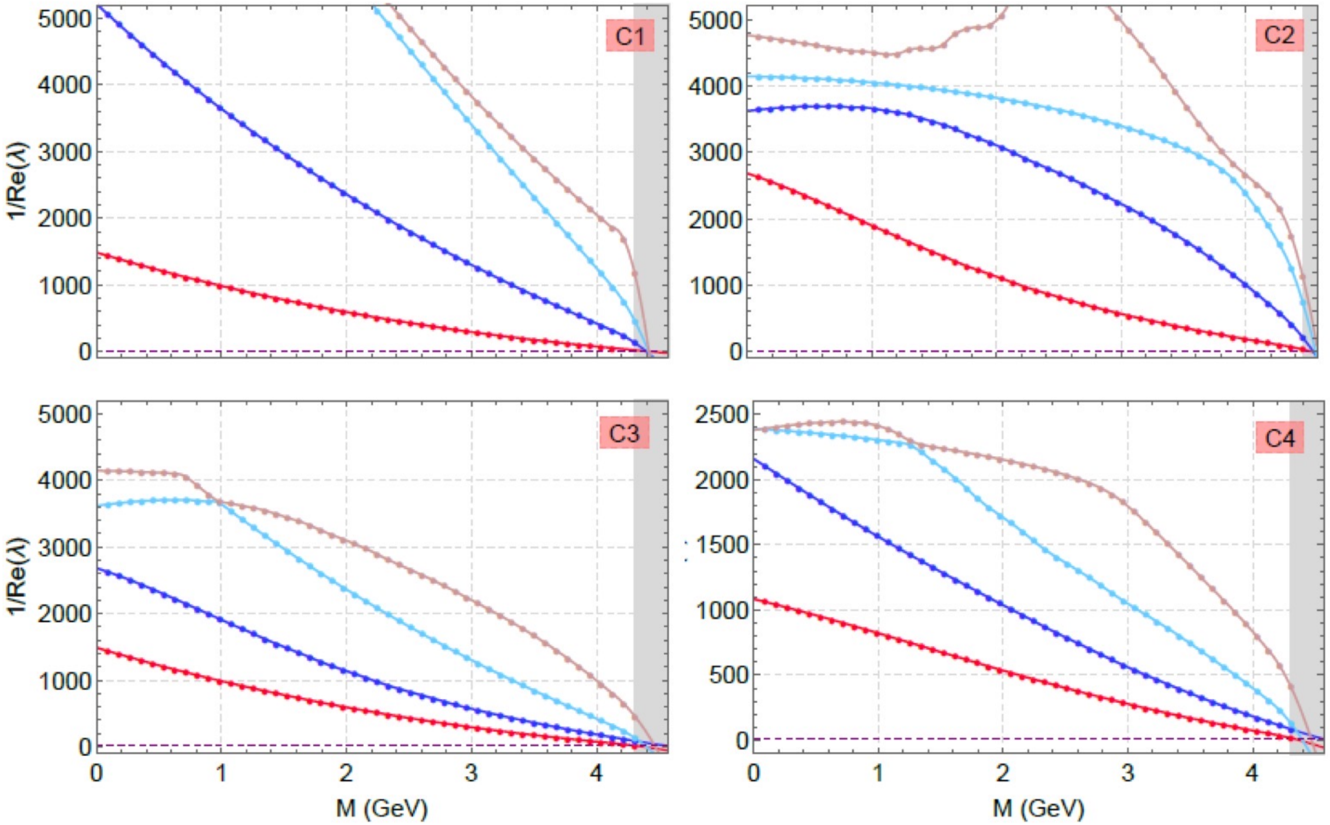
$$(c\bar{c}) + (qqq) \quad or \quad (\bar{c}q) + (cqq)$$

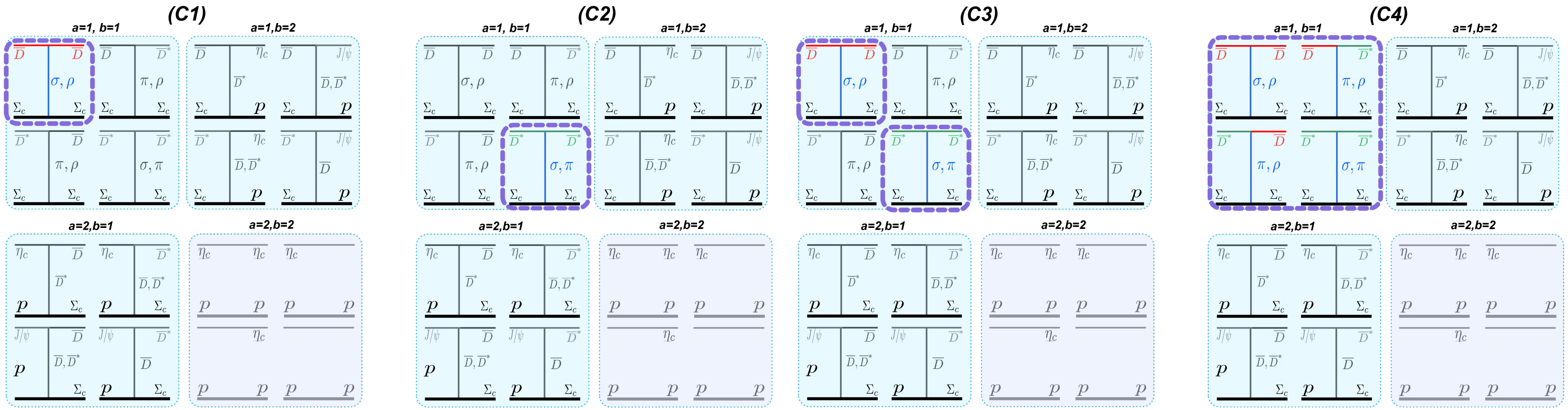
- The quantum numbers did bound states made of $J^P = 1^+/2$ baryons (Σ_c, p) and mesons with $J^P = 0^-$ $(\bar{D}, \eta_c)$ or 1^- $(\bar{D}, J/\Psi)$ are $J = 1^-/2$ and $3^-/2$. We considered only the $1^-/2$ case, for simplicity.

Eigenvalues: $\Sigma_c\{\bar{D},\bar{D}^*\} \leftrightarrow \Sigma_c\{\bar{D},\bar{D}^*\}$

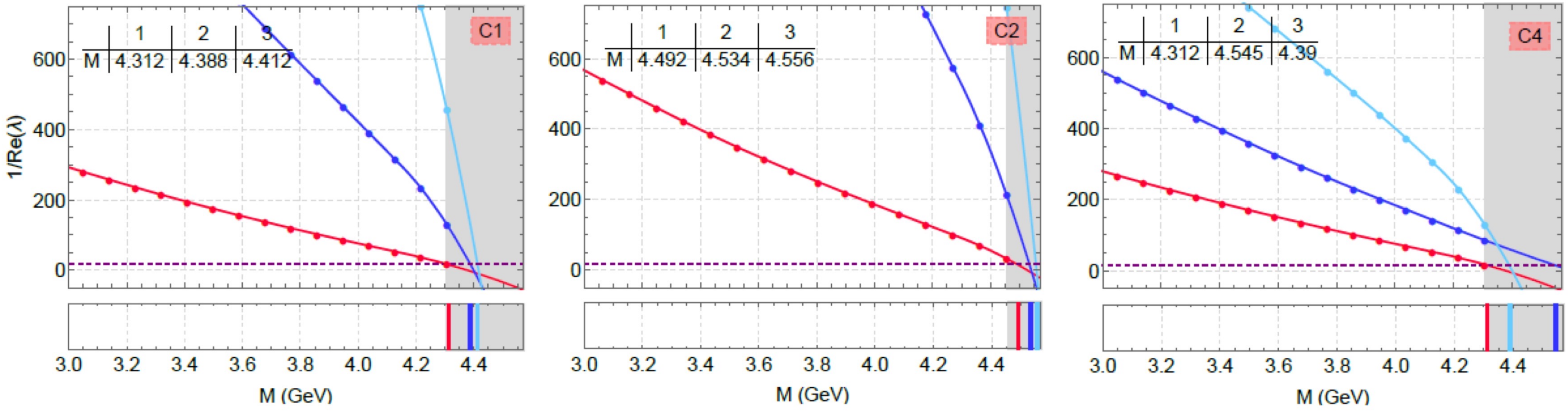
- The relevant channels are:



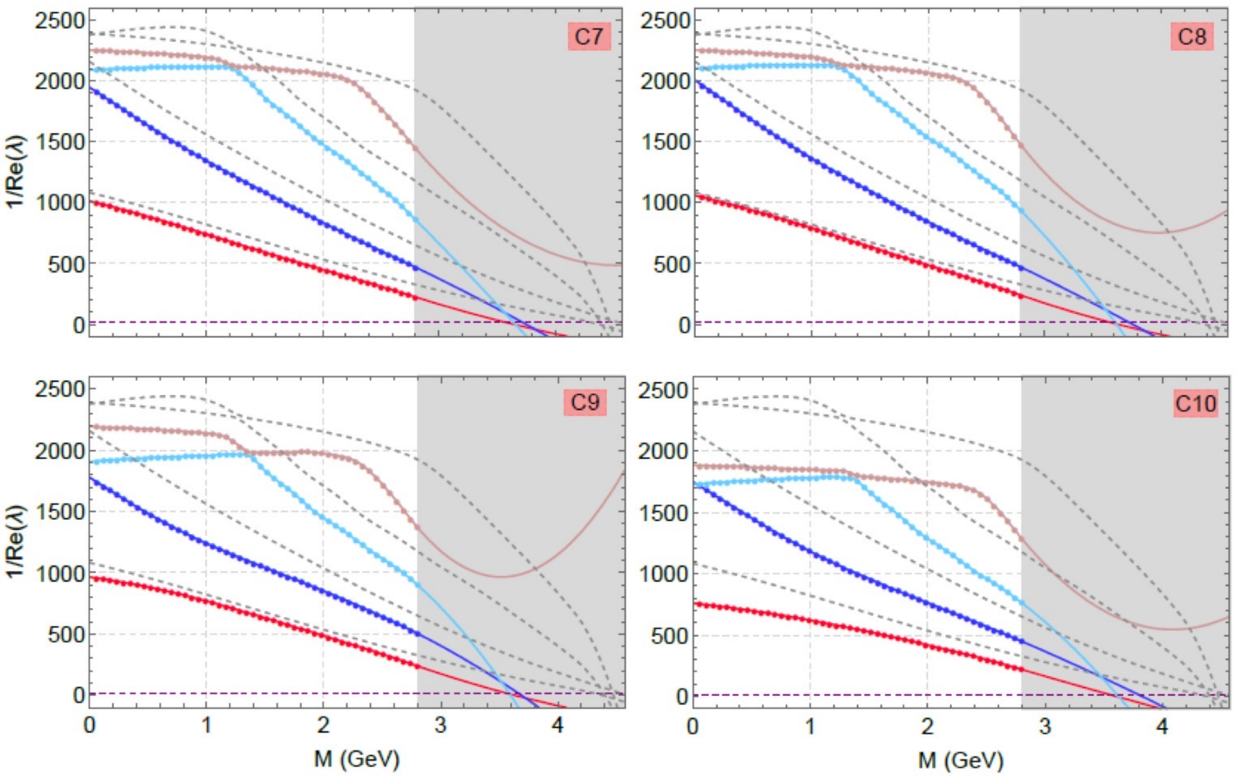


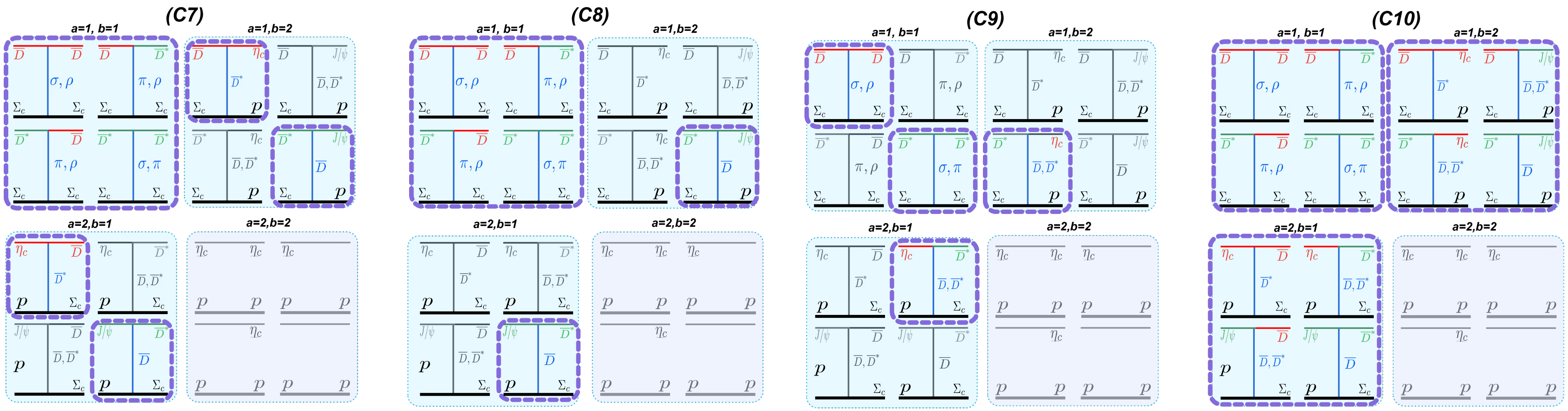


- We started by adding diagrams to <u>the first channel</u> $\Sigma_c\{\overline{D},\overline{D}*\} \leftrightarrow \Sigma_c\{\overline{D},\overline{D}*\}$

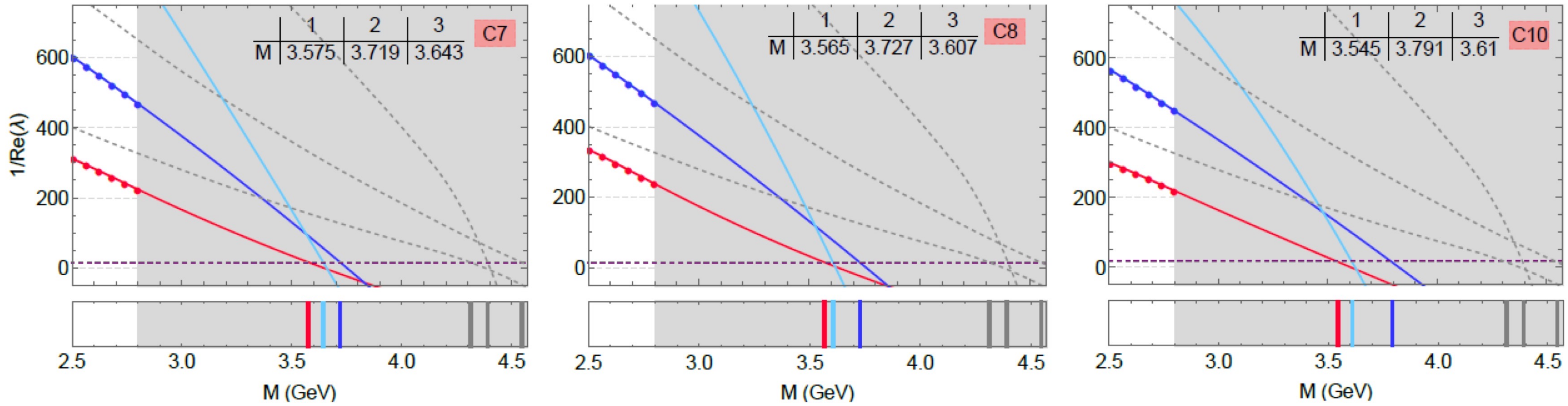


- An overall coupling constant is chosen such that the ground state reproduces the mass of the lightest pentaquark state detected at the LHC, the $P_c(4312)^+$, in our reference calculation (C4) .





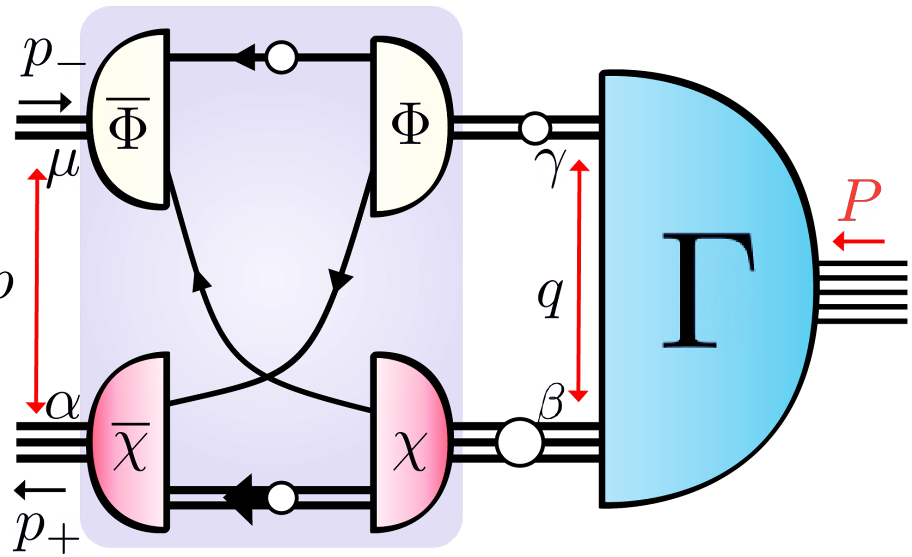
- Beyond the 1st channel, we added the 2nd $p\{\eta_c, J/\psi\} \leftrightarrow p\{\eta_c, J/\psi\}$



- We also calculate the bound state masses when including the <u>second channel</u> .
- In this case, it is more difficult to get the masses of the bound states since the eigenvalues are extrapolated further.

Outlook - Hadronic Exchange

- The pentaquark could be generally described by the $\Sigma_c\{\overline{D}, \overline{D}^*\} \leftrightarrow \Sigma_c\{\overline{D}, \overline{D}^*\}$, especially the diagonal diagrams.
- The $p\{\eta_c, J/\psi\} \leftrightarrow p\{\eta_c, J/\psi\}$ channel does not affect the system fundamentally.
- This model predicts more states near the threshold than the Pentaquark states discovered so far.



Genuine quark exchange

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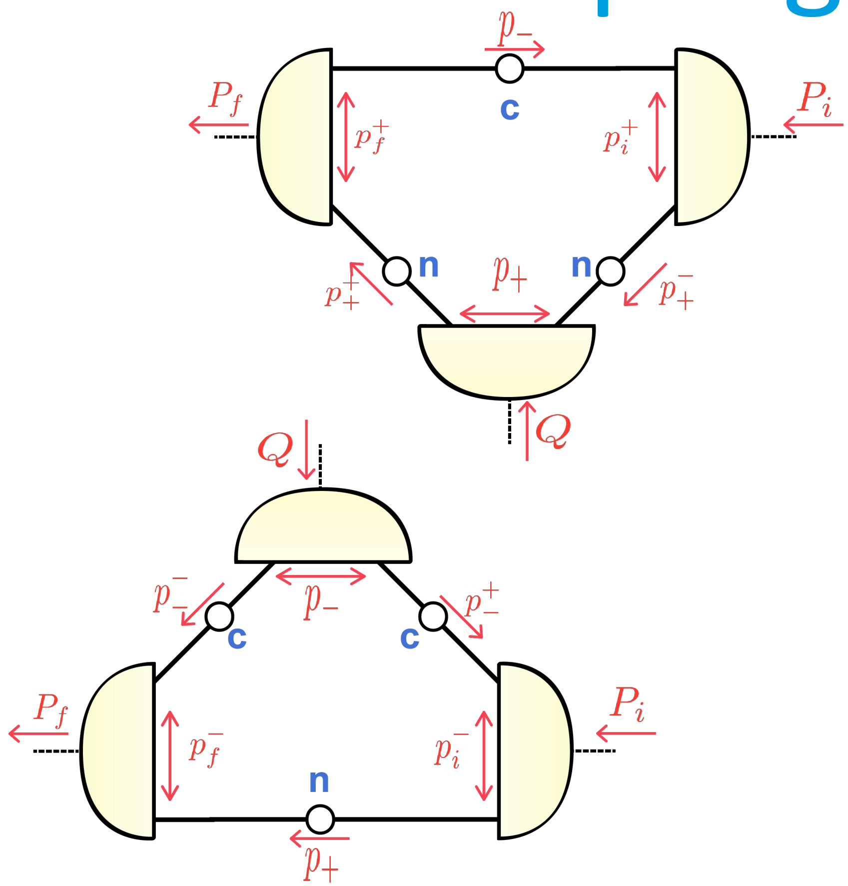
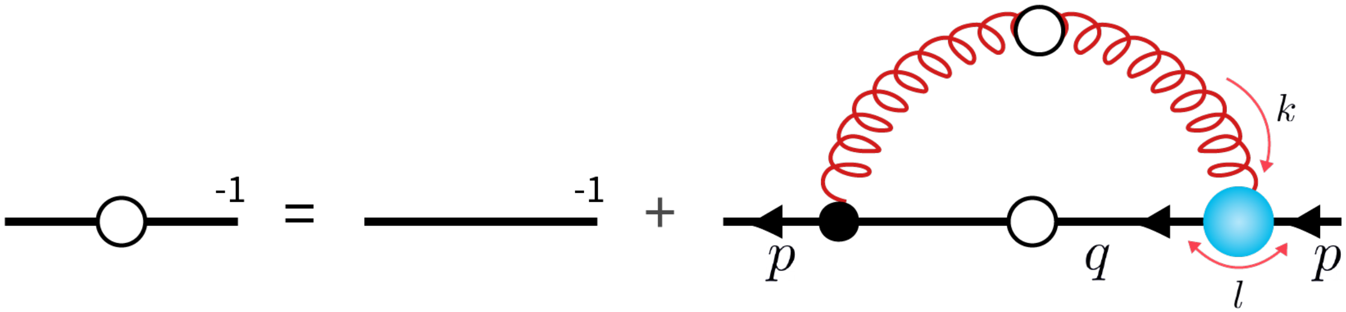
Meson couplings



- All meson coupling appearing in the Pentaquark have the general structure $D^{(*)}D^{(*)}M$ where M stand for a light $n\bar{n}$ meson (π, ρ, σ) or a heavy $c\bar{c}$ meson $(\eta_c, J/\psi)$. We compute them with triangle diagrams:
- The ingredients are the dressed light- and charm- quark propagators and the meson BS amplitude.

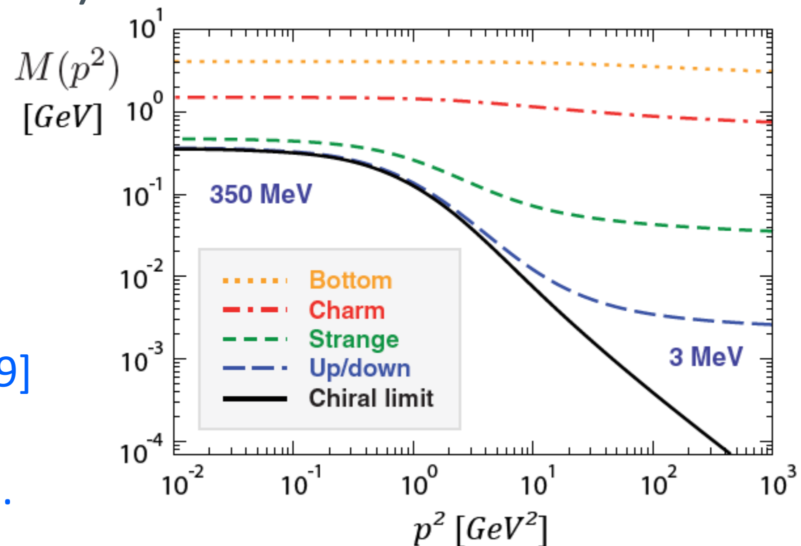
Quark DSE



$$S(p)^{-1} = Z_2(i\gamma_\mu p^\mu + m_0) + \Sigma(p), \quad D^{\mu\nu} = \frac{Z(k^2)}{k^2} T^{\mu\nu}$$

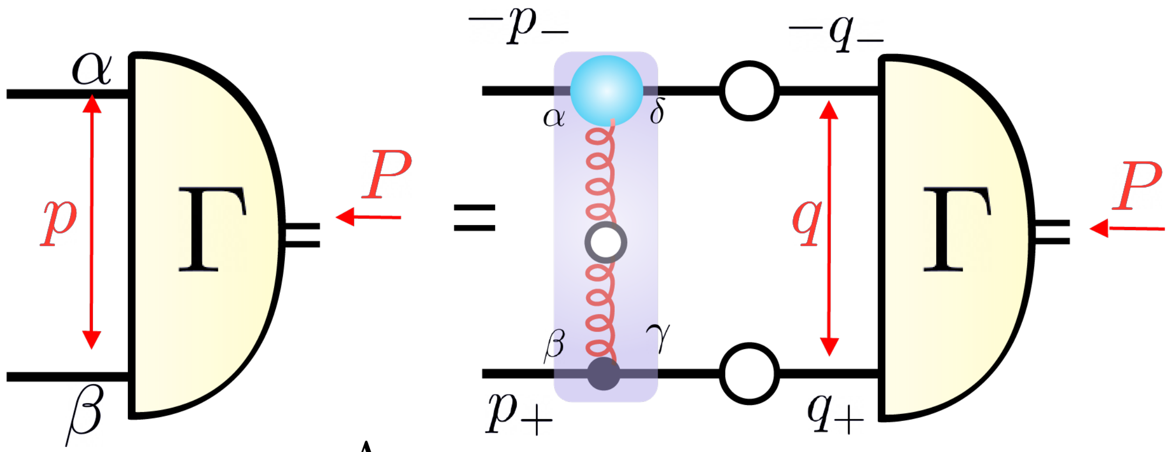
$$\Sigma(p) = -\frac{4g^2}{3} Z_\Gamma \int_q i\gamma^\mu S(q) D^{\mu\nu}(k) \Gamma^\nu(l,k) \tag{10}$$

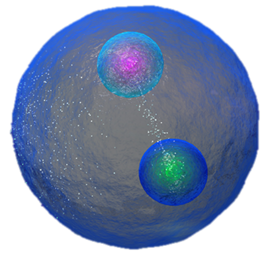
- Using rainbow ladder truncation.
- Effective interaction (Maris-Tandy model).
- DSE reproduces every diagram in perturbation theory.



- Int. J. Mod. Phys. E 12 (2003) 297 [nucl-th/0301049]
- J.Phys.G32(2006)R253 [hep-ph/0605173].
- Commun.Theor.Phys.58(2012)79 [arXiv:1201.3366].

Mesons BSE





$$\Gamma_{\alpha\beta}(p,P) = \int_q^{\Lambda} K_{\alpha\gamma,\delta\beta}(p,q,P)\left\{S(q_+)\Gamma(q,P)S(-q_-)\right\}_{\gamma\delta}\,, \tag{11}$$

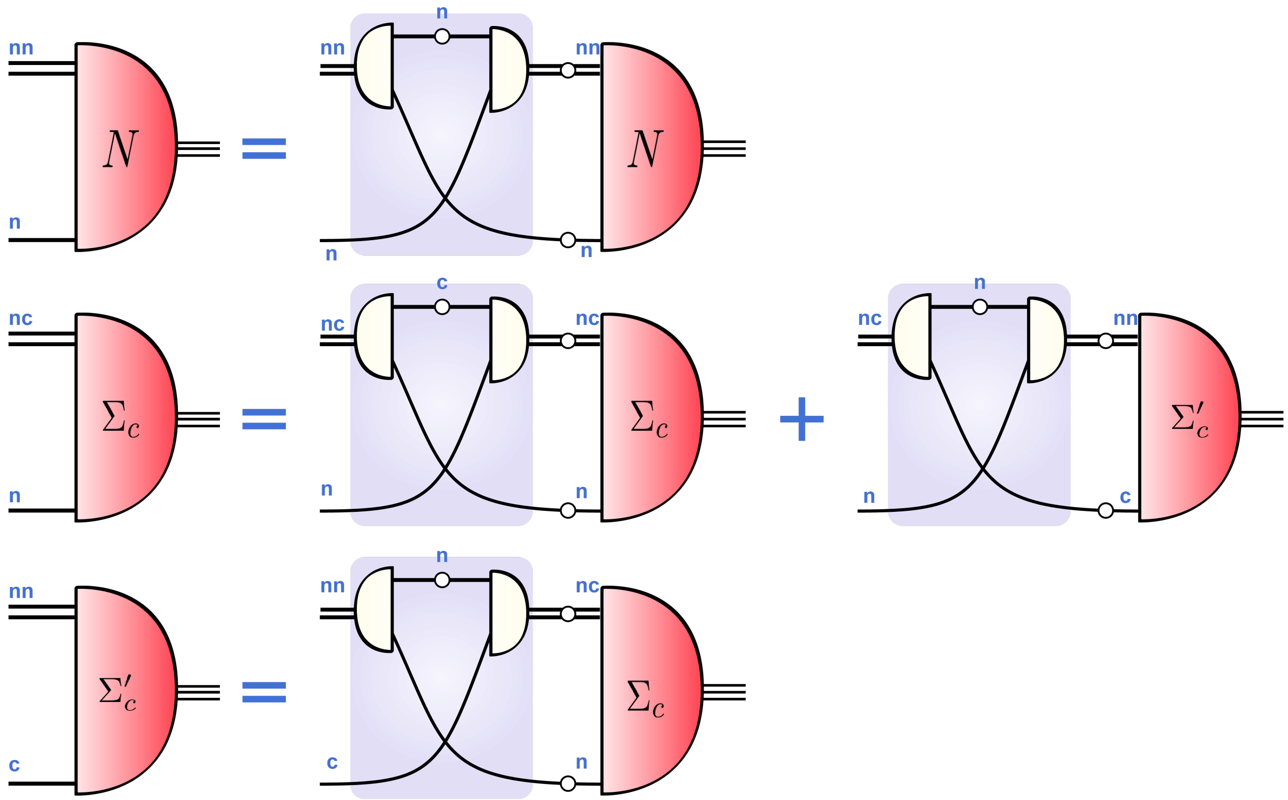
- The quark -antiquark bound state amplitude $\Gamma(p,P)$with relative momentum p, total momentum P, and mass M (at $P^2 = -M^2$)

$$\Gamma(q,P) = \sum_{k=1}^{4} f_k^{ps}(q^2,z,P^2)\{i\gamma^5\tau_k(q,P)\}_{\alpha\beta} \otimes \frac{\delta_{AB}}{\sqrt{3}} \otimes \mathrm{r}^e_{ab}$$

$$\Gamma^{\mu}(q,P) = \sum_{k=1}^{12} f_k^{vc}(q^2,z,P^2)\{i\tau_k^{\mu}(q,P)\}_{\alpha,\beta} \otimes \frac{\delta_{AB}}{\sqrt{3}} \otimes \mathrm{r}^e_{ab} \tag{12}$$

- Maris, Roberts,Tandy, PLB 420 (1998) 267

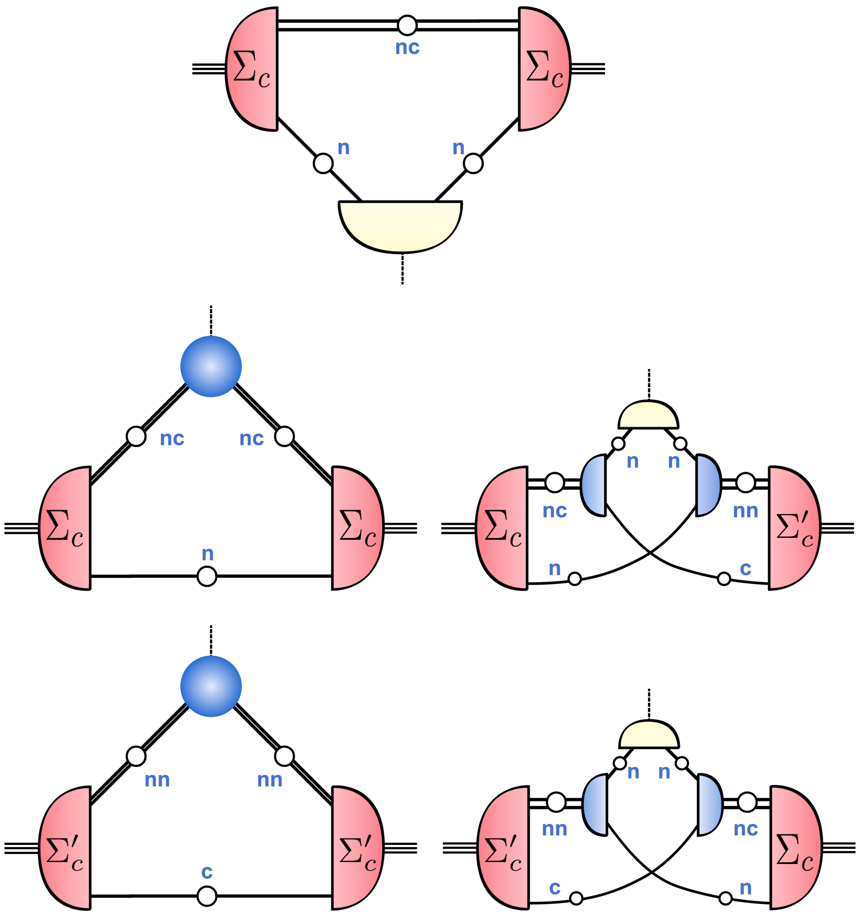
Baryon - Quark-Diquark approach



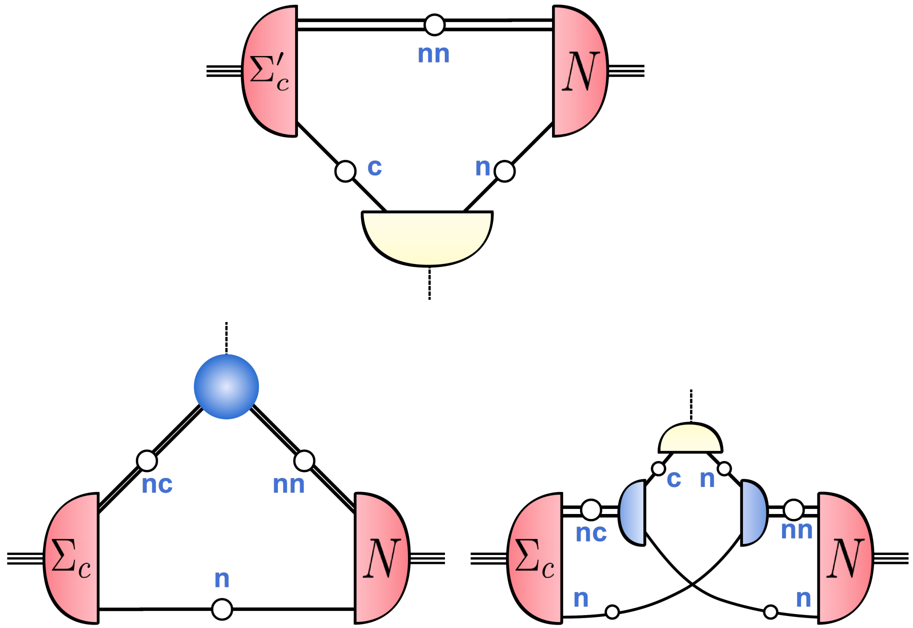
- We need the coupling of the light $n\bar{n}$ mesons to the Σ_c baryon, together with the heavy-light meson induced transition between Σ_c and proton.

- Eichmann, Sanchis-Alepuz, Williams, Alkofer, & Fischer, (2016). Baryons as relativistic three-quark bound states. *Prog. Part. Nucl. Phys, 91*, 1-100. arXiv:1606.09602v

Baryon couplings



- Coupling of light mesons to the Σ_c .



- Coupling of $D^{(*)}$mesons mediating a transition between Σ_c and proton.

- The systematic calculation of the baryon-meson couplings in the quark-diquark approach leads to the following diagrams:

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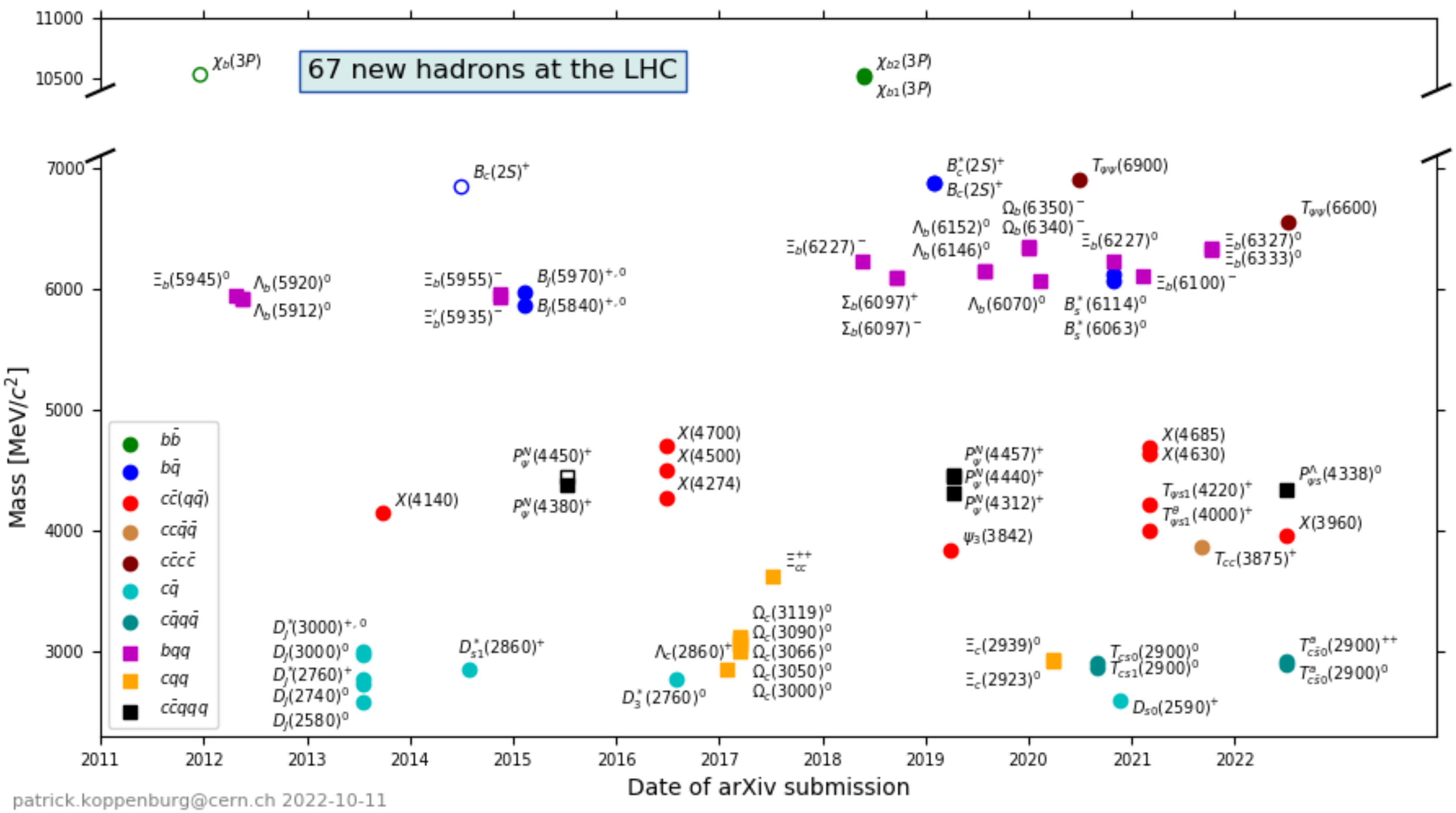
Particle Zoo 2.0



- LHCb collaboration, P. Koppenburg, List of hadrons observed at the LHC, LHCb-FIGURE-2021-001, 2021, and 2022 updates. (See BibTeX snippet). © CC BY 4.0 Patrick Koppenburg, 2022.

Thank you!

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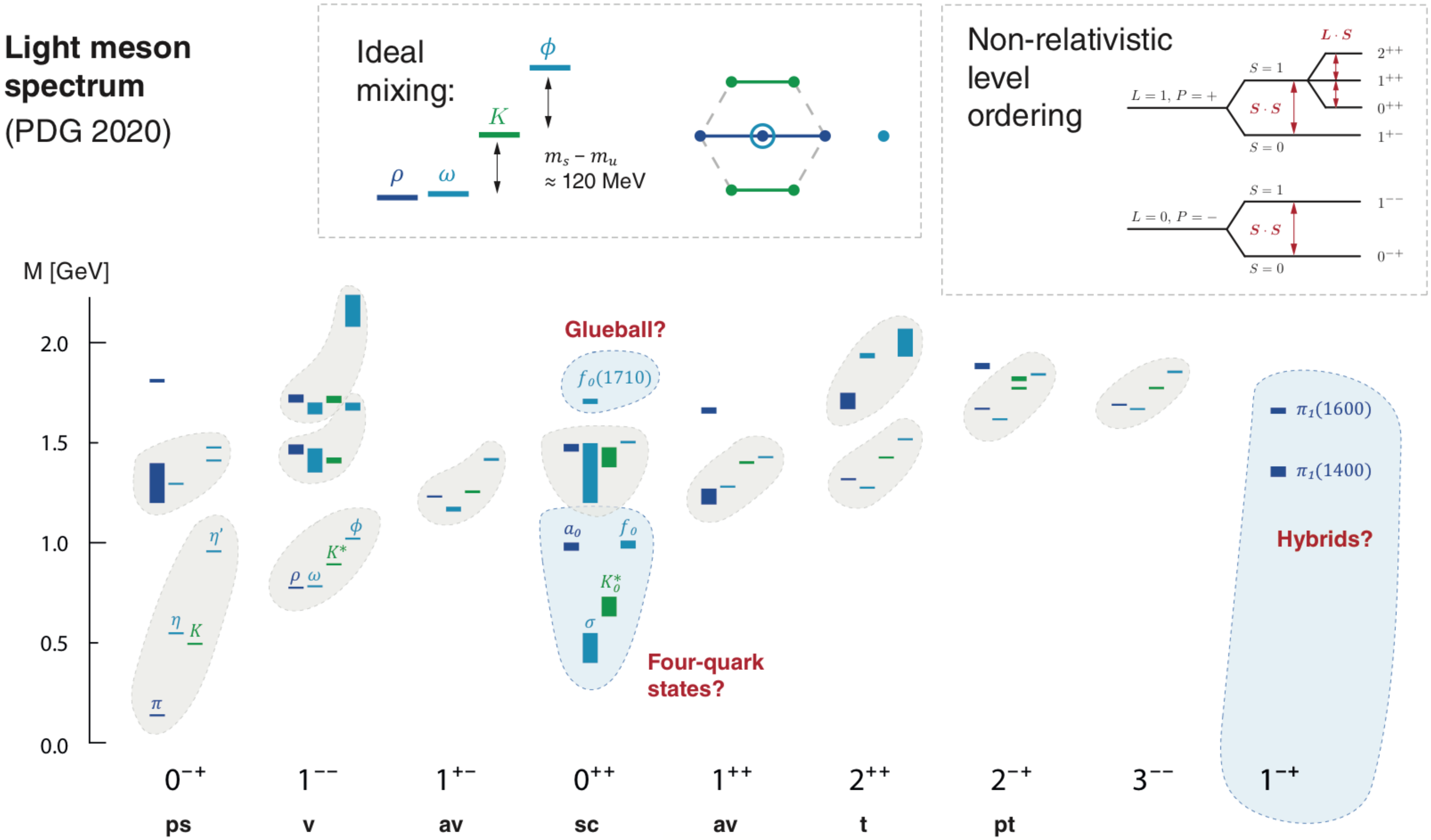
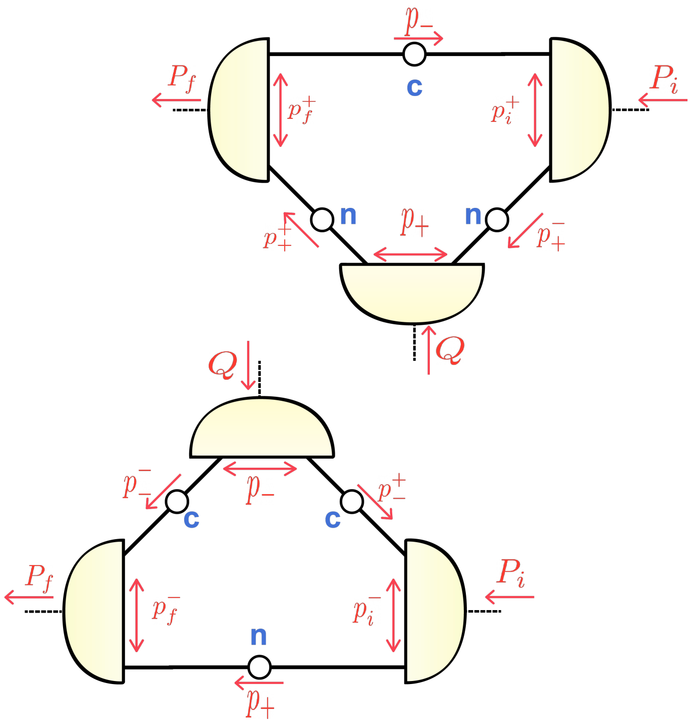
Backup slides

PDG spectrum



Meson couplings

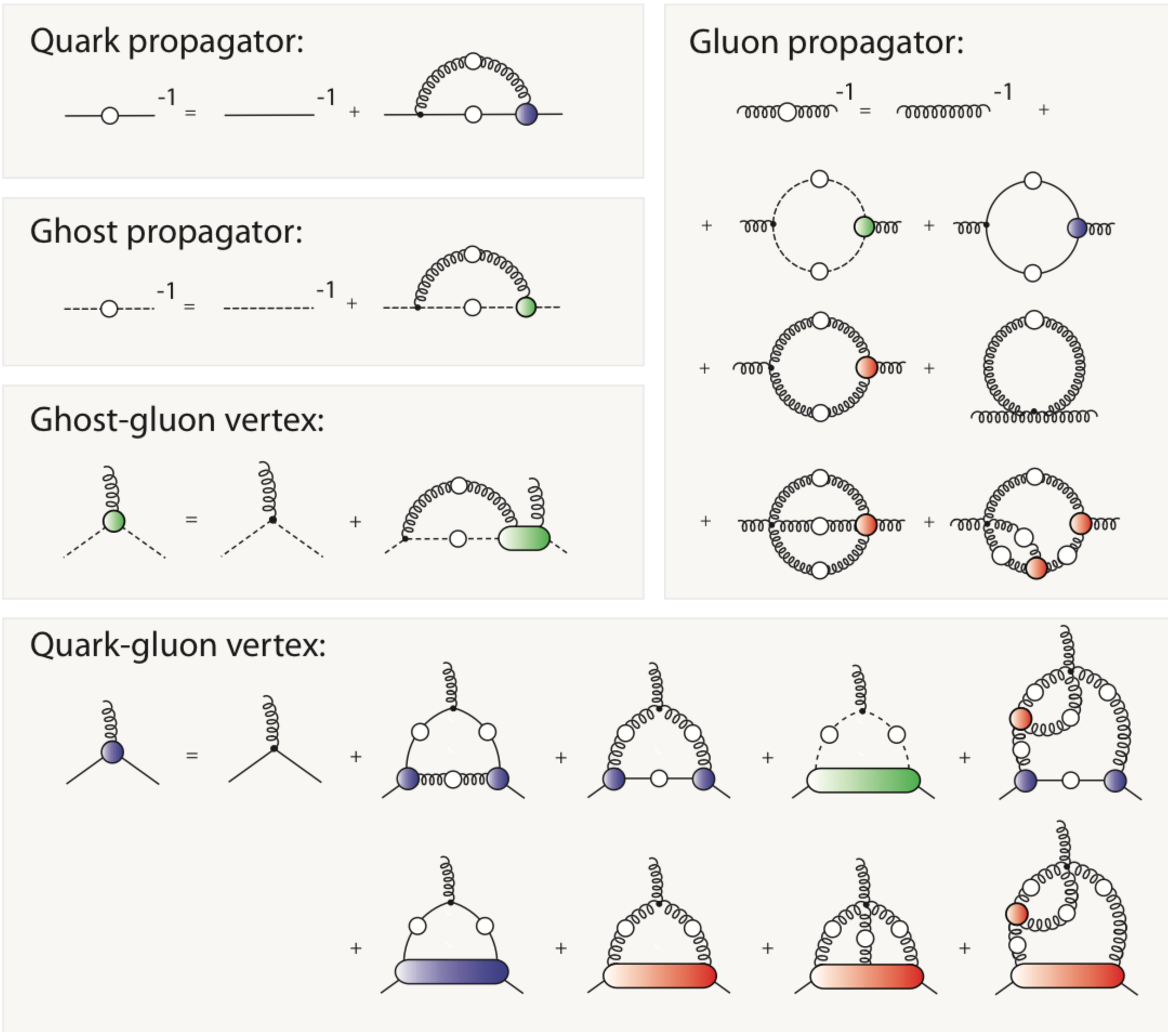


$$\mathscr{M}_1(P,Q)^{(\mu\nu\beta)} = \int_p Tr\left(\overline{\Gamma}^{(\alpha)}_{cn}(p_f^+, P_f) S_n(p_+^+) \Gamma^{(\mu)}_{n\bar{n}}(p_+, Q) S_n(p_+^-) \Gamma^{(\beta)}_{n\bar{c}}(p_i^+, P_i) S_c(p_-)\right)$$

$$\mathscr{M}_2(P,Q)^{(\mu\nu\beta)} = \int_p Tr\left(\overline{\Gamma}^{(\alpha)}_{cn}(p_f^-, P_f) S_n(p_+) \Gamma^{(\beta)}_{n\bar{c}}(p_i^-, P_i) S_c(p_-^+) \Gamma^{(\mu)}_{n\bar{c}}(p_i^-, Q) S_c(p_-^-)\right)$$

Dyson-Schwinger equations

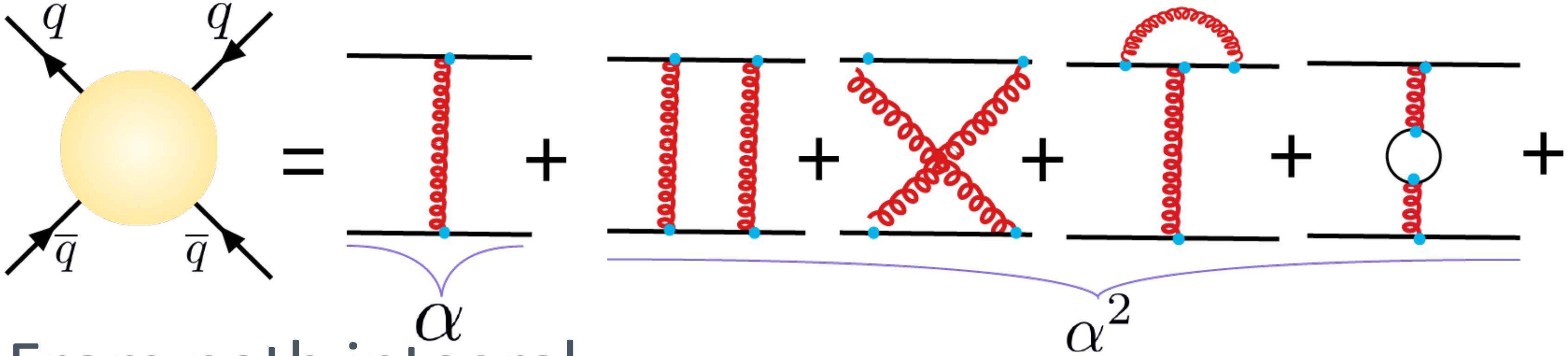
- The DSE for quark, gluon and ghost propagators and quark-gluon and ghost-gluon vertex are illustrated:



Correlation functions

Correlation functions can be calculated…

- In <u>perturbation theory</u>:



- From <u>path integral:</u>

$$\langle 0|\, \mathsf{T}\, \psi(x_1) \ldots \bar{\psi}(x_n)\, |0\rangle \;=\; \int \mathcal{D}[\psi, \bar{\psi}, A]\, e^{iS}\, \psi(x_1) \ldots \bar{\psi}(x_n) \qquad (4)$$

- Self-consistent from <u>each other</u>: functional methods